Computer algebra independent integration tests

8-Special-functions/8.1-Error-functions

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [311]. This is test number [204].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (311)	% 0. (0)
Mathematica	% 96.46 (300)	% 3.54 (11)
Maple	% 57.56 (179)	% 42.44 (132)
Maxima	% 44.69 (139)	% 55.31 (172)
Fricas	% 82.96 (258)	% 17.04 (53)
Sympy	% 48.55 (151)	% 51.45 (160)
Giac	% 42.77 (133)	% 57.23 (178)

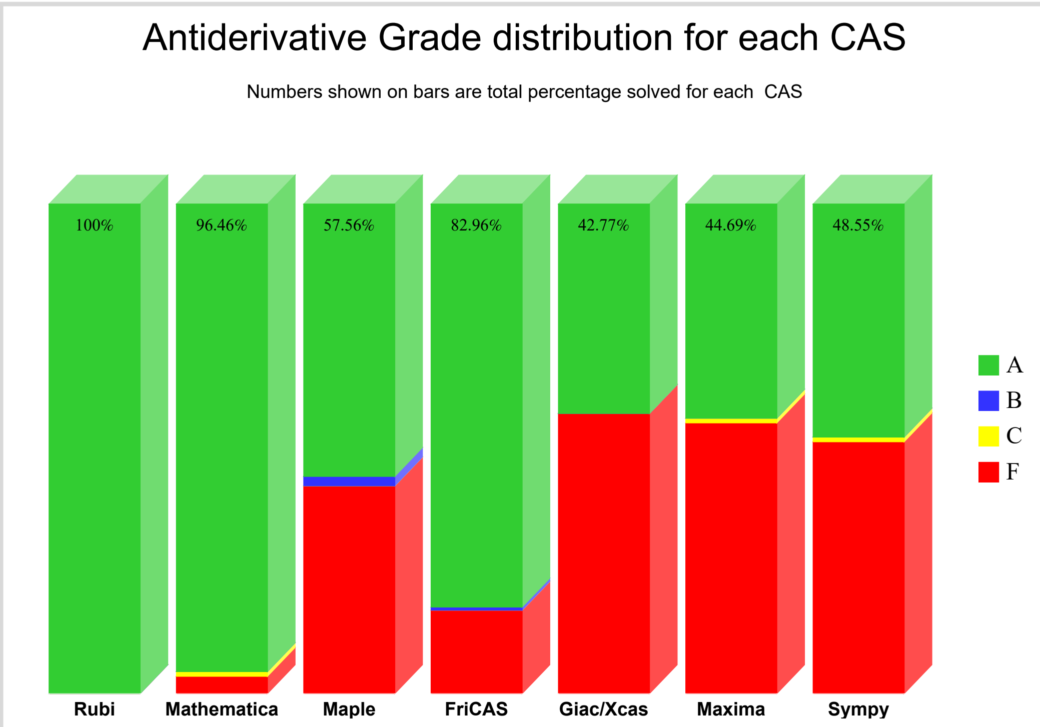
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

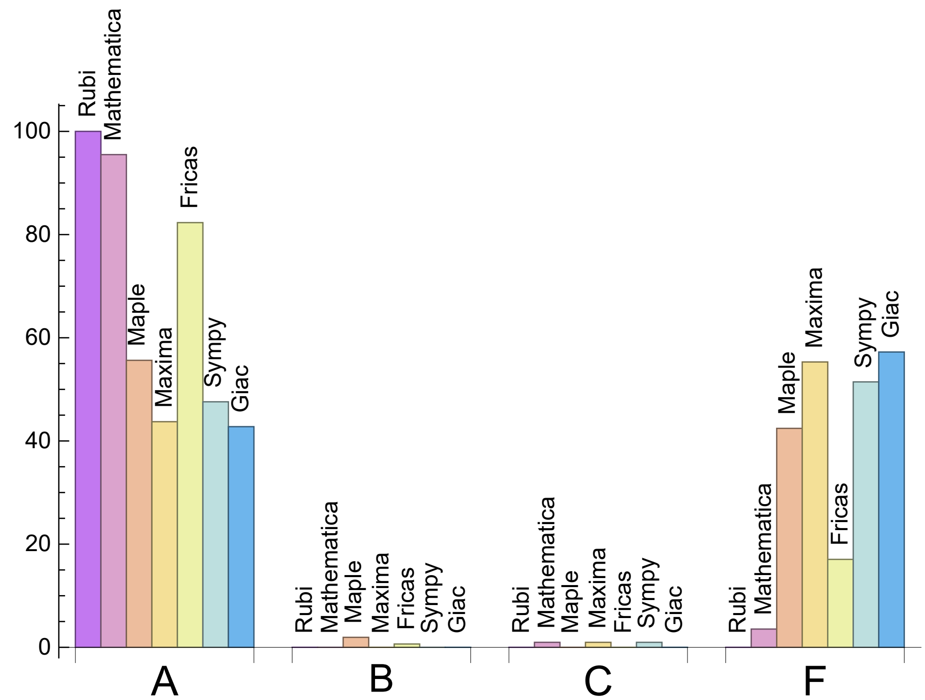
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	95.5	0.	0.96	3.54
Maple	55.63	1.93	0.	42.44
Maxima	43.73	0.	0.96	55.31
Fricas	82.32	0.64	0.	17.04
Sympy	47.59	0.	0.96	51.45
Giac	42.77	0.	0.	57.23

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	68.68	0.74	59.	1.
Mathematica	0.27	56.21	0.64	51.	0.76
Maple	0.15	56.31	0.6	26.	0.81
Maxima	0.47	22.4	0.41	0.	0.
Fricas	1.72	150.58	1.63	119.5	1.9
Sympy	5.23	58.66	0.69	24.	0.87
Giac	0.53	57.41	0.52	0.	0.

1.4 list of integrals that has no closed form antiderivative

{19, 20, 21, 25, 32, 33, 34, 38, 39, 56, 57, 58, 59, 60, 61, 62, 63, 78, 79, 80, 88, 89, 90, 91, 92, 93, 94, 122, 123, 124, 128, 135, 136, 137, 141, 142, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 225, 226, 227, 231, 238, 239, 240, 244, 245, 262, 263, 264, 265, 266, 267, 268, 269, 286, 287, 288, 296, 297, 298, 299, 300, 301, 302}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {125, 126, 127, 138, 139, 156, 171, 172, 173, 174, 176, 177, 184, 185, 189, 199, 200, 203, 204, 205, 206, 276, 277, 278, 280, 281, 282}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

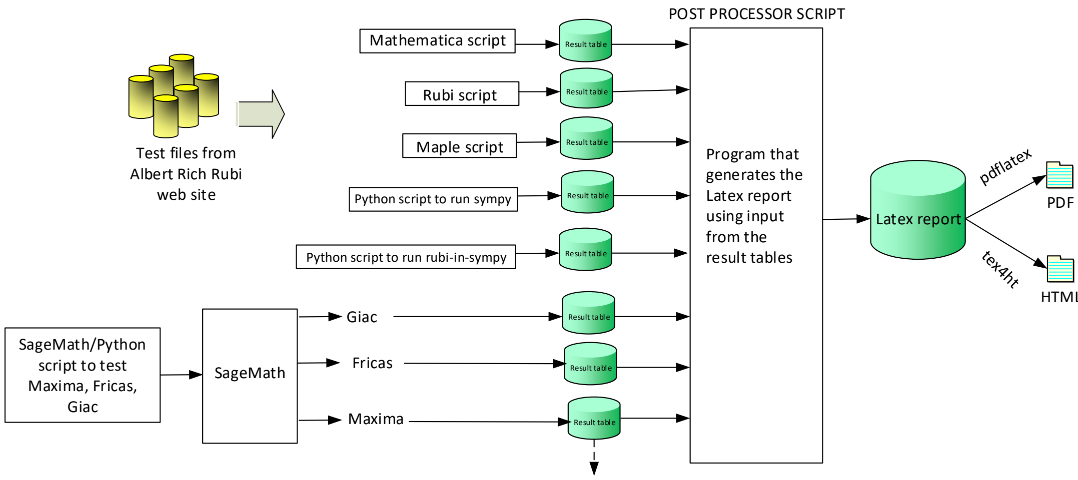
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 308, 309, 310, 311 }

B grade: { }

C grade: { 280, 281, 282 }

F grade: { 72, 98, 99, 175, 201, 202, 241, 304, 305, 306, 307}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 29, 30, 31, 32, 33, 34, 37, 38, 39, 43, 47, 48, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 132, 133, 134, 135, 136, 137, 140, 141, 142, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 178, 179, 180, 181, 182, 183, 186, 191, 192, 193, 194, 195, 196, 197, 198, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 231, 238, 239, 240, 244, 245, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 286, 287, 288, 296, 297, 298, 299, 300, 301, 302, 303 }

B grade: { 118, 119, 150, 190, 221, 222 }

C grade: { }

F grade: { 22, 23, 24, 26, 27, 28, 35, 36, 40, 41, 42, 44, 45, 46, 49, 52, 67, 68, 69, 70, 71, 72, 73, 74, 81, 82, 84, 85, 86, 96, 97, 98, 99, 100, 101, 102, 103, 107, 125, 126, 127, 129, 130, 131, 138, 139, 143, 144, 145, 147, 148, 149, 152, 153, 154, 155, 170, 171, 172, 173, 174, 175, 176, 177, 184, 185, 187, 188, 189, 199, 200, 201, 202, 203, 204, 205, 206, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 304, 305, 306, 307, 308, 309, 310, 311 }

2.1.4 Maxima

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 25, 31, 32, 33, 38, 39, 43, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 77, 78, 79, 80, 83, 87, 88, 89, 90, 91, 92, 93, 94, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 128, 135, 136, 137, 141, 142, 146, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 186, 191, 192, 193, 194, 195, 196, 197, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 231, 238, 239, 240, 244, 245, 249, 262, 263, 264, 265, 266, 267, 268, 269, 286, 287, 288, 296, 297, 298, 299, 300, 301, 302

B grade: { }

C grade: { 207, 208, 209 }

F grade: { 4, 15, 16, 17, 22, 23, 24, 26, 27, 28, 29, 30, 34, 35, 36, 37, 40, 41, 42, 44, 45, 46, 52, 53, 54, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 82, 84, 85, 86, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 118, 119, 120, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 210, 221, 222, 223, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303, 304, 305, 306, 307, 308, 309, 310, 311 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303 }

B grade: { 140, 146 }

C grade: { }

F grade: { 4, 67, 68, 69, 70, 71, 72, 73, 74, 96, 97, 98, 99, 100, 101, 102, 103, 107, 170, 171, 172, 173, 174, 175, 176, 177, 199, 200, 201, 202, 203, 204, 205, 206, 210, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 304, 305, 306, 307, 308, 309, 310, 311 }

2.1.6 Sympy

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 32, 33, 34, 38, 39, 47, 48, 49, 50, 51, 52, 56, 57, 60, 61, 62, 63, 66, 78, 79, 80, 81, 82, 83, 88, 92, 93, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 135, 136, 137, 141, 142, 150, 151, 152, 153, 154, 155, 159, 160, 163, 164, 165, 166, 169, 181, 182, 183, 184, 185, 186, 191, 195, 196, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 238, 239, 240, 244, 245, 253, 254, 255, 256, 257, 258, 262, 263, 266, 267, 268, 269, 272, 286, 287, 288, 289, 290, 291, 296, 300, 301, 303 }

B grade: { }

C grade: { 218, 219, 220 }

F grade: { 4, 21, 26, 27, 28, 29, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 58, 59, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 124, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 161, 162, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 187, 188, 189, 190, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 210, 227, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261, 264, 265, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 292, 293, 294, 295, 297, 298, 299, 302, 304, 305, 306, 307, 308, 309, 310, 311 }

2.1.7 Giac

A grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 91, 92, 93, 94, 104, 105, 106, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 124, 128, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 149, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 225, 226, 227, 231, 238, 239, 240, 244, 245, 262, 263, 264, 265, 266, 267, 268, 269, 286, 287, 288, 296, 297, 298, 299, 300, 301, 302 }

B grade: { }

C grade: { }

F grade: { 4, 5, 6, 7, 22, 23, 24, 26, 27, 28, 35, 36, 37, 44, 45, 47, 48, 49, 50, 51, 52, 67, 68, 69, 70, 71, 72, 73, 74, 81, 82, 83, 84, 85, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 115, 116, 117, 125, 126, 127, 129, 130, 131, 138, 139, 140, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303, 304, 305, 306, 307, 308, 309, 310, 311 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	72	83	85	147	88	86
normalized size	1	1.	0.75	0.86	0.89	1.53	0.92	0.9
time (sec)	N/A	0.096	0.025	0.05	1.097	2.518	4.465	1.175

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	65	74	127	65	76
normalized size	1	1.	0.89	0.92	1.04	1.79	0.92	1.07
time (sec)	N/A	0.062	0.022	0.046	1.125	2.479	1.392	1.269

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	42	47	59	101	39	59
normalized size	1	1.	0.91	1.02	1.28	2.2	0.85	1.28
time (sec)	N/A	0.036	0.033	0.05	1.125	2.567	0.473	1.299

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	23	0	0	0	0
normalized size	1	1.	1.	0.72	0.	0.	0.	0.
time (sec)	N/A	0.014	0.014	0.092	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	50	50	103	36	0
normalized size	1	1.	1.	1.19	1.19	2.45	0.86	0.
time (sec)	N/A	0.039	0.037	0.048	1.238	2.512	0.67	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	69	50	124	60	0
normalized size	1	1.	0.89	0.97	0.7	1.75	0.85	0.
time (sec)	N/A	0.065	0.017	0.046	1.133	2.556	1.665	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	73	87	50	146	87	0
normalized size	1	1.	0.76	0.91	0.52	1.52	0.91	0.
time (sec)	N/A	0.088	0.025	0.046	1.166	2.553	5.126	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	72	90	70	132	99	70
normalized size	1	1.	0.66	0.83	0.64	1.21	0.91	0.64
time (sec)	N/A	0.099	0.018	0.044	1.234	2.52	7.561	1.325

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	72	59	116	75	59
normalized size	1	1.	0.79	0.86	0.7	1.38	0.89	0.7
time (sec)	N/A	0.073	0.016	0.048	1.047	2.525	2.465	1.274

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	41	54	49	100	51	49
normalized size	1	1.	0.69	0.92	0.83	1.69	0.86	0.83
time (sec)	N/A	0.049	0.024	0.055	1.105	2.552	0.804	1.274

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	34	68	24	31
normalized size	1	1.	1.	1.	1.31	2.62	0.92	1.19
time (sec)	N/A	0.005	0.01	0.044	1.086	2.53	0.363	1.238

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	30	32	68	24	32
normalized size	1	1.	1.	1.15	1.23	2.62	0.92	1.23
time (sec)	N/A	0.031	0.039	0.046	1.149	2.631	1.334	1.349

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	53	36	111	54	69
normalized size	1	1.	0.84	0.95	0.64	1.98	0.96	1.23
time (sec)	N/A	0.052	0.061	0.054	1.171	2.583	2.682	1.201

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	71	36	131	76	92
normalized size	1	1.	0.77	0.88	0.44	1.62	0.94	1.14
time (sec)	N/A	0.075	0.044	0.046	1.09	2.606	5.516	1.219

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	248	466	0	579	746	540
normalized size	1	1.	0.86	1.61	0.	2.	2.58	1.87
time (sec)	N/A	0.321	0.305	0.055	0.	2.581	24.802	1.315

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	138	283	0	360	398	365
normalized size	1	1.	0.72	1.47	0.	1.88	2.07	1.9
time (sec)	N/A	0.2	0.208	0.052	0.	2.575	7.06	1.347

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	88	111	0	211	178	201
normalized size	1	1.	0.75	0.94	0.	1.79	1.51	1.7
time (sec)	N/A	0.12	0.085	0.049	0.	2.588	2.571	1.332

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	32	42	107	53	80
normalized size	1	1.	0.97	0.89	1.17	2.97	1.47	2.22
time (sec)	N/A	0.007	0.04	0.047	1.075	2.548	0.82	1.301

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.188	0.39	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.296	0.426	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.53	0.426	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	106	0	0	227	168	0
normalized size	1	1.	0.6	0.	0.	1.28	0.94	0.
time (sec)	N/A	0.294	0.045	0.053	0.	2.579	10.335	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	90	0	0	186	117	0
normalized size	1	1.	0.71	0.	0.	1.48	0.93	0.
time (sec)	N/A	0.181	0.035	0.053	0.	2.961	5.434	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	0	0	142	65	0
normalized size	1	1.	0.9	0.	0.	2.	0.92	0.
time (sec)	N/A	0.086	0.04	0.053	0.	2.927	1.176	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.027	0.047	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	0	153	0	0
normalized size	1	1.	0.94	0.	0.	2.28	0.	0.
time (sec)	N/A	0.1	0.028	0.053	0.	2.612	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	97	0	0	215	0	0
normalized size	1	1.	0.78	0.	0.	1.72	0.	0.
time (sec)	N/A	0.183	0.078	0.052	0.	2.577	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	133	0	0	257	0	0
normalized size	1	1.	0.75	0.	0.	1.45	0.	0.
time (sec)	N/A	0.292	0.041	0.052	0.	2.611	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	106	131	0	277	0	235
normalized size	1	1.	0.64	0.79	0.	1.68	0.	1.42
time (sec)	N/A	0.247	0.09	0.052	0.	2.549	0.	1.391

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	95	0	231	0	151
normalized size	1	1.	0.78	0.84	0.	2.04	0.	1.34
time (sec)	N/A	0.136	0.074	0.046	0.	2.606	0.	1.459

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	48	84	166	0	65
normalized size	1	1.	1.	0.86	1.5	2.96	0.	1.16
time (sec)	N/A	0.05	0.03	0.048	1.746	2.51	0.	1.306

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.037	0.052	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.036	0.051	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.036	0.053	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	226	0	0	645	0	0
normalized size	1	1.	0.6	0.	0.	1.72	0.	0.
time (sec)	N/A	0.422	1.008	0.224	0.	2.631	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	132	0	0	406	0	0
normalized size	1	1.	0.7	0.	0.	2.16	0.	0.
time (sec)	N/A	0.176	0.398	0.053	0.	2.571	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	59	0	227	0	0
normalized size	1	1.	0.93	0.83	0.	3.2	0.	0.
time (sec)	N/A	0.18	0.008	0.044	0.	2.596	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.052	0.39	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.106	0.365	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	88	0	0	302	0	115
normalized size	1	1.	0.86	0.	0.	2.96	0.	1.13
time (sec)	N/A	0.232	0.318	0.158	0.	2.765	0.	1.52

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	0	0	284	0	112
normalized size	1	1.	0.89	0.	0.	3.02	0.	1.19
time (sec)	N/A	0.176	0.275	0.151	0.	2.769	0.	1.553

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	290	0	107
normalized size	1	1.	0.86	0.	0.	3.12	0.	1.15
time (sec)	N/A	0.129	0.245	0.07	0.	2.822	0.	1.341

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	79	79	78	284	0	90
normalized size	1	1.	1.22	1.22	1.2	4.37	0.	1.38
time (sec)	N/A	0.046	0.14	0.108	1.035	2.811	0.	1.315

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	80	0	0	293	0	0
normalized size	1	1.	0.87	0.	0.	3.18	0.	0.
time (sec)	N/A	0.213	0.249	0.227	0.	2.704	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	285	0	0
normalized size	1	1.	0.81	0.	0.	3.	0.	0.
time (sec)	N/A	0.215	0.247	0.241	0.	2.784	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	127	0	0	427	0	211
normalized size	1	1.	1.02	0.	0.	3.42	0.	1.69
time (sec)	N/A	0.313	0.51	0.096	0.	2.873	0.	1.625

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	42	19	0
normalized size	1	1.	1.	0.81	1.05	2.	0.9	0.
time (sec)	N/A	0.03	0.009	0.049	1.021	2.792	4.69	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	42	19	0
normalized size	1	1.	1.	0.81	1.05	2.	0.9	0.
time (sec)	N/A	0.018	0.005	0.111	1.009	2.712	1.185	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	20	46	17	0
normalized size	1	1.	1.	0.	1.	2.3	0.85	0.
time (sec)	N/A	0.029	0.011	180.	1.043	2.675	0.915	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	43	17	0
normalized size	1	1.	1.	0.81	1.05	2.05	0.81	0.
time (sec)	N/A	0.028	0.006	0.043	1.02	2.655	1.994	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	46	19	0
normalized size	1	1.	1.	0.81	1.05	2.19	0.9	0.
time (sec)	N/A	0.029	0.006	0.043	1.021	2.946	2.749	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	65	63	0
normalized size	1	1.	1.	0.	0.	2.32	2.25	0.
time (sec)	N/A	0.037	0.01	0.076	0.	3.319	13.232	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	138	312	0	545	0	347
normalized size	1	1.	0.48	1.09	0.	1.91	0.	1.22
time (sec)	N/A	0.436	0.414	0.268	0.	3.249	0.	1.285

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	99	168	0	328	0	169
normalized size	1	1.	0.64	1.08	0.	2.12	0.	1.09
time (sec)	N/A	0.158	0.279	0.35	0.	3.224	0.	1.314

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	67	63	132	0	65
normalized size	1	1.	0.89	1.18	1.11	2.32	0.	1.14
time (sec)	N/A	0.039	0.038	0.333	1.057	2.904	0.	1.226

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.145	0.089	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.195	0.293	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	0.249	0.309	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	185	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	0.289	0.142	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.215	0.231	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.028	0.112	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.203	0.152	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.306	0.315	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	73	88	111	177	0	139
normalized size	1	1.	0.62	0.75	0.94	1.5	0.	1.18
time (sec)	N/A	0.142	0.046	0.131	1.004	3.028	0.	1.275

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	66	80	131	0	78
normalized size	1	1.	0.72	0.84	1.01	1.66	0.	0.99
time (sec)	N/A	0.08	0.035	0.218	1.012	2.719	0.	1.272

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	51	46	89	34	42
normalized size	1	1.	0.92	1.38	1.24	2.41	0.92	1.14
time (sec)	N/A	0.031	0.02	0.136	1.044	3.005	26.704	1.303

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.086	0.085	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	34	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.127	0.267	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	36	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.14	0.279	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	100	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.311	0.119	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	80	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.214	0.207	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.023	0.108	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	74	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.198	0.117	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.34	0.273	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	119	0	244	0	212
normalized size	1	1.	0.64	0.88	0.	1.81	0.	1.57
time (sec)	N/A	0.195	0.077	0.122	0.	3.005	0.	1.277

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	68	83	0	197	0	128
normalized size	1	1.	0.76	0.92	0.	2.19	0.	1.42
time (sec)	N/A	0.101	0.047	0.247	0.	3.05	0.	1.259

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	39	46	112	0	47
normalized size	1	1.	0.91	0.91	1.07	2.6	0.	1.09
time (sec)	N/A	0.032	0.017	0.109	1.545	2.639	0.	1.307

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.085	0.132	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.133	0.315	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	160	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.16	0.306	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	85	0	0	176	109	0
normalized size	1	1.	0.76	0.	0.	1.57	0.97	0.
time (sec)	N/A	0.146	0.029	0.254	0.	3.089	98.666	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	127	60	0
normalized size	1	1.	0.89	0.	0.	2.02	0.95	0.
time (sec)	N/A	0.068	0.039	0.236	0.	3.099	12.254	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	36	53	0
normalized size	1	1.	1.	0.83	1.06	2.	2.94	0.
time (sec)	N/A	0.017	0.004	0.075	1.019	2.803	1.614	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	130	0	0
normalized size	1	1.	1.	0.	0.	2.5	0.	0.
time (sec)	N/A	0.071	0.015	0.296	0.	2.599	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	85	0	0	192	0	0
normalized size	1	1.	0.79	0.	0.	1.78	0.	0.
time (sec)	N/A	0.148	0.066	0.412	0.	2.68	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	240	0	0	549	0	367
normalized size	1	1.	0.7	0.	0.	1.61	0.	1.07
time (sec)	N/A	0.515	4.596	0.319	0.	2.812	0.	1.297

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	134	113	205	0	117
normalized size	1	1.	0.95	1.56	1.31	2.38	0.	1.36
time (sec)	N/A	0.057	0.105	0.373	1.053	2.74	0.	1.243

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.2	0.1	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	183	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.417	0.364	0.337	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	526	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.969	0.5	0.141	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	163	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.392	0.25	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.042	0.124	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	0.407	0.148	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	351	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.912	0.547	0.356	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	67	0	176	0	0
normalized size	1	1.	1.	1.08	0.	2.84	0.	0.
time (sec)	N/A	0.145	0.109	0.304	0.	2.6	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.069	0.111	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.065	0.066	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.615	0.08	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.644	0.077	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.04	0.059	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.039	0.065	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	93	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.257	0.065	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	91	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.102	0.07	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	62	83	85	167	92	93
normalized size	1	1.	0.65	0.86	0.89	1.74	0.96	0.97
time (sec)	N/A	0.09	0.058	0.046	1.007	2.409	4.051	1.373

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	65	74	147	68	82
normalized size	1	1.	0.76	0.92	1.04	2.07	0.96	1.15
time (sec)	N/A	0.064	0.053	0.042	1.006	2.386	1.281	1.34

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	46	59	122	42	66
normalized size	1	1.	0.93	1.	1.28	2.65	0.91	1.43
time (sec)	N/A	0.035	0.042	0.051	1.142	2.537	0.449	1.322

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	45	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.021	0.062	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	51	50	109	34	0
normalized size	1	1.	1.	1.27	1.25	2.72	0.85	0.
time (sec)	N/A	0.038	0.041	0.045	1.174	2.442	0.609	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	53	69	50	135	60	0
normalized size	1	1.	0.75	0.97	0.7	1.9	0.85	0.
time (sec)	N/A	0.059	0.035	0.042	1.095	2.423	1.587	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	62	87	50	157	87	0
normalized size	1	1.	0.65	0.91	0.52	1.64	0.91	0.
time (sec)	N/A	0.083	0.048	0.042	1.118	2.456	4.637	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	73	90	70	151	102	77
normalized size	1	1.	0.67	0.83	0.64	1.39	0.94	0.71
time (sec)	N/A	0.094	0.017	0.04	1.003	2.645	6.942	1.336

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	72	59	135	78	66
normalized size	1	1.	0.79	0.86	0.7	1.61	0.93	0.79
time (sec)	N/A	0.07	0.016	0.043	1.02	2.399	2.242	1.337

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	54	49	119	54	55
normalized size	1	1.	0.71	0.92	0.83	2.02	0.92	0.93
time (sec)	N/A	0.049	0.024	0.043	1.006	2.422	0.741	1.453

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	35	81	24	35
normalized size	1	1.	1.	1.	1.3	3.	0.89	1.3
time (sec)	N/A	0.005	0.002	0.039	1.031	2.32	0.347	1.321

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	34	76	20	0
normalized size	1	1.	1.	1.07	1.26	2.81	0.74	0.
time (sec)	N/A	0.031	0.015	0.043	1.114	2.355	1.274	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	49	53	36	117	48	0
normalized size	1	1.	0.88	0.95	0.64	2.09	0.86	0.
time (sec)	N/A	0.054	0.04	0.041	1.076	2.127	2.509	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	71	36	140	70	0
normalized size	1	1.	0.9	0.88	0.44	1.73	0.86	0.
time (sec)	N/A	0.074	0.024	0.042	1.092	2.138	5.256	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	268	729	0	689	746	587
normalized size	1	1.	0.92	2.5	0.	2.36	2.55	2.01
time (sec)	N/A	0.272	0.36	0.046	0.	2.099	22.449	1.508

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	159	428	0	435	398	378
normalized size	1	1.	0.82	2.21	0.	2.24	2.05	1.95
time (sec)	N/A	0.181	0.281	0.049	0.	2.157	6.169	1.522

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	104	122	0	254	178	213
normalized size	1	1.	0.87	1.03	0.	2.13	1.5	1.79
time (sec)	N/A	0.118	0.116	0.045	0.	2.042	2.091	1.408

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	33	43	119	53	81
normalized size	1	1.	1.14	0.89	1.16	3.22	1.43	2.19
time (sec)	N/A	0.007	0.041	0.042	1.126	2.072	0.783	1.315

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.122	0.452	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.489	0.375	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	104	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.959	0.399	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	173	0	0	344	172	0
normalized size	1	1.	0.97	0.	0.	1.93	0.97	0.
time (sec)	N/A	0.276	0.391	0.057	0.	2.169	7.471	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	149	0	0	284	121	0
normalized size	1	1.	1.18	0.	0.	2.25	0.96	0.
time (sec)	N/A	0.171	0.376	0.059	0.	2.097	2.505	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	99	0	0	217	68	0
normalized size	1	1.	1.38	0.	0.	3.01	0.94	0.
time (sec)	N/A	0.081	0.128	0.059	0.	2.064	0.834	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.15	0.056	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	0	247	0	0
normalized size	1	1.	0.94	0.	0.	3.69	0.	0.
time (sec)	N/A	0.095	0.034	0.059	0.	2.064	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	97	0	0	329	0	0
normalized size	1	1.	0.78	0.	0.	2.63	0.	0.
time (sec)	N/A	0.178	0.082	0.059	0.	2.15	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	133	0	0	396	0	0
normalized size	1	1.	0.75	0.	0.	2.24	0.	0.
time (sec)	N/A	0.282	0.042	0.059	0.	2.178	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	108	205	0	373	0	300
normalized size	1	1.	0.65	1.24	0.	2.26	0.	1.82
time (sec)	N/A	0.231	0.148	0.052	0.	2.094	0.	1.488

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	151	0	305	0	205
normalized size	1	1.	0.78	1.34	0.	2.7	0.	1.81
time (sec)	N/A	0.126	0.091	0.053	0.	2.151	0.	1.486

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	48	0	219	0	99
normalized size	1	1.	1.	0.86	0.	3.91	0.	1.77
time (sec)	N/A	0.046	0.05	0.044	0.	2.13	0.	1.317

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.146	0.06	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.149	0.059	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.156	0.059	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	375	375	610	0	0	1069	0	0
normalized size	1	1.	1.63	0.	0.	2.85	0.	0.
time (sec)	N/A	0.375	4.625	0.227	0.	2.237	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	301	0	0	647	0	0
normalized size	1	1.	1.59	0.	0.	3.42	0.	0.
time (sec)	N/A	0.189	2.043	0.068	0.	2.178	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	59	0	348	0	0
normalized size	1	1.	0.93	0.83	0.	4.9	0.	0.
time (sec)	N/A	0.156	0.08	0.046	0.	2.114	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.558	0.365	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.341	0.341	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	87	0	0	317	0	122
normalized size	1	1.	0.85	0.	0.	3.11	0.	1.2
time (sec)	N/A	0.215	0.303	0.208	0.	2.181	0.	1.344

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	80	0	0	298	0	119
normalized size	1	1.	0.85	0.	0.	3.17	0.	1.27
time (sec)	N/A	0.166	0.272	0.167	0.	2.188	0.	1.369

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	77	0	0	294	0	111
normalized size	1	1.	0.84	0.	0.	3.2	0.	1.21
time (sec)	N/A	0.101	0.239	0.224	0.	2.202	0.	1.418

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	93	80	80	308	0	112
normalized size	1	1.	1.41	1.21	1.21	4.67	0.	1.7
time (sec)	N/A	0.043	0.14	0.105	1.006	2.136	0.	1.287

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	300	0	0
normalized size	1	1.	0.87	0.	0.	3.23	0.	0.
time (sec)	N/A	0.185	0.262	0.29	0.	2.377	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	0	0	292	0	0
normalized size	1	1.	0.83	0.	0.	3.07	0.	0.
time (sec)	N/A	0.189	0.261	0.243	0.	2.264	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	0	0	466	0	228
normalized size	1	1.	1.	0.	0.	3.7	0.	1.81
time (sec)	N/A	0.254	0.505	0.116	0.	2.33	0.	1.442

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	43	0	82	24	0
normalized size	1	1.	1.	2.05	0.	3.9	1.14	0.
time (sec)	N/A	0.027	0.009	0.226	0.	2.077	2.692	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	30	0	63	24	0
normalized size	1	1.	1.	1.43	0.	3.	1.14	0.
time (sec)	N/A	0.018	0.005	0.107	0.	2.162	0.692	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	53	24	0
normalized size	1	1.	1.	0.	0.	2.65	1.2	0.
time (sec)	N/A	0.028	0.011	0.086	0.	2.076	0.548	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	49	20	0
normalized size	1	1.	1.	0.	0.	2.33	0.95	0.
time (sec)	N/A	0.028	0.006	0.148	0.	2.166	1.427	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	70	22	0
normalized size	1	1.	1.	0.	0.	3.33	1.05	0.
time (sec)	N/A	0.029	0.006	0.352	0.	2.164	3.573	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	82	60	0
normalized size	1	1.	1.	0.	0.	2.93	2.14	0.
time (sec)	N/A	0.033	0.011	0.135	0.	2.198	11.366	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	184	376	0	734	0	0
normalized size	1	1.	0.65	1.33	0.	2.59	0.	0.
time (sec)	N/A	0.368	0.716	0.205	0.	2.196	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	99	206	0	416	0	0
normalized size	1	1.	0.64	1.33	0.	2.68	0.	0.
time (sec)	N/A	0.157	0.297	0.325	0.	2.164	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	92	0	147	0	0
normalized size	1	1.	0.88	1.61	0.	2.58	0.	0.
time (sec)	N/A	0.039	0.039	0.32	0.	2.076	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.488	0.13	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.617	0.3	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.325	0.76	0.327	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	185	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.757	0.141	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.589	0.241	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.031	0.116	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.632	0.134	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.268	0.804	0.309	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	73	135	0	227	0	0
normalized size	1	1.	0.62	1.14	0.	1.92	0.	0.
time (sec)	N/A	0.147	0.047	0.116	0.	2.182	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	58	99	0	157	0	0
normalized size	1	1.	0.72	1.24	0.	1.96	0.	0.
time (sec)	N/A	0.089	0.035	0.209	0.	2.144	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	51	0	97	41	0
normalized size	1	1.	1.	1.42	0.	2.69	1.14	0.
time (sec)	N/A	0.035	0.02	0.138	0.	2.103	22.421	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.134	0.103	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	65	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.207	0.277	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	83	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.224	0.311	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	147	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.341	0.128	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	104	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.287	0.222	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.081	0.111	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	99	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.262	0.128	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	151	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.42	0.286	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	87	172	0	298	0	0
normalized size	1	1.	0.64	1.27	0.	2.21	0.	0.
time (sec)	N/A	0.194	0.132	0.127	0.	2.119	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	69	118	0	225	0	0
normalized size	1	1.	0.77	1.31	0.	2.5	0.	0.
time (sec)	N/A	0.101	0.092	0.245	0.	2.28	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	53	0	120	0	0
normalized size	1	1.	0.91	1.23	0.	2.79	0.	0.
time (sec)	N/A	0.031	0.023	0.113	0.	2.162	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.194	0.133	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.282	0.308	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	161	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.271	0.33	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	112	0	0	235	151	0
normalized size	1	1.	1.	0.	0.	2.1	1.35	0.
time (sec)	N/A	0.166	0.136	0.253	0.	2.083	116.695	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	79	0	0	162	63	0
normalized size	1	1.	1.25	0.	0.	2.57	1.	0.
time (sec)	N/A	0.08	0.116	0.236	0.	2.141	12.568	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	22	19	58	17	0
normalized size	1	1.	1.	1.22	1.06	3.22	0.94	0.
time (sec)	N/A	0.018	0.005	0.071	0.996	2.021	1.497	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	196	0	0
normalized size	1	1.	1.	0.	0.	3.7	0.	0.
time (sec)	N/A	0.086	0.016	0.296	0.	2.068	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	85	0	0	285	0	0
normalized size	1	1.	0.79	0.	0.	2.64	0.	0.
time (sec)	N/A	0.157	0.067	0.317	0.	2.217	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	256	0	0	670	0	0
normalized size	1	1.	0.75	0.	0.	1.96	0.	0.
time (sec)	N/A	0.47	4.223	0.301	0.	2.435	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	175	0	220	0	0
normalized size	1	1.	0.94	2.03	0.	2.56	0.	0.
time (sec)	N/A	0.054	0.105	0.361	0.	2.456	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.691	0.105	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	181	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.411	0.878	0.353	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	526	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.907	1.042	0.146	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	163	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.822	0.253	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.045	0.123	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.949	0.152	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	351	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.869	1.137	0.362	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	84	0	184	0	0
normalized size	1	1.	1.	1.4	0.	3.07	0.	0.
time (sec)	N/A	0.135	0.077	0.302	0.	2.293	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	94	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.568	0.274	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	101	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.437	0.19	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	1.61	0.41	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	1.738	0.384	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	83	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.161	0.166	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	84	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.145	0.178	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	114	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.116	0.348	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	117	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.11	0.342	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	64	77	85	149	88	0
normalized size	1	1.	0.69	0.83	0.91	1.6	0.95	0.
time (sec)	N/A	0.082	0.026	0.043	1.041	2.395	3.702	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	61	74	128	65	0
normalized size	1	1.	0.74	0.88	1.07	1.86	0.94	0.
time (sec)	N/A	0.058	0.028	0.043	0.984	2.123	1.014	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	45	59	103	39	0
normalized size	1	1.	0.87	1.	1.31	2.29	0.87	0.
time (sec)	N/A	0.033	0.028	0.043	1.033	2.297	0.233	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	0	0	0	0
normalized size	1	1.	1.	0.71	0.	0.	0.	0.
time (sec)	N/A	0.013	0.014	0.057	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	47	53	103	34	0
normalized size	1	1.	0.92	1.18	1.32	2.58	0.85	0.
time (sec)	N/A	0.036	0.022	0.042	1.087	2.244	0.558	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	65	53	126	60	0
normalized size	1	1.	0.74	0.94	0.77	1.83	0.87	0.
time (sec)	N/A	0.055	0.025	0.042	1.097	2.32	1.505	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	64	81	53	146	87	0
normalized size	1	1.	0.69	0.87	0.57	1.57	0.94	0.
time (sec)	N/A	0.079	0.024	0.043	1.071	2.459	4.453	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	57	82	69	132	99	0
normalized size	1	1.	0.54	0.78	0.66	1.26	0.94	0.
time (sec)	N/A	0.086	0.036	0.041	1.002	2.348	6.576	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	49	64	58	116	75	0
normalized size	1	1.	0.6	0.79	0.72	1.43	0.93	0.
time (sec)	N/A	0.065	0.028	0.042	1.019	2.352	1.895	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	50	47	100	49	0
normalized size	1	1.	0.72	0.88	0.82	1.75	0.86	0.
time (sec)	N/A	0.044	0.025	0.042	0.974	2.312	0.494	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	34	68	22	0
normalized size	1	1.	1.	1.	1.31	2.62	0.85	0.
time (sec)	N/A	0.005	0.007	0.04	1.018	2.261	0.149	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	31	31	68	32	0
normalized size	1	1.	1.	1.24	1.24	2.72	1.28	0.
time (sec)	N/A	0.029	0.012	0.043	1.296	2.355	1.203	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	52	38	109	63	0
normalized size	1	1.	0.93	0.96	0.7	2.02	1.17	0.
time (sec)	N/A	0.05	0.021	0.043	1.285	2.285	2.354	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	68	38	130	85	0
normalized size	1	1.	0.78	0.87	0.49	1.67	1.09	0.
time (sec)	N/A	0.07	0.027	0.043	1.129	2.392	5.01	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	237	703	0	581	746	0
normalized size	1	1.	0.85	2.52	0.	2.08	2.67	0.
time (sec)	N/A	0.25	0.268	0.049	0.	2.461	18.233	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	142	414	0	362	398	0
normalized size	1	1.	0.76	2.23	0.	1.95	2.14	0.
time (sec)	N/A	0.165	0.175	0.049	0.	2.31	5.381	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	78	117	0	212	178	0
normalized size	1	1.	0.68	1.02	0.	1.84	1.55	0.
time (sec)	N/A	0.105	0.071	0.047	0.	2.325	1.501	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	31	41	107	51	0
normalized size	1	1.	0.94	0.89	1.17	3.06	1.46	0.
time (sec)	N/A	0.006	0.027	0.045	1.081	2.433	0.409	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	1.048	0.506	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.299	0.349	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.522	0.366	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	99	0	0	228	168	0
normalized size	1	1.	0.57	0.	0.	1.3	0.96	0.
time (sec)	N/A	0.256	0.039	0.044	0.	2.286	7.113	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	82	0	0	188	116	0
normalized size	1	1.	0.66	0.	0.	1.52	0.94	0.
time (sec)	N/A	0.159	0.028	0.045	0.	2.412	2.105	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	143	63	0
normalized size	1	1.	0.89	0.	0.	2.01	0.89	0.
time (sec)	N/A	0.077	0.016	0.044	0.	2.306	0.522	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.026	0.043	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	153	0	0
normalized size	1	1.	0.92	0.	0.	2.35	0.	0.
time (sec)	N/A	0.09	0.023	0.042	0.	2.334	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	97	0	0	212	0	0
normalized size	1	1.	0.79	0.	0.	1.72	0.	0.
time (sec)	N/A	0.165	0.028	0.042	0.	2.523	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	114	0	0	255	0	0
normalized size	1	1.	0.66	0.	0.	1.47	0.	0.
time (sec)	N/A	0.279	0.037	0.043	0.	2.554	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	105	0	0	278	0	0
normalized size	1	1.	0.65	0.	0.	1.72	0.	0.
time (sec)	N/A	0.225	0.046	0.045	0.	2.508	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	87	0	0	232	0	0
normalized size	1	1.	0.78	0.	0.	2.09	0.	0.
time (sec)	N/A	0.12	0.03	0.042	0.	2.477	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	169	0	0
normalized size	1	1.	1.	0.	0.	3.13	0.	0.
time (sec)	N/A	0.045	0.011	0.043	0.	2.438	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.035	0.043	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.034	0.044	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.034	0.043	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	366	366	0	0	0	645	0	0
normalized size	1	1.	0.	0.	0.	1.76	0.	0.
time (sec)	N/A	0.361	0.737	0.205	0.	2.555	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	128	0	0	406	0	0
normalized size	1	1.	0.7	0.	0.	2.21	0.	0.
time (sec)	N/A	0.182	0.143	0.046	0.	2.428	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	64	0	0	231	0	0
normalized size	1	1.	0.94	0.	0.	3.4	0.	0.
time (sec)	N/A	0.135	0.041	0.043	0.	2.473	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.051	0.33	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.099	0.323	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	90	0	0	305	0	0
normalized size	1	1.	0.88	0.	0.	2.99	0.	0.
time (sec)	N/A	0.22	0.308	0.142	0.	2.946	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	286	0	0
normalized size	1	1.	0.87	0.	0.	3.08	0.	0.
time (sec)	N/A	0.166	0.276	0.138	0.	3.153	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	78	0	0	293	0	0
normalized size	1	1.	0.86	0.	0.	3.22	0.	0.
time (sec)	N/A	0.133	0.237	0.066	0.	3.386	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	83	78	78	284	0	0
normalized size	1	1.	1.3	1.22	1.22	4.44	0.	0.
time (sec)	N/A	0.038	0.08	0.106	1.099	3.328	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	0	0	296	0	0
normalized size	1	1.	0.87	0.	0.	3.15	0.	0.
time (sec)	N/A	0.217	0.259	0.211	0.	3.304	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	80	0	0	288	0	0
normalized size	1	1.	0.84	0.	0.	3.03	0.	0.
time (sec)	N/A	0.205	0.266	0.217	0.	3.006	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	0	0	429	0	0
normalized size	1	1.	1.	0.	0.	3.4	0.	0.
time (sec)	N/A	0.322	0.442	0.092	0.	3.103	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	43	19	0
normalized size	1	1.	1.	0.	0.	2.05	0.9	0.
time (sec)	N/A	0.027	0.008	0.044	0.	2.681	2.716	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	43	19	0
normalized size	1	1.	1.	0.	0.	2.05	0.9	0.
time (sec)	N/A	0.018	0.005	0.102	0.	2.932	0.728	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	47	24	0
normalized size	1	1.	1.	0.	0.	2.35	1.2	0.
time (sec)	N/A	0.029	0.011	0.061	0.	2.944	0.422	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	45	24	0
normalized size	1	1.	1.	0.	0.	2.14	1.14	0.
time (sec)	N/A	0.026	0.006	0.043	0.	2.98	1.391	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	47	26	0
normalized size	1	1.	1.	0.	0.	2.24	1.24	0.
time (sec)	N/A	0.027	0.006	0.042	0.	2.605	3.677	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	68	63	0
normalized size	1	1.	1.	0.	0.	2.43	2.25	0.
time (sec)	N/A	0.032	0.011	0.043	0.	2.663	10.966	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	131	0	0	547	0	0
normalized size	1	1.	0.51	0.	0.	2.13	0.	0.
time (sec)	N/A	0.409	0.278	0.109	0.	2.808	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	91	0	0	332	0	0
normalized size	1	1.	0.64	0.	0.	2.34	0.	0.
time (sec)	N/A	0.152	0.174	0.24	0.	2.729	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	0	0	135	0	0
normalized size	1	1.	0.89	0.	0.	2.55	0.	0.
time (sec)	N/A	0.039	0.018	0.237	0.	2.709	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.131	0.078	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.184	0.274	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	211	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	0.233	0.297	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.234	0.268	0.11	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.194	0.214	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.024	0.1	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.189	0.135	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	143	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.273	0.291	0.286	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	68	103	0	185	0	0
normalized size	1	1.	0.64	0.96	0.	1.73	0.	0.
time (sec)	N/A	0.115	0.044	0.303	0.	2.586	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	51	72	0	138	0	0
normalized size	1	1.	0.72	1.01	0.	1.94	0.	0.
time (sec)	N/A	0.069	0.026	0.115	0.	2.576	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	0	96	27	0
normalized size	1	1.	1.	1.28	0.	3.	0.84	0.
time (sec)	N/A	0.028	0.014	0.063	0.	2.433	32.308	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.015	0.094	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	32	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.015	0.189	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	34	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.016	0.639	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	52	0	0	0	0	0
normalized size	1	1.	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.023	0.43	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	43	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.02	0.26	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	36	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.014	0.102	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.008	0.066	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	26	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.016	0.119	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	29	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.017	0.349	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	29	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.018	0.645	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	95	0	0	262	0	0
normalized size	1	1.	0.66	0.	0.	1.82	0.	0.
time (sec)	N/A	0.229	0.063	0.112	0.	2.706	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	77	0	0	213	0	0
normalized size	1	1.	0.79	0.	0.	2.2	0.	0.
time (sec)	N/A	0.115	0.038	0.227	0.	2.667	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	0	0	123	0	0
normalized size	1	1.	0.89	0.	0.	2.62	0.	0.
time (sec)	N/A	0.038	0.01	0.227	0.	2.697	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.093	0.079	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.147	0.263	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.219	0.193	0.283	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	78	0	0	180	124	0
normalized size	1	1.	0.64	0.	0.	1.49	1.02	0.
time (sec)	N/A	0.163	0.036	0.112	0.	2.534	32.135	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	131	68	0
normalized size	1	1.	0.84	0.	0.	1.9	0.99	0.
time (sec)	N/A	0.076	0.016	0.225	0.	2.613	5.384	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	43	19	0
normalized size	1	1.	1.	0.	0.	2.05	0.9	0.
time (sec)	N/A	0.018	0.005	0.043	0.	2.582	0.759	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	0	0	135	0	0
normalized size	1	1.	0.95	0.	0.	2.29	0.	0.
time (sec)	N/A	0.082	0.021	0.115	0.	2.694	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	91	0	0	196	0	0
normalized size	1	1.	0.77	0.	0.	1.66	0.	0.
time (sec)	N/A	0.173	0.036	0.279	0.	2.66	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	206	0	0	567	0	0
normalized size	1	1.	0.68	0.	0.	1.87	0.	0.
time (sec)	N/A	0.468	2.117	0.383	0.	2.712	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	0	221	0	0
normalized size	1	1.	0.94	0.	0.	2.83	0.	0.
time (sec)	N/A	0.053	0.071	0.261	0.	2.675	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.17	0.078	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.387	0.306	0.314	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	467	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.892	0.485	0.115	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.351	0.224	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.033	0.107	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.342	0.131	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	319	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.854	0.485	0.327	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	41	0	97	53	0
normalized size	1	1.	1.	1.24	0.	2.94	1.61	0.
time (sec)	N/A	0.119	0.058	0.201	0.	2.639	137.103	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.56	0.052	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.533	0.049	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.534	0.054	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.564	0.056	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	74	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	1.228	0.049	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.768	0.049	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	74	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	10.684	0.049	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	1.588	0.05	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [31] had the largest ratio of [0.6667]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	3	1.	8	0.375
2	A	4	3	1.	8	0.375
3	A	3	3	1.	6	0.5
4	A	1	1	1.	8	0.125
5	A	3	3	1.	8	0.375
6	A	4	3	1.	8	0.375
7	A	5	3	1.	8	0.375
8	A	5	3	1.	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	4	3	1.	8	0.375
10	A	3	3	1.	8	0.375
11	A	1	1	1.	4	0.25
12	A	2	2	1.	8	0.25
13	A	3	3	1.	8	0.375
14	A	4	3	1.	8	0.375
15	A	12	5	1.	14	0.357
16	A	9	5	1.	14	0.357
17	A	7	5	1.	12	0.417
18	A	1	1	1.	6	0.167
19	A	0	0	0.	0	0.
20	A	0	0	0.	0	0.
21	A	0	0	0.	0	0.
22	A	12	6	1.	10	0.6
23	A	8	6	1.	10	0.6
24	A	5	5	1.	8	0.625
25	A	0	0	0.	0	0.
26	A	5	5	1.	10	0.5
27	A	8	6	1.	10	0.6
28	A	12	6	1.	10	0.6
29	A	10	5	1.	10	0.5
30	A	6	5	1.	10	0.5
31	A	4	4	1.	6	0.667
32	A	0	0	0.	0	0.
33	A	0	0	0.	0	0.
34	A	0	0	0.	0	0.
35	A	16	10	1.	16	0.625
36	A	10	9	1.	14	0.643
37	A	4	3	1.	8	0.375
38	A	0	0	0.	0	0.
39	A	0	0	0.	0	0.
40	A	7	7	1.	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
41	A	7	7	1.	15	0.467
42	A	7	7	1.	13	0.538
43	A	3	1	1.	17	0.059
44	A	7	7	1.	17	0.412
45	A	7	7	1.	17	0.412
46	A	8	8	1.	19	0.421
47	A	2	2	1.	19	0.105
48	A	2	2	1.	17	0.118
49	A	2	2	1.	19	0.105
50	A	2	2	1.	19	0.105
51	A	2	2	1.	19	0.105
52	A	2	2	1.	19	0.105
53	A	9	4	1.	17	0.235
54	A	5	4	1.	17	0.235
55	A	2	2	1.	15	0.133
56	A	0	0	0.	0	0.
57	A	0	0	0.	0	0.
58	A	0	0	0.	0	0.
59	A	0	0	0.	0	0.
60	A	0	0	0.	0	0.
61	A	0	0	0.	0	0.
62	A	0	0	0.	0	0.
63	A	0	0	0.	0	0.
64	A	8	5	1.	19	0.263
65	A	5	5	1.	19	0.263
66	A	2	2	1.	17	0.118
67	A	1	1	1.	19	0.053
68	A	4	4	1.	19	0.21
69	A	7	4	1.	19	0.21
70	A	7	4	1.	19	0.21
71	A	4	4	1.	19	0.21
72	A	1	1	1.	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	4	4	1.	19	0.21
74	A	7	5	1.	19	0.263
75	A	9	4	1.	18	0.222
76	A	5	4	1.	18	0.222
77	A	2	2	1.	16	0.125
78	A	0	0	0.	0	0.
79	A	0	0	0.	0	0.
80	A	0	0	0.	0	0.
81	A	7	5	1.	18	0.278
82	A	4	4	1.	18	0.222
83	A	2	2	1.	15	0.133
84	A	4	4	1.	18	0.222
85	A	7	5	1.	18	0.278
86	A	10	6	1.	19	0.316
87	A	3	3	1.	17	0.176
88	A	0	0	0.	0	0.
89	A	0	0	0.	0	0.
90	A	0	0	0.	0	0.
91	A	0	0	0.	0	0.
92	A	0	0	0.	0	0.
93	A	0	0	0.	0	0.
94	A	0	0	0.	0	0.
95	A	4	3	1.	40	0.075
96	A	4	4	1.	18	0.222
97	A	4	4	1.	18	0.222
98	A	4	4	1.	18	0.222
99	A	4	4	1.	18	0.222
100	A	4	4	1.	15	0.267
101	A	4	4	1.	16	0.25
102	A	4	4	1.	15	0.267
103	A	4	4	1.	16	0.25
104	A	5	3	1.	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	4	3	1.	8	0.375
106	A	3	3	1.	6	0.5
107	A	2	2	1.	8	0.25
108	A	3	3	1.	8	0.375
109	A	4	3	1.	8	0.375
110	A	5	3	1.	8	0.375
111	A	5	3	1.	8	0.375
112	A	4	3	1.	8	0.375
113	A	3	3	1.	8	0.375
114	A	1	1	1.	4	0.25
115	A	2	2	1.	8	0.25
116	A	3	3	1.	8	0.375
117	A	4	3	1.	8	0.375
118	A	12	5	1.	14	0.357
119	A	9	5	1.	14	0.357
120	A	7	5	1.	12	0.417
121	A	1	1	1.	6	0.167
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	0	0	0.	0	0.
125	A	12	6	1.	10	0.6
126	A	8	6	1.	10	0.6
127	A	5	5	1.	8	0.625
128	A	0	0	0.	0	0.
129	A	5	5	1.	10	0.5
130	A	8	6	1.	10	0.6
131	A	12	6	1.	10	0.6
132	A	10	5	1.	10	0.5
133	A	6	5	1.	10	0.5
134	A	4	4	1.	6	0.667
135	A	0	0	0.	0	0.
136	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	0	0	0.	0	0.
138	A	16	10	1.	16	0.625
139	A	10	9	1.	14	0.643
140	A	4	3	1.	8	0.375
141	A	0	0	0.	0	0.
142	A	0	0	0.	0	0.
143	A	7	7	1.	17	0.412
144	A	7	7	1.	15	0.467
145	A	7	7	1.	13	0.538
146	A	3	1	1.	17	0.059
147	A	7	7	1.	17	0.412
148	A	7	7	1.	17	0.412
149	A	8	8	1.	19	0.421
150	A	2	2	1.	19	0.105
151	A	2	2	1.	17	0.118
152	A	2	2	1.	19	0.105
153	A	2	2	1.	19	0.105
154	A	2	2	1.	19	0.105
155	A	2	2	1.	19	0.105
156	A	9	4	1.	17	0.235
157	A	5	4	1.	17	0.235
158	A	2	2	1.	15	0.133
159	A	0	0	0.	0	0.
160	A	0	0	0.	0	0.
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	0	0	0.	0	0.
164	A	0	0	0.	0	0.
165	A	0	0	0.	0	0.
166	A	0	0	0.	0	0.
167	A	8	5	1.	19	0.263
168	A	5	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
169	A	2	2	1.	17	0.118
170	A	3	3	1.	19	0.158
171	A	6	6	1.	19	0.316
172	A	9	6	1.	19	0.316
173	A	9	6	1.	19	0.316
174	A	6	6	1.	19	0.316
175	A	3	3	1.	16	0.188
176	A	6	6	1.	19	0.316
177	A	9	7	1.	19	0.368
178	A	9	4	1.	18	0.222
179	A	5	4	1.	18	0.222
180	A	2	2	1.	16	0.125
181	A	0	0	0.	0	0.
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.
184	A	7	5	1.	18	0.278
185	A	4	4	1.	18	0.222
186	A	2	2	1.	15	0.133
187	A	4	4	1.	18	0.222
188	A	7	5	1.	18	0.278
189	A	10	6	1.	19	0.316
190	A	3	3	1.	17	0.176
191	A	0	0	0.	0	0.
192	A	0	0	0.	0	0.
193	A	0	0	0.	0	0.
194	A	0	0	0.	0	0.
195	A	0	0	0.	0	0.
196	A	0	0	0.	0	0.
197	A	0	0	0.	0	0.
198	A	4	3	1.	40	0.075
199	A	6	6	1.	18	0.333
200	A	6	6	1.	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
201	A	6	6	1.	18	0.333
202	A	6	6	1.	18	0.333
203	A	6	6	1.	15	0.4
204	A	6	6	1.	16	0.375
205	A	6	6	1.	15	0.4
206	A	6	6	1.	16	0.375
207	A	5	3	1.	8	0.375
208	A	4	3	1.	8	0.375
209	A	3	3	1.	6	0.5
210	A	1	1	1.	8	0.125
211	A	3	3	1.	8	0.375
212	A	4	3	1.	8	0.375
213	A	5	3	1.	8	0.375
214	A	5	3	1.	8	0.375
215	A	4	3	1.	8	0.375
216	A	3	3	1.	8	0.375
217	A	1	1	1.	4	0.25
218	A	2	2	1.	8	0.25
219	A	3	3	1.	8	0.375
220	A	4	3	1.	8	0.375
221	A	12	5	1.	14	0.357
222	A	9	5	1.	14	0.357
223	A	7	5	1.	12	0.417
224	A	1	1	1.	6	0.167
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.
228	A	12	6	1.	10	0.6
229	A	8	6	1.	10	0.6
230	A	5	5	1.	8	0.625
231	A	0	0	0.	0	0.
232	A	5	5	1.	10	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
233	A	8	6	1.	10	0.6
234	A	12	6	1.	10	0.6
235	A	10	5	1.	10	0.5
236	A	6	5	1.	10	0.5
237	A	4	4	1.	6	0.667
238	A	0	0	0.	0	0.
239	A	0	0	0.	0	0.
240	A	0	0	0.	0	0.
241	A	16	10	1.	16	0.625
242	A	10	9	1.	14	0.643
243	A	4	3	1.	8	0.375
244	A	0	0	0.	0	0.
245	A	0	0	0.	0	0.
246	A	7	7	1.	17	0.412
247	A	7	7	1.	15	0.467
248	A	7	7	1.	13	0.538
249	A	3	1	1.	17	0.059
250	A	7	7	1.	17	0.412
251	A	7	7	1.	17	0.412
252	A	8	8	1.	19	0.421
253	A	2	2	1.	18	0.111
254	A	2	2	1.	16	0.125
255	A	2	2	1.	18	0.111
256	A	2	2	1.	18	0.111
257	A	2	2	1.	18	0.111
258	A	2	2	1.	18	0.111
259	A	9	4	1.	17	0.235
260	A	5	4	1.	17	0.235
261	A	2	2	1.	15	0.133
262	A	0	0	0.	0	0.
263	A	0	0	0.	0	0.
264	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
265	A	0	0	0.	0	0.
266	A	0	0	0.	0	0.
267	A	0	0	0.	0	0.
268	A	0	0	0.	0	0.
269	A	0	0	0.	0	0.
270	A	6	4	1.	18	0.222
271	A	4	4	1.	18	0.222
272	A	2	2	1.	16	0.125
273	A	1	1	1.	18	0.056
274	A	3	3	1.	18	0.167
275	A	5	3	1.	18	0.167
276	A	7	3	1.	18	0.167
277	A	5	3	1.	18	0.167
278	A	3	3	1.	18	0.167
279	A	1	1	1.	15	0.067
280	A	3	3	1.	18	0.167
281	A	5	4	1.	18	0.222
282	A	7	4	1.	18	0.222
283	A	9	4	1.	19	0.21
284	A	5	4	1.	19	0.21
285	A	2	2	1.	17	0.118
286	A	0	0	0.	0	0.
287	A	0	0	0.	0	0.
288	A	0	0	0.	0	0.
289	A	7	5	1.	19	0.263
290	A	4	4	1.	19	0.21
291	A	2	2	1.	16	0.125
292	A	4	4	1.	19	0.21
293	A	7	5	1.	19	0.263
294	A	10	6	1.	19	0.316
295	A	3	3	1.	17	0.176
296	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	0	0	0.	0	0.
298	A	0	0	0.	0	0.
299	A	0	0	0.	0	0.
300	A	0	0	0.	0	0.
301	A	0	0	0.	0	0.
302	A	0	0	0.	0	0.
303	A	5	3	1.	40	0.075
304	A	4	4	1.	18	0.222
305	A	4	4	1.	18	0.222
306	A	4	4	1.	18	0.222
307	A	4	4	1.	18	0.222
308	A	4	4	1.	15	0.267
309	A	4	4	1.	16	0.25
310	A	4	4	1.	15	0.267
311	A	4	4	1.	16	0.25

Chapter 3

Listing of integrals

3.1 $\int x^5 \mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=96

$$-\frac{5\text{Erf}(bx)}{16b^6}+\frac{x^5e^{-b^2x^2}}{6\sqrt{\pi}b}+\frac{5x^3e^{-b^2x^2}}{12\sqrt{\pi}b^3}+\frac{5xe^{-b^2x^2}}{8\sqrt{\pi}b^5}+\frac{1}{6}x^6\text{Erf}(bx)$$

```
[Out] (5*x)/(8*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (5*x^3)/(12*b^3*E^(b^2*x^2)*Sqrt[Pi])
+ x^5/(6*b*E^(b^2*x^2)*Sqrt[Pi]) - (5*Erf[b*x])/(16*b^6) + (x^6*Erf[b*x])/6
```

Rubi [A] time = 0.0961821, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2212, 2205}

$$-\frac{5\text{Erf}(bx)}{16b^6}+\frac{x^5e^{-b^2x^2}}{6\sqrt{\pi}b}+\frac{5x^3e^{-b^2x^2}}{12\sqrt{\pi}b^3}+\frac{5xe^{-b^2x^2}}{8\sqrt{\pi}b^5}+\frac{1}{6}x^6\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Erf[b*x],x]
```

```
[Out] (5*x)/(8*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (5*x^3)/(12*b^3*E^(b^2*x^2)*Sqrt[Pi])
+ x^5/(6*b*E^(b^2*x^2)*Sqrt[Pi]) - (5*Erf[b*x])/(16*b^6) + (x^6*Erf[b*x])/6
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^5\text{erf}(bx)\,dx &= \frac{1}{6}x^6\text{erf}(bx) - \frac{b\int e^{-b^2x^2}x^6\,dx}{3\sqrt{\pi}}\\
&= \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erf}(bx) - \frac{5\int e^{-b^2x^2}x^4\,dx}{6b\sqrt{\pi}}\\
&= \frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erf}(bx) - \frac{5\int e^{-b^2x^2}x^2\,dx}{4b^3\sqrt{\pi}}\\
&= \frac{5e^{-b^2x^2}x}{8b^5\sqrt{\pi}} + \frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erf}(bx) - \frac{5\int e^{-b^2x^2}\,dx}{8b^5\sqrt{\pi}}\\
&= \frac{5e^{-b^2x^2}x}{8b^5\sqrt{\pi}} + \frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} - \frac{5\text{erf}(bx)}{16b^6} + \frac{1}{6}x^6\text{erf}(bx)
\end{aligned}$$

Mathematica [A] time = 0.0254642, size = 72, normalized size = 0.75

$$\frac{e^{-b^2x^2}\left(\sqrt{\pi}e^{b^2x^2}\left(8b^6x^6-15\right)\text{Erf}(bx)+8b^5x^5+20b^3x^3+30bx\right)}{48\sqrt{\pi}b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Erf[b*x],x]
```

```
[Out] (30*b*x + 20*b^3*x^3 + 8*b^5*x^5 + E^(b^2*x^2)*Sqrt[Pi]*(-15 + 8*b^6*x^6)*E
rf[b*x])/(48*b^6*E^(b^2*x^2)*Sqrt[Pi])
```

Maple [A] time = 0.05, size = 83, normalized size = 0.9

$$\frac{1}{b^6}\left(\frac{\mathrm{Erf}\,(bx)\,b^6x^6}{6}-\frac{1}{3\,\sqrt{\pi}}\left(-\frac{b^5x^5}{2\,\mathrm{e}^{b^2x^2}}-\frac{5\,x^3b^3}{4\,\mathrm{e}^{b^2x^2}}-\frac{15\,bx}{8\,\mathrm{e}^{b^2x^2}}+\frac{15\,\sqrt{\pi}\mathrm{Erf}\,(bx)}{16}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erf(b*x),x)
```

```
[Out] 1/b^6*(1/6*erf(b*x)*b^6*x^6-1/3/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^5*x^5-5/4*b^3
*x^3/exp(b^2*x^2)-15/8*b*x/exp(b^2*x^2)+15/16*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.0966, size = 85, normalized size = 0.89

$$\frac{1}{6}\,x^6\,\mathrm{erf}\,(bx)+\frac{b\left(\frac{2\left(4\,b^4x^5+10\,b^2x^3+15\,x\right)e^{\left(-b^2x^2\right)}}{b^6}-\frac{15\,\sqrt{\pi}\,\mathrm{erf}(bx)}{b^7}\right)}{48\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*erf(b*x) + 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-b^2*x^2)/b
^6 - 15*sqrt(pi)*erf(b*x)/b^7)/sqrt(pi)
```

Fricas [A] time = 2.51827, size = 147, normalized size = 1.53

$$\frac{2\,\sqrt{\pi}\left(4\,b^5x^5+10\,b^3x^3+15\,bx\right)e^{\left(-b^2x^2\right)}-\left(15\,\pi-8\,\pi b^6x^6\right)\mathrm{erf}\,(bx)}{48\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/48*(2*sqrt(pi)*(4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*e^(-b^2*x^2) - (15*pi -
8*pi*b^6*x^6)*erf(b*x))/(pi*b^6)
```

Sympy [A] time = 4.46549, size = 88, normalized size = 0.92

$$\begin{cases} \frac{x^6 \operatorname{erf}(bx)}{6} + \frac{x^5 e^{-b^2x^2}}{6\sqrt{\pi}b} + \frac{5x^3 e^{-b^2x^2}}{12\sqrt{\pi}b^3} + \frac{5xe^{-b^2x^2}}{8\sqrt{\pi}b^5} - \frac{5 \operatorname{erf}(bx)}{16b^6} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erf(b*x),x)
```

```
[Out] Piecewise((x**6*erf(b*x)/6 + x**5*exp(-b**2*x**2)/(6*sqrt(pi)*b) + 5*x**3*e
xp(-b**2*x**2)/(12*sqrt(pi)*b**3) + 5*x*exp(-b**2*x**2)/(8*sqrt(pi)*b**5) -
 5*erf(b*x)/(16*b**6), Ne(b, 0)), (0, True))
```

Giac [A] time = 1.17509, size = 86, normalized size = 0.9

$$\frac{1}{6}x^6 \operatorname{erf}(bx) + \frac{b\left(\frac{2\left(4b^4x^5+10b^2x^3+15x\right)e^{\left(-b^2x^2\right)}}{b^6} + \frac{15\sqrt{\pi}\operatorname{erf}(-bx)}{b^7}\right)}{48\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/6*x^6*erf(b*x) + 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-b^2*x^2)/b
^6 + 15*sqrt(pi)*erf(-b*x)/b^7)/sqrt(pi)
```

3.2 $\int x^3 \mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=71

$$-\frac{3\text{Erf}(bx)}{16b^4}+\frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b}+\frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3}+\frac{1}{4}x^4\text{Erf}(bx)$$

```
[Out] (3*x)/(8*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^3/(4*b*E^(b^2*x^2)*Sqrt[Pi]) - (3*Er
f[b*x])/(16*b^4) + (x^4*Erf[b*x])/4
```

Rubi [A] time = 0.0621596, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2212, 2205}

$$-\frac{3\text{Erf}(bx)}{16b^4}+\frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b}+\frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3}+\frac{1}{4}x^4\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Erf[b*x],x]
```

```
[Out] (3*x)/(8*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^3/(4*b*E^(b^2*x^2)*Sqrt[Pi]) - (3*Er
f[b*x])/(16*b^4) + (x^4*Erf[b*x])/4
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^3\text{erf}(bx)\,dx &= \frac{1}{4}x^4\text{erf}(bx) - \frac{b\int e^{-b^2x^2}x^4\,dx}{2\sqrt{\pi}}\\
&= \frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}} + \frac{1}{4}x^4\text{erf}(bx) - \frac{3\int e^{-b^2x^2}x^2\,dx}{4b\sqrt{\pi}}\\
&= \frac{3e^{-b^2x^2}x}{8b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}} + \frac{1}{4}x^4\text{erf}(bx) - \frac{3\int e^{-b^2x^2}\,dx}{8b^3\sqrt{\pi}}\\
&= \frac{3e^{-b^2x^2}x}{8b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}} - \frac{3\text{erf}(bx)}{16b^4} + \frac{1}{4}x^4\text{erf}(bx)
\end{aligned}$$

Mathematica [A] time = 0.0220251, size = 63, normalized size = 0.89

$$-\frac{3\text{Erf}(bx)}{16b^4} + e^{-b^2x^2}\left(\frac{3x}{8\sqrt{\pi}b^3} + \frac{x^3}{4\sqrt{\pi}b}\right) + \frac{1}{4}x^4\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Erf[b*x],x]
```

```
[Out] ((3*x)/(8*b^3*Sqrt[Pi]) + x^3/(4*b*Sqrt[Pi]))/E^(b^2*x^2) - (3*Erf[b*x])/(1
6*b^4) + (x^4*Erf[b*x])/4
```

Maple [A] time = 0.046, size = 65, normalized size = 0.9

$$\frac{1}{b^4}\left(\frac{\text{Erf}\,(bx)\,b^4x^4}{4} - \frac{1}{2\,\sqrt{\pi}}\left(-\frac{x^3b^3}{2\,\text{e}^{b^2x^2}} - \frac{3\,bx}{4\,\text{e}^{b^2x^2}} + \frac{3\,\sqrt{\pi}\text{Erf}\,(bx)}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erf(b*x),x)
```

```
[Out] 1/b^4*(1/4*erf(b*x)*b^4*x^4-1/2/Pi^(1/2)*(-1/2*b^3*x^3/exp(b^2*x^2)-3/4*b*x
/exp(b^2*x^2)+3/8*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.12497, size = 74, normalized size = 1.04

$$\frac{1}{4}\,x^{4}\,\mathrm{erf}\left(bx\right)+\frac{b\left(\frac{2\left(2\,b^{2}x^{3}+3\,x\right)e^{\left(-b^{2}x^{2}\right)}}{b^{4}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}(bx)}{b^{5}}\right)}{16\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*erf(b*x) + 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4 - 3*sqrt(pi
)*erf(b*x)/b^5)/sqrt(pi)
```

Fricas [A] time = 2.47862, size = 127, normalized size = 1.79

$$\frac{2\,\sqrt{\pi}\left(2\,b^{3}x^{3}+3\,bx\right)e^{\left(-b^{2}x^{2}\right)}-\left(3\,\pi-4\,\pi b^{4}x^{4}\right)\mathrm{erf}\left(bx\right)}{16\,\pi b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(pi)*(2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b^4*x^4)*e
rf(b*x))/(pi*b^4)
```

Sympy [A] time = 1.39188, size = 65, normalized size = 0.92

$$\begin{cases}\frac{x^{4}\,\mathrm{erf}\left(bx\right)}{4}+\frac{x^{3}e^{-b^{2}x^{2}}}{4\sqrt{\pi}b}+\frac{3xe^{-b^{2}x^{2}}}{8\sqrt{\pi}b^{3}}-\frac{3\,\mathrm{erf}\left(bx\right)}{16b^{4}} & \text{for}\ b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erf(b*x),x)
```

```
[Out] Piecewise((x**4*erf(b*x)/4 + x**3*exp(-b**2*x**2)/(4*sqrt(pi)*b) + 3*x*exp(
-b**2*x**2)/(8*sqrt(pi)*b**3) - 3*erf(b*x)/(16*b**4), Ne(b, 0)), (0, True))
```

Giac [A] time = 1.26871, size = 76, normalized size = 1.07

$$\frac{1}{4}\,x^4\operatorname{erf}\left(bx\right)+\frac{b\left(\frac{2\left(2\,b^2x^3+3\,x\right)e^{\left(-b^2x^2\right)}}{b^4}+\frac{3\,\sqrt{\pi}\operatorname{erf}(-bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*erf(b*x) + 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4 + 3*sqrt(pi
)*erf(-b*x)/b^5)/sqrt(pi)
```

3.3 $\int x\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\text{Erf}(bx)}{4b^2}+\frac{xe^{-b^2x^2}}{2\sqrt{\pi}b}+\frac{1}{2}x^2\text{Erf}(bx)$$

```
[Out] x/(2*b*E^(b^2*x^2)*Sqrt[Pi]) - Erf[b*x]/(4*b^2) + (x^2*Erf[b*x])/2
```

Rubi [A] time = 0.0361979, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6361, 2212, 2205}

$$-\frac{\text{Erf}(bx)}{4b^2}+\frac{xe^{-b^2x^2}}{2\sqrt{\pi}b}+\frac{1}{2}x^2\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*Erf[b*x],x]
```

```
[Out] x/(2*b*E^(b^2*x^2)*Sqrt[Pi]) - Erf[b*x]/(4*b^2) + (x^2*Erf[b*x])/2
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x\text{erf}(bx)\,dx &= \frac{1}{2}x^2\text{erf}(bx) - \frac{b\int e^{-b^2x^2}x^2\,dx}{\sqrt{\pi}}\\ &= \frac{e^{-b^2x^2}x}{2b\sqrt{\pi}} + \frac{1}{2}x^2\text{erf}(bx) - \frac{\int e^{-b^2x^2}\,dx}{2b\sqrt{\pi}}\\ &= \frac{e^{-b^2x^2}x}{2b\sqrt{\pi}} - \frac{\text{erf}(bx)}{4b^2} + \frac{1}{2}x^2\text{erf}(bx)\end{aligned}$$

Mathematica [A] time = 0.0327715, size = 42, normalized size = 0.91

$$\frac{1}{4}\left(\left(2x^2 - \frac{1}{b^2}\right)\text{Erf}(bx) + \frac{2xe^{-b^2x^2}}{\sqrt{\pi}b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erf[b*x],x]
```

```
[Out] ((2*x)/(b*E^(b^2*x^2)*Sqrt[Pi]) + (-b^(-2) + 2*x^2)*Erf[b*x])/4
```

Maple [A] time = 0.05, size = 47, normalized size = 1.

$$\frac{1}{b^2}\left(\frac{\text{Erf}\,(bx)\,b^2x^2}{2} - \frac{1}{\sqrt{\pi}}\left(-\frac{bx}{2\,e^{b^2x^2}} + \frac{\sqrt{\pi}\text{Erf}\,(bx)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erf(b*x),x)
```

```
[Out] 1/b^2*(1/2*erf(b*x)*b^2*x^2-1/Pi^(1/2)*(-1/2*b*x/exp(b^2*x^2)+1/4*Pi^(1/2)*
erf(b*x)))
```

Maxima [A] time = 1.12473, size = 59, normalized size = 1.28

$$\frac{1}{2}\,x^{2}\operatorname{erf}\left(bx\right)+\frac{b\left(\frac{2\,xe^{\left(-b^{2}x^{2}\right)}}{b^{2}}-\frac{\sqrt{\pi}\operatorname{erf}\left(bx\right)}{b^{3}}\right)}{4\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*erf(b*x) + 1/4*b*(2*x*e^(-b^2*x^2)/b^2 - sqrt(pi)*erf(b*x)/b^3)/sqr
t(pi)
```

Fricas [A] time = 2.56667, size = 101, normalized size = 2.2

$$\frac{2\,\sqrt{\pi}bxe^{\left(-b^{2}x^{2}\right)}-\left(\pi-2\,\pi b^{2}x^{2}\right)\operatorname{erf}\left(bx\right)}{4\,\pi b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(pi)*b*x*e^(-b^2*x^2) - (pi - 2*pi*b^2*x^2)*erf(b*x))/(pi*b^2)
```

Sympy [A] time = 0.472982, size = 39, normalized size = 0.85

$$\begin{cases}\frac{x^{2}\operatorname{erf}\left(bx\right)}{2}+\frac{xe^{-b^{2}x^{2}}}{2\sqrt{\pi}b}-\frac{\operatorname{erf}\left(bx\right)}{4b^{2}} & \text{for}\ b\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x),x)
```

```
[Out] Piecewise((x**2*erf(b*x)/2 + x*exp(-b**2*x**2)/(2*sqrt(pi)*b) - erf(b*x)/(4
*b**2), Ne(b, 0)), (0, True))
```

Giac [A] time = 1.29939, size = 59, normalized size = 1.28

$$\frac{1}{2}\,x^{2}\operatorname{erf}\left(bx\right)+\frac{b\left(\frac{2\,xe^{\left(-b^{2}x^{2}\right)}}{b^{2}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-bx\right)}{b^{3}}\right)}{4\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*erf(b*x) + 1/4*b*(2*x*e^(-b^2*x^2)/b^2 + sqrt(pi)*erf(-b*x)/b^3)/sq
rt(pi)
```

3.4 $\int \frac{\mathbf{Erf}(bx)}{x}\,dx$

Optimal. Leaf size=32

$$\frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0137938, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6358}

$$\frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x,x]
```

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rule 6358

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{\text{erf}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0137354, size = 32, normalized size = 1.

$$\frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x,x]
```

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Maple [A] time = 0.092, size = 23, normalized size = 0.7

$$2\,\frac{bx{}_2F_2(1/2,1/2;\,3/2,3/2;\,-b^2x^2)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x,x)
```

```
[Out] 2/Pi^(1/2)*b*x*hypergeom([1/2,1/2],[3/2,3/2],-b^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erf}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)/x, x)
```

3.5 $\int \frac{\text{Erf}(bx)}{x^3} \, dx$

Optimal. Leaf size=42

$$b^2(-\text{Erf}(bx)) - \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\text{Erf}(bx)}{2x^2}$$

```
[Out] -(b/(E^(b^2*x^2)*Sqrt[Pi]*x)) - b^2*Erf[b*x] - Erf[b*x]/(2*x^2)
```

Rubi [A] time = 0.0386486, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2214, 2205}

$$b^2(-\text{Erf}(bx)) - \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\text{Erf}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x^3,x]
```

```
[Out] -(b/(E^(b^2*x^2)*Sqrt[Pi]*x)) - b^2*Erf[b*x] - Erf[b*x]/(2*x^2)
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = -\frac{\operatorname{erf}(bx)}{2x^2} + \frac{b\int \frac{e^{-b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}$$
$$= -\frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{erf}(bx)}{2x^2} - \frac{\left(2b^3\right)\int e^{-b^2x^2}\,dx}{\sqrt{\pi}}$$
$$= -\frac{be^{-b^2x^2}}{\sqrt{\pi}x} - b^2\operatorname{erf}(bx) - \frac{\operatorname{erf}(bx)}{2x^2}$$

Mathematica [A] time = 0.036902, size = 42, normalized size = 1.

$$b^2(-\operatorname{Erf}(bx)) - \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{Erf}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x^3,x]
```

```
[Out] -(b/(E^(b^2*x^2)*Sqrt[Pi]*x)) - b^2*Erf[b*x] - Erf[b*x]/(2*x^2)
```

Maple [A] time = 0.048, size = 50, normalized size = 1.2

$$b^2\left(-\frac{\operatorname{Erf}(bx)}{2\,b^2x^2} + \frac{1}{\sqrt{\pi}}\left(-\frac{1}{e^{b^2x^2}bx} - \sqrt{\pi}\operatorname{Erf}(bx)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*erf(b*x)/b^2/x^2+1/Pi^(1/2)*(-1/exp(b^2*x^2)/b/x-Pi^(1/2)*erf(b*x
)))
```

Maxima [A] time = 1.23835, size = 50, normalized size = 1.19

$$-\frac{\sqrt{b^2x^2}b\Gamma\left(-\frac{1}{2}, b^2x^2\right)}{2\sqrt{\pi}x} - \frac{\operatorname{erf}(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(b^2*x^2)*b*gamma(-1/2, b^2*x^2)/(sqrt(pi)*x) - 1/2*erf(b*x)/x^2
```

Fricas [A] time = 2.51236, size = 103, normalized size = 2.45

$$-\frac{2\sqrt{\pi}bxe^{\left(-b^2x^2\right)} + \left(\pi + 2\pi b^2x^2\right)\operatorname{erf}(bx)}{2\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(pi)*b*x*e^(-b^2*x^2) + (pi + 2*pi*b^2*x^2)*erf(b*x))/(pi*x^2)
```

Sympy [A] time = 0.670192, size = 36, normalized size = 0.86

$$-b^2\operatorname{erf}(bx) - \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{erf}(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x**3,x)
```

```
[Out] -b**2*erf(b*x) - b*exp(-b**2*x**2)/(sqrt(pi)*x) - erf(b*x)/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)/x^3, x)
```

3.6 $\int \frac{\mathbf{Erf}(bx)}{x^5} dx$

Optimal. Leaf size=71

$$\frac{1}{3}b^4\text{Erf}(bx) + \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} - \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{Erf}(bx)}{4x^4}$$

```
[Out] -b/(6*E^(b^2*x^2)*Sqrt[Pi]*x^3) + b^3/(3*E^(b^2*x^2)*Sqrt[Pi]*x) + (b^4*Erf
[b*x])/3 - Erf[b*x]/(4*x^4)
```

Rubi [A] time = 0.0650876, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2214, 2205}

$$\frac{1}{3}b^4\text{Erf}(bx) + \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} - \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{Erf}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x^5,x]
```

```
[Out] -b/(6*E^(b^2*x^2)*Sqrt[Pi]*x^3) + b^3/(3*E^(b^2*x^2)*Sqrt[Pi]*x) + (b^4*Erf
[b*x])/3 - Erf[b*x]/(4*x^4)
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{erf}(bx)}{x^5}\,dx &= -\frac{\text{erf}(bx)}{4x^4}+\frac{b\int \frac{e^{-b^2x^2}}{x^4}\,dx}{2\sqrt{\pi}}\\
&= -\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3}-\frac{\text{erf}(bx)}{4x^4}-\frac{b^3\int \frac{e^{-b^2x^2}}{x^2}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3}+\frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x}-\frac{\text{erf}(bx)}{4x^4}+\frac{\left(2b^5\right)\int e^{-b^2x^2}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3}+\frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x}+\frac{1}{3}b^4\text{erf}(bx)-\frac{\text{erf}(bx)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0174827, size = 63, normalized size = 0.89

$$\frac{1}{3}b^4\text{Erf}(bx)+e^{-b^2x^2}\left(\frac{b^3}{3\sqrt{\pi}x}-\frac{b}{6\sqrt{\pi}x^3}\right)-\frac{\text{Erf}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x^5,x]
```

```
[Out] (-b/(6*Sqrt[Pi]*x^3) + b^3/(3*Sqrt[Pi]*x))/E^(b^2*x^2) + (b^4*Erf[b*x])/3 -
 Erf[b*x]/(4*x^4)
```

Maple [A] time = 0.046, size = 69, normalized size = 1.

$$b^4\left(-\frac{\text{Erf}\,(bx)}{4\,b^4x^4}+\frac{1}{2\,\sqrt{\pi}}\left(-\frac{1}{3\,\mathrm{e}^{b^2x^2}b^3x^3}+\frac{2}{3\,\mathrm{e}^{b^2x^2}bx}+\frac{2\,\sqrt{\pi}\text{Erf}\,(bx)}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4*erf(b*x)/b^4/x^4+1/2/Pi^(1/2)*(-1/3/exp(b^2*x^2)/b^3/x^3+2/3/exp(
b^2*x^2)/b/x+2/3*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.13255, size = 50, normalized size = 0.7

$$-\frac{\left(b^2x^2\right)^{\frac{3}{2}}b\Gamma\left(-\frac{3}{2},b^2x^2\right)}{4\sqrt{\pi}x^3}-\frac{\operatorname{erf}(bx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*x^2)^(3/2)*b*gamma(-3/2, b^2*x^2)/(sqrt(pi)*x^3) - 1/4*erf(b*x)/x
^4
```

Fricas [A] time = 2.5562, size = 124, normalized size = 1.75

$$\frac{2\sqrt{\pi}\left(2b^3x^3-bx\right)e^{\left(-b^2x^2\right)}-\left(3\pi-4\pi b^4x^4\right)\operatorname{erf}(bx)}{12\pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*sqrt(pi)*(2*b^3*x^3 - b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b^4*x^4)*erf
(b*x))/(pi*x^4)
```

Sympy [A] time = 1.66474, size = 60, normalized size = 0.85

$$\frac{b^4\operatorname{erf}(bx)}{3}+\frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x}-\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3}-\frac{\operatorname{erf}(bx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x**5,x)
```

```
[Out] b**4*erf(b*x)/3 + b**3*exp(-b**2*x**2)/(3*sqrt(pi)*x) - b*exp(-b**2*x**2)/(
6*sqrt(pi)*x**3) - erf(b*x)/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)/x^5, x)
```

3.7 $\int \frac{\mathbf{Erf}(bx)}{x^7} dx$

Optimal. Leaf size=96

$$-\frac{4}{45}b^6\text{Erf}(bx)-\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}+\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{\text{Erf}(bx)}{6x^6}$$

```
[Out] -b/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) + (2*b^3)/(45*E^(b^2*x^2)*Sqrt[Pi]*x^3) -
(4*b^5)/(45*E^(b^2*x^2)*Sqrt[Pi]*x) - (4*b^6*Erf[b*x])/45 - Erf[b*x]/(6*x^6
)
```

Rubi [A] time = 0.0884729, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2214, 2205}

$$-\frac{4}{45}b^6\text{Erf}(bx)-\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}+\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{\text{Erf}(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x^7,x]
```

```
[Out] -b/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) + (2*b^3)/(45*E^(b^2*x^2)*Sqrt[Pi]*x^3) -
(4*b^5)/(45*E^(b^2*x^2)*Sqrt[Pi]*x) - (4*b^6*Erf[b*x])/45 - Erf[b*x]/(6*x^6
)
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
```

```
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{erf}(bx)}{x^7}\,dx &= -\frac{\operatorname{erf}(bx)}{6x^6}+\frac{b\int \frac{e^{-b^2x^2}}{x^6}\,dx}{3\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{\operatorname{erf}(bx)}{6x^6}-\frac{\left(2b^3\right)\int \frac{e^{-b^2x^2}}{x^4}\,dx}{15\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}+\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{\operatorname{erf}(bx)}{6x^6}+\frac{\left(4b^5\right)\int \frac{e^{-b^2x^2}}{x^2}\,dx}{45\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}+\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}-\frac{\operatorname{erf}(bx)}{6x^6}-\frac{\left(8b^7\right)\int e^{-b^2x^2}\,dx}{45\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}+\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}-\frac{4}{45}b^6\operatorname{erf}(bx)-\frac{\operatorname{erf}(bx)}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.0246102, size = 73, normalized size = 0.76

$$\frac{e^{-b^2x^2}\left(-\sqrt{\pi}e^{b^2x^2}\left(8b^6x^6+15\right)\operatorname{Erf}(bx)-8b^5x^5+4b^3x^3-6bx\right)}{90\sqrt{\pi}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x^7,x]
```

```
[Out] (-6*b*x + 4*b^3*x^3 - 8*b^5*x^5 - E^(b^2*x^2)*Sqrt[Pi]*(15 + 8*b^6*x^6)*Erf
[b*x])/(90*E^(b^2*x^2)*Sqrt[Pi]*x^6)
```

Maple [A] time = 0.046, size = 87, normalized size = 0.9

$$b^6\left(-\frac{\operatorname{Erf}(bx)}{6\,b^6x^6}+\frac{1}{3\,\sqrt{\pi}}\left(-\frac{1}{5\,e^{b^2x^2}b^5x^5}+\frac{2}{15\,e^{b^2x^2}b^3x^3}-\frac{4}{15\,e^{b^2x^2}bx}-\frac{4\,\sqrt{\pi}\operatorname{Erf}(bx)}{15}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6*erf(b*x)/b^6/x^6+1/3/Pi^(1/2)*(-1/5/exp(b^2*x^2)/b^5/x^5+2/15/exp
(b^2*x^2)/b^3/x^3-4/15/exp(b^2*x^2)/b/x-4/15*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.16631, size = 50, normalized size = 0.52

$$-\frac{\left(b^2x^2\right)^{\frac{5}{2}}b\Gamma\left(-\frac{5}{2},b^2x^2\right)}{6\sqrt{\pi}x^5}-\frac{\operatorname{erf}(bx)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(b^2*x^2)^(5/2)*b*gamma(-5/2, b^2*x^2)/(sqrt(pi)*x^5) - 1/6*erf(b*x)/x
^6
```

Fricas [A] time = 2.55339, size = 146, normalized size = 1.52

$$-\frac{2\sqrt{\pi}\left(4b^5x^5-2b^3x^3+3bx\right)e^{\left(-b^2x^2\right)}+\left(15\pi+8\pi b^6x^6\right)\operatorname{erf}(bx)}{90\pi x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(2*sqrt(pi)*(4*b^5*x^5 - 2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) + (15*pi + 8
*pi*b^6*x^6)*erf(b*x))/(pi*x^6)
```

Sympy [A] time = 5.12629, size = 87, normalized size = 0.91

$$-\frac{4b^6\operatorname{erf}(bx)}{45}-\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}+\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{\operatorname{erf}(bx)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x**7,x)
```

```
[Out] -4*b**6*erf(b*x)/45 - 4*b**5*exp(-b**2*x**2)/(45*sqrt(pi)*x) + 2*b**3*exp(-
b**2*x**2)/(45*sqrt(pi)*x**3) - b*exp(-b**2*x**2)/(15*sqrt(pi)*x**5) - erf(
b*x)/(6*x**6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)/x^7, x)
```

3.8 $\int x^6 \mathbf{Erf}(bx)\, dx$

Optimal. Leaf size=109

$$\frac{x^6 e^{-b^2x^2}}{7\sqrt{\pi}b}+\frac{3x^4 e^{-b^2x^2}}{7\sqrt{\pi}b^3}+\frac{6x^2 e^{-b^2x^2}}{7\sqrt{\pi}b^5}+\frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7}+\frac{1}{7}x^7\text{Erf}(bx)$$

```
[Out] 6/(7*b^7*E^(b^2*x^2)*Sqrt[Pi]) + (6*x^2)/(7*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (3*
x^4)/(7*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^6/(7*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^7*E
rf[b*x])/7
```

Rubi [A] time = 0.0989079, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2212, 2209}

$$\frac{x^6 e^{-b^2x^2}}{7\sqrt{\pi}b}+\frac{3x^4 e^{-b^2x^2}}{7\sqrt{\pi}b^3}+\frac{6x^2 e^{-b^2x^2}}{7\sqrt{\pi}b^5}+\frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7}+\frac{1}{7}x^7\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Erf[b*x],x]
```

```
[Out] 6/(7*b^7*E^(b^2*x^2)*Sqrt[Pi]) + (6*x^2)/(7*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (3*
x^4)/(7*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^6/(7*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^7*E
rf[b*x])/7
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^6\text{erf}(bx)\,dx &= \frac{1}{7}x^7\text{erf}(bx) - \frac{(2b)\int e^{-b^2x^2}x^7\,dx}{7\sqrt{\pi}}\\
&= \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erf}(bx) - \frac{6\int e^{-b^2x^2}x^5\,dx}{7b\sqrt{\pi}}\\
&= \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erf}(bx) - \frac{12\int e^{-b^2x^2}x^3\,dx}{7b^3\sqrt{\pi}}\\
&= \frac{6e^{-b^2x^2}x^2}{7b^5\sqrt{\pi}} + \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erf}(bx) - \frac{12\int e^{-b^2x^2}x\,dx}{7b^5\sqrt{\pi}}\\
&= \frac{6e^{-b^2x^2}}{7b^7\sqrt{\pi}} + \frac{6e^{-b^2x^2}x^2}{7b^5\sqrt{\pi}} + \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erf}(bx)
\end{aligned}$$

Mathematica [A] time = 0.0177674, size = 72, normalized size = 0.66

$$\frac{e^{-b^2x^2}\left(\sqrt{\pi}b^7x^7e^{b^2x^2}\text{Erf}(bx) + b^6x^6 + 3b^4x^4 + 6b^2x^2 + 6\right)}{7\sqrt{\pi}b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Erf[b*x],x]
```

```
[Out] (6 + 6*b^2*x^2 + 3*b^4*x^4 + b^6*x^6 + b^7*E^(b^2*x^2)*Sqrt[Pi]*x^7*Erf[b*x
])/(7*b^7*E^(b^2*x^2)*Sqrt[Pi])
```

Maple [A] time = 0.044, size = 90, normalized size = 0.8

$$\frac{1}{b^7}\left(\frac{\text{Erf}\,(bx)\,b^7x^7}{7} - \frac{2}{7\sqrt{\pi}}\left(-\frac{b^6x^6}{2\,\text{e}^{b^2x^2}} - \frac{3\,b^4x^4}{2\,\text{e}^{b^2x^2}} - 3\,\frac{b^2x^2}{\text{e}^{b^2x^2}} - 3\,\left(\text{e}^{b^2x^2}\right)^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*erf(b*x),x)
```

```
[Out] 1/b^7*(1/7*erf(b*x)*b^7*x^7-2/7/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^6*x^6-3/2*b^4
*x^4/exp(b^2*x^2)-3*b^2*x^2/exp(b^2*x^2)-3/exp(b^2*x^2)))
```

Maxima [A] time = 1.23354, size = 70, normalized size = 0.64

$$\frac{1}{7}\,x^7\,\mathrm{erf}\,(bx) + \frac{\left(b^6x^6 + 3\,b^4x^4 + 6\,b^2x^2 + 6\right)e^{\left(-b^2x^2\right)}}{7\,\sqrt{\pi}b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*erf(b*x) + 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(-b^2*x^2)/(
sqrt(pi)*b^7)
```

Fricas [A] time = 2.52048, size = 132, normalized size = 1.21

$$\frac{\pi b^7x^7\,\mathrm{erf}\,(bx) + \sqrt{\pi}\left(b^6x^6 + 3\,b^4x^4 + 6\,b^2x^2 + 6\right)e^{\left(-b^2x^2\right)}}{7\,\pi b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/7*(pi*b^7*x^7*erf(b*x) + sqrt(pi)*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e
^(-b^2*x^2))/(pi*b^7)
```

Sympy [A] time = 7.56104, size = 99, normalized size = 0.91

$$\begin{cases} \frac{x^7\,\mathrm{erf}\,(bx)}{7} + \frac{x^6e^{-b^2x^2}}{7\sqrt{\pi}b} + \frac{3x^4e^{-b^2x^2}}{7\sqrt{\pi}b^3} + \frac{6x^2e^{-b^2x^2}}{7\sqrt{\pi}b^5} + \frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*erf(b*x),x)
```

```
[Out] Piecewise((x**7*erf(b*x)/7 + x**6*exp(-b**2*x**2)/(7*sqrt(pi)*b) + 3*x**4*e
xp(-b**2*x**2)/(7*sqrt(pi)*b**3) + 6*x**2*exp(-b**2*x**2)/(7*sqrt(pi)*b**5)
 + 6*exp(-b**2*x**2)/(7*sqrt(pi)*b**7), Ne(b, 0)), (0, True))
```

Giac [A] time = 1.32549, size = 70, normalized size = 0.64

$$\frac{1}{7}\,x^{7}\,\mathrm{erf}\left(bx\right)+\frac{\left(b^{6}x^{6}+3\,b^{4}x^{4}+6\,b^{2}x^{2}+6\right)e^{\left(-b^{2}x^{2}\right)}}{7\,\sqrt{\pi}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/7*x^7*erf(b*x) + 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(-b^2*x^2)/(
sqrt(pi)*b^7)
```

3.9 $\int x^4 \text{Erf}(bx)\,dx$

Optimal. Leaf size=84

$$\frac{x^4 e^{-b^2x^2}}{5\sqrt{\pi}b}+\frac{2x^2e^{-b^2x^2}}{5\sqrt{\pi}b^3}+\frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5}+\frac{1}{5}x^5\text{Erf}(bx)$$

```
[Out] 2/(5*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (2*x^2)/(5*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^4
/(5*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erf[b*x])/5
```

Rubi [A] time = 0.0733966, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2212, 2209}

$$\frac{x^4 e^{-b^2x^2}}{5\sqrt{\pi}b}+\frac{2x^2e^{-b^2x^2}}{5\sqrt{\pi}b^3}+\frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5}+\frac{1}{5}x^5\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Erf[b*x],x]
```

```
[Out] 2/(5*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (2*x^2)/(5*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^4
/(5*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erf[b*x])/5
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\text{erf}(bx)\,dx &= \frac{1}{5}x^5\text{erf}(bx) - \frac{(2b)\int e^{-b^2x^2}x^5\,dx}{5\sqrt{\pi}}\\ &= \frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx) - \frac{4\int e^{-b^2x^2}x^3\,dx}{5b\sqrt{\pi}}\\ &= \frac{2e^{-b^2x^2}x^2}{5b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx) - \frac{4\int e^{-b^2x^2}x\,dx}{5b^3\sqrt{\pi}}\\ &= \frac{2e^{-b^2x^2}}{5b^5\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^2}{5b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx)\end{aligned}$$

Mathematica [A] time = 0.0161336, size = 66, normalized size = 0.79

$$e^{-b^2x^2}\left(\frac{2x^2}{5\sqrt{\pi}b^3} + \frac{2}{5\sqrt{\pi}b^5} + \frac{x^4}{5\sqrt{\pi}b}\right) + \frac{1}{5}x^5\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Erf[b*x],x]
```

```
[Out] (2/(5*b^5*Sqrt[Pi]) + (2*x^2)/(5*b^3*Sqrt[Pi]) + x^4/(5*b*Sqrt[Pi]))/E^(b^2
*x^2) + (x^5*Erf[b*x])/5
```

Maple [A] time = 0.048, size = 72, normalized size = 0.9

$$\frac{1}{b^5}\left(\frac{\text{Erf}\,(bx)\,b^5x^5}{5} - \frac{2}{5\sqrt{\pi}}\left(-\frac{b^4x^4}{2\,e^{b^2x^2}} - \frac{b^2x^2}{e^{b^2x^2}} - \left(e^{b^2x^2}\right)^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erf(b*x),x)
```

```
[Out] 1/b^5*(1/5*erf(b*x)*b^5*x^5-2/5/Pi^(1/2)*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^2
/exp(b^2*x^2)-1/exp(b^2*x^2)))
```

Maxima [A] time = 1.04654, size = 59, normalized size = 0.7

$$\frac{1}{5}\,x^5\,\mathrm{erf}\left(bx\right) + \frac{\left(b^4x^4 + 2\,b^2x^2 + 2\right)e^{\left(-b^2x^2\right)}}{5\,\sqrt{\pi}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*erf(b*x) + 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b^5
)
```

Fricas [A] time = 2.52465, size = 116, normalized size = 1.38

$$\frac{\pi b^5x^5\,\mathrm{erf}\left(bx\right) + \sqrt{\pi}\left(b^4x^4 + 2\,b^2x^2 + 2\right)e^{\left(-b^2x^2\right)}}{5\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/5*(pi*b^5*x^5*erf(b*x) + sqrt(pi)*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2))
/(pi*b^5)
```

Sympy [A] time = 2.46533, size = 75, normalized size = 0.89

$$\begin{cases} \frac{x^5\,\mathrm{erf}\left(bx\right)}{5} + \frac{x^4e^{-b^2x^2}}{5\sqrt{\pi}b} + \frac{2x^2e^{-b^2x^2}}{5\sqrt{\pi}b^3} + \frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erf(b*x),x)
```

```
[Out] Piecewise((x**5*erf(b*x)/5 + x**4*exp(-b**2*x**2)/(5*sqrt(pi)*b) + 2*x**2*e
xp(-b**2*x**2)/(5*sqrt(pi)*b**3) + 2*exp(-b**2*x**2)/(5*sqrt(pi)*b**5), Ne(
b, 0)), (0, True))
```

Giac [A] time = 1.2741, size = 59, normalized size = 0.7

$$\frac{1}{5}\,x^{5}\,\mathrm{erf}\left(bx\right)+\frac{\left(b^{4}x^{4}+2\,b^{2}x^{2}+2\right)e^{\left(-b^{2}x^{2}\right)}}{5\,\sqrt{\pi}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/5*x^5*erf(b*x) + 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b^5
)
```

3.10 $\int x^2 \mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=59

$$\frac{x^2 e^{-b^2x^2}}{3\sqrt{\pi}b}+\frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3}+\frac{1}{3}x^3\mathrm{Erf}(bx)$$

```
[Out] 1/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^2/(3*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf[
b*x])/3
```

Rubi [A] time = 0.0489509, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2212, 2209}

$$\frac{x^2 e^{-b^2x^2}}{3\sqrt{\pi}b}+\frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3}+\frac{1}{3}x^3\mathrm{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erf[b*x],x]
```

```
[Out] 1/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^2/(3*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf[
b*x])/3
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\text{erf}(bx)\,dx &= \frac{1}{3}x^3\text{erf}(bx)-\frac{(2b)\int e^{-b^2x^2}x^3\,dx}{3\sqrt{\pi}}\\ &= \frac{e^{-b^2x^2}x^2}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erf}(bx)-\frac{2\int e^{-b^2x^2}x\,dx}{3b\sqrt{\pi}}\\ &= \frac{e^{-b^2x^2}}{3b^3\sqrt{\pi}}+\frac{e^{-b^2x^2}x^2}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erf}(bx)\end{aligned}$$

Mathematica [A] time = 0.024314, size = 41, normalized size = 0.69

$$\frac{1}{3}\left(\frac{e^{-b^2x^2}\left(b^2x^2+1\right)}{\sqrt{\pi}b^3}+x^3\text{Erf}(bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erf[b*x],x]
```

```
[Out] ((1 + b^2*x^2)/(b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^3*Erf[b*x])/3
```

Maple [A] time = 0.055, size = 54, normalized size = 0.9

$$\frac{1}{b^3}\left(\frac{\text{Erf}\,(bx)\,b^3x^3}{3}-\frac{2}{3\sqrt{\pi}}\left(-\frac{b^2x^2}{2\,e^{b^2x^2}}-\frac{1}{2\,e^{b^2x^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erf(b*x),x)
```

```
[Out] 1/b^3*(1/3*erf(b*x)*b^3*x^3-2/3/Pi^(1/2)*(-1/2*b^2*x^2/exp(b^2*x^2)-1/2/exp
(b^2*x^2)))
```

Maxima [A] time = 1.10549, size = 49, normalized size = 0.83

$$\frac{1}{3}\,x^{3}\,\mathrm{erf}\left(bx\right)+\frac{\left(b^{2}x^{2}+1\right)e^{\left(-b^{2}x^{2}\right)}}{3\,\sqrt{\pi}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*erf(b*x) + 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

Fricas [A] time = 2.55151, size = 100, normalized size = 1.69

$$\frac{\pi b^{3}x^{3}\,\mathrm{erf}\left(bx\right)+\sqrt{\pi}\left(b^{2}x^{2}+1\right)e^{\left(-b^{2}x^{2}\right)}}{3\,\pi b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/3*(pi*b^3*x^3*erf(b*x) + sqrt(pi)*(b^2*x^2 + 1)*e^(-b^2*x^2))/(pi*b^3)
```

Sympy [A] time = 0.804436, size = 51, normalized size = 0.86

$$\begin{cases}\frac{x^{3}\,\mathrm{erf}\left(bx\right)}{3}+\frac{x^{2}e^{-b^{2}x^{2}}}{3\sqrt{\pi}b}+\frac{e^{-b^{2}x^{2}}}{3\sqrt{\pi}b^{3}} & \text{for}\: b\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erf(b*x),x)
```

```
[Out] Piecewise((x**3*erf(b*x)/3 + x**2*exp(-b**2*x**2)/(3*sqrt(pi)*b) + exp(-b**
2*x**2)/(3*sqrt(pi)*b**3), Ne(b, 0)), (0, True))
```

Giac [A] time = 1.27415, size = 49, normalized size = 0.83

$$\frac{1}{3}\,x^3\operatorname{erf}(bx) + \frac{\left(b^2x^2+1\right)e^{\left(-b^2x^2\right)}}{3\,\sqrt{\pi}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*erf(b*x) + 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

3.11 $\int \mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=26

$$\frac{e^{-b^2x^2}}{\sqrt{\pi}b} + x\mathrm{Erf}(bx)$$

```
[Out] 1/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]
```

Rubi [A] time = 0.005257, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6349}

$$\frac{e^{-b^2x^2}}{\sqrt{\pi}b} + x\mathrm{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x],x]

[Out] 1/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \mathrm{erf}(bx)\,dx = \frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\mathrm{erf}(bx)$$

Mathematica [A] time = 0.009682, size = 26, normalized size = 1.

$$\frac{e^{-b^2x^2}}{\sqrt{\pi}b} + x\mathrm{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x],x]
```

```
[Out] 1/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]
```

Maple [A] time = 0.044, size = 26, normalized size = 1.

$$\frac{1}{b}\left(\operatorname{Erf}(bx)\,bx+\frac{\mathrm{e}^{-b^2x^2}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x),x)
```

```
[Out] 1/b*(erf(b*x)*b*x+1/Pi^(1/2)*exp(-b^2*x^2))
```

Maxima [A] time = 1.08567, size = 34, normalized size = 1.31

$$\frac{bx\operatorname{erf}(bx)+\frac{e^{\left(-b^2x^2\right)}}{\sqrt{\pi}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x),x, algorithm="maxima")
```

```
[Out] (b*x*erf(b*x) + e^(-b^2*x^2)/sqrt(pi))/b
```

Fricas [A] time = 2.52973, size = 68, normalized size = 2.62

$$\frac{\pi bx\operatorname{erf}(bx)+\sqrt{\pi}e^{\left(-b^2x^2\right)}}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x),x, algorithm="fricas")
```

```
[Out] (pi*b*x*erf(b*x) + sqrt(pi)*e^(-b^2*x^2))/(pi*b)
```

Sympy [A] time = 0.36341, size = 24, normalized size = 0.92

$$\begin{cases} x\operatorname{erf}(bx) + \frac{e^{-b^2x^2}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x),x)
```

```
[Out] Piecewise((x*erf(b*x) + exp(-b**2*x**2)/(sqrt(pi)*b), Ne(b, 0)), (0, True))
```

Giac [A] time = 1.23808, size = 31, normalized size = 1.19

$$x\operatorname{erf}(bx) + \frac{e^{\left(-b^2x^2\right)}}{\sqrt{\pi}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x),x, algorithm="giac")
```

```
[Out] x*erf(b*x) + e^(-b^2*x^2)/(sqrt(pi)*b)
```

3.12 $\int \frac{\mathbf{Erf}(bx)}{x^2}\,dx$

Optimal. Leaf size=26

$$\frac{b\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}}-\frac{\text{Erf}(bx)}{x}$$

```
[Out] -(Erf[b*x]/x) + (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0313195, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6361, 2210}

$$\frac{b\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}}-\frac{\text{Erf}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x^2,x]
```

```
[Out] -(Erf[b*x]/x) + (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\text{erf}(bx)}{x^2} dx = -\frac{\text{erf}(bx)}{x} + \frac{(2b)\int \frac{e^{-b^2x^2}}{x} dx}{\sqrt{\pi}}$$
$$= -\frac{\text{erf}(bx)}{x} + \frac{b\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0391772, size = 26, normalized size = 1.

$$\frac{b\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{Erf}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x^2,x]
```

```
[Out] -(Erf[b*x]/x) + (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Maple [A] time = 0.046, size = 30, normalized size = 1.2

$$b\left(-\frac{\text{Erf}\,(bx)}{bx} - \frac{\text{Ei}\left(1, b^2x^2\right)}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x^2,x)
```

```
[Out] b*(-erf(b*x)/b/x-1/Pi^(1/2)*Ei(1,b^2*x^2))
```

Maxima [A] time = 1.14936, size = 32, normalized size = 1.23

$$\frac{b\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erf}\,(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] b*Ei(-b^2*x^2)/sqrt(pi) - erf(b*x)/x
```

Fricas [A] time = 2.63074, size = 68, normalized size = 2.62

$$\frac{\sqrt{\pi}bx\text{Ei}\left(-b^2x^2\right) - \pi\operatorname{erf}(bx)}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*b*x*Ei(-b^2*x^2) - pi*erf(b*x))/(pi*x)
```

Sympy [A] time = 1.33391, size = 24, normalized size = 0.92

$$-\frac{b\,\text{E}_1\left(b^2x^2\right)}{\sqrt{\pi}} + \frac{\operatorname{erfc}(bx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x**2,x)
```

```
[Out] -b*expint(1, b**2*x**2)/sqrt(pi) + erfc(b*x)/x - 1/x
```

Giac [A] time = 1.34871, size = 32, normalized size = 1.23

$$\frac{b\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^2,x, algorithm="giac")
```

```
[Out] b*Ei(-b^2*x^2)/sqrt(pi) - erf(b*x)/x
```

3.13 $\int \frac{\mathbf{Erf}(bx)}{x^4}\,dx$

Optimal. Leaf size=56

$$-\frac{b^3\text{ExpIntegralEi}\left(-b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\text{Erf}(bx)}{3x^3}$$

```
[Out] -b/(3*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erf[b*x]/(3*x^3) - (b^3*ExpIntegralEi[-(b
^2*x^2)])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.0521948, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2214, 2210}

$$-\frac{b^3\text{Ei}\left(-b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\text{Erf}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x^4,x]
```

```
[Out] -b/(3*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erf[b*x]/(3*x^3) - (b^3*ExpIntegralEi[-(b
^2*x^2)])/(3*Sqrt[Pi])
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erf}(bx)}{x^4}\,dx &= -\frac{\operatorname{erf}(bx)}{3x^3}+\frac{(2b)\int \frac{e^{-b^2x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\operatorname{erf}(bx)}{3x^3}-\frac{\left(2b^3\right)\int \frac{e^{-b^2x^2}}{x}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\operatorname{erf}(bx)}{3x^3}-\frac{b^3\operatorname{Ei}\left(-b^2x^2\right)}{3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.0610474, size = 47, normalized size = 0.84

$$-\frac{\frac{bx\left(b^2x^2\text{ExpIntegralEi}\left(-b^2x^2\right)+e^{-b^2x^2}\right)}{\sqrt{\pi}}+\operatorname{Erf}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x^4,x]
```

```
[Out] -(Erf[b*x] + (b*x*(E^(-(b^2*x^2)) + b^2*x^2*ExpIntegralEi[-(b^2*x^2)]))/Sqr
t[Pi])/(3*x^3)
```

Maple [A] time = 0.054, size = 53, normalized size = 1.

$$b^3\left(-\frac{\operatorname{Erf}(bx)}{3x^3b^3}+\frac{2}{3\sqrt{\pi}}\left(-\frac{1}{2\,e^{b^2x^2}b^2x^2}+\frac{\operatorname{Ei}\left(1,b^2x^2\right)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x^4,x)
```

```
[Out] b^3*(-1/3*erf(b*x)/b^3/x^3+2/3/Pi^(1/2)*(-1/2/exp(b^2*x^2)/b^2/x^2+1/2*Ei(1
,b^2*x^2)))
```

Maxima [A] time = 1.171, size = 36, normalized size = 0.64

$$-\frac{b^3\Gamma\left(-1,b^2x^2\right)}{3\sqrt{\pi}}-\frac{\operatorname{erf}(bx)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*gamma(-1, b^2*x^2)/sqrt(pi) - 1/3*erf(b*x)/x^3
```

Fricas [A] time = 2.58274, size = 111, normalized size = 1.98

$$-\frac{\pi\operatorname{erf}(bx)+\sqrt{\pi}\left(b^3x^3\operatorname{Ei}\left(-b^2x^2\right)+bxe^{\left(-b^2x^2\right)}\right)}{3\,\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(pi*erf(b*x) + sqrt(pi)*(b^3*x^3*Ei(-b^2*x^2) + b*x*e^(-b^2*x^2)))/(pi
*x^3)
```

Sympy [A] time = 2.68157, size = 54, normalized size = 0.96

$$\frac{b^3\operatorname{E}_1\left(b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}+\frac{\operatorname{erfc}(bx)}{3x^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x**4,x)
```

```
[Out] b**3*expint(1, b**2*x**2)/(3*sqrt(pi)) - b*exp(-b**2*x**2)/(3*sqrt(pi)*x**2
) + erfc(b*x)/(3*x**3) - 1/(3*x**3)
```

Giac [A] time = 1.20057, size = 69, normalized size = 1.23

$$-\frac{\operatorname{erf}(bx)}{3\,x^3}-\frac{b^6x^2\operatorname{Ei}\left(-b^2x^2\right)+b^4e^{\left(-b^2x^2\right)}}{3\,\sqrt{\pi}b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*erf(b*x)/x^3 - 1/3*(b^6*x^2*Ei(-b^2*x^2) + b^4*e^(-b^2*x^2))/(sqrt(pi)
*b^3*x^2)
```

3.14 $\int \frac{\mathbf{Erf}(bx)}{x^6}\,dx$

Optimal. Leaf size=81

$$\frac{b^5\text{ExpIntegralEi}\left(-b^2x^2\right)}{10\sqrt{\pi}}+\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}-\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\text{Erf}(bx)}{5x^5}$$

```
[Out] -b/(10*E^(b^2*x^2)*Sqrt[Pi]*x^4) + b^3/(10*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erf[
b*x]/(5*x^5) + (b^5*ExpIntegralEi[-(b^2*x^2)])/(10*Sqrt[Pi])
```

Rubi [A] time = 0.0745673, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6361, 2214, 2210}

$$\frac{b^5\text{Ei}\left(-b^2x^2\right)}{10\sqrt{\pi}}+\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}-\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\text{Erf}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/x^6,x]
```

```
[Out] -b/(10*E^(b^2*x^2)*Sqrt[Pi]*x^4) + b^3/(10*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erf[
b*x]/(5*x^5) + (b^5*ExpIntegralEi[-(b^2*x^2)])/(10*Sqrt[Pi])
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{erf}(bx)}{x^6}\,dx &= -\frac{\operatorname{erf}(bx)}{5x^5}+\frac{(2b)\int \frac{e^{-b^2x^2}}{x^5}\,dx}{5\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\operatorname{erf}(bx)}{5x^5}-\frac{b^3\int \frac{e^{-b^2x^2}}{x^3}\,dx}{5\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}+\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}-\frac{\operatorname{erf}(bx)}{5x^5}+\frac{b^5\int \frac{e^{-b^2x^2}}{x}\,dx}{5\sqrt{\pi}}\\ &= -\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}+\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}-\frac{\operatorname{erf}(bx)}{5x^5}+\frac{b^5\operatorname{Ei}\left(-b^2x^2\right)}{10\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0441948, size = 62, normalized size = 0.77

$$\frac{b^5x^5\text{ExpIntegralEi}\left(-b^2x^2\right)+bxe^{-b^2x^2}\left(b^2x^2-1\right)-2\sqrt{\pi}\text{Erf}(bx)}{10\sqrt{\pi}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/x^6,x]
```

```
[Out] ((b*x*(-1 + b^2*x^2))/E^(b^2*x^2) - 2*Sqrt[Pi]*Erf[b*x] + b^5*x^5*ExpIntegr
alEi[-(b^2*x^2)])/(10*Sqrt[Pi]*x^5)
```

Maple [A] time = 0.046, size = 71, normalized size = 0.9

$$b^5\left(-\frac{\operatorname{Erf}(bx)}{5\,b^5x^5}+\frac{2}{5\,\sqrt{\pi}}\left(-\frac{1}{4\,e^{b^2x^2}b^4x^4}+\frac{1}{4\,e^{b^2x^2}b^2x^2}-\frac{\operatorname{Ei}\left(1,b^2x^2\right)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5*erf(b*x)/b^5/x^5+2/5/Pi^(1/2)*(-1/4/exp(b^2*x^2)/b^4/x^4+1/4/exp(
b^2*x^2)/b^2/x^2-1/4*Ei(1,b^2*x^2)))
```

Maxima [A] time = 1.09035, size = 36, normalized size = 0.44

$$-\frac{b^5\Gamma\left(-2, b^2x^2\right)}{5\sqrt{\pi}}-\frac{\operatorname{erf}(bx)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*b^5*gamma(-2, b^2*x^2)/sqrt(pi) - 1/5*erf(b*x)/x^5
```

Fricas [A] time = 2.60564, size = 131, normalized size = 1.62

$$-\frac{2\pi\operatorname{erf}(bx)-\sqrt{\pi}\left(b^5x^5\operatorname{Ei}\left(-b^2x^2\right)+\left(b^3x^3-bx\right)e^{\left(-b^2x^2\right)}\right)}{10\pi x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(2*pi*erf(b*x) - sqrt(pi)*(b^5*x^5*Ei(-b^2*x^2) + (b^3*x^3 - b*x)*e^(
-b^2*x^2)))/(pi*x^5)
```

Sympy [A] time = 5.51633, size = 76, normalized size = 0.94

$$-\frac{b^5\operatorname{E}_1\left(b^2x^2\right)}{10\sqrt{\pi}}+\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}-\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}+\frac{\operatorname{erfc}(bx)}{5x^5}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x**6,x)
```

```
[Out] -b**5*expint(1, b**2*x**2)/(10*sqrt(pi)) + b**3*exp(-b**2*x**2)/(10*sqrt(pi
)*x**2) - b*exp(-b**2*x**2)/(10*sqrt(pi)*x**4) + erfc(b*x)/(5*x**5) - 1/(5*
```

```
x**5)
```

Giac [A] time = 1.21881, size = 92, normalized size = 1.14

$$-\frac{\operatorname{erf}(bx)}{5\,x^5}+\frac{b^{10}x^4\operatorname{Ei}\left(-b^2x^2\right)+b^8x^2e^{\left(-b^2x^2\right)}-b^6e^{\left(-b^2x^2\right)}}{10\,\sqrt{\pi}b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/x^6,x, algorithm="giac")
```

```
[Out] -1/5*erf(b*x)/x^5 + 1/10*(b^10*x^4*Ei(-b^2*x^2) + b^8*x^2*e^(-b^2*x^2) - b^
6*e^(-b^2*x^2))/(sqrt(pi)*b^5*x^4)
```

3.15 $\int (c+dx)^3 \mathbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=289

$$\frac{d^2 e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{\sqrt{\pi}b^4}+\frac{d^2 e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^4}-\frac{(bc-ad)^4\mathrm{Erf}(a+bx)}{4b^4d}-\frac{3d(bc-ad)^2\mathrm{Erf}(a+bx)}{4b^4}+\frac{e^{-(a+bx)^2}(bc-a}{\sqrt{\pi}b^4}$$

```
[Out] (d^2*(b*c - a*d))/(b^4*E^(a + b*x)^2*Sqrt[Pi]) + (b*c - a*d)^3/(b^4*E^(a +
b*x)^2*Sqrt[Pi]) + (3*d^3*(a + b*x))/(8*b^4*E^(a + b*x)^2*Sqrt[Pi]) + (3*d*
(b*c - a*d)^2*(a + b*x))/(2*b^4*E^(a + b*x)^2*Sqrt[Pi]) + (d^2*(b*c - a*d)*
(a + b*x)^2)/(b^4*E^(a + b*x)^2*Sqrt[Pi]) + (d^3*(a + b*x)^3)/(4*b^4*E^(a +
 b*x)^2*Sqrt[Pi]) - (3*d^3*Erf[a + b*x])/(16*b^4) - (3*d*(b*c - a*d)^2*Erf[
a + b*x])/(4*b^4) - ((b*c - a*d)^4*Erf[a + b*x])/(4*b^4*d) + ((c + d*x)^4*E
rf[a + b*x])/(4*d)
```

Rubi [A] time = 0.320618, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6361, 2226, 2205, 2209, 2212}

$$\frac{d^2 e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{\sqrt{\pi}b^4}+\frac{d^2 e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^4}-\frac{(bc-ad)^4\mathrm{Erf}(a+bx)}{4b^4d}-\frac{3d(bc-ad)^2\mathrm{Erf}(a+bx)}{4b^4}+\frac{e^{-(a+bx)^2}(bc-a}{\sqrt{\pi}b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Erf[a + b*x],x]
```


```
[Out] (d^2*(b*c - a*d))/(b^4*E^(a + b*x)^2*Sqrt[Pi]) + (b*c - a*d)^3/(b^4*E^(a +
b*x)^2*Sqrt[Pi]) + (3*d^3*(a + b*x))/(8*b^4*E^(a + b*x)^2*Sqrt[Pi]) + (3*d*
(b*c - a*d)^2*(a + b*x))/(2*b^4*E^(a + b*x)^2*Sqrt[Pi]) + (d^2*(b*c - a*d)*
(a + b*x)^2)/(b^4*E^(a + b*x)^2*Sqrt[Pi]) + (d^3*(a + b*x)^3)/(4*b^4*E^(a +
 b*x)^2*Sqrt[Pi]) - (3*d^3*Erf[a + b*x])/(16*b^4) - (3*d*(b*c - a*d)^2*Erf[
a + b*x])/(4*b^4) - ((b*c - a*d)^4*Erf[a + b*x])/(4*b^4*d) + ((c + d*x)^4*E
rf[a + b*x])/(4*d)
```


Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^3 \mathrm{erf}(a+bx)\,dx = \frac{(c+dx)^4 \mathrm{erf}(a+bx)}{4d} - \frac{b\int e^{-(a+bx)^2}(c+dx)^4\,dx}{2d\sqrt{\pi}}$$

$$= \frac{(c+dx)^4 \mathrm{erf}(a+bx)}{4d} - \frac{b\int \left(\frac{(bc-ad)^4 e^{-(a+bx)^2}}{b^4} + \frac{4d(bc-ad)^3 e^{-(a+bx)^2}(a+bx)}{b^4} + \frac{6d^2(bc-ad)^2 e^{-(a+bx)^2}(a+bx)^2}{b^4} + \right.}{2d\sqrt{\pi}}$$

$$= \frac{(c+dx)^4 \mathrm{erf}(a+bx)}{4d} - \frac{d^3\int e^{-(a+bx)^2}(a+bx)^4\,dx}{2b^3\sqrt{\pi}} - \frac{\left(2d^2(bc-ad)\right)\int e^{-(a+bx)^2}(a+bx)^3\,dx}{b^3\sqrt{\pi}} - \Big($$

$$= \frac{(bc-ad)^3 e^{-(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{3d(bc-ad)^2 e^{-(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}} + \frac{d^2(bc-ad)e^{-(a+bx)^2}(a+bx)^2}{b^4\sqrt{\pi}} + \frac{d^3e^{-(a+b}}{4}$$

$$= \frac{d^2(bc-ad)e^{-(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{(bc-ad)^3 e^{-(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{3d^3e^{-(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}} + \frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+}{2b^4\sqrt{\pi}}$$

$$= \frac{d^2(bc-ad)e^{-(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{(bc-ad)^3 e^{-(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{3d^3e^{-(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}} + \frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+}{2b^4\sqrt{\pi}}$$

Mathematica [A] time = 0.304701, size = 248, normalized size = 0.86

$$\frac{e^{-(a+bx)^2}\left(-\sqrt{\pi}e^{(a+bx)^2}\mathrm{Erf}(a+bx)\left(12a^2\left(2b^2c^2d+d^3\right)-16a^3bcd^2+4a^4d^3-8a\left(2b^3c^3+3bcd^2\right)-4b^4x\left(6c^2dx+4c^3+4ca\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Erf[a + b*x],x]
```

```
[Out] (-2*a*(5 + 2*a^2)*d^3 + 2*b*d^2*(8*(1 + a^2)*c + (3 + 2*a^2)*d*x) - 4*a*b^2
*d*(6*c^2 + 4*c*d*x + d^2*x^2) + 4*b^3*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d
^3*x^3) - E^(a + b*x)^2*Sqrt[Pi]*(12*b^2*c^2*d - 16*a^3*b*c*d^2 + 3*d^3 + 4
*a^4*d^3 - 8*a*(2*b^3*c^3 + 3*b*c*d^2) + 12*a^2*(2*b^2*c^2*d + d^3) - 4*b^4
*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Erf[a + b*x])/(16*b^4*E^(a
+ b*x)^2*Sqrt[Pi])
```

Maple [A] time = 0.055, size = 466, normalized size = 1.6

$$\frac{1}{b}\left(\frac{\mathrm{Erf}(bx+a)(d(bx+a)-ad+bc)^4}{4\,db^3} - \frac{1}{2\sqrt{\pi}b^3d}\left(d^4\left(-\frac{(bx+a)^3}{2\,\mathrm{e}^{(bx+a)^2}} - \frac{3\,bx+3\,a}{4\,\mathrm{e}^{(bx+a)^2}} + \frac{3\sqrt{\pi}\mathrm{Erf}(bx+a)}{8}\right) + \frac{a^4d^4\sqrt{\pi}\mathrm{Erf}(bx+}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*erf(b*x+a),x)
```

```
[Out] 1/b*(1/4*erf(b*x+a)*(d*(b*x+a)-a*d+b*c)^4/b^3/d-1/2/Pi^(1/2)/b^3/d*(d^4*(-1
/2*(b*x+a)^3/exp((b*x+a)^2)-3/4*(b*x+a)/exp((b*x+a)^2)+3/8*Pi^(1/2)*erf(b*x
+a))+1/2*a^4*d^4*Pi^(1/2)*erf(b*x+a)+1/2*b^4*c^4*Pi^(1/2)*erf(b*x+a)+2*a^3*
d^4/exp((b*x+a)^2)+6*a^2*d^4*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(
b*x+a))-4*a*d^4*(-1/2*(b*x+a)^2/exp((b*x+a)^2)-1/2/exp((b*x+a)^2))-2*b^3*c^
3*d/exp((b*x+a)^2)+6*b^2*c^2*d^2*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*
erf(b*x+a))+4*b*c*d^3*(-1/2*(b*x+a)^2/exp((b*x+a)^2)-1/2/exp((b*x+a)^2))-2*
a*b^3*c^3*d*Pi^(1/2)*erf(b*x+a)+3*a^2*b^2*c^2*d^2*Pi^(1/2)*erf(b*x+a)-2*a^3
*b*c*d^3*Pi^(1/2)*erf(b*x+a)+6*a*b^2*c^2*d^2/exp((b*x+a)^2)-6*a^2*b*c*d^3/e
xp((b*x+a)^2)-12*a*b*c*d^3*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*
x+a))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(d^3*x^4 + 4*c*d^2*x^3 + 6*c^2*d*x^2 + 4*c^3*x)*erf(b*x + a) - 1/2*inte
grate((b*d^3*x^4 + 4*b*c*d^2*x^3 + 6*b*c^2*d*x^2 + 4*b*c^3*x)*e^(-b^2*x^2 -
 2*a*b*x - a^2), x)/sqrt(pi)
```

Fricas [A] time = 2.58144, size = 579, normalized size = 2.

$$\frac{2\sqrt{\pi}\left(2\,b^3d^3x^3+8\,b^3c^3-12\,ab^2c^2d+8\left(a^2+1\right)bcd^2-\left(2\,a^3+5\,a\right)d^3+2\left(4\,b^3cd^2-ab^2d^3\right)x^2+\left(12\,b^3c^2d-8\,ab^2cd^2+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(pi)*(2*b^3*d^3*x^3 + 8*b^3*c^3 - 12*a*b^2*c^2*d + 8*(a^2 + 1)*
b*c*d^2 - (2*a^3 + 5*a)*d^3 + 2*(4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (12*b^3*c^2
*d - 8*a*b^2*c*d^2 + (2*a^2 + 3)*b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (
4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi*b^4*c^
```

```
3*x + pi*(16*a*b^3*c^3 - 12*(2*a^2 + 1)*b^2*c^2*d + 8*(2*a^3 + 3*a)*b*c*d^2
 - (4*a^4 + 12*a^2 + 3)*d^3))*erf(b*x + a))/(pi*b^4)
```

Sympy [A] time = 24.8021, size = 746, normalized size = 2.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*erf(b*x+a),x)
```

```
[Out] Piecewise((-a**4*d**3*erf(a + b*x)/(4*b**4) + a**3*c*d**2*erf(a + b*x)/b**3
 - a**3*d**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b**4) - 3
*a**2*c**2*d*erf(a + b*x)/(2*b**2) + a**2*c*d**2*exp(-a**2)*exp(-b**2*x**2)
*exp(-2*a*b*x)/(sqrt(pi)*b**3) + a**2*d**3*x*exp(-a**2)*exp(-b**2*x**2)*exp
(-2*a*b*x)/(4*sqrt(pi)*b**3) - 3*a**2*d**3*erf(a + b*x)/(4*b**4) + a*c**3*e
rf(a + b*x)/b - 3*a*c**2*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt
(pi)*b**2) - a*c*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*
b**2) - a*d**3*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b*
*2) + 3*a*c*d**2*erf(a + b*x)/(2*b**3) - 5*a*d**3*exp(-a**2)*exp(-b**2*x**2
)*exp(-2*a*b*x)/(8*sqrt(pi)*b**4) + c**3*x*erf(a + b*x) + 3*c**2*d*x**2*erf
(a + b*x)/2 + c*d**2*x**3*erf(a + b*x) + d**3*x**4*erf(a + b*x)/4 + c**3*ex
p(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) + 3*c**2*d*x*exp(-a**2)
*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) + c*d**2*x**2*exp(-a**2)*exp(
-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) + d**3*x**3*exp(-a**2)*exp(-b**2*x**
2)*exp(-2*a*b*x)/(4*sqrt(pi)*b) - 3*c**2*d*erf(a + b*x)/(4*b**2) + c*d**2*e
xp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b**3) + 3*d**3*x*exp(-a**
2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b**3) - 3*d**3*erf(a + b*x)/(1
6*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
*erf(a), True))
```

Giac [A] time = 1.31544, size = 540, normalized size = 1.87

$$\frac{(dx+c)^4 \operatorname{erf}(bx+a)}{4\,d} + \frac{4\,\pi c^4 \operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right) - 16\,\sqrt{\pi}\left(\frac{\sqrt{\pi}a \operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)c^3 d + \frac{12\,\sqrt{\pi}\left(\frac{\sqrt{\pi}\left(2\,a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(\cdots\right.\right.}{\cdots}\right.}{b}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erf(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(d*x + c)^4*erf(b*x + a)/d + 1/16*(4*pi*c^4*erf(-b*(x + a/b)) - 16*sqrt
(pi)*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c^3*
d + 12*sqrt(pi)*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b)
- 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c^2*d^2/b - 8*sqrt(pi)*(sqrt(pi)*(2*
a^3 + 3*a)*erf(-b*(x + a/b))/b - 2*(b^2*(x + a/b)^2 - 3*a*b*(x + a/b) + 3*a
^2 + 1)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c*d^3/b^2 + sqrt(pi)*(sqrt(pi)*(4*a
^4 + 12*a^2 + 3)*erf(-b*(x + a/b))/b + 2*(2*b^3*(x + a/b)^3 - 8*a*b^2*(x +
a/b)^2 + 12*a^2*b*(x + a/b) - 8*a^3 + 3*b*(x + a/b) - 8*a)*e^(-b^2*x^2 - 2*
a*b*x - a^2)/b)*d^4/b^3)/(pi*d)
```

3.16 $\int (c+dx)^2 \textbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=192

$$-\frac{(bc-ad)^3\text{Erf}(a+bx)}{3b^3d}-\frac{d(bc-ad)\text{Erf}(a+bx)}{2b^3}+\frac{e^{-(a+bx)^2}(bc-ad)^2}{\sqrt{\pi}b^3}+\frac{de^{-(a+bx)^2}(a+bx)(bc-ad)}{\sqrt{\pi}b^3}+\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3\sqrt{\pi}b^3}$$

```
[Out] d^2/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) + (b*c - a*d)^2/(b^3*E^(a + b*x)^2*Sqrt[
Pi]) + (d*(b*c - a*d)*(a + b*x))/(b^3*E^(a + b*x)^2*Sqrt[Pi]) + (d^2*(a + b
*x)^2)/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) - (d*(b*c - a*d)*Erf[a + b*x])/(2*b^3
) - ((b*c - a*d)^3*Erf[a + b*x])/(3*b^3*d) + ((c + d*x)^3*Erf[a + b*x])/(3*
d)
```

Rubi [A] time = 0.199704, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6361, 2226, 2205, 2209, 2212}

$$-\frac{(bc-ad)^3\text{Erf}(a+bx)}{3b^3d}-\frac{d(bc-ad)\text{Erf}(a+bx)}{2b^3}+\frac{e^{-(a+bx)^2}(bc-ad)^2}{\sqrt{\pi}b^3}+\frac{de^{-(a+bx)^2}(a+bx)(bc-ad)}{\sqrt{\pi}b^3}+\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Erf[a + b*x],x]
```

```
[Out] d^2/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) + (b*c - a*d)^2/(b^3*E^(a + b*x)^2*Sqrt[
Pi]) + (d*(b*c - a*d)*(a + b*x))/(b^3*E^(a + b*x)^2*Sqrt[Pi]) + (d^2*(a + b
*x)^2)/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) - (d*(b*c - a*d)*Erf[a + b*x])/(2*b^3
) - ((b*c - a*d)^3*Erf[a + b*x])/(3*b^3*d) + ((c + d*x)^3*Erf[a + b*x])/(3*
d)
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
```

```
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^2\text{erf}(a+bx)\,dx = \frac{(c+dx)^3\text{erf}(a+bx)}{3d} - \frac{(2b)\int e^{-(a+bx)^2}(c+dx)^3\,dx}{3d\sqrt{\pi}}$$

$$= \frac{(c+dx)^3\text{erf}(a+bx)}{3d} - \frac{(2b)\int \left(\frac{(bc-ad)^3e^{-(a+bx)^2}}{b^3} + \frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+bx)}{b^3} + \frac{3d^2(bc-ad)e^{-(a+bx)^2}(a+bx}{b^3}\right.}{3d\sqrt{\pi}}$$

$$= \frac{(c+dx)^3\text{erf}(a+bx)}{3d} - \frac{\left(2d^2\right)\int e^{-(a+bx)^2}(a+bx)^3\,dx}{3b^2\sqrt{\pi}} - \frac{(2d(bc-ad))\int e^{-(a+bx)^2}(a+bx)^2\,dx}{b^2\sqrt{\pi}}$$

$$= \frac{(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}} + \frac{d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} + \frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}} - \frac{(bc-ad)^3\text{erf}(a+}{3b^3d}$$

$$= \frac{d^2e^{-(a+bx)^2}}{3b^3\sqrt{\pi}} + \frac{(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}} + \frac{d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} + \frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}} - \frac{d(bc}{}$$

Mathematica [A] time = 0.208466, size = 138, normalized size = 0.72

$$\frac{\operatorname{Erf}(a+bx)\left(-6a^2bcd+2a^3d^2+3a\left(2b^2c^2+d^2\right)+2b^3x\left(3c^2+3cdx+d^2x^2\right)-3bcd\right)+\frac{2e^{-(a+bx)^2}\left(\left(a^2+1\right)d^2-abd(3c+dx)+b^2\left(3c^2+3c\right.\right.}{\sqrt{\pi}}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Erf[a + b*x],x]
```

```
[Out] ((2*((1 + a^2)*d^2 - a*b*d*(3*c + d*x) + b^2*(3*c^2 + 3*c*d*x + d^2*x^2)))/
(E^(a + b*x)^2*Sqrt[Pi]) + (-3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + 3*a*(2*b^2
*c^2 + d^2) + 2*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Erf[a + b*x])/(6*b^3)
```

Maple [A] time = 0.052, size = 283, normalized size = 1.5

$$\frac{1}{b}\left(\frac{\operatorname{Erf}(bx+a)(d(bx+a)-ad+bc)^3}{3db^2}-\frac{2}{3\sqrt{\pi}b^2d}\left(\frac{b^3c^3\sqrt{\pi}\operatorname{Erf}(bx+a)}{2}+d^3\left(-\frac{(bx+a)^2}{2\,e^{(bx+a)^2}}-\frac{1}{2\,e^{(bx+a)^2}}\right)-\frac{a^3d^3\sqrt{\pi}\operatorname{Erf}(bx\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*erf(b*x+a),x)
```

```
[Out] 1/b*(1/3*erf(b*x+a)*(d*(b*x+a)-a*d+b*c)^3/b^2/d-2/3/Pi^(1/2)/b^2/d*(1/2*b^3
*c^3*Pi^(1/2)*erf(b*x+a)+d^3*(-1/2*(b*x+a)^2/exp((b*x+a)^2)-1/2/exp((b*x+a)
^2))-1/2*a^3*d^3*Pi^(1/2)*erf(b*x+a)-3/2*a^2*d^3/exp((b*x+a)^2)-3*a*d^3*(-1
/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))-3/2*b^2*c^2*d/exp((b*x+a
)^2)+3*b*c*d^2*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))-3/2*a*
b^2*c^2*d*Pi^(1/2)*erf(b*x+a)+3/2*a^2*b*c*d^2*Pi^(1/2)*erf(b*x+a)+3*a*b*c*d
^2/exp((b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(d^2x^3+3\,cdx^2+3\,c^2x\right)\operatorname{erf}(bx+a)-\frac{-\frac{3\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)ab\left(\operatorname{erf}\left(\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}\right)-1\right)}{\left(-b^2\right)^{\frac{3}{2}}\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}}+\frac{b^2e^{\left(-\frac{\left(b^2x+ab\right)^2}{b^2}\right)}}{\left(-b^2\right)^{\frac{3}{2}}}\right)bc^2}{\sqrt{-b^2}}-\frac{3\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)a^2b^2\left(\operatorname{erf}\left(\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}\right)-1\right)}{\left(-b^2\right)^{\frac{5}{2}}\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}}+\frac{2\,ab\cdots}{\cdots}\right.}{\sqrt{\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)*erf(b*x + a) - 1/3*integrate(2*(b*d^2*x
^3 + 3*b*c*d*x^2 + 3*b*c^2*x)*e^(-b^2*x^2 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Fricas [A] time = 2.57547, size = 360, normalized size = 1.88

$$\frac{2\sqrt{\pi}\left(b^2d^2x^2+3\,b^2c^2-3\,abcd+\left(a^2+1\right)d^2+\left(3\,b^2cd-abd^2\right)x\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}+\left(2\,\pi b^3d^2x^3+6\,\pi b^3cdx^2+6\,\pi b^3c^2x+\right.}{6\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(pi)*(b^2*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 + 1)*d^2 + (3*b
^2*c*d - a*b*d^2)*x)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (2*pi*b^3*d^2*x^3 + 6*p
i*b^3*c*d*x^2 + 6*pi*b^3*c^2*x + pi*(6*a*b^2*c^2 - 3*(2*a^2 + 1)*b*c*d + (2
*a^3 + 3*a)*d^2))*erf(b*x + a))/(pi*b^3)
```

Sympy [A] time = 7.06001, size = 398, normalized size = 2.07

$$\begin{cases} \frac{a^3d^2\operatorname{erf}(a+bx)}{3b^3}-\frac{a^2cd\operatorname{erf}(a+bx)}{b^2}+\frac{a^2d^2e^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^3}+\frac{ac^2\operatorname{erf}(a+bx)}{b}-\frac{acde^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b^2}-\frac{ad^2xe^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^2}+\frac{ad^2\operatorname{erf}(a+bx)}{2b^3}+c^2x\,\mathrm{e} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\operatorname{erf}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*erf(b*x+a),x)
```

```
[Out] Piecewise((a**3*d**2*erf(a + b*x)/(3*b**3) - a**2*c*d*erf(a + b*x)/b**2 + a
**2*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b**3) + a*c**
2*erf(a + b*x)/b - a*c*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)
*b**2) - a*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b**2
) + a*d**2*erf(a + b*x)/(2*b**3) + c**2*x*erf(a + b*x) + c*d*x**2*erf(a + b
*x) + d**2*x**3*erf(a + b*x)/3 + c**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b
*x)/(sqrt(pi)*b) + c*d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)
*b) + d**2*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b) - c
```

```
*d*erf(a + b*x)/(2*b**2) + d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3
*sqrt(pi)*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*erf(a), True
))
```

Giac [A] time = 1.34661, size = 365, normalized size = 1.9

$$\frac{(dx+c)^3 \operatorname{erf}(bx+a)}{3d} + \frac{2\pi c^3 \operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right) - 6\sqrt{\pi}\left(\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2abx-a^2\right)}}{b}\right)c^2d + \frac{3\sqrt{\pi}\left(\frac{\sqrt{\pi}\left(2a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\right.\right.}{}\right.}{b}}{6\pi d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erf(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(d*x + c)^3*erf(b*x + a)/d + 1/6*(2*pi*c^3*erf(-b*(x + a/b)) - 6*sqrt(p
i)*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c^2*d
+ 3*sqrt(pi)*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2
*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c*d^2/b - sqrt(pi)*(sqrt(pi)*(2*a^3 + 3
*a)*erf(-b*(x + a/b))/b - 2*(b^2*(x + a/b)^2 - 3*a*b*(x + a/b) + 3*a^2 + 1)
*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*d^3/b^2)/(pi*d)
```

3.17 $\int (c+dx)\mathbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=118

$$-\frac{(bc-ad)^2\mathrm{Erf}(a+bx)}{2b^2d}+\frac{e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^2}-\frac{d\mathrm{Erf}(a+bx)}{4b^2}+\frac{de^{-(a+bx)^2}(a+bx)}{2\sqrt{\pi}b^2}+\frac{(c+dx)^2\mathrm{Erf}(a+bx)}{2d}$$

```
[Out] (b*c - a*d)/(b^2*E^(a + b*x)^2*Sqrt[Pi]) + (d*(a + b*x))/(2*b^2*E^(a + b*x)
^2*Sqrt[Pi]) - (d*Erf[a + b*x])/(4*b^2) - ((b*c - a*d)^2*Erf[a + b*x])/(2*b
^2*d) + ((c + d*x)^2*Erf[a + b*x])/(2*d)
```

Rubi [A] time = 0.119941, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6361, 2226, 2205, 2209, 2212}

$$-\frac{(bc-ad)^2\mathrm{Erf}(a+bx)}{2b^2d}+\frac{e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^2}-\frac{d\mathrm{Erf}(a+bx)}{4b^2}+\frac{de^{-(a+bx)^2}(a+bx)}{2\sqrt{\pi}b^2}+\frac{(c+dx)^2\mathrm{Erf}(a+bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Erf[a + b*x],x]
```

```
[Out] (b*c - a*d)/(b^2*E^(a + b*x)^2*Sqrt[Pi]) + (d*(a + b*x))/(2*b^2*E^(a + b*x)
^2*Sqrt[Pi]) - (d*Erf[a + b*x])/(4*b^2) - ((b*c - a*d)^2*Erf[a + b*x])/(2*b
^2*d) + ((c + d*x)^2*Erf[a + b*x])/(2*d)
```

Rule 6361

```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
(c + d*x)^(m + 1)*Erf[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*(m
 + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d, m
}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\operatorname{erf}(a+bx)\,dx &= \frac{(c+dx)^2\operatorname{erf}(a+bx)}{2d} - \frac{b\int e^{-(a+bx)^2}(c+dx)^2\,dx}{d\sqrt{\pi}} \\
&= \frac{(c+dx)^2\operatorname{erf}(a+bx)}{2d} - \frac{b\int \left(\frac{(bc-ad)^2e^{-(a+bx)^2}}{b^2} + \frac{2d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^2} + \frac{d^2e^{-(a+bx)^2}(a+bx)^2}{b^2}\right)dx}{d\sqrt{\pi}} \\
&= \frac{(c+dx)^2\operatorname{erf}(a+bx)}{2d} - \frac{d\int e^{-(a+bx)^2}(a+bx)^2\,dx}{b\sqrt{\pi}} - \frac{(2(bc-ad))\int e^{-(a+bx)^2}(a+bx)\,dx}{b\sqrt{\pi}} - \frac{(bc-a
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(bc-ad)e^{-(a+bx)^2}}{b^2\sqrt{\pi}} + \frac{de^{-(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}} - \frac{(bc-ad)^2\operatorname{erf}(a+bx)}{2b^2d} + \frac{(c+dx)^2\operatorname{erf}(a+bx)}{2d} - \frac{d\int e
\\
&= \frac{(bc-ad)e^{-(a+bx)^2}}{b^2\sqrt{\pi}} + \frac{de^{-(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}} - \frac{d\operatorname{erf}(a+bx)}{4b^2} - \frac{(bc-ad)^2\operatorname{erf}(a+bx)}{2b^2d} + \frac{(c+dx)^2\operatorname{erf}(}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.0850256, size = 88, normalized size = 0.75

$$\frac{e^{-(a+bx)^2}\left(-\sqrt{\pi}e^{(a+bx)^2}\operatorname{Erf}(a+bx)\left(2a^2d-4abc-4b^2cx-2b^2dx^2+d\right)-2ad+4bc+2bdx\right)}{4\sqrt{\pi}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Erf[a + b*x],x]
```

```
[Out] (4*b*c - 2*a*d + 2*b*d*x - E^(a + b*x)^2*Sqrt[Pi]*(-4*a*b*c + d + 2*a^2*d -
 4*b^2*c*x - 2*b^2*d*x^2)*Erf[a + b*x])/(4*b^2*E^(a + b*x)^2*Sqrt[Pi])
```

Maple [A] time = 0.049, size = 111, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\operatorname{Erf}(bx+a)}{b}\left(\frac{d(bx+a)^2}{2}-ad(bx+a)+bc(bx+a)\right)-\frac{1}{\sqrt{\pi}b}\left(d\left(-\frac{bx+a}{2\,\mathrm{e}^{(bx+a)^2}}+\frac{\sqrt{\pi}\operatorname{Erf}(bx+a)}{4}\right)+\frac{ad}{\mathrm{e}^{(bx+a)^2}}-\frac{bc}{\mathrm{e}^{(bx+a)^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*erf(b*x+a),x)
```

```
[Out] 1/b*(erf(b*x+a)/b*(1/2*d*(b*x+a)^2-a*d*(b*x+a)+b*c*(b*x+a))-1/Pi^(1/2)/b*(d
*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))+a*d/exp((b*x+a)^2)-b
*c/exp((b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(dx^2+2\,cx\right)\operatorname{erf}(bx+a)-\frac{-\frac{\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)ab\left(\operatorname{erf}\left(\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}\right)-1\right)}{\left(-b^2\right)^{\frac{3}{2}}\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}}+\frac{b^2e^{\left(-\frac{\left(b^2x+ab\right)^2}{b^2}\right)}}{\left(-b^2\right)^{\frac{3}{2}}}\right)bc}{\sqrt{-b^2}}-\frac{\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)a^2b^2\left(\operatorname{erf}\left(\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}\right)-1\right)}{\left(-b^2\right)^{\frac{5}{2}}\sqrt{\frac{\left(b^2x+ab\right)^2}{b^2}}}+\frac{2\,ab^3e^{\left(-\frac{\left(b^2x+ab\right)^2}{b^2}\right)}}{\left(-b^2\right)^{\frac{5}{2}}}-\cdots\right.}{2\sqrt{-b^2}}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*erf(b*x + a) - integrate((b*d*x^2 + 2*b*c*x)*e^(-b^2*x^
2 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Fricas [A] time = 2.58843, size = 211, normalized size = 1.79

$$\frac{2\sqrt{\pi}(bdx+2\,bc-ad)e^{\left(-b^2x^2-2\,abx-a^2\right)}+\left(2\,\pi b^2dx^2+4\,\pi b^2cx+\pi\left(4\,abc-\left(2\,a^2+1\right)d\right)\right)\operatorname{erf}(bx+a)}{4\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(pi)*(b*d*x + 2*b*c - a*d)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (2*pi*
b^2*d*x^2 + 4*pi*b^2*c*x + pi*(4*a*b*c - (2*a^2 + 1)*d))*erf(b*x + a))/(pi*
b^2)
```

Sympy [A] time = 2.57094, size = 178, normalized size = 1.51

$$\begin{cases} -\frac{a^2 d \operatorname{erf}(a+bx)}{2b^2} + \frac{ac \operatorname{erf}(a+bx)}{b} - \frac{ade^{-a^2}e^{-b^2x^2}e^{-2abx}}{2\sqrt{\pi}b^2} + cx \operatorname{erf}(a+bx) + \frac{dx^2 \operatorname{erf}(a+bx)}{2} + \frac{ce^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b} + \frac{dxe^{-a^2}e^{-b^2x^2}e^{-2abx}}{2\sqrt{\pi}b} - \frac{d \operatorname{erf}(a+bx)}{4b^2} \\ \left(cx + \frac{dx^2}{2}\right) \operatorname{erf}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a),x)
```

```
[Out] Piecewise((-a**2*d*erf(a + b*x)/(2*b**2) + a*c*erf(a + b*x)/b - a*d*exp(-a*
*2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b**2) + c*x*erf(a + b*x) + d*
x**2*erf(a + b*x)/2 + c*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*
b) + d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) - d*erf(a
+ b*x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*erf(a), True))
```

Giac [A] time = 1.33218, size = 201, normalized size = 1.7

$$\frac{1}{2}\left(dx^2 + 2\,cx\right)\operatorname{erf}(bx+a) - \frac{4\sqrt{\pi}\left(\frac{\sqrt{\pi}a \operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)c - \frac{\sqrt{\pi}\left(\frac{\sqrt{\pi}\left(2\,a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\frac{a}{b}\right)-2\,a\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)d}{b}}{4\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*erf(b*x + a) - 1/4*(4*sqrt(pi)*(sqrt(pi)*a*erf(-b*(x +
a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c - sqrt(pi)*(sqrt(pi)*(2*a^2 + 1
)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/
b)*d/b)/pi
```

3.18 $\int \mathbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)\text{Erf}(a+bx)}{b}+\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

```
[Out] 1/(b*E^(a + b*x)^2*Sqrt[Pi]) + ((a + b*x)*Erf[a + b*x])/b
```

Rubi [A] time = 0.0069472, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6349}

$$\frac{(a+bx)\text{Erf}(a+bx)}{b}+\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[a + b*x],x]
```

```
[Out] 1/(b*E^(a + b*x)^2*Sqrt[Pi]) + ((a + b*x)*Erf[a + b*x])/b
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erf}(a+bx)\,dx=\frac{e^{-(a+bx)^2}}{b\sqrt{\pi}}+\frac{(a+bx)\text{erf}(a+bx)}{b}$$

Mathematica [A] time = 0.0402724, size = 35, normalized size = 0.97

$$\left(\frac{a}{b}+x\right)\text{Erf}(a+bx)+\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[a + b*x],x]
```

```
[Out] 1/(b*E^(a + b*x)^2*Sqrt[Pi]) + (a/b + x)*Erf[a + b*x]
```

Maple [A] time = 0.047, size = 32, normalized size = 0.9

$$\frac{1}{b}\left(\mathrm{Erf}\left(bx+a\right)\left(bx+a\right)+\frac{\mathrm{e}^{-(bx+a)^2}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a),x)
```

```
[Out] 1/b*(erf(b*x+a)*(b*x+a)+1/Pi^(1/2)*exp(-(b*x+a)^2))
```

Maxima [A] time = 1.07452, size = 42, normalized size = 1.17

$$\frac{(bx+a)\,\mathrm{erf}\left(bx+a\right)+\frac{e^{\left(-(bx+a)^2\right)}}{\sqrt{\pi}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*erf(b*x + a) + e^(-(b*x + a)^2)/sqrt(pi))/b
```

Fricas [A] time = 2.5475, size = 107, normalized size = 2.97

$$\frac{(\pi bx+\pi a)\,\mathrm{erf}\left(bx+a\right)+\sqrt{\pi}e^{\left(-b^2x^2-2\,abx-a^2\right)}}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a),x, algorithm="fricas")
```

```
[Out] ((pi*b*x + pi*a)*erf(b*x + a) + sqrt(pi)*e^(-b^2*x^2 - 2*a*b*x - a^2))/(pi*
b)
```

Sympy [A] time = 0.819628, size = 53, normalized size = 1.47

$$\begin{cases} \frac{a \operatorname{erf}(a+bx)}{b} + x \operatorname{erf}(a+bx) + \frac{e^{-a^2} e^{-b^2 x^2} e^{-2abx}}{\sqrt{\pi} b} & \text{for } b \neq 0 \\ x \operatorname{erf}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a),x)
```

```
[Out] Piecewise((a*erf(a + b*x)/b + x*erf(a + b*x) + exp(-a**2)*exp(-b**2*x**2)*e
xp(-2*a*b*x)/(sqrt(pi)*b), Ne(b, 0)), (x*erf(a), True))
```

Giac [A] time = 1.30072, size = 80, normalized size = 2.22

$$x \operatorname{erf}(bx+a) - \frac{\frac{\sqrt{\pi} a \operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2 x^2 - 2\,abx - a^2\right)}}{b}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a),x, algorithm="giac")
```

```
[Out] x*erf(b*x + a) - (sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x -
a^2)/b)/sqrt(pi)
```

3.19 $\int \frac{\textbf{Erf}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{Erf}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Erf[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0147108, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Erf[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{erf}(a+bx)}{c+dx}\,dx=\int \frac{\text{erf}(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 1.18764, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Erf[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.39, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a)/(d*x+c),x)
```

```
[Out] int(erf(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erf}(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(erf(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)/(d*x + c), x)
```

3.20 $\int \frac{\text{Erf}(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=51

$$\frac{2b\text{Unintegrable}\left(\frac{e^{-(a+bx)^2}}{c+dx},x\right)}{\sqrt{\pi}d}-\frac{\text{Erf}(a+bx)}{d(c+dx)}$$

```
[Out] -(Erf[a + b*x]/(d*(c + d*x))) + (2*b*Unintegrable[1/(E^(a + b*x)^2*(c + d*x
)), x])/(d*Sqrt[Pi])
```

Rubi [A] time = 0.0408822, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Erf[a + b*x]/(d*(c + d*x))) + (2*b*Defer[Int][1/(E^(a + b*x)^2*(c + d*x))
, x])/(d*Sqrt[Pi])
```

Rubi steps

$$\int \frac{\text{erf}(a+bx)}{(c+dx)^2} \, dx=-\frac{\text{erf}(a+bx)}{d(c+dx)}+\frac{(2b)\int \frac{e^{-(a+bx)^2}}{c+dx} \, dx}{d\sqrt{\pi}}$$

Mathematica [A] time = 0.296237, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Erf[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.426, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(erf(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2\,b\int \frac{e^{\left(-b^2x^2-2\,abx\right)}}{\sqrt{\pi}d^2xe^{\left(a^2\right)}+\sqrt{\pi}cde^{\left(a^2\right)}}\,dx-\frac{\operatorname{erf}(bx+a)}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*b*integrate(e^(-b^2*x^2 - 2*a*b*x)/(sqrt(pi)*d^2*x*e^(a^2) + sqrt(pi)*c*d
*e^(a^2)), x) - erf(b*x + a)/(d^2*x + c*d)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erf}(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(erf(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)/(d*x + c)^2, x)
```

3.21 $\int \frac{\text{Erf}(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=106

$$\frac{2b^2(bc-ad)\text{Unintegrable}\left(\frac{e^{-(a+bx)^2}}{c+dx},x\right)}{\sqrt{\pi}d^3}-\frac{b^2\text{Erf}(a+bx)}{d^3}-\frac{be^{-(a+bx)^2}}{\sqrt{\pi}d^2(c+dx)}-\frac{\text{Erf}(a+bx)}{2d(c+dx)^2}$$

```
[Out] -(b/(d^2*E^(a + b*x)^2*Sqrt[Pi]*(c + d*x))) - (b^2*Erf[a + b*x])/d^3 - Erf[
a + b*x]/(2*d*(c + d*x)^2) + (2*b^2*(b*c - a*d)*Unintegrable[1/(E^(a + b*x)
^2*(c + d*x)), x])/(d^3*Sqrt[Pi])
```

Rubi [A] time = 0.0839015, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{Erf}(a+bx)}{(c+dx)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*E^(a + b*x)^2*Sqrt[Pi]*(c + d*x))) - (b^2*Erf[a + b*x])/d^3 - Erf[
a + b*x]/(2*d*(c + d*x)^2) + (2*b^2*(b*c - a*d)*Defer[Int][1/(E^(a + b*x)^2
*(c + d*x)), x])/(d^3*Sqrt[Pi])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{erf}(a+bx)}{(c+dx)^3} dx &= -\frac{\text{erf}(a+bx)}{2d(c+dx)^2}+\frac{b\int \frac{e^{-(a+bx)^2}}{(c+dx)^2} dx}{d\sqrt{\pi}}\\
&= -\frac{be^{-(a+bx)^2}}{d^2\sqrt{\pi}(c+dx)}-\frac{\text{erf}(a+bx)}{2d(c+dx)^2}-\frac{\left(2b^3\right)\int e^{-(a+bx)^2} dx}{d^3\sqrt{\pi}}+\frac{\left(2b^2(bc-ad)\right)\int \frac{e^{-(a+bx)^2}}{c+dx} dx}{d^3\sqrt{\pi}}\\
&= -\frac{be^{-(a+bx)^2}}{d^2\sqrt{\pi}(c+dx)}-\frac{b^2\text{erf}(a+bx)}{d^3}-\frac{\text{erf}(a+bx)}{2d(c+dx)^2}+\frac{\left(2b^2(bc-ad)\right)\int \frac{e^{-(a+bx)^2}}{c+dx} dx}{d^3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.530478, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}(a+bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[a + b*x]/(c + d*x)^3,x]
```

```
[Out] Integrate[Erf[a + b*x]/(c + d*x)^3, x]
```

Maple [A] time = 0.426, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a)/(d*x+c)^3,x)
```

```
[Out] int(erf(b*x+a)/(d*x+c)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$b\int \frac{e^{\left(-b^2x^2-2\,abx\right)}}{\sqrt{\pi}d^3x^2e^{\left(a^2\right)}+2\,\sqrt{\pi}cd^2xe^{\left(a^2\right)}+\sqrt{\pi}c^2de^{\left(a^2\right)}}\,dx-\frac{\operatorname{erf}(bx+a)}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b*integrate(e^(-b^2*x^2 - 2*a*b*x)/(sqrt(pi)*d^3*x^2*e^(a^2) + 2*sqrt(pi)*c
*d^2*x*e^(a^2) + sqrt(pi)*c^2*d*e^(a^2)), x) - 1/2*erf(b*x + a)/(d^3*x^2 +
2*c*d^2*x + c^2*d)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erf}\,(bx+a)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}\,(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)/(d*x + c)^3, x)
```

3.22 $\int x^5 \mathbf{Erf}(bx)^2\,dx$

Optimal. Leaf size=178

$$\frac{x^5e^{-b^2x^2}\text{Erf}(bx)}{3\sqrt{\pi}b}+\frac{5x^3e^{-b^2x^2}\text{Erf}(bx)}{6\sqrt{\pi}b^3}+\frac{5xe^{-b^2x^2}\text{Erf}(bx)}{4\sqrt{\pi}b^5}-\frac{5\text{Erf}(bx)^2}{16b^6}+\frac{x^4e^{-2b^2x^2}}{6\pi b^2}+\frac{7x^2e^{-2b^2x^2}}{12\pi b^4}+\frac{11e^{-2b^2x^2}}{12\pi b^6}+\frac{1}{6}x^6\text{Erf}(bx)$$

```
[Out] 11/(12*b^6*E^(2*b^2*x^2)*Pi) + (7*x^2)/(12*b^4*E^(2*b^2*x^2)*Pi) + x^4/(6*b
^2*E^(2*b^2*x^2)*Pi) + (5*x*Erf[b*x])/(4*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (5*x^3
*Erf[b*x])/(6*b^3*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erf[b*x])/(3*b*E^(b^2*x^2)*S
qrt[Pi]) - (5*Erf[b*x]^2)/(16*b^6) + (x^6*Erf[b*x]^2)/6
```

Rubi [A] time = 0.294206, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6364, 6385, 6373, 30, 2209, 2212}

$$\frac{x^5e^{-b^2x^2}\text{Erf}(bx)}{3\sqrt{\pi}b}+\frac{5x^3e^{-b^2x^2}\text{Erf}(bx)}{6\sqrt{\pi}b^3}+\frac{5xe^{-b^2x^2}\text{Erf}(bx)}{4\sqrt{\pi}b^5}-\frac{5\text{Erf}(bx)^2}{16b^6}+\frac{x^4e^{-2b^2x^2}}{6\pi b^2}+\frac{7x^2e^{-2b^2x^2}}{12\pi b^4}+\frac{11e^{-2b^2x^2}}{12\pi b^6}+\frac{1}{6}x^6\text{Erf}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Erf[b*x]^2,x]
```

```
[Out] 11/(12*b^6*E^(2*b^2*x^2)*Pi) + (7*x^2)/(12*b^4*E^(2*b^2*x^2)*Pi) + x^4/(6*b
^2*E^(2*b^2*x^2)*Pi) + (5*x*Erf[b*x])/(4*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (5*x^3
*Erf[b*x])/(6*b^3*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erf[b*x])/(3*b*E^(b^2*x^2)*S
qrt[Pi]) - (5*Erf[b*x]^2)/(16*b^6) + (x^6*Erf[b*x]^2)/6
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int x^5 \mathrm{erf}(bx)^2\,dx = \frac{1}{6}x^6\mathrm{erf}(bx)^2 - \frac{(2b)\int e^{-b^2x^2}x^6\mathrm{erf}(bx)\,dx}{3\sqrt{\pi}}$$
$$= \frac{e^{-b^2x^2}x^5\mathrm{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\mathrm{erf}(bx)^2 - \frac{2\int e^{-2b^2x^2}x^5\,dx}{3\pi} - \frac{5\int e^{-b^2x^2}x^4\mathrm{erf}(bx)\,dx}{3b\sqrt{\pi}}$$
$$= \frac{e^{-2b^2x^2}x^4}{6b^2\pi} + \frac{5e^{-b^2x^2}x^3\mathrm{erf}(bx)}{6b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5\mathrm{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\mathrm{erf}(bx)^2 - \frac{2\int e^{-2b^2x^2}x^3\,dx}{3b^2\pi} - \frac{5\int e^{-2b^2x^2}x^3\,dx}{3b^2\pi}$$
$$= \frac{7e^{-2b^2x^2}x^2}{12b^4\pi} + \frac{e^{-2b^2x^2}x^4}{6b^2\pi} + \frac{5e^{-b^2x^2}x\mathrm{erf}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{-b^2x^2}x^3\mathrm{erf}(bx)}{6b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5\mathrm{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\mathrm{erf}(bx)^2 - \int e$$
$$= \frac{11e^{-2b^2x^2}}{12b^6\pi} + \frac{7e^{-2b^2x^2}x^2}{12b^4\pi} + \frac{e^{-2b^2x^2}x^4}{6b^2\pi} + \frac{5e^{-b^2x^2}x\mathrm{erf}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{-b^2x^2}x^3\mathrm{erf}(bx)}{6b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5\mathrm{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\mathrm{er}$$
$$= \frac{11e^{-2b^2x^2}}{12b^6\pi} + \frac{7e^{-2b^2x^2}x^2}{12b^4\pi} + \frac{e^{-2b^2x^2}x^4}{6b^2\pi} + \frac{5e^{-b^2x^2}x\mathrm{erf}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{-b^2x^2}x^3\mathrm{erf}(bx)}{6b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^5\mathrm{erf}(bx)}{3b\sqrt{\pi}} - \frac{5\mathrm{erf}(b}{16b}$$

Mathematica [A] time = 0.0451967, size = 106, normalized size = 0.6

$$\frac{e^{-2b^2x^2}\left(4\sqrt{\pi}bxe^{b^2x^2}\left(4b^4x^4+10b^2x^2+15\right)\mathrm{Erf}(bx)+\pi e^{2b^2x^2}\left(8b^6x^6-15\right)\mathrm{Erf}(bx)^2+8b^4x^4+28b^2x^2+44\right)}{48\pi b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Erf[b*x]^2,x]
```

```
[Out] (44 + 28*b^2*x^2 + 8*b^4*x^4 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(15 + 10*b^2*x^2
+ 4*b^4*x^4)*Erf[b*x] + E^(2*b^2*x^2)*Pi*(-15 + 8*b^6*x^6)*Erf[b*x]^2)/(48*
b^6*E^(2*b^2*x^2)*Pi)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^5\left(\mathrm{Erf}(bx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erf(b*x)^2,x)
```

```
[Out] int(x^5*erf(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-\frac{\left(2\,b^4x^4+2\,b^2x^2+1\right)e^{\left(-2\,b^2x^2\right)}}{2\,b^2}-\frac{5\left(2\,b^2x^2+1\right)e^{\left(-2\,b^2x^2\right)}}{4\,b^2}-\frac{15\,e^{\left(-2\,b^2x^2\right)}}{4\,b^2}}{6\,\pi b^4}+\frac{\left(8\,\sqrt{\pi}b^6x^6-15\,\sqrt{\pi}\right)\operatorname{erf}\left(bx\right)^2+4\left(4\,b^5x^5+10\,b^3x^3+15\,bx\right)e}{48\,\sqrt{\pi}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/6*integrate((4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-2*b^2*x^2), x)/(pi*b^4)
+ 1/48*((8*sqrt(pi)*b^6*x^6 - 15*sqrt(pi))*erf(b*x)^2 + 4*(4*b^5*x^5 + 10*b
^3*x^3 + 15*b*x)*erf(b*x)*e^(-b^2*x^2))/(sqrt(pi)*b^6)
```

Fricas [A] time = 2.57921, size = 227, normalized size = 1.28

$$\frac{4\,\sqrt{\pi}\left(4\,b^5x^5+10\,b^3x^3+15\,bx\right)\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}-\left(15\,\pi-8\,\pi b^6x^6\right)\operatorname{erf}\left(bx\right)^2+4\left(2\,b^4x^4+7\,b^2x^2+11\right)e^{\left(-2\,b^2x^2\right)}}{48\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(pi)*(4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*erf(b*x)*e^(-b^2*x^2) -
(15*pi - 8*pi*b^6*x^6)*erf(b*x)^2 + 4*(2*b^4*x^4 + 7*b^2*x^2 + 11)*e^(-2*b^
2*x^2))/(pi*b^6)
```

Sympy [A] time = 10.3349, size = 168, normalized size = 0.94

$$\begin{cases} \frac{x^6\operatorname{erf}^2(bx)}{6}+\frac{x^5e^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}b}+\frac{x^4e^{-2b^2x^2}}{6\pi b^2}+\frac{5x^3e^{-b^2x^2}\operatorname{erf}(bx)}{6\sqrt{\pi}b^3}+\frac{7x^2e^{-2b^2x^2}}{12\pi b^4}+\frac{5xe^{-b^2x^2}\operatorname{erf}(bx)}{4\sqrt{\pi}b^5}-\frac{5\operatorname{erf}^2(bx)}{16b^6}+\frac{11e^{-2b^2x^2}}{12\pi b^6} & \text{for}\; b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erf(b*x)**2,x)
```

```
[Out] Piecewise((x**6*erf(b*x)**2/6 + x**5*exp(-b**2*x**2)*erf(b*x)/(3*sqrt(pi)*b
) + x**4*exp(-2*b**2*x**2)/(6*pi*b**2) + 5*x**3*exp(-b**2*x**2)*erf(b*x)/(6
*sqrt(pi)*b**3) + 7*x**2*exp(-2*b**2*x**2)/(12*pi*b**4) + 5*x*exp(-b**2*x**
2)*erf(b*x)/(4*sqrt(pi)*b**5) - 5*erf(b*x)**2/(16*b**6) + 11*exp(-2*b**2*x*
*2)/(12*pi*b**6), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erf}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*erf(b*x)^2, x)
```

3.23 $\int x^3\mathbf{Erf}(bx)^2\,dx$

Optimal. Leaf size=126

$$\frac{x^3e^{-b^2x^2}\mathrm{Erf}(bx)}{2\sqrt{\pi}b}+\frac{3xe^{-b^2x^2}\mathrm{Erf}(bx)}{4\sqrt{\pi}b^3}-\frac{3\mathrm{Erf}(bx)^2}{16b^4}+\frac{x^2e^{-2b^2x^2}}{4\pi b^2}+\frac{e^{-2b^2x^2}}{2\pi b^4}+\frac{1}{4}x^4\mathrm{Erf}(bx)^2$$

```
[Out] 1/(2*b^4*E^(2*b^2*x^2)*Pi) + x^2/(4*b^2*E^(2*b^2*x^2)*Pi) + (3*x*Erf[b*x])/
(4*b^3*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf[b*x])/(2*b*E^(b^2*x^2)*Sqrt[Pi]) -
(3*Erf[b*x]^2)/(16*b^4) + (x^4*Erf[b*x]^2)/4
```

Rubi [A] time = 0.181377, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6364, 6385, 6373, 30, 2209, 2212}

$$\frac{x^3e^{-b^2x^2}\mathrm{Erf}(bx)}{2\sqrt{\pi}b}+\frac{3xe^{-b^2x^2}\mathrm{Erf}(bx)}{4\sqrt{\pi}b^3}-\frac{3\mathrm{Erf}(bx)^2}{16b^4}+\frac{x^2e^{-2b^2x^2}}{4\pi b^2}+\frac{e^{-2b^2x^2}}{2\pi b^4}+\frac{1}{4}x^4\mathrm{Erf}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Erf[b*x]^2,x]
```

```
[Out] 1/(2*b^4*E^(2*b^2*x^2)*Pi) + x^2/(4*b^2*E^(2*b^2*x^2)*Pi) + (3*x*Erf[b*x])/
(4*b^3*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf[b*x])/(2*b*E^(b^2*x^2)*Sqrt[Pi]) -
(3*Erf[b*x]^2)/(16*b^4) + (x^4*Erf[b*x]^2)/4
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3 \operatorname{erf}(bx)^2\,dx &= \frac{1}{4}x^4\operatorname{erf}(bx)^2 - \frac{b\int e^{-b^2x^2}x^4\operatorname{erf}(bx)\,dx}{\sqrt{\pi}} \\
&= \frac{e^{-b^2x^2}x^3\operatorname{erf}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\operatorname{erf}(bx)^2 - \frac{\int e^{-2b^2x^2}x^3\,dx}{\pi} - \frac{3\int e^{-b^2x^2}x^2\operatorname{erf}(bx)\,dx}{2b\sqrt{\pi}} \\
&= \frac{e^{-2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{-b^2x^2}x\operatorname{erf}(bx)}{4b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^3\operatorname{erf}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\operatorname{erf}(bx)^2 - \frac{\int e^{-2b^2x^2}x\,dx}{2b^2\pi} - \frac{3\int e^{-2b^2x^2}x\,dx}{2b^2\pi} - \frac{3\int \cdots}{\cdots} \\
&= \frac{e^{-2b^2x^2}}{2b^4\pi} + \frac{e^{-2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{-b^2x^2}x\operatorname{erf}(bx)}{4b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^3\operatorname{erf}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\operatorname{erf}(bx)^2 - \frac{3\operatorname{Subst}(\int x\,dx, x, \operatorname{erf}(bx))}{8b^4} \\
&= \frac{e^{-2b^2x^2}}{2b^4\pi} + \frac{e^{-2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{-b^2x^2}x\operatorname{erf}(bx)}{4b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^3\operatorname{erf}(bx)}{2b\sqrt{\pi}} - \frac{3\operatorname{erf}(bx)^2}{16b^4} + \frac{1}{4}x^4\operatorname{erf}(bx)^2
\end{aligned}
$$

Mathematica [A] time = 0.0350976, size = 90, normalized size = 0.71

$$\frac{e^{-2b^2x^2}\left(4\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2+3\right)\operatorname{Erf}(bx)+\pi e^{2b^2x^2}\left(4b^4x^4-3\right)\operatorname{Erf}(bx)^2+4b^2x^2+8\right)}{16\pi b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Erf[b*x]^2,x]
```

```
[Out] (8 + 4*b^2*x^2 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x] + E^(2
*b^2*x^2)*Pi*(-3 + 4*b^4*x^4)*Erf[b*x]^2)/(16*b^4*E^(2*b^2*x^2)*Pi)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^3 (\operatorname{Erf}(bx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erf(b*x)^2,x)
```

```
[Out] int(x^3*erf(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-\frac{\left(2\,b^2x^2+1\right)e^{\left(-2\,b^2x^2\right)}}{4\,b^2}-\frac{3\,e^{\left(-2\,b^2x^2\right)}}{4\,b^2}}{2\,\pi b^2}-\frac{\left(3\,\pi-4\,\pi b^4x^4\right)\operatorname{erf}(bx)^2-4\left(2\,\sqrt{\pi}b^3x^3+3\,\sqrt{\pi}bx\right)\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{16\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*integrate((2*b^2*x^3 + 3*x)*e^(-2*b^2*x^2), x)/(pi*b^2) - 1/16*((3*pi
- 4*pi*b^4*x^4)*erf(b*x)^2 - 4*(2*sqrt(pi)*b^3*x^3 + 3*sqrt(pi)*b*x)*erf(b*
x)*e^(-b^2*x^2))/(pi*b^4)
```

Fricas [A] time = 2.96057, size = 186, normalized size = 1.48

$$\frac{4\sqrt{\pi}\left(2\,b^3x^3+3\,bx\right)\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}-\left(3\,\pi-4\,\pi b^4x^4\right)\operatorname{erf}(bx)^2+4\left(b^2x^2+2\right)e^{\left(-2\,b^2x^2\right)}}{16\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(pi)*(2*b^3*x^3 + 3*b*x)*erf(b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b
^4*x^4)*erf(b*x)^2 + 4*(b^2*x^2 + 2)*e^(-2*b^2*x^2))/(pi*b^4)
```

Sympy [A] time = 5.43399, size = 117, normalized size = 0.93

$$\begin{cases} \frac{x^4\operatorname{erf}^2(bx)}{4}+\frac{x^3e^{-b^2x^2}\operatorname{erf}(bx)}{2\sqrt{\pi}b}+\frac{x^2e^{-2b^2x^2}}{4\pi b^2}+\frac{3xe^{-b^2x^2}\operatorname{erf}(bx)}{4\sqrt{\pi}b^3}-\frac{3\operatorname{erf}^2(bx)}{16b^4}+\frac{e^{-2b^2x^2}}{2\pi b^4} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erf(b*x)**2,x)
```

```
[Out] Piecewise((x**4*erf(b*x)**2/4 + x**3*exp(-b**2*x**2)*erf(b*x)/(2*sqrt(pi)*b
) + x**2*exp(-2*b**2*x**2)/(4*pi*b**2) + 3*x*exp(-b**2*x**2)*erf(b*x)/(4*sq
rt(pi)*b**3) - 3*erf(b*x)**2/(16*b**4) + exp(-2*b**2*x**2)/(2*pi*b**4), Ne(
b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{erf}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*erf(b*x)^2, x)
```

3.24 $\int x\text{Erf}(bx)^2\,dx$

Optimal. Leaf size=71

$$\frac{xe^{-b^2x^2}\text{Erf}(bx)}{\sqrt{\pi}b}-\frac{\text{Erf}(bx)^2}{4b^2}+\frac{e^{-2b^2x^2}}{2\pi b^2}+\frac{1}{2}x^2\text{Erf}(bx)^2$$

```
[Out] 1/(2*b^2*E^(2*b^2*x^2)*Pi) + (x*Erf[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) - Erf[b*
x]^2/(4*b^2) + (x^2*Erf[b*x]^2)/2
```

Rubi [A] time = 0.0864978, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6364, 6385, 6373, 30, 2209}

$$\frac{xe^{-b^2x^2}\text{Erf}(bx)}{\sqrt{\pi}b}-\frac{\text{Erf}(bx)^2}{4b^2}+\frac{e^{-2b^2x^2}}{2\pi b^2}+\frac{1}{2}x^2\text{Erf}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Erf[b*x]^2,x]
```

```
[Out] 1/(2*b^2*E^(2*b^2*x^2)*Pi) + (x*Erf[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) - Erf[b*
x]^2/(4*b^2) + (x^2*Erf[b*x]^2)/2
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x\operatorname{erf}(bx)^2\,dx &= \frac{1}{2}x^2\operatorname{erf}(bx)^2 - \frac{(2b)\int e^{-b^2x^2}x^2\operatorname{erf}(bx)\,dx}{\sqrt{\pi}} \\
&= \frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{b\sqrt{\pi}} + \frac{1}{2}x^2\operatorname{erf}(bx)^2 - \frac{2\int e^{-2b^2x^2}x\,dx}{\pi} - \frac{\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx}{b\sqrt{\pi}} \\
&= \frac{e^{-2b^2x^2}}{2b^2\pi} + \frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{b\sqrt{\pi}} + \frac{1}{2}x^2\operatorname{erf}(bx)^2 - \frac{\operatorname{Subst}\left(\int x\,dx, x, \operatorname{erf}(bx)\right)}{2b^2} \\
&= \frac{e^{-2b^2x^2}}{2b^2\pi} + \frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{b\sqrt{\pi}} - \frac{\operatorname{erf}(bx)^2}{4b^2} + \frac{1}{2}x^2\operatorname{erf}(bx)^2
\end{aligned}$$

Mathematica [A] time = 0.0396927, size = 64, normalized size = 0.9

$$\frac{\pi\left(2b^2x^2-1\right)\operatorname{Erf}(bx)^2 + 4\sqrt{\pi}bxe^{-b^2x^2}\operatorname{Erf}(bx) + 2e^{-2b^2x^2}}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erf[b*x]^2,x]
```

```
[Out] (2/E^(2*b^2*x^2) + (4*b*Sqrt[Pi]*x*Erf[b*x])/E^(b^2*x^2) + Pi*(-1 + 2*b^2*x
^2)*Erf[b*x]^2)/(4*b^2*Pi)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x\left(\mathrm{Erf}\left(bx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erf(b*x)^2,x)
```

```
[Out] int(x*erf(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-\frac{e^{\left(-2\,b^{2}x^{2}\right)}}{2\,b^{2}}}{\pi}+\frac{4\,bx\,\mathrm{erf}\left(bx\right)e^{\left(-b^{2}x^{2}\right)}+\left(2\,\sqrt{\pi}b^{2}x^{2}-\sqrt{\pi}\right)\mathrm{erf}\left(bx\right)^{2}}{4\,\sqrt{\pi}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] -2*integrate(x*e^(-2*b^2*x^2), x)/pi + 1/4*(4*b*x*erf(b*x)*e^(-b^2*x^2) + (
2*sqrt(pi)*b^2*x^2 - sqrt(pi))*erf(b*x)^2)/(sqrt(pi)*b^2)
```

Fricas [A] time = 2.92655, size = 142, normalized size = 2.

$$\frac{4\,\sqrt{\pi}bx\,\mathrm{erf}\left(bx\right)e^{\left(-b^{2}x^{2}\right)}-\left(\pi-2\,\pi b^{2}x^{2}\right)\mathrm{erf}\left(bx\right)^{2}+2\,e^{\left(-2\,b^{2}x^{2}\right)}}{4\,\pi b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(pi)*b*x*erf(b*x)*e^(-b^2*x^2) - (pi - 2*pi*b^2*x^2)*erf(b*x)^2
+ 2*e^(-2*b^2*x^2))/(pi*b^2)
```

Sympy [A] time = 1.17646, size = 65, normalized size = 0.92

$$\begin{cases} \frac{x^2 \operatorname{erf}^2(bx)}{2} + \frac{xe^{-b^2x^2} \operatorname{erf}(bx)}{\sqrt{\pi}b} - \frac{\operatorname{erf}^2(bx)}{4b^2} + \frac{e^{-2b^2x^2}}{2\pi b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)**2,x)
```

```
[Out] Piecewise((x**2*erf(b*x)**2/2 + x*exp(-b**2*x**2)*erf(b*x)/(sqrt(pi)*b) - e
rf(b*x)**2/(4*b**2) + exp(-2*b**2*x**2)/(2*pi*b**2), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erf}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*erf(b*x)^2, x)
```

3.25 $\int \frac{\text{Erf}(bx)^2}{x} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erf}(bx)^2}{x},x\right)$$

```
[Out] Unintegrable[Erf[b*x]^2/x, x]
```

Rubi [A] time = 0.0170109, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]^2/x,x]
```

```
[Out] Defer[Int][Erf[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\text{erf}(bx)^2}{x} \, dx = \int \frac{\text{erf}(bx)^2}{x} \, dx$$

Mathematica [A] time = 0.0273292, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]^2/x,x]
```

```
[Out] Integrate[Erf[b*x]^2/x, x]
```

Maple [A] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Erf}(bx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x,x)
```

```
[Out] int(erf(b*x)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)^2/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x,x)
```

```
[Out] Integral(erf(b*x)**2/x, x)
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x, x)
```

3.26 $\int \frac{\mathbf{Erf}(bx)^2}{x^3}\,dx$

Optimal. Leaf size=67

$$-\frac{2be^{-b^2x^2}\text{Erf}(bx)}{\sqrt{\pi}x}+b^2\left(-\text{Erf}(bx)^2\right)+\frac{2b^2\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\pi}-\frac{\text{Erf}(bx)^2}{2x^2}$$

```
[Out] (-2*b*Erf[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) - b^2*Erf[b*x]^2 - Erf[b*x]^2/(2*x
^2) + (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Rubi [A] time = 0.0996888, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6364, 6391, 6373, 30, 2210}

$$-\frac{2be^{-b^2x^2}\text{Erf}(bx)}{\sqrt{\pi}x}+b^2\left(-\text{Erf}(bx)^2\right)+\frac{2b^2\text{Ei}\left(-2b^2x^2\right)}{\pi}-\frac{\text{Erf}(bx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]^2/x^3,x]
```

```
[Out] (-2*b*Erf[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) - b^2*Erf[b*x]^2 - Erf[b*x]^2/(2*x
^2) + (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erf}(bx)^2}{x^3}\,dx &= -\frac{\operatorname{erf}(bx)^2}{2x^2}+\frac{(2b)\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{2be^{-b^2x^2}\operatorname{erf}(bx)}{\sqrt{\pi}x}-\frac{\operatorname{erf}(bx)^2}{2x^2}+\frac{\left(4b^2\right)\int \frac{e^{-2b^2x^2}}{x}\,dx}{\pi}-\frac{\left(4b^3\right)\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx}{\sqrt{\pi}}\\
&= -\frac{2be^{-b^2x^2}\operatorname{erf}(bx)}{\sqrt{\pi}x}-\frac{\operatorname{erf}(bx)^2}{2x^2}+\frac{2b^2\operatorname{Ei}\left(-2b^2x^2\right)}{\pi}-\left(2b^2\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)\\
&= -\frac{2be^{-b^2x^2}\operatorname{erf}(bx)}{\sqrt{\pi}x}-b^2\operatorname{erf}(bx)^2-\frac{\operatorname{erf}(bx)^2}{2x^2}+\frac{2b^2\operatorname{Ei}\left(-2b^2x^2\right)}{\pi}
\end{aligned}$$

Mathematica [A] time = 0.0282474, size = 63, normalized size = 0.94

$$-\frac{2be^{-b^2x^2}\operatorname{Erf}(bx)}{\sqrt{\pi}x}+\left(-b^2-\frac{1}{2x^2}\right)\operatorname{Erf}(bx)^2+\frac{2b^2\operatorname{ExpIntegralEi}\left(-2b^2x^2\right)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]^2/x^3,x]
```

```
[Out] (-2*b*Erf[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) + (-b^2 - 1/(2*x^2))*Erf[b*x]^2 +
(2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Erf}(bx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x^3,x)
```

```
[Out] int(erf(b*x)^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,b\int \frac{\mathrm{erf}(bx)e^{\left(-b^2x^2\right)}}{x^2}\,dx}{\sqrt{\pi}} - \frac{\mathrm{erf}\,(bx)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 2*b*integrate(erf(b*x)*e^(-b^2*x^2)/x^2, x)/sqrt(pi) - 1/2*erf(b*x)^2/x^2
```

Fricas [A] time = 2.6121, size = 153, normalized size = 2.28

$$\frac{4\,b^2x^2\mathrm{Ei}\left(-2\,b^2x^2\right) - 4\,\sqrt{\pi}bx\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)} - \left(\pi + 2\,\pi b^2x^2\right)\mathrm{erf}\,(bx)^2}{2\,\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*Ei(-2*b^2*x^2) - 4*sqrt(pi)*b*x*erf(b*x)*e^(-b^2*x^2) - (pi
+ 2*pi*b^2*x^2)*erf(b*x)^2)/(pi*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x**3,x)
```

```
[Out] Integral(erf(b*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x^3, x)
```

3.27 $\int \frac{\mathbf{Erf}(bx)^2}{x^5}\,dx$

Optimal. Leaf size=125

$$\frac{2b^3e^{-b^2x^2}\mathrm{Erf}(bx)}{3\sqrt{\pi}x}-\frac{be^{-b^2x^2}\mathrm{Erf}(bx)}{3\sqrt{\pi}x^3}+\frac{1}{3}b^4\mathrm{Erf}(bx)^2-\frac{4b^4\mathrm{ExpIntegralEi}\left(-2b^2x^2\right)}{3\pi}-\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{\mathrm{Erf}(bx)^2}{4x^4}$$

```
[Out] -b^2/(3*E^(2*b^2*x^2)*Pi*x^2) - (b*Erf[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x^3) +
 (2*b^3*Erf[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x) + (b^4*Erf[b*x]^2)/3 - Erf[b*x
]^2/(4*x^4) - (4*b^4*ExpIntegralEi[-2*b^2*x^2])/(3*Pi)
```

Rubi [A] time = 0.18297, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6364, 6391, 6373, 30, 2210, 2214}

$$\frac{2b^3e^{-b^2x^2}\mathrm{Erf}(bx)}{3\sqrt{\pi}x}-\frac{be^{-b^2x^2}\mathrm{Erf}(bx)}{3\sqrt{\pi}x^3}+\frac{1}{3}b^4\mathrm{Erf}(bx)^2-\frac{4b^4\mathrm{Ei}\left(-2b^2x^2\right)}{3\pi}-\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{\mathrm{Erf}(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]^2/x^5,x]
```

```
[Out] -b^2/(3*E^(2*b^2*x^2)*Pi*x^2) - (b*Erf[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x^3) +
 (2*b^3*Erf[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x) + (b^4*Erf[b*x]^2)/3 - Erf[b*x
]^2/(4*x^4) - (4*b^4*ExpIntegralEi[-2*b^2*x^2])/(3*Pi)
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx &= -\frac{\operatorname{erf}(bx)^2}{4x^4}+\frac{b\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x^3}-\frac{\operatorname{erf}(bx)^2}{4x^4}+\frac{\left(2b^2\right)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{3\pi}-\frac{\left(2b^3\right)\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx}{3\sqrt{\pi}}\\
&= -\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{be^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x^3}+\frac{2b^3e^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x}-\frac{\operatorname{erf}(bx)^2}{4x^4}-2\frac{\left(4b^4\right)\int \frac{e^{-2b^2x^2}}{x}\,dx}{3\pi}+\frac{\left(4b^5\right)\int e^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}}\\
&= -\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{be^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x^3}+\frac{2b^3e^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x}-\frac{\operatorname{erf}(bx)^2}{4x^4}-\frac{4b^4\operatorname{Ei}\left(-2b^2x^2\right)}{3\pi}+\frac{1}{3}\left(2b^4\right)\operatorname{Subst}\left(\int x\,dx,\right.\\
&= -\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{be^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x^3}+\frac{2b^3e^{-b^2x^2}\operatorname{erf}(bx)}{3\sqrt{\pi}x}+\frac{1}{3}b^4\operatorname{erf}(bx)^2-\frac{\operatorname{erf}(bx)^2}{4x^4}-\frac{4b^4\operatorname{Ei}\left(-2b^2x^2\right)}{3\pi}
\end{aligned}$$

Mathematica [A] time = 0.0784899, size = 97, normalized size = 0.78

$$\frac{\frac{4bxe^{-b^2x^2}(2b^2x^2-1)\operatorname{Erf}(bx)}{\sqrt{\pi}}+(4b^4x^4-3)\operatorname{Erf}(bx)^2-\frac{4b^2x^2\left(4b^2x^2\operatorname{ExpIntegralEi}(-2b^2x^2)+e^{-2b^2x^2}\right)}{\pi}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]^2/x^5,x]
```

```
[Out] ((4*b*x*(-1 + 2*b^2*x^2)*Erf[b*x])/(E^(b^2*x^2)*Sqrt[Pi]) + (-3 + 4*b^4*x^4
)*Erf[b*x]^2 - (4*b^2*x^2*(E^(-2*b^2*x^2) + 4*b^2*x^2*ExpIntegralEi[-2*b^2*
x^2]))/Pi)/(12*x^4)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{Erf}(bx))^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x^5,x)
```

```
[Out] int(erf(b*x)^2/x^5,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.57718, size = 215, normalized size = 1.72

$$-\frac{16\, b^4x^4\operatorname{Ei}\left(-2\, b^2x^2\right)+4\, b^2x^2e^{\left(-2\, b^2x^2\right)}-4\, \sqrt{\pi}\left(2\, b^3x^3-bx\right)\operatorname{erf}(bx)\, e^{\left(-b^2x^2\right)}+\left(3\, \pi-4\, \pi b^4x^4\right)\operatorname{erf}(bx)^2}{12\, \pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(16*b^4*x^4*Ei(-2*b^2*x^2) + 4*b^2*x^2*e^(-2*b^2*x^2) - 4*sqrt(pi)*(2
*b^3*x^3 - b*x)*erf(b*x)*e^(-b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*erf(b*x)^2)/(
pi*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}^2(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x**5,x)
```

```
[Out] Integral(erf(b*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x^5, x)
```

3.28 $\int \frac{\mathbf{Erf}(bx)^2}{x^7}\,dx$

Optimal. Leaf size=177

$$-\frac{8b^5e^{-b^2x^2}\mathrm{Erf}(bx)}{45\sqrt{\pi}x}+\frac{4b^3e^{-b^2x^2}\mathrm{Erf}(bx)}{45\sqrt{\pi}x^3}-\frac{2be^{-b^2x^2}\mathrm{Erf}(bx)}{15\sqrt{\pi}x^5}-\frac{4}{45}b^6\mathrm{Erf}(bx)^2+\frac{28b^6\mathrm{ExpIntegralEi}\left(-2b^2x^2\right)}{45\pi}+\frac{2b^4e^{-2b^2x^2}}{9\pi x^2}$$

```
[Out] -b^2/(15*E^(2*b^2*x^2)*Pi*x^4) + (2*b^4)/(9*E^(2*b^2*x^2)*Pi*x^2) - (2*b*Er
f[b*x])/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) + (4*b^3*Erf[b*x])/(45*E^(b^2*x^2)*Sq
rt[Pi]*x^3) - (8*b^5*Erf[b*x])/(45*E^(b^2*x^2)*Sqrt[Pi]*x) - (4*b^6*Erf[b*x
]^2)/45 - Erf[b*x]^2/(6*x^6) + (28*b^6*ExpIntegralEi[-2*b^2*x^2])/(45*Pi)
```

Rubi [A] time = 0.292238, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6364, 6391, 6373, 30, 2210, 2214}

$$-\frac{8b^5e^{-b^2x^2}\mathrm{Erf}(bx)}{45\sqrt{\pi}x}+\frac{4b^3e^{-b^2x^2}\mathrm{Erf}(bx)}{45\sqrt{\pi}x^3}-\frac{2be^{-b^2x^2}\mathrm{Erf}(bx)}{15\sqrt{\pi}x^5}-\frac{4}{45}b^6\mathrm{Erf}(bx)^2+\frac{28b^6\mathrm{Ei}\left(-2b^2x^2\right)}{45\pi}+\frac{2b^4e^{-2b^2x^2}}{9\pi x^2}-\frac{b^2e^{-2b^2x^2}}{15\pi x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]^2/x^7,x]
```

```
[Out] -b^2/(15*E^(2*b^2*x^2)*Pi*x^4) + (2*b^4)/(9*E^(2*b^2*x^2)*Pi*x^2) - (2*b*Er
f[b*x])/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) + (4*b^3*Erf[b*x])/(45*E^(b^2*x^2)*Sq
rt[Pi]*x^3) - (8*b^5*Erf[b*x])/(45*E^(b^2*x^2)*Sqrt[Pi]*x) - (4*b^6*Erf[b*x
]^2)/45 - Erf[b*x]^2/(6*x^6) + (28*b^6*ExpIntegralEi[-2*b^2*x^2])/(45*Pi)
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
```

```
/; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\int \frac{\text{erf}(bx)^2}{x^7}\,dx = -\frac{\text{erf}(bx)^2}{6x^6} + \frac{(2b)\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^6}\,dx}{3\sqrt{\pi}}$$

$$= -\frac{2be^{-b^2x^2}\text{erf}(bx)}{15\sqrt{\pi}x^5} - \frac{\text{erf}(bx)^2}{6x^6} + \frac{(4b^2)\int \frac{e^{-2b^2x^2}}{x^5}\,dx}{15\pi} - \frac{(4b^3)\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}\,dx}{15\sqrt{\pi}}$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} - \frac{2be^{-b^2x^2}\text{erf}(bx)}{15\sqrt{\pi}x^5} + \frac{4b^3e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x^3} - \frac{\text{erf}(bx)^2}{6x^6} - \frac{(8b^4)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{45\pi} - \frac{(4b^4)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{15\pi} +$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4e^{-2b^2x^2}}{9\pi x^2} - \frac{2be^{-b^2x^2}\text{erf}(bx)}{15\sqrt{\pi}x^5} + \frac{4b^3e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x} - \frac{\text{erf}(bx)^2}{6x^6} + 2\frac{(16b^6$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4e^{-2b^2x^2}}{9\pi x^2} - \frac{2be^{-b^2x^2}\text{erf}(bx)}{15\sqrt{\pi}x^5} + \frac{4b^3e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x} - \frac{\text{erf}(bx)^2}{6x^6} + \frac{28b^6\text{E}$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4e^{-2b^2x^2}}{9\pi x^2} - \frac{2be^{-b^2x^2}\text{erf}(bx)}{15\sqrt{\pi}x^5} + \frac{4b^3e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x} - \frac{4}{45}b^6\text{erf}(bx)^2 - \text{er}$$

Mathematica [A] time = 0.04137, size = 133, normalized size = 0.75

$$\frac{e^{-2b^2x^2}\left(-4\sqrt{\pi}bxe^{b^2x^2}\left(4b^4x^4-2b^2x^2+3\right)\text{Erf}(bx)-\pi e^{2b^2x^2}\left(8b^6x^6+15\right)\text{Erf}(bx)^2+56b^6x^6e^{2b^2x^2}\text{ExpIntegralEi}\left(-2b^2\right.\right.}{90\pi x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]^2/x^7,x]
```

```
[Out] (-6*b^2*x^2 + 20*b^4*x^4 - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 - 2*b^2*x^2 + 4*b^
4*x^4)*Erf[b*x] - E^(2*b^2*x^2)*Pi*(15 + 8*b^6*x^6)*Erf[b*x]^2 + 56*b^6*E^(
2*b^2*x^2)*x^6*ExpIntegralEi[-2*b^2*x^2])/(90*E^(2*b^2*x^2)*Pi*x^6)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\text{Erf}(bx))^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x^7,x)
```

```
[Out] int(erf(b*x)^2/x^7,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.61073, size = 257, normalized size = 1.45

$$\frac{56\,b^6x^6\mathrm{Ei}\left(-2\,b^2x^2\right)-4\,\sqrt{\pi}\left(4\,b^5x^5-2\,b^3x^3+3\,bx\right)\mathrm{erf}\left(bx\right)e^{\left(-b^2x^2\right)}-\left(15\,\pi+8\,\pi b^6x^6\right)\mathrm{erf}\left(bx\right)^2+2\left(10\,b^4x^4-3\,b^2x^2\right)e^{(-2}}{90\,\pi x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/90*(56*b^6*x^6*Ei(-2*b^2*x^2) - 4*sqrt(pi)*(4*b^5*x^5 - 2*b^3*x^3 + 3*b*x
)*erf(b*x)*e^(-b^2*x^2) - (15*pi + 8*pi*b^6*x^6)*erf(b*x)^2 + 2*(10*b^4*x^4
 - 3*b^2*x^2)*e^(-2*b^2*x^2))/(pi*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}^2\left(bx\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x**7,x)
```

```
[Out] Integral(erf(b*x)**2/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x^7, x)
```

3.29 $\int x^4 \mathbf{Erf}(bx)^2\,dx$

Optimal. Leaf size=165

$$\frac{2x^4e^{-b^2x^2}\text{Erf}(bx)}{5\sqrt{\pi}b}+\frac{4x^2e^{-b^2x^2}\text{Erf}(bx)}{5\sqrt{\pi}b^3}+\frac{4e^{-b^2x^2}\text{Erf}(bx)}{5\sqrt{\pi}b^5}-\frac{43\text{Erf}\left(\sqrt{2}bx\right)}{40\sqrt{2\pi}b^5}+\frac{x^3e^{-2b^2x^2}}{5\pi b^2}+\frac{11xe^{-2b^2x^2}}{20\pi b^4}+\frac{1}{5}x^5\text{Erf}(bx)^2$$

```
[Out] (11*x)/(20*b^4*E^(2*b^2*x^2)*Pi) + x^3/(5*b^2*E^(2*b^2*x^2)*Pi) + (4*Erf[b*
x])/(5*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (4*x^2*Erf[b*x])/(5*b^3*E^(b^2*x^2)*Sqrt
[Pi]) + (2*x^4*Erf[b*x])/(5*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erf[b*x]^2)/5 -
(43*Erf[Sqrt[2]*b*x])/(40*b^5*Sqrt[2*Pi])
```

Rubi [A] time = 0.247006, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6364, 6385, 6382, 2205, 2212}

$$\frac{2x^4e^{-b^2x^2}\text{Erf}(bx)}{5\sqrt{\pi}b}+\frac{4x^2e^{-b^2x^2}\text{Erf}(bx)}{5\sqrt{\pi}b^3}+\frac{4e^{-b^2x^2}\text{Erf}(bx)}{5\sqrt{\pi}b^5}-\frac{43\text{Erf}\left(\sqrt{2}bx\right)}{40\sqrt{2\pi}b^5}+\frac{x^3e^{-2b^2x^2}}{5\pi b^2}+\frac{11xe^{-2b^2x^2}}{20\pi b^4}+\frac{1}{5}x^5\text{Erf}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Erf[b*x]^2,x]
```

```
[Out] (11*x)/(20*b^4*E^(2*b^2*x^2)*Pi) + x^3/(5*b^2*E^(2*b^2*x^2)*Pi) + (4*Erf[b*
x])/(5*b^5*E^(b^2*x^2)*Sqrt[Pi]) + (4*x^2*Erf[b*x])/(5*b^3*E^(b^2*x^2)*Sqrt
[Pi]) + (2*x^4*Erf[b*x])/(5*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erf[b*x]^2)/5 -
(43*Erf[Sqrt[2]*b*x])/(40*b^5*Sqrt[2*Pi])
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int x^4\text{erf}(bx)^2\,dx = \frac{1}{5}x^5\text{erf}(bx)^2 - \frac{(4b)\int e^{-b^2x^2}x^5\text{erf}(bx)\,dx}{5\sqrt{\pi}}$$
$$= \frac{2e^{-b^2x^2}x^4\text{erf}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4\int e^{-2b^2x^2}x^4\,dx}{5\pi} - \frac{8\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx}{5b\sqrt{\pi}}$$
$$= \frac{e^{-2b^2x^2}x^3}{5b^2\pi} + \frac{4e^{-b^2x^2}x^2\text{erf}(bx)}{5b^3\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^4\text{erf}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx)^2 - \frac{3\int e^{-2b^2x^2}x^2\,dx}{5b^2\pi} - \frac{8\int e^{-2b^2x^2}x^2\,dx}{5b^2\pi}$$
$$= \frac{11e^{-2b^2x^2}x}{20b^4\pi} + \frac{e^{-2b^2x^2}x^3}{5b^2\pi} + \frac{4e^{-b^2x^2}\text{erf}(bx)}{5b^5\sqrt{\pi}} + \frac{4e^{-b^2x^2}x^2\text{erf}(bx)}{5b^3\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^4\text{erf}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx)^2 - \frac{3\int}{}$$
$$= \frac{11e^{-2b^2x^2}x}{20b^4\pi} + \frac{e^{-2b^2x^2}x^3}{5b^2\pi} + \frac{4e^{-b^2x^2}\text{erf}(bx)}{5b^5\sqrt{\pi}} + \frac{4e^{-b^2x^2}x^2\text{erf}(bx)}{5b^3\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^4\text{erf}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erf}(bx)^2 - \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.0903972, size = 106, normalized size = 0.64

$$\frac{16\pi b^5x^5\text{Erf}(bx)^2 + 32\sqrt{\pi}e^{-b^2x^2}\left(b^4x^4 + 2b^2x^2 + 2\right)\text{Erf}(bx) + 4bxe^{-2b^2x^2}\left(4b^2x^2 + 11\right) - 43\sqrt{2\pi}\text{Erf}\left(\sqrt{2}bx\right)}{80\pi b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Erf[b*x]^2,x]
```

```
[Out] ((4*b*x*(11 + 4*b^2*x^2))/E^(2*b^2*x^2) + (32*Sqrt[Pi]*(2 + 2*b^2*x^2 + b^4
*x^4)*Erf[b*x])/E^(b^2*x^2) + 16*b^5*Pi*x^5*Erf[b*x]^2 - 43*Sqrt[2*Pi]*Erf[
Sqrt[2]*b*x])/(80*b^5*Pi)
```

Maple [A] time = 0.052, size = 131, normalized size = 0.8

$$\frac{1}{b^5}\left(\frac{(\mathrm{Erf}\,(bx))^2\,b^5x^5}{5}-\frac{4\,\mathrm{Erf}\,(bx)}{5\,\sqrt{\pi}}\left(-\frac{b^4x^4}{2\,\mathrm{e}^{b^2x^2}}-\frac{b^2x^2}{\mathrm{e}^{b^2x^2}}-\left(\mathrm{e}^{b^2x^2}\right)^{-1}\right)+\frac{4}{5\,\pi}\left(-\frac{43\,\sqrt{2}\sqrt{\pi}\mathrm{Erf}\left(bx\sqrt{2}\right)}{64}+\frac{11\,bx}{16\,\left(\mathrm{e}^{b^2x^2}\right)^2}+\frac{x^3b^3}{4\,\left(\mathrm{e}^{b^2x^2}\right)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erf(b*x)^2,x)
```

```
[Out] 1/b^5*(1/5*erf(b*x)^2*b^5*x^5-4/5*erf(b*x)/Pi^(1/2)*(-1/2*b^4*x^4/exp(b^2*x
^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))+4/5/Pi*(-43/64*2^(1/2)*Pi^(1/2)*er
f(b*x*2^(1/2))+11/16/exp(b^2*x^2)^2*b*x+1/4/exp(b^2*x^2)^2*b^3*x^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-\frac{1}{16}\,b^4\left(\frac{4\left(4\,b^2x^3+3\,x\right)e^{\left(-2\,b^2x^2\right)}}{b^4}-\frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b^5}\right)-\frac{1}{2}\,b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b^3}\right)+\frac{2\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b}}{5\,\pi b^4}+\frac{\sqrt{\pi}b^5x^5\,\mathrm{erf}\,(bx)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/5*integrate(4*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-2*b^2*x^2), x)/(pi*b^4) + 1/
5*(sqrt(pi)*b^5*x^5*erf(b*x)^2 + 2*(b^4*x^4 + 2*b^2*x^2 + 2)*erf(b*x)*e^(-b
^2*x^2))/(sqrt(pi)*b^5)
```

Fricas [A] time = 2.54934, size = 277, normalized size = 1.68

$$\frac{16\,\pi b^6x^5\,\mathrm{erf}\,(bx)^2+32\,\sqrt{\pi}\left(b^5x^4+2\,b^3x^2+2\,b\right)\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}-43\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)+4\left(4\,b^4x^3+11\,b^2x\right)e^{\left(-2\,b^2\right.}}{80\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*pi*b^6*x^5*erf(b*x)^2 + 32*sqrt(pi)*(b^5*x^4 + 2*b^3*x^2 + 2*b)*er
f(b*x)*e^(-b^2*x^2) - 43*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x
) + 4*(4*b^4*x^3 + 11*b^2*x)*e^(-2*b^2*x^2))/(pi*b^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erf}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erf(b*x)**2,x)
```

```
[Out] Integral(x**4*erf(b*x)**2, x)
```

Giac [A] time = 1.39101, size = 235, normalized size = 1.42

$$\frac{1}{5}x^5\operatorname{erf}(bx)^2 + \frac{b\left(\frac{32\left(b^4x^4+2b^2x^2+2\right)\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{b^6} + \frac{\sqrt{\pi}b^4\left(\frac{4\left(4b^2x^3+3x\right)e^{\left(-2b^2x^2\right)}}{b^4}+\frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^5}\right)+8\sqrt{\pi}b^2\left(\frac{4xe^{\left(-2b^2x^2\right)}}{b^2}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^3}\right)+32\cdots}{\pi b^5}\right)}{80\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5*erf(b*x)^2 + 1/80*b*(32*(b^4*x^4 + 2*b^2*x^2 + 2)*erf(b*x)*e^(-b^2*
x^2)/b^6 + (sqrt(pi)*b^4*(4*(4*b^2*x^3 + 3*x)*e^(-2*b^2*x^2)/b^4 + 3*sqrt(2
)*sqrt(pi)*erf(-sqrt(2)*b*x)/b^5) + 8*sqrt(pi)*b^2*(4*x*e^(-2*b^2*x^2)/b^2
+ sqrt(2)*sqrt(pi)*erf(-sqrt(2)*b*x)/b^3) + 32*sqrt(2)*pi*erf(-sqrt(2)*b*x)
/b)/(pi*b^5))/sqrt(pi)
```

3.30 $\int x^2 \mathbf{Erf}(bx)^2 \, dx$

Optimal. Leaf size=113

$$\frac{2x^2e^{-b^2x^2}\text{Erf}(bx)}{3\sqrt{\pi}b}+\frac{2e^{-b^2x^2}\text{Erf}(bx)}{3\sqrt{\pi}b^3}-\frac{5\text{Erf}\left(\sqrt{2}bx\right)}{6\sqrt{2\pi}b^3}+\frac{xe^{-2b^2x^2}}{3\pi b^2}+\frac{1}{3}x^3\text{Erf}(bx)^2$$

```
[Out] x/(3*b^2*E^(2*b^2*x^2)*Pi) + (2*Erf[b*x])/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) + (2
*x^2*Erf[b*x])/(3*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf[b*x]^2)/3 - (5*Erf[Sqr
t[2]*b*x])/(6*b^3*Sqrt[2*Pi])
```

Rubi [A] time = 0.135931, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6364, 6385, 6382, 2205, 2212}

$$\frac{2x^2e^{-b^2x^2}\text{Erf}(bx)}{3\sqrt{\pi}b}+\frac{2e^{-b^2x^2}\text{Erf}(bx)}{3\sqrt{\pi}b^3}-\frac{5\text{Erf}\left(\sqrt{2}bx\right)}{6\sqrt{2\pi}b^3}+\frac{xe^{-2b^2x^2}}{3\pi b^2}+\frac{1}{3}x^3\text{Erf}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erf[b*x]^2,x]
```


```
[Out] x/(3*b^2*E^(2*b^2*x^2)*Pi) + (2*Erf[b*x])/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) + (2
*x^2*Erf[b*x])/(3*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf[b*x]^2)/3 - (5*Erf[Sqr
t[2]*b*x])/(6*b^3*Sqrt[2*Pi])
```


Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \text{erf}(bx)^2\,dx &= \frac{1}{3}x^3\text{erf}(bx)^2 - \frac{(4b)\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx}{3\sqrt{\pi}} \\
&= \frac{2e^{-b^2x^2}x^2\text{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{3}x^3\text{erf}(bx)^2 - \frac{4\int e^{-2b^2x^2}x^2\,dx}{3\pi} - \frac{4\int e^{-b^2x^2}x\text{erf}(bx)\,dx}{3b\sqrt{\pi}} \\
&= \frac{e^{-2b^2x^2}x}{3b^2\pi} + \frac{2e^{-b^2x^2}\text{erf}(bx)}{3b^3\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^2\text{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{3}x^3\text{erf}(bx)^2 - \frac{\int e^{-2b^2x^2}\,dx}{3b^2\pi} - \frac{4\int e^{-2b^2x^2}\,dx}{3b^2\pi} \\
&= \frac{e^{-2b^2x^2}x}{3b^2\pi} + \frac{2e^{-b^2x^2}\text{erf}(bx)}{3b^3\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^2\text{erf}(bx)}{3b\sqrt{\pi}} + \frac{1}{3}x^3\text{erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}bx\right)}{3b^3} - \frac{\text{erf}\left(\sqrt{2}bx\right)}{6b^3\sqrt{2\pi}}
\end{aligned}$$

Mathematica [A] time = 0.073654, size = 88, normalized size = 0.78

$$\frac{4\pi b^3x^3\text{Erf}(bx)^2 + 8\sqrt{\pi}e^{-b^2x^2}\left(b^2x^2+1\right)\text{Erf}(bx) + 4bxe^{-2b^2x^2} - 5\sqrt{2\pi}\text{Erf}\left(\sqrt{2}bx\right)}{12\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erf[b*x]^2,x]
```

```
[Out] ((4*b*x)/E^(2*b^2*x^2) + (8*Sqrt[Pi]*(1 + b^2*x^2)*Erf[b*x])/E^(b^2*x^2) +
4*b^3*Pi*x^3*Erf[b*x]^2 - 5*Sqrt[2*Pi]*Erf[Sqrt[2]*b*x])/(12*b^3*Pi)
```

Maple [A] time = 0.046, size = 95, normalized size = 0.8

$$\frac{1}{b^3}\left(\frac{(\mathrm{Erf}(bx))^2 b^3x^3}{3} - \frac{4\,\mathrm{Erf}(bx)}{3\sqrt{\pi}}\left(-\frac{b^2x^2}{2\,\mathrm{e}^{b^2x^2}} - \frac{1}{2\,\mathrm{e}^{b^2x^2}}\right) + \frac{4}{3\,\pi}\left(-\frac{5\sqrt{2}\sqrt{\pi}\mathrm{Erf}\left(bx\sqrt{2}\right)}{16} + \frac{bx}{4\left(\mathrm{e}^{b^2x^2}\right)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erf(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*erf(b*x)^2*b^3*x^3-4/3*erf(b*x)/Pi^(1/2)*(-1/2*b^2*x^2/exp(b^2*x
^2)-1/2/exp(b^2*x^2))+4/3/Pi*(-5/16*2^(1/2)*Pi^(1/2)*erf(b*x*2^(1/2))+1/4/e
xp(b^2*x^2)^2*b*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-\frac{1}{4}\,b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b^3}\right) + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b}}{3\,\pi b^2} + \frac{\pi b^3x^3\,\mathrm{erf}(bx)^2 + 2\left(\sqrt{\pi}b^2x^2 + \sqrt{\pi}\right)\mathrm{erf}(bx)\,e^{\left(-b^2x^2\right)}}{3\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*integrate(4*(b^2*x^2 + 1)*e^(-2*b^2*x^2), x)/(pi*b^2) + 1/3*(pi*b^3*x^
3*erf(b*x)^2 + 2*(sqrt(pi)*b^2*x^2 + sqrt(pi))*erf(b*x)*e^(-b^2*x^2))/(pi*b
^3)
```

Fricas [A] time = 2.60602, size = 231, normalized size = 2.04

$$\frac{4\,\pi b^4x^3\,\mathrm{erf}(bx)^2 + 4\,b^2xe^{\left(-2\,b^2x^2\right)} + 8\sqrt{\pi}\left(b^3x^2 + b\right)\mathrm{erf}(bx)\,e^{\left(-b^2x^2\right)} - 5\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)}{12\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi*b^4*x^3*erf(b*x)^2 + 4*b^2*x*e^(-2*b^2*x^2) + 8*sqrt(pi)*(b^3*x^
2 + b)*erf(b*x)*e^(-b^2*x^2) - 5*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqr
t(b^2)*x))/(pi*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erf}^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erf(b*x)**2,x)
```

```
[Out] Integral(x**2*erf(b*x)**2, x)
```

Giac [A] time = 1.45898, size = 151, normalized size = 1.34

$$\frac{1}{3}\,x^3 \operatorname{erf}(bx)^2 + \frac{b\left(\frac{8\,(b^2x^2+1)\operatorname{erf}(bx)e^{(-b^2x^2)}}{b^4} + \frac{\sqrt{\pi}b^2\left(\frac{4\,xe^{(-2\,b^2x^2)}}{b^2} + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}(-\sqrt{2}bx)}{b^3}\right) + \frac{4\,\sqrt{2}\pi\operatorname{erf}(-\sqrt{2}bx)}{b}}{\pi b^3}\right)}{12\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*erf(b*x)^2 + 1/12*b*(8*(b^2*x^2 + 1)*erf(b*x)*e^(-b^2*x^2)/b^4 + (s
qrt(pi)*b^2*(4*x*e^(-2*b^2*x^2)/b^2 + sqrt(2)*sqrt(pi)*erf(-sqrt(2)*b*x)/b^
3) + 4*sqrt(2)*pi*erf(-sqrt(2)*b*x)/b)/(pi*b^3))/sqrt(pi)
```

3.31 $\int \mathbf{Erf}(bx)^2\,dx$

Optimal. Leaf size=56

$$\frac{2e^{-b^2x^2}\text{Erf}(bx)}{\sqrt{\pi}b} + x\text{Erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\text{Erf}\left(\sqrt{2}bx\right)}{b}$$

```
[Out] (2*Erf[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]^2 - (Sqrt[2/Pi]*Erf[Sqrt
[2]*b*x])/b
```

Rubi [A] time = 0.05024, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6352, 12, 6382, 2205}

$$\frac{2e^{-b^2x^2}\text{Erf}(bx)}{\sqrt{\pi}b} + x\text{Erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\text{Erf}\left(\sqrt{2}bx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]^2,x]
```

```
[Out] (2*Erf[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]^2 - (Sqrt[2/Pi]*Erf[Sqrt
[2]*b*x])/b
```

Rule 6352

```
Int[Erf[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x]^2)
/b, x] - Dist[4/Sqrt[Pi], Int[((a + b*x)*Erf[a + b*x])/E^(a + b*x)^2, x], x
] /; FreeQ[{a, b}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erf}(bx)^2\,dx &= x\operatorname{erf}(bx)^2 - \frac{4\int be^{-b^2x^2}x\operatorname{erf}(bx)\,dx}{\sqrt{\pi}}\\ &= x\operatorname{erf}(bx)^2 - \frac{(4b)\int e^{-b^2x^2}x\operatorname{erf}(bx)\,dx}{\sqrt{\pi}}\\ &= \frac{2e^{-b^2x^2}\operatorname{erf}(bx)}{b\sqrt{\pi}} + x\operatorname{erf}(bx)^2 - \frac{4\int e^{-2b^2x^2}\,dx}{\pi}\\ &= \frac{2e^{-b^2x^2}\operatorname{erf}(bx)}{b\sqrt{\pi}} + x\operatorname{erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\operatorname{erf}\left(\sqrt{2}bx\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0300453, size = 56, normalized size = 1.

$$\frac{2e^{-b^2x^2}\operatorname{Erf}(bx)}{\sqrt{\pi}b} + x\operatorname{Erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\operatorname{Erf}\left(\sqrt{2}bx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]^2,x]
```

```
[Out] (2*Erf[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]^2 - (Sqrt[2/Pi]*Erf[Sqrt
[2]*b*x])/b
```

Maple [A] time = 0.048, size = 48, normalized size = 0.9

$$\frac{1}{b}\left(bx\left(\operatorname{Erf}(bx)\right)^2 + 2\,\frac{\operatorname{Erf}(bx)\,e^{-b^2x^2}}{\sqrt{\pi}} - \frac{\sqrt{2}\operatorname{Erf}\left(bx\sqrt{2}\right)}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2,x)
```

```
[Out] 1/b*(b*x*erf(b*x)^2+2*erf(b*x)/Pi^(1/2)*exp(-b^2*x^2)-1/Pi^(1/2)*2^(1/2)*er
f(b*x*2^(1/2)))
```

Maxima [A] time = 1.7455, size = 84, normalized size = 1.5

$$\frac{\left(\sqrt{\pi}bx\,\mathrm{erf}\,(bx)^2\,e^{\left(b^2x^2\right)}+2\,\,\mathrm{erf}\,(bx)\right)e^{\left(-b^2x^2\right)}}{\sqrt{\pi}b}-\frac{\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{\sqrt{\pi}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] (sqrt(pi)*b*x*erf(b*x)^2*e^(b^2*x^2) + 2*erf(b*x))*e^(-b^2*x^2)/(sqrt(pi)*b
) - sqrt(2)*erf(sqrt(2)*b*x)/(sqrt(pi)*b)
```

Fricas [A] time = 2.50966, size = 166, normalized size = 2.96

$$\frac{\pi b^2x\,\mathrm{erf}\,(bx)^2+2\,\sqrt{\pi}b\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}-\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)}{\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] (pi*b^2*x*erf(b*x)^2 + 2*sqrt(pi)*b*erf(b*x)*e^(-b^2*x^2) - sqrt(2)*sqrt(pi
)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{erf}^2\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2,x)
```

```
[Out] Integral(erf(b*x)**2, x)
```

Giac [A] time = 1.30586, size = 65, normalized size = 1.16

$$x \operatorname{erf}(bx)^2 + \frac{b\left(\frac{2\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{b^2} + \frac{\sqrt{2}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^2}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2,x, algorithm="giac")
```

```
[Out] x*erf(b*x)^2 + b*(2*erf(b*x)*e^(-b^2*x^2)/b^2 + sqrt(2)*erf(-sqrt(2)*b*x)/b
^2)/sqrt(pi)
```

3.32 $\int \frac{\mathbf{Erf}(bx)^2}{x^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erf}(bx)^2}{x^2}, x\right)$$

```
[Out] Unintegrable[Erf[b*x]^2/x^2, x]
```

Rubi [A] time = 0.0180231, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Erf[b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\text{erf}(bx)^2}{x^2} dx = \int \frac{\text{erf}(bx)^2}{x^2} dx$$

Mathematica [A] time = 0.0365291, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(bx)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]^2/x^2,x]
```

```
[Out] Integrate[Erf[b*x]^2/x^2, x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Erf}(bx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x^2,x)
```

```
[Out] int(erf(b*x)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4\,b\int \frac{\mathrm{erf}(bx)e^{\left(-b^2x^2\right)}}{x}\,dx}{\sqrt{\pi}} - \frac{\mathrm{erf}\,(bx)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 4*b*integrate(erf(b*x)*e^(-b^2*x^2)/x, x)/sqrt(pi) - erf(b*x)^2/x
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}\,(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x**2,x)
```

```
[Out] Integral(erf(b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x^2, x)
```

3.33 $\int \frac{\textbf{Erf}(bx)^2}{x^4}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erf}(bx)^2}{x^4},x\right)$$

```
[Out] Unintegrable[Erf[b*x]^2/x^4, x]
```

Rubi [A] time = 0.0172838, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]^2/x^4,x]
```

```
[Out] Defer[Int][Erf[b*x]^2/x^4, x]
```

Rubi steps

$$\int \frac{\text{erf}(bx)^2}{x^4}\,dx=\int \frac{\text{erf}(bx)^2}{x^4}\,dx$$

Mathematica [A] time = 0.0356966, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]^2/x^4,x]
```

```
[Out] Integrate[Erf[b*x]^2/x^4, x]
```

Maple [A] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Erf}\,(bx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x^4,x)
```

```
[Out] int(erf(b*x)^2/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4\,b\int \frac{\mathrm{erf}(bx)e^{\left(-b^2x^2\right)}}{x^3}\,dx}{3\,\sqrt{\pi}} - \frac{\mathrm{erf}\,(bx)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 4/3*b*integrate(erf(b*x)*e^(-b^2*x^2)/x^3, x)/sqrt(pi) - 1/3*erf(b*x)^2/x^3
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}\,(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)^2/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}^2(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x**4,x)
```

```
[Out] Integral(erf(b*x)**2/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x^4, x)
```

3.34 $\int \frac{\mathbf{Erf}(bx)^2}{x^6}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erf}(bx)^2}{x^6},x\right)$$

```
[Out] Unintegrable[Erf[b*x]^2/x^6, x]
```

Rubi [A] time = 0.0174087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]^2/x^6,x]
```

```
[Out] Defer[Int][Erf[b*x]^2/x^6, x]
```

Rubi steps

$$\int \frac{\text{erf}(bx)^2}{x^6}\,dx = \int \frac{\text{erf}(bx)^2}{x^6}\,dx$$

Mathematica [A] time = 0.0360329, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]^2/x^6,x]
```

```
[Out] Integrate[Erf[b*x]^2/x^6, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Erf}(bx))^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)^2/x^6,x)
```

```
[Out] int(erf(b*x)^2/x^6,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)^2/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}^2(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)**2/x**6,x)
```

```
[Out] Integral(erf(b*x)**2/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2/x^6, x)
```

3.35 $\int (c+dx)^2 \mathbf{Erf}(a+bx)^2\,dx$

Optimal. Leaf size=375

$$\frac{d(a+bx)^2(bc-ad)\text{Erf}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{Erf}(a+bx)^2}{b^3}+\frac{2de^{-(a+bx)^2}(a+bx)(bc-ad)\text{Erf}(a+bx)}{\sqrt{\pi}b^3}-\frac{d(bc-a}{}$$

```
[Out] (d*(b*c - a*d))/(b^3*E^(2*(a + b*x)^2)*Pi) + (d^2*(a + b*x))/(3*b^3*E^(2*(a
 + b*x)^2)*Pi) + (2*d^2*Erf[a + b*x])/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) + (2*(
b*c - a*d)^2*Erf[a + b*x])/(b^3*E^(a + b*x)^2*Sqrt[Pi]) + (2*d*(b*c - a*d)*
(a + b*x)*Erf[a + b*x])/(b^3*E^(a + b*x)^2*Sqrt[Pi]) + (2*d^2*(a + b*x)^2*E
rf[a + b*x])/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) - (d*(b*c - a*d)*Erf[a + b*x]^2
)/(2*b^3) + ((b*c - a*d)^2*(a + b*x)*Erf[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(
a + b*x)^2*Erf[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*Erf[a + b*x]^2)/(3*b^3) -
 ((b*c - a*d)^2*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b^3 - (5*d^2*Erf[Sqrt[2]
*(a + b*x)])/(6*b^3*Sqrt[2*Pi])
```

Rubi [A] time = 0.421624, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6367, 6352, 6382, 2205, 6364, 6385, 6373, 30, 2209, 2212}

$$\frac{d(a+bx)^2(bc-ad)\text{Erf}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{Erf}(a+bx)^2}{b^3}+\frac{2de^{-(a+bx)^2}(a+bx)(bc-ad)\text{Erf}(a+bx)}{\sqrt{\pi}b^3}-\frac{d(bc-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Erf[a + b*x]^2,x]
```


```
[Out] (d*(b*c - a*d))/(b^3*E^(2*(a + b*x)^2)*Pi) + (d^2*(a + b*x))/(3*b^3*E^(2*(a
 + b*x)^2)*Pi) + (2*d^2*Erf[a + b*x])/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) + (2*(
b*c - a*d)^2*Erf[a + b*x])/(b^3*E^(a + b*x)^2*Sqrt[Pi]) + (2*d*(b*c - a*d)*
(a + b*x)*Erf[a + b*x])/(b^3*E^(a + b*x)^2*Sqrt[Pi]) + (2*d^2*(a + b*x)^2*E
rf[a + b*x])/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) - (d*(b*c - a*d)*Erf[a + b*x]^2
)/(2*b^3) + ((b*c - a*d)^2*(a + b*x)*Erf[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(
a + b*x)^2*Erf[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*Erf[a + b*x]^2)/(3*b^3) -
 ((b*c - a*d)^2*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b^3 - (5*d^2*Erf[Sqrt[2]
*(a + b*x)])/(6*b^3*Sqrt[2*Pi])
```


Rule 6367

```
Int[Erf[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Dist[
1/b^(m + 1), Subst[Int[ExpandIntegrand[Erf[x]^2, (b*c - a*d + d*x)^m, x], x
], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6352

```
Int[Erf[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x]^2)
/b, x] - Dist[4/Sqrt[Pi], Int[((a + b*x)*Erf[a + b*x])/E^(a + b*x)^2, x], x
] /; FreeQ[{a, b}, x]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule steps

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\text{erf}(x)^2+2bcd\left(1-\frac{ad}{bc}\right)x\text{erf}(x)^2+d^2x^2\text{erf}(x)^2\right)dx,x,a+\right.}{b^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int x^2\text{erf}(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\,\text{Subst}\left(\int x\text{erf}(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(b}{}$$

$$= \frac{(bc-ad)^2(a+bx)\text{erf}(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2\text{erf}(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3\text{erf}(a+bx}{3b^3}$$

$$= \frac{2(bc-ad)^2e^{-(a+bx)^2}\text{erf}(a+bx)}{b^3\sqrt{\pi}}+\frac{2d(bc-ad)e^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)}{b^3\sqrt{\pi}}+\frac{2d^2e^{-(a+bx)^2}(a}{3b}$$

$$= \frac{d(bc-ad)e^{-2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{-2(a+bx)^2}(a+bx)}{3b^3\pi}+\frac{2d^2e^{-(a+bx)^2}\text{erf}(a+bx)}{3b^3\sqrt{\pi}}+\frac{2(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}}$$

$$= \frac{d(bc-ad)e^{-2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{-2(a+bx)^2}(a+bx)}{3b^3\pi}+\frac{2d^2e^{-(a+bx)^2}\text{erf}(a+bx)}{3b^3\sqrt{\pi}}+\frac{2(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}}$$

Mathematica [A] time = 1.00768, size = 226, normalized size = 0.6

$$\frac{2\text{Erf}(a+bx)^2\left(-6a^2bcd+2a^3d^2+3a\left(2b^2c^2+d^2\right)+2b^3x\left(3c^2+3cdx+d^2x^2\right)-3bcd\right)+\frac{8e^{-(a+bx)^2}\text{Erf}(a+bx)\left(\left(a^2+1\right)d^2-abd(3c+\right.}{\sqrt{\pi}}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Erf[a + b*x]^2,x]
```

```
[Out] ((4*d*(3*b*c - 2*a*d + b*d*x))/(E^(2*(a + b*x)^2)*Pi) + (8*((1 + a^2)*d^2 -
 a*b*d*(3*c + d*x) + b^2*(3*c^2 + 3*c*d*x + d^2*x^2))*Erf[a + b*x])/(E^(a +
 b*x)^2*Sqrt[Pi]) + 2*(-3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + 3*a*(2*b^2*c^2
+ d^2) + 2*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Erf[a + b*x]^2 - (12*b^2*c^2
- 24*a*b*c*d + (5 + 12*a^2)*d^2)*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/(12*b^3
)
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int (dx+c)^2 (\mathrm{Erf}\,(bx+a))^2 \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*erf(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*erf(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(d^2x^3+3\,cdx^2+3\,c^2x\right)\mathrm{erf}\,(bx+a)^2-\frac{4\int\left(bd^2x^3+3\,bcdx^2+3\,bc^2x\right)\mathrm{erf}\,(bx+a)\,e^{\left(-b^2x^2-2\,abx-a^2\right)}\,dx}{3\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erf(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)*erf(b*x + a)^2 - 1/3*integrate(4*(b*d^2
*x^3 + 3*b*c*d*x^2 + 3*b*c^2*x)*erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x - a^2),
x)/sqrt(pi)
```

Fricas [A] time = 2.63072, size = 645, normalized size = 1.72

$$-\frac{\sqrt{2}\sqrt{\pi}\left(12\,b^2c^2 - 24\,abcd + \left(12\,a^2 + 5\right)d^2\right)\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) - 8\,\sqrt{\pi}\left(b^3d^2x^2 + 3\,b^3c^2 - 3\,ab^2cd + \left(a^2+1\right)bd^2 + \left(3\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erf(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2)*sqrt(pi)*(12*b^2*c^2 - 24*a*b*c*d + (12*a^2 + 5)*d^2)*sqrt(b
^2)*erf(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 8*sqrt(pi)*(b^3*d^2*x^2 + 3*b^3*c^
2 - 3*a*b^2*c*d + (a^2 + 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x)*erf(b*x + a)
*e^(-b^2*x^2 - 2*a*b*x - a^2) - 2*(2*pi*b^4*d^2*x^3 + 6*pi*b^4*c*d*x^2 + 6*
pi*b^4*c^2*x + pi*(6*a*b^3*c^2 - 3*(2*a^2 + 1)*b^2*c*d + (2*a^3 + 3*a)*b*d^
2))*erf(b*x + a)^2 - 4*(b^2*d^2*x + 3*b^2*c*d - 2*a*b*d^2)*e^(-2*b^2*x^2 -
4*a*b*x - 2*a^2))/(pi*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*erf(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\,\mathrm{erf}\,(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erf(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*erf(b*x + a)^2, x)
```

3.36 $\int (c+dx)\,\mathbf{Erf}(a+bx)^2\,dx$

Optimal. Leaf size=188

$$\frac{(a+bx)(bc-ad)\text{Erf}(a+bx)^2}{b^2}+\frac{2e^{-(a+bx)^2}(bc-ad)\text{Erf}(a+bx)}{\sqrt{\pi}b^2}-\frac{\sqrt{\frac{2}{\pi}}(bc-ad)\text{Erf}\left(\sqrt{2}(a+bx)\right)}{b^2}+\frac{d(a+bx)^2\text{Erf}(a+bx)}{2b^2}$$

```
[Out] d/(2*b^2*E^(2*(a + b*x)^2)*Pi) + (2*(b*c - a*d)*Erf[a + b*x])/(b^2*E^(a + b
*x)^2*Sqrt[Pi]) + (d*(a + b*x)*Erf[a + b*x])/(b^2*E^(a + b*x)^2*Sqrt[Pi]) -
 (d*Erf[a + b*x]^2)/(4*b^2) + ((b*c - a*d)*(a + b*x)*Erf[a + b*x]^2)/b^2 +
(d*(a + b*x)^2*Erf[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*Sqrt[2/Pi]*Erf[Sqrt[2
]*(a + b*x)])/b^2
```

Rubi [A] time = 0.1757, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6367, 6352, 6382, 2205, 6364, 6385, 6373, 30, 2209}

$$\frac{(a+bx)(bc-ad)\text{Erf}(a+bx)^2}{b^2}+\frac{2e^{-(a+bx)^2}(bc-ad)\text{Erf}(a+bx)}{\sqrt{\pi}b^2}-\frac{\sqrt{\frac{2}{\pi}}(bc-ad)\text{Erf}\left(\sqrt{2}(a+bx)\right)}{b^2}+\frac{d(a+bx)^2\text{Erf}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Erf[a + b*x]^2,x]
```

```
[Out] d/(2*b^2*E^(2*(a + b*x)^2)*Pi) + (2*(b*c - a*d)*Erf[a + b*x])/(b^2*E^(a + b
*x)^2*Sqrt[Pi]) + (d*(a + b*x)*Erf[a + b*x])/(b^2*E^(a + b*x)^2*Sqrt[Pi]) -
 (d*Erf[a + b*x]^2)/(4*b^2) + ((b*c - a*d)*(a + b*x)*Erf[a + b*x]^2)/b^2 +
(d*(a + b*x)^2*Erf[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*Sqrt[2/Pi]*Erf[Sqrt[2
]*(a + b*x)])/b^2
```

Rule 6367

```
Int[Erf[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Dist[
1/b^(m + 1), Subst[Int[ExpandIntegrand[Erf[x]^2, (b*c - a*d + d*x)^m, x], x
], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6352

```
Int[Erf[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x]^2)
/b, x] - Dist[4/Sqrt[Pi], Int[((a + b*x)*Erf[a + b*x])/E^(a + b*x)^2, x], x
```

```
] /; FreeQ[{a, b}, x]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 6364

```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erf[b*x]^2)/
(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erf[b*x])/E^(b^
2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
```

```
[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\text{erf}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)\text{erf}(x)^2+dx\text{erf}(x)^2\right)dx,x,a+bx\right)}{b^2}\\
&= \frac{d\,\text{Subst}\left(\int x\text{erf}(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int \text{erf}(x)^2\,dx,x,a+bx\right)}{b^2}\\
&= \frac{(bc-ad)(a+bx)\text{erf}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{erf}(a+bx)^2}{2b^2}-\frac{(2d)\,\text{Subst}\left(\int e^{-x^2}x^2\text{erf}(x)\,dx,x,a\right.}{b^2\sqrt{\pi}}\\
&= \frac{2(bc-ad)e^{-(a+bx)^2}\text{erf}(a+bx)}{b^2\sqrt{\pi}}+\frac{de^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)}{b^2\sqrt{\pi}}+\frac{(bc-ad)(a+bx)\text{erf}(a+bx)}{b^2}\\
&= \frac{de^{-2(a+bx)^2}}{2b^2\pi}+\frac{2(bc-ad)e^{-(a+bx)^2}\text{erf}(a+bx)}{b^2\sqrt{\pi}}+\frac{de^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)}{b^2\sqrt{\pi}}+\frac{(bc-ad)(a+b}{b}\\
&= \frac{de^{-2(a+bx)^2}}{2b^2\pi}+\frac{2(bc-ad)e^{-(a+bx)^2}\text{erf}(a+bx)}{b^2\sqrt{\pi}}+\frac{de^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)}{b^2\sqrt{\pi}}-\frac{d\text{erf}(a+bx)^2}{4b^2}
\end{aligned}
$$

Mathematica [A] time = 0.398058, size = 132, normalized size = 0.7

$$
\frac{\pi\text{Erf}(a+bx)^2\left(-2a^2d+4abc+4b^2cx+2b^2dx^2-d\right)+4\sqrt{\pi}e^{-(a+bx)^2}\text{Erf}(a+bx)(-ad+2bc+bdx)+4\sqrt{2\pi}(ad-bc)\text{Erf}\left(\right.}{4\pi b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Erf[a + b*x]^2,x]
```

```
[Out] ((2*d)/E^(2*(a + b*x)^2) + (4*Sqrt[Pi]*(2*b*c - a*d + b*d*x)*Erf[a + b*x])/
E^(a + b*x)^2 + Pi*(4*a*b*c - d - 2*a^2*d + 4*b^2*c*x + 2*b^2*d*x^2)*Erf[a
+ b*x]^2 + 4*(-(b*c) + a*d)*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)])/(4*b^2*Pi)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int (dx+c)\left(\text{Erf}\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*erf(b*x+a)^2,x)
```

```
[Out] int((d*x+c)*erf(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(dx^2+2\,cx\right)\operatorname{erf}(bx+a)^2-\frac{2\int\left(bdx^2+2\,bcx\right)\operatorname{erf}(bx+a)\,e^{\left(-b^2x^2-2\,abx-a^2\right)}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*erf(b*x + a)^2 - integrate(2*(b*d*x^2 + 2*b*c*x)*erf(b*
x + a)*e^(-b^2*x^2 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Fricas [A] time = 2.57145, size = 406, normalized size = 2.16

$$-\frac{4\sqrt{2}\sqrt{\pi}\sqrt{b^2}(bc-ad)\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)-4\sqrt{\pi}\left(b^2dx+2\,b^2c-abd\right)\operatorname{erf}(bx+a)\,e^{\left(-b^2x^2-2\,abx-a^2\right)}-\left(2\,\pi b^3dx^2+4\,\pi b^3cx+\right.}{4\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*sqrt(pi)*sqrt(b^2)*(b*c - a*d)*erf(sqrt(2)*sqrt(b^2)*(b*x +
 a)/b) - 4*sqrt(pi)*(b^2*d*x + 2*b^2*c - a*b*d)*erf(b*x + a)*e^(-b^2*x^2 -
2*a*b*x - a^2) - (2*pi*b^3*d*x^2 + 4*pi*b^3*c*x + pi*(4*a*b^2*c - (2*a^2 +
1)*b*d))*erf(b*x + a)^2 - 2*b*d*e^(-2*b^2*x^2 - 4*a*b*x - 2*a^2))/(pi*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(c+dx)\operatorname{erf}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*erf(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\operatorname{erf}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erf(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*erf(b*x + a)^2, x)
```

3.37 $\int \mathrm{Erf}(a+bx)^2\,dx$

Optimal. Leaf size=71

$$\frac{(a+bx)\mathrm{Erf}(a+bx)^2}{b}+\frac{2e^{-(a+bx)^2}\mathrm{Erf}(a+bx)}{\sqrt{\pi}b}-\frac{\sqrt{\frac{2}{\pi}}\mathrm{Erf}\left(\sqrt{2}(a+bx)\right)}{b}$$

```
[Out] (2*Erf[a + b*x])/(b*E^(a + b*x)^2*Sqrt[Pi]) + ((a + b*x)*Erf[a + b*x]^2)/b
- (Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b
```

Rubi [A] time = 0.179626, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6352, 6382, 2205}

$$\frac{(a+bx)\mathrm{Erf}(a+bx)^2}{b}+\frac{2e^{-(a+bx)^2}\mathrm{Erf}(a+bx)}{\sqrt{\pi}b}-\frac{\sqrt{\frac{2}{\pi}}\mathrm{Erf}\left(\sqrt{2}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[a + b*x]^2,x]
```

```
[Out] (2*Erf[a + b*x])/(b*E^(a + b*x)^2*Sqrt[Pi]) + ((a + b*x)*Erf[a + b*x]^2)/b
- (Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b
```

Rule 6352

```
Int[Erf[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x]^2)
/b, x] - Dist[4/Sqrt[Pi], Int[((a + b*x)*Erf[a + b*x])/E^(a + b*x)^2, x], x
] /; FreeQ[{a, b}, x]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{erf}(a+bx)^2\,dx &= \frac{(a+bx)\operatorname{erf}(a+bx)^2}{b}-\frac{4\int e^{-(a+bx)^2}(a+bx)\operatorname{erf}(a+bx)\,dx}{\sqrt{\pi}}\\
&= \frac{(a+bx)\operatorname{erf}(a+bx)^2}{b}-\frac{4\,\operatorname{Subst}\left(\int e^{-x^2}x\operatorname{erf}(x)\,dx,x,a+bx\right)}{b\sqrt{\pi}}\\
&= \frac{2e^{-(a+bx)^2}\operatorname{erf}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erf}(a+bx)^2}{b}-\frac{4\,\operatorname{Subst}\left(\int e^{-2x^2}\,dx,x,a+bx\right)}{b\pi}\\
&= \frac{2e^{-(a+bx)^2}\operatorname{erf}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erf}(a+bx)^2}{b}-\frac{\sqrt{\frac{2}{\pi}}\operatorname{erf}\left(\sqrt{2}(a+bx)\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.00847, size = 66, normalized size = 0.93

$$
\frac{(a+bx)\operatorname{Erf}(a+bx)^2+\frac{2e^{-(a+bx)^2}\operatorname{Erf}(a+bx)}{\sqrt{\pi}}-\sqrt{\frac{2}{\pi}}\operatorname{Erf}\left(\sqrt{2}(a+bx)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[a + b*x]^2,x]
```

```
[Out] ((2*Erf[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) + (a + b*x)*Erf[a + b*x]^2 - Sqr
t[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b
```

Maple [A] time = 0.044, size = 59, normalized size = 0.8

$$
\frac{1}{b}\left((bx+a)\left(\operatorname{Erf}(bx+a)\right)^2+2\,\frac{\operatorname{Erf}(bx+a)\,e^{-(bx+a)^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{Erf}\left((bx+a)\sqrt{2}\right)}{\sqrt{\pi}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a)^2,x)
```

```
[Out] 1/b*((b*x+a)*erf(b*x+a)^2+2*erf(b*x+a)/Pi^(1/2)*exp(-(b*x+a)^2)-1/Pi^(1/2)*
2^(1/2)*erf((b*x+a)*2^(1/2)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.59558, size = 227, normalized size = 3.2

$$\frac{2\sqrt{\pi}b\,\mathrm{erf}\left(bx+a\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}+\left(\pi b^2x+\pi ab\right)\mathrm{erf}\left(bx+a\right)^2-\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*sqrt(pi)*b*erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (pi*b^2*x + pi*a*
b)*erf(b*x + a)^2 - sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*(b*x +
 a)/b))/(pi*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{erf}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)**2,x)
```

```
[Out] Integral(erf(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)^2, x)
```

3.38 $\int \frac{\mathbf{Erf}(a+bx)^2}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{Erf}(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable[Erf[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0239698, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Erf[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{erf}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{erf}(a+bx)^2}{c+dx} \, dx$$

Mathematica [A] time = 0.0523431, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Erf[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.39, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{Erf}\left(bx+a\right)\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(erf(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left(bx+a\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)^2/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erf}\left(bx+a\right)^{2}}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(erf(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)^2/(d*x + c), x)
```

3.39 $\int \frac{\text{Erf}(a+bx)^2}{(c+dx)^2} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{Erf}(a+bx)^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Erf[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0222502, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erf}(a+bx)^2}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Erf[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\text{erf}(a+bx)^2}{(c+dx)^2} \, dx = \int \frac{\text{erf}(a+bx)^2}{(c+dx)^2} \, dx$$

Mathematica [A] time = 0.105703, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}(a+bx)^2}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Erf[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.365, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Erf}(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(erf(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4\,b\int \frac{\mathrm{erf}(bx+a)\,e^{\left(-b^2x^2-2\,abx\right)}}{\sqrt{\pi}d^2xe^{\left(a^2\right)}+\sqrt{\pi}cde^{\left(a^2\right)}}\,dx-\frac{\mathrm{erf}(bx+a)^2}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 4*b*integrate(erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x)/(sqrt(pi)*d^2*x*e^(a^2) +
 sqrt(pi)*c*d*e^(a^2)), x) - erf(b*x + a)^2/(d^2*x + c*d)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(erf(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)^2/(d*x + c)^2, x)
```

3.40 $\int x^2 \mathbf{Erf}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=102

$$\frac{1}{3}x^3\mathrm{Erf}(d(a+b\log(cx^n)))-\frac{1}{3}x^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\mathrm{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{3}{n}}{2bd}\right)$$

```
[Out] (x^3*Erf[d*(a + b*Log[c*x^n])])/3 - (E^((9 - 12*a*b*d^2*n)/(4*b^2*d^2*n^2))
*x^3*Erf[(2*a*b*d^2 - 3/n + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/(3*(c*x^n)^(3/n
))
```

Rubi [A] time = 0.231717, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6401, 2278, 2274, 15, 2276, 2234, 2205}

$$\frac{1}{3}x^3\mathrm{Erf}(d(a+b\log(cx^n)))-\frac{1}{3}x^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\mathrm{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{3}{n}}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*Erf[d*(a + b*Log[c*x^n])])/3 - (E^((9 - 12*a*b*d^2*n)/(4*b^2*d^2*n^2))
*x^3*Erf[(2*a*b*d^2 - 3/n + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/(3*(c*x^n)^(3/n
))
```

Rule 6401

```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[((e*x)^(m + 1)*Erf[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x] -
 Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(2bdn)\int e^{-d^2\left(a+b\log\left(cx^n\right)\right)^2}x^2\,dx}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(2bdn)\int \exp\left(-a^2d^2-2abd^2\log\left(cx^n\right)-b^2d^2\log^2\left(cx^n\right)\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(2bdn)\int e^{-a^2d^2-b^2d^2\log^2\left(cx^n\right)}x^2\left(cx^n\right)^{-2abd^2}\,dx}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(2bdnx^{2abd^2n}\left(cx^n\right)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2\left(cx^n\right)}x^{2-2abd^2n}\,d}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(2bdx^3\left(cx^n\right)^{-2abd^2-\frac{3-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-a^2d^2+\frac{(3-}{}\right.\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(2bde^{\frac{9-12abd^2n}{4b^2d^2n^2}}x^3\left(cx^n\right)^{-2abd^2-\frac{3-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{\left(3-\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\mathbf{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{3}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}x^3\left(cx^n\right)^{-3/n}\mathbf{erf}\left(\frac{2abd^2-\frac{3}{n}+2b^2d^2\log\left(cx^n\right)}{2bd}\right.$$

Mathematica [A] time = 0.317867, size = 88, normalized size = 0.86

$$\frac{1}{3}\left(x^3\text{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-x^3\text{Erf}\left(ad+bd\log\left(cx^n\right)-\frac{3}{2bdn}\right)\exp\left(-\frac{3\left(\frac{4abn-\frac{3}{d^2}}{b^2}+4n\log\left(cx^n\right)\right)}{4n^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*Erf[d*(a + b*Log[c*x^n])] - (x^3*Erf[a*d - 3/(2*b*d*n) + b*d*Log[c*x^n
]])/E^((3*((-3/d^2 + 4*a*b*n)/b^2 + 4*n*Log[c*x^n]))/(4*n^2)))/3
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int x^2\text{Erf}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erf(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*erf(d*(a+b*ln(c*x^n))),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.76454, size = 302, normalized size = 2.96

$$\frac{1}{3}\,x^3\,\mathrm{erf}\left(bd\log\left(cx^n\right)+ad\right)-\frac{1}{3}\,\sqrt{b^2d^2n^2}\,\mathrm{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n-3\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{3\left(4\,b^2d^2n\log(c)+4\right.}{4\,b^2d^2n^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*erf(b*d*log(c*x^n) + a*d) - 1/3*sqrt(b^2*d^2*n^2)*erf(1/2*(2*b^2*d^
2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 3)*sqrt(b^2*d^2*n^2)/(b^2
*d^2*n^2))*e^(-3/4*(4*b^2*d^2*n*log(c) + 4*a*b*d^2*n - 3)/(b^2*d^2*n^2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erf(d*(a+b*ln(c*x**n))),x)
```

[Out] Timed out

Giac [A] time = 1.51984, size = 115, normalized size = 1.13

$$\frac{1}{3}\,x^{3}\operatorname{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+\frac{\operatorname{erf}\left(-bdn\log\left(x\right)-bd\log\left(c\right)-ad+\frac{3}{2\,bdn}\right)e^{\left(-\frac{3\,a}{bn}+\frac{9}{4\,b^{2}d^{2}n^{2}}\right)}}{3\,c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] 1/3*x^3*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/3*erf(-b*d*n*log(x) - b*d*
log(c) - a*d + 3/2/(b*d*n))*e^(-3*a/(b*n) + 9/4/(b^2*d^2*n^2))/c^(3/n)
```

3.41 $\int x\textbf{Erf}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}x^2\text{Erf}(d(a+b\log(cx^n)))-\frac{1}{2}x^2(cx^n)^{-2/n}e^{\frac{1-2abd^2n}{b^2d^2n^2}}\text{Erf}\left(\frac{abd^2+b^2d^2\log(cx^n)-\frac{1}{n}}{bd}\right)$$

```
[Out] (x^2*Erf[d*(a + b*Log[c*x^n])])/2 - (E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^
2*Erf[(a*b*d^2 - n^(-1) + b^2*d^2*Log[c*x^n])/(b*d)])/(2*(c*x^n)^(2/n))
```

Rubi [A] time = 0.176036, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6401, 2278, 2274, 15, 2276, 2234, 2205}

$$\frac{1}{2}x^2\text{Erf}(d(a+b\log(cx^n)))-\frac{1}{2}x^2(cx^n)^{-2/n}e^{\frac{1-2abd^2n}{b^2d^2n^2}}\text{Erf}\left(\frac{abd^2+b^2d^2\log(cx^n)-\frac{1}{n}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*Erf[d*(a + b*Log[c*x^n])])/2 - (E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^
2*Erf[(a*b*d^2 - n^(-1) + b^2*d^2*Log[c*x^n])/(b*d)])/(2*(c*x^n)^(2/n))
```

Rule 6401

```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[((e*x)^(m + 1)*Erf[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x] -
 Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(bdn)\int e^{-d^2\left(a+b\log\left(cx^n\right)\right)^2}x\,dx}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(bdn)\int \exp\left(-a^2d^2-2abd^2\log\left(cx^n\right)-b^2d^2\log^2\left(cx^n\right)\right)x\,\ldots}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(bdn)\int e^{-a^2d^2-b^2d^2\log^2\left(cx^n\right)}x\left(cx^n\right)^{-2abd^2}\,dx}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(bdnx^{2abd^2n}\left(cx^n\right)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2\left(cx^n\right)}x^{1-2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(bdx^2\left(cx^n\right)^{-2abd^2-\frac{2-2abd^2n}{n}}\right)\mathrm{Subst}\left(\int\exp\left(-a^2d^2+\frac{(2-2abd^2\ldots}{n}\ldots\right.\right.}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(bde^{\frac{1-2abd^2n}{b^2d^2n^2}}x^2\left(cx^n\right)^{-2abd^2-\frac{2-2abd^2n}{n}}\right)\mathrm{Subst}\left(\int\exp\left(-\frac{\left(\frac{2-2abd^2\ldots}{n}\right.}{4\ldots}\right.\right.}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{1}{2}e^{\frac{1-2abd^2n}{b^2d^2n^2}}x^2\left(cx^n\right)^{-2/n}\mathrm{erf}\left(\frac{abd^2-\frac{1}{n}+b^2d^2\log\left(cx^n\right)}{bd}\right)$$

Mathematica [A] time = 0.274896, size = 84, normalized size = 0.89

$$\frac{1}{2}\left(x^2\mathrm{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-x^2e^{-\frac{\frac{2abn-\frac{1}{d^2}}{b^2}+2n\log\left(cx^n\right)}{n^2}}\mathrm{Erf}\left(ad+bd\log\left(cx^n\right)-\frac{1}{bdn}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*Erf[d*(a + b*Log[c*x^n])] - (x^2*Erf[a*d - 1/(b*d*n) + b*d*Log[c*x^n]]
)/E^(((-d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n])/n^2))/2
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int x\mathrm{Erf}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erf(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*erf(d*(a+b*ln(c*x^n))),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.76902, size = 284, normalized size = 3.02

$$\frac{1}{2}x^2\operatorname{erf}(bd\log(cx^n)+ad)-\frac{1}{2}\sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{\left(b^2d^2n^2\log(x)+b^2d^2n\log(c)+abd^2n-1\right)\sqrt{b^2d^2n^2}}{b^2d^2n^2}\right)e^{\left(-\frac{2b^2d^2n\log(c)+2abd^2n-1}{b^2d^2n^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*erf(b*d*log(c*x^n) + a*d) - 1/2*sqrt(b^2*d^2*n^2)*erf((b^2*d^2*n^2*
log(x) + b^2*d^2*n*log(c) + a*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))
*e^(-(2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 1)/(b^2*d^2*n^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{erf}(ad+bd\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*erf(a*d + b*d*log(c*x**n)), x)
```

Giac [A] time = 1.55304, size = 112, normalized size = 1.19

$$\frac{1}{2}x^2\,\mathrm{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+\frac{\mathrm{erf}\left(-bdn\log\left(x\right)-bd\log\left(c\right)-ad+\frac{1}{bdn}\right)e^{\left(-\frac{2a}{bn}+\frac{1}{b^2d^2n^2}\right)}}{2c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] 1/2*x^2*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/2*erf(-b*d*n*log(x) - b*d*
log(c) - a*d + 1/(b*d*n))*e^(-2*a/(b*n) + 1/(b^2*d^2*n^2))/c^(2/n)
```

3.42 $\int \mathrm{Erf}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=93

$$x\mathrm{Erf}(d(a+b\log(cx^n)))-x(cx^n)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\mathrm{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)$$

```
[Out] x*Erf[d*(a + b*Log[c*x^n])] - (E^((1 - 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*x*Erf[
(2*a*b*d^2 - n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/(c*x^n)^n^(-1)
```

Rubi [A] time = 0.128592, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6397, 2277, 2274, 15, 2276, 2234, 2205}

$$x\mathrm{Erf}(d(a+b\log(cx^n)))-x(cx^n)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\mathrm{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Erf[d*(a + b*Log[c*x^n])],x]
```


```
[Out] x*Erf[d*(a + b*Log[c*x^n])] - (E^((1 - 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*x*Erf[
(2*a*b*d^2 - n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/(c*x^n)^n^(-1)
```


Rule 6397

```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*E
rf[d*(a + b*Log[c*x^n])], x] - Dist[(2*b*d*n)/Sqrt[Pi], Int[1/E^(d*(a + b*L
og[c*x^n]))^2, x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(2bdn)\int e^{-d^2(a+b\log(cx^n))^2}\,dx}{\sqrt{\pi}}$$

$$= x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(2bdn)\int \exp\left(-a^2d^2-2abd^2\log\left(cx^n\right)-b^2d^2\log^2\left(cx^n\right)\right)\,dx}{\sqrt{\pi}}$$

$$= x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{(2bdn)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}\left(cx^n\right)^{-2abd^2}\,dx}{\sqrt{\pi}}$$

$$= x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(2bdnx^{2abd^2n}\left(cx^n\right)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(2bdx\left(cx^n\right)^{-2abd^2-\frac{1-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-a^2d^2+\frac{(1-2abd^2n)}{n}\right.\right.}{\sqrt{\pi}}$$

$$= x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - \frac{\left(2bde^{\frac{1-4abd^2n}{4b^2d^2n^2}}x\left(cx^n\right)^{-2abd^2-\frac{1-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{\left(\frac{1-2abd^2n}{n}-\right.}{4b^2d}\right.\right.}{\sqrt{\pi}}$$

$$= x\text{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - e^{\frac{1-4abd^2n}{4b^2d^2n^2}}x\left(cx^n\right)^{-1/n}\text{erf}\left(\frac{2abd^2-\frac{1}{n}+2b^2d^2\log\left(cx^n\right)}{2bd}\right)$$

Mathematica [A] time = 0.244979, size = 80, normalized size = 0.86

$$x\text{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right) - x\text{Erf}\left(ad+bd\log\left(cx^n\right)-\frac{1}{2bdn}\right)\exp\left(-\frac{\frac{4abn-\frac{1}{d^2}}{b^2}+4n\log\left(cx^n\right)}{4n^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*Erf[d*(a + b*Log[c*x^n])] - (x*Erf[a*d - 1/(2*b*d*n) + b*d*Log[c*x^n]])/E
^(((-d^(-2) + 4*a*b*n)/b^2 + 4*n*Log[c*x^n])/(4*n^2))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \text{Erf}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(erf(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,bdne^{\left(-b^2d^2\log(c)^2-a^2d^2\right)}\int\frac{e^{\left(-2\,b^2d^2\log(c)\log\left(x^n\right)-b^2d^2\log\left(x^n\right)^2\right)}}{\left(x^n\right)^{2\,abd^2}}\,dx}{\sqrt{\pi}c^{2\,abd^2}}+x\,\mathrm{erf}\left(bd\log\left(x^n\right)+\left(b\log\left(c\right)+a\right)d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] -2*b*d*n*integrate(e^(-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d
^2*log(x^n)^2 - 2*a*b*d^2*log(x^n) - a^2*d^2), x)/(sqrt(pi)*c^(2*a*b*d^2))
+ x*erf(b*d*log(x^n) + (b*log(c) + a)*d)
```

Fricas [A] time = 2.82209, size = 290, normalized size = 3.12

$$-\sqrt{b^2d^2n^2}\,\mathrm{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n-1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{4\,b^2d^2n\log(c)+4\,abd^2n-1}{4\,b^2d^2n^2}\right)}+x\,\mathrm{erf}\left(bd\log\left(cx^n\right)+ad$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -sqrt(b^2*d^2*n^2)*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a
*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(-1/4*(4*b^2*d^2*n*log(c)
+ 4*a*b*d^2*n - 1)/(b^2*d^2*n^2)) + x*erf(b*d*log(c*x^n) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\mathrm{erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(erf(d*(a + b*log(c*x**n))), x)
```

Giac [A] time = 1.34115, size = 107, normalized size = 1.15

$$x\,\mathrm{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+\frac{\mathrm{erf}\left(-bdn\log\left(x\right)-bd\log\left(c\right)-ad+\frac{1}{2\,bdn}\right)e^{\left(-\frac{a}{bn}+\frac{1}{4\,b^{2}d^{2}n^{2}}\right)}}{c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] x*erf(b*d*n*log(x) + b*d*log(c) + a*d) + erf(-b*d*n*log(x) - b*d*log(c) - a
*d + 1/2/(b*d*n))*e^(-a/(b*n) + 1/4/(b^2*d^2*n^2))/c^(1/n)
```

3.43 $\int \frac{\mathbf{Erf}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{e^{-d^2(a+b\log(cx^n))^2}}{\sqrt{\pi}bdn}+\frac{(a+b\log(cx^n))\,\mathrm{Erf}(d(a+b\log(cx^n)))}{bn}$$

```
[Out] 1/(b*d*E^(d^2*(a + b*Log[c*x^n])^2)*n*Sqrt[Pi]) + (Erf[d*(a + b*Log[c*x^n])
]*(a + b*Log[c*x^n]))/(b*n)
```

Rubi [A] time = 0.0463315, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6349}

$$\frac{e^{-d^2(a+b\log(cx^n))^2}}{\sqrt{\pi}bdn}+\frac{(a+b\log(cx^n))\,\mathrm{Erf}(d(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] 1/(b*d*E^(d^2*(a + b*Log[c*x^n])^2)*n*Sqrt[Pi]) + (Erf[d*(a + b*Log[c*x^n])
]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathbf{erf}(d(a+b\log(cx^n)))}{x}\,dx&=\frac{\mathrm{Subst}\left(\int \mathbf{erf}(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\&=\frac{\mathrm{Subst}\left(\int \mathbf{erf}(x)\,dx,x,ad+bd\log(cx^n)\right)}{bdn}\\&=\frac{e^{-(ad+bd\log(cx^n))^2}}{bdn\sqrt{\pi}}+\frac{\mathbf{erf}(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [A] time = 0.139583, size = 79, normalized size = 1.22

$$\frac{\frac{(cx^n)^{-2abd^2} e^{-d^2\left(a^2+b^2\log^2(cx^n)\right)}}{\sqrt{\pi}bd} + \left(\frac{a}{b} + \log(cx^n)\right)\operatorname{Erf}(d(a + b\log(cx^n)))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (1/(b*d*E^(d^2*(a^2 + b^2*Log[c*x^n]^2))*Sqrt[Pi]*(c*x^n)^(2*a*b*d^2)) + Er
f[d*(a + b*Log[c*x^n])]*(a/b + Log[c*x^n]))/n
```

Maple [A] time = 0.108, size = 79, normalized size = 1.2

$$\frac{\ln(cx^n)\operatorname{Erf}(ad + bd\ln(cx^n))}{n} + \frac{\operatorname{Erf}(ad + bd\ln(cx^n))\,a}{bn} + \frac{e^{-(ad+bd\ln(cx^n))^2}}{bdn\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*erf(a*d+b*d*ln(c*x^n))+1/n/b*erf(a*d+b*d*ln(c*x^n))*a+1/n/b/d
/Pi^(1/2)*exp(-(a*d+b*d*ln(c*x^n))^2)
```

Maxima [A] time = 1.03546, size = 78, normalized size = 1.2

$$\frac{(b\log(cx^n) + a)d\operatorname{erf}((b\log(cx^n) + a)d) + \frac{e^{\left(-\left(b\log(cx^n)+a\right)^2 d^2\right)}}{\sqrt{\pi}}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*erf((b*log(c*x^n) + a)*d) + e^(-(b*log(c*x^n) + a)^2*
d^2)/sqrt(pi))/(b*d*n)
```

Fricas [A] time = 2.81078, size = 284, normalized size = 4.37

$$\frac{(\pi bdn\log(x)+\pi bd\log(c)+\pi ad)\operatorname{erf}(bd\log(cx^n)+ad)+\sqrt{\pi}e^{\left(-b^2d^2n^2\log(x)^2-b^2d^2\log(c)^2-2\,abd^2\log(c)-a^2d^2-2\,\left(b^2d^2n\log(c)+abd^2n\right.\right.}}{\pi bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] ((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*erf(b*d*log(c*x^n) + a*d) + sqr
t(pi)*e^(-b^2*d^2*n^2*log(x)^2 - b^2*d^2*log(c)^2 - 2*a*b*d^2*log(c) - a^2*
d^2 - 2*(b^2*d^2*n*log(c) + a*b*d^2*n)*log(x)))/(pi*b*d*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(ad+bd\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(erf(a*d + b*d*log(c*x**n))/x, x)
```

Giac [A] time = 1.31471, size = 90, normalized size = 1.38

$$\frac{(bdn\log(x)+bd\log(c)+ad)\operatorname{erf}(bdn\log(x)+bd\log(c)+ad)+\frac{e^{\left(-(bdn\log(x)+bd\log(c)+ad)^2\right)}}{\sqrt{\pi}}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] ((b*d*n*log(x) + b*d*log(c) + a*d)*erf(b*d*n*log(x) + b*d*log(c) + a*d) + e
^(-(b*d*n*log(x) + b*d*log(c) + a*d)^2)/sqrt(pi))/(b*d*n)
```

3.44 $\int \frac{\mathbf{Erf}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=92

$$\frac{(cx^n)^{\frac{1}{n}} e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}} \operatorname{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)}{x} - \frac{\operatorname{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

```
[Out] -(Erf[d*(a + b*Log[c*x^n])]/x) + (E^(1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)^n
^(-1)*Erf[(2*a*b*d^2 + n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x
```

Rubi [A] time = 0.213331, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6401, 2278, 2274, 15, 2276, 2234, 2205}

$$\frac{(cx^n)^{\frac{1}{n}} e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}} \operatorname{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)}{x} - \frac{\operatorname{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[d*(a + b*Log[c*x^n])]/x^2,x]
```



```
[Out] -(Erf[d*(a + b*Log[c*x^n])]/x) + (E^(1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)^n
^(-1)*Erf[(2*a*b*d^2 + n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x
```


Rule 6401

```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[((e*x)^(m + 1)*Erf[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x] -
 Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^2}\,dx &= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{(2bdn)\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^2}\,dx}{\sqrt{\pi}} \\
&= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{(2bdn)\int \frac{\exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log^2(cx^n)\right)}{x^2}\,dx}{\sqrt{\pi}} \\
&= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{(2bdn)\int \frac{e^{-a^2d^2-b^2d^2\log^2(cx^n)}(cx^n)^{-2abd^2}}{x^2}\,dx}{\sqrt{\pi}} \\
&= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{\left(2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-2-2abd^2n}\,dx}{\sqrt{\pi}} \\
&= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{\left(2bd(cx^n)^{-2abd^2-\frac{-1-2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-a^2d^2+\frac{(-1-2abd^2n}{n}\right.\right.}{\sqrt{\pi}x} \\
&= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{\left(2bde^{\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}(cx^n)^{-2abd^2-\frac{-1-2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{\left(\frac{-1-2ab}{n}\right.}{}\right.\right.}{\sqrt{\pi}x} \\
&= -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x} + \frac{e^{\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{2abd^2+\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)}{x}
\end{aligned}
$$

Mathematica [A] time = 0.248685, size = 80, normalized size = 0.87

$$
\frac{e^{\frac{\frac{4abn+\frac{1}{d^2}}{b^2}+4n\log(cx^n)}{4n^2}}\operatorname{Erf}\left(ad+bd\log(cx^n)+\frac{1}{2bdn}\right)-\operatorname{Erf}(d(a+b\log(cx^n)))}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (-Erf[d*(a + b*Log[c*x^n])] + E^(((d^(-2) + 4*a*b*n)/b^2 + 4*n*Log[c*x^n])/
(4*n^2))*Erf[a*d + 1/(2*b*d*n) + b*d*Log[c*x^n]])/x
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(erf(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.70359, size = 293, normalized size = 3.18

$$\frac{\sqrt{b^2d^2n^2}\,x\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n+1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(\frac{4\,b^2d^2n\log(c)+4\,abd^2n+1}{4\,b^2d^2n^2}\right)}-\operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(b^2*d^2*n^2)*x*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2
*a*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*n*log(c)
 + 4*a*b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left(ad+bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(erf(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(erf((b*log(c*x^n) + a)*d)/x^2, x)
```

3.45 $\int \frac{\mathbf{Erf}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=95

$$\frac{(cx^n)^{2/n}e^{\frac{2abd^2n+1}{b^2d^2n^2}}\operatorname{Erf}\left(\frac{abd^2n+b^2d^2n\log(cx^n)+1}{bdn}\right)}{2x^2}-\frac{\operatorname{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

```
[Out] -Erf[d*(a + b*Log[c*x^n])]/(2*x^2) + (E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(
c*x^n)^(2/n)*Erf[(1 + a*b*d^2*n + b^2*d^2*n*Log[c*x^n])/(b*d*n)])/(2*x^2)
```

Rubi [A] time = 0.214949, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6401, 2278, 2274, 15, 2276, 2234, 2205}

$$\frac{(cx^n)^{2/n}e^{\frac{2abd^2n+1}{b^2d^2n^2}}\operatorname{Erf}\left(\frac{abd^2n+b^2d^2n\log(cx^n)+1}{bdn}\right)}{2x^2}-\frac{\operatorname{Erf}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -Erf[d*(a + b*Log[c*x^n])]/(2*x^2) + (E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(
c*x^n)^(2/n)*Erf[(1 + a*b*d^2*n + b^2*d^2*n*Log[c*x^n])/(b*d*n)])/(2*x^2)
```

Rule 6401

```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[((e*x)^(m + 1)*Erf[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x] -
 Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{(bdn)\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^3}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{(bdn)\int \frac{\exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log^2(cx^n)\right)}{x^3}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{(bdn)\int \frac{e^{-a^2d^2-b^2d^2\log^2(cx^n)}(cx^n)^{-2abd^2}}{x^3}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-3-2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bd(cx^n)^{-2abd^2-\frac{-2-2abd^2n}{n}}\right)\text{Subst}\left(\int \exp\left(-a^2d^2+\frac{(-2-2abd^2n)x}{n}\right.\right.}{\sqrt{\pi}x^2}$$

$$= -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bde^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{-2abd^2-\frac{-2-2abd^2n}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{\left(\frac{-2-2abd^2n}{n}-2b\right.}{4b^2d^2}\right.\right.}{\sqrt{\pi}x^2}$$

$$= -\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\,\text{erf}\left(\frac{1+abd^2n+b^2d^2n\log(cx^n)}{bdn}\right)}{2x^2}$$

Mathematica [A] time = 0.246959, size = 77, normalized size = 0.81

$$\frac{e^{\frac{\frac{2abn+\frac{1}{d^2}}{b^2}+2n\log(cx^n)}{n^2}}\text{Erf}\left(ad+bd\log(cx^n)+\frac{1}{bdn}\right)-\text{Erf}(d(a+b\log(cx^n)))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (-Erf[d*(a + b*Log[c*x^n])] + E^(((d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n])/
n^2)*Erf[a*d + 1/(b*d*n) + b*d*Log[c*x^n]])/(2*x^2)
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(erf(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.7836, size = 285, normalized size = 3.

$$\frac{\sqrt{b^{2}d^{2}n^{2}}x^{2}\operatorname{erf}\left(\frac{\left(b^{2}d^{2}n^{2}\log(x)+b^{2}d^{2}n\log(c)+abd^{2}n+1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(\frac{2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n+1}{b^{2}d^{2}n^{2}}\right)}-\operatorname{erf}\left(bd\log\left(cx^{n}\right)+ad\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2*d^2*n^2)*x^2*erf((b^2*d^2*n^2*log(x) + b^2*d^2*n*log(c) + a*b
*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^((2*b^2*d^2*n*log(c) + 2*a*b
*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf((b*log(c*x^n) + a)*d)/x^3, x)
```

3.46 $\int (ex)^m \mathbf{Erf}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=125

$$\frac{x(ex)^m (cx^n)^{-\frac{m+1}{n}} \exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right) \text{Erf}\left(\frac{-2abd^2n-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}+\frac{(ex)^{m+1}\text{Erf}(d(a+b\log(cx^n)))}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*Erf[d*(a + b*Log[c*x^n])])/(e*(1 + m)) + (E^(((1 + m)*(1 + m
 - 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*x*(e*x)^m*Erf[(1 + m - 2*a*b*d^2*n - 2*b^
2*d^2*n*Log[c*x^n])/(2*b*d*n)])/((1 + m)*(c*x^n)^((1 + m)/n))
```

Rubi [A] time = 0.312641, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6401, 2278, 2274, 15, 20, 2276, 2234, 2205}

$$\frac{x(ex)^m (cx^n)^{-\frac{m+1}{n}} \exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right) \text{Erf}\left(\frac{-2abd^2n-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}+\frac{(ex)^{m+1}\text{Erf}(d(a+b\log(cx^n)))}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^(1 + m)*Erf[d*(a + b*Log[c*x^n])])/(e*(1 + m)) + (E^(((1 + m)*(1 + m
 - 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*x*(e*x)^m*Erf[(1 + m - 2*a*b*d^2*n - 2*b^
2*d^2*n*Log[c*x^n])/(2*b*d*n)])/((1 + m)*(c*x^n)^((1 + m)/n))
```

Rule 6401

```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[((e*x)^(m + 1)*Erf[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x] -
 Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ex)^m \operatorname{erf}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(2bdn)\int e^{-d^2(a+b\log(cx^n))^2}(ex)^m\,dx}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(2bdn)\int \exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log\right.}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(2bdn)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}(ex)^m(cx^n)^{-2abd^2}\,dx}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-2}}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bdnx^{-m+2abd^2n}(ex)^m(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log}}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bdx(ex)^m(cx^n)^{-2abd^2-\frac{1+m-2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\right.}{(1+}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bd\exp\left(\frac{(1+m)(1+m-4abd^2n)}{4b^2d^2n^2}\right)x(ex)^m(cx^n)^{-2abd^2-\frac{1}{}}\right.}{}$$

$$= \frac{(ex)^{1+m}\operatorname{erf}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{\exp\left(\frac{(1+m)(1+m-4abd^2n)}{4b^2d^2n^2}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{1+}{}\right.}{1+m}$$

<u>Mathematica</u> [A] time = 0.509761, size = 127, normalized size = 1.02

$$\frac{(ex)^m\left(x\operatorname{Erf}(d(a+b\log(cx^n)))-x^{-m}\operatorname{Erf}\left(ad-\frac{-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)\exp\left(\frac{(m+1)\left(-4abd^2n-4b^2d^2n\log(cx^n)+4b^2d^2n^2\log(x)+m+1\right)}{4b^2d^2n^2}\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Erf[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(x*Erf[d*(a + b*Log[c*x^n])] - (E^(((1 + m)*(1 + m - 4*a*b*d^2*n +
 4*b^2*d^2*n^2*Log[x] - 4*b^2*d^2*n*Log[c*x^n]))/(4*b^2*d^2*n^2))*Erf[a*d -
 (1 + m - 2*b^2*d^2*n*Log[c*x^n])/(2*b*d*n)])/x^m))/(1 + m)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{Erf}(d(a + b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*erf(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*erf(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^m x x^m \operatorname{erf}(bd\log(x^n) + (b\log(c) + a)d)}{m+1} - \frac{-\frac{\sqrt{\pi} c^{2\,abd^2} e^m \operatorname{erf}\left(-bdn\log(x) - bd\log(c) - ad + \frac{m}{2\,bdn} + \frac{1}{2\,bdn}\right) e^{\left(-\frac{am}{bn} - \frac{a}{bn} + \frac{m^2}{4\,b^2d^2n^2} + \frac{m}{2\,b^2d^2n^2} + \frac{1}{4\,b^2d^2n^2}\right)}}{c^{\frac{m}{n}} c^{\left(\frac{1}{n}\right)}}}{\sqrt{\pi} c^{2\,abd^2}(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] e^m*x*x^m*erf(b*d*log(x^n) + (b*log(c) + a)*d)/(m + 1) - 2*b*d*e^m*n*integr
ate(e^(-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d^2*log(x^n)^2 -
 2*a*b*d^2*log(x^n) - a^2*d^2 + m*log(x)), x)/(sqrt(pi)*c^(2*a*b*d^2)*(m +
1))
```

Fricas [A] time = 2.87312, size = 427, normalized size = 3.42

$$\frac{x \operatorname{erf}(bd\log(cx^n) + ad)\, e^{(m\log(e) + m\log(x))} - \sqrt{b^2d^2n^2} \operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log(x) + 2\,b^2d^2n\log(c) + 2\,abd^2n - m - 1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right) e^{\left(\frac{4\,b^2d^2mn^2\log(e) - 4\,(b^2d^2m}{}\right.}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] (x*erf(b*d*log(c*x^n) + a*d)*e^(m*log(e) + m*log(x)) - sqrt(b^2*d^2*n^2)*er
f(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - m - 1)*sqr
```

```
t(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*m*n^2*log(e) - 4*(b^2*d^2*m
 + b^2*d^2)*n*log(c) + m^2 - 4*(a*b*d^2*m + a*b*d^2)*n + 2*m + 1)/(b^2*d^2*
n^2)))/(m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*erf(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62525, size = 211, normalized size = 1.69

$$\frac{x^{m+1}\operatorname{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)e^{m}}{m+1}+\frac{\pi\operatorname{erf}\left(-bdn\log\left(x\right)-bd\log\left(c\right)-ad+\frac{m}{2\,bdn}+\frac{1}{2\,bdn}\right)e^{\left(m-\frac{am}{bn}-\frac{a}{bn}+\frac{m^{2}}{4\,b^{2}d^{2}n^{2}}+\frac{m}{2\,b^{2}d^{2}\cdots}\right.}}{(\pi+\pi m)c^{\frac{m}{n}}c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] x^(m + 1)*erf(b*d*n*log(x) + b*d*log(c) + a*d)*e^m/(m + 1) + pi*erf(-b*d*n*
log(x) - b*d*log(c) - a*d + 1/2*m/(b*d*n) + 1/2/(b*d*n))*e^(m - a*m/(b*n) -
 a/(b*n) + 1/4*m^2/(b^2*d^2*n^2) + 1/2*m/(b^2*d^2*n^2) + 1/4/(b^2*d^2*n^2))
/((pi + pi*m)*c^(m/n)*c^(1/n))
```

3.47 $\int e^{c-b^2x^2}\mathbf{Erf}(bx)^2\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^3}{6b}$$

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^3)/(6*b)
```

Rubi [A] time = 0.0297161, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6373, 30}

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)*Erf[b*x]^2,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^3)/(6*b)
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c-b^2x^2}\text{erf}(bx)^2\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x^2\,dx,x,\text{erf}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\text{erf}(bx)^3}{6b}\end{aligned}$$

Mathematica [A] time = 0.0090464, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)*Erf[b*x]^2,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^3)/(6*b)
```

Maple [A] time = 0.049, size = 17, normalized size = 0.8

$$\frac{e^c\left(\text{Erf}\left(bx\right)\right)^3\sqrt{\pi}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)*erf(b*x)^2,x)
```

```
[Out] 1/6*exp(c)*erf(b*x)^3*Pi^(1/2)/b
```

Maxima [A] time = 1.02145, size = 22, normalized size = 1.05

$$\frac{\sqrt{\pi}\,\text{erf}\left(bx\right)^3e^c}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(pi)*erf(b*x)^3*e^c/b
```

Fricas [A] time = 2.79198, size = 42, normalized size = 2.

$$\frac{\sqrt{\pi}\,\text{erf}\left(bx\right)^3e^c}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(pi)*erf(b*x)^3*e^c/b
```

Sympy [A] time = 4.68973, size = 19, normalized size = 0.9

$$\begin{cases} \frac{\sqrt{\pi} e^{c} \operatorname{erf}^{3}{\left(b x \right)}}{6 b} & \text{for}\: b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)*erf(b*x)**2,x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)*erf(b*x)**3/(6*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx)^{2} e^{\left(-b^{2} x^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^2*e^(-b^2*x^2 + c), x)
```

3.48 $\int e^{c-b^2x^2}\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^2}{4b}$$

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Rubi [A] time = 0.01828, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6373, 30}

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)*Erf[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c-b^2x^2}\text{erf}(bx)\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx,x,\text{erf}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\text{erf}(bx)^2}{4b}\end{aligned}$$

Mathematica [A] time = 0.0050945, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)*Erf[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Maple [A] time = 0.111, size = 17, normalized size = 0.8

$$\frac{e^c(\text{Erf}(bx))^2\sqrt{\pi}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)*erf(b*x),x)
```

```
[Out] 1/4*exp(c)*erf(b*x)^2*Pi^(1/2)/b
```

Maxima [A] time = 1.00857, size = 22, normalized size = 1.05

$$\frac{\sqrt{\pi}\,\text{erf}(bx)^2\,e^c}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(b*x)^2*e^c/b
```

Fricas [A] time = 2.71171, size = 42, normalized size = 2.

$$\frac{\sqrt{\pi}\,\text{erf}(bx)^2\,e^c}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*erf(b*x)^2*e^c/b
```

Sympy [A] time = 1.18538, size = 19, normalized size = 0.9

$$\begin{cases} \frac{\sqrt{\pi}e^{c}\operatorname{erf}^{2}(bx)}{4b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)*erf(b*x),x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)*erf(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx)\, e^{\left(-b^{2}x^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2 + c), x)
```

3.49 $\int \frac{e^{c-b^2x^2}}{\mathbf{Erf}(bx)} \, dx$

Optimal. Leaf size=20

$$\frac{\sqrt{\pi}e^c \log(\mathbf{Erf}(bx))}{2b}$$

```
[Out] (E^c*Sqrt[Pi]*Log[Erf[b*x]])/(2*b)
```

Rubi [A] time = 0.0285661, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6373, 29}

$$\frac{\sqrt{\pi}e^c \log(\mathbf{Erf}(bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)/Erf[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Log[Erf[b*x]])/(2*b)
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{c-b^2x^2}}{\text{erf}(bx)} \, dx &= \frac{\left(e^c\sqrt{\pi}\right) \text{Subst}\left(\int \frac{1}{x} \, dx, x, \text{erf}(bx)\right)}{2b} \\ &= \frac{e^c\sqrt{\pi} \log(\text{erf}(bx))}{2b} \end{aligned}$$

Mathematica [A] time = 0.0105597, size = 20, normalized size = 1.

$$\frac{\sqrt{\pi}e^c \log(\mathbf{Erf}(bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)/Erf[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Log[Erf[b*x]])/(2*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2+c}}{\mathrm{Erf}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)/erf(b*x),x)
```

```
[Out] int(exp(-b^2*x^2+c)/erf(b*x),x)
```

Maxima [A] time = 1.0433, size = 20, normalized size = 1.

$$\frac{\sqrt{\pi}e^c \log\left(\mathrm{erf}\,(bx)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(pi)*e^c*log(erf(b*x))/b
```

Fricas [A] time = 2.67479, size = 46, normalized size = 2.3

$$\frac{\sqrt{\pi}e^c \log\left(\mathrm{erf}\,(bx)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*e^c*log(erf(b*x))/b
```

Sympy [A] time = 0.915131, size = 17, normalized size = 0.85

$$\frac{\sqrt{\pi}e^{c}\log\left(\operatorname{erf}\left(bx\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)/erf(b*x),x)
```

```
[Out] sqrt(pi)*exp(c)*log(erf(b*x))/(2*b)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^{2}x^{2}+c\right)}}{\operatorname{erf}\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erf(b*x), x)
```

3.50 $\int \frac{e^{c-b^2x^2}}{\textbf{Erf}(bx)^2}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{\pi}e^c}{2b\text{Erf}(bx)}$$

```
[Out] -(E^c*Sqrt[Pi])/(2*b*Erf[b*x])
```

Rubi [A] time = 0.0276506, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6373, 30}

$$-\frac{\sqrt{\pi}e^c}{2b\text{Erf}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)/Erf[b*x]^2,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(2*b*Erf[b*x])
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c-b^2x^2}}{\text{erf}(bx)^2}\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \text{erf}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}}{2b\text{erf}(bx)}\end{aligned}$$

Mathematica [A] time = 0.0062499, size = 21, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c}{2b\text{Erf}(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)/Erf[b*x]^2,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(2*b*Erf[b*x])
```

Maple [A] time = 0.043, size = 17, normalized size = 0.8

$$-\frac{e^c\sqrt{\pi}}{2\,b\text{Erf}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)/erf(b*x)^2,x)
```

```
[Out] -1/2*exp(c)*Pi^(1/2)/b/erf(b*x)
```

Maxima [A] time = 1.01952, size = 22, normalized size = 1.05

$$-\frac{\sqrt{\pi}e^c}{2\,b\,\text{erf}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(pi)*e^c/(b*erf(b*x))
```

Fricas [A] time = 2.65468, size = 43, normalized size = 2.05

$$-\frac{\sqrt{\pi}e^c}{2\,b\,\text{erf}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(pi)*e^c/(b*erf(b*x))
```

Sympy [A] time = 1.99355, size = 17, normalized size = 0.81

$$-\frac{\sqrt{\pi}e^c}{2b\operatorname{erf}(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)/erf(b*x)**2,x)
```

```
[Out] -sqrt(pi)*exp(c)/(2*b*erf(b*x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^2x^2+c\right)}}{\operatorname{erf}(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erf(b*x)^2, x)
```

3.51 $\int \frac{e^{c-b^2x^2}}{\mathbf{Erf}(bx)^3}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{\pi}e^c}{4b\text{Erf}(bx)^2}$$

```
[Out] -(E^c*Sqrt[Pi])/(4*b*Erf[b*x]^2)
```

Rubi [A] time = 0.0287007, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6373, 30}

$$-\frac{\sqrt{\pi}e^c}{4b\text{Erf}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)/Erf[b*x]^3,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(4*b*Erf[b*x]^2)
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c-b^2x^2}}{\text{erf}(bx)^3}\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, \text{erf}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}}{4b\text{erf}(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0057237, size = 21, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c}{4b\text{Erf}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)/Erf[b*x]^3,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(4*b*Erf[b*x]^2)
```

Maple [A] time = 0.043, size = 17, normalized size = 0.8

$$-\frac{e^c\sqrt{\pi}}{4\,b\left(\text{Erf}\left(bx\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)/erf(b*x)^3,x)
```

```
[Out] -1/4*exp(c)*Pi^(1/2)/b/erf(b*x)^2
```

Maxima [A] time = 1.0211, size = 22, normalized size = 1.05

$$-\frac{\sqrt{\pi}e^c}{4\,b\,\text{erf}\left(bx\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*e^c/(b*erf(b*x)^2)
```

Fricas [A] time = 2.9458, size = 46, normalized size = 2.19

$$-\frac{\sqrt{\pi}e^c}{4\,b\,\text{erf}\left(bx\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*e^c/(b*erf(b*x)^2)
```

Sympy [A] time = 2.74895, size = 19, normalized size = 0.9

$$-\frac{\sqrt{\pi}e^{c}}{4b\operatorname{erf}^{2}(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)/erf(b*x)**3,x)
```

```
[Out] -sqrt(pi)*exp(c)/(4*b*erf(b*x)**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^{2}x^{2}+c\right)}}{\operatorname{erf}(bx)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erf(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erf(b*x)^3, x)
```

3.52 $\int e^{c-b^2x^2}\textbf{Erf}(bx)^n\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^{n+1}}{2b(n+1)}$$

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^(1 + n))/(2*b*(1 + n))
```

Rubi [A] time = 0.0367038, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6373, 30}

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^{n+1}}{2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)*Erf[b*x]^n,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^(1 + n))/(2*b*(1 + n))
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c-b^2x^2}\text{erf}(bx)^n\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x^n\,dx, x, \text{erf}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\text{erf}(bx)^{1+n}}{2b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0101597, size = 28, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\text{Erf}(bx)^{n+1}}{2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)*Erf[b*x]^n,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^(1 + n))/(2*b*(1 + n))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int e^{-b^2x^2+c} (\text{Erf}(bx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)*erf(b*x)^n,x)
```

```
[Out] int(exp(-b^2*x^2+c)*erf(b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erf}(bx)^n e^{(-b^2x^2+c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)^n*e^(-b^2*x^2 + c), x)
```

Fricas [A] time = 3.31869, size = 65, normalized size = 2.32

$$\frac{\sqrt{\pi}\,\text{erf}(bx)^n\,\text{erf}(bx)\,e^c}{2\,(bn+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x)^n,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erf(b*x)^n*erf(b*x)*e^c/(b*n + b)
```

Sympy [A] time = 13.2319, size = 63, normalized size = 2.25

$$\begin{cases} \tilde{\infty} x e^{c} & \text{for}\: b = 0 \wedge n = -1 \\ 0^{n} x e^{c} & \text{for}\: b = 0 \\ \frac{\sqrt{\pi} e^{c} \log{\left(\operatorname{erf}{\left(b x \right)} \right)}}{2 b} & \text{for}\: n = -1 \\ \frac{\sqrt{\pi} e^{c} \operatorname{erf}{\left(b x \right)} \operatorname{erf}^{n}{\left(b x \right)}}{2 b n + 2 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)*erf(b*x)**n,x)
```

```
[Out] Piecewise((zoo*x*exp(c), Eq(b, 0) & Eq(n, -1)), (0**n*x*exp(c), Eq(b, 0)),
(sqrt(pi)*exp(c)*log(erf(b*x))/(2*b), Eq(n, -1)), (sqrt(pi)*exp(c)*erf(b*x)
*erf(b*x)**n/(2*b*n + 2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx)^n e^{\left(-b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erf(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)^n*e^(-b^2*x^2 + c), x)
```

3.53 $\int e^{c+dx^2}x^5\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=285

$$\frac{be^c\mathrm{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\left(b^2-d\right)^{3/2}}-\frac{be^c\mathrm{Erf}\left(x\sqrt{b^2-d}\right)}{d^3\sqrt{b^2-d}}-\frac{bxe^{c-x^2\left(b^2-d\right)}}{\sqrt{\pi}d^2\left(b^2-d\right)}-\frac{3be^c\mathrm{Erf}\left(x\sqrt{b^2-d}\right)}{8d\left(b^2-d\right)^{5/2}}+\frac{bx^3e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{3bxe^{c-x^2\left(b^2-d\right)}}{4\sqrt{\pi}d\left(b^2-d\right)^2}-x$$

```
[Out] -((b*E^(c - (b^2 - d)*x^2)*x)/((b^2 - d)*d^2*Sqrt[Pi])) + (3*b*E^(c - (b^2
- d)*x^2)*x)/(4*(b^2 - d)^2*d*Sqrt[Pi]) + (b*E^(c - (b^2 - d)*x^2)*x^3)/(2*
(b^2 - d)*d*Sqrt[Pi]) + (E^(c + d*x^2)*Erf[b*x])/d^3 - (E^(c + d*x^2)*x^2*E
rf[b*x])/d^2 + (E^(c + d*x^2)*x^4*Erf[b*x])/(2*d) - (b*E^c*Erf[Sqrt[b^2 - d
]*x])/(Sqrt[b^2 - d]*d^3) + (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*(b^2 - d)^(3/2)
*d^2) - (3*b*E^c*Erf[Sqrt[b^2 - d]*x])/(8*(b^2 - d)^(5/2)*d)
```

Rubi [A] time = 0.435986, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6385, 6382, 2205, 2212}

$$\frac{be^c\mathrm{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\left(b^2-d\right)^{3/2}}-\frac{be^c\mathrm{Erf}\left(x\sqrt{b^2-d}\right)}{d^3\sqrt{b^2-d}}-\frac{bxe^{c-x^2\left(b^2-d\right)}}{\sqrt{\pi}d^2\left(b^2-d\right)}-\frac{3be^c\mathrm{Erf}\left(x\sqrt{b^2-d}\right)}{8d\left(b^2-d\right)^{5/2}}+\frac{bx^3e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{3bxe^{c-x^2\left(b^2-d\right)}}{4\sqrt{\pi}d\left(b^2-d\right)^2}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^5*Erf[b*x],x]
```

```
[Out] -((b*E^(c - (b^2 - d)*x^2)*x)/((b^2 - d)*d^2*Sqrt[Pi])) + (3*b*E^(c - (b^2
- d)*x^2)*x)/(4*(b^2 - d)^2*d*Sqrt[Pi]) + (b*E^(c - (b^2 - d)*x^2)*x^3)/(2*
(b^2 - d)*d*Sqrt[Pi]) + (E^(c + d*x^2)*Erf[b*x])/d^3 - (E^(c + d*x^2)*x^2*E
rf[b*x])/d^2 + (E^(c + d*x^2)*x^4*Erf[b*x])/(2*d) - (b*E^c*Erf[Sqrt[b^2 - d
]*x])/(Sqrt[b^2 - d]*d^3) + (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*(b^2 - d)^(3/2)
*d^2) - (3*b*E^c*Erf[Sqrt[b^2 - d]*x])/(8*(b^2 - d)^(5/2)*d)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int e^{c+dx^2}x^5\text{erf}(bx)\,dx = \frac{e^{c+dx^2}x^4\text{erf}(bx)}{2d} - \frac{2\int e^{c+dx^2}x^3\text{erf}(bx)\,dx}{d} - \frac{b\int e^{c-(b^2-d)x^2}x^4\,dx}{d\sqrt{\pi}}$$

$$= \frac{be^{c-(b^2-d)x^2}x^3}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}x^2\text{erf}(bx)}{d^2} + \frac{e^{c+dx^2}x^4\text{erf}(bx)}{2d} + \frac{2\int e^{c+dx^2}x\text{erf}(bx)\,dx}{d^2} + \frac{(2b)\int e^{c-(b^2-d)x^2}}{d^2\sqrt{\pi}}$$

$$= -\frac{be^{c-(b^2-d)x^2}x}{(b^2-d)d^2\sqrt{\pi}} + \frac{3be^{c-(b^2-d)x^2}x}{4(b^2-d)^2d\sqrt{\pi}} + \frac{be^{c-(b^2-d)x^2}x^3}{2(b^2-d)d\sqrt{\pi}} + \frac{e^{c+dx^2}\text{erf}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\text{erf}(bx)}{d^2} + \frac{e^{c+dx}}{}$$

$$= -\frac{be^{c-(b^2-d)x^2}x}{(b^2-d)d^2\sqrt{\pi}} + \frac{3be^{c-(b^2-d)x^2}x}{4(b^2-d)^2d\sqrt{\pi}} + \frac{be^{c-(b^2-d)x^2}x^3}{2(b^2-d)d\sqrt{\pi}} + \frac{e^{c+dx^2}\text{erf}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\text{erf}(bx)}{d^2} + \frac{e^{c+dx}}{}$$

Mathematica [A] time = 0.413958, size = 138, normalized size = 0.48

$$\frac{e^c\left(\frac{b(20b^2d-8b^4-15d^2)\text{Erfi}\left(x\sqrt{d-b^2}\right)}{(d-b^2)^{5/2}} + \frac{2bdxe^{x^2(d-b^2)}\left(2b^2(dx^2-2)+d(7-2dx^2)\right)}{\sqrt{\pi}(b^2-d)^2} + 4e^{dx^2}\left(d^2x^4-2dx^2+2\right)\text{Erf}(bx)\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^5*Erf[b*x],x]
```

```
[Out] (E^c*((2*b*d*E^((-b^2 + d)*x^2)*x*(d*(7 - 2*d*x^2) + 2*b^2*(-2 + d*x^2)))/(
(b^2 - d)^2*Sqrt[Pi]) + 4*E^(d*x^2)*(2 - 2*d*x^2 + d^2*x^4)*Erf[b*x] + (b*(
-8*b^4 + 20*b^2*d - 15*d^2)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^(5/2)))/(8*d
^3)
```

Maple [A] time = 0.268, size = 312, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\mathrm{Erf}(bx)\,\mathrm{e}^{c}}{b^5}\left(\frac{\mathrm{e}^{dx^2}b^6x^4}{2\,d}-2\,\frac{b^2}{d}\left(1/2\,\frac{b^4x^2\mathrm{e}^{dx^2}}{d}-1/2\,\frac{b^4\mathrm{e}^{dx^2}}{d^2}\right)\right)-\frac{\mathrm{e}^{c}}{\sqrt{\pi}b^5}\left(\frac{b^2}{d}\left(\frac{x^3b^3}{2}\mathrm{e}^{\left(-1+\frac{d}{b^2}\right)b^2x^2}\left(-1+\frac{d}{b^2}\right)^{-1}-\frac{3}{2}\left(\frac{bx}{2}\mathrm{e}^{\left(-1+\frac{d}{b^2}\right)}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^5*erf(b*x),x)
```

```
[Out] (erf(b*x)/b^5*exp(c)*(1/2*exp(d*x^2)*b^6*x^4/d-2/d*b^2*(1/2/d*b^4*x^2*exp(d
*x^2)-1/2/d^2*b^4*exp(d*x^2)))-1/Pi^(1/2)/b^5*exp(c)*(1/d*b^2*(1/2/(-1+d/b^
2)*b^3*x^3*exp((-1+d/b^2)*b^2*x^2)-3/2/(-1+d/b^2)*(1/2/(-1+d/b^2)*b*x*exp((
-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1
/2)*b*x)))+1/d^3*b^6*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)-2/d^
2*b^4*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*Pi^(1/2)/(
1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(d^2x^4e^c-2\,dx^2e^c+2\,e^c\right)\mathrm{erf}\left(bx\right)e^{\left(dx^2\right)}}{2\,d^3}-\frac{-\frac{bd^2x^5e^c\Gamma\left(\frac{5}{2},\left(b^2-d\right)x^2\right)}{2\left(\left(b^2-d\right)x^2\right)^{\frac{5}{2}}}+\frac{bdx^3e^c\Gamma\left(\frac{3}{2},\left(b^2-d\right)x^2\right)}{\left(\left(b^2-d\right)x^2\right)^{\frac{3}{2}}}+\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b^2-d}x\right)e^c}{\sqrt{b^2-d}}}{\sqrt{\pi}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*(d^2*x^4*e^c - 2*d*x^2*e^c + 2*e^c)*erf(b*x)*e^(d*x^2)/d^3 - integrate(
(b*d^2*x^4*e^c - 2*b*d*x^2*e^c + 2*b*e^c)*e^(-b^2*x^2 + d*x^2), x)/(sqrt(pi
)*d^3)
```

Fricas [A] time = 3.24942, size = 545, normalized size = 1.91

$$-\frac{\pi\left(8\,b^{5}-20\,b^{3}d+15\,bd^{2}\right)\sqrt{b^{2}-d}\,\mathrm{erf}\left(\sqrt{b^{2}-d}x\right)e^{c}-4\,\left(\pi\left(b^{6}d^{2}-3\,b^{4}d^{3}+3\,b^{2}d^{4}-d^{5}\right)x^{4}-2\,\pi\left(b^{6}d-3\,b^{4}d^{2}+3\,b^{2}d^{3}\right.\right.}{8\,\pi\left(b^{6}d^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erf(b*x),x, algorithm="fricas")
```

```
[Out] -1/8*(pi*(8*b^5 - 20*b^3*d + 15*b*d^2)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e
^c - 4*(pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^2*d^4 - d^5)*x^4 - 2*pi*(b^6*d - 3*b^
4*d^2 + 3*b^2*d^3 - d^4)*x^2 + 2*pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3))*erf(
b*x)*e^(d*x^2 + c) - 2*sqrt(pi)*(2*(b^5*d^2 - 2*b^3*d^3 + b*d^4)*x^3 - (4*b
^5*d - 11*b^3*d^2 + 7*b*d^3)*x)*e^(-b^2*x^2 + d*x^2 + c))/(pi*(b^6*d^3 - 3*
b^4*d^4 + 3*b^2*d^5 - d^6))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**5*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28484, size = 347, normalized size = 1.22

$$-\frac{\left(2\,dx^{2}-\left(dx^{2}+c\right)^{2}+2\,\left(dx^{2}+c\right)c-c^{2}-2\right)\mathrm{erf}\left(bx\right)e^{\left(dx^{2}+c\right)}}{2\,d^{3}}+\frac{\sqrt{\pi}bd^{2}\left(\frac{2\,\left(2\,b^{2}x^{3}-2\,dx^{3}+3\,x\right)e^{\left(-b^{2}x^{2}+dx^{2}+c\right)}}{b^{4}-2\,b^{2}d+d^{2}}+\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b^{2}-d}x\right)e^{c}}{\left(b^{4}-2\,b^{2}d+d^{2}\right)\sqrt{b^{2}-d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erf(b*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x^2 - (d*x^2 + c)^2 + 2*(d*x^2 + c)*c - c^2 - 2)*erf(b*x)*e^(d*x^
2 + c)/d^3 + 1/8*(sqrt(pi)*b*d^2*(2*(2*b^2*x^3 - 2*d*x^3 + 3*x)*e^(-b^2*x^2
 + d*x^2 + c)/(b^4 - 2*b^2*d + d^2) + 3*sqrt(pi)*erf(-sqrt(b^2 - d)*x)*e^c/
((b^4 - 2*b^2*d + d^2)*sqrt(b^2 - d))) - 4*sqrt(pi)*b*d*(2*x*e^(-b^2*x^2 +
d*x^2 + c)/(b^2 - d) + sqrt(pi)*erf(-sqrt(b^2 - d)*x)*e^c/(b^2 - d)^(3/2))
+ 8*pi*b*erf(-sqrt(b^2 - d)*x)*e^c/sqrt(b^2 - d))/(pi*d^3)
```

3.54 $\int e^{c+dx^2} x^3 \textbf{Erf}(bx)\, dx$

Optimal. Leaf size=155

$$\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\sqrt{b^2-d}}-\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{4d\left(b^2-d\right)^{3/2}}+\frac{bxe^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{\text{Erf}(bx)e^{c+dx^2}}{2d^2}+\frac{x^2\text{Erf}(bx)e^{c+dx^2}}{2d}$$

```
[Out] (b*E^(c - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (E^(c + d*x^2)*Erf[b
*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erf[b*x])/(2*d) + (b*E^c*Erf[Sqrt[b^2 - d
]*x])/(2*Sqrt[b^2 - d]*d^2) - (b*E^c*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)^(3/
2)*d)
```

Rubi [A] time = 0.157916, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6385, 6382, 2205, 2212}

$$\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\sqrt{b^2-d}}-\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{4d\left(b^2-d\right)^{3/2}}+\frac{bxe^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{\text{Erf}(bx)e^{c+dx^2}}{2d^2}+\frac{x^2\text{Erf}(bx)e^{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^3*Erf[b*x],x]
```

```
[Out] (b*E^(c - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (E^(c + d*x^2)*Erf[b
*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erf[b*x])/(2*d) + (b*E^c*Erf[Sqrt[b^2 - d
]*x])/(2*Sqrt[b^2 - d]*d^2) - (b*E^c*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)^(3/
2)*d)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
```

```
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{c+dx^2}x^3\operatorname{erf}(bx)\,dx &= \frac{e^{c+dx^2}x^2\operatorname{erf}(bx)}{2d} - \frac{\int e^{c+dx^2}x\operatorname{erf}(bx)\,dx}{d} - \frac{b\int e^{c-\left(b^2-d\right)x^2}x^2\,dx}{d\sqrt{\pi}} \\
&= \frac{be^{c-\left(b^2-d\right)x^2}x}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{e^{c+dx^2}\operatorname{erf}(bx)}{2d^2} + \frac{e^{c+dx^2}x^2\operatorname{erf}(bx)}{2d} + \frac{b\int e^{c-\left(b^2-d\right)x^2}\,dx}{d^2\sqrt{\pi}} - \frac{b\int e^{c+\left(-b^2+d\right)x^2}\,dx}{2\left(b^2-d\right)d\sqrt{\pi}} \\
&= \frac{be^{c-\left(b^2-d\right)x^2}x}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{e^{c+dx^2}\operatorname{erf}(bx)}{2d^2} + \frac{e^{c+dx^2}x^2\operatorname{erf}(bx)}{2d} + \frac{be^c\operatorname{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d^2} - \frac{be^c\operatorname{erf}\left(\sqrt{b^2-d}x\right)}{4\left(b^2-d\right)^{3/2}d}
\end{aligned}
$$

Mathematica [A] time = 0.278587, size = 99, normalized size = 0.64

$$
\frac{e^c\left(\frac{b\left(3d-2b^2\right)\operatorname{Erfi}\left(x\sqrt{d-b^2}\right)}{\left(d-b^2\right)^{3/2}} + \frac{2bdxe^{x^2\left(d-b^2\right)}}{\sqrt{\pi}\left(b^2-d\right)} + 2e^{dx^2}\left(dx^2-1\right)\operatorname{Erf}(bx)\right)}{4d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^3*Erf[b*x],x]
```

```
[Out] (E^c*((2*b*d*E^((-b^2 + d)*x^2)*x)/((b^2 - d)*Sqrt[Pi]) + 2*E^(d*x^2)*(-1 +
 d*x^2)*Erf[b*x] + (b*(-2*b^2 + 3*d)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^(3/
```

```
2)))/(4*d^2)
```

Maple [A] time = 0.35, size = 168, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\mathrm{Erf}\,(bx)\,\mathrm{e}^{c}}{b^{3}}\left(\frac{b^{4}x^{2}\mathrm{e}^{dx^{2}}}{2\,d}-\frac{b^{4}\mathrm{e}^{dx^{2}}}{2\,d^{2}}\right)-\frac{\mathrm{e}^{c}}{b^{3}\sqrt{\pi}}\left(\frac{b^{2}}{d}\left(\frac{bx}{2}\mathrm{e}^{\left(-1+\frac{d}{b^{2}}\right)b^{2}x^{2}}\left(-1+\frac{d}{b^{2}}\right)^{-1}-\frac{\sqrt{\pi}}{4}\mathrm{Erf}\left(\sqrt{1-\frac{d}{b^{2}}}bx\right)\left(-1+\frac{d}{b^{2}}\right)^{-1}\frac{1}{\sqrt{1-\cdots}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^3*erf(b*x),x)
```

```
[Out] (erf(b*x)/b^3*exp(c)*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2))-1/Pi
^(1/2)/b^3*exp(c)*(1/d*b^2*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/
(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))-1/2/d^2*b^4*P
i^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(dx^{2}e^{c}-e^{c}\right)\mathrm{erf}\,(bx)\,e^{\left(dx^{2}\right)}}{2\,d^{2}}-\frac{-\frac{bdx^{3}e^{c}\Gamma\left(\frac{3}{2},\left(b^{2}-d\right)x^{2}\right)}{2\left(\left(b^{2}-d\right)x^{2}\right)^{\frac{3}{2}}}-\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b^{2}-d}x\right)e^{c}}{2\,\sqrt{b^{2}-d}}}{\sqrt{\pi}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2*e^c - e^c)*erf(b*x)*e^(d*x^2)/d^2 - integrate((b*d*x^2*e^c - b*e
^c)*e^(-b^2*x^2 + d*x^2), x)/(sqrt(pi)*d^2)
```

Fricas [A] time = 3.22417, size = 328, normalized size = 2.12

$$\frac{\pi\left(2\,b^{3}-3\,bd\right)\sqrt{b^{2}-d}\,\mathrm{erf}\left(\sqrt{b^{2}-d}x\right)e^{c}+2\,\sqrt{\pi}\left(b^{3}d-bd^{2}\right)xe^{\left(-b^{2}x^{2}+dx^{2}+c\right)}+2\left(\pi\left(b^{4}d-2\,b^{2}d^{2}+d^{3}\right)x^{2}-\pi\left(b^{4}-2\,b^{2}d+\cdots\right.\right.}{4\,\pi\left(b^{4}d^{2}-2\,b^{2}d^{3}+d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(pi*(2*b^3 - 3*b*d)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e^c + 2*sqrt(pi)
*(b^3*d - b*d^2)*x*e^(-b^2*x^2 + d*x^2 + c) + 2*(pi*(b^4*d - 2*b^2*d^2 + d^
3)*x^2 - pi*(b^4 - 2*b^2*d + d^2))*erf(b*x)*e^(d*x^2 + c))/(pi*(b^4*d^2 - 2
*b^2*d^3 + d^4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**3*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31417, size = 169, normalized size = 1.09

$$\frac{\left(dx^2-1\right)\operatorname{erf}\left(bx\right)e^{\left(dx^2+c\right)}}{2\,d^2}+\frac{\sqrt{\pi}bd\left(\frac{2\,xe^{\left(-b^2x^2+dx^2+c\right)}}{b^2-d}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b^2-d}x\right)e^c}{\left(b^2-d\right)^{\frac{3}{2}}}\right)-\frac{2\,\pi b\operatorname{erf}\left(-\sqrt{b^2-d}x\right)e^c}{\sqrt{b^2-d}}}{4\,\pi d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 - 1)*erf(b*x)*e^(d*x^2 + c)/d^2 + 1/4*(sqrt(pi)*b*d*(2*x*e^(-b^2
*x^2 + d*x^2 + c)/(b^2 - d) + sqrt(pi)*erf(-sqrt(b^2 - d)*x)*e^c/(b^2 - d)^
(3/2)) - 2*pi*b*erf(-sqrt(b^2 - d)*x)*e^c/sqrt(b^2 - d))/(pi*d^2)
```

3.55 $\int e^{c+dx^2} x \mathbf{Erf}(bx)\, dx$

Optimal. Leaf size=57

$$\frac{\text{Erf}(bx)e^{c+dx^2}}{2d} - \frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}$$

```
[Out] (E^(c + d*x^2)*Erf[b*x])/(2*d) - (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 -
 d]*d)
```

Rubi [A] time = 0.0387532, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6382, 2205}

$$\frac{\text{Erf}(bx)e^{c+dx^2}}{2d} - \frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x*Erf[b*x],x]
```

```
[Out] (E^(c + d*x^2)*Erf[b*x])/(2*d) - (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 -
 d]*d)
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int e^{c+dx^2}x\text{erf}(bx)\,dx = \frac{e^{c+dx^2}\text{erf}(bx)}{2d} - \frac{b\int e^{c-(b^2-d)x^2}\,dx}{d\sqrt{\pi}}$$
$$= \frac{e^{c+dx^2}\text{erf}(bx)}{2d} - \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d}$$

Mathematica [A] time = 0.0381589, size = 51, normalized size = 0.89

$$\frac{e^c\left(e^{dx^2}\text{Erf}(bx) - \frac{b\text{Erfi}\left(x\sqrt{d-b^2}\right)}{\sqrt{d-b^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x*Erf[b*x],x]
```

```
[Out] (E^c*(E^(d*x^2)*Erf[b*x] - (b*Erfi[Sqrt[-b^2 + d]*x])/Sqrt[-b^2 + d]))/(2*d
)
```

Maple [A] time = 0.333, size = 67, normalized size = 1.2

$$\frac{1}{b}\left(\frac{\text{Erf}\,(bx)\,b}{2\,d}\text{e}^{\frac{b^2dx^2+b^2c}{b^2}} - \frac{b\text{e}^c}{2\,d}\text{Erf}\left(\sqrt{1-\frac{d}{b^2}}bx\right)\frac{1}{\sqrt{1-\frac{d}{b^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x*erf(b*x),x)
```

```
[Out] (1/2*erf(b*x)*b*exp((b^2*d*x^2+b^2*c)/b^2)/d-1/2*b/d*exp(c)/(1-d/b^2)^(1/2)
*erf((1-d/b^2)^(1/2)*b*x))/b
```

Maxima [A] time = 1.0571, size = 63, normalized size = 1.11

$$-\frac{b\,\text{erf}\left(\sqrt{b^2-d}x\right)e^c}{2\,\sqrt{b^2-d}d} + \frac{\text{erf}\,(bx)\,e^{\left(dx^2+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erf(b*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*erf(sqrt(b^2 - d)*x)*e^c/(sqrt(b^2 - d)*d) + 1/2*erf(b*x)*e^(d*x^2 +
 c)/d
```

Fricas [A] time = 2.9042, size = 132, normalized size = 2.32

$$-\frac{\sqrt{b^2-d}b\operatorname{erf}\left(\sqrt{b^2-d}x\right)e^c-\left(b^2-d\right)\operatorname{erf}(bx)e^{\left(dx^2+c\right)}}{2\left(b^2d-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erf(b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(b^2 - d)*b*erf(sqrt(b^2 - d)*x)*e^c - (b^2 - d)*erf(b*x)*e^(d*x^
2 + c))/(b^2*d - d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c\int xe^{dx^2}\operatorname{erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x*erf(b*x),x)
```

```
[Out] exp(c)*Integral(x*exp(d*x**2)*erf(b*x), x)
```

Giac [A] time = 1.22563, size = 65, normalized size = 1.14

$$\frac{b\operatorname{erf}\left(-\sqrt{b^2-d}x\right)e^c}{2\sqrt{b^2-d}d}+\frac{\operatorname{erf}(bx)e^{\left(dx^2+c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/2*b*erf(-sqrt(b^2 - d)*x)*e^c/(sqrt(b^2 - d)*d) + 1/2*erf(b*x)*e^(d*x^2 +
 c)/d
```

3.56 $\int \frac{e^{c+dx^2}\mathbf{Erf}(bx)}{x}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{\text{Erf}(bx)e^{c+dx^2}}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + d*x^2)*Erf[b*x])/x, x]
```

Rubi [A] time = 0.0399124, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + d*x^2)*Erf[b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx$$

Mathematica [A] time = 0.145396, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[b*x])/x,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[b*x])/x, x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{e^{d x^2+c} \mathrm{Erf}(b x)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x)/x,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx) e^{\left(dx^2+c\right)}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erf}(bx) e^{\left(dx^2+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(d*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int \frac{e^{d x^{2}} \operatorname{erf}(b x)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x)/x,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(dx^2+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x, x)
```

3.57 $\int \frac{e^{c+dx^2}\mathbf{Erf}(bx)}{x^3}\,dx$

Optimal. Leaf size=100

$$d\text{Unintegrable}\left(\frac{\text{Erf}(bx)e^{c+dx^2}}{x},x\right)+be^c\left(-\sqrt{b^2-d}\right)\text{Erf}\left(x\sqrt{b^2-d}\right)-\frac{be^{c-x^2\left(b^2-d\right)}}{\sqrt{\pi}x}-\frac{\text{Erf}(bx)e^{c+dx^2}}{2x^2}$$

```
[Out] -((b*E^(c - (b^2 - d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)*Erf[b*x])/(2*x^2
) - b*Sqrt[b^2 - d]*E^c*Erf[Sqrt[b^2 - d]*x] + d*Unintegrable[(E^(c + d*x^2
)*Erf[b*x])/x, x]
```

Rubi [A] time = 0.15383, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[b*x])/x^3,x]
```

```
[Out] -((b*E^(c - (b^2 - d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)*Erf[b*x])/(2*x^2
) - b*Sqrt[b^2 - d]*E^c*Erf[Sqrt[b^2 - d]*x] + d*Defer[Int][(E^(c + d*x^2)*
Erf[b*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^3}\,dx &= -\frac{e^{c+dx^2}\text{erf}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx+\frac{b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{c-\left(b^2-d\right)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\text{erf}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx-\frac{\left(2b\left(b^2-d\right)\right)\int e^{c+\left(-b^2+d\right)x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{c-\left(b^2-d\right)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\text{erf}(bx)}{2x^2}-b\sqrt{b^2-d}e^c\text{erf}\left(\sqrt{b^2-d}x\right)+d\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.195235, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\mathrm{Erf}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^3, x]
```

Maple [A] time = 0.293, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\mathrm{Erf}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x)/x^3,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}\,(bx)\,e^{\left(dx^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}\,(bx)\,e^{\left(dx^2+c\right)}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(d*x^2 + c)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int \frac{e^{dx^{2}}\operatorname{erf}{\left(bx\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x)/x**3,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left(bx\right)e^{\left(dx^{2}+c\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^3, x)
```

3.58 $\int \frac{e^{c+dx^2}\mathbf{Erf}(bx)}{x^5}\,dx$

Optimal. Leaf size=230

$$\frac{1}{2}d^2\text{Unintegrable}\left(\frac{\text{Erf}(bx)e^{c+dx^2}}{x},x\right)-\frac{1}{2}be^cd\sqrt{b^2-d}\text{Erf}\left(x\sqrt{b^2-d}\right)+\frac{1}{3}be^c\left(b^2-d\right)^{3/2}\text{Erf}\left(x\sqrt{b^2-d}\right)-\frac{bde^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}x}$$

```
[Out] -(b*E^(c - (b^2 - d)*x^2))/(6*Sqrt[Pi]*x^3) + (b*(b^2 - d)*E^(c - (b^2 - d)
*x^2))/(3*Sqrt[Pi]*x) - (b*d*E^(c - (b^2 - d)*x^2))/(2*Sqrt[Pi]*x) - (E^(c
+ d*x^2)*Erf[b*x])/(4*x^4) - (d*E^(c + d*x^2)*Erf[b*x])/(4*x^2) + (b*(b^2 -
 d)^(3/2)*E^c*Erf[Sqrt[b^2 - d]*x])/3 - (b*Sqrt[b^2 - d]*d*E^c*Erf[Sqrt[b^2
 - d]*x])/2 + (d^2*Unintegrable[(E^(c + d*x^2)*Erf[b*x])/x, x])/2
```

Rubi [A] time = 0.341617, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[b*x])/x^5,x]
```

```
[Out] -(b*E^(c - (b^2 - d)*x^2))/(6*Sqrt[Pi]*x^3) + (b*(b^2 - d)*E^(c - (b^2 - d)
*x^2))/(3*Sqrt[Pi]*x) - (b*d*E^(c - (b^2 - d)*x^2))/(2*Sqrt[Pi]*x) - (E^(c
+ d*x^2)*Erf[b*x])/(4*x^4) - (d*E^(c + d*x^2)*Erf[b*x])/(4*x^2) + (b*(b^2 -
 d)^(3/2)*E^c*Erf[Sqrt[b^2 - d]*x])/3 - (b*Sqrt[b^2 - d]*d*E^c*Erf[Sqrt[b^2
 - d]*x])/2 + (d^2*Defer[Int][(E^(c + d*x^2)*Erf[b*x])/x, x])/2
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = -\frac{e^{c+dx^2}\text{erf}(bx)}{4x^4} + \frac{1}{2}d\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^3}\,dx + \frac{b\int \frac{e^{c-(b^2-d)x^2}}{x^4}\,dx}{2\sqrt{\pi}}$$

$$= -\frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} - \frac{e^{c+dx^2}\text{erf}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erf}(bx)}{4x^2} + \frac{1}{2}d^2\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx - \frac{\left(b\left(b^2-d\right)\right)\int \frac{e^{c+(-b^2+d)x^2}}{x^2}}{3\sqrt{\pi}}$$

$$= -\frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} + \frac{b\left(b^2-d\right)e^{c-(b^2-d)x^2}}{3\sqrt{\pi}x} - \frac{bde^{c-(b^2-d)x^2}}{2\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erf}(bx)}{4x^2} + \frac{1}{2}d^2\int \frac{e^{c+d}}{}$$

$$= -\frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} + \frac{b\left(b^2-d\right)e^{c-(b^2-d)x^2}}{3\sqrt{\pi}x} - \frac{bde^{c-(b^2-d)x^2}}{2\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erf}(bx)}{4x^2} + \frac{1}{3}b\left(b^2-d\right)$$

Mathematica [A] time = 0.248862, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^5,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^5, x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{Erf}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x)/x^5,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(d*x^2 + c)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^5, x)
```

3.59 $\int e^{c+dx^2} x^4 \mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=185

$$\frac{3\text{Unintegrable}\left(\text{Erf}(bx)e^{c+dx^2},x\right)}{4d^2}-\frac{3be^{c-x^2\left(b^2-d\right)}}{4\sqrt{\pi}d^2\left(b^2-d\right)}+\frac{bx^2e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)^2}-\frac{3x\text{Erf}(bx)e^{c+dx^2}}{4d^2}+\frac{x^3\text{Erf}(bx)}{2d}$$

```
[Out] (-3*b*E^(c - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) + (b*E^(c - (b^2 -
d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) + (b*E^(c - (b^2 - d)*x^2)*x^2)/(2*(b^2
 - d)*d*Sqrt[Pi]) - (3*E^(c + d*x^2)*x*Erf[b*x])/(4*d^2) + (E^(c + d*x^2)*x
^3*Erf[b*x])/(2*d) + (3*Unintegrable[E^(c + d*x^2)*Erf[b*x], x])/(4*d^2)
```

Rubi [A] time = 0.243807, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^4\text{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^4*Erf[b*x],x]
```

```
[Out] (-3*b*E^(c - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) + (b*E^(c - (b^2 -
d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) + (b*E^(c - (b^2 - d)*x^2)*x^2)/(2*(b^2
 - d)*d*Sqrt[Pi]) - (3*E^(c + d*x^2)*x*Erf[b*x])/(4*d^2) + (E^(c + d*x^2)*x
^3*Erf[b*x])/(2*d) + (3*Defer[Int][E^(c + d*x^2)*Erf[b*x], x])/(4*d^2)
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2}x^4\text{erf}(bx)\,dx &= \frac{e^{c+dx^2}x^3\text{erf}(bx)}{2d}-\frac{3\int e^{c+dx^2}x^2\text{erf}(bx)\,dx}{2d}-\frac{b\int e^{c-\left(b^2-d\right)x^2}x^3\,dx}{d\sqrt{\pi}}\\ &= \frac{be^{c-\left(b^2-d\right)x^2}x^2}{2\left(b^2-d\right)d\sqrt{\pi}}-\frac{3e^{c+dx^2}x\text{erf}(bx)}{4d^2}+\frac{e^{c+dx^2}x^3\text{erf}(bx)}{2d}+\frac{3\int e^{c+dx^2}\text{erf}(bx)\,dx}{4d^2}+\frac{(3b)\int e^{c-\left(b^2-d\right)x^2}x\,d}{2d^2\sqrt{\pi}}\\ &= -\frac{3be^{c-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}}+\frac{be^{c-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}}+\frac{be^{c-\left(b^2-d\right)x^2}x^2}{2\left(b^2-d\right)d\sqrt{\pi}}-\frac{3e^{c+dx^2}x\text{erf}(bx)}{4d^2}+\frac{e^{c+dx^2}x^3\text{erf}(bx)}{2d}+\frac{3}{}\end{aligned}$$

Mathematica [A] time = 0.289221, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^4\text{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^4*Erf[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^4*Erf[b*x], x]
```

Maple [A] time = 0.142, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^4\text{Erf}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^4*erf(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^4*erf(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\text{erf}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erf(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erf(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\,\text{erf}\,(bx)\,e^{\left(dx^2+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*erf(b*x)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**4*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erf}(bx) e^{\left(dx^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erf(b*x)*e^(d*x^2 + c), x)
```

3.60 $\int e^{c+dx^2} x^2 \mathbf{Erf}(bx)\, dx$

Optimal. Leaf size=83

$$-\frac{\text{Unintegrable}\left(\text{Erf}(bx)e^{c+dx^2},x\right)}{2d}+\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{x\text{Erf}(bx)e^{c+dx^2}}{2d}$$

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) + (E^(c + d*x^2)*x*Erf[b
*x])/(2*d) - Unintegrable[E^(c + d*x^2)*Erf[b*x], x]/(2*d)
```

Rubi [A] time = 0.0948577, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2} x^2 \text{Erf}(bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^2*Erf[b*x],x]
```

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) + (E^(c + d*x^2)*x*Erf[b
*x])/(2*d) - Defer[Int][E^(c + d*x^2)*Erf[b*x], x]/(2*d)
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2} x^2 \text{erf}(bx)\, dx &= \frac{e^{c+dx^2} x\text{erf}(bx)}{2d}-\frac{\int e^{c+dx^2}\text{erf}(bx)\, dx}{2d}-\frac{b\int e^{c-\left(b^2-d\right)x^2}x\, dx}{d\sqrt{\pi}}\\ &=\frac{be^{c-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)d\sqrt{\pi}}+\frac{e^{c+dx^2}x\text{erf}(bx)}{2d}-\frac{\int e^{c+dx^2}\text{erf}(bx)\, dx}{2d}\end{aligned}$$

Mathematica [A] time = 0.215184, size = 0, normalized size = 0.

$$\int e^{c+dx^2} x^2 \text{Erf}(bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^2*Erf[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^2*Erf[b*x], x]
```

Maple [A] time = 0.231, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^2\mathrm{Erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^2*erf(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^2*erf(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{erf}(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erf(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erf(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\,\mathrm{erf}(bx)\,e^{\left(dx^2+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*erf(b*x)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int x^{2}e^{dx^{2}}\operatorname{erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**2*erf(b*x),x)
```

```
[Out] exp(c)*Integral(x**2*exp(d*x**2)*erf(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\operatorname{erf}(bx)\,e^{\left(dx^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erf(b*x)*e^(d*x^2 + c), x)
```

3.61 $\int e^{c+dx^2}\text{Erf}(bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\text{Erf}(bx)e^{c+dx^2},x\right)$$

```
[Out] Unintegrable[E^(c + d*x^2)*Erf[b*x], x]
```

Rubi [A] time = 0.0146927, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int e^{c+dx^2}\text{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*Erf[b*x],x]
```

```
[Out] Defer[Int][E^(c + d*x^2)*Erf[b*x], x]
```

Rubi steps

$$\int e^{c+dx^2}\text{erf}(bx)\,dx = \int e^{c+dx^2}\text{erf}(bx)\,dx$$

Mathematica [A] time = 0.0279826, size = 0, normalized size = 0.

$$\int e^{c+dx^2}\text{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*Erf[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*Erf[b*x], x]
```

Maple [A] time = 0.112, size = 0, normalized size = 0.

$$\int \mathrm{e}^{d x^{2}+c} \mathrm{Erf}\left(b x\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}\left(bx\right) e^{\left(dx^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{erf}\left(bx\right) e^{\left(dx^{2}+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int e^{dx^{2}} \operatorname{erf}\left(bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x),x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c), x)
```

3.62 $\int \frac{e^{c+dx^2}\mathbf{Erf}(bx)}{x^2}\,dx$

Optimal. Leaf size=61

$$2d\text{Unintegrable}\left(\text{Erf}(bx)e^{c+dx^2},x\right)+\frac{be^c\text{ExpIntegralEi}\left(x^2\left(-\left(b^2-d\right)\right)\right)}{\sqrt{\pi}}-\frac{\text{Erf}(bx)e^{c+dx^2}}{x}$$

```
[Out] -((E^(c + d*x^2)*Erf[b*x])/x) + (b*E^c*ExpIntegralEi[-((b^2 - d)*x^2)])/Sqr
t[Pi] + 2*d*Unintegrable[E^(c + d*x^2)*Erf[b*x], x]
```

Rubi [A] time = 0.116646, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[b*x])/x^2,x]
```

```
[Out] -((E^(c + d*x^2)*Erf[b*x])/x) + (b*E^c*ExpIntegralEi[-((b^2 - d)*x^2)])/Sqr
t[Pi] + 2*d*Defer[Int][E^(c + d*x^2)*Erf[b*x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^2}\,dx &= -\frac{e^{c+dx^2}\text{erf}(bx)}{x}+(2d)\int e^{c+dx^2}\text{erf}(bx)\,dx+\frac{(2b)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+dx^2}\text{erf}(bx)}{x}+\frac{be^c\text{Ei}\left(-\left(b^2-d\right)x^2\right)}{\sqrt{\pi}}+(2d)\int e^{c+dx^2}\text{erf}(bx)\,dx\end{aligned}$$

Mathematica [A] time = 0.202733, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^2,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^2, x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\mathrm{Erf}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x)/x^2,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(d*x^2 + c)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int\frac{e^{dx^{2}}\operatorname{erf}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erf}\left(bx\right)e^{\left(dx^{2}+c\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^2, x)
```

3.63 $\int \frac{e^{c+dx^2}\mathbf{Erf}(bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$\frac{4}{3}d^2\text{Unintegrable}\left(\text{Erf}(bx)e^{c+dx^2},x\right)+\frac{2be^cd\text{ExpIntegralEi}\left(x^2\left(-\left(b^2-d\right)\right)\right)}{3\sqrt{\pi}}-\frac{be^c\left(b^2-d\right)\text{ExpIntegralEi}\left(x^2\left(-\left(b^2-\right.\right.\right.}{3\sqrt{\pi}}$$

```
[Out] -(b*E^(c - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + d*x^2)*Erf[b*x])/(3*x
^3) - (2*d*E^(c + d*x^2)*Erf[b*x])/(3*x) - (b*(b^2 - d)*E^c*ExpIntegralEi[-
((b^2 - d)*x^2)])/(3*Sqrt[Pi]) + (2*b*d*E^c*ExpIntegralEi[-((b^2 - d)*x^2)]
)/(3*Sqrt[Pi]) + (4*d^2*Unintegrable[E^(c + d*x^2)*Erf[b*x], x])/3
```

Rubi [A] time = 0.28526, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[b*x])/x^4,x]
```

```
[Out] -(b*E^(c - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + d*x^2)*Erf[b*x])/(3*x
^3) - (2*d*E^(c + d*x^2)*Erf[b*x])/(3*x) - (b*(b^2 - d)*E^c*ExpIntegralEi[-
((b^2 - d)*x^2)])/(3*Sqrt[Pi]) + (2*b*d*E^c*ExpIntegralEi[-((b^2 - d)*x^2)]
)/(3*Sqrt[Pi]) + (4*d^2*Defer[Int][E^(c + d*x^2)*Erf[b*x], x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^4}\,dx &= -\frac{e^{c+dx^2}\text{erf}(bx)}{3x^3}+\frac{1}{3}(2d)\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^2}\,dx+\frac{(2b)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{c-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\text{erf}(bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erf}(bx)}{3x}+\frac{1}{3}\left(4d^2\right)\int e^{c+dx^2}\text{erf}(bx)\,dx-\frac{\left(2b\left(b^2-d\right)\right)\int \frac{e^{c+(\cdots}}{\cdots}}{3\sqrt{\pi}}\\
&= -\frac{be^{c-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\text{erf}(bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erf}(bx)}{3x}-\frac{b\left(b^2-d\right)e^c\text{Ei}\left(-\left(b^2-d\right)x^2\right)}{3\sqrt{\pi}}+\frac{2bde^c\text{Ei}\left(-\left(b^2-\cdots\right.\right.}{3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.305582, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^4,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[b*x])/x^4, x]
```

Maple [A] time = 0.315, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{Erf}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x)/x^4,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erf}(bx)\,e^{\left(dx^2+c\right)}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(d*x^2 + c)/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^c \int \frac{e^{dx^2} \operatorname{erf}(bx)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x)/x**4,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{(dx^2+c)}}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(d*x^2 + c)/x^4, x)
```

3.64 $\int e^{c+b^2x^2}x^5\textbf{Erf}(bx)\,dx$

Optimal. Leaf size=118

$$\frac{x^4e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{x^2e^{b^2x^2+c}\text{Erf}(bx)}{b^4}+\frac{e^{b^2x^2+c}\text{Erf}(bx)}{b^6}+\frac{2e^cx^3}{3\sqrt{\pi}b^3}-\frac{2e^cx}{\sqrt{\pi}b^5}-\frac{e^cx^5}{5\sqrt{\pi}b}$$

```
[Out] (-2*E^c*x)/(b^5*Sqrt[Pi]) + (2*E^c*x^3)/(3*b^3*Sqrt[Pi]) - (E^c*x^5)/(5*b*S
qrt[Pi]) + (E^(c + b^2*x^2)*Erf[b*x])/b^6 - (E^(c + b^2*x^2)*x^2*Erf[b*x])/
b^4 + (E^(c + b^2*x^2)*x^4*Erf[b*x])/(2*b^2)
```

Rubi [A] time = 0.141572, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6385, 6382, 8, 12, 30}

$$\frac{x^4e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{x^2e^{b^2x^2+c}\text{Erf}(bx)}{b^4}+\frac{e^{b^2x^2+c}\text{Erf}(bx)}{b^6}+\frac{2e^cx^3}{3\sqrt{\pi}b^3}-\frac{2e^cx}{\sqrt{\pi}b^5}-\frac{e^cx^5}{5\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^5*Erf[b*x],x]
```

```
[Out] (-2*E^c*x)/(b^5*Sqrt[Pi]) + (2*E^c*x^3)/(3*b^3*Sqrt[Pi]) - (E^c*x^5)/(5*b*S
qrt[Pi]) + (E^(c + b^2*x^2)*Erf[b*x])/b^6 - (E^(c + b^2*x^2)*x^2*Erf[b*x])/
b^4 + (E^(c + b^2*x^2)*x^4*Erf[b*x])/(2*b^2)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx &= \frac{e^{c+b^2x^2}x^4\text{erf}(bx)}{2b^2}-\frac{2\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx}{b^2}-\frac{\int e^cx^4\,dx}{b\sqrt{\pi}}\\
&= -\frac{e^{c+b^2x^2}x^2\text{erf}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erf}(bx)}{2b^2}+\frac{2\int e^{c+b^2x^2}x\text{erf}(bx)\,dx}{b^4}+\frac{2\int e^cx^2\,dx}{b^3\sqrt{\pi}}-\frac{e^c\int x^4\,dx}{b\sqrt{\pi}}\\
&= -\frac{e^cx^5}{5b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erf}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erf}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erf}(bx)}{2b^2}-\frac{2\int e^c\,dx}{b^5\sqrt{\pi}}+\frac{(2e^c)\int x^2\,dx}{b^3\sqrt{\pi}}\\
&= -\frac{2e^cx}{b^5\sqrt{\pi}}+\frac{2e^cx^3}{3b^3\sqrt{\pi}}-\frac{e^cx^5}{5b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erf}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erf}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erf}(bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0460526, size = 73, normalized size = 0.62

$$\frac{e^c\left(15\sqrt{\pi}e^{b^2x^2}\left(b^4x^4-2b^2x^2+2\right)\text{Erf}(bx)-6b^5x^5+20b^3x^3-60bx\right)}{30\sqrt{\pi}b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^5*Erf[b*x],x]
```

```
[Out] (E^c*(-60*b*x + 20*b^3*x^3 - 6*b^5*x^5 + 15*E^(b^2*x^2)*Sqrt[Pi]*(2 - 2*b^2
*x^2 + b^4*x^4)*Erf[b*x]))/(30*b^6*Sqrt[Pi])
```

Maple [A] time = 0.131, size = 88, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\mathrm{Erf}\,(bx)\,\mathrm{e}^{c}}{b^{5}}\left(\frac{\mathrm{e}^{b^{2}x^{2}}b^{4}x^{4}}{2}-b^{2}x^{2}\mathrm{e}^{b^{2}x^{2}}+\mathrm{e}^{b^{2}x^{2}}\right)-\frac{\mathrm{e}^{c}}{\sqrt{\pi}b^{5}}\left(\frac{b^{5}x^{5}}{5}-\frac{2\,x^{3}b^{3}}{3}+2\,bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^5*erf(b*x),x)
```

```
[Out] (erf(b*x)/b^5*exp(c)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2*exp(b^2*x^2)+exp(b^2
*x^2))-1/Pi^(1/2)/b^5*exp(c)*(1/5*b^5*x^5-2/3*x^3*b^3+2*b*x))/b
```

Maxima [A] time = 1.00364, size = 111, normalized size = 0.94

$$-\frac{6\,b^{5}x^{5}e^{c}-20\,b^{3}x^{3}e^{c}-15\left(\sqrt{\pi}b^{4}x^{4}e^{c}-2\,\sqrt{\pi}b^{2}x^{2}e^{c}+2\,\sqrt{\pi}e^{c}\right)\mathrm{erf}\,(bx)\,e^{\left(b^{2}x^{2}\right)}+60\,bxe^{c}}{30\,\sqrt{\pi}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erf(b*x),x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^5*x^5*e^c - 20*b^3*x^3*e^c - 15*(sqrt(pi)*b^4*x^4*e^c - 2*sqrt(p
i)*b^2*x^2*e^c + 2*sqrt(pi)*e^c)*erf(b*x)*e^(b^2*x^2) + 60*b*x*e^c)/(sqrt(p
i)*b^6)
```

Fricas [A] time = 3.02796, size = 177, normalized size = 1.5

$$\frac{15\left(2\,\pi+\pi b^{4}x^{4}-2\,\pi b^{2}x^{2}\right)\mathrm{erf}\,(bx)\,e^{\left(b^{2}x^{2}+c\right)}-2\,\sqrt{\pi}\left(3\,b^{5}x^{5}-10\,b^{3}x^{3}+30\,bx\right)e^{c}}{30\,\pi b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erf(b*x),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(2*pi + pi*b^4*x^4 - 2*pi*b^2*x^2)*erf(b*x)*e^(b^2*x^2 + c) - 2*sq
rt(pi)*(3*b^5*x^5 - 10*b^3*x^3 + 30*b*x)*e^c)/(pi*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**5*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27515, size = 139, normalized size = 1.18

$$-\frac{\left(2\,b^2x^2-\left(b^2x^2+c\right)^2+2\left(b^2x^2+c\right)c-c^2-2\right)\operatorname{erf}(bx)\,e^{\left(b^2x^2+c\right)}}{2\,b^6}-\frac{3\,\sqrt{\pi}b^4x^5e^c-10\,\sqrt{\pi}b^2x^3e^c+30\,\sqrt{\pi}xe^c}{15\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erf(b*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*x^2 - (b^2*x^2 + c)^2 + 2*(b^2*x^2 + c)*c - c^2 - 2)*erf(b*x)*e
^(b^2*x^2 + c)/b^6 - 1/15*(3*sqrt(pi)*b^4*x^5*e^c - 10*sqrt(pi)*b^2*x^3*e^c
 + 30*sqrt(pi)*x*e^c)/(pi*b^5)
```

3.65 $\int e^{c+b^2x^2}x^3\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=79

$$\frac{x^2e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{2b^4}+\frac{e^cx}{\sqrt{\pi}b^3}-\frac{e^cx^3}{3\sqrt{\pi}b}$$

```
[Out] (E^c*x)/(b^3*Sqrt[Pi]) - (E^c*x^3)/(3*b*Sqrt[Pi]) - (E^(c + b^2*x^2)*Erf[b*
x])/(2*b^4) + (E^(c + b^2*x^2)*x^2*Erf[b*x])/(2*b^2)
```

Rubi [A] time = 0.0799707, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6385, 6382, 8, 12, 30}

$$\frac{x^2e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{2b^4}+\frac{e^cx}{\sqrt{\pi}b^3}-\frac{e^cx^3}{3\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^3*Erf[b*x],x]
```

```
[Out] (E^c*x)/(b^3*Sqrt[Pi]) - (E^c*x^3)/(3*b*Sqrt[Pi]) - (E^(c + b^2*x^2)*Erf[b*
x])/(2*b^4) + (E^(c + b^2*x^2)*x^2*Erf[b*x])/(2*b^2)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{c+b^2x^2}x^3\operatorname{erf}(bx)\,dx &= \frac{e^{c+b^2x^2}x^2\operatorname{erf}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}x\operatorname{erf}(bx)\,dx}{b^2}-\frac{\int e^cx^2\,dx}{b\sqrt{\pi}}\\
&=-\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\operatorname{erf}(bx)}{2b^2}+\frac{\int e^c\,dx}{b^3\sqrt{\pi}}-\frac{e^c\int x^2\,dx}{b\sqrt{\pi}}\\
&=\frac{e^cx}{b^3\sqrt{\pi}}-\frac{e^cx^3}{3b\sqrt{\pi}}-\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\operatorname{erf}(bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0348345, size = 57, normalized size = 0.72

$$\frac{e^c\left(3\sqrt{\pi}e^{b^2x^2}\left(b^2x^2-1\right)\operatorname{Erf}(bx)-2b^3x^3+6bx\right)}{6\sqrt{\pi}b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^3*Erf[b*x],x]
```

```
[Out] (E^c*(6*b*x - 2*b^3*x^3 + 3*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erf[b*x]))/
(6*b^4*Sqrt[Pi])
```

Maple [A] time = 0.218, size = 66, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\operatorname{Erf}(bx)\,e^c}{b^3}\left(\frac{b^2x^2e^{b^2x^2}}{2}-\frac{e^{b^2x^2}}{2}\right)-\frac{e^c}{b^3\sqrt{\pi}}\left(\frac{x^3b^3}{3}-bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^3*erf(b*x),x)
```

```
[Out] (erf(b*x)/b^3*exp(c)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp(b^2*x^2))-1/Pi^(1/2)
/b^3*exp(c)*(1/3*x^3*b^3-b*x))/b
```

Maxima [A] time = 1.01245, size = 80, normalized size = 1.01

$$-\frac{2\,b^3x^3e^c - 3\left(\sqrt{\pi}b^2x^2e^c - \sqrt{\pi}e^c\right)\operatorname{erf}(bx)\,e^{\left(b^2x^2\right)} - 6\,bxe^c}{6\,\sqrt{\pi}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erf(b*x),x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*x^3*e^c - 3*(sqrt(pi)*b^2*x^2*e^c - sqrt(pi)*e^c)*erf(b*x)*e^(b
^2*x^2) - 6*b*x*e^c)/(sqrt(pi)*b^4)
```

Fricas [A] time = 2.71925, size = 131, normalized size = 1.66

$$-\frac{3\left(\pi - \pi b^2x^2\right)\operatorname{erf}(bx)\,e^{\left(b^2x^2+c\right)} + 2\,\sqrt{\pi}\left(b^3x^3 - 3\,bx\right)e^c}{6\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erf(b*x),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(pi - pi*b^2*x^2)*erf(b*x)*e^(b^2*x^2 + c) + 2*sqrt(pi)*(b^3*x^3 -
3*b*x)*e^c)/(pi*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**3*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27213, size = 78, normalized size = 0.99

$$\frac{\left(b^{2}x^{2}-1\right)\operatorname{erf}\left(bx\right)e^{\left(b^{2}x^{2}+c\right)}}{2\,b^{4}}-\frac{\sqrt{\pi}b^{2}x^{3}e^{c}-3\,\sqrt{\pi}xe^{c}}{3\,\pi b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erf(b*x),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2 - 1)*erf(b*x)*e^(b^2*x^2 + c)/b^4 - 1/3*(sqrt(pi)*b^2*x^3*e^c
- 3*sqrt(pi)*x*e^c)/(pi*b^3)
```

3.66 $\int e^{c+b^2x^2} x\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=37

$$\frac{e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{e^cx}{\sqrt{\pi}b}$$

```
[Out] -((E^c*x)/(b*Sqrt[Pi])) + (E^(c + b^2*x^2)*Erf[b*x])/(2*b^2)
```

Rubi [A] time = 0.031491, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6382, 8}

$$\frac{e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{e^cx}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x*Erf[b*x],x]
```

```
[Out] -((E^c*x)/(b*Sqrt[Pi])) + (E^(c + b^2*x^2)*Erf[b*x])/(2*b^2)
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x\text{erf}(bx)\,dx &= \frac{e^{c+b^2x^2}\text{erf}(bx)}{2b^2}-\frac{\int e^c\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^cx}{b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erf}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0201227, size = 34, normalized size = 0.92

$$\frac{e^c\left(e^{b^2x^2}\mathrm{Erf}(bx)-\frac{2bx}{\sqrt{\pi}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x*Erf[b*x],x]
```

```
[Out] (E^c*((-2*b*x)/Sqrt[Pi] + E^(b^2*x^2)*Erf[b*x]))/(2*b^2)
```

Maple [A] time = 0.136, size = 51, normalized size = 1.4

$$\frac{-2\,\mathrm{e}^{b^2x^2+c}\mathrm{e}^{-b^2x^2}xb+\mathrm{e}^{b^2x^2+c}\mathrm{Erf}\,(bx)\,\sqrt{\pi}}{2\,b^2\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x*erf(b*x),x)
```

```
[Out] 1/2*(-2*exp(b^2*x^2+c)*exp(-b^2*x^2)*x*b+exp(b^2*x^2+c)*erf(b*x)*Pi^(1/2))/
b^2/Pi^(1/2)
```

Maxima [A] time = 1.04374, size = 46, normalized size = 1.24

$$-\frac{2\,bxe^c-\sqrt{\pi}\,\mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{2\,\sqrt{\pi}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erf(b*x),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^c - sqrt(pi)*erf(b*x)*e^(b^2*x^2 + c))/(sqrt(pi)*b^2)
```

Fricas [A] time = 3.00517, size = 89, normalized size = 2.41

$$-\frac{2\,\sqrt{\pi}bxe^c-\pi\,\mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{2\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erf(b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(pi)*b*x*e^c - pi*erf(b*x)*e^(b^2*x^2 + c))/(pi*b^2)
```

Sympy [A] time = 26.7039, size = 34, normalized size = 0.92

$$\begin{cases} -\frac{xe^c}{\sqrt{\pi}b} + \frac{e^c e^{b^2x^2}\operatorname{erf}(bx)}{2b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x*erf(b*x),x)
```

```
[Out] Piecewise((-x*exp(c)/(sqrt(pi)*b) + exp(c)*exp(b**2*x**2)*erf(b*x)/(2*b**2)
, Ne(b, 0)), (0, True))
```

Giac [A] time = 1.30347, size = 42, normalized size = 1.14

$$-\frac{xe^c}{\sqrt{\pi}b} + \frac{\operatorname{erf}(bx)\,e^{\left(b^2x^2+c\right)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erf(b*x),x, algorithm="giac")
```

```
[Out] -x*e^c/(sqrt(pi)*b) + 1/2*erf(b*x)*e^(b^2*x^2 + c)/b^2
```

3.67 $\int \frac{e^{c+b^2x^2}\mathbf{Erf}(bx)}{x}\,dx$

Optimal. Leaf size=32

$$\frac{2be^c x\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0434965, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6388}

$$\frac{2be^c x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erf[b*x])/x,x]
```

```
[Out] (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6388

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*b
*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x}\,dx = \frac{2be^c x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0860145, size = 32, normalized size = 1.

$$\frac{2be^c x\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x,x]
```

```
[Out] (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}\mathrm{Erf}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erf(b*x)/x,x)
```

```
[Out] int(exp(b^2*x^2+c)*erf(b*x)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}(bx)\,e^{\left(b^2x^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}(bx)\,e^{\left(b^2x^2+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(b^2*x^2 + c)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erf(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x, x)
```

3.68 $\int \frac{e^{c+b^2x^2}\mathbf{Erf}(bx)}{x^3}\,dx$

Optimal. Leaf size=71

$$\frac{2b^3e^cx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{2x^2}-\frac{be^c}{\sqrt{\pi}x}$$

```
[Out] -((b*E^c)/(Sqrt[Pi]*x)) - (E^(c + b^2*x^2)*Erf[b*x])/(2*x^2) + (2*b^3*E^c*x
*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0891348, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6391, 6388, 12, 30}

$$\frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{2x^2}-\frac{be^c}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erf[b*x])/x^3,x]
```

```
[Out] -((b*E^c)/(Sqrt[Pi]*x)) - (E^(c + b^2*x^2)*Erf[b*x])/(2*x^2) + (2*b^3*E^c*x
*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6388

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*b
*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{2x^2} + b^2\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx + \frac{b\int \frac{e^c}{x^2}\,dx}{\sqrt{\pi}} \\
&= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{2x^2} + \frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}} + \frac{(be^c)\int \frac{1}{x^2}\,dx}{\sqrt{\pi}} \\
&= -\frac{be^c}{\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{2x^2} + \frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.127304, size = 34, normalized size = 0.48

$$
-\frac{2be^c\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},1\right\},\left\{\frac{1}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^3,x]
```

```
[Out] (-2*b*E^c*HypergeometricPFQ[{-1/2, 1}, {1/2, 3/2}, b^2*x^2])/(Sqrt[Pi]*x)
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$
\int \frac{e^{b^2x^2+c}\operatorname{Erf}(bx)}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erf(b*x)/x^3,x)
```

```
[Out] int(exp(b^2*x^2+c)*erf(b*x)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erf(b*x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

3.69 $\int \frac{e^{c+b^2x^2}\mathbf{Erf}(bx)}{x^5}\,dx$

Optimal. Leaf size=115

$$\frac{b^5e^cx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{b^2e^{b^2x^2+c}\text{Erf}(bx)}{4x^2}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{4x^4}-\frac{b^3e^c}{2\sqrt{\pi}x}-\frac{be^c}{6\sqrt{\pi}x^3}$$

```
[Out] -(b*E^c)/(6*Sqrt[Pi]*x^3) - (b^3*E^c)/(2*Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erf
[b*x])/(4*x^4) - (b^2*E^(c + b^2*x^2)*Erf[b*x])/(4*x^2) + (b^5*E^c*x*Hyperg
eometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.134677, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6391, 6388, 12, 30}

$$\frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}-\frac{b^2e^{b^2x^2+c}\text{Erf}(bx)}{4x^2}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{4x^4}-\frac{b^3e^c}{2\sqrt{\pi}x}-\frac{be^c}{6\sqrt{\pi}x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erf[b*x])/x^5,x]
```

```
[Out] -(b*E^c)/(6*Sqrt[Pi]*x^3) - (b^3*E^c)/(2*Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erf
[b*x])/(4*x^4) - (b^2*E^(c + b^2*x^2)*Erf[b*x])/(4*x^2) + (b^5*E^c*x*Hyperg
eometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6388

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*b
*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^5}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^4}+\frac{1}{2}b^2\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx+\frac{b\int \frac{e^c}{x^4}\,dx}{2\sqrt{\pi}}\\
&= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^2}+\frac{1}{2}b^4\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx+\frac{b^3\int \frac{e^c}{x^2}\,dx}{2\sqrt{\pi}}+\frac{(be^c)\int \frac{1}{x^4}\,dx}{2\sqrt{\pi}}\\
&= -\frac{be^c}{6\sqrt{\pi}x^3}-\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^2}+\frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}+\frac{\left(b^3e^c\right)\int \frac{1}{x^2}\,dx}{2\sqrt{\pi}}\\
&= -\frac{be^c}{6\sqrt{\pi}x^3}-\frac{b^3e^c}{2\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\operatorname{erf}(bx)}{4x^2}+\frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.139924, size = 36, normalized size = 0.31

$$
-\frac{2be^c\text{HypergeometricPFQ}\left(\left\{-\frac{3}{2},1\right\},\left\{-\frac{1}{2},\frac{3}{2}\right\},b^2x^2\right)}{3\sqrt{\pi}x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^5,x]
```

```
[Out] (-2*b*E^c*HypergeometricPFQ[{-3/2, 1}, {-1/2, 3/2}, b^2*x^2])/(3*Sqrt[Pi]*x
^3)
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$
\int \frac{e^{b^2x^2+c}\operatorname{Erf}(bx)}{x^5}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erf(b*x)/x^5,x)
```

```
[Out] int(exp(b^2*x^2+c)*erf(b*x)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erf(b*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

3.70 $\int e^{c+b^2x^2}x^4\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=119

$$\frac{3e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{4\sqrt{\pi}b^3}+\frac{x^3e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{3xe^{b^2x^2+c}\text{Erf}(bx)}{4b^4}+\frac{3e^cx^2}{4\sqrt{\pi}b^3}-\frac{e^cx^4}{4\sqrt{\pi}b}$$

```
[Out] (3*E^c*x^2)/(4*b^3*Sqrt[Pi]) - (E^c*x^4)/(4*b*Sqrt[Pi]) - (3*E^(c + b^2*x^2
)*x*Erf[b*x])/(4*b^4) + (E^(c + b^2*x^2)*x^3*Erf[b*x])/(2*b^2) + (3*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(4*b^3*Sqrt[Pi])
```

Rubi [A] time = 0.113313, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6385, 6376, 12, 30}

$$\frac{3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{4\sqrt{\pi}b^3}+\frac{x^3e^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{3xe^{b^2x^2+c}\text{Erf}(bx)}{4b^4}+\frac{3e^cx^2}{4\sqrt{\pi}b^3}-\frac{e^cx^4}{4\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^4*Erf[b*x],x]
```

```
[Out] (3*E^c*x^2)/(4*b^3*Sqrt[Pi]) - (E^c*x^4)/(4*b*Sqrt[Pi]) - (3*E^(c + b^2*x^2
)*x*Erf[b*x])/(4*b^4) + (E^(c + b^2*x^2)*x^3*Erf[b*x])/(2*b^2) + (3*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(4*b^3*Sqrt[Pi])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx &= \frac{e^{c+b^2x^2}x^3\text{erf}(bx)}{2b^2}-\frac{3\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx}{2b^2}-\frac{\int e^cx^3\,dx}{b\sqrt{\pi}}\\ &= -\frac{3e^{c+b^2x^2}x\text{erf}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\text{erf}(bx)}{2b^2}+\frac{3\int e^{c+b^2x^2}\text{erf}(bx)\,dx}{4b^4}+\frac{3\int e^cx\,dx}{2b^3\sqrt{\pi}}-\frac{e^c\int x^3\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^cx^4}{4b\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\text{erf}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\text{erf}(bx)}{2b^2}+\frac{3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{4b^3\sqrt{\pi}}+\frac{(3e^c)\int x\,dx}{2b^3\sqrt{\pi}}\\ &= \frac{3e^cx^2}{4b^3\sqrt{\pi}}-\frac{e^cx^4}{4b\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\text{erf}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\text{erf}(bx)}{2b^2}+\frac{3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{4b^3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.311147, size = 100, normalized size = 0.84

$$\frac{e^c\left(-6b^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)+2\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2-3\right)\text{Erf}(bx)-2b^4x^4+6b^2x^2+3\pi\text{Erf}(bx)\text{Erfi}(bx)\right)}{8\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^4*Erf[b*x],x]
```

```
[Out] (E^c*(6*b^2*x^2 - 2*b^4*x^4 + 2*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*b^2*x^2)*E
rf[b*x] + 3*Pi*Erf[b*x]*Erfi[b*x] - 6*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, -(b^2*x^2)]))/(8*b^5*Sqrt[Pi])
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x^4\text{Erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^4*erf(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^4*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erf(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erf(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*erf(b*x)*e^(b^2*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**4*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erf}(bx) e^{\left(b^2x^2+c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erf(b*x)*e^(b^2*x^2 + c), x)
```

3.71 $\int e^{c+b^2x^2}x^2\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=76

$$-\frac{e^c x^2 \text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{e^c x^2}{2\sqrt{\pi}b}$$

```
[Out] -(E^c*x^2)/(2*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erf[b*x])/(2*b^2) - (E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*b*Sqrt[Pi])
```

Rubi [A] time = 0.0637064, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6385, 6376, 12, 30}

$$-\frac{e^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{Erf}(bx)}{2b^2}-\frac{e^c x^2}{2\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^2*Erf[b*x],x]
```

```
[Out] -(E^c*x^2)/(2*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erf[b*x])/(2*b^2) - (E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*b*Sqrt[Pi])
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx &= \frac{e^{c+b^2x^2}x\text{erf}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}\text{erf}(bx)\,dx}{2b^2}-\frac{\int e^c x\,dx}{b\sqrt{\pi}}\\ &= \frac{e^{c+b^2x^2}x\text{erf}(bx)}{2b^2}-\frac{e^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2b\sqrt{\pi}}-\frac{e^c\int x\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^c x^2}{2b\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\text{erf}(bx)}{2b^2}-\frac{e^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2b\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.214236, size = 80, normalized size = 1.05

$$\frac{e^c\left(2b^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)+\text{Erf}(bx)\left(2\sqrt{\pi}bxe^{b^2x^2}-\pi\text{Erfi}(bx)\right)-2b^2x^2\right)}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^2*Erf[b*x],x]
```

```
[Out] (E^c*(-2*b^2*x^2 + Erf[b*x]*(2*b*E^(b^2*x^2)*Sqrt[Pi]*x - Pi*Erfi[b*x]) + 2
*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]))/(4*b^3*Sqrt[Pi])
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x^2\text{Erf}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^2*erf(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^2*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erf}(bx) e^{(b^2x^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erf(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erf(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \operatorname{erf}(bx) e^{(b^2x^2+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*erf(b*x)*e^(b^2*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**2*erf(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erf}(bx) e^{\left(b^2x^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erf(b*x)*e^(b^2*x^2 + c), x)
```

3.72 $\int e^{c+b^2x^2}\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=29

$$\frac{be^c x^2 \text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0176664, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6376}

$$\frac{be^c x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*Erf[b*x],x]
```

```
[Out] (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\int e^{c+b^2x^2}\text{erf}(bx)\,dx = \frac{be^c x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [F] time = 0.022814, size = 0, normalized size = 0.

$$\int e^{c+b^2x^2}\text{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + b^2*x^2)*Erf[b*x],x]
```

```
[Out] Integrate[E^(c + b^2*x^2)*Erf[b*x], x]
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}\mathrm{Erf}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erf(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(b^2*x^2 + c), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erf(b*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c), x)
```

3.73 $\int \frac{e^{c+b^2x^2}\mathbf{Erf}(bx)}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{2b^3e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{x}+\frac{2be^c\log(x)}{\sqrt{\pi}}$$

```
[Out] -((E^(c + b^2*x^2)*Erf[b*x])/x) + (2*b^3*E^c*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, b^2*x^2])/Sqrt[Pi] + (2*b*E^c*Log[x])/Sqrt[Pi]
```

Rubi [A] time = 0.0643943, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6391, 6376, 12, 29}

$$\frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{x}+\frac{2be^c\log(x)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erf[b*x])/x^2,x]
```

```
[Out] -((E^(c + b^2*x^2)*Erf[b*x])/x) + (2*b^3*E^c*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, b^2*x^2])/Sqrt[Pi] + (2*b*E^c*Log[x])/Sqrt[Pi]
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x} + \left(2b^2\right)\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx + \frac{(2b)\int \frac{e^c}{x}\,dx}{\sqrt{\pi}} \\
&= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x} + \frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}} + \frac{(2be^c)\int \frac{1}{x}\,dx}{\sqrt{\pi}} \\
&= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x} + \frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}} + \frac{2be^c\log(x)}{\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.198308, size = 74, normalized size = 1.12

$$
\frac{e^c\left(-2b^3x^3\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)+\text{Erf}(bx)\left(\pi bx\text{Erfi}(bx)-\sqrt{\pi}e^{b^2x^2}\right)+2bx\log(x)\right)}{\sqrt{\pi}x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^2,x]
```

```
[Out] (E^c*(Erf[b*x]*(-(E^(b^2*x^2)*Sqrt[Pi]) + b*Pi*x*Erfi[b*x]) - 2*b^3*x^3*Hyp
ergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)] + 2*b*x*Log[x]))/(Sqrt[Pi]*x)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$
\int \frac{e^{b^2x^2+c}\operatorname{Erf}(bx)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erf(b*x)/x^2,x)
```

```
[Out] int(exp(b^2*x^2+c)*erf(b*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{(b^2x^2+c)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erf}(bx)\, e^{(b^2x^2+c)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erf(b*x)/x**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(b^2x^2+c\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

3.74 $\int \frac{e^{c+b^2x^2}\textbf{Erf}(bx)}{x^4}\,dx$

Optimal. Leaf size=115

$$\frac{4b^5e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{3\sqrt{\pi}}-\frac{2b^2e^{b^2x^2+c}\text{Erf}(bx)}{3x}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{3x^3}+\frac{4b^3e^c\log(x)}{3\sqrt{\pi}}-\frac{be^c}{3\sqrt{\pi}x^2}$$

```
[Out] -(b*E^c)/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erf[b*x])/(3*x^3) - (2*b^2*E^(
c + b^2*x^2)*Erf[b*x])/(3*x) + (4*b^5*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, b^2*x^2])/(3*Sqrt[Pi]) + (4*b^3*E^c*Log[x])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.10828, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6391, 6376, 12, 29, 30}

$$\frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}}-\frac{2b^2e^{b^2x^2+c}\text{Erf}(bx)}{3x}-\frac{e^{b^2x^2+c}\text{Erf}(bx)}{3x^3}+\frac{4b^3e^c\log(x)}{3\sqrt{\pi}}-\frac{be^c}{3\sqrt{\pi}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erf[b*x])/x^4,x]
```


```
[Out] -(b*E^c)/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erf[b*x])/(3*x^3) - (2*b^2*E^(
c + b^2*x^2)*Erf[b*x])/(3*x) + (4*b^5*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, b^2*x^2])/(3*Sqrt[Pi]) + (4*b^3*E^c*Log[x])/(3*Sqrt[Pi])
```


Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{3x^3}+\frac{1}{3}\left(2b^2\right)\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx+\frac{(2b)\int \frac{e^c}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\operatorname{erf}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx+\frac{\left(4b^3\right)\int \frac{e^c}{x}\,dx}{3\sqrt{\pi}}+\frac{(2be^c)\int \frac{1}{x^3}}{3\sqrt{\pi}}\\
&= -\frac{be^c}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\operatorname{erf}(bx)}{3x}+\frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}}+\frac{\left(4b^3e^c\right)\int \frac{1}{x}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^c}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\operatorname{erf}(bx)}{3x}+\frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}}+\frac{4b^3e^c\log(x)}{3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.340162, size = 100, normalized size = 0.87

$$-\frac{e^c\left(4b^5x^5\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)-2\pi b^3x^3\text{Erf}(bx)\text{Erfi}(bx)+\sqrt{\pi}e^{b^2x^2}\left(2b^2x^2+1\right)\text{Erf}(bx)-4b^3\right.}{3\sqrt{\pi}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^4,x]
```

```
[Out] -(E^c*(b*x + E^(b^2*x^2)*Sqrt[Pi]*(1 + 2*b^2*x^2)*Erf[b*x] - 2*b^3*Pi*x^3*E
rf[b*x]*Erfi[b*x] + 4*b^5*x^5*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2
)] - 4*b^3*x^3*Log[x]))/(3*Sqrt[Pi]*x^3)
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\mathrm{Erf}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erf(b*x)/x^4,x)
```

```
[Out] int(exp(b^2*x^2+c)*erf(b*x)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erf(b*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erf(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

3.75 $\int e^{-b^2x^2}x^5\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=135

$$-\frac{x^4e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^2}-\frac{x^2e^{-b^2x^2}\mathrm{Erf}(bx)}{b^4}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{b^6}+\frac{43\mathrm{Erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{x^3e^{-2b^2x^2}}{4\sqrt{\pi}b^3}-\frac{11xe^{-2b^2x^2}}{16\sqrt{\pi}b^5}$$

```
[Out] (-11*x)/(16*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) - x^3/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]
) - Erf[b*x]/(b^6*E^(b^2*x^2)) - (x^2*Erf[b*x])/(b^4*E^(b^2*x^2)) - (x^4*Er
f[b*x])/(2*b^2*E^(b^2*x^2)) + (43*Erf[Sqrt[2]*b*x])/(32*Sqrt[2]*b^6)
```

Rubi [A] time = 0.195389, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6385, 6382, 2205, 2212}

$$-\frac{x^4e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^2}-\frac{x^2e^{-b^2x^2}\mathrm{Erf}(bx)}{b^4}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{b^6}+\frac{43\mathrm{Erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{x^3e^{-2b^2x^2}}{4\sqrt{\pi}b^3}-\frac{11xe^{-2b^2x^2}}{16\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] (-11*x)/(16*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) - x^3/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]
) - Erf[b*x]/(b^6*E^(b^2*x^2)) - (x^2*Erf[b*x])/(b^4*E^(b^2*x^2)) - (x^4*Er
f[b*x])/(2*b^2*E^(b^2*x^2)) + (43*Erf[Sqrt[2]*b*x])/(32*Sqrt[2]*b^6)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^5\operatorname{erf}(bx)\,dx &= -\frac{e^{-b^2x^2}x^4\operatorname{erf}(bx)}{2b^2}+\frac{2\int e^{-b^2x^2}x^3\operatorname{erf}(bx)\,dx}{b^2}+\frac{\int e^{-2b^2x^2}x^4\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^2\operatorname{erf}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\operatorname{erf}(bx)}{2b^2}+\frac{2\int e^{-b^2x^2}x\operatorname{erf}(bx)\,dx}{b^4}+\frac{3\int e^{-2b^2x^2}x^2\,dx}{4b^3\sqrt{\pi}}+\frac{2\int\\ &= -\frac{11e^{-2b^2x^2}x}{16b^5\sqrt{\pi}}-\frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\operatorname{erf}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\operatorname{erf}(bx)}{2b^2}+\frac{3\int e^{-2b^2x^2}\,dx}{16b^5\sqrt{\pi}}+\\ &= -\frac{11e^{-2b^2x^2}x}{16b^5\sqrt{\pi}}-\frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\operatorname{erf}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\operatorname{erf}(bx)}{2b^2}+\frac{43\operatorname{erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}\end{aligned}$$

Mathematica [A] time = 0.0770188, size = 86, normalized size = 0.64

$$\frac{-32e^{-b^2x^2}\left(b^4x^4+2b^2x^2+2\right)\operatorname{Erf}(bx)-\frac{4bxe^{-2b^2x^2}\left(4b^2x^2+11\right)}{\sqrt{\pi}}+43\sqrt{2}\operatorname{Erf}\left(\sqrt{2}bx\right)}{64b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] ((-4*b*x*(11 + 4*b^2*x^2))/(E^(2*b^2*x^2)*Sqrt[Pi]) - (32*(2 + 2*b^2*x^2 +
b^4*x^4)*Erf[b*x])/E^(b^2*x^2) + 43*Sqrt[2]*Erf[Sqrt[2]*b*x])/(64*b^6)
```

Maple [A] time = 0.122, size = 119, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\mathrm{Erf}\,(bx)}{b^5}\left(-\frac{b^4x^4}{2\,\mathrm{e}^{b^2x^2}}-\frac{b^2x^2}{\mathrm{e}^{b^2x^2}}-\left(\mathrm{e}^{b^2x^2}\right)^{-1}\right)-\frac{1}{\sqrt{\pi}b^5}\left(-\frac{43\,\sqrt{2}\sqrt{\pi}\mathrm{Erf}\left(bx\sqrt{2}\right)}{64}+\frac{11\,bx}{16\,\left(\mathrm{e}^{b^2x^2}\right)^2}+\frac{x^3b^3}{4\,\left(\mathrm{e}^{b^2x^2}\right)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erf(b*x)/exp(b^2*x^2),x)
```

```
[Out] (erf(b*x)/b^5*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2
))-1/Pi^(1/2)/b^5*(-43/64*2^(1/2)*Pi^(1/2)*erf(b*x*2^(1/2))+11/16/exp(b^2*x
^2)^2*b*x+1/4/exp(b^2*x^2)^2*b^3*x^3))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^4x^4+2\,b^2x^2+2\right)\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{2\,b^6}+\frac{-\frac{1}{64}\,b^4\left(\frac{4\left(4\,b^2x^3+3\,x\right)e^{\left(-2\,b^2x^2\right)}}{b^4}-\frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b^5}\right)-\frac{1}{8}\,b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}bx\right)}{b^3}\right)+}{\sqrt{\pi}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*x^4 + 2*b^2*x^2 + 2)*erf(b*x)*e^(-b^2*x^2)/b^6 + integrate((b^4*x
^4 + 2*b^2*x^2 + 2)*e^(-2*b^2*x^2), x)/(sqrt(pi)*b^5)
```

Fricas [A] time = 3.00524, size = 244, normalized size = 1.81

$$\frac{43\,\sqrt{2}\pi\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-32\left(\pi b^5x^4+2\,\pi b^3x^2+2\,\pi b\right)\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}-4\,\sqrt{\pi}\left(4\,b^4x^3+11\,b^2x\right)e^{\left(-2\,b^2x^2\right)}}{64\,\pi b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/64*(43*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 32*(pi*b^5*x^4 + 2
*pi*b^3*x^2 + 2*pi*b)*erf(b*x)*e^(-b^2*x^2) - 4*sqrt(pi)*(4*b^4*x^3 + 11*b^
```

```
2*x)*e^(-2*b^2*x^2))/(pi*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erf(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27722, size = 212, normalized size = 1.57

$$-\frac{\left(b^4x^4+2\,b^2x^2+2\right)\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{2\,b^6}-\frac{\sqrt{\pi}b^4\left(\frac{4\left(4\,b^2x^3+3\,x\right)e^{\left(-2\,b^2x^2\right)}}{b^4}+\frac{3\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^5}\right)+8\,\sqrt{\pi}b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}\right.}{b^3}\right.}{64\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b^4*x^4 + 2*b^2*x^2 + 2)*erf(b*x)*e^(-b^2*x^2)/b^6 - 1/64*(sqrt(pi)*b
^4*(4*(4*b^2*x^3 + 3*x)*e^(-2*b^2*x^2)/b^4 + 3*sqrt(2)*sqrt(pi)*erf(-sqrt(2
)*b*x)/b^5) + 8*sqrt(pi)*b^2*(4*x*e^(-2*b^2*x^2)/b^2 + sqrt(2)*sqrt(pi)*erf
(-sqrt(2)*b*x)/b^3) + 32*sqrt(2)*pi*erf(-sqrt(2)*b*x)/b)/(pi*b^5)
```

3.76 $\int e^{-b^2x^2}x^3\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=90

$$-\frac{x^2e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^2}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^4}+\frac{5\mathrm{Erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{xe^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

```
[Out] -x/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - Erf[b*x]/(2*b^4*E^(b^2*x^2)) - (x^2*Erf
[b*x])/(2*b^2*E^(b^2*x^2)) + (5*Erf[Sqrt[2]*b*x])/(8*Sqrt[2]*b^4)
```

Rubi [A] time = 0.101306, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6385, 6382, 2205, 2212}

$$-\frac{x^2e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^2}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^4}+\frac{5\mathrm{Erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{xe^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] -x/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - Erf[b*x]/(2*b^4*E^(b^2*x^2)) - (x^2*Erf
[b*x])/(2*b^2*E^(b^2*x^2)) + (5*Erf[Sqrt[2]*b*x])/(8*Sqrt[2]*b^4)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx &= -\frac{e^{-b^2x^2}x^2\text{erf}(bx)}{2b^2}+\frac{\int e^{-b^2x^2}x\text{erf}(bx)\,dx}{b^2}+\frac{\int e^{-2b^2x^2}x^2\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{-2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erf}(bx)}{2b^2}+\frac{\int e^{-2b^2x^2}\,dx}{4b^3\sqrt{\pi}}+\frac{\int e^{-2b^2x^2}\,dx}{b^3\sqrt{\pi}}\\ &= -\frac{e^{-2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erf}(bx)}{2b^2}+\frac{5\text{erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}\end{aligned}$$

Mathematica [A] time = 0.0467127, size = 68, normalized size = 0.76

$$\frac{-8e^{-b^2x^2}\left(b^2x^2+1\right)\text{Erf}(bx)-\frac{4bxe^{-2b^2x^2}}{\sqrt{\pi}}+5\sqrt{2}\text{Erf}\left(\sqrt{2}bx\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] ((-4*b*x)/(E^(2*b^2*x^2)*Sqrt[Pi]) - (8*(1 + b^2*x^2)*Erf[b*x])/E^(b^2*x^2)
 + 5*Sqrt[2]*Erf[Sqrt[2]*b*x])/(16*b^4)
```

Maple [A] time = 0.247, size = 83, normalized size = 0.9

$$\frac{1}{b}\left(\frac{\text{Erf}\,(bx)}{b^3}\left(-\frac{b^2x^2}{2\,\mathrm{e}^{b^2x^2}}-\frac{1}{2\,\mathrm{e}^{b^2x^2}}\right)-\frac{1}{b^3\sqrt{\pi}}\left(-\frac{5\,\sqrt{2}\sqrt{\pi}\text{Erf}\left(bx\sqrt{2}\right)}{16}+\frac{bx}{4\,\left(\mathrm{e}^{b^2x^2}\right)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erf(b*x)/exp(b^2*x^2),x)
```

```
[Out] (erf(b*x)/b^3*(-1/2*b^2*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))-1/b^3/Pi^(1/2)*(
-5/16*2^(1/2)*Pi^(1/2)*erf(b*x*2^(1/2))+1/4/exp(b^2*x^2)^2*b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2x^2+1\right)\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{2\,b^4}+\frac{-\frac{1}{16}\,b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}bx\right)}{b^3}\right)+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}bx\right)}{4\,b}}{\sqrt{\pi}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*x^2 + 1)*erf(b*x)*e^(-b^2*x^2)/b^4 + integrate((b^2*x^2 + 1)*e^(-
2*b^2*x^2), x)/(sqrt(pi)*b^3)
```

Fricas [A] time = 3.04953, size = 197, normalized size = 2.19

$$-\frac{4\,\sqrt{\pi}b^2xe^{\left(-2\,b^2x^2\right)}-5\,\sqrt{2}\pi\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)+8\left(\pi b^3x^2+\pi b\right)\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{16\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(pi)*b^2*x*e^(-2*b^2*x^2) - 5*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)
*sqrt(b^2)*x) + 8*(pi*b^3*x^2 + pi*b)*erf(b*x)*e^(-b^2*x^2))/(pi*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3e^{-b^2x^2}\operatorname{erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erf(b*x)/exp(b**2*x**2),x)
```

```
[Out] Integral(x**3*exp(-b**2*x**2)*erf(b*x), x)
```

Giac [A] time = 1.2589, size = 128, normalized size = 1.42

$$-\frac{\left(b^2x^2+1\right)\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{2\,b^4}-\frac{\sqrt{\pi}b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^3}\right)+\frac{4\,\sqrt{2}\pi\operatorname{erf}\left(-\sqrt{2}bx\right)}{b}}{16\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^2 + 1)*erf(b*x)*e^(-b^2*x^2)/b^4 - 1/16*(sqrt(pi)*b^2*(4*x*e^(-
2*b^2*x^2)/b^2 + sqrt(2)*sqrt(pi)*erf(-sqrt(2)*b*x)/b^3) + 4*sqrt(2)*pi*erf
(-sqrt(2)*b*x)/b)/(pi*b^3)
```

3.77 $\int e^{-b^2x^2}x\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=43

$$\frac{\mathrm{Erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^2}$$

```
[Out] -Erf[b*x]/(2*b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2)
```

Rubi [A] time = 0.0324727, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6382, 2205}

$$\frac{\mathrm{Erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] -Erf[b*x]/(2*b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2)
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int e^{-b^2x^2}x\operatorname{erf}(bx)\,dx = -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2b^2} + \frac{\int e^{-2b^2x^2}\,dx}{b\sqrt{\pi}}$$
$$= -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2b^2} + \frac{\operatorname{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

Mathematica [A] time = 0.0171781, size = 39, normalized size = 0.91

$$\frac{\sqrt{2}\operatorname{Erf}\left(\sqrt{2}bx\right) - 2e^{-b^2x^2}\operatorname{Erf}(bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] ((-2*Erf[b*x])/E^(b^2*x^2) + Sqrt[2]*Erf[Sqrt[2]*b*x])/(4*b^2)
```

Maple [A] time = 0.109, size = 39, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{\operatorname{Erf}(bx)\,e^{-b^2x^2}}{2\,b} + \frac{\sqrt{2}\operatorname{Erf}\left(bx\sqrt{2}\right)}{4\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erf(b*x)/exp(b^2*x^2),x)
```

```
[Out] (-1/2*erf(b*x)/b*exp(-b^2*x^2)+1/4/b*2^(1/2)*erf(b*x*2^(1/2)))/b
```

Maxima [A] time = 1.54507, size = 46, normalized size = 1.07

$$-\frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{2\,b^2} + \frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}bx\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*erf(b*x)*e^(-b^2*x^2)/b^2 + 1/4*sqrt(2)*erf(sqrt(2)*b*x)/b^2
```

Fricas [A] time = 2.63868, size = 112, normalized size = 2.6

$$-\frac{2\,b\,\mathrm{erf}\left(bx\right)e^{\left(-b^2x^2\right)} - \sqrt{2}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*erf(b*x)*e^(-b^2*x^2) - sqrt(2)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x
))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{-b^2x^2}\,\mathrm{erf}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)/exp(b**2*x**2),x)
```

```
[Out] Integral(x*exp(-b**2*x**2)*erf(b*x), x)
```

Giac [A] time = 1.30691, size = 47, normalized size = 1.09

$$-\frac{\mathrm{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{2\,b^2} - \frac{\sqrt{2}\,\mathrm{erf}\left(-\sqrt{2}bx\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] -1/2*erf(b*x)*e^(-b^2*x^2)/b^2 - 1/4*sqrt(2)*erf(-sqrt(2)*b*x)/b^2
```

3.78 $\int \frac{e^{-b^2x^2}\textbf{Erf}(bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{e^{-b^2x^2}\text{Erf}(bx)}{x},x\right)$$

```
[Out] Unintegrable[Erf[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi [A] time = 0.0346637, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{-b^2x^2}\text{Erf}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]/(E^(b^2*x^2)*x),x]
```

```
[Out] Defer[Int][Erf[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx = \int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx$$

Mathematica [A] time = 0.0846563, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\text{Erf}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]/(E^(b^2*x^2)*x),x]
```

```
[Out] Integrate[Erf[b*x]/(E^(b^2*x^2)*x), x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{\text{Erf}\,(bx)}{\mathrm{e}^{b^2x^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2)/x,x)
```

```
[Out] int(erf(b*x)/exp(b^2*x^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(-b^2*x^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\,\text{erf}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2)/x,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erf(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\, e^{\left(-b^2x^2\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x, x)
```

3.79 $\int \frac{e^{-b^2x^2}\textbf{Erf}(bx)}{x^3}\,dx$

Optimal. Leaf size=87

$$-b^2\text{Unintegrable}\left(\frac{e^{-b^2x^2}\text{Erf}(bx)}{x},x\right)-\frac{e^{-b^2x^2}\text{Erf}(bx)}{2x^2}-\sqrt{2}b^2\text{Erf}\left(\sqrt{2}bx\right)-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

```
[Out] -(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b^
2*Erf[Sqrt[2]*b*x] - b^2*Unintegrable[Erf[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi [A] time = 0.09838, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{-b^2x^2}\text{Erf}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]/(E^(b^2*x^2)*x^3),x]
```

```
[Out] -(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b^
2*Erf[Sqrt[2]*b*x] - b^2*Defer[Int][Erf[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}\,dx &= -\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}-b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx+\frac{b\int \frac{e^{-2b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}-b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx-\frac{\left(4b^3\right)\int e^{-2b^2x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}-\sqrt{2}b^2\text{erf}\left(\sqrt{2}bx\right)-b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.133205, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\text{Erf}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^3),x]
```

```
[Out] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^3), x]
```

Maple [A] time = 0.315, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}(bx)}{\mathrm{e}^{b^2x^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2)/x^3,x)
```

```
[Out] int(erf(b*x)/exp(b^2*x^2)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2)/x**3,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erf(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

3.80 $\int \frac{e^{-b^2x^2}\mathbf{Erf}(bx)}{x^5}\,dx$

Optimal. Leaf size=160

$$\frac{1}{2}b^4\text{Unintegrable}\left(\frac{e^{-b^2x^2}\text{Erf}(bx)}{x},x\right)+\frac{b^2e^{-b^2x^2}\text{Erf}(bx)}{4x^2}-\frac{e^{-b^2x^2}\text{Erf}(bx)}{4x^4}+\frac{2}{3}\sqrt{2}b^4\text{Erf}\left(\sqrt{2}bx\right)+\frac{b^4\text{Erf}\left(\sqrt{2}bx\right)}{\sqrt{2}}+\frac{7b^3e^{-}}{6\sqrt{}}$$

```
[Out] -b/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x^3) + (7*b^3)/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x) -
Erf[b*x]/(4*E^(b^2*x^2)*x^4) + (b^2*Erf[b*x])/(4*E^(b^2*x^2)*x^2) + (b^4*Er
f[Sqrt[2]*b*x])/Sqrt[2] + (2*Sqrt[2]*b^4*Erf[Sqrt[2]*b*x])/3 + (b^4*Uninteg
rable[Erf[b*x]/(E^(b^2*x^2)*x), x])/2
```

Rubi [A] time = 0.190871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{-b^2x^2}\text{Erf}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erf[b*x]/(E^(b^2*x^2)*x^5),x]
```

```
[Out] -b/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x^3) + (7*b^3)/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x) -
Erf[b*x]/(4*E^(b^2*x^2)*x^4) + (b^2*Erf[b*x])/(4*E^(b^2*x^2)*x^2) + (b^4*Er
f[Sqrt[2]*b*x])/Sqrt[2] + (2*Sqrt[2]*b^4*Erf[Sqrt[2]*b*x])/3 + (b^4*Defer[I
nt][Erf[b*x]/(E^(b^2*x^2)*x), x])/2
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{x^5}\,dx = -\frac{e^{-b^2x^2}\mathrm{erf}(bx)}{4x^4} - \frac{1}{2}b^2\int \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{x^3}\,dx + \frac{b\int \frac{e^{-2b^2x^2}}{x^4}\,dx}{2\sqrt{\pi}}$$

$$= -\frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\mathrm{erf}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{x}\,dx - \frac{b^3\int \frac{e^{-2b^2x^2}}{x^2}\,dx}{2\sqrt{\pi}} - \frac{\left(2b^3\right)\int \cdots}{3\sqrt{\cdots}}$$

$$= -\frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} + \frac{7b^3e^{-2b^2x^2}}{6\sqrt{\pi}x} - \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\mathrm{erf}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{x}\,dx + \frac{\left(2b^5\right)\int e^{-2b^2}\cdots}{\sqrt{\pi}}$$

$$= -\frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} + \frac{7b^3e^{-2b^2x^2}}{6\sqrt{\pi}x} - \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\mathrm{erf}(bx)}{4x^2} + \frac{b^4\mathrm{erf}\left(\sqrt{2}bx\right)}{\sqrt{2}} + \frac{2}{3}\sqrt{2}b^4\mathrm{erf}\left(\sqrt{2}bx\right) + \frac{1}{2}\cdots$$

Mathematica [A] time = 0.15966, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^5),x]
```

```
[Out] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^5), x]
```

Maple [A] time = 0.306, size = 0, normalized size = 0.

$$\int \frac{\mathrm{Erf}\,(bx)}{\mathrm{e}^{b^2x^2}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2)/x^5,x)
```

```
[Out] int(erf(b*x)/exp(b^2*x^2)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*e^(-b^2*x^2)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2)/x**5,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erf(b*x)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^5, x)
```

3.81 $\int e^{-b^2x^2}x^4\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=112

$$-\frac{x^3e^{-b^2x^2}\text{Erf}(bx)}{2b^2}-\frac{3xe^{-b^2x^2}\text{Erf}(bx)}{4b^4}+\frac{3\sqrt{\pi}\text{Erf}(bx)^2}{16b^5}-\frac{x^2e^{-2b^2x^2}}{4\sqrt{\pi}b^3}-\frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5}$$

```
[Out] -1/(2*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) - x^2/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (3
*x*Erf[b*x])/(4*b^4*E^(b^2*x^2)) - (x^3*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (3*
Sqrt[Pi]*Erf[b*x]^2)/(16*b^5)
```

Rubi [A] time = 0.146233, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6385, 6373, 30, 2209, 2212}

$$-\frac{x^3e^{-b^2x^2}\text{Erf}(bx)}{2b^2}-\frac{3xe^{-b^2x^2}\text{Erf}(bx)}{4b^4}+\frac{3\sqrt{\pi}\text{Erf}(bx)^2}{16b^5}-\frac{x^2e^{-2b^2x^2}}{4\sqrt{\pi}b^3}-\frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] -1/(2*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) - x^2/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (3
*x*Erf[b*x])/(4*b^4*E^(b^2*x^2)) - (x^3*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (3*
Sqrt[Pi]*Erf[b*x]^2)/(16*b^5)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx = -\frac{e^{-b^2x^2}x^3\text{erf}(bx)}{2b^2} + \frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)\,dx}{2b^2} + \frac{\int e^{-2b^2x^2}x^3\,dx}{b\sqrt{\pi}}$$

$$= -\frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}} - \frac{3e^{-b^2x^2}x\text{erf}(bx)}{4b^4} - \frac{e^{-b^2x^2}x^3\text{erf}(bx)}{2b^2} + \frac{3\int e^{-b^2x^2}\text{erf}(bx)\,dx}{4b^4} + \frac{\int e^{-2b^2x^2}x\,dx}{2b^3\sqrt{\pi}} + \frac{3\int e^{-2b}}{2b^3}$$

$$= -\frac{e^{-2b^2x^2}}{2b^5\sqrt{\pi}} - \frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}} - \frac{3e^{-b^2x^2}x\text{erf}(bx)}{4b^4} - \frac{e^{-b^2x^2}x^3\text{erf}(bx)}{2b^2} + \frac{\left(3\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx, x, \text{erf}(bx)\right)}{8b^5}$$

$$= -\frac{e^{-2b^2x^2}}{2b^5\sqrt{\pi}} - \frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}} - \frac{3e^{-b^2x^2}x\text{erf}(bx)}{4b^4} - \frac{e^{-b^2x^2}x^3\text{erf}(bx)}{2b^2} + \frac{3\sqrt{\pi}\text{erf}(bx)^2}{16b^5}$$

Mathematica [A] time = 0.0285525, size = 85, normalized size = 0.76

$$\frac{e^{-2b^2x^2}\left(3\pi e^{2b^2x^2}\text{Erf}(bx)^2 - 4\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2+3\right)\text{Erf}(bx) - 4\left(b^2x^2+2\right)\right)}{16\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] (-4*(2 + b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x] + 3
*E^(2*b^2*x^2)*Pi*Erf[b*x]^2)/(16*b^5*E^(2*b^2*x^2)*Sqrt[Pi])
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{Erf}(bx)}{e^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erf(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(x^4*erf(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\frac{(2b^2x^2+1)e^{(-2b^2x^2)}}{4b^2} - \frac{3e^{(-2b^2x^2)}}{4b^2}}{2\sqrt{\pi}b^3} - \frac{4\left(2b^3x^3 + 3bx\right)\operatorname{erf}(bx)e^{(-b^2x^2)} - 3\sqrt{\pi}\operatorname{erf}(bx)^2}{16b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*integrate((2*b^2*x^3 + 3*x)*e^(-2*b^2*x^2), x)/(sqrt(pi)*b^3) - 1/16*(4
*(2*b^3*x^3 + 3*b*x)*erf(b*x)*e^(-b^2*x^2) - 3*sqrt(pi)*erf(b*x)^2)/b^5
```

Fricas [A] time = 3.08856, size = 176, normalized size = 1.57

$$-\frac{4\left(2\pi b^3x^3 + 3\pi bx\right)\operatorname{erf}(bx)e^{(-b^2x^2)} - \sqrt{\pi}\left(3\pi\operatorname{erf}(bx)^2 - 4\left(b^2x^2 + 2\right)e^{(-2b^2x^2)}\right)}{16\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(2*pi*b^3*x^3 + 3*pi*b*x)*erf(b*x)*e^(-b^2*x^2) - sqrt(pi)*(3*pi*e
rf(b*x)^2 - 4*(b^2*x^2 + 2)*e^(-2*b^2*x^2)))/(pi*b^5)
```

Sympy [A] time = 98.6659, size = 109, normalized size = 0.97

$$\begin{cases} -\frac{x^{3}e^{-b^{2}x^{2}}\operatorname{erf}(bx)}{2b^{2}} - \frac{x^{2}e^{-2b^{2}x^{2}}}{4\sqrt{\pi}b^{3}} - \frac{3xe^{-b^{2}x^{2}}\operatorname{erf}(bx)}{4b^{4}} + \frac{3\sqrt{\pi}\operatorname{erf}^{2}(bx)}{16b^{5}} - \frac{e^{-2b^{2}x^{2}}}{2\sqrt{\pi}b^{5}} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erf(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((-x**3*exp(-b**2*x**2)*erf(b*x)/(2*b**2) - x**2*exp(-2*b**2*x**2)
/(4*sqrt(pi)*b**3) - 3*x*exp(-b**2*x**2)*erf(b*x)/(4*b**4) + 3*sqrt(pi)*erf
(b*x)**2/(16*b**5) - exp(-2*b**2*x**2)/(2*sqrt(pi)*b**5), Ne(b, 0)), (0, Tr
ue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{4} \operatorname{erf}(bx)\, e^{\left(-b^{2}x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*erf(b*x)*e^(-b^2*x^2), x)
```

3.82 $\int e^{-b^2x^2}x^2\textbf{Erf}(bx)\,dx$

Optimal. Leaf size=63

$$-\frac{xe^{-b^2x^2}\text{Erf}(bx)}{2b^2}+\frac{\sqrt{\pi}\text{Erf}(bx)^2}{8b^3}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

```
[Out] -1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (Sqr
t[Pi]*Erf[b*x]^2)/(8*b^3)
```

Rubi [A] time = 0.0684107, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6385, 6373, 30, 2209}

$$-\frac{xe^{-b^2x^2}\text{Erf}(bx)}{2b^2}+\frac{\sqrt{\pi}\text{Erf}(bx)^2}{8b^3}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] -1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (Sqr
t[Pi]*Erf[b*x]^2)/(8*b^3)
```

Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^2\operatorname{erf}(bx)\,dx &= -\frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{2b^2}+\frac{\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx}{2b^2}+\frac{\int e^{-2b^2x^2}x\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{-2b^2x^2}}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{2b^2}+\frac{\sqrt{\pi}\,\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b^3}\\ &= -\frac{e^{-2b^2x^2}}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{2b^2}+\frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.0387112, size = 56, normalized size = 0.89

$$-\frac{4bxe^{-b^2x^2}\operatorname{Erf}(bx)+\frac{2e^{-2b^2x^2}}{\sqrt{\pi}}-\sqrt{\pi}\operatorname{Erf}(bx)^2}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Erf[b*x])/E^(b^2*x^2),x]
```

```
[Out] -(2/(E^(2*b^2*x^2)*Sqrt[Pi]) + (4*b*x*Erf[b*x])/E^(b^2*x^2) - Sqrt[Pi]*Erf[
b*x]^2)/(8*b^3)
```

Maple [F] time = 0.236, size = 0, normalized size = 0.

$$\int \frac{x^2\operatorname{Erf}(bx)}{e^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erf(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(x^2*erf(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\frac{e^{\left(-2\,b^2x^2\right)}}{4\,b^2}}{\sqrt{\pi}b} - \frac{4\,bx\,\mathrm{erf}\left(bx\right)e^{\left(-b^2x^2\right)} - \sqrt{\pi}\,\mathrm{erf}\left(bx\right)^2}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*e^(-2*b^2*x^2), x)/(sqrt(pi)*b) - 1/8*(4*b*x*erf(b*x)*e^(-b^2*x
^2) - sqrt(pi)*erf(b*x)^2)/b^3
```

Fricas [A] time = 3.09933, size = 127, normalized size = 2.02

$$-\frac{4\,\pi bx\,\mathrm{erf}\left(bx\right)e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi\,\mathrm{erf}\left(bx\right)^2 - 2\,e^{\left(-2\,b^2x^2\right)}\right)}{8\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*pi*b*x*erf(b*x)*e^(-b^2*x^2) - sqrt(pi)*(pi*erf(b*x)^2 - 2*e^(-2*b^
2*x^2)))/(pi*b^3)
```

Sympy [A] time = 12.2537, size = 60, normalized size = 0.95

$$\begin{cases} -\frac{xe^{-b^2x^2}\,\mathrm{erf}\left(bx\right)}{2b^2} + \frac{\sqrt{\pi}\,\mathrm{erf}^2\left(bx\right)}{8b^3} - \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erf(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((-x*exp(-b**2*x**2)*erf(b*x)/(2*b**2) + sqrt(pi)*erf(b*x)**2/(8*b
**3) - exp(-2*b**2*x**2)/(4*sqrt(pi)*b**3), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erf}(bx)\, e^{\left(-b^2x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*erf(b*x)*e^(-b^2*x^2), x)
```

3.83 $\int e^{-b^2x^2}\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{\pi}\text{Erf}(bx)^2}{4b}$$

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Rubi [A] time = 0.0174227, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6373, 30}

$$\frac{\sqrt{\pi}\text{Erf}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/E^(b^2*x^2),x]
```

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}\text{erf}(bx)\,dx &= \frac{\sqrt{\pi}\,\text{Subst}\left(\int x\,dx, x, \text{erf}(bx)\right)}{2b}\\ &= \frac{\sqrt{\pi}\text{erf}(bx)^2}{4b}\end{aligned}$$

Mathematica [A] time = 0.0040364, size = 18, normalized size = 1.

$$\frac{\sqrt{\pi}\text{Erf}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/E^(b^2*x^2),x]
```

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Maple [A] time = 0.075, size = 15, normalized size = 0.8

$$\frac{(\text{Erf}(bx))^2\sqrt{\pi}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2),x)
```

```
[Out] 1/4*erf(b*x)^2*Pi^(1/2)/b
```

Maxima [A] time = 1.01892, size = 19, normalized size = 1.06

$$\frac{\sqrt{\pi}\,\text{erf}(bx)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(b*x)^2/b
```

Fricas [A] time = 2.80272, size = 36, normalized size = 2.

$$\frac{\sqrt{\pi}\,\text{erf}(bx)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*erf(b*x)^2/b
```

Sympy [A] time = 1.61369, size = 53, normalized size = 2.94

$$\begin{cases} -\frac{\sqrt{\pi}\operatorname{erf}^{2}\left(x\sqrt{b^{2}}\right)}{4b} + \frac{\sqrt{\pi}\sqrt{b^{2}}\operatorname{erf}\left(bx\right)\operatorname{erf}\left(x\sqrt{b^{2}}\right)}{2b^{2}} & \text{for}\: b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((-sqrt(pi)*erf(x*sqrt(b**2))**2/(4*b) + sqrt(pi)*sqrt(b**2)*erf(b
*x)*erf(x*sqrt(b**2))/(2*b**2), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}\left(bx\right)e^{\left(-b^{2}x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2), x)
```

3.84 $\int \frac{e^{-b^2x^2}\textbf{Erf}(bx)}{x^2}\,dx$

Optimal. Leaf size=52

$$-\frac{e^{-b^2x^2}\text{Erf}(bx)}{x}+\frac{b\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\text{Erf}(bx)^2$$

```
[Out] -(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegralEi[
-2*b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0705583, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6391, 6373, 30, 2210}

$$-\frac{e^{-b^2x^2}\text{Erf}(bx)}{x}+\frac{b\text{Ei}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\text{Erf}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/(E^(b^2*x^2)*x^2),x]
```

```
[Out] -(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegralEi[
-2*b^2*x^2])/Sqrt[Pi]
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx &= -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x} - \left(2b^2\right)\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx + \frac{(2b)\int \frac{e^{-2b^2x^2}}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x} + \frac{b\operatorname{Ei}\left(-2b^2x^2\right)}{\sqrt{\pi}} - \left(b\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx, x, \operatorname{erf}(bx)\right)\\ &= -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x} - \frac{1}{2}b\sqrt{\pi}\operatorname{erf}(bx)^2 + \frac{b\operatorname{Ei}\left(-2b^2x^2\right)}{\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0152362, size = 52, normalized size = 1.

$$-\frac{e^{-b^2x^2}\operatorname{Erf}(bx)}{x} + \frac{b\operatorname{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}} - \frac{1}{2}\sqrt{\pi}b\operatorname{Erf}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^2),x]
```

```
[Out] -(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegralEi[
-2*b^2*x^2])/Sqrt[Pi]
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erf}(bx)}{e^{b^2x^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2)/x^2,x)
```

```
[Out] int(erf(b*x)/exp(b^2*x^2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^2, x)
```

Fricas [A] time = 2.59884, size = 130, normalized size = 2.5

$$-\frac{2\,\pi\operatorname{erf}(bx)e^{\left(-b^2x^2\right)} + \sqrt{\pi}\left(\pi bx\operatorname{erf}(bx)^2 - 2\,bx\operatorname{Ei}\left(-2\,b^2x^2\right)\right)}{2\,\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*pi*erf(b*x)*e^(-b^2*x^2) + sqrt(pi)*(pi*b*x*erf(b*x)^2 - 2*b*x*Ei(-
2*b^2*x^2)))/(pi*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2)/x**2,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erf(b*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^2, x)
```

3.85 $\int \frac{e^{-b^2x^2}\mathbf{Erf}(bx)}{x^4}\,dx$

Optimal. Leaf size=108

$$\frac{2b^2e^{-b^2x^2}\mathrm{Erf}(bx)}{3x}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{3x^3}+\frac{1}{3}\sqrt{\pi}b^3\mathrm{Erf}(bx)^2-\frac{4b^3\mathrm{ExpIntegralEi}\left(-2b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}$$

```
[Out] -b/(3*E^(2*b^2*x^2)*Sqrt[Pi]*x^2) - Erf[b*x]/(3*E^(b^2*x^2)*x^3) + (2*b^2*E
rf[b*x])/(3*E^(b^2*x^2)*x) + (b^3*Sqrt[Pi]*Erf[b*x]^2)/3 - (4*b^3*ExpIntegr
alEi[-2*b^2*x^2])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.147618, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6391, 6373, 30, 2210, 2214}

$$\frac{2b^2e^{-b^2x^2}\mathrm{Erf}(bx)}{3x}-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{3x^3}+\frac{1}{3}\sqrt{\pi}b^3\mathrm{Erf}(bx)^2-\frac{4b^3\mathrm{Ei}\left(-2b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/(E^(b^2*x^2)*x^4),x]
```


```
[Out] -b/(3*E^(2*b^2*x^2)*Sqrt[Pi]*x^2) - Erf[b*x]/(3*E^(b^2*x^2)*x^3) + (2*b^2*E
rf[b*x])/(3*E^(b^2*x^2)*x) + (b^3*Sqrt[Pi]*Erf[b*x]^2)/3 - (4*b^3*ExpIntegr
alEi[-2*b^2*x^2])/(3*Sqrt[Pi])
```


Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx &= -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{3x^3}-\frac{1}{3}\left(2b^2\right)\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx+\frac{(2b)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erf}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx-2\frac{\left(4b^3\right)\int \frac{e^{-2b^2x^2}}{x}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erf}(bx)}{3x}-\frac{4b^3\operatorname{Ei}\left(-2b^2x^2\right)}{3\sqrt{\pi}}+\frac{1}{3}\left(2b^3\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\operatorname{er}\\
&= -\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erf}(bx)}{3x}+\frac{1}{3}b^3\sqrt{\pi}\operatorname{erf}(bx)^2-\frac{4b^3\operatorname{Ei}\left(-2b^2x^2\right)}{3\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.0656073, size = 85, normalized size = 0.79

$$
\frac{1}{3}\left(\frac{e^{-b^2x^2}\left(2b^2x^2-1\right)\operatorname{Erf}(bx)}{x^3}+\sqrt{\pi}b^3\operatorname{Erf}(bx)^2+\frac{b\left(-4b^2\operatorname{ExpIntegralEi}\left(-2b^2x^2\right)-\frac{e^{-2b^2x^2}}{x^2}\right)}{\sqrt{\pi}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^4),x]
```

```
[Out] (((-1 + 2*b^2*x^2)*Erf[b*x])/(E^(b^2*x^2)*x^3) + b^3*Sqrt[Pi]*Erf[b*x]^2 +
(b*(-(1/(E^(2*b^2*x^2)*x^2)) - 4*b^2*ExpIntegralEi[-2*b^2*x^2]))/Sqrt[Pi])/
3
```

Maple [F] time = 0.412, size = 0, normalized size = 0.

$$\int \frac{\mathrm{Erf}\,(bx)}{\mathrm{e}^{b^2x^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2)/x^4,x)
```

```
[Out] int(erf(b*x)/exp(b^2*x^2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^4, x)
```

Fricas [A] time = 2.67961, size = 192, normalized size = 1.78

$$-\frac{\left(\pi - 2\,\pi b^2x^2\right)\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi b^3x^3\,\mathrm{erf}\,(bx)^2 - 4\,b^3x^3\mathrm{Ei}\left(-2\,b^2x^2\right) - bxe^{\left(-2\,b^2x^2\right)}\right)}{3\,\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*((pi - 2*pi*b^2*x^2)*erf(b*x)*e^(-b^2*x^2) - sqrt(pi)*(pi*b^3*x^3*erf(
b*x)^2 - 4*b^3*x^3*Ei(-2*b^2*x^2) - b*x*e^(-2*b^2*x^2)))/(pi*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2)/x**4,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erf(b*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*e^(-b^2*x^2)/x^4, x)
```

3.86 $\int e^{c+dx^2}x^3\textbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=342

$$\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d^2\sqrt{b^2-d}}-\frac{a^2b^3e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{5/2}}-\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4d\left(b^2-d\right)^{3/2}}-\frac{ab^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\sqrt{\pi}d\left(b^2-d\right)^2}+\frac{bxe^{-a^2-2abx-\cdots}}{2\sqrt{\pi}d\left(b^2\cdots\right.}$$

```
[Out] -(a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi])
+ (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (
E^(c + d*x^2)*Erf[a + b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erf[a + b*x])/(2*d
) + (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2
*Sqrt[b^2 - d]*d^2) - (a^2*b^3*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 -
d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(5/2)*d) - (b*E^(c + (a^2*d)/(b^2 - d))*
Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(3/2)*d)
```

Rubi [A] time = 0.514996, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6385, 6382, 2234, 2205, 2241, 2240}

$$\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d^2\sqrt{b^2-d}}-\frac{a^2b^3e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{5/2}}-\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4d\left(b^2-d\right)^{3/2}}-\frac{ab^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\sqrt{\pi}d\left(b^2-d\right)^2}+\frac{bxe^{-a^2-2abx-\cdots}}{2\sqrt{\pi}d\left(b^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^3*Erf[a + b*x],x]
```


```
[Out] -(a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi])
+ (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (
E^(c + d*x^2)*Erf[a + b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erf[a + b*x])/(2*d
) + (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2
*Sqrt[b^2 - d]*d^2) - (a^2*b^3*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 -
d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(5/2)*d) - (b*E^(c + (a^2*d)/(b^2 - d))*
Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(3/2)*d)
```


Rule 6385

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m - 1)*E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] + (-Dist[(m - 1)/(2
```

```
*d), Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[b/(d*Sqrt[Pi])
, Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[{a
, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2241

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] +
 (-Dist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x],
 x] - Dist[((m - 1)*e^2)/(2*c*Log[F]), Int[(d + e*x)^(m - 2)*F^(a + b*x + c
*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && Gt
Q[m, 1]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(e*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] - Dist[(b*e - 2*c*d)/(2*
c), Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[
b*e - 2*c*d, 0]
```

Rubi steps

$$\int e^{c+dx^2}x^3\mathrm{erf}(a+bx)\,dx = \frac{e^{c+dx^2}x^2\mathrm{erf}(a+bx)}{2d} - \frac{\int e^{c+dx^2}x\mathrm{erf}(a+bx)\,dx}{d} - \frac{b\int e^{-a^2+c-2abx+(-b^2+d)x^2}x^2\,dx}{d\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\mathrm{erf}(a+bx)}{2d} + \frac{b\int e^{-a^2+c-2abx-(b^2-d)x^2}\,dx}{d^2\sqrt{\pi}}$$

$$= -\frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\mathrm{erf}(a+bx)}{2d} -$$

$$= -\frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\mathrm{erf}(a+bx)}{2d} +$$

$$= -\frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\mathrm{erf}(a+bx)}{2d} +$$

<u>Mathematica</u> [A] time = 4.59637, size = 240, normalized size = 0.7

$$\frac{e^c\left(-\frac{bde^{-a^2-2abx+x^2(d-b^2)}\left(\sqrt{\pi}\sqrt{b^2-d}\left((2a^2+1)b^2-d\right)e^{\frac{(ab+x(b^2-d))^2}{b^2-d}}\mathrm{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)+2(b^2-d)(ab+x(d-b^2))\right)}{\sqrt{\pi}(b^2-d)^3}+\frac{2be^{\frac{a^2d}{b^2-d}}\mathrm{Erfi}\left(\frac{x(d-b^2)-ab}{\sqrt{d-b^2}}\right)}{\sqrt{d-b^2}}+2e^{dx^2}\left(dx^2-1\right.\right.}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^3*Erf[a + b*x],x]
```

```
[Out] (E^c*(2*E^(d*x^2)*(-1 + d*x^2)*Erf[a + b*x] - (b*d*E^(-a^2 - 2*a*b*x + (-b^
2 + d)*x^2)*(2*(b^2 - d)*(a*b + (-b^2 + d)*x) + Sqrt[b^2 - d]*((1 + 2*a^2)*
b^2 - d)*E^((a*b + (b^2 - d)*x)^2/(b^2 - d))*Sqrt[Pi]*Erf[(a*b + (b^2 - d)*
x)/Sqrt[b^2 - d]]))/((b^2 - d)^3*Sqrt[Pi]) + (2*b*E^((a^2*d)/(b^2 - d))*Erf
i[(-(a*b) + (-b^2 + d)*x)/Sqrt[-b^2 + d]])/Sqrt[-b^2 + d]))/(4*d^2)
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int \mathrm{e}^{dx^2+c}x^3\mathrm{Erf}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^3*erf(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^3*erf(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(dx^2e^c-e^c\right)\mathrm{erf}\,(bx+a)\,e^{\left(dx^2\right)}}{2\,d^2}-\frac{-\frac{\left(\frac{\sqrt{\pi}\left(ab+\left(b^2-d\right)x\right)a^2b^2\left(\mathrm{erf}\left(\sqrt{\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}\right)-1\right)}{\left(-b^2+d\right)^{\frac{5}{2}}\sqrt{\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}}-\frac{2\,abe^{\left(-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}\right)}}{\left(-b^2+d\right)^{\frac{3}{2}}}-\frac{\left(ab+\left(b^2-d\right)x\right)^3\Gamma\left(\frac{3}{2},\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}\right)}{\left(-b^2+d\right)^{\frac{5}{2}}\left(\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}\right)^{\frac{3}{2}}}\right)bde^{\left(\frac{a^2b^2}{b^2-d}-a^2\cdots\right.}}{2\sqrt{-b^2+d}}\cdots}{\sqrt{\pi}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2*e^c - e^c)*erf(b*x + a)*e^(d*x^2)/d^2 - integrate((b*d*x^2*e^c -
 b*e^c)*e^(-b^2*x^2 - 2*a*b*x + d*x^2 - a^2), x)/(sqrt(pi)*d^2)
```

Fricas [A] time = 2.81226, size = 549, normalized size = 1.61

$$\frac{\pi\left(2\,b^5-\left(2\,a^2+5\right)b^3d+3\,bd^2\right)\sqrt{b^2-d}\,\mathrm{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)}+2\left(\pi\left(b^6d-3\,b^4d^2+3\,b^2d^3-d^4\right)x^2-\pi\left(b^6-3\,b^4d\cdots\right.\right.}{4\,\pi\left(b^6d^2-3\,b^4d^3+3\,b^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(pi*(2*b^5 - (2*a^2 + 5)*b^3*d + 3*b*d^2)*sqrt(b^2 - d)*erf((a*b + (b^2
 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a^2 - c)*d)/(b^2 - d)) + 2*(pi*(b^6*d
```

```
- 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 - pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3))*
erf(b*x + a)*e^(d*x^2 + c) - 2*sqrt(pi)*(a*b^4*d - a*b^2*d^2 - (b^5*d - 2*b
^3*d^2 + b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x + d*x^2 - a^2 + c))/(pi*(b^6*d^2 -
 3*b^4*d^3 + 3*b^2*d^4 - d^5))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**3*erf(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29738, size = 367, normalized size = 1.07

$$\frac{\left(dx^2-1\right)\operatorname{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{2\,d^2}-\frac{\frac{2\,\pi b\operatorname{erf}\left(-\sqrt{b^2-d}\left(\frac{ab}{b^2-d}+x\right)\right)e^{\left(\frac{b^2c+a^2d-cd}{b^2-d}\right)}}{\sqrt{b^2-d}}-\frac{\sqrt{\pi}\left(\frac{\sqrt{\pi}\left(2\,a^2b^2+b^2-d\right)\operatorname{erf}\left(-\sqrt{b^2-d}\left(\frac{ab}{b^2-d}+x\right)\right)e^{\left(\frac{b^2c+a^2d-cd}{b^2-d}\right)}}{\sqrt{b^2-d}}+2\left(\left(\frac{ab}{b^2-d}+x\right)b^2-\cdots\right.\right.}{b^4-2\,b^2d+d^2}}{4\,\pi d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erf(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 - 1)*erf(b*x + a)*e^(d*x^2 + c)/d^2 - 1/4*(2*pi*b*erf(-sqrt(b^2
- d)*(a*b/(b^2 - d) + x))*e^((b^2*c + a^2*d - c*d)/(b^2 - d))/sqrt(b^2 - d)
 - sqrt(pi)*(sqrt(pi)*(2*a^2*b^2 + b^2 - d)*erf(-sqrt(b^2 - d)*(a*b/(b^2 -
d) + x))*e^((b^2*c + a^2*d - c*d)/(b^2 - d))/sqrt(b^2 - d) + 2*((a*b/(b^2 -
 d) + x)*b^2 - 2*a*b - (a*b/(b^2 - d) + x)*d)*e^(-b^2*x^2 - 2*a*b*x + d*x^2
 - a^2 + c))*b*d/(b^4 - 2*b^2*d + d^2))/(pi*d^2)
```

3.87 $\int e^{c+dx^2} x \mathbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=86

$$\frac{e^{c+dx^2}\text{Erf}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}$$

```
[Out] (E^(c + d*x^2)*Erf[a + b*x])/(2*d) - (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b
+ (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqrt[b^2 - d]*d)
```

Rubi [A] time = 0.0570083, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6382, 2234, 2205}

$$\frac{e^{c+dx^2}\text{Erf}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x*Erf[a + b*x],x]
```

```
[Out] (E^(c + d*x^2)*Erf[a + b*x])/(2*d) - (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b
+ (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqrt[b^2 - d]*d)
```

Rule 6382

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[(E^(c + d*x^2)*Erf[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(-a^2
 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{c+dx^2}x\operatorname{erf}(a+bx)\,dx &= \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{2d} - \frac{b\int e^{-a^2+c-2abx-\left(b^2-d\right)x^2}\,dx}{d\sqrt{\pi}}\\
&= \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{2d} - \frac{\left(be^{\frac{b^2c+a^2d-cd}{b^2-d}}\right)\int \exp\left(\frac{\left(-2ab+2\left(-b^2+d\right)x\right)^2}{4\left(-b^2+d\right)}\right)dx}{d\sqrt{\pi}}\\
&= \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{2d} - \frac{be^{\frac{b^2c+a^2d-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}d}
\end{aligned}$$

Mathematica [A] time = 0.105384, size = 82, normalized size = 0.95

$$\frac{e^c\left(e^{dx^2}\operatorname{Erf}(a+bx)-\frac{be^{\frac{a^2d}{b^2-d}}\operatorname{Erfi}\left(\frac{x\left(d-b^2\right)-ab}{\sqrt{d-b^2}}\right)}{\sqrt{d-b^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x*Erf[a + b*x],x]
```

```
[Out] (E^c*(E^(d*x^2)*Erf[a + b*x] - (b*E^((a^2*d)/(b^2 - d))*Erfi[(-(a*b) + (-b^
2 + d)*x)/Sqrt[-b^2 + d]])/Sqrt[-b^2 + d]))/(2*d)
```

Maple [A] time = 0.373, size = 134, normalized size = 1.6

$$\frac{1}{b}\left(\frac{\operatorname{Erf}(bx+a)\,b}{2\,d}e^{\frac{d(bx+a)^2}{b^2}-2\frac{ad(bx+a)}{b^2}+\frac{a^2d}{b^2}+c}-\frac{b}{2\,d}e^{\frac{a^2d}{b^2}+c-\frac{a^2d^2}{b^4}\left(-1+\frac{d}{b^2}\right)^{-1}}\operatorname{Erf}\left(\sqrt{1-\frac{d}{b^2}}(bx+a)+\frac{ad}{b^2}\frac{1}{\sqrt{1-\frac{d}{b^2}}}\right)\frac{1}{\sqrt{1-\frac{d}{b^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x*erf(b*x+a),x)
```

```
[Out] (1/2*erf(b*x+a)*b/d*exp(d*(b*x+a)^2/b^2-2/b^2*(b*x+a)*a*d+1/b^2*a^2*d+c)-1/
2*b/d*exp(1/b^2*a^2*d+c-a^2*d^2/b^4/(-1+d/b^2))/(1-d/b^2)^(1/2)*erf((1-d/b^
2)^(1/2)*(b*x+a)+a*d/b^2/(1-d/b^2)^(1/2)))/b
```

Maxima [A] time = 1.0529, size = 113, normalized size = 1.31

$$-\frac{b\operatorname{erf}\left(\frac{ab}{\sqrt{b^2-d}}+\sqrt{b^2-d}x\right)e^{\left(\frac{a^2b^2}{b^2-d}-a^2+c\right)}}{2\sqrt{b^2-d}d}+\frac{\operatorname{erf}(bx+a)e^{\left(dx^2+c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*erf(a*b/sqrt(b^2 - d) + sqrt(b^2 - d)*x)*e^(a^2*b^2/(b^2 - d) - a^2
+ c)/(sqrt(b^2 - d)*d) + 1/2*erf(b*x + a)*e^(d*x^2 + c)/d
```

Fricas [A] time = 2.73982, size = 205, normalized size = 2.38

$$-\frac{\sqrt{b^2-d}b\operatorname{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)}-\left(b^2-d\right)\operatorname{erf}(bx+a)e^{\left(dx^2+c\right)}}{2\left(b^2d-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(b^2 - d)*b*erf((a*b + (b^2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a
^2 - c)*d)/(b^2 - d)) - (b^2 - d)*erf(b*x + a)*e^(d*x^2 + c))/(b^2*d - d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x*erf(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24279, size = 117, normalized size = 1.36

$$\frac{b \operatorname{erf}\left(-\sqrt{b^2-d}\left(\frac{ab}{b^2-d}+x\right)\right)e^{\left(\frac{b^2c+a^2d-cd}{b^2-d}\right)}}{2\sqrt{b^2-d}d}+\frac{\operatorname{erf}(bx+a)\,e^{\left(dx^2+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erf(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*b*erf(-sqrt(b^2 - d)*(a*b/(b^2 - d) + x))*e^((b^2*c + a^2*d - c*d)/(b^2
 - d))/(sqrt(b^2 - d)*d) + 1/2*erf(b*x + a)*e^(d*x^2 + c)/d
```

3.88 $\int \frac{e^{c+dx^2}\mathbf{Erf}(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{e^{c+dx^2}\text{Erf}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + d*x^2)*Erf[a + b*x])/x, x]
```

Rubi [A] time = 0.0373856, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[a + b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + d*x^2)*Erf[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.200004, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x, x]
```

Maple [A] time = 0.1, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{d x^{2}+c} \mathrm{Erf}\left(b x+a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left(bx+a\right)e^{\left(dx^{2}+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erf}\left(bx+a\right)e^{\left(dx^{2}+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)*e^(d*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int \frac{e^{dx^{2}} \operatorname{erf}\left(a+bx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x+a)/x,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)\, e^{\left(dx^2+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x, x)
```

3.89 $\int \frac{e^{c+dx^2}\mathbf{Erf}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=183

$$-\frac{2ab^2\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{\sqrt{\pi}}+d\text{Unintegrable}\left(\frac{e^{c+dx^2}\text{Erf}(a+bx)}{x},x\right)-b\sqrt{b^2-d}e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)$$

```
[Out] -((b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)
*Erf[a + b*x])/(2*x^2) - b*Sqrt[b^2 - d]*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b
 + (b^2 - d)*x)/Sqrt[b^2 - d]] - (2*a*b^2*Unintegrable[E^(-a^2 + c - 2*a*b*
x + (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + d*Unintegrable[(E^(c + d*x^2)*Erf[a +
 b*x])/x, x]
```

Rubi [A] time = 0.416555, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[a + b*x])/x^3,x]
```

```
[Out] -((b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)
*Erf[a + b*x])/(2*x^2) - b*Sqrt[b^2 - d]*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b
 + (b^2 - d)*x)/Sqrt[b^2 - d]] - (2*a*b^2*Defer[Int][E^(-a^2 + c - 2*a*b*x
+ (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + d*Defer[Int][(E^(c + d*x^2)*Erf[a + b*x
])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^3}\,dx = -\frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx + \frac{b\int \frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx - \frac{\left(2ab^2\right)\int \frac{e^{-a^2+c-2abx+(-b}}{x}}{\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx - \frac{\left(2ab^2\right)\int \frac{e^{-a^2+c-2abx+(-b}}{x}}{\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2} - b\sqrt{b^2-d}e^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right) + d\int \frac{e^{c}}{}$$

Mathematica [A] time = 0.36426, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^3, x]
```

Maple [A] time = 0.337, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{Erf}\left(bx+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x+a)/x^3,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x+a)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)\,e^{\left(dx^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erf}(bx+a)\,e^{\left(dx^2+c\right)}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x+a)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)\,e^{\left(dx^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

3.90 $\int e^{c+dx^2}x^4\textbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=526

$$\frac{3\text{Unintegrable}\left(e^{c+dx^2}\text{Erf}(a+bx),x\right)}{4d^2}-\frac{3ab^2e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{4d^2\left(b^2-d\right)^{3/2}}-\frac{3be^{-a^2-2abx-x^2(b^2-d)+c}}{4\sqrt{\pi}d^2\left(b^2-d\right)}+\frac{a^3b^4e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{7/2}}$$

```
[Out] (-3*b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) +
(a^2*b^3*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^3*d*Sqrt[Pi])
 + (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) -
(a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)^2*d*Sqrt[Pi])
 + (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x^2)/(2*(b^2 - d)*d*Sqrt[Pi])
- (3*E^(c + d*x^2)*x*Erf[a + b*x])/(4*d^2) + (E^(c + d*x^2)*x^3*Erf[a + b*x
])/(2*d) - (3*a*b^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[
b^2 - d]])/(4*(b^2 - d)^(3/2)*d^2) + (a^3*b^4*E^(c + (a^2*d)/(b^2 - d))*Erf
[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(7/2)*d) + (3*a*b^2*E^(c
+ (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(
5/2)*d) + (3*Unintegrable[E^(c + d*x^2)*Erf[a + b*x], x])/(4*d^2)
```

Rubi [A] time = 0.968988, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^4\text{Erf}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^4*Erf[a + b*x],x]
```

```
[Out] (-3*b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) +
(a^2*b^3*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^3*d*Sqrt[Pi])
 + (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) -
(a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)^2*d*Sqrt[Pi])
 + (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x^2)/(2*(b^2 - d)*d*Sqrt[Pi])
- (3*E^(c + d*x^2)*x*Erf[a + b*x])/(4*d^2) + (E^(c + d*x^2)*x^3*Erf[a + b*x
])/(2*d) - (3*a*b^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[
b^2 - d]])/(4*(b^2 - d)^(3/2)*d^2) + (a^3*b^4*E^(c + (a^2*d)/(b^2 - d))*Erf
[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(7/2)*d) + (3*a*b^2*E^(c
+ (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(
```

```
5/2)*d) + (3*Defer[Int][E^(c + d*x^2)*Erf[a + b*x], x])/(4*d^2)
```

Rubi steps

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx = \frac{e^{c+dx^2}x^3\text{erf}(a+bx)}{2d} - \frac{3\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx}{2d} - \frac{b\int e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}x^3\,dx}{d\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}x^2}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{3e^{c+dx^2}x\text{erf}(a+bx)}{4d^2} + \frac{e^{c+dx^2}x^3\text{erf}(a+bx)}{2d} + \frac{3\int e^{c+dx^2}\text{erf}(a+bx)}{4d^2}$$

$$= -\frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{ab^2e^{-a^2+c-2abx-\left(b^2-d\right)x^2}x}{2\left(b^2-d\right)^2d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-\left(b^2-\right.}}{2\left(b^2-d\right)d\sqrt{}}$$

$$= -\frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} + \frac{a^2b^3e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^3d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{ab^2e^{-a^2+c-2abx-\left(b\right.}}{2\left(b^2-d\right)^2d}$$

$$= -\frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} + \frac{a^2b^3e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^3d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{ab^2e^{-a^2+c-2abx-\left(b\right.}}{2\left(b^2-d\right)^2d}$$

$$= -\frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} + \frac{a^2b^3e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^3d\sqrt{\pi}} + \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{ab^2e^{-a^2+c-2abx-\left(b\right.}}{2\left(b^2-d\right)^2d}$$

Mathematica [A] time = 0.499833, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^4\text{Erf}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^4*Erf[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^4*Erf[a + b*x], x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^4\text{Erf}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^4*erf(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^4*erf(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erf}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erf(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \operatorname{erf}(bx+a)\, e^{\left(dx^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^4*erf(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**4*erf(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erf}(bx + a) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erf(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*erf(b*x + a)*e^(d*x^2 + c), x)
```

3.91 $\int e^{c+dx^2} x^2 \mathbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=163

$$-\frac{\text{Unintegrable}\left(e^{c+dx^2}\text{Erf}(a+bx),x\right)}{2d}+\frac{ab^2e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{3/2}}+\frac{be^{-a^2-2abx-x^2(b^2-d)+c}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{xe^{c+dx^2}\text{Erf}(a+bx)}{2d}$$

```
[Out] (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) + (E^(c
 + d*x^2)*x*Erf[a + b*x])/(2*d) + (a*b^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b
 + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(3/2)*d) - Unintegrable[E^(c +
 d*x^2)*Erf[a + b*x], x]/(2*d)
```

Rubi [A] time = 0.189922, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2} x^2 \text{Erf}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^2*Erf[a + b*x],x]
```

```
[Out] (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) + (E^(c
 + d*x^2)*x*Erf[a + b*x])/(2*d) + (a*b^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b
 + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(3/2)*d) - Defer[Int][E^(c + d
*x^2)*Erf[a + b*x], x]/(2*d)
```

Rubi steps

$$\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx = \frac{e^{c+dx^2}x\text{erf}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erf}(a+bx)\,dx}{2d} - \frac{b\int e^{-a^2+c-2abx+(-b^2+d)x^2}x\,dx}{d\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)d\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erf}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erf}(a+bx)\,dx}{2d} + \frac{(ab^2)\int e^{-a^2+c-2abx+(-b}}{(b^2-d)d\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)d\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erf}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erf}(a+bx)\,dx}{2d} + \frac{\left(ab^2e^{\frac{b^2c+a^2d-cd}{b^2-d}}\right)\int \exp}{(b^2-d}$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)d\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erf}(a+bx)}{2d} + \frac{ab^2e^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}d} - \frac{\int e^{c+dx^2}\text{erf}(a+}{2d}$$

Mathematica [A] time = 0.39214, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^2\text{Erf}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^2*Erf[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^2*Erf[a + b*x], x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^2\text{Erf}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^2*erf(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^2*erf(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erf}(bx + a)\, e^{\left(dx^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erf(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \operatorname{erf}(bx + a)\, e^{\left(dx^2 + c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*erf(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**2*erf(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erf}(bx + a)\, e^{\left(dx^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erf(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*erf(b*x + a)*e^(d*x^2 + c), x)
```

3.92 $\int e^{c+dx^2} \mathbf{Erf}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(e^{c+dx^2}\text{Erf}(a+bx),x\right)$$

```
[Out] Unintegrable[E^(c + d*x^2)*Erf[a + b*x], x]
```

Rubi [A] time = 0.0150303, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}\text{Erf}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*Erf[a + b*x],x]
```

```
[Out] Defer[Int][E^(c + d*x^2)*Erf[a + b*x], x]
```

Rubi steps

$$\int e^{c+dx^2}\text{erf}(a+bx)\,dx=\int e^{c+dx^2}\text{erf}(a+bx)\,dx$$

Mathematica [A] time = 0.0415891, size = 0, normalized size = 0.

$$\int e^{c+dx^2}\text{Erf}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*Erf[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*Erf[a + b*x], x]
```

Maple [A] time = 0.124, size = 0, normalized size = 0.

$$\int \mathrm{e}^{d x^{2}+c} \operatorname{Erf}(b x+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx+a)\, e^{\left(dx^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{erf}(bx+a)\, e^{\left(dx^{2}+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int e^{dx^{2}} \operatorname{erf}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x+a),x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx+a)\,e^{(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c), x)
```

3.93 $\int \frac{e^{c+dx^2}\textbf{Erf}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=81

$$\frac{2b\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{\sqrt{\pi}}+2d\text{Unintegrable}\left(e^{c+dx^2}\text{Erf}(a+bx),x\right)-\frac{e^{c+dx^2}\text{Erf}(a+bx)}{x}$$

```
[Out] -((E^(c + d*x^2)*Erf[a + b*x])/x) + (2*b*Unintegrable[E^(-a^2 + c - 2*a*b*x
 + (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + 2*d*Unintegrable[E^(c + d*x^2)*Erf[a +
 b*x], x]
```

Rubi [A] time = 0.215272, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[a + b*x])/x^2,x]
```

```
[Out] -((E^(c + d*x^2)*Erf[a + b*x])/x) + (2*b*Defer[Int][E^(-a^2 + c - 2*a*b*x +
 (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + 2*d*Defer[Int][E^(c + d*x^2)*Erf[a + b*x
], x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^2}\,dx=-\frac{e^{c+dx^2}\text{erf}(a+bx)}{x}+(2d)\int e^{c+dx^2}\text{erf}(a+bx)\,dx+\frac{(2b)\int \frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x}\,dx}{\sqrt{\pi}}$$

Mathematica [A] time = 0.407365, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^2,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^2, x]
```

Maple [A] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\mathrm{Erf}\left(bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x+a)/x^2,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^c \int \frac{e^{dx^2} \operatorname{erf}(a + bx)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x+a)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erf(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx + a) e^{\left(dx^2 + c\right)}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

3.94 $\int \frac{e^{c+dx^2}\mathbf{Erf}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=351

$$\frac{4a^2b^3\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{3\sqrt{\pi}}-\frac{2b\left(b^2-d\right)\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{3\sqrt{\pi}}+\frac{4bd\text{Unintegrable}\left(\frac{e^{-a^2-2ab}}{}\right.}{3\sqrt{\pi}}$$

```
[Out] -(b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) + (2*a*b^2*E^(
-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x) - (E^(c + d*x^2)*Erf[a
+ b*x])/(3*x^3) - (2*d*E^(c + d*x^2)*Erf[a + b*x])/(3*x) + (2*a*b^2*Sqrt[b^
2 - d]*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/3
+ (4*a^2*b^3*Unintegrable[E^(-a^2 + c - 2*a*b*x + (-b^2 + d)*x^2)/x, x])/(3
*Sqrt[Pi]) - (2*b*(b^2 - d)*Unintegrable[E^(-a^2 + c - 2*a*b*x + (-b^2 + d)
*x^2)/x, x])/(3*Sqrt[Pi]) + (4*b*d*Unintegrable[E^(-a^2 + c - 2*a*b*x + (-b
^2 + d)*x^2)/x, x])/(3*Sqrt[Pi]) + (4*d^2*Unintegrable[E^(c + d*x^2)*Erf[a
+ b*x], x])/3
```

Rubi [A] time = 0.911931, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erf[a + b*x])/x^4,x]
```

```
[Out] -(b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) + (2*a*b^2*E^(
-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x) - (E^(c + d*x^2)*Erf[a
+ b*x])/(3*x^3) - (2*d*E^(c + d*x^2)*Erf[a + b*x])/(3*x) + (2*a*b^2*Sqrt[b^
2 - d]*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/3
+ (4*a^2*b^3*Defer[Int][E^(-a^2 + c - 2*a*b*x + (-b^2 + d)*x^2)/x, x])/(3*S
qrt[Pi]) - (2*b*(b^2 - d)*Defer[Int][E^(-a^2 + c - 2*a*b*x + (-b^2 + d)*x^2
)/x, x])/(3*Sqrt[Pi]) + (4*b*d*Defer[Int][E^(-a^2 + c - 2*a*b*x + (-b^2 + d
)*x^2)/x, x])/(3*Sqrt[Pi]) + (4*d^2*Defer[Int][E^(c + d*x^2)*Erf[a + b*x],
x])/3
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^4}\,dx = -\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3} + \frac{1}{3}(2d)\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^2}\,dx + \frac{(2b)\int \frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x^3}\,dx}{3\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erf}(a+bx)}{3x} + \frac{1}{3}\left(4d^2\right)\int e^{c+dx^2}\text{erf}(a+bx$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2} + \frac{2ab^2e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erf}(a+bx)}{3x} +$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2} + \frac{2ab^2e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erf}(a+bx)}{3x} +$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2} + \frac{2ab^2e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erf}(a+bx)}{3x} +$$

Mathematica [A] time = 0.546689, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erf}(a+bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^4,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^4, x]
```

Maple [A] time = 0.356, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{Erf}\,(bx+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erf(b*x+a)/x^4,x)
```

```
[Out] int(exp(d*x^2+c)*erf(b*x+a)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erf}(bx+a)\,e^{(dx^2+c)}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] integral(erf(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erf(b*x+a)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erf}(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erf(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erf(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

3.95 $\int \left(\frac{e^{-b^2x^2}\mathbf{Erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\mathbf{Erf}(bx)}{x}\right)dx$

Optimal. Leaf size=62

$$-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{2x^2}-\sqrt{2}b^2\mathrm{Erf}\left(\sqrt{2}bx\right)-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

```
[Out] -(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b^
2*Erf[Sqrt[2]*b*x]
```

Rubi [A] time = 0.144876, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6391, 2214, 2205}

$$-\frac{e^{-b^2x^2}\mathrm{Erf}(bx)}{2x^2}-\sqrt{2}b^2\mathrm{Erf}\left(\sqrt{2}bx\right)-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erf[b*x])/(E^(b^2*x^2)*x),x]
```

```
[Out] -(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b^
2*Erf[Sqrt[2]*b*x]
```

Rule 6391

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[(x^(m + 1)*E^(c + d*x^2)*Erf[a + b*x])/(m + 1), x] + (-Dist[(2*d)/(m
 + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Dist[(2*b)/((m +
1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x])
 /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
```

```
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)dx &= b^2\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx+\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx\\ &= -\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2x^2}+\frac{b\int \frac{e^{-2b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2x^2}-\frac{\left(4b^3\right)\int e^{-2b^2x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2x^2}-\sqrt{2}b^2\operatorname{erf}\left(\sqrt{2}bx\right)\end{aligned}$$

Mathematica [A] time = 0.108658, size = 62, normalized size = 1.

$$-\frac{e^{-b^2x^2}\operatorname{Erf}(bx)}{2x^2}-\sqrt{2}b^2\operatorname{Erf}\left(\sqrt{2}bx\right)-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erf[b*x])/(E^(b^2*x^2)*x),x]
```

```
[Out] -(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b^
2*Erf[Sqrt[2]*b*x]
```

Maple [A] time = 0.304, size = 67, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\operatorname{Erf}(bx)\,b}{2\,e^{b^2x^2}x^2}+\frac{b^3}{\sqrt{\pi}}\left(-\frac{1}{\left(e^{b^2x^2}\right)^2bx}-\sqrt{2}\sqrt{\pi}\operatorname{Erf}\left(bx\sqrt{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x)
```

```
[Out] (-1/2*erf(b*x)*b/exp(b^2*x^2)/x^2+1/Pi^(1/2)*b^3*(-1/exp(b^2*x^2)^2/b/x-2^(
1/2)*Pi^(1/2)*erf(b*x*2^(1/2))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{\sqrt{2}\sqrt{b^2x^2}b\Gamma\left(-\frac{1}{2},2\,b^2x^2\right)}{2\,x}}{\sqrt{\pi}}-\frac{\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x, algorith
m="maxima")
```

```
[Out] b*integrate(e^(-2*b^2*x^2)/x^2, x)/sqrt(pi) - 1/2*erf(b*x)*e^(-b^2*x^2)/x^2
```

Fricas [A] time = 2.59963, size = 176, normalized size = 2.84

$$-\frac{2\,\sqrt{2}\pi\sqrt{b^2}bx^2\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)+2\,\sqrt{\pi}bxe^{\left(-2\,b^2x^2\right)}+\pi\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{2\,\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x, algorith
m="fricas")
```

```
[Out] -1/2*(2*sqrt(2)*pi*sqrt(b^2)*b*x^2*erf(sqrt(2)*sqrt(b^2)*x) + 2*sqrt(pi)*b*
x*e^(-2*b^2*x^2) + pi*erf(b*x)*e^(-b^2*x^2))/(pi*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^2+1\right)e^{-b^2x^2}\operatorname{erf}\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b**2*x**2)/x**3+b**2*erf(b*x)/exp(b**2*x**2)/x,x)
```

```
[Out] Integral((b**2*x**2 + 1)*exp(-b**2*x**2)*erf(b*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^2 \operatorname{erf}(bx)\, e^{\left(-b^2x^2\right)}}{x} + \frac{\operatorname{erf}(bx)\, e^{\left(-b^2x^2\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x, algorith
m="giac")
```

```
[Out] integrate(b^2*erf(b*x)*e^(-b^2*x^2)/x + erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

3.96 $\int \mathbf{Erf}(bx)\sin\left(c+ib^2x^2\right)\,dx$

Optimal. Leaf size=66

$$\frac{ibe^{-ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{ic}\text{Erf}(bx)^2}{8b}$$

```
[Out] ((-I/8)*E^(I*c)*Sqrt[Pi]*Erf[b*x]^2)/b + ((I/2)*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*Sqrt[Pi])
```

Rubi [A] time = 0.0630758, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6404, 6376, 6373, 30}

$$\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{ic}\text{Erf}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]*Sin[c + I*b^2*x^2],x]
```

```
[Out] ((-I/8)*E^(I*c)*Sqrt[Pi]*Erf[b*x]^2)/b + ((I/2)*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*Sqrt[Pi])
```

Rule 6404

```
Int[Erf[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[I/2, Int[E
^(-(I*c) - I*d*x^2)*Erf[b*x], x], x] - Dist[I/2, Int[E^(I*c + I*d*x^2)*Erf[
b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx &= -\left(\frac{1}{2}i\int e^{ic-b^2x^2}\operatorname{erf}(bx)\,dx\right)+\frac{1}{2}i\int e^{-ic+b^2x^2}\operatorname{erf}(bx)\,dx\\ &= \frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\left(ie^{ic}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b}\\ &= -\frac{ie^{ic}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0690367, size = 69, normalized size = 1.05

$$\frac{(\cos(c)-i\sin(c))\left(4ib^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+\pi\text{Erf}(bx)^2(\sin(2c)-i\cos(2c))\right)}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]*Sin[c + I*b^2*x^2],x]
```

```
[Out] ((Cos[c] - I*Sin[c])*((4*I)*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2
*x^2] + Pi*Erf[b*x]^2*((-I)*Cos[2*c] + Sin[2*c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int \operatorname{Erf}(bx)\sin\left(c+ib^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)*sin(c+I*b^2*x^2),x)
```

```
[Out] int(erf(b*x)*sin(c+I*b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i\sqrt{\pi}\cos(c)\operatorname{erf}(bx)^2}{8b}+\frac{\sqrt{\pi}\operatorname{erf}(bx)^2\sin(c)}{8b}+\frac{1}{2}i\cos(c)\int\operatorname{erf}(bx)e^{(b^2x^2)}\,dx+\frac{1}{2}\int\operatorname{erf}(bx)e^{(b^2x^2)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sin(c+I*b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/8*I*sqrt(pi)*cos(c)*erf(b*x)^2/b + 1/8*sqrt(pi)*erf(b*x)^2*sin(c)/b + 1/
2*I*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erf(b*x)*e^(b
^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{2}\left(-i\operatorname{erf}(bx)e^{(-2b^2x^2+2ic)}+i\operatorname{erf}(bx)\right)e^{(b^2x^2-ic)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sin(c+I*b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/2*(-I*erf(b*x)*e^(-2*b^2*x^2 + 2*I*c) + I*erf(b*x))*e^(b^2*x^2 -
 I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(ib^2x^2+c\right)\operatorname{erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sin(c+I*b**2*x**2),x)
```

```
[Out] Integral(sin(I*b**2*x**2 + c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx)\sin\left(i\,b^2x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sin(c+I*b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*sin(I*b^2*x^2 + c), x)
```

3.97 $\int \mathbf{Erf}(bx)\sin\left(c-ib^2x^2\right)\,dx$

Optimal. Leaf size=66

$$\frac{i\sqrt{\pi}e^{-ic}\text{Erf}(bx)^2}{8b}-\frac{ibe^{ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}$$

```
[Out] ((I/8)*Sqrt[Pi]*Erf[b*x]^2)/(b*E^(I*c)) - ((I/2)*b*E^(I*c)*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0583387, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6404, 6373, 30, 6376}

$$\frac{i\sqrt{\pi}e^{-ic}\text{Erf}(bx)^2}{8b}-\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]*Sin[c - I*b^2*x^2],x]
```

```
[Out] ((I/8)*Sqrt[Pi]*Erf[b*x]^2)/(b*E^(I*c)) - ((I/2)*b*E^(I*c)*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6404

```
Int[Erf[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[I/2, Int[E
^(-(I*c) - I*d*x^2)*Erf[b*x], x], x] - Dist[I/2, Int[E^(I*c + I*d*x^2)*Erf[
b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)dx &= \frac{1}{2}i\int e^{-ic-b^2x^2}\operatorname{erf}(bx)\,dx-\frac{1}{2}i\int e^{ic+b^2x^2}\operatorname{erf}(bx)\,dx\\ &= -\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(ie^{-ic}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b}\\ &= \frac{ie^{-ic}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}-\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.06519, size = 67, normalized size = 1.02

$$\frac{(\sin(c)+i\cos(c))\left(\pi\operatorname{Erf}(bx)^2-4b^2x^2(\cos(2c)+i\sin(2c))\operatorname{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)\right)}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]*Sin[c - I*b^2*x^2],x]
```

```
[Out] ((I*Cos[c] + Sin[c])*(Pi*Erf[b*x]^2 - 4*b^2*x^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, b^2*x^2]*(Cos[2*c] + I*Sin[2*c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int -\operatorname{Erf}(bx)\sin\left(-c+ib^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-erf(b*x)*sin(-c+I*b^2*x^2),x)
```

```
[Out] int(-erf(b*x)*sin(-c+I*b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{i\sqrt{\pi}\cos(c)\operatorname{erf}(bx)^2}{8b}+\frac{\sqrt{\pi}\operatorname{erf}(bx)^2\sin(c)}{8b}-\frac{1}{2}i\cos(c)\int\operatorname{erf}(bx)e^{(b^2x^2)}\,dx+\frac{1}{2}\int\operatorname{erf}(bx)e^{(b^2x^2)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sin(-c+I*b^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*I*sqrt(pi)*cos(c)*erf(b*x)^2/b + 1/8*sqrt(pi)*erf(b*x)^2*sin(c)/b - 1/2
*I*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erf(b*x)*e^(b^
2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{2}\left(i\operatorname{erf}(bx)e^{(-2b^2x^2-2ic)}-i\operatorname{erf}(bx)\right)e^{(b^2x^2+ic)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sin(-c+I*b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/2*(I*erf(b*x)*e^(-2*b^2*x^2 - 2*I*c) - I*erf(b*x))*e^(b^2*x^2 +
I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\sin\left(ib^2x^2-c\right)\operatorname{erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sin(-c+I*b**2*x**2),x)
```

```
[Out] -Integral(sin(I*b**2*x**2 - c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{erf}(bx)\sin\left(i\,b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sin(-c+I*b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(-erf(b*x)*sin(I*b^2*x^2 - c), x)
```

3.98 $\int \cos\left(c + ib^2x^2\right)\mathbf{Erf}(bx)\,dx$

Optimal. Leaf size=62

$$\frac{be^{-ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{ic}\text{Erf}(bx)^2}{8b}$$

```
[Out] (E^(I*c)*Sqrt[Pi]*Erf[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2
, 2}, b^2*x^2])/(2*E^(I*c)*Sqrt[Pi])
```

Rubi [A] time = 0.0557298, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6407, 6376, 6373, 30}

$$\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{ic}\text{Erf}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + I*b^2*x^2]*Erf[b*x],x]
```

```
[Out] (E^(I*c)*Sqrt[Pi]*Erf[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2
, 2}, b^2*x^2])/(2*E^(I*c)*Sqrt[Pi])
```

Rule 6407

```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[E
^(-(I*c) - I*d*x^2)*Erf[b*x], x], x] + Dist[1/2, Int[E^(I*c + I*d*x^2)*Erf[
b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos\left(c+ib^2x^2\right)\operatorname{erf}(bx)\,dx &= \frac{1}{2}\int e^{ic-b^2x^2}\operatorname{erf}(bx)\,dx+\frac{1}{2}\int e^{-ic+b^2x^2}\operatorname{erf}(bx)\,dx\\
&= \frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^{ic}\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\operatorname{erf}(bx))}{4b}\\
&= \frac{e^{ic}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}
\end{aligned}$$

Mathematica [F] time = 0.614601, size = 0, normalized size = 0.

$$\int \cos\left(c+ib^2x^2\right)\operatorname{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + I*b^2*x^2]*Erf[b*x],x]
```

```
[Out] Integrate[Cos[c + I*b^2*x^2]*Erf[b*x], x]
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int \cos\left(c+ib^2x^2\right)\operatorname{Erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+I*b^2*x^2)*erf(b*x),x)
```

```
[Out] int(cos(c+I*b^2*x^2)*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\cos\left(c\right)\operatorname{erf}\left(bx\right)^{2}}{8\,b}+\frac{i\,\sqrt{\pi}\operatorname{erf}\left(bx\right)^{2}\sin\left(c\right)}{8\,b}+\frac{1}{2}\,\cos\left(c\right)\int\operatorname{erf}\left(bx\right)e^{\left(b^{2}x^{2}\right)}\,dx-\frac{1}{2}i\int\operatorname{erf}\left(bx\right)e^{\left(b^{2}x^{2}\right)}\,dx\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*cos(c)*erf(b*x)^2/b + 1/8*I*sqrt(pi)*erf(b*x)^2*sin(c)/b + 1/2
*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) - 1/2*I*integrate(erf(b*x)*e^(b^
2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{2}\left(\operatorname{erf}\left(bx\right)e^{\left(-2\,b^{2}x^{2}+2i\,c\right)}+\operatorname{erf}\left(bx\right)\right)e^{\left(b^{2}x^{2}-i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(erf(b*x)*e^(-2*b^2*x^2 + 2*I*c) + erf(b*x))*e^(b^2*x^2 - I*c)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(ib^{2}x^{2}+c\right)\operatorname{erf}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b**2*x**2)*erf(b*x),x)
```

```
[Out] Integral(cos(I*b**2*x**2 + c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(i\, b^2 x^2 + c\right) \operatorname{erf}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(I*b^2*x^2 + c)*erf(b*x), x)
```

3.99 $\int \cos\left(c - ib^2x^2\right) \mathbf{Erf}(bx)\, dx$

Optimal. Leaf size=62

$$\frac{be^{ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-ic}\text{Erf}(bx)^2}{8b}$$

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^(I*c)) + (b*E^(I*c)*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0547896, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6407, 6373, 30, 6376}

$$\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-ic}\text{Erf}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c - I*b^2*x^2]*Erf[b*x],x]
```

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^(I*c)) + (b*E^(I*c)*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rule 6407

```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[E
^(-(I*c) - I*d*x^2)*Erf[b*x], x], x] + Dist[1/2, Int[E^(I*c + I*d*x^2)*Erf[
b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \cos\left(c-ib^2x^2\right)\operatorname{erf}(bx)\,dx &= \frac{1}{2}\int e^{-ic-b^2x^2}\operatorname{erf}(bx)\,dx+\frac{1}{2}\int e^{ic+b^2x^2}\operatorname{erf}(bx)\,dx\\ &= \frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^{-ic}\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\operatorname{erf}(bx))}{4b}\\ &= \frac{e^{-ic}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 0.64351, size = 0, normalized size = 0.

$$\int \cos\left(c-ib^2x^2\right)\operatorname{Erf}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c - I*b^2*x^2]*Erf[b*x],x]
```

```
[Out] Integrate[Cos[c - I*b^2*x^2]*Erf[b*x], x]
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \cos\left(-c+ib^2x^2\right)\operatorname{Erf}(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(-c+I*b^2*x^2)*erf(b*x),x)
```

```
[Out] int(cos(-c+I*b^2*x^2)*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\cos\left(c\right)\operatorname{erf}\left(bx\right)^{2}}{8\,b}-\frac{i\,\sqrt{\pi}\operatorname{erf}\left(bx\right)^{2}\sin\left(c\right)}{8\,b}+\frac{1}{2}\,\cos\left(c\right)\int\operatorname{erf}\left(bx\right)e^{\left(b^{2}x^{2}\right)}\,dx+\frac{1}{2}i\,\int\operatorname{erf}\left(bx\right)e^{\left(b^{2}x^{2}\right)}\,dx\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*cos(c)*erf(b*x)^2/b - 1/8*I*sqrt(pi)*erf(b*x)^2*sin(c)/b + 1/2
*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) + 1/2*I*integrate(erf(b*x)*e^(b^
2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2}\left(\operatorname{erf}\left(bx\right)e^{\left(-2\,b^{2}x^{2}-2i\,c\right)}+\operatorname{erf}\left(bx\right)\right)e^{\left(b^{2}x^{2}+i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(erf(b*x)*e^(-2*b^2*x^2 - 2*I*c) + erf(b*x))*e^(b^2*x^2 + I*c)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(ib^{2}x^{2}-c\right)\operatorname{erf}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b**2*x**2)*erf(b*x),x)
```

```
[Out] Integral(cos(I*b**2*x**2 - c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(i\,b^{2}x^{2}-c\right)\operatorname{erf}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(I*b^2*x^2 - c)*erf(b*x), x)
```

3.100 $\int \mathbf{Erf}(bx) \sinh\left(c + b^2x^2\right) dx$

Optimal. Leaf size=56

$$\frac{be^c x^2 \text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}} - \frac{\sqrt{\pi}e^{-c}\text{Erf}(bx)^2}{8b}$$

```
[Out] -(Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0556531, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6410, 6376, 6373, 30}

$$\frac{be^c x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}} - \frac{\sqrt{\pi}e^{-c}\text{Erf}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]*Sinh[c + b^2*x^2],x]
```

```
[Out] -(Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rule 6410

```
Int[Erf[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[1/2, Int[
E^(c + d*x^2)*Erf[b*x], x], x] - Dist[1/2, Int[E^(-c - d*x^2)*Erf[b*x], x],
 x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx = -\left(\frac{1}{2}\int e^{-c-b^2x^2}\operatorname{erf}(bx)\,dx\right)+\frac{1}{2}\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx$$
$$=\frac{be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\left(e^{-c}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b}$$
$$=-\frac{e^{-c}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

Mathematica [A] time = 0.0397702, size = 57, normalized size = 1.02

$$\frac{4b^2x^2(\sinh(c)+\cosh(c))\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+\pi\text{Erf}(bx)^2(\sinh(c)-\cosh(c))}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]*Sinh[c + b^2*x^2],x]
```

```
[Out] (Pi*Erf[b*x]^2*(-Cosh[c] + Sinh[c]) + 4*b^2*x^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c]))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \operatorname{Erf}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(b*x)*sinh(b^2*x^2+c),x)
```

```
[Out] int(erf(b*x)*sinh(b^2*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\,(bx)^2\,e^{(-c)}}{8\,b}+\frac{1}{2}\int \mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sinh(b^2*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*erf(b*x)^2*e^(-c)/b + 1/2*integrate(erf(b*x)*e^(b^2*x^2 + c),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{erf}\,(bx)\sinh\left(b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sinh(b^2*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(erf(b*x)*sinh(b^2*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(b^2x^2+c\right)\mathrm{erf}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sinh(b**2*x**2+c),x)
```

```
[Out] Integral(sinh(b**2*x**2 + c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erf}(bx) \sinh\left(b^2x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(b*x)*sinh(b^2*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(erf(b*x)*sinh(b^2*x^2 + c), x)
```

3.101 $\int \mathbf{Erf}(bx)\sinh\left(c-b^2x^2\right)\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{\pi}e^c\mathrm{Erf}(bx)^2}{8b}-\frac{be^{-c}x^2\mathrm{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}$$

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(8*b) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rubi [A] time = 0.0561107, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6410, 6373, 30, 6376}

$$\frac{\sqrt{\pi}e^c\mathrm{Erf}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[b*x]*Sinh[c - b^2*x^2],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(8*b) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rule 6410

```
Int[Erf[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[1/2, Int[
E^(c + d*x^2)*Erf[b*x], x], x] - Dist[1/2, Int[E^(-c - d*x^2)*Erf[b*x], x],
 x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx &= \frac{1}{2}\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx-\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erf}(bx)\,dx\\ &= -\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b}\\ &= \frac{e^c\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0394838, size = 61, normalized size = 1.09

$$\frac{(\cosh(c)-\sinh(c))\left(\pi\operatorname{Erf}(bx)^2(\sinh(2c)+\cosh(2c))-4b^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)\right)}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[b*x]*Sinh[c - b^2*x^2],x]
```

```
[Out] ((Cosh[c] - Sinh[c])*(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^
2] + Pi*Erf[b*x]^2*(Cosh[2*c] + Sinh[2*c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int -\operatorname{Erf}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-erf(b*x)*sinh(b^2*x^2-c),x)
```

```
[Out] int(-erf(b*x)*sinh(b^2*x^2-c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\,\mathrm{erf}\,(bx)^2\,e^c}{8\,b}-\frac{1}{2}\int \mathrm{erf}\,(bx)\,e^{\left(b^2x^2-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sinh(b^2*x^2-c),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*erf(b*x)^2*e^c/b - 1/2*integrate(erf(b*x)*e^(b^2*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\,\mathrm{erf}\,(bx)\sinh\left(b^2x^2-c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sinh(b^2*x^2-c),x, algorithm="fricas")
```

```
[Out] integral(-erf(b*x)*sinh(b^2*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \sinh\left(b^2x^2-c\right)\mathrm{erf}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sinh(b**2*x**2-c),x)
```

```
[Out] -Integral(sinh(b**2*x**2 - c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\,\mathrm{erf}\,(bx)\sinh\left(b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erf(b*x)*sinh(b^2*x^2-c),x, algorithm="giac")
```

```
[Out] integrate(-erf(b*x)*sinh(b^2*x^2 - c), x)
```

3.102 $\int \cosh\left(c+b^2x^2\right) \mathbf{Erf}(bx)\, dx$

Optimal. Leaf size=56

$$\frac{be^c x^2 \text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{Erf}(bx)^2}{8b}$$

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2
, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0531506, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6413, 6376, 6373, 30}

$$\frac{be^c x^2 {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{Erf}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + b^2*x^2]*Erf[b*x],x]
```

```
[Out] (Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2
, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rule 6413

```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[
E^(c + d*x^2)*Erf[b*x], x], x] + Dist[1/2, Int[E^(-c - d*x^2)*Erf[b*x], x],
 x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cosh\left(c+b^2x^2\right)\operatorname{erf}(bx)\,dx &= \frac{1}{2}\int e^{-c-b^2x^2}\operatorname{erf}(bx)\,dx+\frac{1}{2}\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx\\
&= \frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^{-c}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b}\\
&= \frac{e^{-c}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.256855, size = 93, normalized size = 1.66

$$\frac{4b^2x^2\sinh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)-4b^2x^2\cosh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + b^2*x^2]*Erf[b*x],x]
```

```
[Out] (-4*b^2*x^2*Cosh[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)] + Pi*Er
f[b*x]*(2*Cosh[c]*Erfi[b*x] + Erf[b*x]*(Cosh[c] - Sinh[c])) + 4*b^2*x^2*Hyp
ergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*Sinh[c])/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right)\operatorname{Erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b^2*x^2+c)*erf(b*x),x)
```

```
[Out] int(cosh(b^2*x^2+c)*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\operatorname{erf}(bx)^2 e^{(-c)}}{8\,b} + \frac{1}{2}\int \operatorname{erf}(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*erf(b*x)^2*e^(-c)/b + 1/2*integrate(erf(b*x)*e^(b^2*x^2 + c),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\cosh\left(b^2x^2+c\right)\operatorname{erf}(bx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(b^2*x^2 + c)*erf(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right)\operatorname{erf}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b**2*x**2+c)*erf(b*x),x)
```

```
[Out] Integral(cosh(b**2*x**2 + c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2 + c\right) \operatorname{erf}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(b^2*x^2 + c)*erf(b*x), x)
```

3.103 $\int \cosh\left(c - b^2x^2\right) \mathbf{Erf}(bx)\, dx$

Optimal. Leaf size=56

$$\frac{be^{-c}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^c\text{Erf}(bx)^2}{8b}$$

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rubi [A] time = 0.0524462, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6413, 6373, 30, 6376}

$$\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^c\text{Erf}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c - b^2*x^2]*Erf[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erf[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rule 6413

```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[
E^(c + d*x^2)*Erf[b*x], x], x] + Dist[1/2, Int[E^(-c - d*x^2)*Erf[b*x], x],
 x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$
\begin{aligned}
\int \cosh\left(c-b^2x^2\right)\operatorname{erf}(bx)\,dx &= \frac{1}{2}\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx+\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erf}(bx)\,dx\\
&= \frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}(bx)\right)}{4b}\\
&= \frac{e^c\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.101778, size = 91, normalized size = 1.62

$$
\frac{-4b^2x^2\sinh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)-4b^2x^2\cosh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right) \ldots}{8\sqrt{\pi}b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c - b^2*x^2]*Erf[b*x],x]
```

```
[Out] (-4*b^2*x^2*Cosh[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)] - 4*b^2
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*Sinh[c] + Pi*Erf[b*x]*(2*
Cosh[c]*Erfi[b*x] + Erf[b*x]*(Cosh[c] + Sinh[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$
\int \cosh\left(b^2x^2-c\right)\operatorname{Erf}(bx)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b^2*x^2-c)*erf(b*x),x)
```

```
[Out] int(cosh(b^2*x^2-c)*erf(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\,\text{erf}\,(bx)^2\,e^c}{8\,b} + \frac{1}{2}\int \text{erf}\,(bx)\,e^{\left(b^2x^2-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erf(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*erf(b*x)^2*e^c/b + 1/2*integrate(erf(b*x)*e^(b^2*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(b^2x^2-c\right)\text{erf}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erf(b*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(b^2*x^2 - c)*erf(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2-c\right)\text{erf}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b**2*x**2-c)*erf(b*x),x)
```

```
[Out] Integral(cosh(b**2*x**2 - c)*erf(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2 - c\right) \operatorname{erf}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erf(b*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(b^2*x^2 - c)*erf(b*x), x)
```

3.104 $\int x^5 \mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=96

$$\frac{5\text{Erf}(bx)}{16b^6}-\frac{x^5e^{-b^2x^2}}{6\sqrt{\pi}b}-\frac{5x^3e^{-b^2x^2}}{12\sqrt{\pi}b^3}-\frac{5xe^{-b^2x^2}}{8\sqrt{\pi}b^5}+\frac{1}{6}x^6\text{Erfc}(bx)$$

```
[Out] (-5*x)/(8*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (5*x^3)/(12*b^3*E^(b^2*x^2)*Sqrt[Pi])
 - x^5/(6*b*E^(b^2*x^2)*Sqrt[Pi]) + (5*Erf[b*x])/(16*b^6) + (x^6*Erfc[b*x])
/6
```

Rubi [A] time = 0.0897058, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2212, 2205}

$$\frac{5\text{Erf}(bx)}{16b^6}-\frac{x^5e^{-b^2x^2}}{6\sqrt{\pi}b}-\frac{5x^3e^{-b^2x^2}}{12\sqrt{\pi}b^3}-\frac{5xe^{-b^2x^2}}{8\sqrt{\pi}b^5}+\frac{1}{6}x^6\text{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Erfc[b*x],x]
```

```
[Out] (-5*x)/(8*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (5*x^3)/(12*b^3*E^(b^2*x^2)*Sqrt[Pi])
 - x^5/(6*b*E^(b^2*x^2)*Sqrt[Pi]) + (5*Erf[b*x])/(16*b^6) + (x^6*Erfc[b*x])
/6
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^5\text{erfc}(bx)\,dx &= \frac{1}{6}x^6\text{erfc}(bx) + \frac{b\int e^{-b^2x^2}x^6\,dx}{3\sqrt{\pi}} \\
&= -\frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfc}(bx) + \frac{5\int e^{-b^2x^2}x^4\,dx}{6b\sqrt{\pi}} \\
&= -\frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfc}(bx) + \frac{5\int e^{-b^2x^2}x^2\,dx}{4b^3\sqrt{\pi}} \\
&= -\frac{5e^{-b^2x^2}x}{8b^5\sqrt{\pi}} - \frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfc}(bx) + \frac{5\int e^{-b^2x^2}\,dx}{8b^5\sqrt{\pi}} \\
&= -\frac{5e^{-b^2x^2}x}{8b^5\sqrt{\pi}} - \frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{5\text{erf}(bx)}{16b^6} + \frac{1}{6}x^6\text{erfc}(bx)
\end{aligned}$$

Mathematica [A] time = 0.0581878, size = 62, normalized size = 0.65

$$\frac{1}{48}\left(\frac{15\text{Erf}(bx)}{b^6} - \frac{2xe^{-b^2x^2}\left(4b^4x^4+10b^2x^2+15\right)}{\sqrt{\pi}b^5} + 8x^6\text{Erfc}(bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Erfc[b*x],x]
```

```
[Out] ((-2*x*(15 + 10*b^2*x^2 + 4*b^4*x^4))/(b^5*E^(b^2*x^2)*Sqrt[Pi]) + (15*Erf[
b*x])/b^6 + 8*x^6*Erfc[b*x])/48
```

Maple [A] time = 0.046, size = 83, normalized size = 0.9

$$\frac{1}{b^6}\left(\frac{b^6x^6\text{erfc}\,(bx)}{6} + \frac{1}{3\sqrt{\pi}}\left(-\frac{b^5x^5}{2\,\mathrm{e}^{b^2x^2}} - \frac{5\,x^3b^3}{4\,\mathrm{e}^{b^2x^2}} - \frac{15\,bx}{8\,\mathrm{e}^{b^2x^2}} + \frac{15\,\sqrt{\pi}\text{Erf}\,(bx)}{16}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erfc(b*x),x)
```

```
[Out] 1/b^6*(1/6*b^6*x^6*erfc(b*x)+1/3/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^5*x^5-5/4*b^
3*x^3/exp(b^2*x^2)-15/8*b*x/exp(b^2*x^2)+15/16*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.00688, size = 85, normalized size = 0.89

$$\frac{1}{6}\,x^{6}\operatorname{erfc}\left(bx\right)-\frac{b\left(\frac{2\left(4\,b^{4}x^{5}+10\,b^{2}x^{3}+15\,x\right)e^{\left(-b^{2}x^{2}\right)}}{b^{6}}-\frac{15\,\sqrt{\pi}\operatorname{erf}(bx)}{b^{7}}\right)}{48\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*erfc(b*x) - 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-b^2*x^2)/
b^6 - 15*sqrt(pi)*erf(b*x)/b^7)/sqrt(pi)
```

Fricas [A] time = 2.40923, size = 167, normalized size = 1.74

$$\frac{8\,\pi b^{6}x^{6}-2\,\sqrt{\pi}\left(4\,b^{5}x^{5}+10\,b^{3}x^{3}+15\,bx\right)e^{\left(-b^{2}x^{2}\right)}+\left(15\,\pi-8\,\pi b^{6}x^{6}\right)\operatorname{erf}\left(bx\right)}{48\,\pi b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/48*(8*pi*b^6*x^6 - 2*sqrt(pi)*(4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*e^(-b^2*x
^2) + (15*pi - 8*pi*b^6*x^6)*erf(b*x))/(pi*b^6)
```

Sympy [A] time = 4.05136, size = 92, normalized size = 0.96

$$\begin{cases}\frac{x^{6}\operatorname{erfc}\left(bx\right)}{6}-\frac{x^{5}e^{-b^{2}x^{2}}}{6\sqrt{\pi}b}-\frac{5x^{3}e^{-b^{2}x^{2}}}{12\sqrt{\pi}b^{3}}-\frac{5xe^{-b^{2}x^{2}}}{8\sqrt{\pi}b^{5}}-\frac{5\operatorname{erfc}\left(bx\right)}{16b^{6}} & \text{for}\: b\neq 0\\\frac{x^{6}}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erfc(b*x),x)
```

```
[Out] Piecewise((x**6*erfc(b*x)/6 - x**5*exp(-b**2*x**2)/(6*sqrt(pi)*b) - 5*x**3*
exp(-b**2*x**2)/(12*sqrt(pi)*b**3) - 5*x*exp(-b**2*x**2)/(8*sqrt(pi)*b**5)
- 5*erfc(b*x)/(16*b**6), Ne(b, 0)), (x**6/6, True))
```

Giac [A] time = 1.37257, size = 93, normalized size = 0.97

$$-\frac{1}{6}\,x^{6}\,\mathrm{erf}\left(bx\right)+\frac{1}{6}\,x^{6}-\frac{b\left(\frac{2\left(4\,b^{4}x^{5}+10\,b^{2}x^{3}+15\,x\right)e^{\left(-b^{2}x^{2}\right)}}{b^{6}}+\frac{15\,\sqrt{\pi}\,\mathrm{erf}(-bx)}{b^{7}}\right)}{48\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x),x, algorithm="giac")
```

```
[Out] -1/6*x^6*erf(b*x) + 1/6*x^6 - 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(
-b^2*x^2)/b^6 + 15*sqrt(pi)*erf(-b*x)/b^7)/sqrt(pi)
```

3.105 $\int x^3 \mathbf{Erfc}(bx)\, dx$

Optimal. Leaf size=71

$$\frac{3\mathrm{Erf}(bx)}{16b^4} - \frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b} - \frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3} + \frac{1}{4}x^4\mathrm{Erfc}(bx)$$

```
[Out] (-3*x)/(8*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^3/(4*b*E^(b^2*x^2)*Sqrt[Pi]) + (3*E
rf[b*x])/(16*b^4) + (x^4*Erfc[b*x])/4
```

Rubi [A] time = 0.0635941, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2212, 2205}

$$\frac{3\mathrm{Erf}(bx)}{16b^4} - \frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b} - \frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3} + \frac{1}{4}x^4\mathrm{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Erfc[b*x],x]
```

```
[Out] (-3*x)/(8*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^3/(4*b*E^(b^2*x^2)*Sqrt[Pi]) + (3*E
rf[b*x])/(16*b^4) + (x^4*Erfc[b*x])/4
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^3\text{erfc}(bx)\,dx &= \frac{1}{4}x^4\text{erfc}(bx)+\frac{b\int e^{-b^2x^2}x^4\,dx}{2\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)+\frac{3\int e^{-b^2x^2}x^2\,dx}{4b\sqrt{\pi}}\\ &= -\frac{3e^{-b^2x^2}x}{8b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)+\frac{3\int e^{-b^2x^2}\,dx}{8b^3\sqrt{\pi}}\\ &= -\frac{3e^{-b^2x^2}x}{8b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}}+\frac{3\text{erf}(bx)}{16b^4}+\frac{1}{4}x^4\text{erfc}(bx)\end{aligned}$$

Mathematica [A] time = 0.0525482, size = 54, normalized size = 0.76

$$\frac{1}{16}\left(\frac{3\text{Erf}(bx)}{b^4}-\frac{2xe^{-b^2x^2}\left(2b^2x^2+3\right)}{\sqrt{\pi}b^3}+4x^4\text{Erfc}(bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Erfc[b*x],x]
```

```
[Out] ((-2*x*(3 + 2*b^2*x^2))/(b^3*E^(b^2*x^2)*Sqrt[Pi]) + (3*Erf[b*x])/b^4 + 4*x
^4*Erfc[b*x])/16
```

Maple [A] time = 0.042, size = 65, normalized size = 0.9

$$\frac{1}{b^4}\left(\frac{b^4x^4\text{erfc}\,(bx)}{4}+\frac{1}{2\sqrt{\pi}}\left(-\frac{x^3b^3}{2\,e^{b^2x^2}}-\frac{3\,bx}{4\,e^{b^2x^2}}+\frac{3\sqrt{\pi}\text{Erf}\,(bx)}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erfc(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*erfc(b*x)+1/2/Pi^(1/2)*(-1/2*b^3*x^3/exp(b^2*x^2)-3/4*b*
x/exp(b^2*x^2)+3/8*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.00585, size = 74, normalized size = 1.04

$$\frac{1}{4}\,x^4\operatorname{erfc}(bx)-\frac{b\left(\frac{2\left(2\,b^2x^3+3\,x\right)e^{\left(-b^2x^2\right)}}{b^4}-\frac{3\sqrt{\pi}\operatorname{erf}(bx)}{b^5}\right)}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*erfc(b*x) - 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4 - 3*sqrt(p
i)*erf(b*x)/b^5)/sqrt(pi)
```

Fricas [A] time = 2.38594, size = 147, normalized size = 2.07

$$\frac{4\,\pi b^4x^4-2\sqrt{\pi}\left(2\,b^3x^3+3\,bx\right)e^{\left(-b^2x^2\right)}+\left(3\,\pi-4\,\pi b^4x^4\right)\operatorname{erf}(bx)}{16\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/16*(4*pi*b^4*x^4 - 2*sqrt(pi)*(2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) + (3*pi -
4*pi*b^4*x^4)*erf(b*x))/(pi*b^4)
```

Sympy [A] time = 1.28072, size = 68, normalized size = 0.96

$$\begin{cases}\frac{x^4\operatorname{erfc}(bx)}{4}-\frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b}-\frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3}-\frac{3\operatorname{erfc}(bx)}{16b^4} & \text{for } b\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erfc(b*x),x)
```

```
[Out] Piecewise((x**4*erfc(b*x)/4 - x**3*exp(-b**2*x**2)/(4*sqrt(pi)*b) - 3*x*exp
(-b**2*x**2)/(8*sqrt(pi)*b**3) - 3*erfc(b*x)/(16*b**4), Ne(b, 0)), (x**4/4,
 True))
```

Giac [A] time = 1.33988, size = 82, normalized size = 1.15

$$-\frac{1}{4}\,x^4\,\mathrm{erf}\,(bx)+\frac{1}{4}\,x^4-\frac{b\left(\frac{2\left(2\,b^2x^3+3\,x\right)e^{\left(-b^2x^2\right)}}{b^4}+\frac{3\,\sqrt{\pi}\,\mathrm{erf}(-bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x),x, algorithm="giac")
```

```
[Out] -1/4*x^4*erf(b*x) + 1/4*x^4 - 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4
+ 3*sqrt(pi)*erf(-b*x)/b^5)/sqrt(pi)
```

3.106 $\int x\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=46

$$\frac{\text{Erf}(bx)}{4b^2}-\frac{xe^{-b^2x^2}}{2\sqrt{\pi}b}+\frac{1}{2}x^2\text{Erfc}(bx)$$

```
[Out] -x/(2*b*E^(b^2*x^2)*Sqrt[Pi]) + Erf[b*x]/(4*b^2) + (x^2*Erfc[b*x])/2
```

Rubi [A] time = 0.0346855, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6362, 2212, 2205}

$$\frac{\text{Erf}(bx)}{4b^2}-\frac{xe^{-b^2x^2}}{2\sqrt{\pi}b}+\frac{1}{2}x^2\text{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*Erfc[b*x],x]
```

```
[Out] -x/(2*b*E^(b^2*x^2)*Sqrt[Pi]) + Erf[b*x]/(4*b^2) + (x^2*Erfc[b*x])/2
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\text{erfc}(bx)\,dx = \frac{1}{2}x^2\text{erfc}(bx) + \frac{b\int e^{-b^2x^2}x^2\,dx}{\sqrt{\pi}}$$
$$= -\frac{e^{-b^2x^2}x}{2b\sqrt{\pi}} + \frac{1}{2}x^2\text{erfc}(bx) + \frac{\int e^{-b^2x^2}\,dx}{2b\sqrt{\pi}}$$
$$= -\frac{e^{-b^2x^2}x}{2b\sqrt{\pi}} + \frac{\text{erf}(bx)}{4b^2} + \frac{1}{2}x^2\text{erfc}(bx)$$

Mathematica [A] time = 0.0417538, size = 43, normalized size = 0.93

$$\frac{1}{4}\left(\frac{\text{Erf}(bx)}{b^2} + 2x\left(x\text{Erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erfc[b*x],x]
```

```
[Out] (Erf[b*x]/b^2 + 2*x*(-(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]))/4
```

Maple [A] time = 0.051, size = 46, normalized size = 1.

$$\frac{1}{b^2}\left(\frac{b^2x^2\text{erfc}\,(bx)}{2} + \frac{1}{\sqrt{\pi}}\left(-\frac{bx}{2\,e^{b^2x^2}} + \frac{\sqrt{\pi}\text{Erf}\,(bx)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfc(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*erfc(b*x)+1/Pi^(1/2)*(-1/2*b*x/exp(b^2*x^2)+1/4*Pi^(1/2)
*erf(b*x)))
```

Maxima [A] time = 1.14194, size = 59, normalized size = 1.28

$$\frac{1}{2}\,x^2\,\mathrm{erfc}\,(bx) - \frac{b\left(\frac{2\,xe^{\left(-b^2x^2\right)}}{b^2} - \frac{\sqrt{\pi}\,\mathrm{erf}(bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*erfc(b*x) - 1/4*b*(2*x*e^(-b^2*x^2)/b^2 - sqrt(pi)*erf(b*x)/b^3)/sq
rt(pi)
```

Fricas [A] time = 2.53695, size = 122, normalized size = 2.65

$$\frac{2\,\pi b^2x^2 - 2\,\sqrt{\pi}bxe^{\left(-b^2x^2\right)} + \left(\pi - 2\,\pi b^2x^2\right)\mathrm{erf}\,(bx)}{4\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*pi*b^2*x^2 - 2*sqrt(pi)*b*x*e^(-b^2*x^2) + (pi - 2*pi*b^2*x^2)*erf(b
*x))/(pi*b^2)
```

Sympy [A] time = 0.448672, size = 42, normalized size = 0.91

$$\begin{cases} \frac{x^2\,\mathrm{erfc}\,(bx)}{2} - \frac{xe^{-b^2x^2}}{2\sqrt{\pi}b} - \frac{\mathrm{erfc}\,(bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x),x)
```

```
[Out] Piecewise((x**2*erfc(b*x)/2 - x*exp(-b**2*x**2)/(2*sqrt(pi)*b) - erfc(b*x)/
(4*b**2), Ne(b, 0)), (x**2/2, True))
```

Giac [A] time = 1.32194, size = 66, normalized size = 1.43

$$-\frac{1}{2}x^2\operatorname{erf}(bx)+\frac{1}{2}x^2-\frac{b\left(\frac{2xe^{\left(-b^2x^2\right)}}{b^2}+\frac{\sqrt{\pi}\operatorname{erf}(-bx)}{b^3}\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x),x, algorithm="giac")
```

```
[Out] -1/2*x^2*erf(b*x) + 1/2*x^2 - 1/4*b*(2*x*e^(-b^2*x^2)/b^2 + sqrt(pi)*erf(-b
*x)/b^3)/sqrt(pi)
```

3.107 $\int \frac{\textbf{Erfc}(bx)}{x} \, dx$

Optimal. Leaf size=35

$$\log(x) - \frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (-2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi] + L
og[x]
```

Rubi [A] time = 0.0234426, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6359, 6358}

$$\log(x) - \frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x,x]
```

```
[Out] (-2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi] + L
og[x]
```

Rule 6359

```
Int[Erfc[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[Log[x], x] - Int[Erf[b*x]/x, x
] /; FreeQ[b, x]
```

Rule 6358

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{\operatorname{erfc}(bx)}{x}\,dx = \log(x) - \int \frac{\operatorname{erf}(bx)}{x}\,dx$$
$$= -\frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}} + \log(x)$$

Mathematica [A] time = 0.0205093, size = 45, normalized size = 1.29

$$\log(x)(\operatorname{Erf}(bx)+\operatorname{Erfc}(bx)) - \frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x,x]
```

```
[Out] (-2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi] + (
Erf[b*x] + Erfc[b*x])*Log[x]
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x,x)
```

```
[Out] int(erfc(b*x)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{erf}\,(bx)-1}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x, x)
```

3.108 $\int \frac{\textbf{Erfc}(bx)}{x^3} dx$

Optimal. Leaf size=40

$$b^2\text{Erf}(bx) + \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\text{Erfc}(bx)}{2x^2}$$

```
[Out] b/(E^(b^2*x^2)*Sqrt[Pi]*x) + b^2*Erf[b*x] - Erfc[b*x]/(2*x^2)
```

Rubi [A] time = 0.0378237, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2214, 2205}

$$b^2\text{Erf}(bx) + \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\text{Erfc}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x^3,x]
```

```
[Out] b/(E^(b^2*x^2)*Sqrt[Pi]*x) + b^2*Erf[b*x] - Erfc[b*x]/(2*x^2)
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{erfc}(bx)}{x^3}\,dx &= -\frac{\operatorname{erfc}(bx)}{2x^2}-\frac{b\int \frac{e^{-b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{\sqrt{\pi}x}-\frac{\operatorname{erfc}(bx)}{2x^2}+\frac{\left(2b^3\right)\int e^{-b^2x^2}\,dx}{\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{\sqrt{\pi}x}+b^2\operatorname{erf}(bx)-\frac{\operatorname{erfc}(bx)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0410057, size = 40, normalized size = 1.

$$b^2\operatorname{Erf}(bx)+\frac{be^{-b^2x^2}}{\sqrt{\pi}x}-\frac{\operatorname{Erfc}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x^3,x]
```

```
[Out] b/(E^(b^2*x^2)*Sqrt[Pi]*x) + b^2*Erf[b*x] - Erfc[b*x]/(2*x^2)
```

Maple [A] time = 0.045, size = 51, normalized size = 1.3

$$b^2\left(-\frac{\operatorname{erfc}(bx)}{2\,b^2x^2}-\frac{1}{\sqrt{\pi}}\left(-\frac{1}{e^{b^2x^2}bx}-\sqrt{\pi}\operatorname{Erf}(bx)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2/b^2/x^2*erfc(b*x)-1/Pi^(1/2)*(-1/exp(b^2*x^2)/b/x-Pi^(1/2)*erf(b*
x)))
```

Maxima [A] time = 1.17421, size = 50, normalized size = 1.25

$$\frac{\sqrt{b^2x^2}b\Gamma\left(-\frac{1}{2},b^2x^2\right)}{2\sqrt{\pi}x}-\frac{\operatorname{erfc}(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^2)*b*gamma(-1/2, b^2*x^2)/(sqrt(pi)*x) - 1/2*erfc(b*x)/x^2
```

Fricas [A] time = 2.44195, size = 109, normalized size = 2.72

$$-\frac{\pi-2\sqrt{\pi}bxe^{\left(-b^2x^2\right)}-\left(\pi+2\pi b^2x^2\right)\operatorname{erf}(bx)}{2\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(pi - 2*sqrt(pi)*b*x*e^(-b^2*x^2) - (pi + 2*pi*b^2*x^2)*erf(b*x))/(pi*
x^2)
```

Sympy [A] time = 0.60934, size = 34, normalized size = 0.85

$$-b^2\operatorname{erfc}(bx)+\frac{be^{-b^2x^2}}{\sqrt{\pi}x}-\frac{\operatorname{erfc}(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x**3,x)
```

```
[Out] -b**2*erfc(b*x) + b*exp(-b**2*x**2)/(sqrt(pi)*x) - erfc(b*x)/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erfc}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x^3, x)
```

3.109 $\int \frac{\textbf{Erfc}(bx)}{x^5}\, dx$

Optimal. Leaf size=71

$$-\frac{1}{3}b^4\text{Erf}(bx)-\frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x}+\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3}-\frac{\text{Erfc}(bx)}{4x^4}$$

```
[Out] b/(6*E^(b^2*x^2)*Sqrt[Pi]*x^3) - b^3/(3*E^(b^2*x^2)*Sqrt[Pi]*x) - (b^4*Erf[
b*x])/3 - Erfc[b*x]/(4*x^4)
```

Rubi [A] time = 0.059194, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2214, 2205}

$$-\frac{1}{3}b^4\text{Erf}(bx)-\frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x}+\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3}-\frac{\text{Erfc}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x^5,x]
```

```
[Out] b/(6*E^(b^2*x^2)*Sqrt[Pi]*x^3) - b^3/(3*E^(b^2*x^2)*Sqrt[Pi]*x) - (b^4*Erf[
b*x])/3 - Erfc[b*x]/(4*x^4)
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{erfc}(bx)}{x^5}\,dx &= -\frac{\operatorname{erfc}(bx)}{4x^4} - \frac{b\int \frac{e^{-b^2x^2}}{x^4}\,dx}{2\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\operatorname{erfc}(bx)}{4x^4} + \frac{b^3\int \frac{e^{-b^2x^2}}{x^2}\,dx}{3\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)}{4x^4} - \frac{\left(2b^5\right)\int e^{-b^2x^2}\,dx}{3\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} - \frac{1}{3}b^4\operatorname{erf}(bx) - \frac{\operatorname{erfc}(bx)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0351263, size = 53, normalized size = 0.75

$$\frac{1}{12}\left(-4b^4\operatorname{Erf}(bx) + \frac{2e^{-b^2x^2}\left(b-2b^3x^2\right)}{\sqrt{\pi}x^3} - \frac{3\operatorname{Erfc}(bx)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x^5,x]
```

```
[Out] ((2*(b - 2*b^3*x^2))/(E^(b^2*x^2)*Sqrt[Pi]*x^3) - 4*b^4*Erf[b*x] - (3*Erfc[
b*x])/x^4)/12
```

Maple [A] time = 0.042, size = 69, normalized size = 1.

$$b^4\left(-\frac{\operatorname{erfc}(bx)}{4\,b^4x^4} - \frac{1}{2\,\sqrt{\pi}}\left(-\frac{1}{3\,e^{b^2x^2}b^3x^3} + \frac{2}{3\,e^{b^2x^2}bx} + \frac{2\,\sqrt{\pi}\operatorname{Erf}(bx)}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4/b^4/x^4*erfc(b*x)-1/2/Pi^(1/2)*(-1/3/exp(b^2*x^2)/b^3/x^3+2/3/exp
(b^2*x^2)/b/x+2/3*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.09469, size = 50, normalized size = 0.7

$$\frac{\left(b^2x^2\right)^{\frac{3}{2}} b\Gamma\left(-\frac{3}{2}, b^2x^2\right)}{4\sqrt{\pi}x^3} - \frac{\text{erfc}\,(bx)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*x^2)^(3/2)*b*gamma(-3/2, b^2*x^2)/(sqrt(pi)*x^3) - 1/4*erfc(b*x)/x
^4
```

Fricas [A] time = 2.42292, size = 135, normalized size = 1.9

$$-\frac{3\,\pi + 2\,\sqrt{\pi}\left(2\,b^3x^3 - bx\right)e^{\left(-b^2x^2\right)} - \left(3\,\pi - 4\,\pi b^4x^4\right)\text{erf}\,(bx)}{12\,\pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*pi + 2*sqrt(pi)*(2*b^3*x^3 - b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b^4*
x^4)*erf(b*x))/(pi*x^4)
```

Sympy [A] time = 1.58735, size = 60, normalized size = 0.85

$$\frac{b^4\,\text{erfc}\,(bx)}{3} - \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} + \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{erfc}\,(bx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x**5,x)
```

```
[Out] b**4*erfc(b*x)/3 - b**3*exp(-b**2*x**2)/(3*sqrt(pi)*x) + b*exp(-b**2*x**2)/
(6*sqrt(pi)*x**3) - erfc(b*x)/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x^5, x)
```

3.110 $\int \frac{\text{Erfc}(bx)}{x^7} dx$

Optimal. Leaf size=96

$$\frac{4}{45}b^6\text{Erf}(bx) + \frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3} + \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\text{Erfc}(bx)}{6x^6}$$

```
[Out] b/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) - (2*b^3)/(45*E^(b^2*x^2)*Sqrt[Pi]*x^3) + (
4*b^5)/(45*E^(b^2*x^2)*Sqrt[Pi]*x) + (4*b^6*Erf[b*x])/45 - Erfc[b*x]/(6*x^6
)
```

Rubi [A] time = 0.0829265, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2214, 2205}

$$\frac{4}{45}b^6\text{Erf}(bx) + \frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3} + \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\text{Erfc}(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x^7,x]
```

```
[Out] b/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) - (2*b^3)/(45*E^(b^2*x^2)*Sqrt[Pi]*x^3) + (
4*b^5)/(45*E^(b^2*x^2)*Sqrt[Pi]*x) + (4*b^6*Erf[b*x])/45 - Erfc[b*x]/(6*x^6
)
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
```

```
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{erfc}(bx)}{x^7}\,dx &= -\frac{\operatorname{erfc}(bx)}{6x^6}-\frac{b\int \frac{e^{-b^2x^2}}{x^6}\,dx}{3\sqrt{\pi}}\\
&= \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{\operatorname{erfc}(bx)}{6x^6}+\frac{\left(2b^3\right)\int \frac{e^{-b^2x^2}}{x^4}\,dx}{15\sqrt{\pi}}\\
&= \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}-\frac{\operatorname{erfc}(bx)}{6x^6}-\frac{\left(4b^5\right)\int \frac{e^{-b^2x^2}}{x^2}\,dx}{45\sqrt{\pi}}\\
&= \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}+\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}-\frac{\operatorname{erfc}(bx)}{6x^6}+\frac{\left(8b^7\right)\int e^{-b^2x^2}\,dx}{45\sqrt{\pi}}\\
&= \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}+\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}+\frac{4}{45}b^6\operatorname{erf}(bx)-\frac{\operatorname{erfc}(bx)}{6x^6}
\end{aligned}
$$

Mathematica [A] time = 0.0476964, size = 62, normalized size = 0.65

$$
\frac{1}{90}\left(8b^6\operatorname{Erf}(bx)+\frac{2be^{-b^2x^2}\left(4b^4x^4-2b^2x^2+3\right)}{\sqrt{\pi}x^5}-\frac{15\operatorname{Erfc}(bx)}{x^6}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x^7,x]
```

```
[Out] ((2*b*(3 - 2*b^2*x^2 + 4*b^4*x^4))/(E^(b^2*x^2)*Sqrt[Pi]*x^5) + 8*b^6*Erf[b
*x] - (15*Erfc[b*x])/x^6)/90
```

Maple [A] time = 0.042, size = 87, normalized size = 0.9

$$b^{6}\left(-\frac{\operatorname{erfc}\left(bx\right)}{6\,b^{6}x^{6}}-\frac{1}{3\,\sqrt{\pi}}\left(-\frac{1}{5\,\mathrm{e}^{b^{2}x^{2}}b^{5}x^{5}}+\frac{2}{15\,\mathrm{e}^{b^{2}x^{2}}b^{3}x^{3}}-\frac{4}{15\,\mathrm{e}^{b^{2}x^{2}}bx}-\frac{4\,\sqrt{\pi}\mathrm{Erf}\left(bx\right)}{15}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6/b^6/x^6*erfc(b*x)-1/3/Pi^(1/2)*(-1/5/exp(b^2*x^2)/b^5/x^5+2/15/ex
p(b^2*x^2)/b^3/x^3-4/15/exp(b^2*x^2)/b/x-4/15*Pi^(1/2)*erf(b*x)))
```

Maxima [A] time = 1.11779, size = 50, normalized size = 0.52

$$\frac{\left(b^{2}x^{2}\right)^{\frac{5}{2}}b\Gamma\left(-\frac{5}{2},b^{2}x^{2}\right)}{6\,\sqrt{\pi}x^{5}}-\frac{\operatorname{erfc}\left(bx\right)}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*x^2)^(5/2)*b*gamma(-5/2, b^2*x^2)/(sqrt(pi)*x^5) - 1/6*erfc(b*x)/x
^6
```

Fricas [A] time = 2.45557, size = 157, normalized size = 1.64

$$-\frac{15\,\pi-2\,\sqrt{\pi}\left(4\,b^{5}x^{5}-2\,b^{3}x^{3}+3\,bx\right)e^{\left(-b^{2}x^{2}\right)}-\left(15\,\pi+8\,\pi b^{6}x^{6}\right)\operatorname{erf}\left(bx\right)}{90\,\pi x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(15*pi - 2*sqrt(pi)*(4*b^5*x^5 - 2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) - (1
5*pi + 8*pi*b^6*x^6)*erf(b*x))/(pi*x^6)
```

Sympy [A] time = 4.63689, size = 87, normalized size = 0.91

$$-\frac{4b^6 \operatorname{erfc}(bx)}{45}+\frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x}-\frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3}+\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5}-\frac{\operatorname{erfc}(bx)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x**7,x)
```

```
[Out] -4*b**6*erfc(b*x)/45 + 4*b**5*exp(-b**2*x**2)/(45*sqrt(pi)*x) - 2*b**3*exp(
-b**2*x**2)/(45*sqrt(pi)*x**3) + b*exp(-b**2*x**2)/(15*sqrt(pi)*x**5) - erf
c(b*x)/(6*x**6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x^7, x)
```

3.111 $\int x^6 \mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=109

$$-\frac{x^6 e^{-b^2x^2}}{7\sqrt{\pi}b}-\frac{3x^4e^{-b^2x^2}}{7\sqrt{\pi}b^3}-\frac{6x^2e^{-b^2x^2}}{7\sqrt{\pi}b^5}-\frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7}+\frac{1}{7}x^7\mathrm{Erfc}(bx)$$

```
[Out] -6/(7*b^7*E^(b^2*x^2)*Sqrt[Pi]) - (6*x^2)/(7*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (3
*x^4)/(7*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^6/(7*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^7*
Erfc[b*x])/7
```

Rubi [A] time = 0.0940924, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2212, 2209}

$$-\frac{x^6 e^{-b^2x^2}}{7\sqrt{\pi}b}-\frac{3x^4e^{-b^2x^2}}{7\sqrt{\pi}b^3}-\frac{6x^2e^{-b^2x^2}}{7\sqrt{\pi}b^5}-\frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7}+\frac{1}{7}x^7\mathrm{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Erfc[b*x],x]
```

```
[Out] -6/(7*b^7*E^(b^2*x^2)*Sqrt[Pi]) - (6*x^2)/(7*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (3
*x^4)/(7*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^6/(7*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^7*
Erfc[b*x])/7
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^6 \text{erfc}(bx)\,dx &= \frac{1}{7}x^7\text{erfc}(bx) + \frac{(2b)\int e^{-b^2x^2}x^7\,dx}{7\sqrt{\pi}} \\
&= -\frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erfc}(bx) + \frac{6\int e^{-b^2x^2}x^5\,dx}{7b\sqrt{\pi}} \\
&= -\frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erfc}(bx) + \frac{12\int e^{-b^2x^2}x^3\,dx}{7b^3\sqrt{\pi}} \\
&= -\frac{6e^{-b^2x^2}x^2}{7b^5\sqrt{\pi}} - \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erfc}(bx) + \frac{12\int e^{-b^2x^2}x\,dx}{7b^5\sqrt{\pi}} \\
&= -\frac{6e^{-b^2x^2}}{7b^7\sqrt{\pi}} - \frac{6e^{-b^2x^2}x^2}{7b^5\sqrt{\pi}} - \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erfc}(bx)
\end{aligned}$$

Mathematica [A] time = 0.0174086, size = 73, normalized size = 0.67

$$\frac{e^{-b^2x^2}\left(\sqrt{\pi}b^7x^7e^{b^2x^2}\text{Erfc}(bx) - b^6x^6 - 3b^4x^4 - 6b^2x^2 - 6\right)}{7\sqrt{\pi}b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Erfc[b*x],x]
```

```
[Out] (-6 - 6*b^2*x^2 - 3*b^4*x^4 - b^6*x^6 + b^7*E^(b^2*x^2)*Sqrt[Pi]*x^7*Erfc[b
*x])/(7*b^7*E^(b^2*x^2)*Sqrt[Pi])
```

Maple [A] time = 0.04, size = 90, normalized size = 0.8

$$\frac{1}{b^7}\left(\frac{b^7x^7\text{erfc}\,(bx)}{7} + \frac{2}{7\sqrt{\pi}}\left(-\frac{b^6x^6}{2\,e^{b^2x^2}} - \frac{3\,b^4x^4}{2\,e^{b^2x^2}} - 3\,\frac{b^2x^2}{e^{b^2x^2}} - 3\,\left(e^{b^2x^2}\right)^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*erfc(b*x),x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*erfc(b*x)+2/7/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^6*x^6-3/2*b^
4*x^4/exp(b^2*x^2)-3*b^2*x^2/exp(b^2*x^2)-3/exp(b^2*x^2)))
```

Maxima [A] time = 1.00334, size = 70, normalized size = 0.64

$$\frac{1}{7}\,x^{7}\,\mathrm{erfc}\left(bx\right)-\frac{\left(b^{6}x^{6}+3\,b^{4}x^{4}+6\,b^{2}x^{2}+6\right)e^{\left(-b^{2}x^{2}\right)}}{7\,\sqrt{\pi}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfc(b*x),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*erfc(b*x) - 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(-b^2*x^2)/
(sqrt(pi)*b^7)
```

Fricas [A] time = 2.64507, size = 151, normalized size = 1.39

$$-\frac{\pi b^{7}x^{7}\,\mathrm{erf}\left(bx\right)-\pi b^{7}x^{7}+\sqrt{\pi}\left(b^{6}x^{6}+3\,b^{4}x^{4}+6\,b^{2}x^{2}+6\right)e^{\left(-b^{2}x^{2}\right)}}{7\,\pi b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfc(b*x),x, algorithm="fricas")
```

```
[Out] -1/7*(pi*b^7*x^7*erf(b*x) - pi*b^7*x^7 + sqrt(pi)*(b^6*x^6 + 3*b^4*x^4 + 6*
b^2*x^2 + 6)*e^(-b^2*x^2))/(pi*b^7)
```

Sympy [A] time = 6.9419, size = 102, normalized size = 0.94

$$\begin{cases}\frac{x^{7}\,\mathrm{erfc}\left(bx\right)}{7}-\frac{x^{6}e^{-b^{2}x^{2}}}{7\sqrt{\pi}b}-\frac{3x^{4}e^{-b^{2}x^{2}}}{7\sqrt{\pi}b^{3}}-\frac{6x^{2}e^{-b^{2}x^{2}}}{7\sqrt{\pi}b^{5}}-\frac{6e^{-b^{2}x^{2}}}{7\sqrt{\pi}b^{7}} & \text{for } b\neq 0\\ \frac{x^{7}}{7} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*erfc(b*x),x)
```

```
[Out] Piecewise((x**7*erfc(b*x)/7 - x**6*exp(-b**2*x**2)/(7*sqrt(pi)*b) - 3*x**4*
exp(-b**2*x**2)/(7*sqrt(pi)*b**3) - 6*x**2*exp(-b**2*x**2)/(7*sqrt(pi)*b**5
) - 6*exp(-b**2*x**2)/(7*sqrt(pi)*b**7), Ne(b, 0)), (x**7/7, True))
```

Giac [A] time = 1.33595, size = 77, normalized size = 0.71

$$-\frac{1}{7}x^7\operatorname{erf}(bx)+\frac{1}{7}x^7-\frac{\left(b^6x^6+3\,b^4x^4+6\,b^2x^2+6\right)e^{\left(-b^2x^2\right)}}{7\sqrt{\pi}b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfc(b*x),x, algorithm="giac")
```

```
[Out] -1/7*x^7*erf(b*x) + 1/7*x^7 - 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(
-b^2*x^2)/(sqrt(pi)*b^7)
```

3.112 $\int x^4 \mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=84

$$-\frac{x^4 e^{-b^2x^2}}{5\sqrt{\pi}b}-\frac{2x^2 e^{-b^2x^2}}{5\sqrt{\pi}b^3}-\frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5}+\frac{1}{5}x^5\mathrm{Erfc}(bx)$$

```
[Out] -2/(5*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (2*x^2)/(5*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^
4/(5*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erfc[b*x])/5
```

Rubi [A] time = 0.0703607, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2212, 2209}

$$-\frac{x^4 e^{-b^2x^2}}{5\sqrt{\pi}b}-\frac{2x^2 e^{-b^2x^2}}{5\sqrt{\pi}b^3}-\frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5}+\frac{1}{5}x^5\mathrm{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Erfc[b*x],x]
```

```
[Out] -2/(5*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (2*x^2)/(5*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^
4/(5*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erfc[b*x])/5
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\text{erfc}(bx)\,dx &= \frac{1}{5}x^5\text{erfc}(bx)+\frac{(2b)\int e^{-b^2x^2}x^5\,dx}{5\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)+\frac{4\int e^{-b^2x^2}x^3\,dx}{5b\sqrt{\pi}}\\ &= -\frac{2e^{-b^2x^2}x^2}{5b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)+\frac{4\int e^{-b^2x^2}x\,dx}{5b^3\sqrt{\pi}}\\ &= -\frac{2e^{-b^2x^2}}{5b^5\sqrt{\pi}}-\frac{2e^{-b^2x^2}x^2}{5b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)\end{aligned}$$

Mathematica [A] time = 0.0158322, size = 66, normalized size = 0.79

$$e^{-b^2x^2}\left(-\frac{2x^2}{5\sqrt{\pi}b^3}-\frac{2}{5\sqrt{\pi}b^5}-\frac{x^4}{5\sqrt{\pi}b}\right)+\frac{1}{5}x^5\text{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Erfc[b*x],x]
```

```
[Out] (-2/(5*b^5*Sqrt[Pi]) - (2*x^2)/(5*b^3*Sqrt[Pi]) - x^4/(5*b*Sqrt[Pi]))/E^(b^
2*x^2) + (x^5*Erfc[b*x])/5
```

Maple [A] time = 0.043, size = 72, normalized size = 0.9

$$\frac{1}{b^5}\left(\frac{b^5x^5\text{erfc}\,(bx)}{5}+\frac{2}{5\sqrt{\pi}}\left(-\frac{b^4x^4}{2\,\text{e}^{b^2x^2}}-\frac{b^2x^2}{\text{e}^{b^2x^2}}-\left(\text{e}^{b^2x^2}\right)^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erfc(b*x),x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*erfc(b*x)+2/5/Pi^(1/2)*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^
2/exp(b^2*x^2)-1/exp(b^2*x^2)))
```

Maxima [A] time = 1.01978, size = 59, normalized size = 0.7

$$\frac{1}{5}x^5\operatorname{erfc}(bx) - \frac{\left(b^4x^4 + 2\,b^2x^2 + 2\right)e^{\left(-b^2x^2\right)}}{5\,\sqrt{\pi}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*erfc(b*x) - 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b^
5)
```

Fricas [A] time = 2.3993, size = 135, normalized size = 1.61

$$-\frac{\pi b^5x^5\operatorname{erf}(bx) - \pi b^5x^5 + \sqrt{\pi}\left(b^4x^4 + 2\,b^2x^2 + 2\right)e^{\left(-b^2x^2\right)}}{5\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x),x, algorithm="fricas")
```

```
[Out] -1/5*(pi*b^5*x^5*erf(b*x) - pi*b^5*x^5 + sqrt(pi)*(b^4*x^4 + 2*b^2*x^2 + 2)
*e^(-b^2*x^2))/(pi*b^5)
```

Sympy [A] time = 2.24198, size = 78, normalized size = 0.93

$$\begin{cases} \frac{x^5\operatorname{erfc}(bx)}{5} - \frac{x^4e^{-b^2x^2}}{5\sqrt{\pi}b} - \frac{2x^2e^{-b^2x^2}}{5\sqrt{\pi}b^3} - \frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5} & \text{for}\ b \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erfc(b*x),x)
```

```
[Out] Piecewise((x**5*erfc(b*x)/5 - x**4*exp(-b**2*x**2)/(5*sqrt(pi)*b) - 2*x**2*
exp(-b**2*x**2)/(5*sqrt(pi)*b**3) - 2*exp(-b**2*x**2)/(5*sqrt(pi)*b**5), Ne
(b, 0)), (x**5/5, True))
```

Giac [A] time = 1.33656, size = 66, normalized size = 0.79

$$-\frac{1}{5}\,x^5\operatorname{erf}\left(bx\right)+\frac{1}{5}\,x^5-\frac{\left(b^4x^4+2\,b^2x^2+2\right)e^{\left(-b^2x^2\right)}}{5\,\sqrt{\pi}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x),x, algorithm="giac")
```

```
[Out] -1/5*x^5*erf(b*x) + 1/5*x^5 - 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(s
qrt(pi)*b^5)
```

3.113 $\int x^2\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=59

$$-\frac{x^2e^{-b^2x^2}}{3\sqrt{\pi}b}-\frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3}+\frac{1}{3}x^3\text{Erfc}(bx)$$

```
[Out] -1/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^2/(3*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf
c[b*x])/3
```

Rubi [A] time = 0.0485909, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2212, 2209}

$$-\frac{x^2e^{-b^2x^2}}{3\sqrt{\pi}b}-\frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3}+\frac{1}{3}x^3\text{Erfc}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erfc[b*x],x]
```

```
[Out] -1/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) - x^2/(3*b*E^(b^2*x^2)*Sqrt[Pi]) + (x^3*Erf
c[b*x])/3
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\text{erfc}(bx)\,dx &= \frac{1}{3}x^3\text{erfc}(bx)+\frac{(2b)\int e^{-b^2x^2}x^3\,dx}{3\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}x^2}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)+\frac{2\int e^{-b^2x^2}x\,dx}{3b\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}}{3b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^2}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)\end{aligned}$$

Mathematica [A] time = 0.024247, size = 42, normalized size = 0.71

$$\frac{1}{3}\left(x^3\text{Erfc}(bx)-\frac{e^{-b^2x^2}\left(b^2x^2+1\right)}{\sqrt{\pi}b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erfc[b*x],x]
```

```
[Out] (-((1 + b^2*x^2)/(b^3*E^(b^2*x^2)*Sqrt[Pi])) + x^3*Erfc[b*x])/3
```

Maple [A] time = 0.043, size = 54, normalized size = 0.9

$$\frac{1}{b^3}\left(\frac{b^3x^3\text{erfc}\,(bx)}{3}+\frac{2}{3\,\sqrt{\pi}}\left(-\frac{b^2x^2}{2\,\text{e}^{b^2x^2}}-\frac{1}{2\,\text{e}^{b^2x^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfc(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*erfc(b*x)+2/3/Pi^(1/2)*(-1/2*b^2*x^2/exp(b^2*x^2)-1/2/ex
p(b^2*x^2)))
```

Maxima [A] time = 1.00612, size = 49, normalized size = 0.83

$$\frac{1}{3}x^3\operatorname{erfc}(bx) - \frac{\left(b^2x^2+1\right)e^{\left(-b^2x^2\right)}}{3\sqrt{\pi}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*erfc(b*x) - 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

Fricas [A] time = 2.42238, size = 119, normalized size = 2.02

$$-\frac{\pi b^3x^3\operatorname{erf}(bx) - \pi b^3x^3 + \sqrt{\pi}\left(b^2x^2+1\right)e^{\left(-b^2x^2\right)}}{3\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x),x, algorithm="fricas")
```

```
[Out] -1/3*(pi*b^3*x^3*erf(b*x) - pi*b^3*x^3 + sqrt(pi)*(b^2*x^2 + 1)*e^(-b^2*x^2
))/(pi*b^3)
```

Sympy [A] time = 0.741049, size = 54, normalized size = 0.92

$$\begin{cases} \frac{x^3\operatorname{erfc}(bx)}{3} - \frac{x^2e^{-b^2x^2}}{3\sqrt{\pi}b} - \frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfc(b*x),x)
```

```
[Out] Piecewise((x**3*erfc(b*x)/3 - x**2*exp(-b**2*x**2)/(3*sqrt(pi)*b) - exp(-b*
*2*x**2)/(3*sqrt(pi)*b**3), Ne(b, 0)), (x**3/3, True))
```

Giac [A] time = 1.4527, size = 55, normalized size = 0.93

$$-\frac{1}{3}x^3\operatorname{erf}(bx)+\frac{1}{3}x^3-\frac{\left(b^2x^2+1\right)e^{\left(-b^2x^2\right)}}{3\sqrt{\pi}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x),x, algorithm="giac")
```

```
[Out] -1/3*x^3*erf(b*x) + 1/3*x^3 - 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

3.114 $\int \text{Erfc}(bx)\,dx$

Optimal. Leaf size=27

$$x\text{Erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b}$$

```
[Out] -(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]
```

Rubi [A] time = 0.0049246, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6350}

$$x\text{Erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x],x]
```

```
[Out] -(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]
```

Rule 6350

```
Int[Erfc[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x])/b
, x] - Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erfc}(bx)\,dx = -\frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\text{erfc}(bx)$$

Mathematica [A] time = 0.0021854, size = 27, normalized size = 1.

$$x\text{Erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x],x]
```

```
[Out] -(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]
```

Maple [A] time = 0.039, size = 27, normalized size = 1.

$$\frac{1}{b}\left(bx\mathrm{erfc}\left(bx\right)-\frac{\mathrm{e}^{-b^2x^2}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x),x)
```

```
[Out] 1/b*(b*x*erfc(b*x)-1/Pi^(1/2)*exp(-b^2*x^2))
```

Maxima [A] time = 1.03127, size = 35, normalized size = 1.3

$$\frac{bx\,\mathrm{erfc}\left(bx\right)-\frac{e^{\left(-b^2x^2\right)}}{\sqrt{\pi}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x),x, algorithm="maxima")
```

```
[Out] (b*x*erfc(b*x) - e^(-b^2*x^2)/sqrt(pi))/b
```

Fricas [A] time = 2.31983, size = 81, normalized size = 3.

$$-\frac{\pi bx\,\mathrm{erf}\left(bx\right)-\pi bx+\sqrt{\pi}e^{\left(-b^2x^2\right)}}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x),x, algorithm="fricas")
```

```
[Out] -(pi*b*x*erf(b*x) - pi*b*x + sqrt(pi)*e^(-b^2*x^2))/(pi*b)
```

Sympy [A] time = 0.346812, size = 24, normalized size = 0.89

$$\begin{cases} x \operatorname{erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x),x)
```

```
[Out] Piecewise((x*erfc(b*x) - exp(-b**2*x**2)/(sqrt(pi)*b), Ne(b, 0)), (x, True)
)
```

Giac [A] time = 1.32128, size = 35, normalized size = 1.3

$$-x \operatorname{erf}(bx) + x - \frac{e^{\left(-b^2x^2\right)}}{\sqrt{\pi}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x),x, algorithm="giac")
```

```
[Out] -x*erf(b*x) + x - e^(-b^2*x^2)/(sqrt(pi)*b)
```

3.115 $\int \frac{\textbf{Erfc}(bx)}{x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{b\,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}}-\frac{\text{Erfc}(bx)}{x}$$

```
[Out] -(Erfc[b*x]/x) - (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0312142, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6362, 2210}

$$-\frac{b\,\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}}-\frac{\text{Erfc}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x^2,x]
```

```
[Out] -(Erfc[b*x]/x) - (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\text{erfc}(bx)}{x^2}\,dx = -\frac{\text{erfc}(bx)}{x} - \frac{(2b)\int \frac{e^{-b^2x^2}}{x}\,dx}{\sqrt{\pi}}$$
$$= -\frac{\text{erfc}(bx)}{x} - \frac{b\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0152504, size = 27, normalized size = 1.

$$-\frac{b\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{Erfc}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x^2,x]
```

```
[Out] -(Erfc[b*x]/x) - (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Maple [A] time = 0.043, size = 29, normalized size = 1.1

$$b\left(-\frac{\text{erfc}\,(bx)}{bx} + \frac{\text{Ei}\left(1, b^2x^2\right)}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x^2,x)
```

```
[Out] b*(-1/b/x*erfc(b*x)+1/Pi^(1/2)*Ei(1,b^2*x^2))
```

Maxima [A] time = 1.11359, size = 34, normalized size = 1.26

$$-\frac{b\text{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erfc}\,(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] -b*Ei(-b^2*x^2)/sqrt(pi) - erfc(b*x)/x
```

Fricas [A] time = 2.35477, size = 76, normalized size = 2.81

$$-\frac{\pi + \sqrt{\pi}bx\text{Ei}\left(-b^2x^2\right) - \pi\,\text{erf}\,(bx)}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(pi + sqrt(pi)*b*x*Ei(-b^2*x^2) - pi*erf(b*x))/(pi*x)
```

Sympy [A] time = 1.27443, size = 20, normalized size = 0.74

$$\frac{b\,\text{E}_1\left(b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erfc}\,(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x**2,x)
```

```
[Out] b*expint(1, b**2*x**2)/sqrt(pi) - erfc(b*x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x^2, x)
```

3.116 $\int \frac{\mathbf{Erfc}(bx)}{x^4}\,dx$

Optimal. Leaf size=56

$$\frac{b^3\text{ExpIntegralEi}\left(-b^2x^2\right)}{3\sqrt{\pi}}+\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\text{Erfc}(bx)}{3x^3}$$

```
[Out] b/(3*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erfc[b*x]/(3*x^3) + (b^3*ExpIntegralEi[-(b
^2*x^2)])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.0540663, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2214, 2210}

$$\frac{b^3\text{Ei}\left(-b^2x^2\right)}{3\sqrt{\pi}}+\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\text{Erfc}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x^4,x]
```

```
[Out] b/(3*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erfc[b*x]/(3*x^3) + (b^3*ExpIntegralEi[-(b
^2*x^2)])/(3*Sqrt[Pi])
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{erfc}(bx)}{x^4}\,dx &= -\frac{\text{erfc}(bx)}{3x^3} - \frac{(2b)\int \frac{e^{-b^2x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{erfc}(bx)}{3x^3} + \frac{\left(2b^3\right)\int \frac{e^{-b^2x^2}}{x}\,dx}{3\sqrt{\pi}}\\ &= \frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{erfc}(bx)}{3x^3} + \frac{b^3\text{Ei}\left(-b^2x^2\right)}{3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0395833, size = 49, normalized size = 0.88

$$\frac{1}{3}\left(\frac{b\left(b^2\text{ExpIntegralEi}\left(-b^2x^2\right)+\frac{e^{-b^2x^2}}{x^2}\right)}{\sqrt{\pi}}-\frac{\text{Erfc}(bx)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x^4,x]
```

```
[Out] (-(Erfc[b*x]/x^3) + (b*(1/(E^(b^2*x^2)*x^2) + b^2*ExpIntegralEi[-(b^2*x^2)]
))/Sqrt[Pi])/3
```

Maple [A] time = 0.041, size = 53, normalized size = 1.

$$b^3\left(-\frac{\text{erfc}\,(bx)}{3\,x^3b^3}-\frac{2}{3\,\sqrt{\pi}}\left(-\frac{1}{2\,b^2x^2\text{e}^{b^2x^2}}+\frac{\text{Ei}\left(1,b^2x^2\right)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x^4,x)
```

```
[Out] b^3*(-1/3/b^3/x^3*erfc(b*x)-2/3/Pi^(1/2)*(-1/2/exp(b^2*x^2)/b^2/x^2+1/2*Ei(
1,b^2*x^2)))
```

Maxima [A] time = 1.07609, size = 36, normalized size = 0.64

$$\frac{b^3\Gamma\left(-1, b^2x^2\right)}{3\sqrt{\pi}} - \frac{\operatorname{erfc}(bx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*gamma(-1, b^2*x^2)/sqrt(pi) - 1/3*erfc(b*x)/x^3
```

Fricas [A] time = 2.12675, size = 117, normalized size = 2.09

$$-\frac{\pi - \pi\operatorname{erf}(bx) - \sqrt{\pi}\left(b^3x^3\operatorname{Ei}\left(-b^2x^2\right) + bxe^{\left(-b^2x^2\right)}\right)}{3\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(pi - pi*erf(b*x) - sqrt(pi)*(b^3*x^3*Ei(-b^2*x^2) + b*x*e^(-b^2*x^2))
)/(pi*x^3)
```

Sympy [A] time = 2.50946, size = 48, normalized size = 0.86

$$-\frac{b^3\operatorname{E}_1\left(b^2x^2\right)}{3\sqrt{\pi}} + \frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\operatorname{erfc}(bx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x**4,x)
```

```
[Out] -b**3*expint(1, b**2*x**2)/(3*sqrt(pi)) + b*exp(-b**2*x**2)/(3*sqrt(pi)*x**
2) - erfc(b*x)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x^4, x)
```

3.117 $\int \frac{\textbf{Erfc}(bx)}{x^6}\,dx$

Optimal. Leaf size=81

$$-\frac{b^5\text{ExpIntegralEi}\left(-b^2x^2\right)}{10\sqrt{\pi}}-\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}+\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\text{Erfc}(bx)}{5x^5}$$

```
[Out] b/(10*E^(b^2*x^2)*Sqrt[Pi]*x^4) - b^3/(10*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erfc[
b*x]/(5*x^5) - (b^5*ExpIntegralEi[-(b^2*x^2)])/(10*Sqrt[Pi])
```

Rubi [A] time = 0.0740645, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6362, 2214, 2210}

$$-\frac{b^5\text{Ei}\left(-b^2x^2\right)}{10\sqrt{\pi}}-\frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2}+\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\text{Erfc}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/x^6,x]
```

```
[Out] b/(10*E^(b^2*x^2)*Sqrt[Pi]*x^4) - b^3/(10*E^(b^2*x^2)*Sqrt[Pi]*x^2) - Erfc[
b*x]/(5*x^5) - (b^5*ExpIntegralEi[-(b^2*x^2)])/(10*Sqrt[Pi])
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx &= -\frac{\operatorname{erfc}(bx)}{5x^5} - \frac{(2b)\int \frac{e^{-b^2x^2}}{x^5}\,dx}{5\sqrt{\pi}} \\
&= \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} - \frac{\operatorname{erfc}(bx)}{5x^5} + \frac{b^3\int \frac{e^{-b^2x^2}}{x^3}\,dx}{5\sqrt{\pi}} \\
&= \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} - \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} - \frac{\operatorname{erfc}(bx)}{5x^5} - \frac{b^5\int \frac{e^{-b^2x^2}}{x}\,dx}{5\sqrt{\pi}} \\
&= \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} - \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} - \frac{\operatorname{erfc}(bx)}{5x^5} - \frac{b^5\operatorname{Ei}\left(-b^2x^2\right)}{10\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.0239886, size = 73, normalized size = 0.9

$$
-\frac{b^5\text{ExpIntegralEi}\left(-b^2x^2\right)}{10\sqrt{\pi}} + e^{-b^2x^2}\left(\frac{b}{10\sqrt{\pi}x^4} - \frac{b^3}{10\sqrt{\pi}x^2}\right) - \frac{\operatorname{Erfc}(bx)}{5x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/x^6,x]
```

```
[Out] (b/(10*Sqrt[Pi]*x^4) - b^3/(10*Sqrt[Pi]*x^2))/E^(b^2*x^2) - Erfc[b*x]/(5*x^
5) - (b^5*ExpIntegralEi[-(b^2*x^2)])/(10*Sqrt[Pi])
```

Maple [A] time = 0.042, size = 71, normalized size = 0.9

$$
b^5\left(-\frac{\operatorname{erfc}(bx)}{5\,b^5x^5} - \frac{2}{5\sqrt{\pi}}\left(-\frac{1}{4\,e^{b^2x^2}b^4x^4} + \frac{1}{4\,b^2x^2e^{b^2x^2}} - \frac{\operatorname{Ei}\left(1,b^2x^2\right)}{4}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5/b^5/x^5*erfc(b*x)-2/5/Pi^(1/2)*(-1/4/exp(b^2*x^2)/b^4/x^4+1/4/exp
(b^2*x^2)/b^2/x^2-1/4*Ei(1,b^2*x^2)))
```

Maxima [A] time = 1.09173, size = 36, normalized size = 0.44

$$\frac{b^5\Gamma\left(-2, b^2x^2\right)}{5\sqrt{\pi}} - \frac{\operatorname{erfc}(bx)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*gamma(-2, b^2*x^2)/sqrt(pi) - 1/5*erfc(b*x)/x^5
```

Fricas [A] time = 2.13838, size = 140, normalized size = 1.73

$$-\frac{2\pi - 2\pi \operatorname{erf}(bx) + \sqrt{\pi}\left(b^5x^5\operatorname{Ei}\left(-b^2x^2\right) + \left(b^3x^3 - bx\right)e^{\left(-b^2x^2\right)}\right)}{10\,\pi x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(2*pi - 2*pi*erf(b*x) + sqrt(pi)*(b^5*x^5*Ei(-b^2*x^2) + (b^3*x^3 - b
*x)*e^(-b^2*x^2)))/(pi*x^5)
```

Sympy [A] time = 5.2556, size = 70, normalized size = 0.86

$$\frac{b^5\operatorname{E}_1\left(b^2x^2\right)}{10\sqrt{\pi}} - \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} + \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} - \frac{\operatorname{erfc}(bx)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x**6,x)
```

```
[Out] b**5*expint(1, b**2*x**2)/(10*sqrt(pi)) - b**3*exp(-b**2*x**2)/(10*sqrt(pi)
*x**2) + b*exp(-b**2*x**2)/(10*sqrt(pi)*x**4) - erfc(b*x)/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)/x^6, x)
```

3.118 $\int (c+dx)^3 \mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=292

$$-\frac{d^2e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{\sqrt{\pi}b^4}-\frac{d^2e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^4}+\frac{(bc-ad)^4\text{Erf}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2\text{Erf}(a+bx)}{4b^4}-\frac{e^{-(a+bx)^2}(bc}{\sqrt{\pi}b^4}$$

```
[Out] -((d^2*(b*c - a*d))/(b^4*E^(a + b*x)^2*Sqrt[Pi])) - (b*c - a*d)^3/(b^4*E^(a
 + b*x)^2*Sqrt[Pi]) - (3*d^3*(a + b*x))/(8*b^4*E^(a + b*x)^2*Sqrt[Pi]) - (3
*d*(b*c - a*d)^2*(a + b*x))/(2*b^4*E^(a + b*x)^2*Sqrt[Pi]) - (d^2*(b*c - a*
d)*(a + b*x)^2)/(b^4*E^(a + b*x)^2*Sqrt[Pi]) - (d^3*(a + b*x)^3)/(4*b^4*E^(
a + b*x)^2*Sqrt[Pi]) + (3*d^3*Erf[a + b*x])/(16*b^4) + (3*d*(b*c - a*d)^2*E
rf[a + b*x])/(4*b^4) + ((b*c - a*d)^4*Erf[a + b*x])/(4*b^4*d) + ((c + d*x)^
4*Erfc[a + b*x])/(4*d)
```

Rubi [A] time = 0.272314, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6362, 2226, 2205, 2209, 2212}

$$-\frac{d^2e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{\sqrt{\pi}b^4}-\frac{d^2e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^4}+\frac{(bc-ad)^4\text{Erf}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2\text{Erf}(a+bx)}{4b^4}-\frac{e^{-(a+bx)^2}(bc}{\sqrt{\pi}b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Erfc[a + b*x],x]
```



```
[Out] -((d^2*(b*c - a*d))/(b^4*E^(a + b*x)^2*Sqrt[Pi])) - (b*c - a*d)^3/(b^4*E^(a
 + b*x)^2*Sqrt[Pi]) - (3*d^3*(a + b*x))/(8*b^4*E^(a + b*x)^2*Sqrt[Pi]) - (3
*d*(b*c - a*d)^2*(a + b*x))/(2*b^4*E^(a + b*x)^2*Sqrt[Pi]) - (d^2*(b*c - a*
d)*(a + b*x)^2)/(b^4*E^(a + b*x)^2*Sqrt[Pi]) - (d^3*(a + b*x)^3)/(4*b^4*E^(
a + b*x)^2*Sqrt[Pi]) + (3*d^3*Erf[a + b*x])/(16*b^4) + (3*d*(b*c - a*d)^2*E
rf[a + b*x])/(4*b^4) + ((b*c - a*d)^4*Erf[a + b*x])/(4*b^4*d) + ((c + d*x)^
4*Erfc[a + b*x])/(4*d)
```


Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^3\text{erfc}(a+bx)\,dx = \frac{(c+dx)^4\text{erfc}(a+bx)}{4d} + \frac{b\int e^{-(a+bx)^2}(c+dx)^4\,dx}{2d\sqrt{\pi}}$$

$$= \frac{(c+dx)^4\text{erfc}(a+bx)}{4d} + \frac{b\int\left(\frac{(bc-ad)^4e^{-(a+bx)^2}}{b^4} + \frac{4d(bc-ad)^3e^{-(a+bx)^2}(a+bx)}{b^4} + \frac{6d^2(bc-ad)^2e^{-(a+bx)^2}(a+bx}{b^4}\right.}{2d\sqrt{\pi}}$$

$$= \frac{(c+dx)^4\text{erfc}(a+bx)}{4d} + \frac{d^3\int e^{-(a+bx)^2}(a+bx)^4\,dx}{2b^3\sqrt{\pi}} + \frac{\left(2d^2(bc-ad)\right)\int e^{-(a+bx)^2}(a+bx)^3\,dx}{b^3\sqrt{\pi}}$$

$$= -\frac{(bc-ad)^3e^{-(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}} - \frac{d^2(bc-ad)e^{-(a+bx)^2}(a+bx)^2}{b^4\sqrt{\pi}} - \frac{d^3e}{}$$

$$= -\frac{d^2(bc-ad)e^{-(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{(bc-ad)^3e^{-(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{3d^3e^{-(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}} - \frac{3d(bc-ad)^2e^{-(a+bx)^2}(}{2b^4\sqrt{\pi}}$$

$$= -\frac{d^2(bc-ad)e^{-(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{(bc-ad)^3e^{-(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{3d^3e^{-(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}} - \frac{3d(bc-ad)^2e^{-(a+bx)^2}(}{2b^4\sqrt{\pi}}$$

Mathematica [A] time = 0.359714, size = 268, normalized size = 0.92

$$\frac{e^{-(a+bx)^2}\left(\sqrt{\pi}e^{(a+bx)^2}\left(12a^2\left(2b^2c^2d+d^3\right)-16a^3bcd^2+4a^4d^3-8a\left(2b^3c^3+3bcd^2\right)+3\left(4b^2c^2d+d^3\right)\right)\text{Erf}(a+bx)-2bd}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Erfc[a + b*x],x]
```

```
[Out] (2*a*(5 + 2*a^2)*d^3 - 2*b*d^2*(8*(1 + a^2)*c + (3 + 2*a^2)*d*x) + 4*a*b^2*
d*(6*c^2 + 4*c*d*x + d^2*x^2) - 4*b^3*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^
3*x^3) + (-16*a^3*b*c*d^2 + 4*a^4*d^3 - 8*a*(2*b^3*c^3 + 3*b*c*d^2) + 12*a^
2*(2*b^2*c^2*d + d^3) + 3*(4*b^2*c^2*d + d^3))*E^(a + b*x)^2*Sqrt[Pi]*Erf[a
 + b*x] + 4*b^4*E^(a + b*x)^2*Sqrt[Pi]*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 +
 d^3*x^3)*Erfc[a + b*x])/(16*b^4*E^(a + b*x)^2*Sqrt[Pi])
```

Maple [B] time = 0.046, size = 729, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*erfc(b*x+a),x)
```

```
[Out] 1/b*(1/4/b^3*d^3*erfc(b*x+a)*(b*x+a)^4-1/b^3*d^3*erfc(b*x+a)*(b*x+a)^3*a+1/
b^2*d^2*erfc(b*x+a)*(b*x+a)^3*c+3/2/b^3*d^3*erfc(b*x+a)*(b*x+a)^2*a^2-3/b^2
*d^2*erfc(b*x+a)*(b*x+a)^2*a*c+3/2/b*d*erfc(b*x+a)*(b*x+a)^2*c^2-1/b^3*d^3*
erfc(b*x+a)*(b*x+a)*a^3+3/b^2*d^2*erfc(b*x+a)*(b*x+a)*a^2*c-3/b*d*erfc(b*x+
a)*(b*x+a)*a*c^2+erfc(b*x+a)*(b*x+a)*c^3+1/4/b^3*d^3*erfc(b*x+a)*a^4-1/b^2*
d^2*erfc(b*x+a)*a^3*c+3/2/b*d*erfc(b*x+a)*a^2*c^2-erfc(b*x+a)*a*c^3+1/4*b/d
*erfc(b*x+a)*c^4+1/2/Pi^(1/2)/b^3/d*(d^4*(-1/2*(b*x+a)^3/exp((b*x+a)^2)-3/4
*(b*x+a)/exp((b*x+a)^2)+3/8*Pi^(1/2)*erf(b*x+a))+1/2*a^4*d^4*Pi^(1/2)*erf(b
*x+a)+1/2*b^4*c^4*Pi^(1/2)*erf(b*x+a)+2*a^3*d^4/exp((b*x+a)^2)+6*a^2*d^4*(-
1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))-4*a*d^4*(-1/2*(b*x+a)^2
/exp((b*x+a)^2)-1/2/exp((b*x+a)^2))-2*b^3*c^3*d/exp((b*x+a)^2)+6*b^2*c^2*d^
2*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))+4*b*c*d^3*(-1/2*(b*
x+a)^2/exp((b*x+a)^2)-1/2/exp((b*x+a)^2))-2*a*b^3*c^3*d*Pi^(1/2)*erf(b*x+a)
+3*a^2*b^2*c^2*d^2*Pi^(1/2)*erf(b*x+a)-2*a^3*b*c*d^3*Pi^(1/2)*erf(b*x+a)+6*
a*b^2*c^2*d^2/exp((b*x+a)^2)-6*a^2*b*c*d^3/exp((b*x+a)^2)-12*a*b*c*d^3*(-1/
2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \operatorname{erfc}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3*erfc(b*x + a), x)
```

Fricas [A] time = 2.09864, size = 689, normalized size = 2.36

$$\frac{4\,\pi b^4 d^3 x^4 + 16\,\pi b^4 c d^2 x^3 + 24\,\pi b^4 c^2 d x^2 + 16\,\pi b^4 c^3 x - 2\,\sqrt{\pi}\Big(2\,b^3 d^3 x^3 + 8\,b^3 c^3 - 12\,a b^2 c^2 d + 8\,\big(a^2+1\big) b c d^2 - \big(2\,a^3 + 5\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*(4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi*
b^4*c^3*x - 2*sqrt(pi)*(2*b^3*d^3*x^3 + 8*b^3*c^3 - 12*a*b^2*c^2*d + 8*(a^2
 + 1)*b*c*d^2 - (2*a^3 + 5*a)*d^3 + 2*(4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (12*b
^3*c^2*d - 8*a*b^2*c*d^2 + (2*a^2 + 3)*b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x - a^
2) - (4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi*
b^4*c^3*x + pi*(16*a*b^3*c^3 - 12*(2*a^2 + 1)*b^2*c^2*d + 8*(2*a^3 + 3*a)*b
*c*d^2 - (4*a^4 + 12*a^2 + 3)*d^3))*erf(b*x + a))/(pi*b^4)
```

Sympy [A] time = 22.4494, size = 746, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*erfc(b*x+a),x)
```

```
[Out] Piecewise((-a**4*d**3*erfc(a + b*x)/(4*b**4) + a**3*c*d**2*erfc(a + b*x)/b*
*3 + a**3*d**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b**4) -
 3*a**2*c**2*d*erfc(a + b*x)/(2*b**2) - a**2*c*d**2*exp(-a**2)*exp(-b**2*x*
*2)*exp(-2*a*b*x)/(sqrt(pi)*b**3) - a**2*d**3*x*exp(-a**2)*exp(-b**2*x**2)*
exp(-2*a*b*x)/(4*sqrt(pi)*b**3) - 3*a**2*d**3*erfc(a + b*x)/(4*b**4) + a*c*
*3*erfc(a + b*x)/b + 3*a*c**2*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2
*sqrt(pi)*b**2) + a*c*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt
(pi)*b**2) + a*d**3*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(p
i)*b**2) + 3*a*c*d**2*erfc(a + b*x)/(2*b**3) + 5*a*d**3*exp(-a**2)*exp(-b**
2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b**4) + c**3*x*erfc(a + b*x) + 3*c**2*d*x
**2*erfc(a + b*x)/2 + c*d**2*x**3*erfc(a + b*x) + d**3*x**4*erfc(a + b*x)/4
 - c**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) - 3*c**2*d*x*
exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) - c*d**2*x**2*exp(-
a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) - d**3*x**3*exp(-a**2)*exp
(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b) - 3*c**2*d*erfc(a + b*x)/(4*b**2)
 - c*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b**3) - 3*d**3
*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b**3) - 3*d**3*erfc
(a + b*x)/(16*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 +
d**3*x**4/4)*erfc(a), True))
```

Giac [A] time = 1.50791, size = 587, normalized size = 2.01

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 - \left(x\operatorname{erf}(bx+a) - \frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}\right)c^3 - \frac{3}{4}\left(2\,x^2\operatorname{erf}(bx+a) + \frac{\frac{\sqrt{\pi}\left(2\,a^2+1\right)\operatorname{erf}\left(-b\left(x+\right.\right.}{b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 - (x*erf(b*x + a) - (sqrt(pi)*a*erf
(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/sqrt(pi))*c^3 - 3/4*(2*x
^2*erf(b*x + a) + (sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b
) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(sqrt(pi)*b))*c^2*d - 1/2*(2*x^3*e
rf(b*x + a) - (sqrt(pi)*(2*a^3 + 3*a)*erf(-b*(x + a/b))/b - 2*(b^2*(x + a/b
)^2 - 3*a*b*(x + a/b) + 3*a^2 + 1)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(sqrt(pi
)*b^2))*c*d^2 - 1/16*(4*x^4*erf(b*x + a) + (sqrt(pi)*(4*a^4 + 12*a^2 + 3)*e
rf(-b*(x + a/b))/b + 2*(2*b^3*(x + a/b)^3 - 8*a*b^2*(x + a/b)^2 + 12*a^2*b*
(x + a/b) - 8*a^3 + 3*b*(x + a/b) - 8*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(s
qrt(pi)*b^3))*d^3 + c^3*x
```

3.119 $\int (c+dx)^2 \textbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=194

$$\frac{(bc-ad)^3\operatorname{Erf}(a+bx)}{3b^3d}+\frac{d(bc-ad)\operatorname{Erf}(a+bx)}{2b^3}-\frac{e^{-(a+bx)^2}(bc-ad)^2}{\sqrt{\pi}b^3}-\frac{de^{-(a+bx)^2}(a+bx)(bc-ad)}{\sqrt{\pi}b^3}-\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3\sqrt{\pi}b^3}$$

```
[Out] -d^2/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) - (b*c - a*d)^2/(b^3*E^(a + b*x)^2*Sqrt
[Pi]) - (d*(b*c - a*d)*(a + b*x))/(b^3*E^(a + b*x)^2*Sqrt[Pi]) - (d^2*(a +
b*x)^2)/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) + (d*(b*c - a*d)*Erf[a + b*x])/(2*b^
3) + ((b*c - a*d)^3*Erf[a + b*x])/(3*b^3*d) + ((c + d*x)^3*Erfc[a + b*x])/(
3*d)
```

Rubi [A] time = 0.180591, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6362, 2226, 2205, 2209, 2212}

$$\frac{(bc-ad)^3\operatorname{Erf}(a+bx)}{3b^3d}+\frac{d(bc-ad)\operatorname{Erf}(a+bx)}{2b^3}-\frac{e^{-(a+bx)^2}(bc-ad)^2}{\sqrt{\pi}b^3}-\frac{de^{-(a+bx)^2}(a+bx)(bc-ad)}{\sqrt{\pi}b^3}-\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Erfc[a + b*x],x]
```

```
[Out] -d^2/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) - (b*c - a*d)^2/(b^3*E^(a + b*x)^2*Sqrt
[Pi]) - (d*(b*c - a*d)*(a + b*x))/(b^3*E^(a + b*x)^2*Sqrt[Pi]) - (d^2*(a +
b*x)^2)/(3*b^3*E^(a + b*x)^2*Sqrt[Pi]) + (d*(b*c - a*d)*Erf[a + b*x])/(2*b^
3) + ((b*c - a*d)^3*Erf[a + b*x])/(3*b^3*d) + ((c + d*x)^3*Erfc[a + b*x])/(
3*d)
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
```

```
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^2\mathrm{erfc}(a+bx)\,dx = \frac{(c+dx)^3\mathrm{erfc}(a+bx)}{3d} + \frac{(2b)\int e^{-(a+bx)^2}(c+dx)^3\,dx}{3d\sqrt{\pi}}$$

$$= \frac{(c+dx)^3\mathrm{erfc}(a+bx)}{3d} + \frac{(2b)\int\left(\frac{(bc-ad)^3e^{-(a+bx)^2}}{b^3} + \frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+bx)}{b^3} + \frac{3d^2(bc-ad)e^{-(a+bx)^2}(a+b}{b^3}\right.}{3d\sqrt{\pi}}$$

$$= \frac{(c+dx)^3\mathrm{erfc}(a+bx)}{3d} + \frac{\left(2d^2\right)\int e^{-(a+bx)^2}(a+bx)^3\,dx}{3b^2\sqrt{\pi}} + \frac{(2d(bc-ad))\int e^{-(a+bx)^2}(a+bx)^2\,dx}{b^2\sqrt{\pi}}$$

$$= -\frac{(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}} - \frac{d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} - \frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}} + \frac{(bc-ad)^3\mathrm{erf}(a}{3b^3d}$$

$$= -\frac{d^2e^{-(a+bx)^2}}{3b^3\sqrt{\pi}} - \frac{(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}} - \frac{d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} - \frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}} + \frac{d(b}{}$$

Mathematica [A] time = 0.28104, size = 159, normalized size = 0.82

$$\frac{\frac{2e^{-(a+bx)^2}\left(-\left(a^2+1\right)d^2+\sqrt{\pi}b^3xe^{(a+bx)^2}\left(3c^2+3cdx+d^2x^2\right)\text{Erfc}(a+bx)+abd(3c+dx)-b^2\left(3c^2+3cdx+d^2x^2\right)\right)}{\sqrt{\pi}}-\left(-6a^2bcd+2a^3d^2+3a\left(2b^2c^2+d^2\right)-3\right.}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Erfc[a + b*x],x]
```

```
[Out] (-((-3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + 3*a*(2*b^2*c^2 + d^2))*Erf[a + b*x
]) + (2*(-((1 + a^2)*d^2) + a*b*d*(3*c + d*x) - b^2*(3*c^2 + 3*c*d*x + d^2*
x^2) + b^3*E^(a + b*x)^2*Sqrt[Pi]*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Erfc[a + b*
x]))/(E^(a + b*x)^2*Sqrt[Pi]))/(6*b^3)
```

Maple [B] time = 0.049, size = 428, normalized size = 2.2

$$\frac{1}{b}\left(\frac{d^2\text{erfc}\left(bx+a\right)\left(bx+a\right)^3}{3\,b^2}-\frac{d^2\text{erfc}\left(bx+a\right)\left(bx+a\right)^2a}{b^2}+\frac{d\text{erfc}\left(bx+a\right)\left(bx+a\right)^2c}{b}+\frac{d^2\text{erfc}\left(bx+a\right)\left(bx+a\right)a^2}{b^2}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*erfc(b*x+a),x)
```

```
[Out] 1/b*(1/3/b^2*d^2*erfc(b*x+a)*(b*x+a)^3-1/b^2*d^2*erfc(b*x+a)*(b*x+a)^2*a+1/
b*d*erfc(b*x+a)*(b*x+a)^2*c+1/b^2*d^2*erfc(b*x+a)*(b*x+a)*a^2-2/b*d*erfc(b*
x+a)*(b*x+a)*a*c+erfc(b*x+a)*(b*x+a)*c^2-1/3/b^2*d^2*erfc(b*x+a)*a^3+1/b*d*
erfc(b*x+a)*a^2*c-erfc(b*x+a)*a*c^2+1/3*b/d*erfc(b*x+a)*c^3+2/3/Pi^(1/2)/b^
2/d*(1/2*b^3*c^3*Pi^(1/2)*erf(b*x+a)+d^3*(-1/2*(b*x+a)^2/exp((b*x+a)^2)-1/2
/exp((b*x+a)^2))-1/2*a^3*d^3*Pi^(1/2)*erf(b*x+a)-3/2*a^2*d^3/exp((b*x+a)^2)
-3*a*d^3*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))-3/2*b^2*c^2*
d/exp((b*x+a)^2)+3*b*c*d^2*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*
x+a))-3/2*a*b^2*c^2*d*Pi^(1/2)*erf(b*x+a)+3/2*a^2*b*c*d^2*Pi^(1/2)*erf(b*x+
a)+3*a*b*c*d^2/exp((b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\,\text{erfc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*erfc(b*x + a), x)
```

Fricas [A] time = 2.15712, size = 435, normalized size = 2.24

$$\frac{2\,\pi b^3 d^2 x^3 + 6\,\pi b^3 c d x^2 + 6\,\pi b^3 c^2 x - 2\,\sqrt{\pi}\left(b^2 d^2 x^2 + 3\,b^2 c^2 - 3\,abcd + \left(a^2+1\right)d^2 + \left(3\,b^2 cd - abd^2\right)x\right)e^{\left(-b^2x^2-2\,abx-a^2\right)} - \left(2\right.}{6\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*pi*b^3*d^2*x^3 + 6*pi*b^3*c*d*x^2 + 6*pi*b^3*c^2*x - 2*sqrt(pi)*(b^2
*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 + 1)*d^2 + (3*b^2*c*d - a*b*d^2)*x)
*e^(-b^2*x^2 - 2*a*b*x - a^2) - (2*pi*b^3*d^2*x^3 + 6*pi*b^3*c*d*x^2 + 6*pi
*b^3*c^2*x + pi*(6*a*b^2*c^2 - 3*(2*a^2 + 1)*b*c*d + (2*a^3 + 3*a)*d^2))*er
f(b*x + a))/(pi*b^3)
```

Sympy [A] time = 6.16949, size = 398, normalized size = 2.05

$$\begin{cases} \frac{a^3d^2\,\mathrm{erfc}\,(a+bx)}{3b^3} - \frac{a^2cd\,\mathrm{erfc}\,(a+bx)}{b^2} - \frac{a^2d^2e^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^3} + \frac{ac^2\,\mathrm{erfc}\,(a+bx)}{b} + \frac{acde^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b^2} + \frac{ad^2xe^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^2} + \frac{ad^2\,\mathrm{erfc}\,(a+bx)}{2b^3} + c^2 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\mathrm{erfc}\,(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*erfc(b*x+a),x)
```

```
[Out] Piecewise((a**3*d**2*erfc(a + b*x)/(3*b**3) - a**2*c*d*erfc(a + b*x)/b**2 -
 a**2*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b**3) + a*c
**2*erfc(a + b*x)/b + a*c*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(
pi)*b**2) + a*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b
**2) + a*d**2*erfc(a + b*x)/(2*b**3) + c**2*x*erfc(a + b*x) + c*d*x**2*erfc
(a + b*x) + d**2*x**3*erfc(a + b*x)/3 - c**2*exp(-a**2)*exp(-b**2*x**2)*exp
(-2*a*b*x)/(sqrt(pi)*b) - c*d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(s
qrt(pi)*b) - d**2*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)
```

```
*b) - c*d*erfc(a + b*x)/(2*b**2) - d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a
*b*x)/(3*sqrt(pi)*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*erfc
(a), True))
```

Giac [A] time = 1.52206, size = 378, normalized size = 1.95

$$\frac{1}{3}d^2x^3+cdx^2-\left(x\,\mathrm{erf}\,(bx+a)-\frac{\frac{\sqrt{\pi}a\,\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}\right)c^2-\frac{1}{2}\left(2\,x^2\,\mathrm{erf}\,(bx+a)+\frac{\frac{\sqrt{\pi}\left(2\,a^2+1\right)\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(b\left(\right.\right.}{}}{\sqrt{\pi}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 - (x*erf(b*x + a) - (sqrt(pi)*a*erf(-b*(x + a/b))/b -
 e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/sqrt(pi))*c^2 - 1/2*(2*x^2*erf(b*x + a) +
(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x
^2 - 2*a*b*x - a^2)/b)/(sqrt(pi)*b))*c*d - 1/6*(2*x^3*erf(b*x + a) - (sqrt(
pi)*(2*a^3 + 3*a)*erf(-b*(x + a/b))/b - 2*(b^2*(x + a/b)^2 - 3*a*b*(x + a/b
) + 3*a^2 + 1)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(sqrt(pi)*b^2))*d^2 + c^2*x
```

3.120 $\int (c+dx)\mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^2\text{Erf}(a+bx)}{2b^2d}-\frac{e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^2}+\frac{d\text{Erf}(a+bx)}{4b^2}-\frac{de^{-(a+bx)^2}(a+bx)}{2\sqrt{\pi}b^2}+\frac{(c+dx)^2\text{Erfc}(a+bx)}{2d}$$

```
[Out] -((b*c - a*d)/(b^2*E^(a + b*x)^2*Sqrt[Pi])) - (d*(a + b*x))/(2*b^2*E^(a + b
*x)^2*Sqrt[Pi]) + (d*Erf[a + b*x])/(4*b^2) + ((b*c - a*d)^2*Erf[a + b*x])/(
2*b^2*d) + ((c + d*x)^2*Erfc[a + b*x])/(2*d)
```

Rubi [A] time = 0.117892, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6362, 2226, 2205, 2209, 2212}

$$\frac{(bc-ad)^2\text{Erf}(a+bx)}{2b^2d}-\frac{e^{-(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^2}+\frac{d\text{Erf}(a+bx)}{4b^2}-\frac{de^{-(a+bx)^2}(a+bx)}{2\sqrt{\pi}b^2}+\frac{(c+dx)^2\text{Erfc}(a+bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Erfc[a + b*x],x]
```

```
[Out] -((b*c - a*d)/(b^2*E^(a + b*x)^2*Sqrt[Pi])) - (d*(a + b*x))/(2*b^2*E^(a + b
*x)^2*Sqrt[Pi]) + (d*Erf[a + b*x])/(4*b^2) + ((b*c - a*d)^2*Erf[a + b*x])/(
2*b^2*d) + ((c + d*x)^2*Erfc[a + b*x])/(2*d)
```

Rule 6362

```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfc[a + b*x])/(d*(m + 1)), x] + Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\text{erfc}(a+bx)\,dx &= \frac{(c+dx)^2\text{erfc}(a+bx)}{2d}+\frac{b\int e^{-(a+bx)^2}(c+dx)^2\,dx}{d\sqrt{\pi}}\\
&= \frac{(c+dx)^2\text{erfc}(a+bx)}{2d}+\frac{b\int \left(\frac{(bc-ad)^2e^{-(a+bx)^2}}{b^2}+\frac{2d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^2}+\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{b^2}\right)dx}{d\sqrt{\pi}}\\
&= \frac{(c+dx)^2\text{erfc}(a+bx)}{2d}+\frac{d\int e^{-(a+bx)^2}(a+bx)^2\,dx}{b\sqrt{\pi}}+\frac{(2(bc-ad))\int e^{-(a+bx)^2}(a+bx)\,dx}{b\sqrt{\pi}}+\frac{(b
\\
&= -\frac{(bc-ad)e^{-(a+bx)^2}}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}}+\frac{(bc-ad)^2\text{erf}(a+bx)}{2b^2d}+\frac{(c+dx)^2\text{erfc}(a+bx)}{2d}+
\\
&= -\frac{(bc-ad)e^{-(a+bx)^2}}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}}+\frac{d\text{erf}(a+bx)}{4b^2}+\frac{(bc-ad)^2\text{erf}(a+bx)}{2b^2d}+\frac{(c+dx)
\end{aligned}$$

Mathematica [A] time = 0.115806, size = 104, normalized size = 0.87

$$\frac{e^{-(a+bx)^2}\left(\sqrt{\pi}e^{(a+bx)^2}\left(2a^2d-4abc+d\right)\text{Erf}(a+bx)+2\sqrt{\pi}b^2xe^{(a+bx)^2}(2c+dx)\text{Erfc}(a+bx)+2ad-4bc-2bdx\right)}{4\sqrt{\pi}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Erfc[a + b*x],x]
```

```
[Out] (-4*b*c + 2*a*d - 2*b*d*x + (-4*a*b*c + d + 2*a^2*d)*E^(a + b*x)^2*Sqrt[Pi]
*Erf[a + b*x] + 2*b^2*E^(a + b*x)^2*Sqrt[Pi]*x*(2*c + d*x)*Erfc[a + b*x])/(
4*b^2*E^(a + b*x)^2*Sqrt[Pi])
```

Maple [A] time = 0.045, size = 122, normalized size = 1.

$$\frac{1}{b}\left(\frac{d\operatorname{erfc}(bx+a)(bx+a)^2}{2b}-\frac{\operatorname{erfc}(bx+a)(bx+a)ad}{b}+\operatorname{erfc}(bx+a)c(bx+a)+\frac{1}{\sqrt{\pi}b}\left(d\left(-\frac{bx+a}{2\,e^{(bx+a)^2}}+\frac{\sqrt{\pi}\operatorname{Erf}(bx+a)}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*erfc(b*x+a),x)
```

```
[Out] 1/b*(1/2/b*erfc(b*x+a)*d*(b*x+a)^2-1/b*erfc(b*x+a)*(b*x+a)*a*d+erfc(b*x+a)*
c*(b*x+a)+1/Pi^(1/2)/b*(d*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x
+a))+a*d/exp((b*x+a)^2)-b*c/exp((b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\operatorname{erfc}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*erfc(b*x + a), x)
```

Fricas [A] time = 2.042, size = 254, normalized size = 2.13

$$\frac{2\,\pi b^2dx^2+4\,\pi b^2cx-2\,\sqrt{\pi}(bdx+2\,bc-ad)e^{\left(-b^2x^2-2\,abx-a^2\right)}-\left(2\,\pi b^2dx^2+4\,\pi b^2cx+\pi\left(4\,abc-\left(2\,a^2+1\right)d\right)\right)\operatorname{erf}(bx+a)}{4\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*pi*b^2*d*x^2 + 4*pi*b^2*c*x - 2*sqrt(pi)*(b*d*x + 2*b*c - a*d)*e^(-b
^2*x^2 - 2*a*b*x - a^2) - (2*pi*b^2*d*x^2 + 4*pi*b^2*c*x + pi*(4*a*b*c - (2
*a^2 + 1)*d))*erf(b*x + a))/(pi*b^2)
```

Sympy [A] time = 2.09058, size = 178, normalized size = 1.5

$$\begin{cases} -\frac{a^2 d \operatorname{erfc}(a+bx)}{2b^2} + \frac{ac \operatorname{erfc}(a+bx)}{b} + \frac{ade^{-a^2}e^{-b^2x^2}e^{-2abx}}{2\sqrt{\pi}b^2} + cx\operatorname{erfc}(a+bx) + \frac{dx^2\operatorname{erfc}(a+bx)}{2} - \frac{ce^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b} - \frac{dxe^{-a^2}e^{-b^2x^2}e^{-2abx}}{2\sqrt{\pi}b} - \frac{d\operatorname{erf} \\ \left(cx + \frac{dx^2}{2}\right)\operatorname{erfc}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a),x)
```

```
[Out] Piecewise((-a**2*d*erfc(a + b*x)/(2*b**2) + a*c*erfc(a + b*x)/b + a*d*exp(-
a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b**2) + c*x*erfc(a + b*x) +
 d*x**2*erfc(a + b*x)/2 - c*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(
pi)*b) - d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) - d*er
fc(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*erfc(a), True))
```

Giac [A] time = 1.40821, size = 213, normalized size = 1.79

$$\frac{1}{2}dx^2 - \left(x\operatorname{erf}(bx+a) - \frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}\right)c - \frac{1}{4}\left(2\,x^2\operatorname{erf}(bx+a) + \frac{\frac{\sqrt{\pi}\left(2\,a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\frac{a}{b}\right)-2\,a\right)e^{(}}{b}}{\sqrt{\pi}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 - (x*erf(b*x + a) - (sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2
 - 2*a*b*x - a^2)/b)/sqrt(pi))*c - 1/4*(2*x^2*erf(b*x + a) + (sqrt(pi)*(2*a
^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x -
 a^2)/b)/(sqrt(pi)*b))*d + c*x
```

3.121 $\int \mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\text{Erfc}(a+bx)}{b}-\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

```
[Out] -(1/(b*E^(a + b*x)^2*Sqrt[Pi])) + ((a + b*x)*Erfc[a + b*x])/b
```

Rubi [A] time = 0.0067298, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6350}

$$\frac{(a+bx)\text{Erfc}(a+bx)}{b}-\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[a + b*x],x]
```

```
[Out] -(1/(b*E^(a + b*x)^2*Sqrt[Pi])) + ((a + b*x)*Erfc[a + b*x])/b
```

Rule 6350

```
Int[Erfc[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x])/b
, x] - Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erfc}(a+bx)\,dx=-\frac{e^{-(a+bx)^2}}{b\sqrt{\pi}}+\frac{(a+bx)\text{erfc}(a+bx)}{b}$$

Mathematica [A] time = 0.0407458, size = 42, normalized size = 1.14

$$-\frac{a\text{Erf}(a+bx)}{b}+x\text{Erfc}(a+bx)-\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[a + b*x],x]
```

```
[Out] -(1/(b*E^(a + b*x)^2*Sqrt[Pi])) - (a*Erf[a + b*x])/b + x*Erfc[a + b*x]
```

Maple [A] time = 0.042, size = 33, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\operatorname{erfc}(bx+a)-\frac{\mathrm{e}^{-(bx+a)^2}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*erfc(b*x+a)-1/Pi^(1/2)*exp(-(b*x+a)^2))
```

Maxima [A] time = 1.12613, size = 43, normalized size = 1.16

$$\frac{(bx+a)\operatorname{erfc}(bx+a)-\frac{e^{\left(-(bx+a)^2\right)}}{\sqrt{\pi}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*erfc(b*x + a) - e^(-(b*x + a)^2)/sqrt(pi))/b
```

Fricas [A] time = 2.07231, size = 119, normalized size = 3.22

$$\frac{\pi bx-(\pi bx+\pi a)\operatorname{erf}(bx+a)-\sqrt{\pi}e^{\left(-b^2x^2-2\,abx-a^2\right)}}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] (pi*b*x - (pi*b*x + pi*a)*erf(b*x + a) - sqrt(pi)*e^(-b^2*x^2 - 2*a*b*x - a
^2))/(pi*b)
```

Sympy [A] time = 0.782964, size = 53, normalized size = 1.43

$$\begin{cases} \frac{a \operatorname{erfc}(a+bx)}{b} + x \operatorname{erfc}(a+bx) - \frac{e^{-a^2} e^{-b^2 x^2} e^{-2abx}}{\sqrt{\pi} b} & \text{for } b \neq 0 \\ x \operatorname{erfc}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a),x)
```

```
[Out] Piecewise((a*erfc(a + b*x)/b + x*erfc(a + b*x) - exp(-a**2)*exp(-b**2*x**2)
*exp(-2*a*b*x)/(sqrt(pi)*b), Ne(b, 0)), (x*erfc(a), True))
```

Giac [A] time = 1.31475, size = 81, normalized size = 2.19

$$-x \operatorname{erf}(bx+a) + x + \frac{\frac{\sqrt{\pi} a \operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2 x^2 - 2\,abx - a^2\right)}}{b}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a),x, algorithm="giac")
```

```
[Out] -x*erf(b*x + a) + x + (sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b
*x - a^2)/b)/sqrt(pi)
```

3.122 $\int \frac{\text{Erfc}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{Erfc}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Erfc[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0150038, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Erfc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{erfc}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfc}(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 0.122384, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Erfc[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.452, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(bx+a\right)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a)/(d*x+c),x)
```

```
[Out] int(erfc(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(bx+a\right)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\operatorname{erf}\left(bx+a\right)-1}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(a+bx\right)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(erfc(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)/(d*x + c), x)
```

3.123 $\int \frac{\text{Erfc}(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=51

$$-\frac{2b\text{Unintegrable}\left(\frac{e^{-(a+bx)^2}}{c+dx},x\right)}{\sqrt{\pi}d}-\frac{\text{Erfc}(a+bx)}{d(c+dx)}$$

```
[Out] -(Erfc[a + b*x]/(d*(c + d*x))) - (2*b*Unintegrable[1/(E^(a + b*x)^2*(c + d*
x)), x])/(d*Sqrt[Pi])
```

Rubi [A] time = 0.038967, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Erfc[a + b*x]/(d*(c + d*x))) - (2*b*Defer[Int][1/(E^(a + b*x)^2*(c + d*x)
), x])/(d*Sqrt[Pi])
```

Rubi steps

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2} dx = -\frac{\text{erfc}(a+bx)}{d(c+dx)} - \frac{(2b)\int \frac{e^{-(a+bx)^2}}{c+dx} dx}{d\sqrt{\pi}}$$

Mathematica [A] time = 0.488978, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Erfc[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.375, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(erfc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\operatorname{erf}(bx+a)-1}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(erfc(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)/(d*x + c)^2, x)
```

3.124 $\int \frac{\textbf{Erfc}(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{2b^2(bc-ad)\text{Unintegrable}\left(\frac{e^{-(a+bx)^2}}{c+dx},x\right)}{\sqrt{\pi}d^3}+\frac{b^2\text{Erf}(a+bx)}{d^3}+\frac{be^{-(a+bx)^2}}{\sqrt{\pi}d^2(c+dx)}-\frac{\text{Erfc}(a+bx)}{2d(c+dx)^2}$$

```
[Out] b/(d^2*E^(a + b*x)^2*Sqrt[Pi]*(c + d*x)) + (b^2*Erf[a + b*x])/d^3 - Erfc[a
+ b*x]/(2*d*(c + d*x)^2) - (2*b^2*(b*c - a*d)*Unintegrable[1/(E^(a + b*x)^2
*(c + d*x)), x])/(d^3*Sqrt[Pi])
```

Rubi [A] time = 0.0792894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(a+bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[a + b*x]/(c + d*x)^3,x]
```

```
[Out] b/(d^2*E^(a + b*x)^2*Sqrt[Pi]*(c + d*x)) + (b^2*Erf[a + b*x])/d^3 - Erfc[a
+ b*x]/(2*d*(c + d*x)^2) - (2*b^2*(b*c - a*d)*Defer[Int][1/(E^(a + b*x)^2*(
c + d*x)), x])/(d^3*Sqrt[Pi])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{erfc}(a+bx)}{(c+dx)^3}\,dx &= -\frac{\text{erfc}(a+bx)}{2d(c+dx)^2}-\frac{b\int \frac{e^{-(a+bx)^2}}{(c+dx)^2}\,dx}{d\sqrt{\pi}}\\
&= \frac{be^{-(a+bx)^2}}{d^2\sqrt{\pi}(c+dx)}-\frac{\text{erfc}(a+bx)}{2d(c+dx)^2}+\frac{\left(2b^3\right)\int e^{-(a+bx)^2}\,dx}{d^3\sqrt{\pi}}-\frac{\left(2b^2(bc-ad)\right)\int \frac{e^{-(a+bx)^2}}{c+dx}\,dx}{d^3\sqrt{\pi}}\\
&= \frac{be^{-(a+bx)^2}}{d^2\sqrt{\pi}(c+dx)}+\frac{b^2\text{erf}(a+bx)}{d^3}-\frac{\text{erfc}(a+bx)}{2d(c+dx)^2}-\frac{\left(2b^2(bc-ad)\right)\int \frac{e^{-(a+bx)^2}}{c+dx}\,dx}{d^3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.959073, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(a+bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[a + b*x]/(c + d*x)^3,x]
```

```
[Out] Integrate[Erfc[a + b*x]/(c + d*x)^3, x]
```

Maple [A] time = 0.399, size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a)/(d*x+c)^3,x)
```

```
[Out] int(erfc(b*x+a)/(d*x+c)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)/(d*x + c)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{erf}\,(bx+a)-1}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)/(d*x + c)^3, x)
```

3.125 $\int x^5 \mathbf{Erfc}(bx)^2 \, dx$

Optimal. Leaf size=178

$$-\frac{x^5 e^{-b^2x^2}\text{Erfc}(bx)}{3\sqrt{\pi}b}-\frac{5x^3e^{-b^2x^2}\text{Erfc}(bx)}{6\sqrt{\pi}b^3}-\frac{5xe^{-b^2x^2}\text{Erfc}(bx)}{4\sqrt{\pi}b^5}-\frac{5\text{Erfc}(bx)^2}{16b^6}+\frac{x^4e^{-2b^2x^2}}{6\pi b^2}+\frac{7x^2e^{-2b^2x^2}}{12\pi b^4}+\frac{11e^{-2b^2x^2}}{12\pi b^6}+\frac{1}{6}x^6\text{Er}$$

```
[Out] 11/(12*b^6*E^(2*b^2*x^2)*Pi) + (7*x^2)/(12*b^4*E^(2*b^2*x^2)*Pi) + x^4/(6*b
^2*E^(2*b^2*x^2)*Pi) - (5*x*Erfc[b*x])/(4*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (5*x^
3*Erfc[b*x])/(6*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (x^5*Erfc[b*x])/(3*b*E^(b^2*x^2
)*Sqrt[Pi]) - (5*Erfc[b*x]^2)/(16*b^6) + (x^6*Erfc[b*x]^2)/6
```

Rubi [A] time = 0.276389, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6365, 6386, 6374, 30, 2209, 2212}

$$-\frac{x^5 e^{-b^2x^2}\text{Erfc}(bx)}{3\sqrt{\pi}b}-\frac{5x^3e^{-b^2x^2}\text{Erfc}(bx)}{6\sqrt{\pi}b^3}-\frac{5xe^{-b^2x^2}\text{Erfc}(bx)}{4\sqrt{\pi}b^5}-\frac{5\text{Erfc}(bx)^2}{16b^6}+\frac{x^4e^{-2b^2x^2}}{6\pi b^2}+\frac{7x^2e^{-2b^2x^2}}{12\pi b^4}+\frac{11e^{-2b^2x^2}}{12\pi b^6}+\frac{1}{6}x^6\text{Er}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Erfc[b*x]^2,x]
```

```
[Out] 11/(12*b^6*E^(2*b^2*x^2)*Pi) + (7*x^2)/(12*b^4*E^(2*b^2*x^2)*Pi) + x^4/(6*b
^2*E^(2*b^2*x^2)*Pi) - (5*x*Erfc[b*x])/(4*b^5*E^(b^2*x^2)*Sqrt[Pi]) - (5*x^
3*Erfc[b*x])/(6*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (x^5*Erfc[b*x])/(3*b*E^(b^2*x^2
)*Sqrt[Pi]) - (5*Erfc[b*x]^2)/(16*b^6) + (x^6*Erfc[b*x]^2)/6
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int x^5\text{erfc}(bx)^2\,dx = \frac{1}{6}x^6\text{erfc}(bx)^2 + \frac{(2b)\int e^{-b^2x^2}x^6\text{erfc}(bx)\,dx}{3\sqrt{\pi}}$$
$$= -\frac{e^{-b^2x^2}x^5\text{erfc}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfc}(bx)^2 - \frac{2\int e^{-2b^2x^2}x^5\,dx}{3\pi} + \frac{5\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx}{3b\sqrt{\pi}}$$
$$= \frac{e^{-2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{-b^2x^2}x^3\text{erfc}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5\text{erfc}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfc}(bx)^2 - \frac{2\int e^{-2b^2x^2}x^3\,dx}{3b^2\pi} - \frac{5\int e^{-2b^2x^2}x^3\,d}{3b^2\pi}$$
$$= \frac{7e^{-2b^2x^2}x^2}{12b^4\pi} + \frac{e^{-2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{-b^2x^2}x\text{erfc}(bx)}{4b^5\sqrt{\pi}} - \frac{5e^{-b^2x^2}x^3\text{erfc}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5\text{erfc}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfc}(bx)^2 -$$
$$= \frac{11e^{-2b^2x^2}}{12b^6\pi} + \frac{7e^{-2b^2x^2}x^2}{12b^4\pi} + \frac{e^{-2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{-b^2x^2}x\text{erfc}(bx)}{4b^5\sqrt{\pi}} - \frac{5e^{-b^2x^2}x^3\text{erfc}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5\text{erfc}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6$$
$$= \frac{11e^{-2b^2x^2}}{12b^6\pi} + \frac{7e^{-2b^2x^2}x^2}{12b^4\pi} + \frac{e^{-2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{-b^2x^2}x\text{erfc}(bx)}{4b^5\sqrt{\pi}} - \frac{5e^{-b^2x^2}x^3\text{erfc}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5\text{erfc}(bx)}{3b\sqrt{\pi}} - \frac{5\text{er}}{1}$$

Mathematica [A] time = 0.391022, size = 173, normalized size = 0.97

$$\frac{1}{24}\left(\left(\frac{15}{b^6} - 8x^6\right)\text{Erf}(bx) + \frac{e^{-2b^2x^2}\left(4\sqrt{\pi}bxe^{b^2x^2}\left(4b^4x^4 + 10b^2x^2 + 15\right)\text{Erf}(bx) - 15\pi e^{2b^2x^2}\text{Erf}(bx)^2 + 8b^4x^4 + 28b^2x^2 + 44\right.}{2\pi b^6}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*Erfc[b*x]^2,x]
```

```
[Out] (4*x^6 - (2*x*(15 + 10*b^2*x^2 + 4*b^4*x^4))/(b^5*E^(b^2*x^2)*Sqrt[Pi]) + (
15/b^6 - 8*x^6)*Erf[b*x] + 4*x^6*Erf[b*x]^2 + (44 + 28*b^2*x^2 + 8*b^4*x^4
+ 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(15 + 10*b^2*x^2 + 4*b^4*x^4)*Erf[b*x] - 15*E^
(2*b^2*x^2)*Pi*Erf[b*x]^2)/(2*b^6*E^(2*b^2*x^2)*Pi))/24
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x^5\left(\text{erfc}\,(bx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erfc(b*x)^2,x)
```

```
[Out] int(x^5*erfc(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfc(b*x)^2, x)
```

Fricas [A] time = 2.16863, size = 344, normalized size = 1.93

$$\frac{8\,\pi b^6 x^6 - \left(15\,\pi - 8\,\pi b^6 x^6\right) \operatorname{erf}(bx)^2 - 4\,\sqrt{\pi}\left(4\,b^5 x^5 + 10\,b^3 x^3 + 15\,bx - \left(4\,b^5 x^5 + 10\,b^3 x^3 + 15\,bx\right) \operatorname{erf}(bx)\right) e^{\left(-b^2 x^2\right)} + 2\,(}{48\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(8*pi*b^6*x^6 - (15*pi - 8*pi*b^6*x^6)*erf(b*x)^2 - 4*sqrt(pi)*(4*b^5*
x^5 + 10*b^3*x^3 + 15*b*x - (4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*erf(b*x))*e^(
-b^2*x^2) + 2*(15*pi - 8*pi*b^6*x^6)*erf(b*x) + 4*(2*b^4*x^4 + 7*b^2*x^2 +
11)*e^(-2*b^2*x^2))/(pi*b^6)
```

Sympy [A] time = 7.47145, size = 172, normalized size = 0.97

$$\begin{cases} \frac{x^6 \operatorname{erfc}^2(bx)}{6} - \frac{x^5 e^{-b^2x^2} \operatorname{erfc}(bx)}{3\sqrt{\pi} b} + \frac{x^4 e^{-2b^2x^2}}{6\pi b^2} - \frac{5x^3 e^{-b^2x^2} \operatorname{erfc}(bx)}{6\sqrt{\pi} b^3} + \frac{7x^2 e^{-2b^2x^2}}{12\pi b^4} - \frac{5x e^{-b^2x^2} \operatorname{erfc}(bx)}{4\sqrt{\pi} b^5} - \frac{5 \operatorname{erfc}^2(bx)}{16 b^6} + \frac{11 e^{-2b^2x^2}}{12\pi b^6} & \text{for } b \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erfc(b*x)**2,x)
```

```
[Out] Piecewise((x**6*erfc(b*x)**2/6 - x**5*exp(-b**2*x**2)*erfc(b*x)/(3*sqrt(pi)
*b) + x**4*exp(-2*b**2*x**2)/(6*pi*b**2) - 5*x**3*exp(-b**2*x**2)*erfc(b*x)
/(6*sqrt(pi)*b**3) + 7*x**2*exp(-2*b**2*x**2)/(12*pi*b**4) - 5*x*exp(-b**2*
x**2)*erfc(b*x)/(4*sqrt(pi)*b**5) - 5*erfc(b*x)**2/(16*b**6) + 11*exp(-2*b*
*2*x**2)/(12*pi*b**6), Ne(b, 0)), (x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*erfc(b*x)^2, x)
```

3.126 $\int x^3 \textbf{Erfc}(bx)^2\,dx$

Optimal. Leaf size=126

$$-\frac{x^3e^{-b^2x^2}\mathrm{Erfc}(bx)}{2\sqrt{\pi}b}-\frac{3xe^{-b^2x^2}\mathrm{Erfc}(bx)}{4\sqrt{\pi}b^3}-\frac{3\mathrm{Erfc}(bx)^2}{16b^4}+\frac{x^2e^{-2b^2x^2}}{4\pi b^2}+\frac{e^{-2b^2x^2}}{2\pi b^4}+\frac{1}{4}x^4\mathrm{Erfc}(bx)^2$$

```
[Out] 1/(2*b^4*E^(2*b^2*x^2)*Pi) + x^2/(4*b^2*E^(2*b^2*x^2)*Pi) - (3*x*Erfc[b*x])
/(4*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (x^3*Erfc[b*x])/(2*b*E^(b^2*x^2)*Sqrt[Pi])
- (3*Erfc[b*x]^2)/(16*b^4) + (x^4*Erfc[b*x]^2)/4
```

Rubi [A] time = 0.171174, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6365, 6386, 6374, 30, 2209, 2212}

$$-\frac{x^3e^{-b^2x^2}\mathrm{Erfc}(bx)}{2\sqrt{\pi}b}-\frac{3xe^{-b^2x^2}\mathrm{Erfc}(bx)}{4\sqrt{\pi}b^3}-\frac{3\mathrm{Erfc}(bx)^2}{16b^4}+\frac{x^2e^{-2b^2x^2}}{4\pi b^2}+\frac{e^{-2b^2x^2}}{2\pi b^4}+\frac{1}{4}x^4\mathrm{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Erfc[b*x]^2,x]
```

```
[Out] 1/(2*b^4*E^(2*b^2*x^2)*Pi) + x^2/(4*b^2*E^(2*b^2*x^2)*Pi) - (3*x*Erfc[b*x])
/(4*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (x^3*Erfc[b*x])/(2*b*E^(b^2*x^2)*Sqrt[Pi])
- (3*Erfc[b*x]^2)/(16*b^4) + (x^4*Erfc[b*x]^2)/4
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int x^3\text{erfc}(bx)^2\,dx = \frac{1}{4}x^4\text{erfc}(bx)^2 + \frac{b\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx}{\sqrt{\pi}}$$

$$= -\frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfc}(bx)^2 - \frac{\int e^{-2b^2x^2}x^3\,dx}{\pi} + \frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx}{2b\sqrt{\pi}}$$

$$= \frac{e^{-2b^2x^2}x^2}{4b^2\pi} - \frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfc}(bx)^2 - \frac{\int e^{-2b^2x^2}x\,dx}{2b^2\pi} - \frac{3\int e^{-2b^2x^2}x\,dx}{2b^2\pi} + $$

$$= \frac{e^{-2b^2x^2}}{2b^4\pi} + \frac{e^{-2b^2x^2}x^2}{4b^2\pi} - \frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfc}(bx)^2 - \frac{3\,\text{Subst}(\int x\,dx, x, \text{erfc}(}{8b^4}$$

$$= \frac{e^{-2b^2x^2}}{2b^4\pi} + \frac{e^{-2b^2x^2}x^2}{4b^2\pi} - \frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b\sqrt{\pi}} - \frac{3\text{erfc}(bx)^2}{16b^4} + \frac{1}{4}x^4\text{erfc}(bx)^2$$

Mathematica [A] time = 0.376289, size = 149, normalized size = 1.18

$$\frac{1}{8}\left(\left(\frac{3}{b^4}-4x^4\right)\text{Erf}(bx)+\frac{e^{-2b^2x^2}\left(4\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2+3\right)\text{Erf}(bx)-3\pi e^{2b^2x^2}\text{Erf}(bx)^2+4b^2x^2+8\right)}{2\pi b^4}-\frac{2xe^{-b^2x^2}\left(2b^2x^2+\right.}{\sqrt{\pi}b^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Erfc[b*x]^2,x]
```

```
[Out] (2*x^4 - (2*x*(3 + 2*b^2*x^2))/(b^3*E^(b^2*x^2)*Sqrt[Pi]) + (3/b^4 - 4*x^4)
*Erf[b*x] + 2*x^4*Erf[b*x]^2 + (8 + 4*b^2*x^2 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*
(3 + 2*b^2*x^2)*Erf[b*x] - 3*E^(2*b^2*x^2)*Pi*Erf[b*x]^2)/(2*b^4*E^(2*b^2*x
^2)*Pi))/8
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x^3 (\text{erfc}\,(bx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erfc(b*x)^2,x)
```

```
[Out] int(x^3*erfc(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\text{erfc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfc(b*x)^2, x)
```

Fricas [A] time = 2.09719, size = 284, normalized size = 2.25

$$\frac{4\,\pi b^4 x^4 - \left(3\,\pi - 4\,\pi b^4 x^4\right)\operatorname{erf}\left(bx\right)^2 - 4\,\sqrt{\pi}\left(2\,b^3 x^3 + 3\,bx - \left(2\,b^3 x^3 + 3\,bx\right)\operatorname{erf}\left(bx\right)\right)e^{\left(-b^2 x^2\right)} + 2\left(3\,\pi - 4\,\pi b^4 x^4\right)\operatorname{erf}\left(bx\right) +}{16\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*pi*b^4*x^4 - (3*pi - 4*pi*b^4*x^4)*erf(b*x)^2 - 4*sqrt(pi)*(2*b^3*x
^3 + 3*b*x - (2*b^3*x^3 + 3*b*x)*erf(b*x))*e^(-b^2*x^2) + 2*(3*pi - 4*pi*b^
4*x^4)*erf(b*x) + 4*(b^2*x^2 + 2)*e^(-2*b^2*x^2))/(pi*b^4)
```

Sympy [A] time = 2.50494, size = 121, normalized size = 0.96

$$\begin{cases} \frac{x^4 \operatorname{erfc}^2(bx)}{4} - \frac{x^3 e^{-b^2x^2} \operatorname{erfc}(bx)}{2\sqrt{\pi}b} + \frac{x^2 e^{-2b^2x^2}}{4\pi b^2} - \frac{3x e^{-b^2x^2} \operatorname{erfc}(bx)}{4\sqrt{\pi}b^3} - \frac{3\operatorname{erfc}^2(bx)}{16b^4} + \frac{e^{-2b^2x^2}}{2\pi b^4} & \text{for } b \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erfc(b*x)**2,x)
```

```
[Out] Piecewise((x**4*erfc(b*x)**2/4 - x**3*exp(-b**2*x**2)*erfc(b*x)/(2*sqrt(pi)
*b) + x**2*exp(-2*b**2*x**2)/(4*pi*b**2) - 3*x*exp(-b**2*x**2)*erfc(b*x)/(4
*sqrt(pi)*b**3) - 3*erfc(b*x)**2/(16*b**4) + exp(-2*b**2*x**2)/(2*pi*b**4),
 Ne(b, 0)), (x**4/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*erfc(b*x)^2, x)
```

3.127 $\int x\mathbf{Erfc}(bx)^2\,dx$

Optimal. Leaf size=72

$$-\frac{xe^{-b^2x^2}\text{Erfc}(bx)}{\sqrt{\pi}b}-\frac{\text{Erfc}(bx)^2}{4b^2}+\frac{e^{-2b^2x^2}}{2\pi b^2}+\frac{1}{2}x^2\text{Erfc}(bx)^2$$

```
[Out] 1/(2*b^2*E^(2*b^2*x^2)*Pi) - (x*Erfc[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) - Erfc[
b*x]^2/(4*b^2) + (x^2*Erfc[b*x]^2)/2
```

Rubi [A] time = 0.0810356, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6365, 6386, 6374, 30, 2209}

$$-\frac{xe^{-b^2x^2}\text{Erfc}(bx)}{\sqrt{\pi}b}-\frac{\text{Erfc}(bx)^2}{4b^2}+\frac{e^{-2b^2x^2}}{2\pi b^2}+\frac{1}{2}x^2\text{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Erfc[b*x]^2,x]
```


```
[Out] 1/(2*b^2*E^(2*b^2*x^2)*Pi) - (x*Erfc[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) - Erfc[
b*x]^2/(4*b^2) + (x^2*Erfc[b*x]^2)/2
```


Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
```

```
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x\text{erfc}(bx)^2\,dx &= \frac{1}{2}x^2\text{erfc}(bx)^2+\frac{(2b)\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}x\text{erfc}(bx)}{b\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2-\frac{2\int e^{-2b^2x^2}x\,dx}{\pi}+\frac{\int e^{-b^2x^2}\text{erfc}(bx)\,dx}{b\sqrt{\pi}}\\ &= \frac{e^{-2b^2x^2}}{2b^2\pi}-\frac{e^{-b^2x^2}x\text{erfc}(bx)}{b\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2-\frac{\text{Subst}(\int x\,dx,x,\text{erfc}(bx))}{2b^2}\\ &= \frac{e^{-2b^2x^2}}{2b^2\pi}-\frac{e^{-b^2x^2}x\text{erfc}(bx)}{b\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{4b^2}+\frac{1}{2}x^2\text{erfc}(bx)^2\end{aligned}$$

Mathematica [A] time = 0.128451, size = 99, normalized size = 1.38

$$\frac{\pi\left(2b^2x^2-1\right)\text{Erf}(bx)^2+\left(4\sqrt{\pi}bxe^{-b^2x^2}+\pi\left(2-4b^2x^2\right)\right)\text{Erf}(bx)+2e^{-2b^2x^2}\left(\sqrt{\pi}bxe^{b^2x^2}-1\right)^2}{4\pi b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Erfc[b*x]^2,x]
```

```
[Out] ((2*(-1 + b*E^(b^2*x^2)*Sqrt[Pi]*x)^2)/E^(2*b^2*x^2) + ((4*b*Sqrt[Pi]*x)/E^
(b^2*x^2) + Pi*(2 - 4*b^2*x^2))*Erf[b*x] + Pi*(-1 + 2*b^2*x^2)*Erf[b*x]^2)/
(4*b^2*Pi)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x\left(\mathrm{erfc}\left(bx\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfc(b*x)^2,x)
```

```
[Out] int(x*erfc(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{erfc}\left(bx\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*erfc(b*x)^2, x)
```

Fricas [A] time = 2.06366, size = 217, normalized size = 3.01

$$\frac{2\,\pi b^{2}x^{2}-\left(\pi-2\,\pi b^{2}x^{2}\right)\mathrm{erf}\left(bx\right)^{2}+4\,\sqrt{\pi}(bx\,\mathrm{erf}\left(bx\right)-bx)e^{\left(-b^{2}x^{2}\right)}+2\left(\pi-2\,\pi b^{2}x^{2}\right)\mathrm{erf}\left(bx\right)+2\,e^{\left(-2\,b^{2}x^{2}\right)}}{4\,\pi b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*pi*b^2*x^2 - (pi - 2*pi*b^2*x^2)*erf(b*x)^2 + 4*sqrt(pi)*(b*x*erf(b*
x) - b*x)*e^(-b^2*x^2) + 2*(pi - 2*pi*b^2*x^2)*erf(b*x) + 2*e^(-2*b^2*x^2))
/(pi*b^2)
```

Sympy [A] time = 0.833607, size = 68, normalized size = 0.94

$$\begin{cases} \frac{x^{2}\,\mathrm{erfc}^{2}\left(bx\right)}{2}-\frac{xe^{-b^{2}x^{2}}\,\mathrm{erfc}\left(bx\right)}{\sqrt{\pi}b}-\frac{\mathrm{erfc}^{2}\left(bx\right)}{4b^{2}}+\frac{e^{-2b^{2}x^{2}}}{2\pi b^{2}} & \text{for}\ b\neq 0\\ \frac{x^{2}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)**2,x)
```

```
[Out] Piecewise((x**2*erfc(b*x)**2/2 - x*exp(-b**2*x**2)*erfc(b*x)/(sqrt(pi)*b) -
 erfc(b*x)**2/(4*b**2) + exp(-2*b**2*x**2)/(2*pi*b**2), Ne(b, 0)), (x**2/2,
 True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*erfc(b*x)^2, x)
```

3.128 $\int \frac{\operatorname{Erfc}(bx)^2}{x} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\operatorname{Erfc}(bx)^2}{x}, x\right)$$

```
[Out] Unintegrable[Erfc[b*x]^2/x, x]
```

Rubi [A] time = 0.0164441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{Erfc}(bx)^2}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]^2/x,x]
```

```
[Out] Defer[Int][Erfc[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\operatorname{erfc}(bx)^2}{x} dx = \int \frac{\operatorname{erfc}(bx)^2}{x} dx$$

Mathematica [A] time = 0.149953, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Erfc}(bx)^2}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]^2/x,x]
```

```
[Out] Integrate[Erfc[b*x]^2/x, x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfc}(bx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x,x)
```

```
[Out] int(erfc(b*x)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erf}(bx)^2 - 2\operatorname{erf}(bx) + 1}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2/x,x)
```

```
[Out] Integral(erfc(b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x, x)
```

3.129 $\int \frac{\mathbf{Erfc}(bx)^2}{x^3}\,dx$

Optimal. Leaf size=67

$$\frac{2be^{-b^2x^2}\text{Erfc}(bx)}{\sqrt{\pi}x}+b^2\left(-\text{Erfc}(bx)^2\right)+\frac{2b^2\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\pi}-\frac{\text{Erfc}(bx)^2}{2x^2}$$

```
[Out] (2*b*Erfc[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) - b^2*Erfc[b*x]^2 - Erfc[b*x]^2/(2
*x^2) + (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Rubi [A] time = 0.0946859, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6365, 6392, 6374, 30, 2210}

$$\frac{2be^{-b^2x^2}\text{Erfc}(bx)}{\sqrt{\pi}x}+b^2\left(-\text{Erfc}(bx)^2\right)+\frac{2b^2\text{Ei}\left(-2b^2x^2\right)}{\pi}-\frac{\text{Erfc}(bx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]^2/x^3,x]
```

```
[Out] (2*b*Erfc[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) - b^2*Erfc[b*x]^2 - Erfc[b*x]^2/(2
*x^2) + (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx &= -\frac{\operatorname{erfc}(bx)^2}{2x^2} - \frac{(2b)\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^2}\,dx}{\sqrt{\pi}}\\
&= \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)^2}{2x^2} + \frac{\left(4b^2\right)\int \frac{e^{-2b^2x^2}}{x}\,dx}{\pi} + \frac{\left(4b^3\right)\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx}{\sqrt{\pi}}\\
&= \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)^2}{2x^2} + \frac{2b^2\operatorname{Ei}\left(-2b^2x^2\right)}{\pi} - \left(2b^2\right)\operatorname{Subst}\left(\int x\,dx, x, \operatorname{erfc}(bx)\right)\\
&= \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{\sqrt{\pi}x} - b^2\operatorname{erfc}(bx)^2 - \frac{\operatorname{erfc}(bx)^2}{2x^2} + \frac{2b^2\operatorname{Ei}\left(-2b^2x^2\right)}{\pi}
\end{aligned}$$

Mathematica [A] time = 0.0343234, size = 63, normalized size = 0.94

$$\frac{2be^{-b^2x^2}\operatorname{Erfc}(bx)}{\sqrt{\pi}x} + \left(-b^2 - \frac{1}{2x^2}\right)\operatorname{Erfc}(bx)^2 + \frac{2b^2\operatorname{ExpIntegralEi}\left(-2b^2x^2\right)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]^2/x^3,x]
```

```
[Out] (2*b*Erfc[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) + (-b^2 - 1/(2*x^2))*Erfc[b*x]^2 +
 (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfc}(bx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x^3,x)
```

```
[Out] int(erfc(b*x)^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2/x^3, x)
```

Fricas [A] time = 2.06378, size = 247, normalized size = 3.69

$$-\frac{\pi - 4\,\pi\sqrt{b^2}bx^2\operatorname{erf}\left(\sqrt{b^2}x\right) - 4\,b^2x^2\operatorname{Ei}\left(-2\,b^2x^2\right) + \left(\pi + 2\,\pi b^2x^2\right)\operatorname{erf}(bx)^2 + 4\,\sqrt{\pi}(bx\operatorname{erf}(bx) - bx)e^{\left(-b^2x^2\right)} - 2\,\pi\operatorname{erf}(bx)}{2\,\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(pi - 4*pi*sqrt(b^2)*b*x^2*erf(sqrt(b^2)*x) - 4*b^2*x^2*Ei(-2*b^2*x^2)
 + (pi + 2*pi*b^2*x^2)*erf(b*x)^2 + 4*sqrt(pi)*(b*x*erf(b*x) - b*x)*e^(-b^2
*x^2) - 2*pi*erf(b*x))/(pi*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2/x**3,x)
```

```
[Out] Integral(erfc(b*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x^3, x)
```

3.130 $\int \frac{\textbf{Erfc}(bx)^2}{x^5}\,dx$

Optimal. Leaf size=125

$$-\frac{2b^3e^{-b^2x^2}\mathrm{Erfc}(bx)}{3\sqrt{\pi}x}+\frac{be^{-b^2x^2}\mathrm{Erfc}(bx)}{3\sqrt{\pi}x^3}+\frac{1}{3}b^4\mathrm{Erfc}(bx)^2-\frac{4b^4\mathrm{ExpIntegralEi}\left(-2b^2x^2\right)}{3\pi}-\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{\mathrm{Erfc}(bx)^2}{4x^4}$$

```
[Out] -b^2/(3*E^(2*b^2*x^2)*Pi*x^2) + (b*Erfc[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x^3)
- (2*b^3*Erfc[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x) + (b^4*Erfc[b*x]^2)/3 - Erfc
[b*x]^2/(4*x^4) - (4*b^4*ExpIntegralEi[-2*b^2*x^2])/(3*Pi)
```

Rubi [A] time = 0.177593, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6365, 6392, 6374, 30, 2210, 2214}

$$-\frac{2b^3e^{-b^2x^2}\mathrm{Erfc}(bx)}{3\sqrt{\pi}x}+\frac{be^{-b^2x^2}\mathrm{Erfc}(bx)}{3\sqrt{\pi}x^3}+\frac{1}{3}b^4\mathrm{Erfc}(bx)^2-\frac{4b^4\mathrm{Ei}\left(-2b^2x^2\right)}{3\pi}-\frac{b^2e^{-2b^2x^2}}{3\pi x^2}-\frac{\mathrm{Erfc}(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]^2/x^5,x]
```


```
[Out] -b^2/(3*E^(2*b^2*x^2)*Pi*x^2) + (b*Erfc[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x^3)
- (2*b^3*Erfc[b*x])/(3*E^(b^2*x^2)*Sqrt[Pi]*x) + (b^4*Erfc[b*x]^2)/3 - Erfc
[b*x]^2/(4*x^4) - (4*b^4*ExpIntegralEi[-2*b^2*x^2])/(3*Pi)
```


Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\int \frac{\text{erfc}(bx)^2}{x^5}\,dx = -\frac{\text{erfc}(bx)^2}{4x^4} - \frac{b\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx}{\sqrt{\pi}}$$

$$= \frac{be^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x^3} - \frac{\text{erfc}(bx)^2}{4x^4} + \frac{\left(2b^2\right)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{3\pi} + \frac{\left(2b^3\right)\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}\,dx}{3\sqrt{\pi}}$$

$$= -\frac{b^2e^{-2b^2x^2}}{3\pi x^2} + \frac{be^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x^3} - \frac{2b^3e^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x} - \frac{\text{erfc}(bx)^2}{4x^4} - 2\frac{\left(4b^4\right)\int \frac{e^{-2b^2x^2}}{x}\,dx}{3\pi} - \frac{\left(4b^5\right)\int e^{-b^2x^2}\text{e}\ldots}{3\sqrt{\pi}\ldots}$$

$$= -\frac{b^2e^{-2b^2x^2}}{3\pi x^2} + \frac{be^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x^3} - \frac{2b^3e^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x} - \frac{\text{erfc}(bx)^2}{4x^4} - \frac{4b^4\text{Ei}\left(-2b^2x^2\right)}{3\pi} + \frac{1}{3}\left(2b^4\right)\text{Subst}\left(\int\ldots\right.$$

$$= -\frac{b^2e^{-2b^2x^2}}{3\pi x^2} + \frac{be^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x^3} - \frac{2b^3e^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x} + \frac{1}{3}b^4\text{erfc}(bx)^2 - \frac{\text{erfc}(bx)^2}{4x^4} - \frac{4b^4\text{Ei}\left(-2b^2x^2\right)}{3\pi}$$

Mathematica [A] time = 0.0819507, size = 97, normalized size = 0.78

$$\frac{-\frac{4bxe^{-b^2x^2}(2b^2x^2-1)\text{Erfc}(bx)}{\sqrt{\pi}}+\left(4b^4x^4-3\right)\text{Erfc}(bx)^2-\frac{4b^2x^2\left(4b^2x^2\text{ExpIntegralEi}(-2b^2x^2)+e^{-2b^2x^2}\right)}{\pi}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]^2/x^5,x]
```

```
[Out] ((-4*b*x*(-1 + 2*b^2*x^2)*Erfc[b*x])/(E^(b^2*x^2)*Sqrt[Pi]) + (-3 + 4*b^4*x
^4)*Erfc[b*x]^2 - (4*b^2*x^2*(E^(-2*b^2*x^2) + 4*b^2*x^2*ExpIntegralEi[-2*b
^2*x^2]))/Pi)/(12*x^4)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(\text{erfc}\,(bx))^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x^5,x)
```

```
[Out] int(erfc(b*x)^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2/x^5, x)
```

Fricas [A] time = 2.14996, size = 329, normalized size = 2.63

$$-\frac{3\,\pi+8\,\pi\sqrt{b^2}b^3x^4\operatorname{erf}\left(\sqrt{b^2}x\right)+16\,b^4x^4\operatorname{Ei}\left(-2\,b^2x^2\right)+4\,b^2x^2e^{\left(-2\,b^2x^2\right)}+\left(3\,\pi-4\,\pi b^4x^4\right)\operatorname{erf}\left(bx\right)^2+4\,\sqrt{\pi}\left(2\,b^3x^3-bx-\right.}{12\,\pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*pi + 8*pi*sqrt(b^2)*b^3*x^4*erf(sqrt(b^2)*x) + 16*b^4*x^4*Ei(-2*b^
2*x^2) + 4*b^2*x^2*e^(-2*b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*erf(b*x)^2 + 4*sq
rt(pi)*(2*b^3*x^3 - b*x - (2*b^3*x^3 - b*x)*erf(b*x))*e^(-b^2*x^2) - 6*pi*e
rf(b*x))/(pi*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2/x**5,x)
```

```
[Out] Integral(erfc(b*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(bx\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x^5, x)
```

3.131 $\int \frac{\mathbf{Erfc}(bx)^2}{x^7}\,dx$

Optimal. Leaf size=177

$$\frac{8b^5e^{-b^2x^2}\text{Erfc}(bx)}{45\sqrt{\pi}x}-\frac{4b^3e^{-b^2x^2}\text{Erfc}(bx)}{45\sqrt{\pi}x^3}+\frac{2be^{-b^2x^2}\text{Erfc}(bx)}{15\sqrt{\pi}x^5}-\frac{4}{45}b^6\text{Erfc}(bx)^2+\frac{28b^6\text{ExpIntegralEi}\left(-2b^2x^2\right)}{45\pi}+\frac{2b^4e^{-2b^2x^2}}{9\pi x^2}$$

```
[Out] -b^2/(15*E^(2*b^2*x^2)*Pi*x^4) + (2*b^4)/(9*E^(2*b^2*x^2)*Pi*x^2) + (2*b*Er
fc[b*x])/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) - (4*b^3*Erfc[b*x])/(45*E^(b^2*x^2)*
Sqrt[Pi]*x^3) + (8*b^5*Erfc[b*x])/(45*E^(b^2*x^2)*Sqrt[Pi]*x) - (4*b^6*Erfc
[b*x]^2)/45 - Erfc[b*x]^2/(6*x^6) + (28*b^6*ExpIntegralEi[-2*b^2*x^2])/(45*
Pi)
```

Rubi [A] time = 0.281998, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6365, 6392, 6374, 30, 2210, 2214}

$$\frac{8b^5e^{-b^2x^2}\text{Erfc}(bx)}{45\sqrt{\pi}x}-\frac{4b^3e^{-b^2x^2}\text{Erfc}(bx)}{45\sqrt{\pi}x^3}+\frac{2be^{-b^2x^2}\text{Erfc}(bx)}{15\sqrt{\pi}x^5}-\frac{4}{45}b^6\text{Erfc}(bx)^2+\frac{28b^6\text{Ei}\left(-2b^2x^2\right)}{45\pi}+\frac{2b^4e^{-2b^2x^2}}{9\pi x^2}-\frac{b^2e^{-2b^2x^2}}{15\pi x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]^2/x^7,x]
```

```
[Out] -b^2/(15*E^(2*b^2*x^2)*Pi*x^4) + (2*b^4)/(9*E^(2*b^2*x^2)*Pi*x^2) + (2*b*Er
fc[b*x])/(15*E^(b^2*x^2)*Sqrt[Pi]*x^5) - (4*b^3*Erfc[b*x])/(45*E^(b^2*x^2)*
Sqrt[Pi]*x^3) + (8*b^5*Erfc[b*x])/(45*E^(b^2*x^2)*Sqrt[Pi]*x) - (4*b^6*Erfc
[b*x]^2)/45 - Erfc[b*x]^2/(6*x^6) + (28*b^6*ExpIntegralEi[-2*b^2*x^2])/(45*
Pi)
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
```

```
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7}\,dx = -\frac{\operatorname{erfc}(bx)^2}{6x^6} - \frac{(2b)\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^6}\,dx}{3\sqrt{\pi}}$$

$$= \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{15\sqrt{\pi}x^5} - \frac{\operatorname{erfc}(bx)^2}{6x^6} + \frac{\left(4b^2\right)\int \frac{e^{-2b^2x^2}}{x^5}\,dx}{15\pi} + \frac{\left(4b^3\right)\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx}{15\sqrt{\pi}}$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x^3} - \frac{\operatorname{erfc}(bx)^2}{6x^6} - \frac{\left(8b^4\right)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{45\pi} - \frac{\left(4b^4\right)\int \frac{e^{-2b^2x^2}}{x^3}\,d}{15\pi}$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4e^{-2b^2x^2}}{9\pi x^2} + \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x^3} + \frac{8b^5e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)^2}{6x^6} + 2$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4e^{-2b^2x^2}}{9\pi x^2} + \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x^3} + \frac{8b^5e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)^2}{6x^6} + \frac{28}{}$$

$$= -\frac{b^2e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4e^{-2b^2x^2}}{9\pi x^2} + \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x^3} + \frac{8b^5e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x} - \frac{4}{45}b^6\operatorname{erfc}(bx)^2$$

Mathematica [A] time = 0.0418421, size = 133, normalized size = 0.75

$$\frac{e^{-2b^2x^2}\left(4\sqrt{\pi}bxe^{b^2x^2}\left(4b^4x^4-2b^2x^2+3\right)\operatorname{Erfc}(bx)-\pi e^{2b^2x^2}\left(8b^6x^6+15\right)\operatorname{Erfc}(bx)^2+56b^6x^6e^{2b^2x^2}\operatorname{ExpIntegralEi}\left(-2b^2x\right.}{90\pi x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]^2/x^7,x]
```

```
[Out] (-6*b^2*x^2 + 20*b^4*x^4 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 - 2*b^2*x^2 + 4*b^
4*x^4)*Erfc[b*x] - E^(2*b^2*x^2)*Pi*(15 + 8*b^6*x^6)*Erfc[b*x]^2 + 56*b^6*E
^(2*b^2*x^2)*x^6*ExpIntegralEi[-2*b^2*x^2])/(90*E^(2*b^2*x^2)*Pi*x^6)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{erfc}(bx)\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x^7,x)
```

[Out] int(erfc(b*x)^2/x^7,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(erfc(b*x)^2/x^7,x, algorithm="maxima")

[Out] integrate(erfc(b*x)^2/x^7, x)

Fricas [A] time = 2.17821, size = 396, normalized size = 2.24

$$-\frac{15\,\pi - 16\,\pi\sqrt{b^2}b^5x^6\operatorname{erf}\left(\sqrt{b^2}x\right) - 56\,b^6x^6\operatorname{Ei}\left(-2\,b^2x^2\right) + \left(15\,\pi + 8\,\pi b^6x^6\right)\operatorname{erf}(bx)^2 - 4\,\sqrt{\pi}\left(4\,b^5x^5 - 2\,b^3x^3 + 3\,bx - (4\right.}{90\,\pi x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(erfc(b*x)^2/x^7,x, algorithm="fricas")

```
[Out] -1/90*(15*pi - 16*pi*sqrt(b^2)*b^5*x^6*erf(sqrt(b^2)*x) - 56*b^6*x^6*Ei(-2*
b^2*x^2) + (15*pi + 8*pi*b^6*x^6)*erf(b*x)^2 - 4*sqrt(pi)*(4*b^5*x^5 - 2*b^
3*x^3 + 3*b*x - (4*b^5*x^5 - 2*b^3*x^3 + 3*b*x)*erf(b*x))*e^(-b^2*x^2) - 30
*pi*erf(b*x) - 2*(10*b^4*x^4 - 3*b^2*x^2)*e^(-2*b^2*x^2))/(pi*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(erfc(b*x)**2/x**7,x)

```
[Out] Integral(erfc(b*x)**2/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x^7, x)
```

3.132 $\int x^4 \mathbf{Erfc}(bx)^2 \, dx$

Optimal. Leaf size=165

$$-\frac{43\text{Erf}\left(\sqrt{2}bx\right)}{40\sqrt{2\pi}b^5}-\frac{2x^4e^{-b^2x^2}\text{Erfc}(bx)}{5\sqrt{\pi}b}-\frac{4x^2e^{-b^2x^2}\text{Erfc}(bx)}{5\sqrt{\pi}b^3}-\frac{4e^{-b^2x^2}\text{Erfc}(bx)}{5\sqrt{\pi}b^5}+\frac{x^3e^{-2b^2x^2}}{5\pi b^2}+\frac{11xe^{-2b^2x^2}}{20\pi b^4}+\frac{1}{5}x^5\text{Erfc}(bx)^2$$

```
[Out] (11*x)/(20*b^4*E^(2*b^2*x^2)*Pi) + x^3/(5*b^2*E^(2*b^2*x^2)*Pi) - (43*Erf[S
qrt[2]*b*x])/(40*b^5*Sqrt[2*Pi]) - (4*Erfc[b*x])/(5*b^5*E^(b^2*x^2)*Sqrt[Pi
]) - (4*x^2*Erfc[b*x])/(5*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (2*x^4*Erfc[b*x])/(5*
b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erfc[b*x]^2)/5
```

Rubi [A] time = 0.230568, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6365, 6386, 6383, 2205, 2212}

$$-\frac{43\text{Erf}\left(\sqrt{2}bx\right)}{40\sqrt{2\pi}b^5}-\frac{2x^4e^{-b^2x^2}\text{Erfc}(bx)}{5\sqrt{\pi}b}-\frac{4x^2e^{-b^2x^2}\text{Erfc}(bx)}{5\sqrt{\pi}b^3}-\frac{4e^{-b^2x^2}\text{Erfc}(bx)}{5\sqrt{\pi}b^5}+\frac{x^3e^{-2b^2x^2}}{5\pi b^2}+\frac{11xe^{-2b^2x^2}}{20\pi b^4}+\frac{1}{5}x^5\text{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Erfc[b*x]^2,x]
```

```
[Out] (11*x)/(20*b^4*E^(2*b^2*x^2)*Pi) + x^3/(5*b^2*E^(2*b^2*x^2)*Pi) - (43*Erf[S
qrt[2]*b*x])/(40*b^5*Sqrt[2*Pi]) - (4*Erfc[b*x])/(5*b^5*E^(b^2*x^2)*Sqrt[Pi
]) - (4*x^2*Erfc[b*x])/(5*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (2*x^4*Erfc[b*x])/(5*
b*E^(b^2*x^2)*Sqrt[Pi]) + (x^5*Erfc[b*x]^2)/5
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int x^4\text{erfc}(bx)^2\,dx = \frac{1}{5}x^5\text{erfc}(bx)^2 + \frac{(4b)\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx}{5\sqrt{\pi}}$$

$$= -\frac{2e^{-b^2x^2}x^4\text{erfc}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfc}(bx)^2 - \frac{4\int e^{-2b^2x^2}x^4\,dx}{5\pi} + \frac{8\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx}{5b\sqrt{\pi}}$$

$$= \frac{e^{-2b^2x^2}x^3}{5b^2\pi} - \frac{4e^{-b^2x^2}x^2\text{erfc}(bx)}{5b^3\sqrt{\pi}} - \frac{2e^{-b^2x^2}x^4\text{erfc}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfc}(bx)^2 - \frac{3\int e^{-2b^2x^2}x^2\,dx}{5b^2\pi} - \frac{8\int e^{-2b^2x^2}x^2}{5b^2\pi}$$

$$= \frac{11e^{-2b^2x^2}x}{20b^4\pi} + \frac{e^{-2b^2x^2}x^3}{5b^2\pi} - \frac{4e^{-b^2x^2}\text{erfc}(bx)}{5b^5\sqrt{\pi}} - \frac{4e^{-b^2x^2}x^2\text{erfc}(bx)}{5b^3\sqrt{\pi}} - \frac{2e^{-b^2x^2}x^4\text{erfc}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfc}(bx)^2 -$$

$$= \frac{11e^{-2b^2x^2}x}{20b^4\pi} + \frac{e^{-2b^2x^2}x^3}{5b^2\pi} - \frac{2\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}bx\right)}{5b^5} - \frac{11\text{erf}\left(\sqrt{2}bx\right)}{40b^5\sqrt{2\pi}} - \frac{4e^{-b^2x^2}\text{erfc}(bx)}{5b^5\sqrt{\pi}} - \frac{4e^{-b^2x^2}x^2\text{erfc}(bx)}{5b^3\sqrt{\pi}} -$$

Mathematica [A] time = 0.148396, size = 108, normalized size = 0.65

$$\frac{4\left(4\pi b^5x^5\text{Erfc}(bx)^2 - 8\sqrt{\pi}e^{-b^2x^2}\left(b^4x^4 + 2b^2x^2 + 2\right)\text{Erfc}(bx) + bxe^{-2b^2x^2}\left(4b^2x^2 + 11\right)\right) - 43\sqrt{2\pi}\text{Erf}\left(\sqrt{2}bx\right)}{80\pi b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Erfc[b*x]^2,x]
```

```
[Out] (-43*Sqrt[2*Pi]*Erf[Sqrt[2]*b*x] + 4*((b*x*(11 + 4*b^2*x^2))/E^(2*b^2*x^2)
- (8*Sqrt[Pi]*(2 + 2*b^2*x^2 + b^4*x^4)*Erfc[b*x])/E^(b^2*x^2) + 4*b^5*Pi*x
^5*Erfc[b*x]^2))/(80*b^5*Pi)
```

Maple [A] time = 0.052, size = 205, normalized size = 1.2

$$\frac{1}{b^5}\left(\frac{b^5x^5}{5}-\frac{2\,\mathrm{Erf}\,(bx)\,b^5x^5}{5}+\frac{4}{5\sqrt{\pi}}\left(-\frac{b^4x^4}{2\,\mathrm{e}^{b^2x^2}}-\frac{b^2x^2}{\mathrm{e}^{b^2x^2}}-\left(\mathrm{e}^{b^2x^2}\right)^{-1}\right)+\frac{(\mathrm{Erf}\,(bx))^2\,b^5x^5}{5}-\frac{4\,\mathrm{Erf}\,(bx)}{5\sqrt{\pi}}\left(-\frac{b^4x^4}{2\,\mathrm{e}^{b^2x^2}}-\frac{b^2x^2}{\mathrm{e}^{b^2x^2}}-\left(\mathrm{e}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erfc(b*x)^2,x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5-2/5*erf(b*x)*b^5*x^5+4/5/Pi^(1/2)*(-1/2*b^4*x^4/exp(b^2*
x^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))+1/5*erf(b*x)^2*b^5*x^5-4/5*erf(b*
x)/Pi^(1/2)*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))
+4/5/Pi*(-43/64*2^(1/2)*Pi^(1/2)*erf(b*x*2^(1/2))+11/16/exp(b^2*x^2)^2*b*x+
1/4/exp(b^2*x^2)^2*b^3*x^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\mathrm{erfc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfc(b*x)^2, x)
```

Fricas [A] time = 2.09448, size = 373, normalized size = 2.26

$$\frac{16\,\pi b^6x^5\,\mathrm{erf}\,(bx)^2-32\,\pi b^6x^5\,\mathrm{erf}\,(bx)+16\,\pi b^6x^5-43\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-32\,\sqrt{\pi}\left(b^5x^4+2\,b^3x^2-\left(b^5x^4+2\,b^3x^{\cdots}\right.\right.}{80\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*pi*b^6*x^5*erf(b*x)^2 - 32*pi*b^6*x^5*erf(b*x) + 16*pi*b^6*x^5 - 4
3*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 32*sqrt(pi)*(b^5*x^
4 + 2*b^3*x^2 - (b^5*x^4 + 2*b^3*x^2 + 2*b)*erf(b*x) + 2*b)*e^(-b^2*x^2) +
4*(4*b^4*x^3 + 11*b^2*x)*e^(-2*b^2*x^2))/(pi*b^6)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erfc(b*x)**2,x)
```

```
[Out] Integral(x**4*erfc(b*x)**2, x)
```

Giac [A]   time = 1.48752, size = 300, normalized size = 1.82

$$\frac{1}{5}x^5 \operatorname{erf}(bx)^2 - \frac{2}{5}x^5 \operatorname{erf}(bx) + \frac{1}{5}x^5 + \frac{b\left(\frac{32\left(b^4x^4+2b^2x^2+2\right)\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{b^6} + \frac{\sqrt{\pi}b^4\left(\frac{4\left(4b^2x^3+3x\right)e^{\left(-2b^2x^2\right)}}{b^4} + \frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^5}\right) + 8\sqrt{\pi}b^2\left(\frac{4xe^{\left(-2b^2x^2\right)}}{b^2} \cdots\right.}{\pi b^5}\cdots\right)}{80\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5*erf(b*x)^2 - 2/5*x^5*erf(b*x) + 1/5*x^5 + 1/80*b*(32*(b^4*x^4 + 2*b
^2*x^2 + 2)*erf(b*x)*e^(-b^2*x^2)/b^6 + (sqrt(pi)*b^4*(4*(4*b^2*x^3 + 3*x)*
e^(-2*b^2*x^2)/b^4 + 3*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*b*x)/b^5) + 8*sqrt(pi)
*b^2*(4*x*e^(-2*b^2*x^2)/b^2 + sqrt(2)*sqrt(pi)*erf(-sqrt(2)*b*x)/b^3) + 32
*sqrt(2)*pi*erf(-sqrt(2)*b*x)/b)/(pi*b^5))/sqrt(pi) - 2/5*(b^4*x^4 + 2*b^2*
x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b^5)
```

3.133 $\int x^2 \textbf{Erfc}(bx)^2\,dx$

Optimal. Leaf size=113

$$-\frac{5\text{Erf}\left(\sqrt{2}bx\right)}{6\sqrt{2\pi}b^3}-\frac{2x^2e^{-b^2x^2}\text{Erfc}(bx)}{3\sqrt{\pi}b}-\frac{2e^{-b^2x^2}\text{Erfc}(bx)}{3\sqrt{\pi}b^3}+\frac{xe^{-2b^2x^2}}{3\pi b^2}+\frac{1}{3}x^3\text{Erfc}(bx)^2$$

```
[Out] x/(3*b^2*E^(2*b^2*x^2)*Pi) - (5*Erf[Sqrt[2]*b*x])/(6*b^3*Sqrt[2*Pi]) - (2*E
rfc[b*x])/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (2*x^2*Erfc[b*x])/(3*b*E^(b^2*x^2)
*Sqrt[Pi]) + (x^3*Erfc[b*x]^2)/3
```

Rubi [A] time = 0.125594, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6365, 6386, 6383, 2205, 2212}

$$-\frac{5\text{Erf}\left(\sqrt{2}bx\right)}{6\sqrt{2\pi}b^3}-\frac{2x^2e^{-b^2x^2}\text{Erfc}(bx)}{3\sqrt{\pi}b}-\frac{2e^{-b^2x^2}\text{Erfc}(bx)}{3\sqrt{\pi}b^3}+\frac{xe^{-2b^2x^2}}{3\pi b^2}+\frac{1}{3}x^3\text{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erfc[b*x]^2,x]
```

```
[Out] x/(3*b^2*E^(2*b^2*x^2)*Pi) - (5*Erf[Sqrt[2]*b*x])/(6*b^3*Sqrt[2*Pi]) - (2*E
rfc[b*x])/(3*b^3*E^(b^2*x^2)*Sqrt[Pi]) - (2*x^2*Erfc[b*x])/(3*b*E^(b^2*x^2)
*Sqrt[Pi]) + (x^3*Erfc[b*x]^2)/3
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\mathrm{erfc}(bx)^2\,dx &= \frac{1}{3}x^3\mathrm{erfc}(bx)^2+\frac{(4b)\int e^{-b^2x^2}x^3\mathrm{erfc}(bx)\,dx}{3\sqrt{\pi}}\\
&= -\frac{2e^{-b^2x^2}x^2\mathrm{erfc}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\mathrm{erfc}(bx)^2-\frac{4\int e^{-2b^2x^2}x^2\,dx}{3\pi}+\frac{4\int e^{-b^2x^2}x\mathrm{erfc}(bx)\,dx}{3b\sqrt{\pi}}\\
&= \frac{e^{-2b^2x^2}x}{3b^2\pi}-\frac{2e^{-b^2x^2}\mathrm{erfc}(bx)}{3b^3\sqrt{\pi}}-\frac{2e^{-b^2x^2}x^2\mathrm{erfc}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\mathrm{erfc}(bx)^2-\frac{\int e^{-2b^2x^2}\,dx}{3b^2\pi}-\frac{4\int e^{-2b^2x^2}\,dx}{3b^2\pi}\\
&= \frac{e^{-2b^2x^2}x}{3b^2\pi}-\frac{\sqrt{\frac{2}{\pi}}\mathrm{erf}\left(\sqrt{2}bx\right)}{3b^3}-\frac{\mathrm{erf}\left(\sqrt{2}bx\right)}{6b^3\sqrt{2\pi}}-\frac{2e^{-b^2x^2}\mathrm{erfc}(bx)}{3b^3\sqrt{\pi}}-\frac{2e^{-b^2x^2}x^2\mathrm{erfc}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\mathrm{erfc}(bx)^2
\end{aligned}$$

Mathematica [A] time = 0.0909281, size = 88, normalized size = 0.78

$$\frac{4\pi b^3x^3\mathrm{Erfc}(bx)^2-8\sqrt{\pi}e^{-b^2x^2}\left(b^2x^2+1\right)\mathrm{Erfc}(bx)+4bxe^{-2b^2x^2}-5\sqrt{2\pi}\mathrm{Erf}\left(\sqrt{2}bx\right)}{12\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erfc[b*x]^2,x]
```

```
[Out] ((4*b*x)/E^(2*b^2*x^2) - 5*Sqrt[2*Pi]*Erf[Sqrt[2]*b*x] - (8*Sqrt[Pi]*(1 + b
^2*x^2)*Erfc[b*x])/E^(b^2*x^2) + 4*b^3*Pi*x^3*Erfc[b*x]^2)/(12*b^3*Pi)
```

Maple [A] time = 0.053, size = 151, normalized size = 1.3

$$\frac{1}{b^3}\left(\frac{x^3b^3}{3}-\frac{2\,\mathrm{Erf}\,(bx)\,b^3x^3}{3}+\frac{4}{3\sqrt{\pi}}\left(-\frac{b^2x^2}{2\,\mathrm{e}^{b^2x^2}}-\frac{1}{2\,\mathrm{e}^{b^2x^2}}\right)+\frac{b^3x^3\,(\mathrm{Erf}\,(bx))^2}{3}-\frac{4\,\mathrm{Erf}\,(bx)}{3\sqrt{\pi}}\left(-\frac{b^2x^2}{2\,\mathrm{e}^{b^2x^2}}-\frac{1}{2\,\mathrm{e}^{b^2x^2}}\right)+\frac{4}{3\,\pi}\left(-\frac{5}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfc(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*x^3*b^3-2/3*erf(b*x)*b^3*x^3+4/3/Pi^(1/2)*(-1/2*b^2*x^2/exp(b^2*
x^2)-1/2/exp(b^2*x^2))+1/3*b^3*x^3*erf(b*x)^2-4/3*erf(b*x)/Pi^(1/2)*(-1/2*b
^2*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))+4/3/Pi*(-5/16*2^(1/2)*Pi^(1/2)*erf(b*
x*2^(1/2))+1/4/exp(b^2*x^2)^2*b*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{erfc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfc(b*x)^2, x)
```

Fricas [A] time = 2.15058, size = 305, normalized size = 2.7

$$\frac{4\,\pi b^4x^3\,\mathrm{erf}\,(bx)^2-8\,\pi b^4x^3\,\mathrm{erf}\,(bx)+4\,\pi b^4x^3+4\,b^2xe^{\left(-2\,b^2x^2\right)}-5\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-8\,\sqrt{\pi}\left(b^3x^2-\left(b^3x^2+b\right)\mathrm{er}\cdots\right.}{12\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi*b^4*x^3*erf(b*x)^2 - 8*pi*b^4*x^3*erf(b*x) + 4*pi*b^4*x^3 + 4*b^
2*x*e^(-2*b^2*x^2) - 5*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x)
- 8*sqrt(pi)*(b^3*x^2 - (b^3*x^2 + b)*erf(b*x) + b)*e^(-b^2*x^2))/(pi*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfc(b*x)**2,x)
```

```
[Out] Integral(x**2*erfc(b*x)**2, x)
```

Giac [A] time = 1.486, size = 205, normalized size = 1.81

$$\frac{1}{3}\,x^3 \operatorname{erf}(bx)^2 - \frac{2}{3}\,x^3 \operatorname{erf}(bx) + \frac{1}{3}\,x^3 + \frac{b\left(\frac{8\left(b^2x^2+1\right)\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{b^4} + \frac{\sqrt{\pi}b^2\left(\frac{4\,xe^{\left(-2\,b^2x^2\right)}}{b^2} + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^3}\right) + \frac{4\,\sqrt{2}\pi\operatorname{erf}\left(-\sqrt{2}bx\right)}{b}}{\pi b^3}\right)}{12\,\sqrt{\pi}} - \frac{2\left(b^2x^2+1\right.}{3\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*erf(b*x)^2 - 2/3*x^3*erf(b*x) + 1/3*x^3 + 1/12*b*(8*(b^2*x^2 + 1)*e
rf(b*x)*e^(-b^2*x^2)/b^4 + (sqrt(pi)*b^2*(4*x*e^(-2*b^2*x^2)/b^2 + sqrt(2)*
sqrt(pi)*erf(-sqrt(2)*b*x)/b^3) + 4*sqrt(2)*pi*erf(-sqrt(2)*b*x)/b)/(pi*b^3
))/sqrt(pi) - 2/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

3.134 $\int \mathbf{Erfc}(bx)^2 \, dx$

Optimal. Leaf size=56

$$-\frac{2e^{-b^2x^2}\text{Erfc}(bx)}{\sqrt{\pi}b}-\frac{\sqrt{\frac{2}{\pi}}\text{Erf}\left(\sqrt{2}bx\right)}{b}+x\text{Erfc}(bx)^2$$

```
[Out] -((Sqrt[2/Pi]*Erf[Sqrt[2]*b*x])/b) - (2*Erfc[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi])
 + x*Erfc[b*x]^2
```

Rubi [A] time = 0.0458552, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6353, 12, 6383, 2205}

$$-\frac{2e^{-b^2x^2}\text{Erfc}(bx)}{\sqrt{\pi}b}-\frac{\sqrt{\frac{2}{\pi}}\text{Erf}\left(\sqrt{2}bx\right)}{b}+x\text{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]^2,x]
```

```
[Out] -((Sqrt[2/Pi]*Erf[Sqrt[2]*b*x])/b) - (2*Erfc[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi])
 + x*Erfc[b*x]^2
```

Rule 6353

```
Int[Erfc[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x]^
2)/b, x] + Dist[4/Sqrt[Pi], Int[((a + b*x)*Erfc[a + b*x])/E^(a + b*x)^2, x]
, x] /; FreeQ[{a, b}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfc}(bx)^2\,dx &= x\operatorname{erfc}(bx)^2+\frac{4\int be^{-b^2x^2}x\operatorname{erfc}(bx)\,dx}{\sqrt{\pi}}\\ &= x\operatorname{erfc}(bx)^2+\frac{(4b)\int e^{-b^2x^2}x\operatorname{erfc}(bx)\,dx}{\sqrt{\pi}}\\ &= -\frac{2e^{-b^2x^2}\operatorname{erfc}(bx)}{b\sqrt{\pi}}+x\operatorname{erfc}(bx)^2-\frac{4\int e^{-2b^2x^2}\,dx}{\pi}\\ &= -\frac{\sqrt{\frac{2}{\pi}}\operatorname{erf}\left(\sqrt{2}bx\right)}{b}-\frac{2e^{-b^2x^2}\operatorname{erfc}(bx)}{b\sqrt{\pi}}+x\operatorname{erfc}(bx)^2\end{aligned}$$

Mathematica [A] time = 0.0497155, size = 56, normalized size = 1.

$$-\frac{2e^{-b^2x^2}\operatorname{Erfc}(bx)}{\sqrt{\pi}b}-\frac{\sqrt{\frac{2}{\pi}}\operatorname{Erf}\left(\sqrt{2}bx\right)}{b}+x\operatorname{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]^2,x]
```

```
[Out] -((Sqrt[2/Pi]*Erf[Sqrt[2]*b*x])/b) - (2*Erfc[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi])
 + x*Erfc[b*x]^2
```

Maple [A] time = 0.044, size = 48, normalized size = 0.9

$$\frac{1}{b}\left(bx\left(\operatorname{Erf}(bx)\right)^2+2\,\frac{\operatorname{Erf}(bx)\,e^{-b^2x^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{Erf}\left(bx\sqrt{2}\right)}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2,x)
```

```
[Out] 1/b*(b*x*erf(b*x)^2+2*erf(b*x)/Pi^(1/2)*exp(-b^2*x^2)-1/Pi^(1/2)*2^(1/2)*er
f(b*x*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2, x)
```

Fricas [A] time = 2.12992, size = 219, normalized size = 3.91

$$\frac{\pi b^2 x \operatorname{erf}(bx)^2 - 2\,\pi b^2 x \operatorname{erf}(bx) + \pi b^2 x - \sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right) + 2\,\sqrt{\pi}(b\operatorname{erf}(bx) - b)e^{\left(-b^2x^2\right)}}{\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] (pi*b^2*x*erf(b*x)^2 - 2*pi*b^2*x*erf(b*x) + pi*b^2*x - sqrt(2)*sqrt(pi)*sq
rt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) + 2*sqrt(pi)*(b*erf(b*x) - b)*e^(-b^2*x^2)
)/(pi*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2,x)
```

```
[Out] Integral(erfc(b*x)**2, x)
```

Giac [A] time = 1.31717, size = 99, normalized size = 1.77

$$x\operatorname{erf}(bx)^2 - 2\,x\operatorname{erf}(bx) + \frac{b\left(\frac{2\,\operatorname{erf}(bx)e^{\left(-b^2x^2\right)}}{b^2} + \frac{\sqrt{2}\operatorname{erf}\left(-\sqrt{2}bx\right)}{b^2}\right)}{\sqrt{\pi}} + x - \frac{2\,e^{\left(-b^2x^2\right)}}{\sqrt{\pi}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] x*erf(b*x)^2 - 2*x*erf(b*x) + b*(2*erf(b*x)*e^(-b^2*x^2)/b^2 + sqrt(2)*erf(
-sqrt(2)*b*x)/b^2)/sqrt(pi) + x - 2*e^(-b^2*x^2)/(sqrt(pi)*b)
```

3.135 $\int \frac{\mathbf{Erfc}(bx)^2}{x^2}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfc}(bx)^2}{x^2},x\right)$$

```
[Out] Unintegrable[Erfc[b*x]^2/x^2, x]
```

Rubi [A] time = 0.0171936, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Erfc[b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\text{erfc}(bx)^2}{x^2}\,dx = \int \frac{\text{erfc}(bx)^2}{x^2}\,dx$$

Mathematica [A] time = 0.145891, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]^2/x^2,x]
```

```
[Out] Integrate[Erfc[b*x]^2/x^2, x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfc}(bx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x^2,x)
```

```
[Out] int(erfc(b*x)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erf}(bx)^2 - 2\,\operatorname{erf}(bx) + 1}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2/x**2,x)
```

```
[Out] Integral(erfc(b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x^2, x)
```

3.136 $\int \frac{\mathbf{Erfc}(bx)^2}{x^4}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfc}(bx)^2}{x^4},x\right)$$

```
[Out] Unintegrable[Erfc[b*x]^2/x^4, x]
```

Rubi [A] time = 0.0169246, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]^2/x^4,x]
```

```
[Out] Defer[Int][Erfc[b*x]^2/x^4, x]
```

Rubi steps

$$\int \frac{\text{erfc}(bx)^2}{x^4}\,dx=\int \frac{\text{erfc}(bx)^2}{x^4}\,dx$$

Mathematica [A] time = 0.148502, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]^2/x^4,x]
```

```
[Out] Integrate[Erfc[b*x]^2/x^4, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{erfc}\,(bx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x^4,x)
```

```
[Out] int(erfc(b*x)^2/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}\,(bx)^2 - 2\,\mathrm{erf}\,(bx) + 1}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}^2\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2/x**4,x)
```

```
[Out] Integral(erfc(b*x)**2/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x^4, x)
```

3.137 $\int \frac{\mathbf{Erfc}(bx)^2}{x^6}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfc}(bx)^2}{x^6},x\right)$$

```
[Out] Unintegrable[Erfc[b*x]^2/x^6, x]
```

Rubi [A] time = 0.0168902, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]^2/x^6,x]
```

```
[Out] Defer[Int][Erfc[b*x]^2/x^6, x]
```

Rubi steps

$$\int \frac{\text{erfc}(bx)^2}{x^6}\,dx = \int \frac{\text{erfc}(bx)^2}{x^6}\,dx$$

Mathematica [A] time = 0.155884, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]^2/x^6,x]
```

```
[Out] Integrate[Erfc[b*x]^2/x^6, x]
```

Maple [A] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{erfc}\,(bx))^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)^2/x^6,x)
```

```
[Out] int(erfc(b*x)^2/x^6,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2/x^6, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erf}\,(bx)^2 - 2\,\mathrm{erf}\,(bx) + 1}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}^2\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)**2/x**6,x)
```

```
[Out] Integral(erfc(b*x)**2/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2/x^6, x)
```

3.138 $\int (c+dx)^2 \mathbf{Erfc}(a+bx)^2 \, dx$

Optimal. Leaf size=375

$$-\frac{\sqrt{\frac{2}{\pi}}(bc-ad)^2\text{Erf}\left(\sqrt{2}(a+bx)\right)}{b^3}+\frac{d(a+bx)^2(bc-ad)\text{Erfc}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{Erfc}(a+bx)^2}{b^3}-\frac{2de^{-(a+bx)^2}(a-}{}$$

```
[Out] (d*(b*c - a*d))/(b^3*E^(2*(a + b*x)^2)*Pi) + (d^2*(a + b*x))/(3*b^3*E^(2*(a
 + b*x)^2)*Pi) - ((b*c - a*d)^2*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b^3 - (5
*d^2*Erf[Sqrt[2]*(a + b*x)])/(6*b^3*Sqrt[2*Pi]) - (2*d^2*Erfc[a + b*x])/(3*
b^3*E^(a + b*x)^2*Sqrt[Pi]) - (2*(b*c - a*d)^2*Erfc[a + b*x])/(b^3*E^(a + b
*x)^2*Sqrt[Pi]) - (2*d*(b*c - a*d)*(a + b*x)*Erfc[a + b*x])/(b^3*E^(a + b*x
)^2*Sqrt[Pi]) - (2*d^2*(a + b*x)^2*Erfc[a + b*x])/(3*b^3*E^(a + b*x)^2*Sqrt
[Pi]) - (d*(b*c - a*d)*Erfc[a + b*x]^2)/(2*b^3) + ((b*c - a*d)^2*(a + b*x)*
Erfc[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*Erfc[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*Erfc[a + b*x]^2)/(3*b^3)
```

Rubi [A] time = 0.374947, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6368, 6353, 6383, 2205, 6365, 6386, 6374, 30, 2209, 2212}

$$-\frac{\sqrt{\frac{2}{\pi}}(bc-ad)^2\text{Erf}\left(\sqrt{2}(a+bx)\right)}{b^3}+\frac{d(a+bx)^2(bc-ad)\text{Erfc}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{Erfc}(a+bx)^2}{b^3}-\frac{2de^{-(a+bx)^2}(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Erfc[a + b*x]^2,x]
```


```
[Out] (d*(b*c - a*d))/(b^3*E^(2*(a + b*x)^2)*Pi) + (d^2*(a + b*x))/(3*b^3*E^(2*(a
 + b*x)^2)*Pi) - ((b*c - a*d)^2*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b^3 - (5
*d^2*Erf[Sqrt[2]*(a + b*x)])/(6*b^3*Sqrt[2*Pi]) - (2*d^2*Erfc[a + b*x])/(3*
b^3*E^(a + b*x)^2*Sqrt[Pi]) - (2*(b*c - a*d)^2*Erfc[a + b*x])/(b^3*E^(a + b
*x)^2*Sqrt[Pi]) - (2*d*(b*c - a*d)*(a + b*x)*Erfc[a + b*x])/(b^3*E^(a + b*x
)^2*Sqrt[Pi]) - (2*d^2*(a + b*x)^2*Erfc[a + b*x])/(3*b^3*E^(a + b*x)^2*Sqrt
[Pi]) - (d*(b*c - a*d)*Erfc[a + b*x]^2)/(2*b^3) + ((b*c - a*d)^2*(a + b*x)*
Erfc[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*Erfc[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*Erfc[a + b*x]^2)/(3*b^3)
```


Rule 6368

```
Int[Erfc[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Dist
[1/b^(m + 1), Subst[Int[ExpandIntegrand[Erfc[x]^2, (b*c - a*d + d*x)^m, x],
 x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6353

```
Int[Erfc[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x]^
2)/b, x] + Dist[4/Sqrt[Pi], Int[((a + b*x)*Erfc[a + b*x])/E^(a + b*x)^2, x]
, x] /; FreeQ[{a, b}, x]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\text{erfc}(x)^2+2bcd\left(1-\frac{ad}{bc}\right)x\text{erfc}(x)^2+d^2x^2\text{erfc}(x)^2\right)dx,x,\right.}{b^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int x^2\text{erfc}(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\,\text{Subst}\left(\int x\text{erfc}(x)^2\,dx,x,a+bx\right)}{b^3}+$$

$$= \frac{(bc-ad)^2(a+bx)\text{erfc}(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2\text{erfc}(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3\text{erfc}(a+}{3b^3}$$

$$= -\frac{2(bc-ad)^2e^{-(a+bx)^2}\text{erfc}(a+bx)}{b^3\sqrt{\pi}}-\frac{2d(bc-ad)e^{-(a+bx)^2}(a+bx)\text{erfc}(a+bx)}{b^3\sqrt{\pi}}-\frac{2d^2e^{-(a+bx)}}{}$$

$$= \frac{d(bc-ad)e^{-2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{-2(a+bx)^2}(a+bx)}{3b^3\pi}-\frac{(bc-ad)^2\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^3}-\frac{2d^2e^{-(a+bx)^2}}{3b^3}$$

$$= \frac{d(bc-ad)e^{-2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{-2(a+bx)^2}(a+bx)}{3b^3\pi}-\frac{d^2\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{3b^3}-\frac{(bc-ad)^2\sqrt{\frac{2}{\pi}}\text{erf}\left(\right.}{b^3}$$

Mathematica [A] time = 4.62498, size = 610, normalized size = 1.63

$$\frac{d^2\left(-12\sqrt{\pi}\text{ExpIntegralE}\left(\frac{3}{2},(a+bx)^2\right)-12\sqrt{2\pi}a^2\text{Erf}\left(\sqrt{2}(a+bx)\right)+12\pi a^2bx+12\pi ab^2x^2+12\pi bx\text{Erf}(a+bx)-12\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Erfc[a + b*x]^2,x]
```

```
[Out] (-12*b^2*Sqrt[Pi]*(c + d*x)^2*(Sqrt[2]*Erf[Sqrt[2]*(a + b*x)] + Erfc[a + b*
x]*(2/E^(a + b*x)^2 - Sqrt[Pi]*(a + b*x)*Erfc[a + b*x])) + 6*b*d*(c + d*x)*
(2/E^(2*(a + b*x)^2) + (4*Sqrt[Pi]*(a + b*x))/E^(a + b*x)^2 - 2*Pi*(a + b*x
)^2 - 2*Pi*Erf[a + b*x] - (4*Sqrt[Pi]*(a + b*x)*Erf[a + b*x])/E^(a + b*x)^2
 + 4*Pi*(a + b*x)^2*Erf[a + b*x] + Pi*Erf[a + b*x]^2 - 2*Pi*(a + b*x)^2*Erf
[a + b*x]^2 + 4*a*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)] + 4*b*Sqrt[2*Pi]*x*Erf[
Sqrt[2]*(a + b*x)] + 2*Pi*(2 + Erfc[-a - b*x]*Erfc[a + b*x]) - 4*Sqrt[Pi]*(
a + b*x)*ExpIntegralE[1/2, (a + b*x)^2]) + d^2*((24*Sqrt[Pi])/E^(a + b*x)^2
 - 36*b*Pi*x + 12*a^2*b*Pi*x + 12*a*b^2*Pi*x^2 + 4*b^3*Pi*x^3 - (8*(a + b*x
))/E^(2*(a + b*x)^2) - (8*Sqrt[Pi]*(1 + (a + b*x)^2))/E^(a + b*x)^2 + 12*a*
Pi*Erf[a + b*x] + 12*b*Pi*x*Erf[a + b*x] - 8*Pi*(a + b*x)^3*Erf[a + b*x] +
(8*Sqrt[Pi]*(1 + (a + b*x)^2)*Erf[a + b*x])/E^(a + b*x)^2 + 6*Pi*(a + b*x)*
Erf[a + b*x]^2 + 4*Pi*(a + b*x)^3*Erf[a + b*x]^2 - 5*Sqrt[2*Pi]*Erf[Sqrt[2]
*(a + b*x)] - 12*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)] - 12*b*Sqrt[2*Pi]*x*
(2*a + b*x)*Erf[Sqrt[2]*(a + b*x)] - 12*Sqrt[Pi]*ExpIntegralE[3/2, (a + b*x
)^2]))/(12*b^3*Pi)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int (dx + c)^2 (\mathrm{erfc}(bx + a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*erfc(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*erfc(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \mathrm{erfc}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*erfc(b*x + a)^2, x)
```

Fricas [A] time = 2.23668, size = 1069, normalized size = 2.85

$$\frac{4\,\pi b^4 d^2 x^3 + 12\,\pi b^4 c d x^2 + 12\,\pi b^4 c^2 x - \sqrt{2}\sqrt{\pi}\left(12\,b^2c^2 - 24\,abcd + \left(12\,a^2+5\right)d^2\right)\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) - 4\,\pi\left(6\,ab^2c^2 - 3\,\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi*b^4*d^2*x^3 + 12*pi*b^4*c*d*x^2 + 12*pi*b^4*c^2*x - sqrt(2)*sqrt
(pi)*(12*b^2*c^2 - 24*a*b*c*d + (12*a^2 + 5)*d^2)*sqrt(b^2)*erf(sqrt(2)*sqr
t(b^2)*(b*x + a)/b) - 4*pi*(6*a*b^2*c^2 - 3*(2*a^2 + 1)*b*c*d + (2*a^3 + 3*
a)*d^2)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b) + 2*(2*pi*b^4*d^2*x^3 + 6*pi*b
^4*c*d*x^2 + 6*pi*b^4*c^2*x + pi*(6*a*b^3*c^2 - 3*(2*a^2 + 1)*b^2*c*d + (2*
a^3 + 3*a)*b*d^2))*erf(b*x + a)^2 - 8*sqrt(pi)*(b^3*d^2*x^2 + 3*b^3*c^2 - 3
*a*b^2*c*d + (a^2 + 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x - (b^3*d^2*x^2 + 3
*b^3*c^2 - 3*a*b^2*c*d + (a^2 + 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x)*erf(b
*x + a))*e^(-b^2*x^2 - 2*a*b*x - a^2) - 8*(pi*b^4*d^2*x^3 + 3*pi*b^4*c*d*x^
2 + 3*pi*b^4*c^2*x)*erf(b*x + a) + 4*(b^2*d^2*x + 3*b^2*c*d - 2*a*b*d^2)*e^
(-2*b^2*x^2 - 4*a*b*x - 2*a^2))/(pi*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \operatorname{erfc}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*erfc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*erfc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \operatorname{erfc}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*erfc(b*x + a)^2, x)
```

3.139 $\int (c+dx)\textbf{Erfc}(a+bx)^2\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{\frac{2}{\pi}}(bc-ad)\text{Erf}\left(\sqrt{2}(a+bx)\right)}{b^2}+\frac{(a+bx)(bc-ad)\text{Erfc}(a+bx)^2}{b^2}-\frac{2e^{-(a+bx)^2}(bc-ad)\text{Erfc}(a+bx)}{\sqrt{\pi}b^2}+\frac{d(a+bx)^2\text{Erfc}(a+}{2b^2}$$

```
[Out] d/(2*b^2*E^(2*(a + b*x)^2)*Pi) - ((b*c - a*d)*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b
*x)])/b^2 - (2*(b*c - a*d)*Erfc[a + b*x])/(b^2*E^(a + b*x)^2*Sqrt[Pi]) - (d
*(a + b*x)*Erfc[a + b*x])/(b^2*E^(a + b*x)^2*Sqrt[Pi]) - (d*Erfc[a + b*x]^2
)/(4*b^2) + ((b*c - a*d)*(a + b*x)*Erfc[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Er
fc[a + b*x]^2)/(2*b^2)
```

Rubi [A] time = 0.189064, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6368, 6353, 6383, 2205, 6365, 6386, 6374, 30, 2209}

$$-\frac{\sqrt{\frac{2}{\pi}}(bc-ad)\text{Erf}\left(\sqrt{2}(a+bx)\right)}{b^2}+\frac{(a+bx)(bc-ad)\text{Erfc}(a+bx)^2}{b^2}-\frac{2e^{-(a+bx)^2}(bc-ad)\text{Erfc}(a+bx)}{\sqrt{\pi}b^2}+\frac{d(a+bx)^2\text{Erfc}(a+}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Erfc[a + b*x]^2,x]
```


```
[Out] d/(2*b^2*E^(2*(a + b*x)^2)*Pi) - ((b*c - a*d)*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b
*x)])/b^2 - (2*(b*c - a*d)*Erfc[a + b*x])/(b^2*E^(a + b*x)^2*Sqrt[Pi]) - (d
*(a + b*x)*Erfc[a + b*x])/(b^2*E^(a + b*x)^2*Sqrt[Pi]) - (d*Erfc[a + b*x]^2
)/(4*b^2) + ((b*c - a*d)*(a + b*x)*Erfc[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Er
fc[a + b*x]^2)/(2*b^2)
```


Rule 6368

```
Int[Erfc[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Dist
[1/b^(m + 1), Subst[Int[ExpandIntegrand[Erfc[x]^2, (b*c - a*d + d*x)^m, x],
 x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6353

```
Int[Erfc[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x]^
2)/b, x] + Dist[4/Sqrt[Pi], Int[((a + b*x)*Erfc[a + b*x])/E^(a + b*x)^2, x]
```

```
, x] /; FreeQ[{a, b}, x]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 6365

```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfc[b*x]^2
)/(m + 1), x] + Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[(x^(m + 1)*Erfc[b*x])/E^
(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
```

```
[d*e - c*f, 0]
```

Rubi steps

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)\text{erfc}(x)^2+dx\text{erfc}(x)^2\right)dx,x,a+bx\right)}{b^2}$$

$$= \frac{d\,\text{Subst}\left(\int x\text{erfc}(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int \text{erfc}(x)^2\,dx,x,a+bx\right)}{b^2}$$

$$= \frac{(bc-ad)(a+bx)\text{erfc}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{erfc}(a+bx)^2}{2b^2}+\frac{(2d)\,\text{Subst}\left(\int e^{-x^2}x^2\text{erfc}(x)\,dx\right.}{b^2\sqrt{\pi}}$$

$$= -\frac{2(bc-ad)e^{-(a+bx)^2}\text{erfc}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a+bx)\text{erfc}(a+bx)}{b^2\sqrt{\pi}}+\frac{(bc-ad)(a+bx)\text{erfc}(a}{b^2}$$

$$= \frac{de^{-2(a+bx)^2}}{2b^2\pi}-\frac{(bc-ad)\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^2}-\frac{2(bc-ad)e^{-(a+bx)^2}\text{erfc}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a}{}$$

$$= \frac{de^{-2(a+bx)^2}}{2b^2\pi}-\frac{(bc-ad)\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^2}-\frac{2(bc-ad)e^{-(a+bx)^2}\text{erfc}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a}{}$$

Mathematica [A] time = 2.04342, size = 301, normalized size = 1.59

$$\frac{d\left(-4\sqrt{\pi}(a+bx)\text{ExpIntegralE}\left(\frac{1}{2},(a+bx)^2\right)-2\pi(a+bx)^2\text{Erf}(a+bx)^2+4\pi(a+bx)^2\text{Erf}(a+bx)-4\sqrt{\pi}e^{-(a+bx)^2}(a+bx)\text{Erf}(a+bx)+\pi\text{Erf}(a+bx)^2-2\pi\text{Erf}(a+bx)+4\sqrt{2\pi}bx\text{Er}\right.}{\pi}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Erfc[a + b*x]^2,x]
```

```
[Out] (4*b*(c + d*x)*(-(Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)]) + Erfc[a + b*x]*(-2/(E
^(a + b*x)^2*Sqrt[Pi]) + (a + b*x)*Erfc[a + b*x])) + (d*(2/E^(2*(a + b*x)^2
) + (4*Sqrt[Pi]*(a + b*x))/E^(a + b*x)^2 - 2*Pi*(a + b*x)^2 - 2*Pi*Erf[a +
b*x] - (4*Sqrt[Pi]*(a + b*x)*Erf[a + b*x])/E^(a + b*x)^2 + 4*Pi*(a + b*x)^2
*Erf[a + b*x] + Pi*Erf[a + b*x]^2 - 2*Pi*(a + b*x)^2*Erf[a + b*x]^2 + 4*a*S
qrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)] + 4*b*Sqrt[2*Pi]*x*Erf[Sqrt[2]*(a + b*x)]
+ 2*Pi*(2 + Erfc[-a - b*x]*Erfc[a + b*x]) - 4*Sqrt[Pi]*(a + b*x)*ExpIntegra
lE[1/2, (a + b*x)^2]))/Pi)/(4*b^2)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int (dx + c) (\text{erfc}(bx + a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*erfc(b*x+a)^2,x)
```

```
[Out] int((d*x+c)*erfc(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \,\text{erfc}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*erfc(b*x + a)^2, x)
```

Fricas [A] time = 2.17829, size = 647, normalized size = 3.42

$$\frac{2\,\pi b^3 dx^2 + 4\,\pi b^3 cx - 4\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}(bc - ad)\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) - 2\,\pi\left(4\,abc - \left(2\,a^2 + 1\right)d\right)\sqrt{b^2}\,\text{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + \left(2\,\pi b^3 dx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*pi*b^3*d*x^2 + 4*pi*b^3*c*x - 4*sqrt(2)*sqrt(pi)*sqrt(b^2)*(b*c - a*
d)*erf(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 2*pi*(4*a*b*c - (2*a^2 + 1)*d)*sqrt
(b^2)*erf(sqrt(b^2)*(b*x + a)/b) + (2*pi*b^3*d*x^2 + 4*pi*b^3*c*x + pi*(4*a
*b^2*c - (2*a^2 + 1)*b*d))*erf(b*x + a)^2 + 2*b*d*e^(-2*b^2*x^2 - 4*a*b*x -
 2*a^2) - 4*sqrt(pi)*(b^2*d*x + 2*b^2*c - a*b*d - (b^2*d*x + 2*b^2*c - a*b*
d)*erf(b*x + a))*e^(-b^2*x^2 - 2*a*b*x - a^2) - 4*(pi*b^3*d*x^2 + 2*pi*b^3*
c*x)*erf(b*x + a))/(pi*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\operatorname{erfc}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*erfc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\operatorname{erfc}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*erfc(b*x + a)^2, x)
```

3.140 $\int \textbf{Erfc}(a+bx)^2\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{\frac{2}{\pi}}\operatorname{Erf}\left(\sqrt{2}(a+bx)\right)}{b}+\frac{(a+bx)\operatorname{Erfc}(a+bx)^2}{b}-\frac{2e^{-(a+bx)^2}\operatorname{Erfc}(a+bx)}{\sqrt{\pi}b}$$

```
[Out] -((Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b) - (2*Erfc[a + b*x])/(b*E^(a + b*x)
^2*Sqrt[Pi]) + ((a + b*x)*Erfc[a + b*x]^2)/b
```

Rubi [A] time = 0.156236, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6353, 6383, 2205}

$$-\frac{\sqrt{\frac{2}{\pi}}\operatorname{Erf}\left(\sqrt{2}(a+bx)\right)}{b}+\frac{(a+bx)\operatorname{Erfc}(a+bx)^2}{b}-\frac{2e^{-(a+bx)^2}\operatorname{Erfc}(a+bx)}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[a + b*x]^2,x]
```

```
[Out] -((Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b) - (2*Erfc[a + b*x])/(b*E^(a + b*x)
^2*Sqrt[Pi]) + ((a + b*x)*Erfc[a + b*x]^2)/b
```

Rule 6353

```
Int[Erfc[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x]^
2)/b, x] + Dist[4/Sqrt[Pi], Int[((a + b*x)*Erfc[a + b*x])/E^(a + b*x)^2, x]
, x] /; FreeQ[{a, b}, x]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfc}(a+bx)^2\,dx &= \frac{(a+bx)\operatorname{erfc}(a+bx)^2}{b}+\frac{4\int e^{-(a+bx)^2}(a+bx)\operatorname{erfc}(a+bx)\,dx}{\sqrt{\pi}}\\ &= \frac{(a+bx)\operatorname{erfc}(a+bx)^2}{b}+\frac{4\,\mathrm{Subst}\left(\int e^{-x^2}x\operatorname{erfc}(x)\,dx,x,a+bx\right)}{b\sqrt{\pi}}\\ &= -\frac{2e^{-(a+bx)^2}\operatorname{erfc}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erfc}(a+bx)^2}{b}-\frac{4\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx,x,a+bx\right)}{b\pi}\\ &= -\frac{\sqrt{\frac{2}{\pi}}\operatorname{erf}\left(\sqrt{2}(a+bx)\right)}{b}-\frac{2e^{-(a+bx)^2}\operatorname{erfc}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erfc}(a+bx)^2}{b}\end{aligned}$$

Mathematica [A] time = 0.0796041, size = 66, normalized size = 0.93

$$\frac{\operatorname{Erfc}(a+bx)\left((a+bx)\operatorname{Erfc}(a+bx)-\frac{2e^{-(a+bx)^2}}{\sqrt{\pi}}\right)-\sqrt{\frac{2}{\pi}}\operatorname{Erf}\left(\sqrt{2}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[a + b*x]^2,x]
```

```
[Out] (-(Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)]) + Erfc[a + b*x]*(-2/(E^(a + b*x)^2*Sq
rt[Pi]) + (a + b*x)*Erfc[a + b*x]))/b
```

Maple [A] time = 0.046, size = 59, normalized size = 0.8

$$\frac{1}{b}\left((bx+a)\left(\operatorname{Erf}(bx+a)\right)^2+2\,\frac{\operatorname{Erf}(bx+a)\,\mathrm{e}^{-(bx+a)^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{Erf}\left((bx+a)\sqrt{2}\right)}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a)^2,x)
```

```
[Out] 1/b*((b*x+a)*erf(b*x+a)^2+2*erf(b*x+a)/Pi^(1/2)*exp(-(b*x+a)^2)-1/Pi^(1/2)*
2^(1/2)*erf((b*x+a)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)^2, x)
```

Fricas [B] time = 2.11445, size = 348, normalized size = 4.9

$$-\frac{2\,\pi b^2 x \operatorname{erf}(bx+a) - \pi b^2 x + 2\,\pi a\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \left(\pi b^2 x + \pi ab\right)\operatorname{erf}(bx+a)^2 + \sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) - 2\sqrt{}}{\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*pi*b^2*x*erf(b*x + a) - pi*b^2*x + 2*pi*a*sqrt(b^2)*erf(sqrt(b^2)*(b*x
+ a)/b) - (pi*b^2*x + pi*a*b)*erf(b*x + a)^2 + sqrt(2)*sqrt(pi)*sqrt(b^2)*e
rf(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 2*sqrt(pi)*(b*erf(b*x + a) - b)*e^(-b^2
*x^2 - 2*a*b*x - a^2))/(pi*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)**2,x)
```

```
[Out] Integral(erfc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)^2, x)
```

3.141 $\int \frac{\mathbf{Erfc}(a+bx)^2}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{Erfc}(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable[Erfc[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0226769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Erfc[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{erfc}(a+bx)^2}{c+dx} \, dx$$

Mathematica [A] time = 0.557632, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Erfc[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.365, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfc}(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(erfc(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)^2/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erf}(bx+a)^2 - 2\,\operatorname{erf}(bx+a) + 1}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((erf(b*x + a)^2 - 2*erf(b*x + a) + 1)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(erfc(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)^2/(d*x + c), x)
```

3.142 $\int \frac{\mathbf{Erfc}(a+bx)^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{Erfc}(a+bx)^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Erfc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0230374, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfc}(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Erfc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx$$

Mathematica [A] time = 0.341426, size = 0, normalized size = 0.

$$\int \frac{\text{Erfc}(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Erfc[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.341, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfc}(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(erfc(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)^2/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erf}(bx+a)^2 - 2\operatorname{erf}(bx+a)+1}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((erf(b*x + a)^2 - 2*erf(b*x + a) + 1)/(d^2*x^2 + 2*c*d*x + c^2), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(erfc(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)^2/(d*x + c)^2, x)
```

3.143 $\int x^2 \mathbf{Erfc}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=102

$$\frac{1}{3}x^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\text{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{3}{n}}{2bd}\right)+\frac{1}{3}x^3\text{Erfc}(d(a+b\log(cx^n)))$$

```
[Out] (E^((9 - 12*a*b*d^2*n)/(4*b^2*d^2*n^2))*x^3*Erf[(2*a*b*d^2 - 3/n + 2*b^2*d^
2*Log[c*x^n])/(2*b*d)])/(3*(c*x^n)^(3/n)) + (x^3*Erfc[d*(a + b*Log[c*x^n])]
)/3
```

Rubi [A] time = 0.215069, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6402, 2278, 2274, 15, 2276, 2234, 2205}

$$\frac{1}{3}x^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\text{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{3}{n}}{2bd}\right)+\frac{1}{3}x^3\text{Erfc}(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^((9 - 12*a*b*d^2*n)/(4*b^2*d^2*n^2))*x^3*Erf[(2*a*b*d^2 - 3/n + 2*b^2*d^
2*Log[c*x^n])/(2*b*d)])/(3*(c*x^n)^(3/n)) + (x^3*Erfc[d*(a + b*Log[c*x^n])]
)/3
```

Rule 6402

```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfc[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 + Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^2\text{erfc}\,(d\,(a+b\log{(cx^n)}))\;dx = \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n)})) + \frac{(2bdn)\int e^{-d^2(a+b\log(cx^n))^2}x^2\,dx}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n)})) + \frac{(2bdn)\int \exp\left(-a^2d^2-2abd^2\log{(cx^n)}-b^2d^2\log^2{(c}\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n)})) + \frac{(2bdn)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^2\,(cx^n)^{-2abd^2}\;dx}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n)})) + \frac{\left(2bdnx^{2abd^2n}\,(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{2-2abd^2}}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n)})) + \frac{\left(2bdx^3\,(cx^n)^{-2abd^2-\frac{3-2abd^2n}{n}}\right)\text{Subst}\left(\int \exp\left(-a^2d^2+\right.\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n)})) + \frac{\left(2bde^{\frac{9-12abd^2n}{4b^2d^2n^2}}x^3\,(cx^n)^{-2abd^2-\frac{3-2abd^2n}{n}}\right)\text{Subst}\left(\int \exp\left(-\right.\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}x^3\,(cx^n)^{-3/n}\,\text{erf}\left(\frac{2abd^2-\frac{3}{n}+2b^2d^2\log{(cx^n)}}{2bd}\right) + \frac{1}{3}x^3\text{erfc}\,(d\,(a+b\log{(cx^n}$$

Mathematica [A] time = 0.303027, size = 87, normalized size = 0.85

$$\frac{1}{3}\left(x^3\text{Erf}\left(ad+bd\log{(cx^n)}-\frac{3}{2bdn}\right)\exp\left(\frac{3\left(\frac{\frac{3}{d^2}-4abn}{b^2}-4n\log{(cx^n)}\right)}{4n^2}\right)+x^3\text{Erfc}\,(d\,(a+b\log{(cx^n)}))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^((3*((3/d^2 - 4*a*b*n)/b^2 - 4*n*Log[c*x^n]))/(4*n^2))*x^3*Erf[a*d - 3/(
2*b*d*n) + b*d*Log[c*x^n]] + x^3*Erfc[d*(a + b*Log[c*x^n])])/3
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int x^2\text{erfc}\,(d\,(a+b\ln{(cx^n)}))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfc(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*erfc(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}\left((b \log\left(cx^n\right) + a)d\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfc((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 2.18052, size = 317, normalized size = 3.11

$$-\frac{1}{3}\,x^3 \operatorname{erf}\left(bd\log\left(cx^n\right) + ad\right) + \frac{1}{3}\,x^3 + \frac{1}{3}\,\sqrt{b^2d^2n^2} \operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right) + 2\,b^2d^2n\log\left(c\right) + 2\,abd^2n - 3\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{3\left(4\,b^2\right.}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/3*x^3*erf(b*d*log(c*x^n) + a*d) + 1/3*x^3 + 1/3*sqrt(b^2*d^2*n^2)*erf(1/
2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 3)*sqrt(b^2*d^
2*n^2)/(b^2*d^2*n^2))*e^(-3/4*(4*b^2*d^2*n*log(c) + 4*a*b*d^2*n - 3)/(b^2*d
^2*n^2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34418, size = 122, normalized size = 1.2

$$-\frac{1}{3}\,x^3\,\mathrm{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+\frac{1}{3}\,x^3-\frac{\mathrm{erf}\left(-bdn\log\left(x\right)-bd\log\left(c\right)-ad+\frac{3}{2\,bdn}\right)e^{\left(-\frac{3\,a}{bn}+\frac{9}{4\,b^2d^2n^2}\right)}}{3\,c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] -1/3*x^3*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/3*x^3 - 1/3*erf(-b*d*n*lo
g(x) - b*d*log(c) - a*d + 3/2/(b*d*n))*e^(-3*a/(b*n) + 9/4/(b^2*d^2*n^2))/c
^(3/n)
```

3.144 $\int x\text{Erfc}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}x^2\left(cx^n\right)^{-2/n}e^{\frac{1-2abd^2n}{b^2d^2n^2}}\text{Erf}\left(\frac{abd^2+b^2d^2\log\left(cx^n\right)-\frac{1}{n}}{bd}\right)+\frac{1}{2}x^2\text{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)$$

```
[Out] (E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^2*Erf[(a*b*d^2 - n^(-1) + b^2*d^2*Lo
g[c*x^n])/(b*d)])/(2*(c*x^n)^(2/n)) + (x^2*Erfc[d*(a + b*Log[c*x^n])])/2
```

Rubi [A] time = 0.165809, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6402, 2278, 2274, 15, 2276, 2234, 2205}

$$\frac{1}{2}x^2\left(cx^n\right)^{-2/n}e^{\frac{1-2abd^2n}{b^2d^2n^2}}\text{Erf}\left(\frac{abd^2+b^2d^2\log\left(cx^n\right)-\frac{1}{n}}{bd}\right)+\frac{1}{2}x^2\text{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^2*Erf[(a*b*d^2 - n^(-1) + b^2*d^2*Lo
g[c*x^n])/(b*d)])/(2*(c*x^n)^(2/n)) + (x^2*Erfc[d*(a + b*Log[c*x^n])])/2
```

Rule 6402

```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfc[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 + Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x\text{erfc}(d(a+b\log(cx^n)))\,dx = \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n))) + \frac{(bdn)\int e^{-d^2(a+b\log(cx^n))^2}x\,dx}{\sqrt{\pi}}$$
$$= \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n))) + \frac{(bdn)\int \exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log^2(cx^n)\right)}{\sqrt{\pi}}$$
$$= \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n))) + \frac{(bdn)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x(cx^n)^{-2abd^2}\,dx}{\sqrt{\pi}}$$
$$= \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n))) + \frac{\left(bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{1-2abd^2n}\,dx}{\sqrt{\pi}}$$
$$= \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n))) + \frac{\left(bdx^2(cx^n)^{-2abd^2-\frac{2-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-a^2d^2+\frac{(2-2a}{}\right.\right.}{\sqrt{\pi}}$$
$$= \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n))) + \frac{\left(bde^{\frac{1-2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{-2abd^2-\frac{2-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{\left(\frac{2-2a}{n}\right.}{}\right.\right.}{\sqrt{\pi}}$$
$$= \frac{1}{2}e^{\frac{1-2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{-2/n}\,\text{erf}\left(\frac{abd^2-\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right) + \frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n)))$$

Mathematica [A] time = 0.271673, size = 80, normalized size = 0.85

$$\frac{1}{2}\left(x^2e^{\frac{\frac{\frac{1}{d^2}-2abn}{b^2}-2n\log(cx^n)}{n^2}}\text{Erf}\left(ad+bd\log(cx^n)-\frac{1}{bdn}\right)+x^2\text{Erfc}(d(a+b\log(cx^n)))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^(((d^(-2) - 2*a*b*n)/b^2 - 2*n*Log[c*x^n])/n^2)*x^2*Erf[a*d - 1/(b*d*n)
+ b*d*Log[c*x^n]] + x^2*Erfc[d*(a + b*Log[c*x^n])])/2
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int x\text{erfc}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfc(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*erfc(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfc}\left((b \log \left(cx^{n}\right) + a)d\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*erfc((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 2.18754, size = 298, normalized size = 3.17

$$-\frac{1}{2}\,x^{2} \operatorname{erf}\left(bd \log \left(cx^{n}\right) + ad\right) + \frac{1}{2}\,\sqrt{b^{2}d^{2}n^{2}} \operatorname{erf}\left(\frac{\left(b^{2}d^{2}n^{2} \log \left(x\right) + b^{2}d^{2}n \log \left(c\right) + abd^{2}n - 1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{2\,b^{2}d^{2}n \log(c)+2\,abd^{2}n}{b^{2}d^{2}n^{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*erf(b*d*log(c*x^n) + a*d) + 1/2*sqrt(b^2*d^2*n^2)*erf((b^2*d^2*n^2
*log(x) + b^2*d^2*n*log(c) + a*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2)
)*e^(-(2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 1)/(b^2*d^2*n^2)) + 1/2*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfc}\left(ad + bd \log \left(cx^{n}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*erfc(a*d + b*d*log(c*x**n)), x)
```

Giac [A] time = 1.36917, size = 119, normalized size = 1.27

$$-\frac{1}{2}x^2\operatorname{erf}(bdn\log(x) + bd\log(c) + ad) + \frac{1}{2}x^2 - \frac{\operatorname{erf}\left(-bdn\log(x) - bd\log(c) - ad + \frac{1}{bdn}\right)e^{\left(-\frac{2a}{bn} + \frac{1}{b^2d^2n^2}\right)}}{2c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] -1/2*x^2*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/2*x^2 - 1/2*erf(-b*d*n*lo
g(x) - b*d*log(c) - a*d + 1/(b*d*n))*e^(-2*a/(b*n) + 1/(b^2*d^2*n^2))/c^(2/
n)
```

3.145 $\int \mathbf{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=92

$$x\left(cx^n\right)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\operatorname{Erf}\left(\frac{2abd^2+2b^2d^2\log\left(cx^n\right)-\frac{1}{n}}{2bd}\right)+x\operatorname{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)$$

```
[Out] (E^((1 - 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*x*Erf[(2*a*b*d^2 - n^(-1) + 2*b^2*d^
2*Log[c*x^n])/(2*b*d)])/(c*x^n)^n^(-1) + x*Erfc[d*(a + b*Log[c*x^n])]
```

Rubi [A] time = 0.101062, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6398, 2277, 2274, 15, 2276, 2234, 2205}

$$x\left(cx^n\right)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\operatorname{Erf}\left(\frac{2abd^2+2b^2d^2\log\left(cx^n\right)-\frac{1}{n}}{2bd}\right)+x\operatorname{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^((1 - 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*x*Erf[(2*a*b*d^2 - n^(-1) + 2*b^2*d^
2*Log[c*x^n])/(2*b*d)])/(c*x^n)^n^(-1) + x*Erfc[d*(a + b*Log[c*x^n])]
```

Rule 6398

```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*
Erfc[d*(a + b*Log[c*x^n])], x] + Dist[(2*b*d*n)/Sqrt[Pi], Int[1/E^(d*(a + b
*Log[c*x^n]))^2, x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = x\operatorname{erfc}(d(a+b\log(cx^n))) + \frac{(2bdn)\int e^{-d^2(a+b\log(cx^n))^2}\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfc}(d(a+b\log(cx^n))) + \frac{(2bdn)\int \exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log^2(cx^n)\right)\,d}{\sqrt{\pi}}$$

$$= x\operatorname{erfc}(d(a+b\log(cx^n))) + \frac{(2bdn)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}(cx^n)^{-2abd^2}\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfc}(d(a+b\log(cx^n))) + \frac{\left(2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfc}(d(a+b\log(cx^n))) + \frac{\left(2bdx(cx^n)^{-2abd^2-\frac{1-2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-a^2d^2+\frac{(1-2abd^2}{n}\right.\right.}{\sqrt{\pi}}$$

$$= x\operatorname{erfc}(d(a+b\log(cx^n))) + \frac{\left(2bde^{\frac{1-4abd^2n}{4b^2d^2n^2}}x(cx^n)^{-2abd^2-\frac{1-2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{\left(\frac{1-2abd^2}{n}\right.}{4}\right.\right.}{\sqrt{\pi}}$$

$$= e^{\frac{1-4abd^2n}{4b^2d^2n^2}}x(cx^n)^{-1/n}\operatorname{erf}\left(\frac{2abd^2-\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right) + x\operatorname{erfc}(d(a+b\log(cx^n)))$$

Mathematica [A] time = 0.239139, size = 77, normalized size = 0.84

$$xe^{\frac{\frac{\frac{1}{d^2}-4abn}{b^2}-4n\log(cx^n)}{4n^2}}\operatorname{Erf}\left(ad+bd\log(cx^n)-\frac{1}{2bdn}\right)+x\operatorname{Erfc}(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] E^(((d^(-2) - 4*a*b*n)/b^2 - 4*n*Log[c*x^n])/(4*n^2))*x*Erf[a*d - 1/(2*b*d*
n) + b*d*Log[c*x^n]] + x*Erfc[d*(a + b*Log[c*x^n])]
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \operatorname{erfc}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(erfc(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(erfc((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 2.20213, size = 294, normalized size = 3.2

$$\sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n-1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{4\,b^2d^2n\log(c)+4\,abd^2n-1}{4\,b^2d^2n^2}\right)}-x\operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] sqrt(b^2*d^2*n^2)*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*
b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(-1/4*(4*b^2*d^2*n*log(c) +
 4*a*b*d^2*n - 1)/(b^2*d^2*n^2)) - x*erf(b*d*log(c*x^n) + a*d) + x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(erfc(d*(a + b*log(c*x**n))), x)
```

Giac [A] time = 1.41814, size = 111, normalized size = 1.21

$$-x\,\mathrm{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+x-\frac{\mathrm{erf}\left(-bdn\log\left(x\right)-bd\log\left(c\right)-ad+\frac{1}{2\,bdn}\right)e^{\left(-\frac{a}{bn}+\frac{1}{4\,b^{2}d^{2}n^{2}}\right)}}{c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] -x*erf(b*d*n*log(x) + b*d*log(c) + a*d) + x - erf(-b*d*n*log(x) - b*d*log(c
) - a*d + 1/2/(b*d*n))*e^(-a/(b*n) + 1/4/(b^2*d^2*n^2))/c^(1/n)
```

3.146 $\int \frac{\text{Erfc}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=66

$$\frac{(a+b\log(cx^n))\,\text{Erfc}(d(a+b\log(cx^n)))}{bn}-\frac{e^{-d^2(a+b\log(cx^n))^2}}{\sqrt{\pi}bdn}$$

```
[Out] -(1/(b*d*E^(d^2*(a + b*Log[c*x^n])^2)*n*Sqrt[Pi])) + (Erfc[d*(a + b*Log[c*x
^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rubi [A] time = 0.0430606, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6350}

$$\frac{(a+b\log(cx^n))\,\text{Erfc}(d(a+b\log(cx^n)))}{bn}-\frac{e^{-d^2(a+b\log(cx^n))^2}}{\sqrt{\pi}bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] -(1/(b*d*E^(d^2*(a + b*Log[c*x^n])^2)*n*Sqrt[Pi])) + (Erfc[d*(a + b*Log[c*x
^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6350

```
Int[Erfc[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erfc[a + b*x])/b
, x] - Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \text{erfc}(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int \text{erfc}(x)\,dx,x,ad+bd\log(cx^n)\right)}{bdn}\\ &= -\frac{e^{-(ad+bd\log(cx^n))^2}}{bdn\sqrt{\pi}}+\frac{\text{erfc}(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [A] time = 0.139772, size = 93, normalized size = 1.41

$$\frac{-\frac{(cx^n)^{-2abd^2}e^{-d^2\left(a^2+b^2\log^2(cx^n)\right)}}{\sqrt{\pi}bd}-\frac{a\text{Erf}(d(a+b\log(cx^n)))}{b}+\log(cx^n)\,\text{Erfc}\left(d\left(a+b\log(cx^n)\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (-(1/(b*d*E^(d^2*(a^2 + b^2*Log[c*x^n]^2))*Sqrt[Pi]*(c*x^n)^(2*a*b*d^2))) -
 (a*Erf[d*(a + b*Log[c*x^n])])/b + Erfc[d*(a + b*Log[c*x^n])]*Log[c*x^n])/n
```

Maple [A] time = 0.105, size = 80, normalized size = 1.2

$$\frac{\ln(cx^n)\operatorname{erfc}(ad+bd\ln(cx^n))}{n}+\frac{\operatorname{erfc}(ad+bd\ln(cx^n))\,a}{bn}-\frac{e^{-(ad+bd\ln(cx^n))^2}}{bdn\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*erfc(a*d+b*d*ln(c*x^n))+1/n/b*erfc(a*d+b*d*ln(c*x^n))*a-1/n/b
/d/Pi^(1/2)*exp(-(a*d+b*d*ln(c*x^n))^2)
```

Maxima [A] time = 1.00637, size = 80, normalized size = 1.21

$$\frac{(b\log(cx^n)+a)d\operatorname{erfc}((b\log(cx^n)+a)d)-\frac{e^{\left(-\left(b\log(cx^n)+a\right)^2d^2\right)}}{\sqrt{\pi}}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*erfc((b*log(c*x^n) + a)*d) - e^(-(b*log(c*x^n) + a)^2
*d^2)/sqrt(pi))/(b*d*n)
```

Fricas [B] time = 2.13556, size = 308, normalized size = 4.67

$$\frac{\pi bdn\log\left(x\right)-\left(\pi bdn\log\left(x\right)+\pi bd\log\left(c\right)+\pi ad\right)\operatorname{erf}\left(bd\log\left(cx^{n}\right)+ad\right)-\sqrt{\pi}e^{\left(-b^{2}d^{2}n^{2}\log(x)^{2}-b^{2}d^{2}\log(c)^{2}-2\,abd^{2}\log(c)-a^{2}d^{2}-2\,(\right.}}{\pi bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] (pi*b*d*n*log(x) - (pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*erf(b*d*log(c
*x^n) + a*d) - sqrt(pi)*e^(-b^2*d^2*n^2*log(x)^2 - b^2*d^2*log(c)^2 - 2*a*b
*d^2*log(c) - a^2*d^2 - 2*(b^2*d^2*n*log(c) + a*b*d^2*n)*log(x)))/(pi*b*d*n
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(erfc(a*d + b*d*log(c*x**n))/x, x)
```

Giac [A] time = 1.28677, size = 112, normalized size = 1.7

$$\frac{bdn\log\left(x\right)+bd\log\left(c\right)+ad-\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)\operatorname{erf}\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)-\frac{e^{\left(-(bdn\log(x)+bd\log(c)+ad)^{2}\right)}}{\sqrt{\pi}}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] (b*d*n*log(x) + b*d*log(c) + a*d - (b*d*n*log(x) + b*d*log(c) + a*d)*erf(b*
d*n*log(x) + b*d*log(c) + a*d) - e^(-(b*d*n*log(x) + b*d*log(c) + a*d)^2)/s
qrt(pi))/(b*d*n)
```

3.147 $\int \frac{\mathbf{Erfc}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=93

$$-\frac{(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}}\operatorname{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)}{x}-\frac{\operatorname{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

```
[Out] -((E^(1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)^n^(-1)*Erf[(2*a*b*d^2 + n^(-1) +
 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x) - Erfc[d*(a + b*Log[c*x^n])]/x
```

Rubi [A] time = 0.184518, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6402, 2278, 2274, 15, 2276, 2234, 2205}

$$-\frac{(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}}\operatorname{Erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)}{x}-\frac{\operatorname{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -((E^(1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)^n^(-1)*Erf[(2*a*b*d^2 + n^(-1) +
 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x) - Erfc[d*(a + b*Log[c*x^n])]/x
```

Rule 6402

```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfc[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 + Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx = -\frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} - \frac{(2bdn)\int \frac{e^{-d^2\left(a+b\log\left(cx^n\right)\right)^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} - \frac{(2bdn)\int \frac{\exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log^2(cx^n)\right)}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} - \frac{(2bdn)\int \frac{e^{-a^2d^2-b^2d^2\log^2(cx^n)}(cx^n)^{-2abd^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} - \frac{\left(2bdnx^{2abd^2n}\left(cx^n\right)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-2-2abd^2n}\,d}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} - \frac{\left(2bd\left(cx^n\right)^{-2abd^2-\frac{-1-2abd^2n}{n}}\right)\text{Subst}\left(\int \exp\left(-a^2d^2+\frac{(-1-2ab}{n}\right.\right.}{\sqrt{\pi}x}$$

$$= -\frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} - \frac{\left(2bde^{\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}\left(cx^n\right)^{-2abd^2-\frac{-1-2abd^2n}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{(-1-}{}\right.\right.}{\sqrt{\pi}x}$$

$$= -\frac{e^{\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\text{erf}\left(\frac{2abd^2+\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)}{x} - \frac{\text{erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

Mathematica [A] time = 0.262326, size = 81, normalized size = 0.87

$$-\frac{\left(cx^n\right)^{\frac{1}{n}}e^{\frac{4abd^2n+1}{4b^2d^2n^2}}\text{Erf}\left(ad+bd\log\left(cx^n\right)+\frac{1}{2bdn}\right)+\text{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -((E^((1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x^n)^n^(-1)*Erf[a*d + 1/(2*b*d*
n) + b*d*Log[c*x^n]] + Erfc[d*(a + b*Log[c*x^n])])/x)
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(erfc(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\left((b\log\left(cx^n\right)+a)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [A] time = 2.37687, size = 300, normalized size = 3.23

$$-\frac{\sqrt{b^2d^2n^2}x\,\text{erf}\left(\frac{\left(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n+1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(\frac{4\,b^2d^2n\log(c)+4\,abd^2n+1}{4\,b^2d^2n^2}\right)}-\text{erf}\left(bd\log\left(cx^n\right)+ad\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(b^2*d^2*n^2)*x*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) +
2*a*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*n*log(c
) + 4*a*b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d) + 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\left(ad+bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(erfc(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc((b*log(c*x^n) + a)*d)/x^2, x)
```

3.148 $\int \frac{\mathbf{Erfc}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=95

$$-\frac{(cx^n)^{2/n}e^{\frac{2abd^2n+1}{b^2d^2n^2}}\operatorname{Erf}\left(\frac{abd^2n+b^2d^2n\log(cx^n)+1}{bdn}\right)}{2x^2}-\frac{\operatorname{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

```
[Out] -(E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(2/n)*Erf[(1 + a*b*d^2*n + b^
2*d^2*n*Log[c*x^n])/(b*d*n)])/(2*x^2) - Erfc[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rubi [A] time = 0.188596, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6402, 2278, 2274, 15, 2276, 2234, 2205}

$$-\frac{(cx^n)^{2/n}e^{\frac{2abd^2n+1}{b^2d^2n^2}}\operatorname{Erf}\left(\frac{abd^2n+b^2d^2n\log(cx^n)+1}{bdn}\right)}{2x^2}-\frac{\operatorname{Erfc}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -(E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(2/n)*Erf[(1 + a*b*d^2*n + b^
2*d^2*n*Log[c*x^n])/(b*d*n)])/(2*x^2) - Erfc[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 6402

```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfc[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 + Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2} - \frac{(bdn)\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^3}dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2} - \frac{(bdn)\int \frac{\exp\left(-a^2d^2-2abd^2\log(cx^n)-b^2d^2\log^2(cx^n)\right)}{x^3}dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2} - \frac{(bdn)\int \frac{e^{-a^2d^2-b^2d^2\log^2(cx^n)}(cx^n)^{-2abd^2}}{x^3}dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2} - \frac{\left(bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x^{-3-2abd^2n}dx}{\sqrt{\pi}}$$

$$= -\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2} - \frac{\left(bd(cx^n)^{-2abd^2-\frac{-2-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-a^2d^2+\frac{(-2-2abd^2n}{n}\right.\right.}{\sqrt{\pi}x^2}$$

$$= -\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2} - \frac{\left(bde^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{-2abd^2-\frac{-2-2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{\left(\frac{-2-2abd^2n}{n}\right.}{4b^2}\right.\right.}{\sqrt{\pi}x^2}$$

$$= -\frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\,\text{erf}\left(\frac{1+abd^2n+b^2d^2n\log(cx^n)}{bdn}\right)}{2x^2} - \frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

Mathematica [A] time = 0.260842, size = 79, normalized size = 0.83

$$-\frac{(cx^n)^{2/n}e^{\frac{2abd^2n+1}{b^2d^2n^2}}\text{Erf}\left(ad+bd\log(cx^n)+\frac{1}{bdn}\right)+\text{Erfc}(d(a+b\log(cx^n)))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -(E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(2/n)*Erf[a*d + 1/(b*d*n) + b
*d*Log[c*x^n]] + Erfc[d*(a + b*Log[c*x^n])])/(2*x^2)
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{\text{erfc}(d(a+b\ln(cx^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(erfc(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left((b\log\left(cx^n\right) + a)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [A] time = 2.26407, size = 292, normalized size = 3.07

$$-\frac{\sqrt{b^2d^2n^2}x^2\operatorname{erf}\left(\frac{\left(b^2d^2n^2\log(x)+b^2d^2n\log(c)+abd^2n+1\right)\sqrt{b^2d^2n^2}}{b^2d^2n^2}\right)e^{\left(\frac{2\,b^2d^2n\log(c)+2\,abd^2n+1}{b^2d^2n^2}\right)} - \operatorname{erf}\left(bd\log\left(cx^n\right) + ad\right) + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(b^2*d^2*n^2)*x^2*erf((b^2*d^2*n^2*log(x) + b^2*d^2*n*log(c) + a*
b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^((2*b^2*d^2*n*log(c) + 2*a*
b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d) + 1)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc((b*log(c*x^n) + a)*d)/x^3, x)
```

3.149 $\int (ex)^m \textbf{Erfc}\,(d\,(a+b\log\,(cx^n)))\;dx$

Optimal. Leaf size=126

$$\frac{(ex)^{m+1}\mathrm{Erfc}\,(d\,(a+b\log\,(cx^n)))}{e(m+1)}-\frac{x(ex)^m\,(cx^n)^{-\frac{m+1}{n}}\exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right)\mathrm{Erf}\left(\frac{-2abd^2n-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}$$

```
[Out] -((E^(((1 + m)*(1 + m - 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*x*(e*x)^m*Erf[(1 + m
 - 2*a*b*d^2*n - 2*b^2*d^2*n*Log[c*x^n])/(2*b*d*n)])/((1 + m)*(c*x^n)^((1 +
 m)/n))) + ((e*x)^(1 + m)*Erfc[d*(a + b*Log[c*x^n])])/(e*(1 + m))
```

Rubi [A] time = 0.254358, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6402, 2278, 2274, 15, 20, 2276, 2234, 2205}

$$\frac{(ex)^{m+1}\mathrm{Erfc}\,(d\,(a+b\log\,(cx^n)))}{e(m+1)}-\frac{x(ex)^m\,(cx^n)^{-\frac{m+1}{n}}\exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right)\mathrm{Erf}\left(\frac{-2abd^2n-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Erfc[d*(a + b*Log[c*x^n])],x]
```


```
[Out] -((E^(((1 + m)*(1 + m - 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*x*(e*x)^m*Erf[(1 + m
 - 2*a*b*d^2*n - 2*b^2*d^2*n*Log[c*x^n])/(2*b*d*n)])/((1 + m)*(c*x^n)^((1 +
 m)/n))) + ((e*x)^(1 + m)*Erfc[d*(a + b*Log[c*x^n])])/(e*(1 + m))
```


Rule 6402

```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfc[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 + Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{(2bdn)\int e^{-d^2(a+b\log(cx^n))^2}(ex)^m\,dx}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{(2bdn)\int \exp\left(-a^2d^2 - 2abd^2\log(cx^n) - b^2d^2 \right.}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{(2bdn)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}(ex)^m(cx^n)^{-2abd^2}\,dx}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{\left(2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d^2\log^2(cx^n)}x}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{\left(2bdnx^{-m+2abd^2n}(ex)^m(cx^n)^{-2abd^2}\right)\int e^{-a^2d^2-b^2d}}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{\left(2bdx(ex)^m(cx^n)^{-2abd^2-\frac{1+m-2abd^2n}{n}}\right)\text{Subst}\left(\int \text{ex}\right.}{(1}$$

$$= \frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{\left(2bd\exp\left(\frac{(1+m)(1+m-4abd^2n)}{4b^2d^2n^2}\right)x(ex)^m(cx^n)^{-2abd}\right.}{}$$

$$= -\frac{\exp\left(\frac{(1+m)(1+m-4abd^2n)}{4b^2d^2n^2}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\text{erf}\left(\frac{1+m-2abd^2n-2b^2d^2n\log(cx^n)}{2bdn}\right)}{1+m} + \frac{(ex)^{1+}}{}$$

Mathematica [A] time = 0.504831, size = 126, normalized size = 1.

$$\frac{(ex)^m\left(x^{-m}\text{Erf}\left(ad-\frac{-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)\exp\left(\frac{(m+1)(-4abd^2n-4b^2d^2n\log(cx^n)+4b^2d^2n^2\log(x)+m+1)}{4b^2d^2n^2}\right)+x\text{Erfc}(d(a+b\log(cx^n)))\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Erfc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*((E^(((1 + m)*(1 + m - 4*a*b*d^2*n + 4*b^2*d^2*n^2*Log[x] - 4*b^2*
d^2*n*Log[c*x^n]))/(4*b^2*d^2*n^2))*Erf[a*d - (1 + m - 2*b^2*d^2*n*Log[c*x^
n])/(2*b*d*n)])/x^m + x*Erfc[d*(a + b*Log[c*x^n])]))/(1 + m)
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfc}\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*erfc(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*erfc(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfc}\left((b\log\left(cx^n\right)+a)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*erfc((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 2.33048, size = 466, normalized size = 3.7

$$-\frac{x \operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)e^{(m\log(e)+m\log(x))} - \sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n-m-1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(\frac{4\,b^2d^2mn^2\log(e)-4\left(b^2d^2\right.}{}\right.}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -(x*erf(b*d*log(c*x^n) + a*d)*e^(m*log(e) + m*log(x)) - sqrt(b^2*d^2*n^2)*e
rf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - m - 1)*sq
rt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*m*n^2*log(e) - 4*(b^2*d^2*
m + b^2*d^2)*n*log(c) + m^2 - 4*(a*b*d^2*m + a*b*d^2)*n + 2*m + 1)/(b^2*d^2
*n^2)) - x*e^(m*log(e) + m*log(x)))/(m + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfc}(ad + bd\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*erfc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*erfc(a*d + b*d*log(c*x**n)), x)
```

Giac [A] time = 1.44204, size = 228, normalized size = 1.81

$$-\frac{xx^m \operatorname{erf}(bdn\log(x) + bd\log(c) + ad)e^m}{m+1} + \frac{xx^m e^m}{m+1} - \frac{\pi \operatorname{erf}\left(-bdn\log(x) - bd\log(c) - ad + \frac{m}{2\,bdn} + \frac{1}{2\,bdn}\right)e^{\left(m - \frac{am}{bn} - \frac{a}{bn} + \frac{\cdots}{4\cdots}\right.}}{(\pi + \pi m)c^{\frac{m}{n}}c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] -x*x^m*erf(b*d*n*log(x) + b*d*log(c) + a*d)*e^m/(m + 1) + x*x^m*e^m/(m + 1)
 - pi*erf(-b*d*n*log(x) - b*d*log(c) - a*d + 1/2*m/(b*d*n) + 1/2/(b*d*n))*e
^(m - a*m/(b*n) - a/(b*n) + 1/4*m^2/(b^2*d^2*n^2) + 1/2*m/(b^2*d^2*n^2) + 1
/4/(b^2*d^2*n^2))/((pi + pi*m)*c^(m/n)*c^(1/n))
```

3.150 $\int e^{c-b^2x^2}\mathbf{Erfc}(bx)^2\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^3}{6b}$$

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^3)/(6*b)
```

Rubi [A] time = 0.0273624, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6374, 30}

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)*Erfc[b*x]^2,x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^3)/(6*b)
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c-b^2x^2}\text{erfc}(bx)^2\,dx &= -\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x^2\,dx, x, \text{erfc}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}\text{erfc}(bx)^3}{6b}\end{aligned}$$

Mathematica [A] time = 0.0093958, size = 21, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)*Erfc[b*x]^2,x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^3)/(6*b)
```

Maple [B] time = 0.226, size = 43, normalized size = 2.1

$$\frac{1}{b}\left(\frac{e^c\sqrt{\pi}\text{Erf}(bx)}{2}-\frac{e^c\sqrt{\pi}\,(\text{Erf}(bx))^2}{2}+\frac{e^c\sqrt{\pi}\,(\text{Erf}(bx))^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)*erfc(b*x)^2,x)
```

```
[Out] (1/2*exp(c)*Pi^(1/2)*erf(b*x)-1/2*exp(c)*Pi^(1/2)*erf(b*x)^2+1/6*exp(c)*Pi^
(1/2)*erf(b*x)^3)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfc}\,(bx)^2\,e^{\left(-b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^2*e^(-b^2*x^2 + c), x)
```

Fricas [A] time = 2.07739, size = 82, normalized size = 3.9

$$\frac{\sqrt{\pi}\left(\text{erf}\,(bx)^3-3\,\text{erf}\,(bx)^2+3\,\text{erf}\,(bx)\right)e^c}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(pi)*(erf(b*x)^3 - 3*erf(b*x)^2 + 3*erf(b*x))*e^c/b
```

Sympy [A] time = 2.69231, size = 24, normalized size = 1.14

$$\begin{cases} -\frac{\sqrt{\pi}e^{c}\operatorname{erfc}^{3}(bx)}{6b} & \text{for } b \neq 0 \\ xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)*erfc(b*x)**2,x)
```

```
[Out] Piecewise((-sqrt(pi)*exp(c)*erfc(b*x)**3/(6*b), Ne(b, 0)), (x*exp(c), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)^{2} e^{\left(-b^{2}x^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^2*e^(-b^2*x^2 + c), x)
```

3.151 $\int e^{c-b^2x^2}\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{4b}$$

[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(4*b)

Rubi [A] time = 0.0177382, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6374, 30}

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{4b}$$

Antiderivative was successfully verified.

[In] Int[E^(c - b^2*x^2)*Erfc[b*x],x]

[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(4*b)

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c-b^2x^2}\text{erfc}(bx)\,dx &= -\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx,x,\text{erfc}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}\text{erfc}(bx)^2}{4b}\end{aligned}$$

Mathematica [A] time = 0.0053808, size = 21, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)*Erfc[b*x],x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(4*b)
```

Maple [A] time = 0.107, size = 30, normalized size = 1.4

$$\frac{1}{b}\left(\frac{\mathrm{e}^c\sqrt{\pi}\text{Erf}\,(bx)}{2}-\frac{\mathrm{e}^c\sqrt{\pi}\,(\text{Erf}\,(bx))^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)*erfc(b*x),x)
```

```
[Out] (1/2*exp(c)*Pi^(1/2)*erf(b*x)-1/4*exp(c)*Pi^(1/2)*erf(b*x)^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfc}\,(bx)\,e^{\left(-b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2 + c), x)
```

Fricas [A] time = 2.16173, size = 63, normalized size = 3.

$$-\frac{\sqrt{\pi}\left(\text{erf}\,(bx)^2-2\,\text{erf}\,(bx)\right)e^c}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(b*x)^2 - 2*erf(b*x))*e^c/b
```

Sympy [A] time = 0.691919, size = 24, normalized size = 1.14

$$\begin{cases} -\frac{\sqrt{\pi}e^{c}\operatorname{erfc}^{2}(bx)}{4b} & \text{for } b \neq 0 \\ xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)*erfc(b*x),x)
```

```
[Out] Piecewise((-sqrt(pi)*exp(c)*erfc(b*x)**2/(4*b), Ne(b, 0)), (x*exp(c), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\,e^{\left(-b^{2}x^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2 + c), x)
```

3.152 $\int \frac{e^{c-b^2x^2}}{\mathbf{Erfc}(bx)}\,dx$

Optimal. Leaf size=20

$$-\frac{\sqrt{\pi}e^c \log(\mathbf{Erfc}(bx))}{2b}$$

```
[Out] -(E^c*Sqrt[Pi]*Log[Erfc[b*x]])/(2*b)
```

Rubi [A] time = 0.02752, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6374, 29}

$$-\frac{\sqrt{\pi}e^c \log(\mathbf{Erfc}(bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)/Erfc[b*x],x]
```

```
[Out] -(E^c*Sqrt[Pi]*Log[Erfc[b*x]])/(2*b)
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\,dx &= -\frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \operatorname{erfc}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}\log(\operatorname{erfc}(bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.0107057, size = 20, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c \log(\mathbf{Erfc}(bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)/Erfc[b*x],x]
```

```
[Out] -(E^c*Sqrt[Pi]*Log[Erfc[b*x]])/(2*b)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{-b^2x^2+c}}{\mathrm{erfc}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)/erfc(b*x),x)
```

```
[Out] int(exp(-b^2*x^2+c)/erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^2x^2+c\right)}}{\mathrm{erfc}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erfc(b*x), x)
```

Fricas [A] time = 2.07644, size = 53, normalized size = 2.65

$$-\frac{\sqrt{\pi}e^c \log\left(\mathrm{erf}\,(bx) - 1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(pi)*e^c*log(erf(b*x) - 1)/b
```

Sympy [A] time = 0.548293, size = 24, normalized size = 1.2

$$\begin{cases} -\frac{\sqrt{\pi}e^{c}\log\left(\operatorname{erfc}\left(bx\right)\right)}{2b} & \text{for}\: b \neq 0 \\ xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)/erfc(b*x),x)
```

```
[Out] Piecewise((-sqrt(pi)*exp(c)*log(erfc(b*x))/(2*b), Ne(b, 0)), (x*exp(c), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^{2}x^{2}+c\right)}}{\operatorname{erfc}\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erfc(b*x), x)
```

3.153 $\int \frac{e^{c-b^2x^2}}{\mathbf{Erfc}(bx)^2}\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c}{2b\text{Erfc}(bx)}$$

```
[Out] (E^c*Sqrt[Pi])/(2*b*Erfc[b*x])
```

Rubi [A] time = 0.0277962, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6374, 30}

$$\frac{\sqrt{\pi}e^c}{2b\text{Erfc}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)/Erfc[b*x]^2,x]
```

```
[Out] (E^c*Sqrt[Pi])/(2*b*Erfc[b*x])
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)^2}\,dx &= -\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \text{erfc}(bx)\right)}{2b}\\
&= \frac{e^c\sqrt{\pi}}{2b\text{erfc}(bx)}
\end{aligned}$$

Mathematica [A] time = 0.0060713, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c}{2b\text{Erfc}(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)/Erfc[b*x]^2,x]
```

```
[Out] (E^c*Sqrt[Pi])/(2*b*Erfc[b*x])
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{-b^2x^2+c}}{(\mathrm{erfc}\,(bx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)/erfc(b*x)^2,x)
```

```
[Out] int(exp(-b^2*x^2+c)/erfc(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^2x^2+c\right)}}{\mathrm{erfc}\,(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erfc(b*x)^2, x)
```

Fricas [A] time = 2.16569, size = 49, normalized size = 2.33

$$-\frac{\sqrt{\pi}e^c}{2\,(b\,\mathrm{erf}\,(bx)-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(pi)*e^c/(b*erf(b*x) - b)
```

Sympy [A] time = 1.42661, size = 20, normalized size = 0.95

$$\begin{cases} \frac{\sqrt{\pi}e^{c}}{2b\operatorname{erfc}(bx)} & \text{for } b \neq 0 \\ xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)/erfc(b*x)**2,x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)/(2*b*erfc(b*x)), Ne(b, 0)), (x*exp(c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^{2}x^{2}+c\right)}}{\operatorname{erfc}(bx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erfc(b*x)^2, x)
```

3.154 $\int \frac{e^{c-b^2x^2}}{\mathbf{Erfc}(bx)^3}\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c}{4b\text{Erfc}(bx)^2}$$

```
[Out] (E^c*Sqrt[Pi])/(4*b*Erfc[b*x]^2)
```

Rubi [A] time = 0.0293194, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6374, 30}

$$\frac{\sqrt{\pi}e^c}{4b\text{Erfc}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)/Erfc[b*x]^3,x]
```

```
[Out] (E^c*Sqrt[Pi])/(4*b*Erfc[b*x]^2)
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)^3}\,dx &= -\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, \text{erfc}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}}{4b\text{erfc}(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.005656, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c}{4b\text{Erfc}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)/Erfc[b*x]^3,x]
```

```
[Out] (E^c*Sqrt[Pi])/(4*b*Erfc[b*x]^2)
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2+c}}{(\text{erfc}\,(bx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)/erfc(b*x)^3,x)
```

```
[Out] int(exp(-b^2*x^2+c)/erfc(b*x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^2x^2+c\right)}}{\text{erfc}\,(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erfc(b*x)^3, x)
```

Fricas [A] time = 2.16436, size = 70, normalized size = 3.33

$$\frac{\sqrt{\pi}e^c}{4\left(b\,\text{erf}\,(bx)^2-2\,b\,\text{erf}\,(bx)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*e^c/(b*erf(b*x)^2 - 2*b*erf(b*x) + b)
```

Sympy [A] time = 3.57281, size = 22, normalized size = 1.05

$$\begin{cases} \frac{\sqrt{\pi}e^{c}}{4b\operatorname{erfc}^{2}(bx)} & \text{for } b \neq 0 \\ xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)/erfc(b*x)**3,x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)/(4*b*erfc(b*x)**2), Ne(b, 0)), (x*exp(c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(-b^{2}x^{2}+c\right)}}{\operatorname{erfc}(bx)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)/erfc(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-b^2*x^2 + c)/erfc(b*x)^3, x)
```

3.155 $\int e^{c-b^2x^2}\mathbf{Erfc}(bx)^n\,dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^{n+1}}{2b(n+1)}$$

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^(1 + n))/(2*b*(1 + n))
```

Rubi [A] time = 0.0326613, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6374, 30}

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^{n+1}}{2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - b^2*x^2)*Erfc[b*x]^n,x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^(1 + n))/(2*b*(1 + n))
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c-b^2x^2}\text{erfc}(bx)^n\,dx &= -\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x^n\,dx,x,\text{erfc}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}\text{erfc}(bx)^{1+n}}{2b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0106263, size = 28, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^{n+1}}{2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c - b^2*x^2)*Erfc[b*x]^n,x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^(1 + n))/(2*b*(1 + n))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int e^{-b^2x^2+c} (\text{erfc}\,(bx))^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b^2*x^2+c)*erfc(b*x)^n,x)
```

```
[Out] int(exp(-b^2*x^2+c)*erfc(b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfc}\,(bx)^n \, e^{\left(-b^2x^2+c\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)^n*e^(-b^2*x^2 + c), x)
```

Fricas [A] time = 2.19796, size = 82, normalized size = 2.93

$$\frac{\sqrt{\pi}(-\text{erf}\,(bx)+1)^n(\text{erf}\,(bx)-1)e^c}{2\,(bn+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x)^n,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*(-erf(b*x) + 1)^n*(erf(b*x) - 1)*e^c/(b*n + b)
```

Sympy [A] time = 11.366, size = 60, normalized size = 2.14

$$\begin{cases} xe^{c} & \text{for } b = 0 \wedge (b = 0 \vee n = -1) \\ -\frac{\sqrt{\pi}e^{c} \log(\operatorname{erfc}(bx))}{2b} & \text{for } n = -1 \\ -\frac{\sqrt{\pi}e^{c} \operatorname{erfc}(bx) \operatorname{erfc}^{n}(bx)}{2bn+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b**2*x**2+c)*erfc(b*x)**n,x)
```

```
[Out] Piecewise((x*exp(c), Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (-sqrt(pi)*exp(c)*
log(erfc(b*x))/(2*b), Eq(n, -1)), (-sqrt(pi)*exp(c)*erfc(b*x)*erfc(b*x)**n/
(2*b*n + 2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)^{n} e^{(-b^{2}x^{2}+c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b^2*x^2+c)*erfc(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)^n*e^(-b^2*x^2 + c), x)
```

3.156 $\int e^{c+dx^2}x^5\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=283

$$-\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\left(b^2-d\right)^{3/2}}+\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{d^3\sqrt{b^2-d}}+\frac{bxe^{c-x^2\left(b^2-d\right)}}{\sqrt{\pi}d^2\left(b^2-d\right)}+\frac{3be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{8d\left(b^2-d\right)^{5/2}}-\frac{bx^3e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{3bxe^{c-x^2\left(b^2-d\right)}}{4\sqrt{\pi}d\left(b^2-d\right)^2}-$$

```
[Out] (b*E^(c - (b^2 - d)*x^2)*x)/((b^2 - d)*d^2*Sqrt[Pi]) - (3*b*E^(c - (b^2 - d
)*x^2)*x)/(4*(b^2 - d)^2*d*Sqrt[Pi]) - (b*E^(c - (b^2 - d)*x^2)*x^3)/(2*(b^
2 - d)*d*Sqrt[Pi]) + (b*E^c*Erf[Sqrt[b^2 - d]*x])/(Sqrt[b^2 - d]*d^3) - (b*
E^c*Erf[Sqrt[b^2 - d]*x])/(2*(b^2 - d)^(3/2)*d^2) + (3*b*E^c*Erf[Sqrt[b^2 -
 d]*x])/(8*(b^2 - d)^(5/2)*d) + (E^(c + d*x^2)*Erfc[b*x])/d^3 - (E^(c + d*x
^2)*x^2*Erfc[b*x])/d^2 + (E^(c + d*x^2)*x^4*Erfc[b*x])/(2*d)
```

Rubi [A] time = 0.367885, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6386, 6383, 2205, 2212}

$$-\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\left(b^2-d\right)^{3/2}}+\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{d^3\sqrt{b^2-d}}+\frac{bxe^{c-x^2\left(b^2-d\right)}}{\sqrt{\pi}d^2\left(b^2-d\right)}+\frac{3be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{8d\left(b^2-d\right)^{5/2}}-\frac{bx^3e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{3bxe^{c-x^2\left(b^2-d\right)}}{4\sqrt{\pi}d\left(b^2-d\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^5*Erfc[b*x],x]
```


```
[Out] (b*E^(c - (b^2 - d)*x^2)*x)/((b^2 - d)*d^2*Sqrt[Pi]) - (3*b*E^(c - (b^2 - d
)*x^2)*x)/(4*(b^2 - d)^2*d*Sqrt[Pi]) - (b*E^(c - (b^2 - d)*x^2)*x^3)/(2*(b^
2 - d)*d*Sqrt[Pi]) + (b*E^c*Erf[Sqrt[b^2 - d]*x])/(Sqrt[b^2 - d]*d^3) - (b*
E^c*Erf[Sqrt[b^2 - d]*x])/(2*(b^2 - d)^(3/2)*d^2) + (3*b*E^c*Erf[Sqrt[b^2 -
 d]*x])/(8*(b^2 - d)^(5/2)*d) + (E^(c + d*x^2)*Erfc[b*x])/d^3 - (E^(c + d*x
^2)*x^2*Erfc[b*x])/d^2 + (E^(c + d*x^2)*x^4*Erfc[b*x])/(2*d)
```


Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx = \frac{e^{c+dx^2}x^4\text{erfc}(bx)}{2d} - \frac{2\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx}{d} + \frac{b\int e^{c-(b^2-d)x^2}x^4\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{c-(b^2-d)x^2}x^3}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{e^{c+dx^2}x^2\text{erfc}(bx)}{d^2} + \frac{e^{c+dx^2}x^4\text{erfc}(bx)}{2d} + \frac{2\int e^{c+dx^2}x\text{erfc}(bx)\,dx}{d^2} - \frac{(2b)\int e^{c-(}}{d^2}$$

$$= \frac{be^{c-(b^2-d)x^2}x}{\left(b^2-d\right)d^2\sqrt{\pi}} - \frac{3be^{c-(b^2-d)x^2}x}{4\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{be^{c-(b^2-d)x^2}x^3}{2\left(b^2-d\right)d\sqrt{\pi}} + \frac{e^{c+dx^2}\text{erfc}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\text{erfc}(bx)}{d^2} + \frac{e^{c+}}{}$$

$$= \frac{be^{c-(b^2-d)x^2}x}{\left(b^2-d\right)d^2\sqrt{\pi}} - \frac{3be^{c-(b^2-d)x^2}x}{4\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{be^{c-(b^2-d)x^2}x^3}{2\left(b^2-d\right)d\sqrt{\pi}} + \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{\sqrt{b^2-d}d^3} - \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{2\left(b^2-d\right)^{3/2}d^2}$$

Mathematica [A] time = 0.715748, size = 184, normalized size = 0.65

$$\frac{e^c\left(-\frac{bd\left(4b^2-7d\right)\text{Erf}\left(x\sqrt{b^2-d}\right)}{\left(b^2-d\right)^{5/2}} + \frac{8b\text{Erfi}\left(x\sqrt{d-b^2}\right)}{\sqrt{d-b^2}} - \frac{2bdxe^{x^2\left(d-b^2\right)}\left(2b^2\left(dx^2-2\right)+d\left(7-2dx^2\right)\right)}{\sqrt{\pi}\left(b^2-d\right)^2} - 4e^{dx^2}\left(d^2x^4-2dx^2+2\right)\text{Erf}(bx) + 4e^{dx^2}\left(d^2x\right.}{8d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c + d*x^2)*x^5*Erfc[b*x],x]
```

```
[Out] (E^c*(4*E^(d*x^2)*(2 - 2*d*x^2 + d^2*x^4) - (2*b*d*E^((-b^2 + d)*x^2)*x*(d*
(7 - 2*d*x^2) + 2*b^2*(-2 + d*x^2)))/((b^2 - d)^2*Sqrt[Pi]) - 4*E^(d*x^2)*(
2 - 2*d*x^2 + d^2*x^4)*Erf[b*x] - (b*(4*b^2 - 7*d)*d*Erf[Sqrt[b^2 - d]*x])/
(b^2 - d)^(5/2) + (8*b*Erfi[Sqrt[-b^2 + d]*x])/Sqrt[-b^2 + d]))/(8*d^3)
```

Maple [A] time = 0.205, size = 376, normalized size = 1.3

$$\frac{1}{b}\left(\frac{e^c}{b^5}\left(\frac{e^{dx^2}b^6x^4}{2d}-2\frac{b^2}{d}\left(1/2\frac{b^4x^2e^{dx^2}}{d}-1/2\frac{b^4e^{dx^2}}{d^2}\right)\right)-\frac{\mathrm{Erf}(bx)e^c}{b^5}\left(\frac{e^{dx^2}b^6x^4}{2d}-2\frac{b^2}{d}\left(1/2\frac{b^4x^2e^{dx^2}}{d}-1/2\frac{b^4e^{dx^2}}{d^2}\right)\right)+\frac{e^c}{\sqrt{\pi}b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^5*erfc(b*x),x)
```

```
[Out] (1/b^5*exp(c)*(1/2*exp(d*x^2)*b^6*x^4/d-2/d*b^2*(1/2/d*b^4*x^2*exp(d*x^2)-1
/2/d^2*b^4*exp(d*x^2)))-erf(b*x)/b^5*exp(c)*(1/2*exp(d*x^2)*b^6*x^4/d-2/d*b
^2*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2)))+1/Pi^(1/2)/b^5*exp(c)
*(1/d*b^2*(1/2/(-1+d/b^2)*b^3*x^3*exp((-1+d/b^2)*b^2*x^2)-3/2/(-1+d/b^2)*(1
/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)
^(1/2)*erf((1-d/b^2)^(1/2)*b*x)))+1/d^3*b^6*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1
-d/b^2)^(1/2)*b*x)-2/d^2*b^4*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/
4/(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\,\mathrm{erfc}(bx)\,e^{(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfc(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.19624, size = 734, normalized size = 2.59

$$\frac{\pi\left(8\,b^5-20\,b^3d+15\,bd^2\right)\sqrt{b^2-d}\,\mathrm{erf}\left(\sqrt{b^2-d}x\right)e^c-2\,\sqrt{\pi}\left(2\,\left(b^5d^2-2\,b^3d^3+bd^4\right)x^3-\left(4\,b^5d-11\,b^3d^2+7\,bd^3\right)x\right)e^{\left(-b^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*(8*b^5 - 20*b^3*d + 15*b*d^2)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e^
c - 2*sqrt(pi)*(2*(b^5*d^2 - 2*b^3*d^3 + b*d^4)*x^3 - (4*b^5*d - 11*b^3*d^2
 + 7*b*d^3)*x)*e^(-b^2*x^2 + d*x^2 + c) + 4*(pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^
2*d^4 - d^5)*x^4 - 2*pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 + 2*pi*(b
^6 - 3*b^4*d + 3*b^2*d^2 - d^3) - (pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^2*d^4 - d^
5)*x^4 - 2*pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 + 2*pi*(b^6 - 3*b^4
*d + 3*b^2*d^2 - d^3))*erf(b*x))*e^(d*x^2 + c))/(pi*(b^6*d^3 - 3*b^4*d^4 +
3*b^2*d^5 - d^6))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**5*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\,\mathrm{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfc(b*x)*e^(d*x^2 + c), x)
```

3.157 $\int e^{c+dx^2}x^3\text{Erfc}(bx)\,dx$

Optimal. Leaf size=155

$$-\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\sqrt{b^2-d}}+\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{4d\left(b^2-d\right)^{3/2}}-\frac{bxe^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{\text{Erfc}(bx)e^{c+dx^2}}{2d^2}+\frac{x^2\text{Erfc}(bx)e^{c+dx^2}}{2d}$$

```
[Out] -(b*E^(c - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (b*E^c*Erf[Sqrt[b^2
 - d]*x])/(2*Sqrt[b^2 - d]*d^2) + (b*E^c*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)
^(3/2)*d) - (E^(c + d*x^2)*Erfc[b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfc[b*x
])/(2*d)
```

Rubi [A] time = 0.156891, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6386, 6383, 2205, 2212}

$$-\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{2d^2\sqrt{b^2-d}}+\frac{be^c\text{Erf}\left(x\sqrt{b^2-d}\right)}{4d\left(b^2-d\right)^{3/2}}-\frac{bxe^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{\text{Erfc}(bx)e^{c+dx^2}}{2d^2}+\frac{x^2\text{Erfc}(bx)e^{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^3*Erfc[b*x],x]
```

```
[Out] -(b*E^(c - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (b*E^c*Erf[Sqrt[b^2
 - d]*x])/(2*Sqrt[b^2 - d]*d^2) + (b*E^c*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)
^(3/2)*d) - (E^(c + d*x^2)*Erfc[b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfc[b*x
])/(2*d)
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
```

```
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx = \frac{e^{c+dx^2}x^2\text{erfc}(bx)}{2d} - \frac{\int e^{c+dx^2}x\text{erfc}(bx)\,dx}{d} + \frac{b\int e^{c-\left(b^2-d\right)x^2}x^2\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{c-\left(b^2-d\right)x^2}x}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfc}(bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfc}(bx)}{2d} - \frac{b\int e^{c-\left(b^2-d\right)x^2}\,dx}{d^2\sqrt{\pi}} + \frac{b\int e^{c+\left(-b^2+d\right)x^2}\,dx}{2\left(b^2-d\right)d\sqrt{\pi}}$$

$$= -\frac{be^{c-\left(b^2-d\right)x^2}x}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d^2} + \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{4\left(b^2-d\right)^{3/2}d} - \frac{e^{c+dx^2}\text{erfc}(bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfc}(bx)}{2d}$$

Mathematica [A] time = 0.296925, size = 99, normalized size = 0.64

$$\frac{e^c\left(\frac{\left(2b^3-3bd\right)\text{Erfi}\left(x\sqrt{d-b^2}\right)}{\left(d-b^2\right)^{3/2}} + \frac{2bdxe^{x^2\left(d-b^2\right)}}{\sqrt{\pi}\left(d-b^2\right)} + 2e^{dx^2}\left(dx^2-1\right)\text{Erfc}(bx)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^3*Erfc[b*x],x]
```

```
[Out] (E^c*((2*b*d*E^((-b^2 + d)*x^2)*x)/((-b^2 + d)*Sqrt[Pi]) + 2*E^(d*x^2)*(-1
+ d*x^2)*Erfc[b*x] + ((2*b^3 - 3*b*d)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^(3
```

```
/2)))/(4*d^2)
```

Maple [A] time = 0.325, size = 206, normalized size = 1.3

$$\frac{1}{b}\left(\frac{e^c}{b^3}\left(\frac{b^4x^2e^{dx^2}}{2d}-\frac{b^4e^{dx^2}}{2d^2}\right)-\frac{\mathrm{Erf}(bx)\,e^c}{b^3}\left(\frac{b^4x^2e^{dx^2}}{2d}-\frac{b^4e^{dx^2}}{2d^2}\right)+\frac{e^c}{b^3\sqrt{\pi}}\left(\frac{b^2}{d}\left(\frac{bx}{2}e^{\left(-1+\frac{d}{b^2}\right)b^2x^2}\left(-1+\frac{d}{b^2}\right)^{-1}-\frac{\sqrt{\pi}}{4}\mathrm{Erf}\left(\sqrt{1-\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^3*erfc(b*x),x)
```

```
[Out] (1/b^3*exp(c)*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2))-erf(b*x)/b^
3*exp(c)*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2))+1/Pi^(1/2)/b^3*e
xp(c)*(1/d*b^2*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*P
i^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))-1/2/d^2*b^4*Pi^(1/2)/(1-d
/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\mathrm{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfc(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.16406, size = 416, normalized size = 2.68

$$-\frac{\pi\left(2\,b^3-3\,bd\right)\sqrt{b^2-d}\,\mathrm{erf}\left(\sqrt{b^2-d}x\right)e^c+2\,\sqrt{\pi}\left(b^3d-bd^2\right)xe^{\left(-b^2x^2+dx^2+c\right)}-2\left(\pi\left(b^4d-2\,b^2d^2+d^3\right)x^2-\pi\left(b^4-2\,b^2d+\right.\right.}{4\,\pi\left(b^4d^2-2\,b^2d^3+d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfc(b*x),x, algorithm="fricas")
```

```
[Out] -1/4*(pi*(2*b^3 - 3*b*d)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e^c + 2*sqrt(pi
)*(b^3*d - b*d^2)*x*e^(-b^2*x^2 + d*x^2 + c) - 2*(pi*(b^4*d - 2*b^2*d^2 + d
^3)*x^2 - pi*(b^4 - 2*b^2*d + d^2) - (pi*(b^4*d - 2*b^2*d^2 + d^3)*x^2 - pi
*(b^4 - 2*b^2*d + d^2))*erf(b*x))*e^(d*x^2 + c))/(pi*(b^4*d^2 - 2*b^2*d^3 +
 d^4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**3*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}(bx) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfc(b*x)*e^(d*x^2 + c), x)
```

3.158 $\int e^{c+dx^2} x \mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=57

$$\frac{be^c \text{Erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}+\frac{\text{Erfc}(bx)e^{c+dx^2}}{2d}$$

```
[Out] (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d) + (E^(c + d*x^2)*Erfc[b*x]
)/(2*d)
```

Rubi [A] time = 0.0392718, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6383, 2205}

$$\frac{be^c \text{Erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}+\frac{\text{Erfc}(bx)e^{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x*Erfc[b*x],x]
```


```
[Out] (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d) + (E^(c + d*x^2)*Erfc[b*x]
)/(2*d)
```


Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int e^{c+dx^2}x\text{erfc}(bx)\,dx=\frac{e^{c+dx^2}\text{erfc}(bx)}{2d}+\frac{b\int e^{c-(b^2-d)x^2}\,dx}{d\sqrt{\pi}}$$
$$=\frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d}+\frac{e^{c+dx^2}\text{erfc}(bx)}{2d}$$

Mathematica [A] time = 0.0385994, size = 50, normalized size = 0.88

$$\frac{e^c\left(\frac{b\text{Erfi}\left(x\sqrt{d-b^2}\right)}{\sqrt{d-b^2}}+e^{dx^2}\text{Erfc}(bx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x*Erfc[b*x],x]
```

```
[Out] (E^c*(E^(d*x^2)*Erfc[b*x] + (b*Erfi[Sqrt[-b^2 + d]*x])/Sqrt[-b^2 + d]))/(2*
d)
```

Maple [A] time = 0.32, size = 92, normalized size = 1.6

$$\frac{1}{b}\left(\frac{b}{2\,d}\text{e}^{\frac{b^2dx^2+b^2c}{b^2}}-\frac{\text{Erf}\,(bx)\,b}{2\,d}\text{e}^{\frac{b^2dx^2+b^2c}{b^2}}+\frac{b\text{e}^c}{2\,d}\text{Erf}\left(\sqrt{1-\frac{d}{b^2}}bx\right)\frac{1}{\sqrt{1-\frac{d}{b^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x*erfc(b*x),x)
```

```
[Out] (1/2*b*exp((b^2*d*x^2+b^2*c)/b^2)/d-1/2*erf(b*x)*b*exp((b^2*d*x^2+b^2*c)/b^
2)/d+1/2*b/d*exp(c)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*erfc(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.07589, size = 147, normalized size = 2.58

$$\frac{\sqrt{b^2-d}\,b\,\mathrm{erf}\left(\sqrt{b^2-d}\,x\right)e^c+\left(b^2-\left(b^2-d\right)\mathrm{erf}\left(bx\right)-d\right)e^{\left(dx^2+c\right)}}{2\left(b^2d-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2 - d)*b*erf(sqrt(b^2 - d)*x)*e^c + (b^2 - (b^2 - d)*erf(b*x) -
 d)*e^(d*x^2 + c))/(b^2*d - d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c\int xe^{dx^2}\,\mathrm{erfc}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x*erfc(b*x),x)
```

```
[Out] exp(c)*Integral(x*exp(d*x**2)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{erfc}\left(bx\right)e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*erfc(b*x)*e^(d*x^2 + c), x)
```

3.159 $\int \frac{e^{c+dx^2}\textbf{Erfc}(bx)}{x}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{\text{Erfc}(bx)e^{c+dx^2}}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + d*x^2)*Erfc[b*x])/x, x]
```

Rubi [A] time = 0.0373325, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + d*x^2)*Erfc[b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx$$

Mathematica [A] time = 0.487532, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x, x]
```

Maple [A] time = 0.13, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{d x^{2}+c} \operatorname{erfc}\left(b x\right)}{x}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x)/x,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(b x\right) e^{\left(d x^{2}+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\operatorname{erf}\left(b x\right) - 1)e^{\left(d x^{2}+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int \frac{e^{d x^{2}} \operatorname{erfc}\left(b x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x)/x,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(dx^2+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x, x)
```

3.160 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(bx)}{x^3}\,dx$

Optimal. Leaf size=98

$$d\text{Unintegrable}\left(\frac{\text{Erfc}(bx)e^{c+dx^2}}{x},x\right)+be^c\sqrt{b^2-d}\,\text{Erf}\left(x\sqrt{b^2-d}\right)+\frac{be^{c-x^2\left(b^2-d\right)}}{\sqrt{\pi}x}-\frac{\text{Erfc}(bx)e^{c+dx^2}}{2x^2}$$

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(Sqrt[Pi]*x) + b*Sqrt[b^2 - d]*E^c*Erf[Sqrt[b^2 -
 d]*x] - (E^(c + d*x^2)*Erfc[b*x])/(2*x^2) + d*Unintegrable[(E^(c + d*x^2)*
Erfc[b*x])/x, x]
```

Rubi [A] time = 0.147338, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[b*x])/x^3,x]
```

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(Sqrt[Pi]*x) + b*Sqrt[b^2 - d]*E^c*Erf[Sqrt[b^2 -
 d]*x] - (E^(c + d*x^2)*Erfc[b*x])/(2*x^2) + d*Defer[Int][(E^(c + d*x^2)*Er
fc[b*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx &= -\frac{e^{c+dx^2}\text{erfc}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx-\frac{b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^2}\,dx}{\sqrt{\pi}}\\
&= \frac{be^{c-\left(b^2-d\right)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\text{erfc}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx+\frac{\left(2b\left(b^2-d\right)\right)\int e^{c+\left(-b^2+d\right)x^2}\,dx}{\sqrt{\pi}}\\
&= \frac{be^{c-\left(b^2-d\right)x^2}}{\sqrt{\pi}x}+b\sqrt{b^2-d}e^c\text{erf}\left(\sqrt{b^2-d}x\right)-\frac{e^{c+dx^2}\text{erfc}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.617439, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^3, x]
```

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfc}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x)/x^3,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\text{erf}\,(bx)-1)e^{\left(dx^2+c\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int\frac{e^{dx^{2}}\operatorname{erfc}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x)/x**3,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erfc}\left(bx\right)e^{\left(dx^{2}+c\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^3, x)
```

3.161 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(bx)}{x^5}\,dx$

Optimal. Leaf size=230

$$\frac{1}{2}d^2\text{Unintegrable}\left(\frac{\text{Erfc}(bx)e^{c+dx^2}}{x},x\right)+\frac{1}{2}be^c d\sqrt{b^2-d}\,\text{Erf}\left(x\sqrt{b^2-d}\right)-\frac{1}{3}be^c\left(b^2-d\right)^{3/2}\text{Erf}\left(x\sqrt{b^2-d}\right)+\frac{bde^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}x}$$

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(6*Sqrt[Pi]*x^3) - (b*(b^2 - d)*E^(c - (b^2 - d)*
x^2))/(3*Sqrt[Pi]*x) + (b*d*E^(c - (b^2 - d)*x^2))/(2*Sqrt[Pi]*x) - (b*(b^2
 - d)^(3/2)*E^c*Erf[Sqrt[b^2 - d]*x])/3 + (b*Sqrt[b^2 - d]*d*E^c*Erf[Sqrt[b
^2 - d]*x])/2 - (E^(c + d*x^2)*Erfc[b*x])/(4*x^4) - (d*E^(c + d*x^2)*Erfc[b
*x])/(4*x^2) + (d^2*Unintegrable[(E^(c + d*x^2)*Erfc[b*x])/x, x])/2
```

Rubi [A] time = 0.325333, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[b*x])/x^5,x]
```

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(6*Sqrt[Pi]*x^3) - (b*(b^2 - d)*E^(c - (b^2 - d)*
x^2))/(3*Sqrt[Pi]*x) + (b*d*E^(c - (b^2 - d)*x^2))/(2*Sqrt[Pi]*x) - (b*(b^2
 - d)^(3/2)*E^c*Erf[Sqrt[b^2 - d]*x])/3 + (b*Sqrt[b^2 - d]*d*E^c*Erf[Sqrt[b
^2 - d]*x])/2 - (E^(c + d*x^2)*Erfc[b*x])/(4*x^4) - (d*E^(c + d*x^2)*Erfc[b
*x])/(4*x^2) + (d^2*Defer[Int][(E^(c + d*x^2)*Erfc[b*x])/x, x])/2
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^5}\,dx = -\frac{e^{c+dx^2}\text{erfc}(bx)}{4x^4} + \frac{1}{2}d\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx - \frac{b\int \frac{e^{c-(b^2-d)x^2}}{x^4}\,dx}{2\sqrt{\pi}}$$

$$= \frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} - \frac{e^{c+dx^2}\text{erfc}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erfc}(bx)}{4x^2} + \frac{1}{2}d^2\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx + \frac{\left(b\left(b^2-d\right)\right)\int \frac{e^{c+(-}}{}}{3\sqrt{\pi}}$$

$$= \frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} - \frac{b\left(b^2-d\right)e^{c-(b^2-d)x^2}}{3\sqrt{\pi}x} + \frac{bde^{c-(b^2-d)x^2}}{2\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfc}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erfc}(bx)}{4x^2} + \frac{1}{2}d^2\int$$

$$= \frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} - \frac{b\left(b^2-d\right)e^{c-(b^2-d)x^2}}{3\sqrt{\pi}x} + \frac{bde^{c-(b^2-d)x^2}}{2\sqrt{\pi}x} - \frac{1}{3}b\left(b^2-d\right)^{3/2}e^c\text{erf}\left(\sqrt{b^2-d}x\right) + \frac{1}{2}b\sqrt{b^2-}$$

Mathematica [A] time = 0.760107, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^5,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^5, x]
```

Maple [A] time = 0.327, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfc}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x)/x^5,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\text{erf}(bx)-1)e^{(dx^2+c)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^5, x)
```

3.162 $\int e^{c+dx^2} x^4 \textbf{Erfc}(bx)\, dx$

Optimal. Leaf size=185

$$\frac{3\text{Unintegrable}\left(\text{Erfc}(bx)e^{c+dx^2},x\right)}{4d^2}+\frac{3be^{c-x^2(b^2-d)}}{4\sqrt{\pi}d^2\left(b^2-d\right)}-\frac{bx^2e^{c-x^2(b^2-d)}}{2\sqrt{\pi}d\left(b^2-d\right)}-\frac{be^{c-x^2(b^2-d)}}{2\sqrt{\pi}d\left(b^2-d\right)^2}-\frac{3x\text{Erfc}(bx)e^{c+dx^2}}{4d^2}+\frac{x^3\text{Erf}}{}$$

```
[Out] (3*b*E^(c - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) - (b*E^(c - (b^2 - d
)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) - (b*E^(c - (b^2 - d)*x^2)*x^2)/(2*(b^2
- d)*d*Sqrt[Pi]) - (3*E^(c + d*x^2)*x*Erfc[b*x])/(4*d^2) + (E^(c + d*x^2)*x
^3*Erfc[b*x])/(2*d) + (3*Unintegrable[E^(c + d*x^2)*Erfc[b*x], x])/(4*d^2)
```

Rubi [A] time = 0.233365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2} x^4 \text{Erfc}(bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^4*Erfc[b*x],x]
```

```
[Out] (3*b*E^(c - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) - (b*E^(c - (b^2 - d
)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) - (b*E^(c - (b^2 - d)*x^2)*x^2)/(2*(b^2
- d)*d*Sqrt[Pi]) - (3*E^(c + d*x^2)*x*Erfc[b*x])/(4*d^2) + (E^(c + d*x^2)*x
^3*Erfc[b*x])/(2*d) + (3*Defer[Int][E^(c + d*x^2)*Erfc[b*x], x])/(4*d^2)
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2} x^4 \text{erfc}(bx)\, dx &= \frac{e^{c+dx^2}x^3\text{erfc}(bx)}{2d}-\frac{3\int e^{c+dx^2}x^2\text{erfc}(bx)\, dx}{2d}+\frac{b\int e^{c-\left(b^2-d\right)x^2}x^3\, dx}{d\sqrt{\pi}}\\ &= -\frac{be^{c-\left(b^2-d\right)x^2}x^2}{2\left(b^2-d\right)d\sqrt{\pi}}-\frac{3e^{c+dx^2}x\text{erfc}(bx)}{4d^2}+\frac{e^{c+dx^2}x^3\text{erfc}(bx)}{2d}+\frac{3\int e^{c+dx^2}\text{erfc}(bx)\, dx}{4d^2}-\frac{(3b)\int e^{c-(b}}{2d^2}\\ &= \frac{3be^{c-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}}-\frac{be^{c-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}}-\frac{be^{c-\left(b^2-d\right)x^2}x^2}{2\left(b^2-d\right)d\sqrt{\pi}}-\frac{3e^{c+dx^2}x\text{erfc}(bx)}{4d^2}+\frac{e^{c+dx^2}x^3\text{erfc}(bx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.757467, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^4\text{Erfc}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^4*Erfc[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^4*Erfc[b*x], x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^4\text{erfc}(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^4*erfc(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^4*erfc(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\text{erfc}(bx)\,e^{(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfc(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^4\,\text{erf}(bx) - x^4\right)e^{(dx^2+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-(x^4*erf(b*x) - x^4)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**4*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}(bx) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfc(b*x)*e^(d*x^2 + c), x)
```

3.163 $\int e^{c+dx^2} x^2 \textbf{Erfc}(bx)\, dx$

Optimal. Leaf size=83

$$-\frac{\text{Unintegrable}\left(\text{Erfc}(bx)e^{c+dx^2},x\right)}{2d}-\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{x\text{Erfc}(bx)e^{c+dx^2}}{2d}$$

```
[Out] -(b*E^(c - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) + (E^(c + d*x^2)*x*Erfc
[b*x])/(2*d) - Unintegrable[E^(c + d*x^2)*Erfc[b*x], x]/(2*d)
```

Rubi [A] time = 0.0936751, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int e^{c+dx^2} x^2 \text{Erfc}(bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^2*Erfc[b*x],x]
```

```
[Out] -(b*E^(c - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) + (E^(c + d*x^2)*x*Erfc
[b*x])/(2*d) - Defer[Int][E^(c + d*x^2)*Erfc[b*x], x]/(2*d)
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2} x^2 \text{erfc}(bx)\, dx &= \frac{e^{c+dx^2} x\text{erfc}(bx)}{2d}-\frac{\int e^{c+dx^2}\text{erfc}(bx)\, dx}{2d}+\frac{b\int e^{c-\left(b^2-d\right)x^2} x\, dx}{d\sqrt{\pi}}\\ &= -\frac{be^{c-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)d\sqrt{\pi}}+\frac{e^{c+dx^2} x\text{erfc}(bx)}{2d}-\frac{\int e^{c+dx^2}\text{erfc}(bx)\, dx}{2d}\end{aligned}$$

Mathematica [A] time = 0.589352, size = 0, normalized size = 0.

$$\int e^{c+dx^2} x^2 \text{Erfc}(bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^2*Erfc[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^2*Erfc[b*x], x]
```

Maple [A] time = 0.241, size = 0, normalized size = 0.

$$\int e^{d x^{2}+c} x^{2} \operatorname{erfc}\left(b x\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^2*erfc(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^2*erfc(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{2} \operatorname{erfc}\left(b x\right) e^{\left(d x^{2}+c\right)}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfc(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^{2} \operatorname{erf}\left(b x\right) - x^{2}\right) e^{\left(d x^{2}+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-(x^2*erf(b*x) - x^2)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int x^{2}e^{dx^{2}}\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**2*erfc(b*x),x)
```

```
[Out] exp(c)*Integral(x**2*exp(d*x**2)*erfc(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\operatorname{erfc}(bx)\,e^{\left(dx^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfc(b*x)*e^(d*x^2 + c), x)
```

3.164 $\int e^{c+dx^2}\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\text{Erfc}(bx)e^{c+dx^2},x\right)$$

```
[Out] Unintegrable[E^(c + d*x^2)*Erfc[b*x], x]
```

Rubi [A] time = 0.0151178, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}\text{Erfc}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*Erfc[b*x],x]

[Out] Defer[Int][E^(c + d*x^2)*Erfc[b*x], x]
```

Rubi steps

$$\int e^{c+dx^2}\text{erfc}(bx)\,dx=\int e^{c+dx^2}\text{erfc}(bx)\,dx$$

Mathematica [A] time = 0.0312052, size = 0, normalized size = 0.

$$\int e^{c+dx^2}\text{Erfc}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*Erfc[b*x],x]

[Out] Integrate[E^(c + d*x^2)*Erfc[b*x], x]
```

Maple [A] time = 0.116, size = 0, normalized size = 0.

$$\int \mathrm{e}^{d x^{2}+c} \operatorname{erfc}(b x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(b x)\, e^{\left(d x^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(\operatorname{erf}(b x)-1) e^{\left(d x^{2}+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int e^{d x^{2}} \operatorname{erfc}(b x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x),x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c), x)
```

3.165 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(bx)}{x^2}\,dx$

Optimal. Leaf size=62

$$2d\text{Unintegrable}\left(\text{Erfc}(bx)e^{c+dx^2},x\right)-\frac{be^c\text{ExpIntegralEi}\left(x^2\left(-\left(b^2-d\right)\right)\right)}{\sqrt{\pi}}-\frac{\text{Erfc}(bx)e^{c+dx^2}}{x}$$

```
[Out] -((E^(c + d*x^2)*Erfc[b*x])/x) - (b*E^c*ExpIntegralEi[-((b^2 - d)*x^2)])/Sq
rt[Pi] + 2*d*Unintegrable[E^(c + d*x^2)*Erfc[b*x], x]
```

Rubi [A] time = 0.114185, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[b*x])/x^2,x]
```

```
[Out] -((E^(c + d*x^2)*Erfc[b*x])/x) - (b*E^c*ExpIntegralEi[-((b^2 - d)*x^2)])/Sq
rt[Pi] + 2*d*Defer[Int][E^(c + d*x^2)*Erfc[b*x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx &= -\frac{e^{c+dx^2}\text{erfc}(bx)}{x}+(2d)\int e^{c+dx^2}\text{erfc}(bx)\,dx-\frac{(2b)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+dx^2}\text{erfc}(bx)}{x}-\frac{be^c\text{Ei}\left(-\left(b^2-d\right)x^2\right)}{\sqrt{\pi}}+(2d)\int e^{c+dx^2}\text{erfc}(bx)\,dx\end{aligned}$$

Mathematica [A] time = 0.632476, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^2,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^2, x]
```

Maple [A] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\operatorname{erfc}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x)/x^2,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(dx^2+c\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(\operatorname{erf}(bx)-1)e^{\left(dx^2+c\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int \frac{e^{dx^{2}}\operatorname{erfc}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(bx\right)e^{\left(dx^{2}+c\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^2, x)
```

3.166 $\int \frac{e^{c+dx^2}\textbf{Erfc}(bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$\frac{4}{3}d^2\text{Unintegrable}\left(\text{Erfc}(bx)e^{c+dx^2},x\right)-\frac{2be^cd\text{ExpIntegralEi}\left(x^2\left(-\left(b^2-d\right)\right)\right)}{3\sqrt{\pi}}+\frac{be^c\left(b^2-d\right)\text{ExpIntegralEi}\left(x^2\left(-\left(b\right.\right.\right.}{3\sqrt{\pi}}$$

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + d*x^2)*Erfc[b*x])/(3*x
^3) - (2*d*E^(c + d*x^2)*Erfc[b*x])/(3*x) + (b*(b^2 - d)*E^c*ExpIntegralEi[
-((b^2 - d)*x^2)])/(3*Sqrt[Pi]) - (2*b*d*E^c*ExpIntegralEi[-((b^2 - d)*x^2)
])/(3*Sqrt[Pi]) + (4*d^2*Unintegrable[E^(c + d*x^2)*Erfc[b*x], x])/3
```

Rubi [A] time = 0.268319, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[b*x])/x^4,x]
```

```
[Out] (b*E^(c - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + d*x^2)*Erfc[b*x])/(3*x
^3) - (2*d*E^(c + d*x^2)*Erfc[b*x])/(3*x) + (b*(b^2 - d)*E^c*ExpIntegralEi[
-((b^2 - d)*x^2)])/(3*Sqrt[Pi]) - (2*b*d*E^c*ExpIntegralEi[-((b^2 - d)*x^2)
])/(3*Sqrt[Pi]) + (4*d^2*Defer[Int][E^(c + d*x^2)*Erfc[b*x], x])/3
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^4}\,dx=-\frac{e^{c+dx^2}\text{erfc}(bx)}{3x^3}+\frac{1}{3}(2d)\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx-\frac{(2b)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^3}\,dx}{3\sqrt{\pi}}$$

$$=\frac{be^{c-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\text{erfc}(bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erfc}(bx)}{3x}+\frac{1}{3}\left(4d^2\right)\int e^{c+dx^2}\text{erfc}(bx)\,dx+\frac{\left(2b\left(b^2-d\right)\right)\ldots}{3\ldots}$$

$$=\frac{be^{c-\left(b^2-d\right)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\text{erfc}(bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erfc}(bx)}{3x}+\frac{b\left(b^2-d\right)e^c\text{Ei}\left(-\left(b^2-d\right)x^2\right)}{3\sqrt{\pi}}-\frac{2bde^c\text{Ei}\left(-\left(\ldots\right.\right.}{3\ldots}$$

Mathematica [A] time = 0.803698, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\mathrm{Erfc}(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^4,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^4, x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\mathrm{erfc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x)/x^4,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)\,e^{\left(dx^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\mathrm{erf}\,(bx)-1)e^{\left(dx^2+c\right)}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int\frac{e^{dx^{2}}\operatorname{erfc}\left(bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x)/x**4,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erfc}\left(bx\right)e^{\left(dx^{2}+c\right)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(d*x^2 + c)/x^4, x)
```

3.167 $\int e^{c+b^2x^2} x^5 \textbf{Erfc}(bx)\, dx$

Optimal. Leaf size=118

$$\frac{x^4 e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2} - \frac{x^2 e^{b^2x^2+c}\text{Erfc}(bx)}{b^4} + \frac{e^{b^2x^2+c}\text{Erfc}(bx)}{b^6} - \frac{2e^c x^3}{3\sqrt{\pi} b^3} + \frac{2e^c x}{\sqrt{\pi} b^5} + \frac{e^c x^5}{5\sqrt{\pi} b}$$

```
[Out] (2*E^c*x)/(b^5*Sqrt[Pi]) - (2*E^c*x^3)/(3*b^3*Sqrt[Pi]) + (E^c*x^5)/(5*b*Sq
rt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/b^6 - (E^(c + b^2*x^2)*x^2*Erfc[b*x])
/b^4 + (E^(c + b^2*x^2)*x^4*Erfc[b*x])/(2*b^2)
```

Rubi [A] time = 0.146585, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6386, 6383, 8, 12, 30}

$$\frac{x^4 e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2} - \frac{x^2 e^{b^2x^2+c}\text{Erfc}(bx)}{b^4} + \frac{e^{b^2x^2+c}\text{Erfc}(bx)}{b^6} - \frac{2e^c x^3}{3\sqrt{\pi} b^3} + \frac{2e^c x}{\sqrt{\pi} b^5} + \frac{e^c x^5}{5\sqrt{\pi} b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^5*Erfc[b*x],x]
```

```
[Out] (2*E^c*x)/(b^5*Sqrt[Pi]) - (2*E^c*x^3)/(3*b^3*Sqrt[Pi]) + (E^c*x^5)/(5*b*Sq
rt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/b^6 - (E^(c + b^2*x^2)*x^2*Erfc[b*x])
/b^4 + (E^(c + b^2*x^2)*x^4*Erfc[b*x])/(2*b^2)
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx &= \frac{e^{c+b^2x^2}x^4\text{erfc}(bx)}{2b^2}-\frac{2\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx}{b^2}+\frac{\int e^cx^4\,dx}{b\sqrt{\pi}}\\
&=-\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfc}(bx)}{2b^2}+\frac{2\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx}{b^4}-\frac{2\int e^cx^2\,dx}{b^3\sqrt{\pi}}+\frac{e^c\int x^4\,dx}{b\sqrt{\pi}}\\
&=\frac{e^cx^5}{5b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfc}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfc}(bx)}{2b^2}+\frac{2\int e^c\,dx}{b^5\sqrt{\pi}}-\frac{(2e^c)\int x^2\,dx}{b^3\sqrt{\pi}}\\
&=\frac{2e^cx}{b^5\sqrt{\pi}}-\frac{2e^cx^3}{3b^3\sqrt{\pi}}+\frac{e^cx^5}{5b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfc}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfc}(bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0470334, size = 73, normalized size = 0.62

$$\frac{e^c\left(15\sqrt{\pi}e^{b^2x^2}\left(b^4x^4-2b^2x^2+2\right)\text{Erfc}(bx)+6b^5x^5-20b^3x^3+60bx\right)}{30\sqrt{\pi}b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^5*Erfc[b*x],x]
```

```
[Out] (E^c*(60*b*x - 20*b^3*x^3 + 6*b^5*x^5 + 15*E^(b^2*x^2)*Sqrt[Pi]*(2 - 2*b^2*
x^2 + b^4*x^4)*Erfc[b*x]))/(30*b^6*Sqrt[Pi])
```

Maple [A] time = 0.116, size = 135, normalized size = 1.1

$$\frac{1}{b}\left(\frac{e^c}{b^5}\left(\frac{e^{b^2x^2}b^4x^4}{2}-b^2x^2e^{b^2x^2}+e^{b^2x^2}\right)-\frac{\text{Erf}\,(bx)\,e^c}{b^5}\left(\frac{e^{b^2x^2}b^4x^4}{2}-b^2x^2e^{b^2x^2}+e^{b^2x^2}\right)+\frac{e^c}{\sqrt{\pi}b^5}\left(\frac{b^5x^5}{5}-\frac{2\,x^3b^3}{3}+2\,bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^5*erfc(b*x),x)
```

```
[Out] (1/b^5*exp(c)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2*exp(b^2*x^2)+exp(b^2*x^2))-
erf(b*x)/b^5*exp(c)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2*exp(b^2*x^2)+exp(b^2*
x^2))+1/Pi^(1/2)/b^5*exp(c)*(1/5*b^5*x^5-2/3*x^3*b^3+2*b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfc}(bx) e^{\left(b^2x^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.18223, size = 227, normalized size = 1.92

$$\frac{2\sqrt{\pi}\left(3\,b^5x^5 - 10\,b^3x^3 + 30\,bx\right)e^c + 15\left(2\,\pi + \pi b^4x^4 - 2\,\pi b^2x^2 - \left(2\,\pi + \pi b^4x^4 - 2\,\pi b^2x^2\right)\operatorname{erf}(bx)\right)e^{\left(b^2x^2+c\right)}}{30\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/30*(2*sqrt(pi)*(3*b^5*x^5 - 10*b^3*x^3 + 30*b*x)*e^c + 15*(2*pi + pi*b^4*
x^4 - 2*pi*b^2*x^2 - (2*pi + pi*b^4*x^4 - 2*pi*b^2*x^2)*erf(b*x))*e^(b^2*x^
2 + c))/(pi*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**5*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfc}(bx) e^{(b^2x^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfc(b*x)*e^(b^2*x^2 + c), x)
```

3.168 $\int e^{c+b^2x^2}x^3\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=80

$$\frac{x^2e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2b^4}-\frac{e^cx}{\sqrt{\pi}b^3}+\frac{e^cx^3}{3\sqrt{\pi}b}$$

```
[Out] -((E^c*x)/(b^3*Sqrt[Pi])) + (E^c*x^3)/(3*b*Sqrt[Pi]) - (E^(c + b^2*x^2)*Erf
c[b*x])/(2*b^4) + (E^(c + b^2*x^2)*x^2*Erfc[b*x])/(2*b^2)
```

Rubi [A] time = 0.0892668, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6386, 6383, 8, 12, 30}

$$\frac{x^2e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2b^4}-\frac{e^cx}{\sqrt{\pi}b^3}+\frac{e^cx^3}{3\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^3*Erfc[b*x],x]
```


```
[Out] -((E^c*x)/(b^3*Sqrt[Pi])) + (E^c*x^3)/(3*b*Sqrt[Pi]) - (E^(c + b^2*x^2)*Erf
c[b*x])/(2*b^4) + (E^(c + b^2*x^2)*x^2*Erfc[b*x])/(2*b^2)
```


Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx &= \frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx}{b^2}+\frac{\int e^cx^2\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{2b^2}-\frac{\int e^c\,dx}{b^3\sqrt{\pi}}+\frac{e^c\int x^2\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^cx}{b^3\sqrt{\pi}}+\frac{e^cx^3}{3b\sqrt{\pi}}-\frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0345227, size = 58, normalized size = 0.72

$$\frac{e^c\left(3\sqrt{\pi}e^{b^2x^2}\left(b^2x^2-1\right)\text{Erfc}(bx)+2bx\left(b^2x^2-3\right)\right)}{6\sqrt{\pi}b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^3*Erfc[b*x],x]
```

```
[Out] (E^c*(2*b*x*(-3 + b^2*x^2) + 3*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erfc[b*x
]))/(6*b^4*Sqrt[Pi])
```

Maple [A] time = 0.209, size = 99, normalized size = 1.2

$$\frac{1}{b}\left(\frac{e^c}{b^3}\left(\frac{b^2x^2e^{b^2x^2}}{2}-\frac{e^{b^2x^2}}{2}\right)-\frac{\text{Erf}\,(bx)\,e^c}{b^3}\left(\frac{b^2x^2e^{b^2x^2}}{2}-\frac{e^{b^2x^2}}{2}\right)+\frac{e^c}{b^3\sqrt{\pi}}\left(\frac{x^3b^3}{3}-bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^3*erfc(b*x),x)
```

```
[Out] (1/b^3*exp(c)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp(b^2*x^2))-erf(b*x)/b^3*exp(
c)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp(b^2*x^2))+1/Pi^(1/2)/b^3*exp(c)*(1/3*x
^3*b^3-b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}(bx)\, e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.14394, size = 157, normalized size = 1.96

$$\frac{2\sqrt{\pi}\left(b^3x^3 - 3\,bx\right)e^c - 3\left(\pi - \pi b^2x^2 - \left(\pi - \pi b^2x^2\right)\operatorname{erf}(bx)\right)e^{\left(b^2x^2+c\right)}}{6\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(pi)*(b^3*x^3 - 3*b*x)*e^c - 3*(pi - pi*b^2*x^2 - (pi - pi*b^2*x
^2)*erf(b*x))*e^(b^2*x^2 + c))/(pi*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**3*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}(bx) e^{(b^2x^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfc(b*x)*e^(b^2*x^2 + c), x)
```

3.169 $\int e^{c+b^2x^2} x\textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=36

$$\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}+\frac{e^cx}{\sqrt{\pi}b}$$

```
[Out] (E^c*x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2)
```

Rubi [A] time = 0.0347616, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6383, 8}

$$\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}+\frac{e^cx}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x*Erfc[b*x],x]
```

```
[Out] (E^c*x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2)
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx &= \frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{\int e^c\,dx}{b\sqrt{\pi}}\\ &= \frac{e^cx}{b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0199603, size = 36, normalized size = 1.

$$\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}+\frac{e^cx}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x*Erfc[b*x],x]
```

```
[Out] (E^c*x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2)
```

Maple [A] time = 0.138, size = 51, normalized size = 1.4

$$\frac{2\,e^{b^2x^2+c}e^{-b^2x^2}xb+e^{b^2x^2+c}\text{erfc}\,(bx)\,\sqrt{\pi}}{2\,b^2\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x*erfc(b*x),x)
```

```
[Out] 1/2*(2*exp(b^2*x^2+c)*exp(-b^2*x^2)*x*b+exp(b^2*x^2+c)*erfc(b*x)*Pi^(1/2))/
Pi^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.10251, size = 97, normalized size = 2.69

$$\frac{2\,\sqrt{\pi}bxe^c+(\pi-\pi\,\text{erf}\,(bx))e^{\left(b^2x^2+c\right)}}{2\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erfc(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(pi)*b*x*e^c + (pi - pi*erf(b*x))*e^(b^2*x^2 + c))/(pi*b^2)
```

Sympy [A] time = 22.4207, size = 41, normalized size = 1.14

$$\begin{cases} \frac{xe^c}{\sqrt{\pi}b} + \frac{e^c e^{b^2x^2} \operatorname{erfc}(bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2e^c}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x*erfc(b*x),x)
```

```
[Out] Piecewise((x*exp(c)/(sqrt(pi)*b) + exp(c)*exp(b**2*x**2)*erfc(b*x)/(2*b**2)
, Ne(b, 0)), (x**2*exp(c)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfc}(bx) e^{(b^2x^2+c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*erfc(b*x)*e^(b^2*x^2 + c), x)
```

3.170 $\int \frac{e^{c+b^2x^2}\mathbf{Erfc}(bx)}{x}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}e^c\text{ExpIntegralEi}\left(b^2x^2\right)-\frac{2be^cx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (E^c*ExpIntegralEi[b^2*x^2])/2 - (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/
2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.120363, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6389, 2210, 6388}

$$\frac{1}{2}e^c\text{Ei}\left(b^2x^2\right)-\frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfc[b*x])/x,x]
```

```
[Out] (E^c*ExpIntegralEi[b^2*x^2])/2 - (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/
2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6389

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)])/(x_), x_Symbol] :> Int[E^(c
 + d*x^2)/x, x] - Int[(E^(c + d*x^2)*Erf[b*x])/x, x] /; FreeQ[{b, c, d}, x]
 && EqQ[d, b^2]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 6388

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*b
*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; Fr
```

```
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x}\,dx = \int \frac{e^{c+b^2x^2}}{x}\,dx - \int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x}\,dx$$
$$= \frac{1}{2}e^c\text{Ei}\left(b^2x^2\right) - \frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.133736, size = 45, normalized size = 0.94

$$\frac{1}{2}e^c\left(\text{ExpIntegralEi}\left(b^2x^2\right) - \frac{4bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x,x]
```

```
[Out] (E^c*(ExpIntegralEi[b^2*x^2] - (4*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2
}, b^2*x^2])/Sqrt[Pi]))/2
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\text{erfc}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfc(b*x)/x,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfc(b*x)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\left(bx\right)e^{\left(b^2x^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\text{erf}\,(bx)-1)e^{\left(b^2x^2+c\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfc(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x, x)
```

3.171 $\int \frac{e^{c+b^2x^2}\mathbf{Erfc}(bx)}{x^3}\,dx$

Optimal. Leaf size=88

$$-\frac{2b^3e^cx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2x^2}+\frac{1}{2}b^2e^c\text{ExpIntegralEi}\left(b^2x^2\right)+\frac{be^c}{\sqrt{\pi}x}$$

```
[Out] (b*E^c)/(Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erfc[b*x])/(2*x^2) + (b^2*E^c*ExpIn
tegralEi[b^2*x^2])/2 - (2*b^3*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2},
 b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.165603, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6392, 6389, 2210, 6388, 12, 30}

$$-\frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{2x^2}+\frac{1}{2}b^2e^c\text{Ei}\left(b^2x^2\right)+\frac{be^c}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^3,x]
```

```
[Out] (b*E^c)/(Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erfc[b*x])/(2*x^2) + (b^2*E^c*ExpIn
tegralEi[b^2*x^2])/2 - (2*b^3*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2},
 b^2*x^2])/Sqrt[Pi]
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6389

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)])/(x_), x_Symbol] :> Int[E^(c
 + d*x^2)/x, x] - Int[(E^(c + d*x^2)*Erf[b*x])/x, x] /; FreeQ[{b, c, d}, x]
 && EqQ[d, b^2]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 6388

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*b
*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{2x^2}+b^2\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx-\frac{b\int \frac{e^c}{x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{2x^2}+b^2\int \frac{e^{c+b^2x^2}}{x}\,dx-b^2\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx-\frac{(be^c)\int \frac{1}{x^2}\,dx}{\sqrt{\pi}}\\ &= \frac{be^c}{\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{2x^2}+\frac{1}{2}b^2e^c\operatorname{Ei}\left(b^2x^2\right)-\frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.207471, size = 65, normalized size = 0.74

$$-\frac{e^c\left(-\frac{4bx\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},1\right\},\left\{\frac{1}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}-b^2x^2\text{ExpIntegralEi}\left(b^2x^2\right)+e^{b^2x^2}\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^3,x]
```

```
[Out] -(E^c*(E^(b^2*x^2) - b^2*x^2*ExpIntegralEi[b^2*x^2] - (4*b*x*Hypergeometric
PFQ[{-1/2, 1}, {1/2, 3/2}, b^2*x^2])/Sqrt[Pi]))/(2*x^2)
```

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\mathrm{erfc}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfc(b*x)/x^3,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfc(b*x)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\mathrm{erf}\,(bx)-1)e^{\left(b^2x^2+c\right)}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfc(b*x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(b^2x^2+c\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

3.172 $\int \frac{e^{c+b^2x^2}\textbf{Erfc}(bx)}{x^5}\,dx$

Optimal. Leaf size=134

$$-\frac{b^5e^cx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{b^2e^{b^2x^2+c}\text{Erfc}(bx)}{4x^2}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{4x^4}+\frac{1}{4}b^4e^c\text{ExpIntegralEi}\left(b^2x\right.$$

```
[Out] (b*E^c)/(6*Sqrt[Pi]*x^3) + (b^3*E^c)/(2*Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erfc
[b*x])/(4*x^4) - (b^2*E^(c + b^2*x^2)*Erfc[b*x])/(4*x^2) + (b^4*E^c*ExpInte
gralEi[b^2*x^2])/4 - (b^5*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2
*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.201698, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6392, 6389, 2210, 6388, 12, 30}

$$-\frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}-\frac{b^2e^{b^2x^2+c}\text{Erfc}(bx)}{4x^2}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{4x^4}+\frac{1}{4}b^4e^c\text{Ei}\left(b^2x^2\right)+\frac{b^3e^c}{2\sqrt{\pi}x}+\frac{be^c}{6\sqrt{\pi}x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^5,x]
```

```
[Out] (b*E^c)/(6*Sqrt[Pi]*x^3) + (b^3*E^c)/(2*Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erfc
[b*x])/(4*x^4) - (b^2*E^(c + b^2*x^2)*Erfc[b*x])/(4*x^2) + (b^4*E^c*ExpInte
gralEi[b^2*x^2])/4 - (b^5*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2
*x^2])/Sqrt[Pi]
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6389

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)])/(x_), x_Symbol] :> Int[E^(c
 + d*x^2)/x, x] - Int[(E^(c + d*x^2)*Erf[b*x])/x, x] /; FreeQ[{b, c, d}, x]
```

```
&& EqQ[d, b^2]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 6388

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*b
*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^4}+\frac{1}{2}b^2\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx-\frac{b\int \frac{e^c}{x^4}\,dx}{2\sqrt{\pi}}\\
&= -\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^2}+\frac{1}{2}b^4\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx-\frac{b^3\int \frac{e^c}{x^2}\,dx}{2\sqrt{\pi}}-\frac{(be^c)\int \frac{1}{x^4}\,dx}{2\sqrt{\pi}}\\
&= \frac{be^c}{6\sqrt{\pi}x^3}-\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^2}+\frac{1}{2}b^4\int \frac{e^{c+b^2x^2}}{x}\,dx-\frac{1}{2}b^4\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx-\\
&= \frac{be^c}{6\sqrt{\pi}x^3}+\frac{b^3e^c}{2\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\operatorname{erfc}(bx)}{4x^2}+\frac{1}{4}b^4e^c\operatorname{Ei}\left(b^2x^2\right)-\frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};\right.}{\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.223616, size = 83, normalized size = 0.62

$$-\frac{e^c\left(3\sqrt{\pi}\left(e^{b^2x^2}\left(b^2x^2+1\right)-b^4x^4\text{ExpIntegralEi}\left(b^2x^2\right)\right)-8bx\text{HypergeometricPFQ}\left(\left\{-\frac{3}{2},1\right\},\left\{-\frac{1}{2},\frac{3}{2}\right\},b^2x^2\right)\right)}{12\sqrt{\pi}x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^5,x]
```

```
[Out] -(E^c*(3*Sqrt[Pi]*(E^(b^2*x^2)*(1 + b^2*x^2) - b^4*x^4*ExpIntegralEi[b^2*x^
2]) - 8*b*x*HypergeometricPFQ[{-3/2, 1}, {-1/2, 3/2}, b^2*x^2]))/(12*Sqrt[P
i]*x^4)
```

Maple [F] time = 0.311, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\operatorname{erfc}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfc(b*x)/x^5,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfc(b*x)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\,e^{\left(b^2x^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(\operatorname{erf}(bx)-1)e^{\left(b^2x^2+c\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfc(b*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(b^2x^2+c\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

3.173 $\int e^{c+b^2x^2}x^4\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=138

$$-\frac{3e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{4\sqrt{\pi}b^3}+\frac{x^3e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}-\frac{3xe^{b^2x^2+c}\text{Erfc}(bx)}{4b^4}+\frac{3\sqrt{\pi}e^c\text{Erfi}(bx)}{8b^5}-\frac{3e^cx^2}{4\sqrt{\pi}b^3}$$

```
[Out] (-3*E^c*x^2)/(4*b^3*Sqrt[Pi]) + (E^c*x^4)/(4*b*Sqrt[Pi]) - (3*E^(c + b^2*x^
2)*x*Erfc[b*x])/(4*b^4) + (E^(c + b^2*x^2)*x^3*Erfc[b*x])/(2*b^2) + (3*E^c*
Sqrt[Pi]*Erfi[b*x])/(8*b^5) - (3*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, b^2*x^2])/(4*b^3*Sqrt[Pi])
```

Rubi [A] time = 0.138433, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6386, 6377, 2204, 6376, 12, 30}

$$-\frac{3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{4\sqrt{\pi}b^3}+\frac{x^3e^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}-\frac{3xe^{b^2x^2+c}\text{Erfc}(bx)}{4b^4}+\frac{3\sqrt{\pi}e^c\text{Erfi}(bx)}{8b^5}-\frac{3e^cx^2}{4\sqrt{\pi}b^3}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^4*Erfc[b*x],x]
```

```
[Out] (-3*E^c*x^2)/(4*b^3*Sqrt[Pi]) + (E^c*x^4)/(4*b*Sqrt[Pi]) - (3*E^(c + b^2*x^
2)*x*Erfc[b*x])/(4*b^4) + (E^(c + b^2*x^2)*x^3*Erfc[b*x])/(2*b^2) + (3*E^c*
Sqrt[Pi]*Erfi[b*x])/(8*b^5) - (3*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, b^2*x^2])/(4*b^3*Sqrt[Pi])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
```

```
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int e^{c+b^2x^2}x^4\text{erfc}(bx)\,dx = \frac{e^{c+b^2x^2}x^3\text{erfc}(bx)}{2b^2} - \frac{3\int e^{c+b^2x^2}x^2\text{erfc}(bx)\,dx}{2b^2} + \frac{\int e^c x^3\,dx}{b\sqrt{\pi}}$$

$$= -\frac{3e^{c+b^2x^2}x\text{erfc}(bx)}{4b^4} + \frac{e^{c+b^2x^2}x^3\text{erfc}(bx)}{2b^2} + \frac{3\int e^{c+b^2x^2}\text{erfc}(bx)\,dx}{4b^4} - \frac{3\int e^c x\,dx}{2b^3\sqrt{\pi}} + \frac{e^c\int x^3\,dx}{b\sqrt{\pi}}$$

$$= \frac{e^c x^4}{4b\sqrt{\pi}} - \frac{3e^{c+b^2x^2}x\text{erfc}(bx)}{4b^4} + \frac{e^{c+b^2x^2}x^3\text{erfc}(bx)}{2b^2} + \frac{3\int e^{c+b^2x^2}\,dx}{4b^4} - \frac{3\int e^{c+b^2x^2}\text{erf}(bx)\,dx}{4b^4} - \frac{(3e\ldots}{2\ldots}$$

$$= -\frac{3e^c x^2}{4b^3\sqrt{\pi}} + \frac{e^c x^4}{4b\sqrt{\pi}} - \frac{3e^{c+b^2x^2}x\text{erfc}(bx)}{4b^4} + \frac{e^{c+b^2x^2}x^3\text{erfc}(bx)}{2b^2} + \frac{3e^c\sqrt{\pi}\text{erfi}(bx)}{8b^5} - \frac{3e^c x^2\,{}_2F_2\left(1,1\ldots\right.}{4b^3\sqrt{\ldots}}$$

Mathematica [A] time = 0.341129, size = 147, normalized size = 1.07

$$-\frac{e^c\left(-6b^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)+2\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2-3\right)\text{Erf}(bx)-2b^4x^4-4\sqrt{\pi}b^3x^3e^{b^2x^2}+\ldots\right.}{8\sqrt{\pi}b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c + b^2*x^2)*x^4*Erfc[b*x],x]
```

```
[Out] -(E^c*(6*b*E^(b^2*x^2)*Sqrt[Pi]*x + 6*b^2*x^2 - 4*b^3*E^(b^2*x^2)*Sqrt[Pi]*
x^3 - 2*b^4*x^4 + 2*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*b^2*x^2)*Erf[b*x] - 3*
Pi*Erfi[b*x] + 3*Pi*Erf[b*x]*Erfi[b*x] - 6*b^2*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(b^2*x^2)]))/(8*b^5*Sqrt[Pi])
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \mathrm{e}^{b^2x^2+c} x^4 \mathrm{erfc}\left(bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^4*erfc(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^4*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \,\mathrm{erfc}\left(bx\right) e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(x^4 \,\mathrm{erf}\left(bx\right) - x^4\right)e^{\left(b^2x^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-(x^4*erf(b*x) - x^4)*e^(b^2*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**4*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}(bx) e^{(b^2x^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfc(b*x)*e^(b^2*x^2 + c), x)
```

3.174 $\int e^{c+b^2x^2}x^2\textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=95

$$\frac{e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{4b^3}+\frac{e^cx^2}{2\sqrt{\pi}b}$$

```
[Out] (E^c*x^2)/(2*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfc[b*x])/(2*b^2) - (E^c*Sqr
t[Pi]*Erfi[b*x])/(4*b^3) + (E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2
*x^2])/(2*b*Sqrt[Pi])
```

Rubi [A] time = 0.0823174, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6386, 6377, 2204, 6376, 12, 30}

$$\frac{e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{Erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{4b^3}+\frac{e^cx^2}{2\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^2*Erfc[b*x],x]
```

```
[Out] (E^c*x^2)/(2*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfc[b*x])/(2*b^2) - (E^c*Sqr
t[Pi]*Erfi[b*x])/(4*b^3) + (E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2
*x^2])/(2*b*Sqrt[Pi])
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^2\operatorname{erfc}(bx)\,dx &= \frac{e^{c+b^2x^2}x\operatorname{erfc}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx}{2b^2}+\frac{\int e^c x\,dx}{b\sqrt{\pi}}\\ &= \frac{e^{c+b^2x^2}x\operatorname{erfc}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}\,dx}{2b^2}+\frac{\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx}{2b^2}+\frac{e^c\int x\,dx}{b\sqrt{\pi}}\\ &= \frac{e^cx^2}{2b\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\operatorname{erfc}(bx)}{2b^2}-\frac{e^c\sqrt{\pi}\operatorname{erfi}(bx)}{4b^3}+\frac{e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2b\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.287222, size = 104, normalized size = 1.09

$$-\frac{e^c\left(2b^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)+\operatorname{Erf}(bx)\left(2\sqrt{\pi}bxe^{b^2x^2}-\pi\operatorname{Erfi}(bx)\right)-2b^2x^2-2\sqrt{\pi}bxe^{b^2x^2}+\pi\right.}{4\sqrt{\pi}b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c + b^2*x^2)*x^2*Erfc[b*x],x]
```

```
[Out] -(E^c*(-2*b*E^(b^2*x^2)*Sqrt[Pi]*x - 2*b^2*x^2 + Pi*Erfi[b*x] + Erf[b*x]*(2
*b*E^(b^2*x^2)*Sqrt[Pi]*x - Pi*Erfi[b*x]) + 2*b^2*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, -(b^2*x^2)]))/(4*b^3*Sqrt[Pi])
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int \mathrm{e}^{b^2x^2+c}x^2\mathrm{erfc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^2*erfc(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^2*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(x^2\,\mathrm{erf}\,(bx)-x^2\right)e^{\left(b^2x^2+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-(x^2*erf(b*x) - x^2)*e^(b^2*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**2*erfc(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}(bx) e^{\left(b^2x^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfc(b*x)*e^(b^2*x^2 + c), x)
```

3.175 $\int e^{c+b^2x^2}\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{2b}-\frac{be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) - (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0346488, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6377, 2204, 6376}

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{2b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*Erfc[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) - (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
```

```
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\int e^{c+b^2x^2}\text{erfc}(bx)\,dx = \int e^{c+b^2x^2}\,dx - \int e^{c+b^2x^2}\text{erf}(bx)\,dx$$
$$= \frac{e^c\sqrt{\pi}\,\text{erfi}(bx)}{2b} - \frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [F] time = 0.0810612, size = 0, normalized size = 0.

$$\int e^{c+b^2x^2}\text{Erfc}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + b^2*x^2)*Erfc[b*x],x]
```

```
[Out] Integrate[E^(c + b^2*x^2)*Erfc[b*x], x]
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}\text{erfc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfc(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(\text{erf}(bx) - 1)e^{\left(b^2x^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfc(b*x),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfc}(bx)\, e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c), x)
```

3.176 $\int \frac{e^{c+b^2x^2}\textbf{Erfc}(bx)}{x^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2b^3e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{x}+\sqrt{\pi}be^c\text{Erfi}(bx)-\frac{2be^c\log(x)}{\sqrt{\pi}}$$

```
[Out] -((E^(c + b^2*x^2)*Erfc[b*x])/x) + b*E^c*Sqrt[Pi]*Erfi[b*x] - (2*b^3*E^c*x^
2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi] - (2*b*E^c*Log[x])
/Sqrt[Pi]
```

Rubi [A] time = 0.0787574, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6392, 6377, 2204, 6376, 12, 29}

$$-\frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{x}+\sqrt{\pi}be^c\text{Erfi}(bx)-\frac{2be^c\log(x)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^2,x]
```

```
[Out] -((E^(c + b^2*x^2)*Erfc[b*x])/x) + b*E^c*Sqrt[Pi]*Erfi[b*x] - (2*b^3*E^c*x^
2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi] - (2*b*E^c*Log[x])
/Sqrt[Pi]
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2}\,dx &= -\frac{e^{c+b^2x^2}\text{erfc}(bx)}{x} + \left(2b^2\right)\int e^{c+b^2x^2}\text{erfc}(bx)\,dx - \frac{(2b)\int \frac{e^c}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+b^2x^2}\text{erfc}(bx)}{x} + \left(2b^2\right)\int e^{c+b^2x^2}\,dx - \left(2b^2\right)\int e^{c+b^2x^2}\text{erf}(bx)\,dx - \frac{\left(2be^c\right)\int \frac{1}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+b^2x^2}\text{erfc}(bx)}{x} + be^c\sqrt{\pi}\,\text{erfi}(bx) - \frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}} - \frac{2be^c\log(x)}{\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.262034, size = 99, normalized size = 1.24

$$-\frac{e^c\left(-2b^3x^3\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)+\text{Erf}(bx)\left(\pi bx\text{Erfi}(bx)-\sqrt{\pi}e^{b^2x^2}\right)+\sqrt{\pi}e^{b^2x^2}-\pi bx\text{Erfi}(bx)+\right.}{\sqrt{\pi}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^2,x]
```

```
[Out] -((E^c*(E^(b^2*x^2)*Sqrt[Pi] - b*Pi*x*Erfi[b*x] + Erf[b*x]*(-(E^(b^2*x^2)*S
qrt[Pi]) + b*Pi*x*Erfi[b*x]) - 2*b^3*x^3*HypergeometricPFQ[{1, 1}, {3/2, 2}
, -(b^2*x^2)] + 2*b*x*Log[x]))/(Sqrt[Pi]*x))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\operatorname{erfc}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfc(b*x)/x^2,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfc(b*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\,e^{\left(b^2x^2+c\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(\operatorname{erf}(bx)-1)e^{\left(b^2x^2+c\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfc(b*x)/x**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(b^2x^2+c\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

3.177 $\int \frac{e^{c+b^2x^2}\mathbf{Erfc}(bx)}{x^4}\,dx$

Optimal. Leaf size=134

$$-\frac{4b^5e^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{3\sqrt{\pi}}-\frac{2b^2e^{b^2x^2+c}\text{Erfc}(bx)}{3x}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{3x^3}+\frac{2}{3}\sqrt{\pi}b^3e^c\text{Erfi}(bx)-\frac{4b$$

```
[Out] (b*E^c)/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erfc[b*x])/(3*x^3) - (2*b^2*E^(
c + b^2*x^2)*Erfc[b*x])/(3*x) + (2*b^3*E^c*Sqrt[Pi]*Erfi[b*x])/3 - (4*b^5*E
^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(3*Sqrt[Pi]) - (4*b^3*
E^c*Log[x])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.128984, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6392, 6377, 2204, 6376, 12, 29, 30}

$$-\frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}}-\frac{2b^2e^{b^2x^2+c}\text{Erfc}(bx)}{3x}-\frac{e^{b^2x^2+c}\text{Erfc}(bx)}{3x^3}+\frac{2}{3}\sqrt{\pi}b^3e^c\text{Erfi}(bx)-\frac{4b^3e^c\log(x)}{3\sqrt{\pi}}+\frac{be^c}{3\sqrt{\pi}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^4,x]
```


```
[Out] (b*E^c)/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erfc[b*x])/(3*x^3) - (2*b^2*E^(
c + b^2*x^2)*Erfc[b*x])/(3*x) + (2*b^3*E^c*Sqrt[Pi]*Erfi[b*x])/3 - (4*b^5*E
^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(3*Sqrt[Pi]) - (4*b^3*
E^c*Log[x])/(3*Sqrt[Pi])
```


Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
```

```
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x^3}+\frac{1}{3}\left(2b^2\right)\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^2}\,dx-\frac{(2b)\int \frac{e^c}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx-\frac{\left(4b^3\right)\int \frac{e^c}{x}\,dx}{3\sqrt{\pi}}-\frac{\left(2be^c\right)\int}{3\sqrt{\pi}}\\
&= \frac{be^c}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{c+b^2x^2}\,dx-\frac{1}{3}\left(4b^4\right)\int e^{c+b^2x^2}\operatorname{erf}(bx)\\
&= \frac{be^c}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\operatorname{erfc}(bx)}{3x}+\frac{2}{3}b^3e^c\sqrt{\pi}\operatorname{erfi}(bx)-\frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.420005, size = 151, normalized size = 1.13

$$\frac{e^c\left(4b^5x^5\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)-2\pi b^3x^3\text{Erf}(bx)\text{Erfi}(bx)+\sqrt{\pi}e^{b^2x^2}\left(2b^2x^2+1\right)\text{Erf}(bx)+2\pi b^3\right.}{3\sqrt{\pi}x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^4,x]
```

```
[Out] (E^c*(-(E^(b^2*x^2)*Sqrt[Pi]) + b*x - 2*b^2*E^(b^2*x^2)*Sqrt[Pi]*x^2 + E^(b
^2*x^2)*Sqrt[Pi]*(1 + 2*b^2*x^2)*Erf[b*x] + 2*b^3*Pi*x^3*Erfi[b*x] - 2*b^3*
Pi*x^3*Erf[b*x]*Erfi[b*x] + 4*b^5*x^5*HypergeometricPFQ[{1, 1}, {3/2, 2}, -
(b^2*x^2)] - 4*b^3*x^3*Log[x]))/(3*Sqrt[Pi]*x^3)
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}\text{erfc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfc(b*x)/x^4,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfc(b*x)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\text{erf}\,(bx)-1)e^{\left(b^2x^2+c\right)}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfc(b*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfc(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

3.178 $\int e^{-b^2x^2}x^5\textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=135

$$-\frac{43\text{Erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{x^4e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{x^2e^{-b^2x^2}\text{Erfc}(bx)}{b^4}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{b^6}+\frac{x^3e^{-2b^2x^2}}{4\sqrt{\pi}b^3}+\frac{11xe^{-2b^2x^2}}{16\sqrt{\pi}b^5}$$

```
[Out] (11*x)/(16*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) + x^3/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi])
 - (43*Erf[Sqrt[2]*b*x])/(32*Sqrt[2]*b^6) - Erfc[b*x]/(b^6*E^(b^2*x^2)) - (
x^2*Erfc[b*x])/(b^4*E^(b^2*x^2)) - (x^4*Erfc[b*x])/(2*b^2*E^(b^2*x^2))
```

Rubi [A] time = 0.194401, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6386, 6383, 2205, 2212}

$$-\frac{43\text{Erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{x^4e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{x^2e^{-b^2x^2}\text{Erfc}(bx)}{b^4}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{b^6}+\frac{x^3e^{-2b^2x^2}}{4\sqrt{\pi}b^3}+\frac{11xe^{-2b^2x^2}}{16\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Erfc[b*x])/E^(b^2*x^2),x]
```


```
[Out] (11*x)/(16*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) + x^3/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi])
 - (43*Erf[Sqrt[2]*b*x])/(32*Sqrt[2]*b^6) - Erfc[b*x]/(b^6*E^(b^2*x^2)) - (
x^2*Erfc[b*x])/(b^4*E^(b^2*x^2)) - (x^4*Erfc[b*x])/(2*b^2*E^(b^2*x^2))
```


Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx &= -\frac{e^{-b^2x^2}x^4\text{erfc}(bx)}{2b^2}+\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx}{b^2}-\frac{\int e^{-2b^2x^2}x^4\,dx}{b\sqrt{\pi}}\\ &= \frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfc}(bx)}{2b^2}+\frac{2\int e^{-b^2x^2}x\text{erfc}(bx)\,dx}{b^4}-\frac{3\int e^{-2b^2x^2}x^2\,dx}{4b^3\sqrt{\pi}}-2\\ &= \frac{11e^{-2b^2x^2}x}{16b^5\sqrt{\pi}}+\frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfc}(bx)}{2b^2}-\frac{3\int e^{-2b^2x^2}\,dx}{16b^5\sqrt{\pi}}\\ &= \frac{11e^{-2b^2x^2}x}{16b^5\sqrt{\pi}}+\frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{43\text{erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfc}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.13191, size = 87, normalized size = 0.64

$$\frac{4e^{-2b^2x^2}\left(\frac{bx\left(4b^2x^2+11\right)}{\sqrt{\pi}}-8e^{b^2x^2}\left(b^4x^4+2b^2x^2+2\right)\text{Erfc}(bx)\right)-43\sqrt{2}\text{Erf}\left(\sqrt{2}bx\right)}{64b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] (-43*Sqrt[2]*Erf[Sqrt[2]*b*x] + (4*((b*x*(11 + 4*b^2*x^2))/Sqrt[Pi] - 8*E^(
b^2*x^2)*(2 + 2*b^2*x^2 + b^4*x^4)*Erfc[b*x]))/E^(2*b^2*x^2))/(64*b^6)
```

Maple [A] time = 0.127, size = 172, normalized size = 1.3

$$\frac{1}{b}\left(\frac{1}{b^5}\left(-\frac{b^4x^4}{2\,e^{b^2x^2}}-\frac{b^2x^2}{e^{b^2x^2}}-\left(e^{b^2x^2}\right)^{-1}\right)-\frac{\mathrm{Erf}\,(bx)}{b^5}\left(-\frac{b^4x^4}{2\,e^{b^2x^2}}-\frac{b^2x^2}{e^{b^2x^2}}-\left(e^{b^2x^2}\right)^{-1}\right)+\frac{1}{\sqrt{\pi}b^5}\left(-\frac{43\sqrt{2}\sqrt{\pi}\mathrm{Erf}\left(bx\sqrt{2}\right)}{64}+\frac{11}{16\,(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erfc(b*x)/exp(b^2*x^2),x)
```

```
[Out] (1/b^5*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))-erf(
b*x)/b^5*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))+1/
Pi^(1/2)/b^5*(-43/64*2^(1/2)*Pi^(1/2)*erf(b*x*2^(1/2))+11/16/exp(b^2*x^2)^2
*b*x+1/4/exp(b^2*x^2)^2*b^3*x^3))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\,\mathrm{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfc(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.11923, size = 298, normalized size = 2.21

$$-\frac{43\sqrt{2}\pi\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-4\sqrt{\pi}\left(4\,b^4x^3+11\,b^2x\right)e^{\left(-2\,b^2x^2\right)}+32\left(\pi b^5x^4+2\,\pi b^3x^2+2\,\pi b-\left(\pi b^5x^4+2\,\pi b^3x^2+2\,\pi b\right)\right.}{64\,\pi b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/64*(43*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 4*sqrt(pi)*(4*b^4
*x^3 + 11*b^2*x)*e^(-2*b^2*x^2) + 32*(pi*b^5*x^4 + 2*pi*b^3*x^2 + 2*pi*b -
(pi*b^5*x^4 + 2*pi*b^3*x^2 + 2*pi*b)*erf(b*x))*e^(-b^2*x^2))/(pi*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erfc(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfc}(bx)\, e^{(-b^2x^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfc(b*x)*e^(-b^2*x^2), x)
```

3.179 $\int e^{-b^2x^2}x^3\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=90

$$-\frac{5\text{Erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{x^2e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2b^4}+\frac{xe^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

```
[Out] x/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (5*Erf[Sqrt[2]*b*x])/(8*Sqrt[2]*b^4) - E
rfc[b*x]/(2*b^4*E^(b^2*x^2)) - (x^2*Erfc[b*x])/(2*b^2*E^(b^2*x^2))
```

Rubi [A] time = 0.10145, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6386, 6383, 2205, 2212}

$$-\frac{5\text{Erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{x^2e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2b^4}+\frac{xe^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Erfc[b*x])/E^(b^2*x^2),x]
```


```
[Out] x/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (5*Erf[Sqrt[2]*b*x])/(8*Sqrt[2]*b^4) - E
rfc[b*x]/(2*b^4*E^(b^2*x^2)) - (x^2*Erfc[b*x])/(2*b^2*E^(b^2*x^2))
```


Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx &= -\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{2b^2}+\frac{\int e^{-b^2x^2}x\text{erfc}(bx)\,dx}{b^2}-\frac{\int e^{-2b^2x^2}x^2\,dx}{b\sqrt{\pi}}\\
&= \frac{e^{-2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{2b^2}-\frac{\int e^{-2b^2x^2}\,dx}{4b^3\sqrt{\pi}}-\frac{\int e^{-2b^2x^2}\,dx}{b^3\sqrt{\pi}}\\
&= \frac{e^{-2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{5\text{erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0924486, size = 69, normalized size = 0.77

$$\frac{4e^{-2b^2x^2}\left(\frac{bx}{\sqrt{\pi}}-2e^{b^2x^2}\left(b^2x^2+1\right)\text{Erfc}(bx)\right)-5\sqrt{2}\text{Erf}\left(\sqrt{2}bx\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] (-5*Sqrt[2]*Erf[Sqrt[2]*b*x] + (4*((b*x)/Sqrt[Pi] - 2*E^(b^2*x^2)*(1 + b^2*
x^2)*Erfc[b*x]))/E^(2*b^2*x^2))/(16*b^4)
```

Maple [A] time = 0.245, size = 118, normalized size = 1.3

$$\frac{1}{b}\left(\frac{1}{b^3}\left(-\frac{b^2x^2}{2\,e^{b^2x^2}}-\frac{1}{2\,e^{b^2x^2}}\right)-\frac{\text{Erf}\,(bx)}{b^3}\left(-\frac{b^2x^2}{2\,e^{b^2x^2}}-\frac{1}{2\,e^{b^2x^2}}\right)+\frac{1}{b^3\sqrt{\pi}}\left(-\frac{5\,\sqrt{2}\sqrt{\pi}\text{Erf}\left(bx\sqrt{2}\right)}{16}+\frac{bx}{4\,\left(e^{b^2x^2}\right)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erfc(b*x)/exp(b^2*x^2),x)
```

```
[Out] (1/b^3*(-1/2*b^2*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))-erf(b*x)/b^3*(-1/2*b^2*
x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))+1/b^3/Pi^(1/2)*(-5/16*2^(1/2)*Pi^(1/2)*e
rf(b*x*2^(1/2))+1/4/exp(b^2*x^2)^2*b*x))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}(bx)e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfc(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.28048, size = 225, normalized size = 2.5

$$\frac{4\sqrt{\pi}b^2xe^{\left(-2\,b^2x^2\right)} - 5\sqrt{2}\pi\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right) - 8\left(\pi b^3x^2 + \pi b - \left(\pi b^3x^2 + \pi b\right)\operatorname{erf}(bx)\right)e^{\left(-b^2x^2\right)}}{16\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(pi)*b^2*x*e^(-2*b^2*x^2) - 5*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)*
sqrt(b^2)*x) - 8*(pi*b^3*x^2 + pi*b - (pi*b^3*x^2 + pi*b)*erf(b*x))*e^(-b^2
*x^2))/(pi*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3e^{-b^2x^2}\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erfc(b*x)/exp(b**2*x**2),x)
```

```
[Out] Integral(x**3*exp(-b**2*x**2)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}(bx) e^{\left(-b^2x^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfc(b*x)*e^(-b^2*x^2), x)
```

3.180 $\int e^{-b^2x^2} x \,\mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\text{Erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}$$

```
[Out] -Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2) - Erfc[b*x]/(2*b^2*E^(b^2*x^2))
```

Rubi [A] time = 0.0310541, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6383, 2205}

$$-\frac{\text{Erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] -Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2) - Erfc[b*x]/(2*b^2*E^(b^2*x^2))
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int e^{-b^2x^2}x\text{erfc}(bx)\,dx = -\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2} - \frac{\int e^{-2b^2x^2}\,dx}{b\sqrt{\pi}}$$
$$= -\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

Mathematica [A] time = 0.0225946, size = 39, normalized size = 0.91

$$-\frac{2e^{-b^2x^2}\text{Erfc}(bx) + \sqrt{2}\text{Erf}\left(\sqrt{2}bx\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] -(Sqrt[2]*Erf[Sqrt[2]*b*x] + (2*Erfc[b*x])/E^(b^2*x^2))/(4*b^2)
```

Maple [A] time = 0.113, size = 53, normalized size = 1.2

$$\frac{1}{b}\left(-\frac{e^{-b^2x^2}}{2\,b} + \frac{\text{Erf}\,(bx)\,e^{-b^2x^2}}{2\,b} - \frac{\sqrt{2}\text{Erf}\left(bx\sqrt{2}\right)}{4\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfc(b*x)/exp(b^2*x^2),x)
```

```
[Out] (-1/2/b*exp(-b^2*x^2)+1/2*erf(b*x)/b*exp(-b^2*x^2)-1/4/b*2^(1/2)*erf(b*x*2^
(1/2)))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*erfc(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.16247, size = 120, normalized size = 2.79

$$-\frac{\sqrt{2}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-2\,(b\,\mathrm{erf}\,(bx)-b)e^{\left(-b^2x^2\right)}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 2*(b*erf(b*x) - b)*e^(-b
^2*x^2))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{-b^2x^2}\,\mathrm{erfc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)/exp(b**2*x**2),x)
```

```
[Out] Integral(x*exp(-b**2*x**2)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*erfc(b*x)*e^(-b^2*x^2), x)
```

3.181 $\int \frac{e^{-b^2x^2}\mathbf{Erfc}(bx)}{x}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{e^{-b^2x^2}\text{Erfc}(bx)}{x},x\right)$$

```
[Out] Unintegrable[Erfc[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi [A] time = 0.0324618, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{-b^2x^2}\text{Erfc}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]/(E^(b^2*x^2)*x),x]
```

```
[Out] Defer[Int][Erfc[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx = \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx$$

Mathematica [A] time = 0.193638, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\text{Erfc}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x),x]
```

```
[Out] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x), x]
```

Maple [A] time = 0.133, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{e^{b^{2}x^{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2)/x,x)
```

```
[Out] int(erfc(b*x)/exp(b^2*x^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(-b^{2}x^{2}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(\operatorname{erf}(bx)-1)e^{\left(-b^{2}x^{2}\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(-b^2*x^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^{2}x^{2}}\operatorname{erfc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2)/x,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erfc(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x, x)
```

3.182 $\int \frac{e^{-b^2x^2}\mathbf{Erfc}(bx)}{x^3}\,dx$

Optimal. Leaf size=85

$$-b^2\text{Unintegrable}\left(\frac{e^{-b^2x^2}\text{Erfc}(bx)}{x},x\right)+\sqrt{2}b^2\text{Erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2x^2}+\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

```
[Out] b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2*
E^(b^2*x^2)*x^2) - b^2*Unintegrable[Erfc[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi [A] time = 0.0938761, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{-b^2x^2}\text{Erfc}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]/(E^(b^2*x^2)*x^3),x]
```

```
[Out] b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2*
E^(b^2*x^2)*x^2) - b^2*Defer[Int][Erfc[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx &= -\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}-b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx-\frac{b\int \frac{e^{-2b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\
&= \frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}-b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx+\frac{\left(4b^3\right)\int e^{-2b^2x^2}\,dx}{\sqrt{\pi}}\\
&= \frac{be^{-2b^2x^2}}{\sqrt{\pi}x}+\sqrt{2}b^2\text{erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}-b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.282345, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\text{Erfc}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^3),x]
```

```
[Out] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^3), x]
```

Maple [A] time = 0.308, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{\mathrm{e}^{b^2x^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2)/x^3,x)
```

```
[Out] int(erfc(b*x)/exp(b^2*x^2)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(\operatorname{erf}(bx)-1)e^{\left(-b^2x^2\right)}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(-b^2*x^2)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2)/x**3,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erfc(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

3.183 $\int \frac{e^{-b^2x^2}\mathbf{Erfc}(bx)}{x^5}\,dx$

Optimal. Leaf size=161

$$\frac{1}{2}b^4\text{Unintegrable}\left(\frac{e^{-b^2x^2}\text{Erfc}(bx)}{x},x\right)-\frac{2}{3}\sqrt{2}b^4\text{Erf}\left(\sqrt{2}bx\right)-\frac{b^4\text{Erf}\left(\sqrt{2}bx\right)}{\sqrt{2}}+\frac{b^2e^{-b^2x^2}\text{Erfc}(bx)}{4x^2}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{4x^4}-\frac{7b^3e}{6\sqrt{}}$$

```
[Out] b/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x^3) - (7*b^3)/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x) - (
b^4*Erf[Sqrt[2]*b*x])/Sqrt[2] - (2*Sqrt[2]*b^4*Erf[Sqrt[2]*b*x])/3 - Erfc[b
*x]/(4*E^(b^2*x^2)*x^4) + (b^2*Erfc[b*x])/(4*E^(b^2*x^2)*x^2) + (b^4*Uninte
grable[Erfc[b*x]/(E^(b^2*x^2)*x), x])/2
```

Rubi [A] time = 0.182261, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{-b^2x^2}\text{Erfc}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfc[b*x]/(E^(b^2*x^2)*x^5),x]
```

```
[Out] b/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x^3) - (7*b^3)/(6*E^(2*b^2*x^2)*Sqrt[Pi]*x) - (
b^4*Erf[Sqrt[2]*b*x])/Sqrt[2] - (2*Sqrt[2]*b^4*Erf[Sqrt[2]*b*x])/3 - Erfc[b
*x]/(4*E^(b^2*x^2)*x^4) + (b^2*Erfc[b*x])/(4*E^(b^2*x^2)*x^2) + (b^4*Defer[
Int][Erfc[b*x]/(E^(b^2*x^2)*x), x])/2
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x^5}\,dx = -\frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^4} - \frac{1}{2}b^2\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x^3}\,dx - \frac{b\int \frac{e^{-2b^2x^2}}{x^4}\,dx}{2\sqrt{\pi}}$$

$$= \frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x}\,dx + \frac{b^3\int \frac{e^{-2b^2x^2}}{x^2}\,dx}{2\sqrt{\pi}} + \frac{(2b^3}{}$$

$$= \frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{7b^3e^{-2b^2x^2}}{6\sqrt{\pi}x} - \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x}\,dx - \frac{(2b^5)\int}{\sqrt{}}$$

$$= \frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{7b^3e^{-2b^2x^2}}{6\sqrt{\pi}x} - \frac{b^4\mathrm{erf}\left(\sqrt{2}bx\right)}{\sqrt{2}} - \frac{2}{3}\sqrt{2}b^4\mathrm{erf}\left(\sqrt{2}bx\right) - \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\mathrm{erfc}(bx)}{4x^2}$$

Mathematica [A] time = 0.271161, size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\mathrm{Erfc}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^5),x]
```

```
[Out] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^5), x]
```

Maple [A] time = 0.33, size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)}{e^{b^2x^2}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2)/x^5,x)
```

```
[Out] int(erfc(b*x)/exp(b^2*x^2)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)\,e^{(-b^2x^2)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\text{erf}(bx)-1)e^{\left(-b^2x^2\right)}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*e^(-b^2*x^2)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\,\text{erfc}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2)/x**5,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erfc(b*x)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^5, x)
```

3.184 $\int e^{-b^2x^2}x^4\textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=112

$$-\frac{x^3e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{3xe^{-b^2x^2}\text{Erfc}(bx)}{4b^4}-\frac{3\sqrt{\pi}\text{Erfc}(bx)^2}{16b^5}+\frac{x^2e^{-2b^2x^2}}{4\sqrt{\pi}b^3}+\frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5}$$

```
[Out] 1/(2*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) + x^2/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (3*
x*Erfc[b*x])/(4*b^4*E^(b^2*x^2)) - (x^3*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (3
*Sqrt[Pi]*Erfc[b*x]^2)/(16*b^5)
```

Rubi [A] time = 0.166182, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6386, 6374, 30, 2209, 2212}

$$-\frac{x^3e^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{3xe^{-b^2x^2}\text{Erfc}(bx)}{4b^4}-\frac{3\sqrt{\pi}\text{Erfc}(bx)^2}{16b^5}+\frac{x^2e^{-2b^2x^2}}{4\sqrt{\pi}b^3}+\frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] 1/(2*b^5*E^(2*b^2*x^2)*Sqrt[Pi]) + x^2/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (3*
x*Erfc[b*x])/(4*b^4*E^(b^2*x^2)) - (x^3*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (3
*Sqrt[Pi]*Erfc[b*x]^2)/(16*b^5)
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx &= -\frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b^2}+\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx}{2b^2}-\frac{\int e^{-2b^2x^2}x^3\,dx}{b\sqrt{\pi}}\\
&= \frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b^2}+\frac{3\int e^{-b^2x^2}\text{erfc}(bx)\,dx}{4b^4}-\frac{\int e^{-2b^2x^2}x\,dx}{2b^3\sqrt{\pi}}-\frac{3\int
\\
&= \frac{e^{-2b^2x^2}}{2b^5\sqrt{\pi}}+\frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b^2}-\frac{\left(3\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx,x,\text{erfc}(bx)\right)}{8b^5}\\
&= \frac{e^{-2b^2x^2}}{2b^5\sqrt{\pi}}+\frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b^2}-\frac{3\sqrt{\pi}\text{erfc}(bx)^2}{16b^5}
\end{aligned}
$$

Mathematica [A] time = 0.136288, size = 112, normalized size = 1.

$$
-\frac{-4\sqrt{\pi}bxe^{-b^2x^2}\left(2b^2x^2+3\right)\text{Erf}(bx)-4e^{-2b^2x^2}\left(b^2x^2+2\right)+4\sqrt{\pi}bxe^{-b^2x^2}\left(2b^2x^2+3\right)+3\pi\text{Erf}(bx)^2-6\pi\text{Erf}(bx)}{16\sqrt{\pi}b^5}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] -((-4*(2 + b^2*x^2))/E^(2*b^2*x^2) + (4*b*Sqrt[Pi]*x*(3 + 2*b^2*x^2))/E^(b^
2*x^2) - 6*Pi*Erf[b*x] - (4*b*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x])/E^(b^2*x
^2) + 3*Pi*Erf[b*x]^2)/(16*b^5*Sqrt[Pi])
```

Maple [F] time = 0.253, size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{erfc}(bx)}{e^{b^2x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erfc(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(x^4*erfc(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfc(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.08305, size = 235, normalized size = 2.1

$$-\frac{4\left(2\,\pi b^3x^3+3\,\pi bx-\left(2\,\pi b^3x^3+3\,\pi bx\right)\operatorname{erf}(bx)\right)e^{\left(-b^2x^2\right)}+\sqrt{\pi}\left(3\,\pi\operatorname{erf}(bx)^2-6\,\pi\operatorname{erf}(bx)-4\left(b^2x^2+2\right)e^{\left(-2\,b^2x^2\right)}\right)}{16\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(2*pi*b^3*x^3 + 3*pi*b*x - (2*pi*b^3*x^3 + 3*pi*b*x)*erf(b*x))*e^(
-b^2*x^2) + sqrt(pi)*(3*pi*erf(b*x)^2 - 6*pi*erf(b*x) - 4*(b^2*x^2 + 2)*e^(
```

```
-2*b^2*x^2)))/(pi*b^5)
```

Sympy [A] time = 116.695, size = 151, normalized size = 1.35

$$\begin{cases} -\frac{x^3 e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2} + \frac{x^2 e^{-2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{3xe^{-b^2x^2}\operatorname{erfc}(bx)}{4b^4} + \frac{3\sqrt{\pi}\operatorname{erf}^2\left(x\sqrt{b^2}\right)}{16b^5} + \frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5} + \frac{3\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(x\sqrt{b^2}\right)\operatorname{erfc}(bx)}{8b^6} & \text{for } b \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erfc(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((-x**3*exp(-b**2*x**2)*erfc(b*x)/(2*b**2) + x**2*exp(-2*b**2*x**2
)/(4*sqrt(pi)*b**3) - 3*x*exp(-b**2*x**2)*erfc(b*x)/(4*b**4) + 3*sqrt(pi)*e
rf(x*sqrt(b**2))**2/(16*b**5) + exp(-2*b**2*x**2)/(2*sqrt(pi)*b**5) + 3*sqr
t(pi)*sqrt(b**2)*erf(x*sqrt(b**2))*erfc(b*x)/(8*b**6), Ne(b, 0)), (x**5/5,
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfc(b*x)*e^(-b^2*x^2), x)
```

3.185 $\int e^{-b^2x^2}x^2\textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=63

$$-\frac{xe^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}\text{Erfc}(bx)^2}{8b^3}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

```
[Out] 1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (Sqr
t[Pi]*Erfc[b*x]^2)/(8*b^3)
```

Rubi [A] time = 0.079704, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6386, 6374, 30, 2209}

$$-\frac{xe^{-b^2x^2}\text{Erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}\text{Erfc}(bx)^2}{8b^3}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] 1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (Sqr
t[Pi]*Erfc[b*x]^2)/(8*b^3)
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx &= -\frac{e^{-b^2x^2}x\text{erfc}(bx)}{2b^2}+\frac{\int e^{-b^2x^2}\text{erfc}(bx)\,dx}{2b^2}-\frac{\int e^{-2b^2x^2}x\,dx}{b\sqrt{\pi}}\\ &= \frac{e^{-2b^2x^2}}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x\text{erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}\,\text{Subst}\left(\int x\,dx,x,\text{erfc}(bx)\right)}{4b^3}\\ &= \frac{e^{-2b^2x^2}}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x\text{erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.115911, size = 79, normalized size = 1.25

$$\frac{\left(4bxe^{-b^2x^2}+2\sqrt{\pi}\right)\text{Erf}(bx)+2e^{-2b^2x^2}\left(\frac{1}{\sqrt{\pi}}-2bxe^{b^2x^2}\right)-\sqrt{\pi}\text{Erf}(bx)^2}{8b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Erfc[b*x])/E^(b^2*x^2),x]
```

```
[Out] ((2*(1/Sqrt[Pi] - 2*b*E^(b^2*x^2)*x))/E^(2*b^2*x^2) + (2*Sqrt[Pi] + (4*b*x)
/E^(b^2*x^2))*Erf[b*x] - Sqrt[Pi]*Erf[b*x]^2)/(8*b^3)
```

Maple [F] time = 0.236, size = 0, normalized size = 0.

$$\int \frac{x^2\text{erfc}\,(bx)}{e^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfc(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(x^2*erfc(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfc(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.14149, size = 162, normalized size = 2.57

$$\frac{4\left(\pi bx \operatorname{erf}(bx) - \pi bx\right)e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi \operatorname{erf}(bx)^2 - 2\,\pi \operatorname{erf}(bx) - 2\,e^{\left(-2\,b^2x^2\right)}\right)}{8\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(pi*b*x*erf(b*x) - pi*b*x)*e^(-b^2*x^2) - sqrt(pi)*(pi*erf(b*x)^2 -
2*pi*erf(b*x) - 2*e^(-2*b^2*x^2)))/(pi*b^3)
```

Sympy [A] time = 12.5679, size = 63, normalized size = 1.

$$\begin{cases} -\frac{xe^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2} - \frac{\sqrt{\pi}\operatorname{erfc}^2(bx)}{8b^3} + \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfc(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((-x*exp(-b**2*x**2)*erfc(b*x)/(2*b**2) - sqrt(pi)*erfc(b*x)**2/(8
*b**3) + exp(-2*b**2*x**2)/(4*sqrt(pi)*b**3), Ne(b, 0)), (x**3/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}(bx) e^{\left(-b^2x^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfc(b*x)*e^(-b^2*x^2), x)
```

3.186 $\int e^{-b^2x^2}\textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{\pi}\text{Erfc}(bx)^2}{4b}$$

[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(4*b)

Rubi [A] time = 0.0183673, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6374, 30}

$$-\frac{\sqrt{\pi}\text{Erfc}(bx)^2}{4b}$$

Antiderivative was successfully verified.

[In] Int[Erfc[b*x]/E^(b^2*x^2),x]

[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(4*b)

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}\text{erfc}(bx)\,dx &= -\frac{\sqrt{\pi}\,\text{Subst}\left(\int x\,dx, x, \text{erfc}(bx)\right)}{2b}\\ &= -\frac{\sqrt{\pi}\text{erfc}(bx)^2}{4b}\end{aligned}$$

Mathematica [A] time = 0.0045196, size = 18, normalized size = 1.

$$-\frac{\sqrt{\pi}\mathrm{Erfc}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/E^(b^2*x^2),x]
```

```
[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(4*b)
```

Maple [A] time = 0.071, size = 22, normalized size = 1.2

$$\frac{\sqrt{\pi}}{2\,b}\left(-\frac{\left(\mathrm{Erf}\left(bx\right)\right)^{2}}{2}+\mathrm{Erf}\left(bx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2),x)
```

```
[Out] 1/2*Pi^(1/2)/b*(-1/2*erf(b*x)^2+erf(b*x))
```

Maxima [A] time = 0.995533, size = 19, normalized size = 1.06

$$-\frac{\sqrt{\pi}\,\mathrm{erfc}\left(bx\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*erfc(b*x)^2/b
```

Fricas [A] time = 2.02123, size = 58, normalized size = 3.22

$$-\frac{\sqrt{\pi}\left(\mathrm{erf}\left(bx\right)^{2}-2\,\mathrm{erf}\left(bx\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(b*x)^2 - 2*erf(b*x))/b
```

Sympy [A] time = 1.49723, size = 17, normalized size = 0.94

$$\begin{cases} -\frac{\sqrt{\pi}\operatorname{erfc}^{2}(bx)}{4b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((-sqrt(pi)*erfc(b*x)**2/(4*b), Ne(b, 0)), (x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\, e^{\left(-b^{2}x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2), x)
```

3.187 $\int \frac{e^{-b^2x^2}\mathbf{Erfc}(bx)}{x^2}\,dx$

Optimal. Leaf size=53

$$-\frac{e^{-b^2x^2}\mathrm{Erfc}(bx)}{x}-\frac{b\mathrm{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\mathrm{Erfc}(bx)^2$$

```
[Out] -(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpIntegralE
i[-2*b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0860244, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6392, 6374, 30, 2210}

$$-\frac{e^{-b^2x^2}\mathrm{Erfc}(bx)}{x}-\frac{b\mathrm{Ei}\left(-2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\mathrm{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/(E^(b^2*x^2)*x^2),x]
```

```
[Out] -(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpIntegralE
i[-2*b^2*x^2])/Sqrt[Pi]
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^2}\,dx &= -\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x} - \left(2b^2\right)\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx - \frac{(2b)\int \frac{e^{-2b^2x^2}}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x} - \frac{b\operatorname{Ei}\left(-2b^2x^2\right)}{\sqrt{\pi}} + \left(b\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx, x, \operatorname{erfc}(bx)\right)\\ &= -\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x} + \frac{1}{2}b\sqrt{\pi}\operatorname{erfc}(bx)^2 - \frac{b\operatorname{Ei}\left(-2b^2x^2\right)}{\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0157171, size = 53, normalized size = 1.

$$-\frac{e^{-b^2x^2}\operatorname{Erfc}(bx)}{x} - \frac{b\operatorname{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}} + \frac{1}{2}\sqrt{\pi}b\operatorname{Erfc}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^2),x]
```

```
[Out] -(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpIntegralE
i[-2*b^2*x^2])/Sqrt[Pi]
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{e^{b^2x^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2)/x^2,x)
```

```
[Out] int(erfc(b*x)/exp(b^2*x^2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^2, x)
```

Fricas [A] time = 2.06845, size = 196, normalized size = 3.7

$$-\frac{2\,\pi^{\frac{3}{2}}\sqrt{b^2}x\operatorname{erf}\left(\sqrt{b^2}x\right) + 2\left(\pi - \pi\operatorname{erf}(bx)\right)e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi bx\operatorname{erf}(bx)^2 - 2\,bx\operatorname{Ei}\left(-2\,b^2x^2\right)\right)}{2\,\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*pi^(3/2)*sqrt(b^2)*x*erf(sqrt(b^2)*x) + 2*(pi - pi*erf(b*x))*e^(-b^
2*x^2) - sqrt(pi)*(pi*b*x*erf(b*x)^2 - 2*b*x*Ei(-2*b^2*x^2)))/(pi*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2)/x**2,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erfc(b*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^2, x)
```

3.188 $\int \frac{e^{-b^2x^2}\mathbf{Erfc}(bx)}{x^4}\,dx$

Optimal. Leaf size=108

$$\frac{2b^2e^{-b^2x^2}\text{Erfc}(bx)}{3x}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{3x^3}-\frac{1}{3}\sqrt{\pi}b^3\text{Erfc}(bx)^2+\frac{4b^3\text{ExpIntegralEi}\left(-2b^2x^2\right)}{3\sqrt{\pi}}+\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}$$

```
[Out] b/(3*E^(2*b^2*x^2)*Sqrt[Pi]*x^2) - Erfc[b*x]/(3*E^(b^2*x^2)*x^3) + (2*b^2*E
rfc[b*x])/(3*E^(b^2*x^2)*x) - (b^3*Sqrt[Pi]*Erfc[b*x]^2)/3 + (4*b^3*ExpInte
gralEi[-2*b^2*x^2])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.156677, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6392, 6374, 30, 2210, 2214}

$$\frac{2b^2e^{-b^2x^2}\text{Erfc}(bx)}{3x}-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{3x^3}-\frac{1}{3}\sqrt{\pi}b^3\text{Erfc}(bx)^2+\frac{4b^3\text{Ei}\left(-2b^2x^2\right)}{3\sqrt{\pi}}+\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/(E^(b^2*x^2)*x^4),x]
```


```
[Out] b/(3*E^(2*b^2*x^2)*Sqrt[Pi]*x^2) - Erfc[b*x]/(3*E^(b^2*x^2)*x^3) + (2*b^2*E
rfc[b*x])/(3*E^(b^2*x^2)*x) - (b^3*Sqrt[Pi]*Erfc[b*x]^2)/3 + (4*b^3*ExpInte
gralEi[-2*b^2*x^2])/(3*Sqrt[Pi])
```


Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx &= -\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{3x^3}-\frac{1}{3}\left(2b^2\right)\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^2}\,dx-\frac{(2b)\int \frac{e^{-2b^2x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\
&= \frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erfc}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx+2\frac{\left(4b^3\right)\int \frac{e^{-2b^2x^2}}{x}\,dx}{3\sqrt{\pi}}\\
&= \frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erfc}(bx)}{3x}+\frac{4b^3\operatorname{Ei}\left(-2b^2x^2\right)}{3\sqrt{\pi}}-\frac{1}{3}\left(2b^3\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\mathrm{e}\\
&= \frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erfc}(bx)}{3x}-\frac{1}{3}b^3\sqrt{\pi}\operatorname{erfc}(bx)^2+\frac{4b^3\operatorname{Ei}\left(-2b^2x^2\right)}{3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0671575, size = 85, normalized size = 0.79

$$\frac{1}{3}\left(\frac{e^{-b^2x^2}\left(2b^2x^2-1\right)\operatorname{Erfc}(bx)}{x^3}-\sqrt{\pi}b^3\operatorname{Erfc}(bx)^2+\frac{b\left(4b^2\operatorname{ExpIntegralEi}\left(-2b^2x^2\right)+\frac{e^{-2b^2x^2}}{x^2}\right)}{\sqrt{\pi}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^4),x]
```

```
[Out] (((-1 + 2*b^2*x^2)*Erfc[b*x])/(E^(b^2*x^2)*x^3) - b^3*Sqrt[Pi]*Erfc[b*x]^2
+ (b*(1/(E^(2*b^2*x^2)*x^2) + 4*b^2*ExpIntegralEi[-2*b^2*x^2]))/Sqrt[Pi])/3
```

Maple [F] time = 0.317, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)}{e^{b^2x^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2)/x^4,x)
```

```
[Out] int(erfc(b*x)/exp(b^2*x^2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx)\,e^{\left(-b^2x^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^4, x)
```

Fricas [A] time = 2.21655, size = 285, normalized size = 2.64

$$\frac{2\,\pi^{\frac{3}{2}}\sqrt{b^2}b^2x^3\operatorname{erf}\left(\sqrt{b^2}x\right)-\left(\pi-2\,\pi b^2x^2-\left(\pi-2\,\pi b^2x^2\right)\operatorname{erf}(bx)\right)e^{\left(-b^2x^2\right)}-\sqrt{\pi}\left(\pi b^3x^3\operatorname{erf}(bx)^2-4\,b^3x^3\operatorname{Ei}\left(-2\,b^2x^2\right)-b\right.}{3\,\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*pi^(3/2)*sqrt(b^2)*b^2*x^3*erf(sqrt(b^2)*x) - (pi - 2*pi*b^2*x^2 - (
pi - 2*pi*b^2*x^2)*erf(b*x))*e^(-b^2*x^2) - sqrt(pi)*(pi*b^3*x^3*erf(b*x)^2
 - 4*b^3*x^3*Ei(-2*b^2*x^2) - b*x*e^(-2*b^2*x^2)))/(pi*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-b^2x^2}\,\mathrm{erfc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2)/x**4,x)
```

```
[Out] Integral(exp(-b**2*x**2)*erfc(b*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*e^(-b^2*x^2)/x^4, x)
```

3.189 $\int e^{c+dx^2}x^3\mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=342

$$-\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d^2\sqrt{b^2-d}}+\frac{a^2b^3e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{5/2}}+\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{4d\left(b^2-d\right)^{3/2}}+\frac{ab^2e^{-a^2-2abx-x^2(b^2-d)+c}}{2\sqrt{\pi}d\left(b^2-d\right)^2}-\frac{bxe^{-a^2-2a\cdots}}{2\sqrt{\pi}\cdots}$$

```
[Out] (a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) -
 (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (b
*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqrt[
b^2 - d]*d^2) + (a^2*b^3*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/
Sqrt[b^2 - d]])/(2*(b^2 - d)^(5/2)*d) + (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a
*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(3/2)*d) - (E^(c + d*x^2)*Er
fc[a + b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfc[a + b*x])/(2*d)
```

Rubi [A] time = 0.469905, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6386, 6383, 2234, 2205, 2241, 2240}

$$-\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d^2\sqrt{b^2-d}}+\frac{a^2b^3e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{5/2}}+\frac{be^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{4d\left(b^2-d\right)^{3/2}}+\frac{ab^2e^{-a^2-2abx-x^2(b^2-d)+c}}{2\sqrt{\pi}d\left(b^2-d\right)^2}-\frac{bxe^{-a^2-2a\cdots}}{2\sqrt{\pi}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^3*Erfc[a + b*x],x]
```

```
[Out] (a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) -
 (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)*d*Sqrt[Pi]) - (b
*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqrt[
b^2 - d]*d^2) + (a^2*b^3*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/
Sqrt[b^2 - d]])/(2*(b^2 - d)^(5/2)*d) + (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a
*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(3/2)*d) - (E^(c + d*x^2)*Er
fc[a + b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfc[a + b*x])/(2*d)
```

Rule 6386

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
```

```
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ
[{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2241

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] +
 (-Dist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x],
 x] - Dist[((m - 1)*e^2)/(2*c*Log[F]), Int[(d + e*x)^(m - 2)*F^(a + b*x + c
*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && Gt
Q[m, 1]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(e*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] - Dist[(b*e - 2*c*d)/(2*
c), Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[
b*e - 2*c*d, 0]
```

Rubi steps

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx = \frac{e^{c+dx^2}x^2\text{erfc}(a+bx)}{2d} - \frac{\int e^{c+dx^2}x\text{erfc}(a+bx)\,dx}{d} + \frac{b\int e^{-a^2+c-2abx+(-b^2+d)x^2}x^2\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfc}(a+bx)}{2d} - \frac{b\int e^{-a^2+c-2abx-(b^2-d}}{d^2\sqrt{\pi}}$$

$$= \frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfc}(a+bx}{2d}$$

$$= \frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{be^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}d^2} + \frac{be^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{er}}{4(b^2-}$$

$$= \frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}} - \frac{be^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}d^2} + \frac{a^2b^3e^{\frac{b^2c+a^2d-}{b^2-d}}}{2(b^2}$$

Mathematica [A] time = 4.22265, size = 256, normalized size = 0.75

$$-\frac{e^c\left(-\frac{bde^{-a^2-2abx+x^2(d-b^2)}\left(\sqrt{\pi}\sqrt{b^2-d}\left((2a^2+1)b^2-d\right)e^{\frac{(ab+x(b^2-d))^2}{b^2-d}}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)+2(b^2-d)(ab+x(d-b^2))\right)}{\sqrt{\pi}(b^2-d)^3} + \frac{2be^{\frac{a^2d}{b^2-d}}\text{Erfi}\left(\frac{x(d-b^2)-ab}{\sqrt{d-b^2}}\right)}{\sqrt{d-b^2}} + 2e^{dx^2}(dx^2}{4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c + d*x^2)*x^3*Erfc[a + b*x],x]
```

```
[Out] -(E^c*(-2*E^(d*x^2)*(-1 + d*x^2) + 2*E^(d*x^2)*(-1 + d*x^2)*Erf[a + b*x] -
(b*d*E^(-a^2 - 2*a*b*x + (-b^2 + d)*x^2)*(2*(b^2 - d)*(a*b + (-b^2 + d)*x)
+ Sqrt[b^2 - d]*((1 + 2*a^2)*b^2 - d)*E^((a*b + (b^2 - d)*x)^2/(b^2 - d))*S
qrt[Pi]*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]]))/((b^2 - d)^3*Sqrt[Pi]) + (
2*b*E^((a^2*d)/(b^2 - d))*Erfi[(-(a*b) + (-b^2 + d)*x)/Sqrt[-b^2 + d]])/Sqr
t[-b^2 + d]))/(4*d^2)
```

Maple [F] time = 0.301, size = 0, normalized size = 0.

$$\int \mathrm{e}^{dx^2+c}x^3\mathrm{erfc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^3*erfc(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^3*erfc(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\mathrm{erfc}\,(bx+a)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.43509, size = 670, normalized size = 1.96

$$-\frac{\pi\left(2\,b^5-\left(2\,a^2+5\right)b^3d+3\,bd^2\right)\sqrt{b^2-d}\,\mathrm{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)}-2\,\sqrt{\pi}\left(ab^4d-ab^2d^2-\left(b^5d-2\,b^3d^2+bd^3\right)x\right)e^{\left(-b^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(pi*(2*b^5 - (2*a^2 + 5)*b^3*d + 3*b*d^2)*sqrt(b^2 - d)*erf((a*b + (b^
2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a^2 - c)*d)/(b^2 - d)) - 2*sqrt(pi)*(
a*b^4*d - a*b^2*d^2 - (b^5*d - 2*b^3*d^2 + b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x
+ d*x^2 - a^2 + c) - 2*(pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 - pi*(
b^6 - 3*b^4*d + 3*b^2*d^2 - d^3) - (pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4
)*x^2 - pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3))*erf(b*x + a))*e^(d*x^2 + c))/
(pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^2*d^4 - d^5))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**3*erfc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfc}(bx + a) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfc(b*x + a)*e^(d*x^2 + c), x)
```

3.190 $\int e^{c+dx^2} x \,\mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=86

$$\frac{be^{\frac{a^2d}{b^2-d}+c}\operatorname{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}+\frac{e^{c+dx^2}\operatorname{Erfc}(a+bx)}{2d}$$

```
[Out] (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqr
t[b^2 - d]*d) + (E^(c + d*x^2)*Erfc[a + b*x])/(2*d)
```

Rubi [A] time = 0.0535091, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6383, 2234, 2205}

$$\frac{be^{\frac{a^2d}{b^2-d}+c}\operatorname{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}+\frac{e^{c+dx^2}\operatorname{Erfc}(a+bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x*Erfc[a + b*x],x]
```

```
[Out] (b*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqr
t[b^2 - d]*d) + (E^(c + d*x^2)*Erfc[a + b*x])/(2*d)
```

Rule 6383

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfc[a + b*x])/(2*d), x] + Dist[b/(d*Sqrt[Pi]), Int[E^(-a
^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int e^{c+dx^2}x\text{erfc}(a+bx)\,dx=\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d}+\frac{b\int e^{-a^2+c-2abx-\left(b^2-d\right)x^2}\,dx}{d\sqrt{\pi}}$$
$$=\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d}+\frac{\left(be^{\frac{b^2c+a^2d-cd}{b^2-d}}\right)\int\exp\left(\frac{\left(-2ab+2\left(-b^2+d\right)x\right)^2}{4\left(-b^2+d\right)}\right)dx}{d\sqrt{\pi}}$$
$$=\frac{be^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}d}+\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

Mathematica [A] time = 0.105111, size = 81, normalized size = 0.94

$$\frac{e^c\left(\frac{be^{\frac{a^2d}{b^2-d}}\text{Erfi}\left(\frac{x\left(d-b^2\right)-ab}{\sqrt{d-b^2}}\right)}{\sqrt{d-b^2}}+e^{dx^2}\text{Erfc}(a+bx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x*Erfc[a + b*x],x]
```

```
[Out] (E^c*(E^(d*x^2)*Erfc[a + b*x] + (b*E^((a^2*d)/(b^2 - d))*Erfi[(-(a*b) + (-b
^2 + d)*x)/Sqrt[-b^2 + d]])/Sqrt[-b^2 + d]))/(2*d)
```

Maple [B] time = 0.361, size = 175, normalized size = 2.

$$\frac{1}{b}\left(\frac{b}{2\,d}\text{e}^{\frac{d(bx+a)^2}{b^2}-2\frac{ad(bx+a)}{b^2}+\frac{a^2d}{b^2}+c}-\frac{\text{Erf}\,(bx+a)\,b}{2\,d}\text{e}^{\frac{d(bx+a)^2}{b^2}-2\frac{ad(bx+a)}{b^2}+\frac{a^2d}{b^2}+c}+\frac{b}{2\,d}\text{e}^{\frac{a^2d}{b^2}+c-\frac{a^2d^2}{b^4}\left(-1+\frac{d}{b^2}\right)^{-1}}\text{Erf}\left(\sqrt{1-\frac{d}{b^2}}\,(bx+a)+\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x*erfc(b*x+a),x)
```

```
[Out] (1/2*b/d*exp(d*(b*x+a)^2/b^2-2/b^2*(b*x+a)*a*d+1/b^2*a^2*d+c)-1/2*erf(b*x+a
)*b/d*exp(d*(b*x+a)^2/b^2-2/b^2*(b*x+a)*a*d+1/b^2*a^2*d+c)+1/2*b/d*exp(1/b^
2*a^2*d+c-a^2*d^2/b^4/(-1+d/b^2))/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*(b*x+
a)+a*d/b^2/(1-d/b^2)^(1/2)))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.45632, size = 220, normalized size = 2.56

$$\frac{\sqrt{b^2-d}\, b \operatorname{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right) e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)} + \left(b^2-\left(b^2-d\right)\operatorname{erf}(bx+a)-d\right) e^{\left(dx^2+c\right)}}{2\left(b^2d-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2 - d)*b*erf((a*b + (b^2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a^
2 - c)*d)/(b^2 - d)) + (b^2 - (b^2 - d)*erf(b*x + a) - d)*e^(d*x^2 + c))/(b
^2*d - d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x*erfc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfc}(bx + a) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*erfc(b*x + a)*e^(d*x^2 + c), x)
```

3.191 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + d*x^2)*Erfc[a + b*x])/x, x]
```

Rubi [A] time = 0.0376461, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[a + b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + d*x^2)*Erfc[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.691216, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x, x]
```

Maple [A] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{e^{d x^{2}+c} \operatorname{erfc}\left(b x+a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}\left(bx+a\right) e^{\left(dx^{2}+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\operatorname{erf}\left(bx+a\right)-1)e^{\left(dx^{2}+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int \frac{e^{dx^{2}} \operatorname{erfc}\left(a+bx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x+a)/x,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)\,e^{\left(dx^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x, x)
```

3.192 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=181

$$\frac{2ab^2\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{\sqrt{\pi}}+d\text{Unintegrable}\left(\frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x},x\right)+b\sqrt{b^2-d}e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-\right.}{\sqrt{b^2-d}}\right.$$

```
[Out] (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(Sqrt[Pi]*x) + b*Sqrt[b^2 - d]*E
^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]] - (E^(c + d
*x^2)*Erfc[a + b*x])/(2*x^2) + (2*a*b^2*Unintegrable[E^(-a^2 + c - 2*a*b*x
+ (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + d*Unintegrable[(E^(c + d*x^2)*Erfc[a +
b*x])/x, x]
```

Rubi [A] time = 0.411235, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[a + b*x])/x^3,x]
```

```
[Out] (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(Sqrt[Pi]*x) + b*Sqrt[b^2 - d]*E
^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]] - (E^(c + d
*x^2)*Erfc[a + b*x])/(2*x^2) + (2*a*b^2*Defer[Int][E^(-a^2 + c - 2*a*b*x +
(-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + d*Defer[Int][(E^(c + d*x^2)*Erfc[a + b*x]
)/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^3}\,dx = -\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx - \frac{b\int \frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx + \frac{\left(2ab^2\right)\int \frac{e^{-a^2+c-2abx+\left(-\right.}}{x}}{\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx + \frac{\left(2ab^2\right)\int \frac{e^{-a^2+c-2abx+\left(-\right.}}{x}}{\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{\sqrt{\pi}x} + b\sqrt{b^2-d}e^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right) - \frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2} + d\int \frac{e^{c+}}{}$$

Mathematica [A] time = 0.877985, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^3, x]
```

Maple [A] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfc}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x+a)/x^3,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x+a)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\operatorname{erf}(bx+a)-1)e^{\left(dx^2+c\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x+a)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

3.193 $\int e^{c+dx^2}x^4\mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=526

$$\frac{3\text{Unintegrable}\left(e^{c+dx^2}\text{Erfc}(a+bx),x\right)}{4d^2}+\frac{3ab^2e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{4d^2\left(b^2-d\right)^{3/2}}+\frac{3be^{-a^2-2abx-x^2(b^2-d)+c}}{4\sqrt{\pi}d^2\left(b^2-d\right)}-\frac{a^3b^4e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x(b^2}{\sqrt{b^2-}}\right.}{2d\left(b^2-d\right)^{7/2}}$$

```
[Out] (3*b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) - (
a^2*b^3*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^3*d*Sqrt[Pi])
- (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) + (
a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)^2*d*Sqrt[Pi])
- (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x^2)/(2*(b^2 - d)*d*Sqrt[Pi]) +
 (3*a*b^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])
/(4*(b^2 - d)^(3/2)*d^2) - (a^3*b^4*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b
^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(7/2)*d) - (3*a*b^2*E^(c + (a^2*d)/
(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(5/2)*d) -
(3*E^(c + d*x^2)*x*Erfc[a + b*x])/(4*d^2) + (E^(c + d*x^2)*x^3*Erfc[a + b*x
])/(2*d) + (3*Unintegrable[E^(c + d*x^2)*Erfc[a + b*x], x])/(4*d^2)
```

Rubi [A] time = 0.907243, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int e^{c+dx^2}x^4\text{Erfc}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^4*Erfc[a + b*x],x]
```

```
[Out] (3*b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(4*(b^2 - d)*d^2*Sqrt[Pi]) - (
a^2*b^3*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^3*d*Sqrt[Pi])
- (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)^2*d*Sqrt[Pi]) + (
a*b^2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(2*(b^2 - d)^2*d*Sqrt[Pi])
- (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x^2)/(2*(b^2 - d)*d*Sqrt[Pi]) +
 (3*a*b^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])
/(4*(b^2 - d)^(3/2)*d^2) - (a^3*b^4*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b
^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(7/2)*d) - (3*a*b^2*E^(c + (a^2*d)/
(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(5/2)*d) -
(3*E^(c + d*x^2)*x*Erfc[a + b*x])/(4*d^2) + (E^(c + d*x^2)*x^3*Erfc[a + b*x
```

```
])/(2*d) + (3*Defer[Int][E^(c + d*x^2)*Erfc[a + b*x], x])/(4*d^2)
```

Rubi steps

$$\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx = \frac{e^{c+dx^2}x^3\text{erfc}(a+bx)}{2d} - \frac{3\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx}{2d} + \frac{b\int e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}x^3\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}x^2}{2\left(b^2-d\right)d\sqrt{\pi}} - \frac{3e^{c+dx^2}x\text{erfc}(a+bx)}{4d^2} + \frac{e^{c+dx^2}x^3\text{erfc}(a+bx)}{2d} + \frac{3\int e^{c+dx^2}\text{erfc}(a+}{4d^2}$$

$$= \frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} + \frac{ab^2e^{-a^2+c-2abx-\left(b^2-d\right)x^2}x}{2\left(b^2-d\right)^2d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x}}{2\left(b^2-d\right)d\sqrt{\pi}}$$

$$= \frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} - \frac{a^2b^3e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^3d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} + \frac{ab^2e^{-a^2+c-2abx-\left(b^2-\right.}}{2\left(b^2-d\right)^2d\sqrt{}}$$

$$= \frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} - \frac{a^2b^3e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^3d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} + \frac{ab^2e^{-a^2+c-2abx-\left(b^2-\right.}}{2\left(b^2-d\right)^2d\sqrt{}}$$

$$= \frac{3be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{4\left(b^2-d\right)d^2\sqrt{\pi}} - \frac{a^2b^3e^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^3d\sqrt{\pi}} - \frac{be^{-a^2+c-2abx-\left(b^2-d\right)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}} + \frac{ab^2e^{-a^2+c-2abx-\left(b^2-\right.}}{2\left(b^2-d\right)^2d\sqrt{}}$$

Mathematica [A] time = 1.04229, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^4\text{Erfc}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^4*Erfc[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^4*Erfc[a + b*x], x]
```

Maple [A] time = 0.146, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^4\text{erfc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^4*erfc(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^4*erfc(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}(bx + a) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^4 \operatorname{erf}(bx + a) - x^4\right)e^{(dx^2+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(-(x^4*erf(b*x + a) - x^4)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**4*erfc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfc(b*x + a)*e^(d*x^2 + c), x)
```

3.194 $\int e^{c+dx^2}x^2\mathbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=163

$$-\frac{\text{Unintegrable}\left(e^{c+dx^2}\text{Erfc}(a+bx),x\right)}{2d}-\frac{ab^2e^{\frac{a^2d}{b^2-d}+c}\text{Erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d\left(b^2-d\right)^{3/2}}-\frac{be^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\sqrt{\pi}d\left(b^2-d\right)}+\frac{xe^{c+dx^2}\text{Erfc}(a+bx)}{2d}$$

```
[Out] -(b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) - (a*b
^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^
2 - d)^(3/2)*d) + (E^(c + d*x^2)*x*Erfc[a + b*x])/(2*d) - Unintegrable[E^(c
 + d*x^2)*Erfc[a + b*x], x]/(2*d)
```

Rubi [A] time = 0.18238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^2\text{Erfc}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^2*Erfc[a + b*x],x]
```

```
[Out] -(b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(2*(b^2 - d)*d*Sqrt[Pi]) - (a*b
^2*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^
2 - d)^(3/2)*d) + (E^(c + d*x^2)*x*Erfc[a + b*x])/(2*d) - Defer[Int][E^(c +
 d*x^2)*Erfc[a + b*x], x]/(2*d)
```

Rubi steps

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx = \frac{e^{c+dx^2}x\text{erfc}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfc}(a+bx)\,dx}{2d} + \frac{b\int e^{-a^2+c-2abx+(-b^2+d)x^2}x\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)\,d\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erfc}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfc}(a+bx)\,dx}{2d} - \frac{(ab^2)\int e^{-a^2+c-2abx}}{(b^2-d)\,d}$$

$$= -\frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)\,d\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erfc}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfc}(a+bx)\,dx}{2d} - \frac{\left(ab^2e^{\frac{b^2c+a^2d-cd}{b^2-d}}\right)\int e}{(b^2}$$

$$= -\frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)\,d\sqrt{\pi}} - \frac{ab^2e^{\frac{b^2c+a^2d-cd}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}\,d} + \frac{e^{c+dx^2}x\text{erfc}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfc}(a}{2d}$$

Mathematica [A] time = 0.821834, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^2\text{Erfc}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^2*Erfc[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^2*Erfc[a + b*x], x]
```

Maple [A] time = 0.253, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^2\text{erfc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^2*erfc(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^2*erfc(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(x^2 \operatorname{erf}(bx+a) - x^2\right)e^{\left(dx^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(-(x^2*erf(b*x + a) - x^2)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**2*erfc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfc(b*x + a)*e^(d*x^2 + c), x)
```

3.195 $\int e^{c+dx^2} \textbf{Erfc}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(e^{c+dx^2}\text{Erfc}(a+bx),x\right)$$

```
[Out] Unintegrable[E^(c + d*x^2)*Erfc[a + b*x], x]
```

Rubi [A] time = 0.0141731, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int e^{c+dx^2}\text{Erfc}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*Erfc[a + b*x],x]
```

```
[Out] Defer[Int][E^(c + d*x^2)*Erfc[a + b*x], x]
```

Rubi steps

$$\int e^{c+dx^2}\text{erfc}(a+bx)\,dx = \int e^{c+dx^2}\text{erfc}(a+bx)\,dx$$

Mathematica [A] time = 0.0446625, size = 0, normalized size = 0.

$$\int e^{c+dx^2}\text{Erfc}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*Erfc[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*Erfc[a + b*x], x]
```

Maple [A] time = 0.123, size = 0, normalized size = 0.

$$\int \mathrm{e}^{d x^2+c} \operatorname{erfc}(b x+a)\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(b x+a) e^{\left(d x^2+c\right)}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-(\operatorname{erf}(b x+a)-1) e^{\left(d x^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int e^{d x^{2}} \operatorname{erfc}(a+b x)\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x+a),x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c), x)
```

3.196 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=81

$$-\frac{2b\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{\sqrt{\pi}}+2d\text{Unintegrable}\left(e^{c+dx^2}\text{Erfc}(a+bx),x\right)-\frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x}$$

```
[Out] -((E^(c + d*x^2)*Erfc[a + b*x])/x) - (2*b*Unintegrable[E^(-a^2 + c - 2*a*b*
x + (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + 2*d*Unintegrable[E^(c + d*x^2)*Erfc[a
 + b*x], x]
```

Rubi [A] time = 0.204536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[a + b*x])/x^2,x]
```

```
[Out] -((E^(c + d*x^2)*Erfc[a + b*x])/x) - (2*b*Defer[Int][E^(-a^2 + c - 2*a*b*x
+ (-b^2 + d)*x^2)/x, x])/Sqrt[Pi] + 2*d*Defer[Int][E^(c + d*x^2)*Erfc[a + b
*x], x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^2}\,dx=-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}+(2d)\int e^{c+dx^2}\text{erfc}(a+bx)\,dx-\frac{(2b)\int \frac{e^{-a^2+c-2abx+(-b^2+d)x^2}}{x}\,dx}{\sqrt{\pi}}$$

Mathematica [A] time = 0.949121, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^2,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^2, x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{d x^2+c} \operatorname{erfc}(b x+a)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x+a)/x^2,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(b x+a)\, e^{\left(d x^2+c\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\operatorname{erf}(b x+a)-1) e^{\left(d x^2+c\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^c \int \frac{e^{dx^2} \operatorname{erfc}(a + bx)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x+a)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfc(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx + a) e^{\left(dx^2 + c\right)}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

3.197 $\int \frac{e^{c+dx^2}\mathbf{Erfc}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=351

$$-\frac{4a^2b^3\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{3\sqrt{\pi}}+\frac{2b\left(b^2-d\right)\text{Unintegrable}\left(\frac{e^{-a^2-2abx+x^2(d-b^2)+c}}{x},x\right)}{3\sqrt{\pi}}-\frac{4bd\text{Unintegrable}\left(\frac{e^{-a^2}}{}\right.}{3\sqrt{\pi}}$$

```
[Out] (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) - (2*a*b^2*E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x) - (2*a*b^2*Sqrt[b^2 - d]
*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/3 - (E^(
c + d*x^2)*Erfc[a + b*x])/(3*x^3) - (2*d*E^(c + d*x^2)*Erfc[a + b*x])/(3*x)
 - (4*a^2*b^3*Unintegrable[E^(-a^2 + c - 2*a*b*x + (-b^2 + d)*x^2)/x, x])/(
3*Sqrt[Pi]) + (2*b*(b^2 - d)*Unintegrable[E^(-a^2 + c - 2*a*b*x + (-b^2 + d
)*x^2)/x, x])/(3*Sqrt[Pi]) - (4*b*d*Unintegrable[E^(-a^2 + c - 2*a*b*x + (-
b^2 + d)*x^2)/x, x])/(3*Sqrt[Pi]) + (4*d^2*Unintegrable[E^(c + d*x^2)*Erfc[
a + b*x], x])/3
```

Rubi [A] time = 0.868551, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfc}(a+bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfc[a + b*x])/x^4,x]
```

```
[Out] (b*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x^2) - (2*a*b^2*E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2))/(3*Sqrt[Pi]*x) - (2*a*b^2*Sqrt[b^2 - d]
*E^(c + (a^2*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/3 - (E^(
c + d*x^2)*Erfc[a + b*x])/(3*x^3) - (2*d*E^(c + d*x^2)*Erfc[a + b*x])/(3*x)
 - (4*a^2*b^3*Defer[Int][E^(-a^2 + c - 2*a*b*x + (-b^2 + d)*x^2)/x, x])/(3*
Sqrt[Pi]) + (2*b*(b^2 - d)*Defer[Int][E^(-a^2 + c - 2*a*b*x + (-b^2 + d)*x^
2)/x, x])/(3*Sqrt[Pi]) - (4*b*d*Defer[Int][E^(-a^2 + c - 2*a*b*x + (-b^2 +
d)*x^2)/x, x])/(3*Sqrt[Pi]) + (4*d^2*Defer[Int][E^(c + d*x^2)*Erfc[a + b*x]
, x])/3
```

Rubi steps

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^4}\,dx = -\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{3x^3} + \frac{1}{3}(2d)\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx - \frac{(2b)\int \frac{e^{-a^2+c-2abx+(-b^2+d)x^2}}{x^3}\,dx}{3\sqrt{\pi}}$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x^2} - \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\operatorname{erfc}(a+bx)}{3x} + \frac{1}{3}\left(4d^2\right)\int e^{c+dx^2}\operatorname{erfc}(a+$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x^2} - \frac{2ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\operatorname{erfc}(a+bx)}{3x} +$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x^2} - \frac{2ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\operatorname{erfc}(a+bx)}{3x} +$$

$$= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x^2} - \frac{2ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x} - \frac{2}{3}ab^2\sqrt{b^2-d}e^{\frac{b^2c+a^2d-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+\left(b^2-d\right)x}{\sqrt{b^2-d}}\right) -$$

Mathematica [A] time = 1.13716, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\operatorname{Erfc}(a+bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^4,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^4, x]
```

Maple [A] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\operatorname{erfc}(bx+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfc(b*x+a)/x^4,x)
```

```
[Out] int(exp(d*x^2+c)*erfc(b*x+a)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(\operatorname{erf}(bx+a)-1)e^{(dx^2+c)}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfc(b*x+a)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfc}(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfc(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

3.198 $\int \left(\frac{e^{-b^2x^2}\mathbf{Erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\mathbf{Erfc}(bx)}{x}\right)dx$

Optimal. Leaf size=60

$$\sqrt{2}b^2\text{Erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2x^2}+\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

```
[Out] b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2*
E^(b^2*x^2)*x^2)
```

Rubi [A] time = 0.134725, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6392, 2214, 2205}

$$\sqrt{2}b^2\text{Erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2x^2}+\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfc[b*x])/(E^(b^2*x^2)*x),x]
```

```
[Out] b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2*
E^(b^2*x^2)*x^2)
```

Rule 6392

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfc[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x],
x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
```

```
] && LeQ[-n, m + 1]))
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)dx&=b^2\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx+\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx\\&=-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}-\frac{b\int\frac{e^{-2b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\&=\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}+\frac{\left(4b^3\right)\int e^{-2b^2x^2}\,dx}{\sqrt{\pi}}\\&=\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}+\sqrt{2}b^2\text{erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0774352, size = 60, normalized size = 1.

$$\sqrt{2}b^2\text{Erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{Erfc}(bx)}{2x^2}+\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfc[b*x])/(E^(b^2*x^2)*x),x]
```

```
[Out] b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2*
E^(b^2*x^2)*x^2)
```

Maple [A] time = 0.302, size = 84, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b}{2\,\mathrm{e}^{b^2x^2}x^2}+\frac{\text{Erf}\,(bx)\,b}{2\,\mathrm{e}^{b^2x^2}x^2}-\frac{b^3}{\sqrt{\pi}}\left(-\frac{1}{\left(\mathrm{e}^{b^2x^2}\right)^2bx}-\sqrt{2}\sqrt{\pi}\text{Erf}\left(bx\sqrt{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x)
```

```
[Out] (-1/2*b/exp(b^2*x^2)/x^2+1/2*erf(b*x)*b/exp(b^2*x^2)/x^2-1/Pi^(1/2)*b^3*(-1
/exp(b^2*x^2)^2/b/x-2^(1/2)*Pi^(1/2)*erf(b*x*2^(1/2))))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^2 \operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}}{x} + \frac{\operatorname{erfc}(bx)\, e^{\left(-b^2x^2\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x, algori
thm="maxima")
```

```
[Out] integrate(b^2*erfc(b*x)*e^(-b^2*x^2)/x + erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

Fricas [A] time = 2.29337, size = 184, normalized size = 3.07

$$\frac{2\sqrt{2}\pi\sqrt{b^2}bx^2 \operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right) + 2\sqrt{\pi}bxe^{\left(-2\,b^2x^2\right)} - (\pi - \pi \operatorname{erf}(bx))e^{\left(-b^2x^2\right)}}{2\,\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x, algori
thm="fricas")
```

```
[Out] 1/2*(2*sqrt(2)*pi*sqrt(b^2)*b*x^2*erf(sqrt(2)*sqrt(b^2)*x) + 2*sqrt(pi)*b*x
*e^(-2*b^2*x^2) - (pi - pi*erf(b*x))*e^(-b^2*x^2))/(pi*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 1\right) e^{-b^2x^2} \operatorname{erfc}(bx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b**2*x**2)/x**3+b**2*erfc(b*x)/exp(b**2*x**2)/x,x)
```

```
[Out] Integral((b**2*x**2 + 1)*exp(-b**2*x**2)*erfc(b*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^2 \operatorname{erfc}(bx) e^{\left(-b^2x^2\right)}}{x} + \frac{\operatorname{erfc}(bx) e^{\left(-b^2x^2\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x, algori
thm="giac")
```

```
[Out] integrate(b^2*erfc(b*x)*e^(-b^2*x^2)/x + erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

3.199 $\int \text{Erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx$

Optimal. Leaf size=91

$$-\frac{ibe^{-ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}+\frac{i\sqrt{\pi}e^{ic}\text{Erfc}(bx)^2}{8b}+\frac{i\sqrt{\pi}e^{-ic}\text{Erfi}(bx)}{4b}$$

```
[Out] ((I/8)*E^(I*c)*Sqrt[Pi]*Erfc[b*x]^2)/b + ((I/4)*Sqrt[Pi]*Erfi[b*x])/(b*E^(I
*c)) - ((I/2)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*
Sqrt[Pi])
```

Rubi [A] time = 0.0772999, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6405, 6377, 2204, 6376, 6374, 30}

$$-\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{i\sqrt{\pi}e^{ic}\text{Erfc}(bx)^2}{8b}+\frac{i\sqrt{\pi}e^{-ic}\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]*Sin[c + I*b^2*x^2],x]
```


```
[Out] ((I/8)*E^(I*c)*Sqrt[Pi]*Erfc[b*x]^2)/b + ((I/4)*Sqrt[Pi]*Erfi[b*x])/(b*E^(I
*c)) - ((I/2)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*
Sqrt[Pi])
```


Rule 6405

```
Int[Erfc[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[I/2, Int[
E^(-(I*c) - I*d*x^2)*Erfc[b*x], x], x] - Dist[I/2, Int[E^(I*c + I*d*x^2)*Er
fc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx &= -\left(\frac{1}{2}i\int e^{ic-b^2x^2}\operatorname{erfc}(bx)\,dx\right)+\frac{1}{2}i\int e^{-ic+b^2x^2}\operatorname{erfc}(bx)\,dx\\ &= \frac{1}{2}i\int e^{-ic+b^2x^2}\,dx-\frac{1}{2}i\int e^{-ic+b^2x^2}\operatorname{erf}(bx)\,dx+\frac{\left(ie^{ic}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfc}(bx)\right)}{4b}\\ &= \frac{ie^{ic}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}+\frac{ie^{-ic}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}-\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.567593, size = 94, normalized size = 1.03

$$\frac{(\sin(c)+i\cos(c))\left(-4b^2x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+\pi\left(\text{Erf}(bx)^2(\cos(2c)+i\sin(2c))-2\text{Erf}(bx)(\cos\right.\right.}{8\sqrt{\pi}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfc[b*x]*Sin[c + I*b^2*x^2],x]
```

```
[Out] ((I*Cos[c] + Sin[c])*(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^
2] + Pi*(2*Erfi[b*x] - 2*Erf[b*x]*(Cos[2*c] + I*Sin[2*c]) + Erf[b*x]^2*(Cos
[2*c] + I*Sin[2*c]))))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)*sin(c+I*b^2*x^2),x)
```

```
[Out] int(erfc(b*x)*sin(c+I*b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{i\sqrt{\pi}\cos(c)\operatorname{erfc}(bx)^2}{8b}-\frac{\sqrt{\pi}\operatorname{erfc}(bx)^2\sin(c)}{8b}+\frac{1}{2}i\cos(c)\int \operatorname{erfc}(bx)e^{\left(b^2x^2\right)}\,dx+\frac{1}{2}\int \operatorname{erfc}(bx)e^{\left(b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sin(c+I*b^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*I*sqrt(pi)*cos(c)*erfc(b*x)^2/b - 1/8*sqrt(pi)*erfc(b*x)^2*sin(c)/b + 1
/2*I*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erfc(b*x)*e
^(b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{2}\left((i\operatorname{erf}(bx)-i)e^{\left(-2b^2x^2+2ic\right)}-i\operatorname{erf}(bx)+i\right)e^{\left(b^2x^2-ic\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sin(c+I*b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/2*((I*erf(b*x) - I)*e^(-2*b^2*x^2 + 2*I*c) - I*erf(b*x) + I)*e^(
b^2*x^2 - I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(ib^2x^2 + c\right)\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sin(c+I*b**2*x**2),x)
```

```
[Out] Integral(sin(I*b**2*x**2 + c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\sin\left(i\,b^2x^2 + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sin(c+I*b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*sin(I*b^2*x^2 + c), x)
```

3.200 $\int \mathbf{Erfc}(bx)\sin\left(c - ib^2x^2\right)\,dx$

Optimal. Leaf size=91

$$\frac{ibe^{ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{-ic}\text{Erfc}(bx)^2}{8b}-\frac{i\sqrt{\pi}e^{ic}\text{Erfi}(bx)}{4b}$$

```
[Out] ((-I/8)*Sqrt[Pi]*Erfc[b*x]^2)/(b*E^(I*c)) - ((I/4)*E^(I*c)*Sqrt[Pi]*Erfi[b*
x])/b + ((I/2)*b*E^(I*c)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/
Sqrt[Pi]
```

Rubi [A] time = 0.0754201, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6405, 6374, 30, 6377, 2204, 6376}

$$\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{-ic}\text{Erfc}(bx)^2}{8b}-\frac{i\sqrt{\pi}e^{ic}\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]*Sin[c - I*b^2*x^2],x]
```

```
[Out] ((-I/8)*Sqrt[Pi]*Erfc[b*x]^2)/(b*E^(I*c)) - ((I/4)*E^(I*c)*Sqrt[Pi]*Erfi[b*
x])/b + ((I/2)*b*E^(I*c)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/
Sqrt[Pi]
```

Rule 6405

```
Int[Erfc[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[I/2, Int[
E^(-(I*c) - I*d*x^2)*Erfc[b*x], x], x] - Dist[I/2, Int[E^(I*c + I*d*x^2)*Er
fc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \text{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx &= \frac{1}{2}i\int e^{-ic-b^2x^2}\text{erfc}(bx)\,dx-\frac{1}{2}i\int e^{ic+b^2x^2}\text{erfc}(bx)\,dx\\ &= -\left(\frac{1}{2}i\int e^{ic+b^2x^2}\,dx\right)+\frac{1}{2}i\int e^{ic+b^2x^2}\text{erf}(bx)\,dx-\frac{\left(ie^{-ic}\sqrt{\pi}\right)\text{Subst}(\int x\,dx,x,\text{erfc}(bx))}{4b}\\ &= -\frac{ie^{-ic}\sqrt{\pi}\text{erfc}(bx)^2}{8b}-\frac{ie^{ic}\sqrt{\pi}\text{erfi}(bx)}{4b}+\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.437127, size = 101, normalized size = 1.11

$$\frac{1}{2}i\left(\frac{bx^2(\cos(c)+i\sin(c))\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{\sqrt{\pi}}-\frac{\sqrt{\pi}\left(\text{Erf}(bx)^2(\cos(c)-i\sin(c))-2\text{Erf}(bx)(\cos(c\right.}{4b}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfc[b*x]*Sin[c - I*b^2*x^2],x]
```

```
[Out] (I/2)*(-(Sqrt[Pi]*(-2*Erf[b*x]*(Cos[c] - I*Sin[c]) + Erf[b*x]^2*(Cos[c] - I
*Sin[c]) + 2*Erfi[b*x]*(Cos[c] + I*Sin[c])))/(4*b) + (b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cos[c] + I*Sin[c]))/Sqrt[Pi])
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int -\mathrm{erfc}\,(bx)\sin\left(-c+ib^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-erfc(b*x)*sin(-c+I*b^2*x^2),x)
```

```
[Out] int(-erfc(b*x)*sin(-c+I*b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i\sqrt{\pi}\cos(c)\,\mathrm{erfc}\,(bx)^2}{8\,b}-\frac{\sqrt{\pi}\,\mathrm{erfc}\,(bx)^2\sin(c)}{8\,b}-\frac{1}{2}i\,\cos(c)\int\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2\right)}\,dx+\frac{1}{2}\int\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sin(-c+I*b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/8*I*sqrt(pi)*cos(c)*erfc(b*x)^2/b - 1/8*sqrt(pi)*erfc(b*x)^2*sin(c)/b -
1/2*I*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erfc(b*x)*
e^(b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{2}\left((-i\,\mathrm{erf}\,(bx)+i)e^{\left(-2\,b^2x^2-2i\,c\right)}+i\,\mathrm{erf}\,(bx)-i\right)e^{\left(b^2x^2+i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sin(-c+I*b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/2*((-I*erf(b*x) + I)*e^(-2*b^2*x^2 - 2*I*c) + I*erf(b*x) - I)*e^
(b^2*x^2 + I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \sin\left(ib^2x^2 - c\right)\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sin(-c+I*b**2*x**2),x)
```

```
[Out] -Integral(sin(I*b**2*x**2 - c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{erfc}(bx)\sin\left(i\,b^2x^2 - c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sin(-c+I*b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(-erfc(b*x)*sin(I*b^2*x^2 - c), x)
```

3.201 $\int \cos\left(c+ib^2x^2\right) \mathbf{Erfc}(bx)\,dx$

Optimal. Leaf size=85

$$-\frac{be^{-ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{ic}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^{-ic}\text{Erfi}(bx)}{4b}$$

```
[Out] -(E^(I*c)*Sqrt[Pi]*Erfc[b*x]^2)/(8*b) + (Sqrt[Pi]*Erfi[b*x])/(4*b*E^(I*c))
- (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*E^(I*c)*Sqrt[Pi])
```

Rubi [A] time = 0.0734826, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6408, 6377, 2204, 6376, 6374, 30}

$$-\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{ic}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^{-ic}\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + I*b^2*x^2]*Erfc[b*x],x]
```



```
[Out] -(E^(I*c)*Sqrt[Pi]*Erfc[b*x]^2)/(8*b) + (Sqrt[Pi]*Erfi[b*x])/(4*b*E^(I*c))
- (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*E^(I*c)*Sqrt[Pi])
```


Rule 6408

```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[
E^(-(I*c) - I*d*x^2)*Erfc[b*x], x], x] + Dist[1/2, Int[E^(I*c + I*d*x^2)*Er
fc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos\left(c+ib^2x^2\right)\operatorname{erfc}(bx)\,dx &= \frac{1}{2}\int e^{ic-b^2x^2}\operatorname{erfc}(bx)\,dx+\frac{1}{2}\int e^{-ic+b^2x^2}\operatorname{erfc}(bx)\,dx\\ &= \frac{1}{2}\int e^{-ic+b^2x^2}\,dx-\frac{1}{2}\int e^{-ic+b^2x^2}\operatorname{erf}(bx)\,dx-\frac{\left(e^{ic}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfc}(bx)\right)}{4b}\\ &= -\frac{e^{ic}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}+\frac{e^{-ic}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}-\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 1.61014, size = 0, normalized size = 0.

$$\int \cos\left(c+ib^2x^2\right)\operatorname{Erfc}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + I*b^2*x^2]*Erfc[b*x],x]
```

```
[Out] Integrate[Cos[c + I*b^2*x^2]*Erfc[b*x], x]
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \cos \left(c + i b^2 x^2\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+I*b^2*x^2)*erfc(b*x),x)
```

```
[Out] int(cos(c+I*b^2*x^2)*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{\pi}\cos(c)\operatorname{erfc}(bx)^2}{8\,b} - \frac{i\sqrt{\pi}\operatorname{erfc}(bx)^2\sin(c)}{8\,b} + \frac{1}{2}\cos(c)\int \operatorname{erfc}(bx)e^{\left(b^2x^2\right)}\,dx - \frac{1}{2}i\int \operatorname{erfc}(bx)e^{\left(b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*cos(c)*erfc(b*x)^2/b - 1/8*I*sqrt(pi)*erfc(b*x)^2*sin(c)/b +
1/2*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) - 1/2*I*integrate(erfc(b*x)*
e^(b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{1}{2}\left((\operatorname{erf}(bx)-1)e^{\left(-2\,b^2x^2+2\,i\,c\right)} + \operatorname{erf}(bx)-1\right)e^{\left(b^2x^2-i\,c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-1/2*((erf(b*x) - 1)*e^(-2*b^2*x^2 + 2*I*c) + erf(b*x) - 1)*e^(b^2
*x^2 - I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(ib^2x^2 + c\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b**2*x**2)*erfc(b*x),x)
```

```
[Out] Integral(cos(I*b**2*x**2 + c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(i\, b^2x^2 + c\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(I*b^2*x^2 + c)*erfc(b*x), x)
```

3.202 $\int \cos\left(c - ib^2x^2\right) \mathbf{Erfc}(bx)\, dx$

Optimal. Leaf size=85

$$-\frac{be^{ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-ic}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^{ic}\text{Erfi}(bx)}{4b}$$

```
[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^(I*c)) + (E^(I*c)*Sqrt[Pi]*Erfi[b*x])/(4*b)
- (b*E^(I*c)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0738777, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6408, 6374, 30, 6377, 2204, 6376}

$$-\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-ic}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^{ic}\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c - I*b^2*x^2]*Erfc[b*x],x]
```

```
[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^(I*c)) + (E^(I*c)*Sqrt[Pi]*Erfi[b*x])/(4*b)
- (b*E^(I*c)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rule 6408

```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[
E^(-(I*c) - I*d*x^2)*Erfc[b*x], x], x] + Dist[1/2, Int[E^(I*c + I*d*x^2)*Er
fc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \cos\left(c-ib^2x^2\right)\operatorname{erfc}(bx)\,dx &= \frac{1}{2}\int e^{-ic-b^2x^2}\operatorname{erfc}(bx)\,dx+\frac{1}{2}\int e^{ic+b^2x^2}\operatorname{erfc}(bx)\,dx\\ &= \frac{1}{2}\int e^{ic+b^2x^2}\,dx-\frac{1}{2}\int e^{ic+b^2x^2}\operatorname{erf}(bx)\,dx-\frac{\left(e^{-ic}\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\operatorname{erfc}(bx))}{4b}\\ &= -\frac{e^{-ic}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}+\frac{e^{ic}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}-\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 1.73841, size = 0, normalized size = 0.

$$\int \cos\left(c-ib^2x^2\right)\operatorname{Erfc}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c - I*b^2*x^2]*Erfc[b*x],x]
```

```
[Out] Integrate[Cos[c - I*b^2*x^2]*Erfc[b*x], x]
```

Maple [F] time = 0.384, size = 0, normalized size = 0.

$$\int \cos\left(-c+ib^2x^2\right)\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(-c+I*b^2*x^2)*erfc(b*x),x)
```

```
[Out] int(cos(-c+I*b^2*x^2)*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{\pi}\cos(c)\operatorname{erfc}(bx)^2}{8\,b}+\frac{i\sqrt{\pi}\operatorname{erfc}(bx)^2\sin(c)}{8\,b}+\frac{1}{2}\cos(c)\int \operatorname{erfc}(bx)\,e^{\left(b^2x^2\right)}\,dx+\frac{1}{2}i\int \operatorname{erfc}(bx)\,e^{\left(b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*cos(c)*erfc(b*x)^2/b + 1/8*I*sqrt(pi)*erfc(b*x)^2*sin(c)/b +
1/2*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) + 1/2*I*integrate(erfc(b*x)*
e^(b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{1}{2}\left((\operatorname{erf}(bx)-1)e^{\left(-2\,b^2x^2-2i\,c\right)}+\operatorname{erf}(bx)-1\right)e^{\left(b^2x^2+i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-1/2*((erf(b*x) - 1)*e^(-2*b^2*x^2 - 2*I*c) + erf(b*x) - 1)*e^(b^2
*x^2 + I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(ib^2x^2 - c\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b**2*x**2)*erfc(b*x),x)
```

```
[Out] Integral(cos(I*b**2*x**2 - c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(i\, b^2x^2 - c\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(I*b^2*x^2 - c)*erfc(b*x), x)
```

3.203 $\int \textbf{Erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx$

Optimal. Leaf size=75

$$-\frac{be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{4b}$$

```
[Out] (Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^c) + (E^c*Sqrt[Pi]*Erfi[b*x])/(4*b) - (b*E^c*
x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.069854, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6411, 6377, 2204, 6376, 6374, 30}

$$-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]*Sinh[c + b^2*x^2],x]
```


```
[Out] (Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^c) + (E^c*Sqrt[Pi]*Erfi[b*x])/(4*b) - (b*E^c*
x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```


Rule 6411

```
Int[Erfc[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfc[b*x], x], x] - Dist[1/2, Int[E^(-c - d*x^2)*Erfc[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx &= -\left(\frac{1}{2}\int e^{-c-b^2x^2}\operatorname{erfc}(bx)\,dx\right)+\frac{1}{2}\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx\\ &= \frac{1}{2}\int e^{c+b^2x^2}\,dx-\frac{1}{2}\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx+\frac{\left(e^{-c}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfc}(bx)\right)}{4b}\\ &= \frac{e^{-c}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}+\frac{e^{c}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.160899, size = 83, normalized size = 1.11

$$\frac{\pi\left(\operatorname{Erf}(bx)^2(\cosh(c)-\sinh(c))-2\operatorname{Erf}(bx)(\cosh(c)-\sinh(c))+2\operatorname{Erfi}(bx)(\sinh(c)+\cosh(c))\right)-4b^2x^2(\sinh(c)+\cosh(c)}{8\sqrt{\pi}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfc[b*x]*Sinh[c + b^2*x^2],x]
```

```
[Out] (-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c]
) + Pi*(-2*Erf[b*x]*(Cosh[c] - Sinh[c]) + Erf[b*x]^2*(Cosh[c] - Sinh[c]) +
```

```
2*Erfi[b*x]*(Cosh[c] + Sinh[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\sinh\left(b^2x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfc(b*x)*sinh(b^2*x^2+c),x)
```

```
[Out] int(erfc(b*x)*sinh(b^2*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\sinh\left(b^2x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sinh(b^2*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(erfc(b*x)*sinh(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-(\operatorname{erf}(bx)-1)\sinh\left(b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sinh(b^2*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(erf(b*x) - 1)*sinh(b^2*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(b^2x^2+c\right)\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sinh(b**2*x**2+c),x)
```

```
[Out] Integral(sinh(b**2*x**2 + c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfc}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfc(b*x)*sinh(b^2*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(erfc(b*x)*sinh(b^2*x^2 + c), x)
```

3.204 $\int \mathbf{Erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx$

Optimal. Leaf size=77

$$\frac{be^{-c}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{8b}-\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)}{4b}$$

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(8*b) - (Sqrt[Pi]*Erfi[b*x])/(4*b*E^c) + (b*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rubi [A] time = 0.0707612, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6411, 6374, 30, 6377, 2204, 6376}

$$\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{8b}-\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfc[b*x]*Sinh[c - b^2*x^2],x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(8*b) - (Sqrt[Pi]*Erfi[b*x])/(4*b*E^c) + (b*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rule 6411

```
Int[Erfc[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfc[b*x], x], x] - Dist[1/2, Int[E^(-c - d*x^2)*Erfc[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx &= \frac{1}{2}\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx-\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erfc}(bx)\,dx\\ &= -\left(\frac{1}{2}\int e^{-c+b^2x^2}\,dx\right)+\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erf}(bx)\,dx-\frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\operatorname{erfc}(bx))}{4b}\\ &= -\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}-\frac{e^{-c}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}+\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.145196, size = 84, normalized size = 1.09

$$-\frac{(\cosh(c)-\sinh(c))\left(\pi\left(\operatorname{Erf}(bx)^2(\sinh(2c)+\cosh(2c))-2\operatorname{Erf}(bx)(\sinh(2c)+\cosh(2c))+2\operatorname{Erfi}(bx)\right)-4b^2x^2\text{Hyperge}\right.}{8\sqrt{\pi}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfc[b*x]*Sinh[c - b^2*x^2],x]
```

```
[Out] -((Cosh[c] - Sinh[c])*(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x
^2] + Pi*(2*Erfi[b*x] - 2*Erf[b*x]*(Cosh[2*c] + Sinh[2*c]) + Erf[b*x]^2*(Co
sh[2*c] + Sinh[2*c]))))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int -\mathrm{erfc}\,(bx)\sinh\left(b^{2}x^{2}-c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-erfc(b*x)*sinh(b^2*x^2-c),x)
```

```
[Out] int(-erfc(b*x)*sinh(b^2*x^2-c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \mathrm{erfc}\,(bx)\sinh\left(b^{2}x^{2}-c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sinh(b^2*x^2-c),x, algorithm="maxima")
```

```
[Out] -integrate(erfc(b*x)*sinh(b^2*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((\mathrm{erf}\,(bx)-1)\sinh\left(b^{2}x^{2}-c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sinh(b^2*x^2-c),x, algorithm="fricas")
```

```
[Out] integral((erf(b*x) - 1)*sinh(b^2*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \sinh\left(b^{2}x^{2}-c\right)\mathrm{erfc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sinh(b**2*x**2-c),x)
```

```
[Out] -Integral(sinh(b**2*x**2 - c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{erfc}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfc(b*x)*sinh(b^2*x^2-c),x, algorithm="giac")
```

```
[Out] integrate(-erfc(b*x)*sinh(b^2*x^2 - c), x)
```

3.205 $\int \cosh\left(c+b^2x^2\right) \mathbf{Erfc}(bx)\, dx$

Optimal. Leaf size=75

$$-\frac{be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{4b}$$

```
[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^c) + (E^c*Sqrt[Pi]*Erfi[b*x])/(4*b) - (b*E^c
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0716178, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6414, 6377, 2204, 6376, 6374, 30}

$$-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^c\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + b^2*x^2]*Erfc[b*x],x]
```


```
[Out] -(Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^c) + (E^c*Sqrt[Pi]*Erfi[b*x])/(4*b) - (b*E^c
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```


Rule 6414

```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfc[b*x], x], x] + Dist[1/2, Int[E^(-c - d*x^2)*Erfc[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh\left(c+b^2x^2\right)\text{erfc}(bx)\,dx &= \frac{1}{2}\int e^{-c-b^2x^2}\text{erfc}(bx)\,dx+\frac{1}{2}\int e^{c+b^2x^2}\text{erfc}(bx)\,dx\\ &= \frac{1}{2}\int e^{c+b^2x^2}\,dx-\frac{1}{2}\int e^{c+b^2x^2}\text{erf}(bx)\,dx-\frac{\left(e^{-c}\sqrt{\pi}\right)\text{Subst}(\int x\,dx,x,\text{erfc}(bx))}{4b}\\ &= -\frac{e^{-c}\sqrt{\pi}\text{erfc}(bx)^2}{8b}+\frac{e^c\sqrt{\pi}\text{erfi}(bx)}{4b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.116303, size = 114, normalized size = 1.52

$$\frac{-4b^2x^2\sinh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+4b^2x^2\cosh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{8\sqrt{\pi}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + b^2*x^2]*Erfc[b*x],x]
```

```
[Out] (4*b^2*x^2*Cosh[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)] - 4*b^2*
x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*Sinh[c] + Pi*(Erf[b*x]^2*(
```

```
-Cosh[c] + Sinh[c]) + 2*Erfi[b*x]*(Cosh[c] + Sinh[c]) - 2*Erf[b*x]*(-Cosh[c
] + Cosh[c]*Erfi[b*x] + Sinh[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b^2*x^2+c)*erfc(b*x),x)
```

```
[Out] int(cosh(b^2*x^2+c)*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right) \operatorname{erfc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b^2*x^2 + c)*erfc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\cosh\left(b^2x^2+c\right)\operatorname{erf}(bx)+\cosh\left(b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-cosh(b^2*x^2 + c)*erf(b*x) + cosh(b^2*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right)\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b**2*x**2+c)*erfc(b*x),x)
```

```
[Out] Integral(cosh(b**2*x**2 + c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right)\operatorname{erfc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(b^2*x^2 + c)*erfc(b*x), x)
```

3.206 $\int \cosh\left(c-b^2x^2\right) \textbf{Erfc}(bx)\,dx$

Optimal. Leaf size=77

$$-\frac{be^{-c}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)}{4b}$$

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(8*b) + (Sqrt[Pi]*Erfi[b*x])/(4*b*E^c) - (b*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rubi [A] time = 0.0714556, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6414, 6374, 30, 6377, 2204, 6376}

$$-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^c\text{Erfc}(bx)^2}{8b}+\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c - b^2*x^2]*Erfc[b*x],x]
```

```
[Out] -(E^c*Sqrt[Pi]*Erfc[b*x]^2)/(8*b) + (Sqrt[Pi]*Erfi[b*x])/(4*b*E^c) - (b*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Rule 6414

```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfc[b*x], x], x] + Dist[1/2, Int[E^(-c - d*x^2)*Erfc[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6374

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(E^
c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6377

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rubi steps

$$\begin{aligned}\int \cosh\left(c-b^2x^2\right)\operatorname{erfc}(bx)\,dx &= \frac{1}{2}\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx+\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erfc}(bx)\,dx\\ &= \frac{1}{2}\int e^{-c+b^2x^2}\,dx-\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erf}(bx)\,dx-\frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfc}(bx)\right)}{4b}\\ &= -\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}+\frac{e^{-c}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.11046, size = 117, normalized size = 1.52

$$\frac{4b^2x^2\sinh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+4b^2x^2\cosh(c)\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)\cdots}{8\sqrt{\pi}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c - b^2*x^2]*Erfc[b*x],x]
```

```
[Out] (4*b^2*x^2*Cosh[c]*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)] + 4*b^2*
x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*Sinh[c] - Pi*(2*Erf[b*x]*(
-Cosh[c] + Cosh[c]*Erfi[b*x] - Sinh[c]) + 2*Erfi[b*x]*(-Cosh[c] + Sinh[c])
```

```
+ Erf[b*x]^2*(Cosh[c] + Sinh[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2-c\right) \operatorname{erfc}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b^2*x^2-c)*erfc(b*x),x)
```

```
[Out] int(cosh(b^2*x^2-c)*erfc(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2-c\right) \operatorname{erfc}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erfc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b^2*x^2 - c)*erfc(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\cosh \left(b^2x^2-c\right) \operatorname{erf}(bx)+\cosh \left(b^2x^2-c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erfc(b*x),x, algorithm="fricas")
```

```
[Out] integral(-cosh(b^2*x^2 - c)*erf(b*x) + cosh(b^2*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2-c\right) \operatorname{erfc}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b**2*x**2-c)*erfc(b*x),x)
```

```
[Out] Integral(cosh(b**2*x**2 - c)*erfc(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2 - c\right) \operatorname{erfc}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erfc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(b^2*x^2 - c)*erfc(b*x), x)
```

3.207 $\int x^5 \mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=93

$$\frac{5\text{Erfi}(bx)}{16b^6}-\frac{x^5e^{b^2x^2}}{6\sqrt{\pi}b}+\frac{5x^3e^{b^2x^2}}{12\sqrt{\pi}b^3}-\frac{5xe^{b^2x^2}}{8\sqrt{\pi}b^5}+\frac{1}{6}x^6\text{Erfi}(bx)$$

```
[Out] (-5*E^(b^2*x^2)*x)/(8*b^5*Sqrt[Pi]) + (5*E^(b^2*x^2)*x^3)/(12*b^3*Sqrt[Pi])
 - (E^(b^2*x^2)*x^5)/(6*b*Sqrt[Pi]) + (5*Erfi[b*x])/(16*b^6) + (x^6*Erfi[b*
x])/6
```

Rubi [A] time = 0.0823144, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2212, 2204}

$$\frac{5\text{Erfi}(bx)}{16b^6}-\frac{x^5e^{b^2x^2}}{6\sqrt{\pi}b}+\frac{5x^3e^{b^2x^2}}{12\sqrt{\pi}b^3}-\frac{5xe^{b^2x^2}}{8\sqrt{\pi}b^5}+\frac{1}{6}x^6\text{Erfi}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Erfi[b*x],x]
```

```
[Out] (-5*E^(b^2*x^2)*x)/(8*b^5*Sqrt[Pi]) + (5*E^(b^2*x^2)*x^3)/(12*b^3*Sqrt[Pi])
 - (E^(b^2*x^2)*x^5)/(6*b*Sqrt[Pi]) + (5*Erfi[b*x])/(16*b^6) + (x^6*Erfi[b*
x])/6
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^5\text{erfi}(bx)\,dx &= \frac{1}{6}x^6\text{erfi}(bx) - \frac{b\int e^{b^2x^2}x^6\,dx}{3\sqrt{\pi}}\\
&= -\frac{e^{b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx) + \frac{5\int e^{b^2x^2}x^4\,dx}{6b\sqrt{\pi}}\\
&= \frac{5e^{b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx) - \frac{5\int e^{b^2x^2}x^2\,dx}{4b^3\sqrt{\pi}}\\
&= -\frac{5e^{b^2x^2}x}{8b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx) + \frac{5\int e^{b^2x^2}\,dx}{8b^5\sqrt{\pi}}\\
&= -\frac{5e^{b^2x^2}x}{8b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{5\text{erfi}(bx)}{16b^6} + \frac{1}{6}x^6\text{erfi}(bx)\end{aligned}$$

Mathematica [A] time = 0.0264514, size = 64, normalized size = 0.69

$$\frac{\sqrt{\pi}\left(8b^6x^6+15\right)\text{Erfi}(bx) - 2bxe^{b^2x^2}\left(4b^4x^4-10b^2x^2+15\right)}{48\sqrt{\pi}b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Erfi[b*x],x]
```

```
[Out] (-2*b*E^(b^2*x^2)*x*(15 - 10*b^2*x^2 + 4*b^4*x^4) + Sqrt[Pi]*(15 + 8*b^6*x^
6)*Erfi[b*x])/(48*b^6*Sqrt[Pi])
```

Maple [A] time = 0.043, size = 77, normalized size = 0.8

$$\frac{1}{b^6}\left(\frac{b^6x^6\text{erfi}\,(bx)}{6} - \frac{1}{3\sqrt{\pi}}\left(\frac{b^5x^5\text{e}^{b^2x^2}}{2} - \frac{5\,\text{e}^{b^2x^2}b^3x^3}{4} + \frac{15\,\text{e}^{b^2x^2}bx}{8} - \frac{15\sqrt{\pi}\text{erfi}\,(bx)}{16}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erfi(b*x),x)
```

```
[Out] 1/b^6*(1/6*b^6*x^6*erfi(b*x)-1/3/Pi^(1/2)*(1/2*b^5*x^5*exp(b^2*x^2)-5/4*exp
(b^2*x^2)*b^3*x^3+15/8*exp(b^2*x^2)*b*x-15/16*Pi^(1/2)*erfi(b*x)))
```

Maxima [C] time = 1.04066, size = 85, normalized size = 0.91

$$\frac{1}{6}\,x^{6}\,\mathrm{erfi}\left(bx\right)-\frac{b\left(\frac{2\left(4\,b^{4}x^{5}-10\,b^{2}x^{3}+15\,x\right)e^{\left(b^{2}x^{2}\right)}}{b^{6}}+\frac{15i\,\sqrt{\pi}\,\mathrm{erf}\left(i\,bx\right)}{b^{7}}\right)}{48\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*erfi(b*x) - 1/48*b*(2*(4*b^4*x^5 - 10*b^2*x^3 + 15*x)*e^(b^2*x^2)/b
^6 + 15*I*sqrt(pi)*erf(I*b*x)/b^7)/sqrt(pi)
```

Fricas [A] time = 2.39547, size = 149, normalized size = 1.6

$$-\frac{2\,\sqrt{\pi}\left(4\,b^{5}x^{5}-10\,b^{3}x^{3}+15\,bx\right)e^{\left(b^{2}x^{2}\right)}-\left(15\,\pi+8\,\pi b^{6}x^{6}\right)\mathrm{erfi}\left(bx\right)}{48\,\pi b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x),x, algorithm="fricas")
```

```
[Out] -1/48*(2*sqrt(pi)*(4*b^5*x^5 - 10*b^3*x^3 + 15*b*x)*e^(b^2*x^2) - (15*pi +
8*pi*b^6*x^6)*erfi(b*x))/(pi*b^6)
```

Sympy [A] time = 3.70205, size = 88, normalized size = 0.95

$$\begin{cases}\frac{x^{6}\,\mathrm{erfi}\left(bx\right)}{6}-\frac{x^{5}e^{b^{2}x^{2}}}{6\sqrt{\pi}b}+\frac{5x^{3}e^{b^{2}x^{2}}}{12\sqrt{\pi}b^{3}}-\frac{5xe^{b^{2}x^{2}}}{8\sqrt{\pi}b^{5}}+\frac{5\,\mathrm{erfi}\left(bx\right)}{16b^{6}} & \text{for}\ b\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erfi(b*x),x)
```

```
[Out] Piecewise((x**6*erfi(b*x)/6 - x**5*exp(b**2*x**2)/(6*sqrt(pi)*b) + 5*x**3*e
xp(b**2*x**2)/(12*sqrt(pi)*b**3) - 5*x*exp(b**2*x**2)/(8*sqrt(pi)*b**5) + 5
*erfi(b*x)/(16*b**6), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfi(b*x), x)
```

3.208 $\int x^3 \mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=69

$$-\frac{3\text{Erfi}(bx)}{16b^4}-\frac{x^3e^{b^2x^2}}{4\sqrt{\pi}b}+\frac{3xe^{b^2x^2}}{8\sqrt{\pi}b^3}+\frac{1}{4}x^4\text{Erfi}(bx)$$

```
[Out] (3*E^(b^2*x^2)*x)/(8*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^3)/(4*b*Sqrt[Pi]) - (3*
Erfi[b*x])/(16*b^4) + (x^4*Erfi[b*x])/4
```

Rubi [A] time = 0.0578031, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2212, 2204}

$$-\frac{3\text{Erfi}(bx)}{16b^4}-\frac{x^3e^{b^2x^2}}{4\sqrt{\pi}b}+\frac{3xe^{b^2x^2}}{8\sqrt{\pi}b^3}+\frac{1}{4}x^4\text{Erfi}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Erfi[b*x],x]
```

```
[Out] (3*E^(b^2*x^2)*x)/(8*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^3)/(4*b*Sqrt[Pi]) - (3*
Erfi[b*x])/(16*b^4) + (x^4*Erfi[b*x])/4
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^3\text{erfi}(bx)\,dx &= \frac{1}{4}x^4\text{erfi}(bx) - \frac{b\int e^{b^2x^2}x^4\,dx}{2\sqrt{\pi}}\\ &= -\frac{e^{b^2x^2}x^3}{4b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfi}(bx) + \frac{3\int e^{b^2x^2}x^2\,dx}{4b\sqrt{\pi}}\\ &= \frac{3e^{b^2x^2}x}{8b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3}{4b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfi}(bx) - \frac{3\int e^{b^2x^2}\,dx}{8b^3\sqrt{\pi}}\\ &= \frac{3e^{b^2x^2}x}{8b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3}{4b\sqrt{\pi}} - \frac{3\text{erfi}(bx)}{16b^4} + \frac{1}{4}x^4\text{erfi}(bx)\end{aligned}$$

Mathematica [A] time = 0.0280703, size = 51, normalized size = 0.74

$$\frac{\left(4b^4x^4-3\right)\text{Erfi}(bx)-\frac{2bxe^{b^2x^2}\left(2b^2x^2-3\right)}{\sqrt{\pi}}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Erfi[b*x],x]
```

```
[Out] ((-2*b*E^(b^2*x^2)*x*(-3 + 2*b^2*x^2))/Sqrt[Pi] + (-3 + 4*b^4*x^4)*Erfi[b*x
])/(16*b^4)
```

Maple [A] time = 0.043, size = 61, normalized size = 0.9

$$\frac{1}{b^4}\left(\frac{b^4x^4\text{erfi}\,(bx)}{4}-\frac{1}{2\sqrt{\pi}}\left(\frac{e^{b^2x^2}b^3x^3}{2}-\frac{3\,e^{b^2x^2}bx}{4}+\frac{3\sqrt{\pi}\text{erfi}\,(bx)}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erfi(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*erfi(b*x)-1/2/Pi^(1/2)*(1/2*exp(b^2*x^2)*b^3*x^3-3/4*exp
(b^2*x^2)*b*x+3/8*Pi^(1/2)*erfi(b*x)))
```

Maxima [C] time = 0.983883, size = 74, normalized size = 1.07

$$\frac{1}{4}\,x^4\,\mathrm{erfi}\,(bx)-\frac{b\left(\frac{2\,(2\,b^2x^3-3\,x)e^{\left(b^2x^2\right)}}{b^4}-\frac{3i\,\sqrt{\pi}\,\mathrm{erf}(i\,bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*erfi(b*x) - 1/16*b*(2*(2*b^2*x^3 - 3*x)*e^(b^2*x^2)/b^4 - 3*I*sqrt(
pi)*erf(I*b*x)/b^5)/sqrt(pi)
```

Fricas [A] time = 2.12278, size = 128, normalized size = 1.86

$$-\frac{2\,\sqrt{\pi}\left(2\,b^3x^3-3\,bx\right)e^{\left(b^2x^2\right)}+\left(3\,\pi-4\,\pi b^4x^4\right)\mathrm{erfi}\,(bx)}{16\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x),x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(pi)*(2*b^3*x^3 - 3*b*x)*e^(b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*e
rfi(b*x))/(pi*b^4)
```

Sympy [A] time = 1.01405, size = 65, normalized size = 0.94

$$\begin{cases} \frac{x^4\,\mathrm{erfi}\,(bx)}{4}-\frac{x^3e^{b^2x^2}}{4\sqrt{\pi}b}+\frac{3xe^{b^2x^2}}{8\sqrt{\pi}b^3}-\frac{3\,\mathrm{erfi}\,(bx)}{16b^4} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erfi(b*x),x)
```

```
[Out] Piecewise((x**4*erfi(b*x)/4 - x**3*exp(b**2*x**2)/(4*sqrt(pi)*b) + 3*x*exp(
b**2*x**2)/(8*sqrt(pi)*b**3) - 3*erfi(b*x)/(16*b**4), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfi(b*x), x)
```

3.209 $\int x\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=45

$$\frac{\mathrm{Erfi}(bx)}{4b^2}-\frac{xe^{b^2x^2}}{2\sqrt{\pi}b}+\frac{1}{2}x^2\mathrm{Erfi}(bx)$$

```
[Out] -(E^(b^2*x^2)*x)/(2*b*Sqrt[Pi]) + Erfi[b*x]/(4*b^2) + (x^2*Erfi[b*x])/2
```

Rubi [A] time = 0.0331474, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6363, 2212, 2204}

$$\frac{\mathrm{Erfi}(bx)}{4b^2}-\frac{xe^{b^2x^2}}{2\sqrt{\pi}b}+\frac{1}{2}x^2\mathrm{Erfi}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*Erfi[b*x],x]
```

```
[Out] -(E^(b^2*x^2)*x)/(2*b*Sqrt[Pi]) + Erfi[b*x]/(4*b^2) + (x^2*Erfi[b*x])/2
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\text{erfi}(bx)\,dx &= \frac{1}{2}x^2\text{erfi}(bx) - \frac{b\int e^{b^2x^2}x^2\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{b^2x^2}x}{2b\sqrt{\pi}} + \frac{1}{2}x^2\text{erfi}(bx) + \frac{\int e^{b^2x^2}\,dx}{2b\sqrt{\pi}}\\ &= -\frac{e^{b^2x^2}x}{2b\sqrt{\pi}} + \frac{\text{erfi}(bx)}{4b^2} + \frac{1}{2}x^2\text{erfi}(bx)\end{aligned}$$

Mathematica [A] time = 0.0276535, size = 39, normalized size = 0.87

$$\frac{1}{4}\left(\left(\frac{1}{b^2}+2x^2\right)\text{Erfi}(bx) - \frac{2xe^{b^2x^2}}{\sqrt{\pi}b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erfi[b*x],x]
```

```
[Out] ((-2*E^(b^2*x^2)*x)/(b*Sqrt[Pi]) + (b^(-2) + 2*x^2)*Erfi[b*x])/4
```

Maple [A] time = 0.043, size = 45, normalized size = 1.

$$\frac{1}{b^2}\left(\frac{b^2x^2\text{erfi}\,(bx)}{2} - \frac{1}{\sqrt{\pi}}\left(\frac{e^{b^2x^2}bx}{2} - \frac{\sqrt{\pi}\text{erfi}\,(bx)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfi(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*erfi(b*x)-1/Pi^(1/2)*(1/2*exp(b^2*x^2)*b*x-1/4*Pi^(1/2)*
erfi(b*x)))
```

Maxima [C] time = 1.03254, size = 59, normalized size = 1.31

$$\frac{1}{2}\,x^2\,\mathrm{erfi}\,(bx) - \frac{b\left(\frac{2\,xe^{\left(b^2x^2\right)}}{b^2} + \frac{i\,\sqrt{\pi}\,\mathrm{erf}(i\,bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*erfi(b*x) - 1/4*b*(2*x*e^(b^2*x^2)/b^2 + I*sqrt(pi)*erf(I*b*x)/b^3)
/sqrt(pi)
```

Fricas [A] time = 2.29701, size = 103, normalized size = 2.29

$$-\frac{2\,\sqrt{\pi}bxe^{\left(b^2x^2\right)} - \left(\pi + 2\,\pi b^2x^2\right)\mathrm{erfi}\,(bx)}{4\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(pi)*b*x*e^(b^2*x^2) - (pi + 2*pi*b^2*x^2)*erfi(b*x))/(pi*b^2)
```

Sympy [A] time = 0.23252, size = 39, normalized size = 0.87

$$\begin{cases} \frac{x^2\,\mathrm{erfi}\,(bx)}{2} - \frac{xe^{b^2x^2}}{2\sqrt{\pi}b} + \frac{\mathrm{erfi}\,(bx)}{4b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x),x)
```

```
[Out] Piecewise((x**2*erfi(b*x)/2 - x*exp(b**2*x**2)/(2*sqrt(pi)*b) + erfi(b*x)/(
4*b**2), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*erfi(b*x), x)
```

3.210 $\int \frac{\mathbf{Erfi}(bx)}{x}\,dx$

Optimal. Leaf size=31

$$\frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0125711, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6360}

$$\frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/x,x]
```

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rule 6360

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{\text{erfi}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0138789, size = 31, normalized size = 1.

$$\frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x,x]
```

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Maple [A] time = 0.057, size = 22, normalized size = 0.7

$$2\,{\frac{bx{\mbox{$_2$F$_2$}(1/2,1/2;\,3/2,3/2;\,{b}^{2}{x}^{2})}}{\sqrt{\pi}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x,x)
```

```
[Out] 2/Pi^(1/2)*b*x*hypergeom([1/2,1/2],[3/2,3/2],b^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erfi}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x, x)
```

3.211 $\int \frac{\text{Erfi}(bx)}{x^3}\,dx$

Optimal. Leaf size=40

$$b^2\text{Erfi}(bx) - \frac{be^{b^2x^2}}{\sqrt{\pi}x} - \frac{\text{Erfi}(bx)}{2x^2}$$

```
[Out] -((b*E^(b^2*x^2))/(Sqrt[Pi]*x)) + b^2*Erfi[b*x] - Erfi[b*x]/(2*x^2)
```

Rubi [A] time = 0.0356932, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2214, 2204}

$$b^2\text{Erfi}(bx) - \frac{be^{b^2x^2}}{\sqrt{\pi}x} - \frac{\text{Erfi}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/x^3,x]
```

```
[Out] -((b*E^(b^2*x^2))/(Sqrt[Pi]*x)) + b^2*Erfi[b*x] - Erfi[b*x]/(2*x^2)
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{erfi}(bx)}{x^3}\,dx &= -\frac{\text{erfi}(bx)}{2x^2}+\frac{b\int \frac{e^{b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{be^{b^2x^2}}{\sqrt{\pi}x}-\frac{\text{erfi}(bx)}{2x^2}+\frac{\left(2b^3\right)\int e^{b^2x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{be^{b^2x^2}}{\sqrt{\pi}x}+b^2\text{erfi}(bx)-\frac{\text{erfi}(bx)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0224858, size = 37, normalized size = 0.92

$$\left(b^2-\frac{1}{2x^2}\right)\text{Erfi}(bx)-\frac{be^{b^2x^2}}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x^3,x]
```

```
[Out] -((b*E^(b^2*x^2))/(Sqrt[Pi]*x)) + (b^2 - 1/(2*x^2))*Erfi[b*x]
```

Maple [A] time = 0.042, size = 47, normalized size = 1.2

$$b^2\left(-\frac{\text{erfi}\,(bx)}{2\,b^2x^2}+\frac{1}{\sqrt{\pi}}\left(-\frac{e^{b^2x^2}}{bx}+\sqrt{\pi}\text{erfi}\,(bx)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2/b^2/x^2*erfi(b*x)+1/Pi^(1/2)*(-exp(b^2*x^2)/b/x+Pi^(1/2)*erfi(b*x
)))
```

Maxima [A] time = 1.08743, size = 53, normalized size = 1.32

$$-\frac{\sqrt{-b^2x^2}b\Gamma\left(-\frac{1}{2}, -b^2x^2\right)}{2\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-b^2*x^2)*b*gamma(-1/2, -b^2*x^2)/(sqrt(pi)*x) - 1/2*erfi(b*x)/x^
2
```

Fricas [A] time = 2.24377, size = 103, normalized size = 2.58

$$-\frac{2\sqrt{\pi}bxe^{\left(b^2x^2\right)} + \left(\pi - 2\pi b^2x^2\right)\operatorname{erfi}(bx)}{2\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(pi)*b*x*e^(b^2*x^2) + (pi - 2*pi*b^2*x^2)*erfi(b*x))/(pi*x^2)
```

Sympy [A] time = 0.557861, size = 34, normalized size = 0.85

$$b^2\operatorname{erfi}(bx) - \frac{be^{b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x**3,x)
```

```
[Out] b**2*erfi(b*x) - b*exp(b**2*x**2)/(sqrt(pi)*x) - erfi(b*x)/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x^3, x)
```

3.212 $\int \frac{\textbf{Erfi}(bx)}{x^5} dx$

Optimal. Leaf size=69

$$\frac{1}{3}b^4\text{Erfi}(bx) - \frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x} - \frac{be^{b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{Erfi}(bx)}{4x^4}$$

```
[Out] -(b*E^(b^2*x^2))/(6*Sqrt[Pi]*x^3) - (b^3*E^(b^2*x^2))/(3*Sqrt[Pi]*x) + (b^4
*Erfi[b*x])/3 - Erfi[b*x]/(4*x^4)
```

Rubi [A] time = 0.0550196, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2214, 2204}

$$\frac{1}{3}b^4\text{Erfi}(bx) - \frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x} - \frac{be^{b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{Erfi}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/x^5,x]
```

```
[Out] -(b*E^(b^2*x^2))/(6*Sqrt[Pi]*x^3) - (b^3*E^(b^2*x^2))/(3*Sqrt[Pi]*x) + (b^4
*Erfi[b*x])/3 - Erfi[b*x]/(4*x^4)
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erfi}(bx)}{x^5}\,dx &= -\frac{\operatorname{erfi}(bx)}{4x^4}+\frac{b\int \frac{e^{b^2x^2}}{x^4}\,dx}{2\sqrt{\pi}}\\
&= -\frac{be^{b^2x^2}}{6\sqrt{\pi}x^3}-\frac{\operatorname{erfi}(bx)}{4x^4}+\frac{b^3\int \frac{e^{b^2x^2}}{x^2}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{b^2x^2}}{6\sqrt{\pi}x^3}-\frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x}-\frac{\operatorname{erfi}(bx)}{4x^4}+\frac{\left(2b^5\right)\int e^{b^2x^2}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{b^2x^2}}{6\sqrt{\pi}x^3}-\frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x}+\frac{1}{3}b^4\operatorname{erfi}(bx)-\frac{\operatorname{erfi}(bx)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0246377, size = 51, normalized size = 0.74

$$\frac{\left(4b^4x^4-3\right)\operatorname{Erfi}(bx)-\frac{2bxe^{b^2x^2}\left(2b^2x^2+1\right)}{\sqrt{\pi}}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x^5,x]
```

```
[Out] ((-2*b*E^(b^2*x^2)*x*(1 + 2*b^2*x^2))/Sqrt[Pi] + (-3 + 4*b^4*x^4)*Erfi[b*x]
)/(12*x^4)
```

Maple [A] time = 0.042, size = 65, normalized size = 0.9

$$b^4\left(-\frac{\operatorname{erfi}(bx)}{4x^4b^4}+\frac{1}{2\sqrt{\pi}}\left(-\frac{e^{b^2x^2}}{3x^3b^3}-\frac{2e^{b^2x^2}}{3bx}+\frac{2\sqrt{\pi}\operatorname{erfi}(bx)}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4/b^4/x^4*erfi(b*x)+1/2/Pi^(1/2)*(-1/3*exp(b^2*x^2)/b^3/x^3-2/3*exp
(b^2*x^2)/b/x+2/3*Pi^(1/2)*erfi(b*x)))
```

Maxima [A] time = 1.09734, size = 53, normalized size = 0.77

$$-\frac{\left(-b^2x^2\right)^{\frac{3}{2}}b\Gamma\left(-\frac{3}{2},-b^2x^2\right)}{4\sqrt{\pi}x^3}-\frac{\operatorname{erfi}(bx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(-b^2*x^2)^(3/2)*b*gamma(-3/2, -b^2*x^2)/(sqrt(pi)*x^3) - 1/4*erfi(b*x
)/x^4
```

Fricas [A] time = 2.32028, size = 126, normalized size = 1.83

$$-\frac{2\sqrt{\pi}\left(2b^3x^3+bx\right)e^{\left(b^2x^2\right)}+\left(3\pi-4\pi b^4x^4\right)\operatorname{erfi}(bx)}{12\pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(pi)*(2*b^3*x^3 + b*x)*e^(b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*erf
i(b*x))/(pi*x^4)
```

Sympy [A] time = 1.50513, size = 60, normalized size = 0.87

$$\frac{b^4\operatorname{erfi}(bx)}{3}-\frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x}-\frac{be^{b^2x^2}}{6\sqrt{\pi}x^3}-\frac{\operatorname{erfi}(bx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x**5,x)
```

```
[Out] b**4*erfi(b*x)/3 - b**3*exp(b**2*x**2)/(3*sqrt(pi)*x) - b*exp(b**2*x**2)/(6
*sqrt(pi)*x**3) - erfi(b*x)/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x^5, x)
```

3.213 $\int \frac{\text{Erfi}(bx)}{x^7}\,dx$

Optimal. Leaf size=93

$$\frac{4}{45}b^6\text{Erfi}(bx) - \frac{4b^5e^{b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\text{Erfi}(bx)}{6x^6}$$

```
[Out] -(b*E^(b^2*x^2))/(15*Sqrt[Pi]*x^5) - (2*b^3*E^(b^2*x^2))/(45*Sqrt[Pi]*x^3)
- (4*b^5*E^(b^2*x^2))/(45*Sqrt[Pi]*x) + (4*b^6*Erfi[b*x])/45 - Erfi[b*x]/(6
*x^6)
```

Rubi [A] time = 0.0785057, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2214, 2204}

$$\frac{4}{45}b^6\text{Erfi}(bx) - \frac{4b^5e^{b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\text{Erfi}(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/x^7,x]
```

```
[Out] -(b*E^(b^2*x^2))/(15*Sqrt[Pi]*x^5) - (2*b^3*E^(b^2*x^2))/(45*Sqrt[Pi]*x^3)
- (4*b^5*E^(b^2*x^2))/(45*Sqrt[Pi]*x) + (4*b^6*Erfi[b*x])/45 - Erfi[b*x]/(6
*x^6)
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
```

```
] && LeQ[-n, m + 1]))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{erfi}(bx)}{x^7}\,dx &= -\frac{\operatorname{erfi}(bx)}{6x^6} + \frac{b\int \frac{e^{b^2x^2}}{x^6}\,dx}{3\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\operatorname{erfi}(bx)}{6x^6} + \frac{\left(2b^3\right)\int \frac{e^{b^2x^2}}{x^4}\,dx}{15\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{\operatorname{erfi}(bx)}{6x^6} + \frac{\left(4b^5\right)\int \frac{e^{b^2x^2}}{x^2}\,dx}{45\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{4b^5e^{b^2x^2}}{45\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)}{6x^6} + \frac{\left(8b^7\right)\int e^{b^2x^2}\,dx}{45\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{4b^5e^{b^2x^2}}{45\sqrt{\pi}x} + \frac{4}{45}b^6\operatorname{erfi}(bx) - \frac{\operatorname{erfi}(bx)}{6x^6}
\end{aligned}
$$

Mathematica [A] time = 0.0239837, size = 64, normalized size = 0.69

$$
\frac{\sqrt{\pi}\left(8b^6x^6-15\right)\operatorname{Erfi}(bx)-2bxe^{b^2x^2}\left(4b^4x^4+2b^2x^2+3\right)}{90\sqrt{\pi}x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x^7,x]
```

```
[Out] (-2*b*E^(b^2*x^2)*x*(3 + 2*b^2*x^2 + 4*b^4*x^4) + Sqrt[Pi]*(-15 + 8*b^6*x^6
)*Erfi[b*x])/(90*Sqrt[Pi]*x^6)
```

Maple [A] time = 0.043, size = 81, normalized size = 0.9

$$b^{6}\left(-\frac{\operatorname{erfi}\left(bx\right)}{6\,b^{6}x^{6}}+\frac{1}{3\,\sqrt{\pi}}\left(-\frac{\mathrm{e}^{b^{2}x^{2}}}{5\,b^{5}x^{5}}-\frac{2\,\mathrm{e}^{b^{2}x^{2}}}{15\,x^{3}b^{3}}-\frac{4\,\mathrm{e}^{b^{2}x^{2}}}{15\,bx}+\frac{4\,\sqrt{\pi}\operatorname{erfi}\left(bx\right)}{15}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6/b^6/x^6*erfi(b*x)+1/3/Pi^(1/2)*(-1/5*exp(b^2*x^2)/b^5/x^5-2/15*ex
p(b^2*x^2)/b^3/x^3-4/15*exp(b^2*x^2)/b/x+4/15*Pi^(1/2)*erfi(b*x)))
```

Maxima [A] time = 1.07059, size = 53, normalized size = 0.57

$$-\frac{\left(-b^{2}x^{2}\right)^{\frac{5}{2}}b\Gamma\left(-\frac{5}{2},-b^{2}x^{2}\right)}{6\,\sqrt{\pi}x^{5}}-\frac{\operatorname{erfi}\left(bx\right)}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(-b^2*x^2)^(5/2)*b*gamma(-5/2, -b^2*x^2)/(sqrt(pi)*x^5) - 1/6*erfi(b*x
)/x^6
```

Fricas [A] time = 2.4588, size = 146, normalized size = 1.57

$$-\frac{2\,\sqrt{\pi}\left(4\,b^{5}x^{5}+2\,b^{3}x^{3}+3\,bx\right)e^{\left(b^{2}x^{2}\right)}+\left(15\,\pi-8\,\pi b^{6}x^{6}\right)\operatorname{erfi}\left(bx\right)}{90\,\pi x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(2*sqrt(pi)*(4*b^5*x^5 + 2*b^3*x^3 + 3*b*x)*e^(b^2*x^2) + (15*pi - 8*
pi*b^6*x^6)*erfi(b*x))/(pi*x^6)
```

Sympy [A] time = 4.45284, size = 87, normalized size = 0.94

$$\frac{4b^6 \operatorname{erfi}(bx)}{45} - \frac{4b^5 e^{b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3 e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\operatorname{erfi}(bx)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x**7,x)
```

```
[Out] 4*b**6*erfi(b*x)/45 - 4*b**5*exp(b**2*x**2)/(45*sqrt(pi)*x) - 2*b**3*exp(b*
*2*x**2)/(45*sqrt(pi)*x**3) - b*exp(b**2*x**2)/(15*sqrt(pi)*x**5) - erfi(b*
x)/(6*x**6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x^7, x)
```

3.214 $\int x^6 \mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=105

$$-\frac{x^6 e^{b^2x^2}}{7\sqrt{\pi}b}+\frac{3x^4 e^{b^2x^2}}{7\sqrt{\pi}b^3}-\frac{6x^2 e^{b^2x^2}}{7\sqrt{\pi}b^5}+\frac{6e^{b^2x^2}}{7\sqrt{\pi}b^7}+\frac{1}{7}x^7\mathrm{Erfi}(bx)$$

```
[Out] (6*E^(b^2*x^2))/(7*b^7*Sqrt[Pi]) - (6*E^(b^2*x^2)*x^2)/(7*b^5*Sqrt[Pi]) + (
3*E^(b^2*x^2)*x^4)/(7*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^6)/(7*b*Sqrt[Pi]) + (x
^7*Erfi[b*x])/7
```

Rubi [A] time = 0.0859314, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2212, 2209}

$$-\frac{x^6 e^{b^2x^2}}{7\sqrt{\pi}b}+\frac{3x^4 e^{b^2x^2}}{7\sqrt{\pi}b^3}-\frac{6x^2 e^{b^2x^2}}{7\sqrt{\pi}b^5}+\frac{6e^{b^2x^2}}{7\sqrt{\pi}b^7}+\frac{1}{7}x^7\mathrm{Erfi}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Erfi[b*x],x]
```

```
[Out] (6*E^(b^2*x^2))/(7*b^7*Sqrt[Pi]) - (6*E^(b^2*x^2)*x^2)/(7*b^5*Sqrt[Pi]) + (
3*E^(b^2*x^2)*x^4)/(7*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^6)/(7*b*Sqrt[Pi]) + (x
^7*Erfi[b*x])/7
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^6 \operatorname{erfi}(bx)\,dx &= \frac{1}{7}x^7\operatorname{erfi}(bx) - \frac{(2b)\int e^{b^2x^2}x^7\,dx}{7\sqrt{\pi}} \\
&= -\frac{e^{b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\operatorname{erfi}(bx) + \frac{6\int e^{b^2x^2}x^5\,dx}{7b\sqrt{\pi}} \\
&= \frac{3e^{b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\operatorname{erfi}(bx) - \frac{12\int e^{b^2x^2}x^3\,dx}{7b^3\sqrt{\pi}} \\
&= -\frac{6e^{b^2x^2}x^2}{7b^5\sqrt{\pi}} + \frac{3e^{b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\operatorname{erfi}(bx) + \frac{12\int e^{b^2x^2}x\,dx}{7b^5\sqrt{\pi}} \\
&= \frac{6e^{b^2x^2}}{7b^7\sqrt{\pi}} - \frac{6e^{b^2x^2}x^2}{7b^5\sqrt{\pi}} + \frac{3e^{b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\operatorname{erfi}(bx)
\end{aligned}
$$

Mathematica [A] time = 0.0363018, size = 57, normalized size = 0.54

$$
\frac{1}{7}\left(\frac{e^{b^2x^2}\left(-b^6x^6+3b^4x^4-6b^2x^2+6\right)}{\sqrt{\pi}b^7}+x^7\operatorname{Erfi}(bx)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Erfi[b*x],x]
```

```
[Out] ((E^(b^2*x^2)*(6 - 6*b^2*x^2 + 3*b^4*x^4 - b^6*x^6))/(b^7*Sqrt[Pi]) + x^7*E
rfi[b*x])/7
```

Maple [A] time = 0.041, size = 82, normalized size = 0.8

$$
\frac{1}{b^7}\left(\frac{b^7x^7\operatorname{erfi}(bx)}{7} - \frac{2}{7\sqrt{\pi}}\left(\frac{b^6x^6e^{b^2x^2}}{2} - \frac{3\,e^{b^2x^2}b^4x^4}{2} + 3\,b^2x^2e^{b^2x^2} - 3\,e^{b^2x^2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*erfi(b*x),x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*erfi(b*x)-2/7/Pi^(1/2)*(1/2*b^6*x^6*exp(b^2*x^2)-3/2*exp
(b^2*x^2)*b^4*x^4+3*b^2*x^2*exp(b^2*x^2)-3*exp(b^2*x^2)))
```

Maxima [A] time = 1.00238, size = 69, normalized size = 0.66

$$\frac{1}{7}x^7\operatorname{erfi}(bx) - \frac{\left(b^6x^6 - 3\,b^4x^4 + 6\,b^2x^2 - 6\right)e^{\left(b^2x^2\right)}}{7\,\sqrt{\pi}b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*erfi(b*x) - 1/7*(b^6*x^6 - 3*b^4*x^4 + 6*b^2*x^2 - 6)*e^(b^2*x^2)/(
sqrt(pi)*b^7)
```

Fricas [A] time = 2.34847, size = 132, normalized size = 1.26

$$\frac{\pi b^7x^7\operatorname{erfi}(bx) - \sqrt{\pi}\left(b^6x^6 - 3\,b^4x^4 + 6\,b^2x^2 - 6\right)e^{\left(b^2x^2\right)}}{7\,\pi b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/7*(pi*b^7*x^7*erfi(b*x) - sqrt(pi)*(b^6*x^6 - 3*b^4*x^4 + 6*b^2*x^2 - 6)*
e^(b^2*x^2))/(pi*b^7)
```

Sympy [A] time = 6.57559, size = 99, normalized size = 0.94

$$\begin{cases} \frac{x^7\operatorname{erfi}(bx)}{7} - \frac{x^6e^{b^2x^2}}{7\sqrt{\pi}b} + \frac{3x^4e^{b^2x^2}}{7\sqrt{\pi}b^3} - \frac{6x^2e^{b^2x^2}}{7\sqrt{\pi}b^5} + \frac{6e^{b^2x^2}}{7\sqrt{\pi}b^7} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*erfi(b*x),x)
```

```
[Out] Piecewise((x**7*erfi(b*x)/7 - x**6*exp(b**2*x**2)/(7*sqrt(pi)*b) + 3*x**4*e
xp(b**2*x**2)/(7*sqrt(pi)*b**3) - 6*x**2*exp(b**2*x**2)/(7*sqrt(pi)*b**5) +
 6*exp(b**2*x**2)/(7*sqrt(pi)*b**7), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \operatorname{erfi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*erfi(b*x), x)
```

3.215 $\int x^4 \mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=81

$$-\frac{x^4e^{b^2x^2}}{5\sqrt{\pi}b}+\frac{2x^2e^{b^2x^2}}{5\sqrt{\pi}b^3}-\frac{2e^{b^2x^2}}{5\sqrt{\pi}b^5}+\frac{1}{5}x^5\mathrm{Erfi}(bx)$$

```
[Out] (-2*E^(b^2*x^2))/(5*b^5*Sqrt[Pi]) + (2*E^(b^2*x^2)*x^2)/(5*b^3*Sqrt[Pi]) -
(E^(b^2*x^2)*x^4)/(5*b*Sqrt[Pi]) + (x^5*Erfi[b*x])/5
```

Rubi [A] time = 0.0646533, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2212, 2209}

$$-\frac{x^4e^{b^2x^2}}{5\sqrt{\pi}b}+\frac{2x^2e^{b^2x^2}}{5\sqrt{\pi}b^3}-\frac{2e^{b^2x^2}}{5\sqrt{\pi}b^5}+\frac{1}{5}x^5\mathrm{Erfi}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Erfi[b*x],x]
```

```
[Out] (-2*E^(b^2*x^2))/(5*b^5*Sqrt[Pi]) + (2*E^(b^2*x^2)*x^2)/(5*b^3*Sqrt[Pi]) -
(E^(b^2*x^2)*x^4)/(5*b*Sqrt[Pi]) + (x^5*Erfi[b*x])/5
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4\operatorname{erfi}(bx)\,dx &= \frac{1}{5}x^5\operatorname{erfi}(bx)-\frac{(2b)\int e^{b^2x^2}x^5\,dx}{5\sqrt{\pi}}\\
&= -\frac{e^{b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\operatorname{erfi}(bx)+\frac{4\int e^{b^2x^2}x^3\,dx}{5b\sqrt{\pi}}\\
&= \frac{2e^{b^2x^2}x^2}{5b^3\sqrt{\pi}}-\frac{e^{b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\operatorname{erfi}(bx)-\frac{4\int e^{b^2x^2}x\,dx}{5b^3\sqrt{\pi}}\\
&= -\frac{2e^{b^2x^2}}{5b^5\sqrt{\pi}}+\frac{2e^{b^2x^2}x^2}{5b^3\sqrt{\pi}}-\frac{e^{b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\operatorname{erfi}(bx)
\end{aligned}
$$

Mathematica [A] time = 0.0283888, size = 49, normalized size = 0.6

$$\frac{1}{5}\left(x^5\operatorname{Erfi}(bx)-\frac{e^{b^2x^2}\left(b^4x^4-2b^2x^2+2\right)}{\sqrt{\pi}b^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Erfi[b*x],x]
```

```
[Out] (-((E^(b^2*x^2)*(2 - 2*b^2*x^2 + b^4*x^4))/(b^5*Sqrt[Pi])) + x^5*Erfi[b*x])
/5
```

Maple [A] time = 0.042, size = 64, normalized size = 0.8

$$\frac{1}{b^5}\left(\frac{b^5x^5\operatorname{erfi}(bx)}{5}-\frac{2}{5\sqrt{\pi}}\left(\frac{e^{b^2x^2}b^4x^4}{2}-b^2x^2e^{b^2x^2}+e^{b^2x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erfi(b*x),x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*erfi(b*x)-2/5/Pi^(1/2)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2
*exp(b^2*x^2)+exp(b^2*x^2)))
```

Maxima [A] time = 1.01852, size = 58, normalized size = 0.72

$$\frac{1}{5}x^5\,\mathrm{erfi}\,(bx) - \frac{\left(b^4x^4 - 2\,b^2x^2 + 2\right)e^{\left(b^2x^2\right)}}{5\,\sqrt{\pi}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*erfi(b*x) - 1/5*(b^4*x^4 - 2*b^2*x^2 + 2)*e^(b^2*x^2)/(sqrt(pi)*b^5
)
```

Fricas [A] time = 2.35194, size = 116, normalized size = 1.43

$$\frac{\pi b^5x^5\,\mathrm{erfi}\,(bx) - \sqrt{\pi}\left(b^4x^4 - 2\,b^2x^2 + 2\right)e^{\left(b^2x^2\right)}}{5\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/5*(pi*b^5*x^5*erfi(b*x) - sqrt(pi)*(b^4*x^4 - 2*b^2*x^2 + 2)*e^(b^2*x^2))
/(pi*b^5)
```

Sympy [A] time = 1.895, size = 75, normalized size = 0.93

$$\begin{cases} \frac{x^5\,\mathrm{erfi}\,(bx)}{5} - \frac{x^4e^{b^2x^2}}{5\sqrt{\pi}b} + \frac{2x^2e^{b^2x^2}}{5\sqrt{\pi}b^3} - \frac{2e^{b^2x^2}}{5\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erfi(b*x),x)
```

```
[Out] Piecewise((x**5*erfi(b*x)/5 - x**4*exp(b**2*x**2)/(5*sqrt(pi)*b) + 2*x**2*e
xp(b**2*x**2)/(5*sqrt(pi)*b**3) - 2*exp(b**2*x**2)/(5*sqrt(pi)*b**5), Ne(b,
 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfi(b*x), x)
```

3.216 $\int x^2 \mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=57

$$-\frac{x^2 e^{b^2 x^2}}{3\sqrt{\pi}b}+\frac{e^{b^2 x^2}}{3\sqrt{\pi}b^3}+\frac{1}{3}x^3\text{Erfi}(bx)$$

```
[Out] E^(b^2*x^2)/(3*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^2)/(3*b*Sqrt[Pi]) + (x^3*Erfi
[b*x])/3
```

Rubi [A] time = 0.0442204, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2212, 2209}

$$-\frac{x^2 e^{b^2 x^2}}{3\sqrt{\pi}b}+\frac{e^{b^2 x^2}}{3\sqrt{\pi}b^3}+\frac{1}{3}x^3\text{Erfi}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erfi[b*x],x]
```

```
[Out] E^(b^2*x^2)/(3*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^2)/(3*b*Sqrt[Pi]) + (x^3*Erfi
[b*x])/3
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\text{erfi}(bx)\,dx &= \frac{1}{3}x^3\text{erfi}(bx) - \frac{(2b)\int e^{b^2x^2}x^3\,dx}{3\sqrt{\pi}}\\ &= -\frac{e^{b^2x^2}x^2}{3b\sqrt{\pi}} + \frac{1}{3}x^3\text{erfi}(bx) + \frac{2\int e^{b^2x^2}x\,dx}{3b\sqrt{\pi}}\\ &= \frac{e^{b^2x^2}}{3b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^2}{3b\sqrt{\pi}} + \frac{1}{3}x^3\text{erfi}(bx)\end{aligned}$$

Mathematica [A] time = 0.0249373, size = 41, normalized size = 0.72

$$\frac{1}{3}\left(\frac{e^{b^2x^2}\left(1-b^2x^2\right)}{\sqrt{\pi}b^3} + x^3\text{Erfi}(bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erfi[b*x],x]
```

```
[Out] ((E^(b^2*x^2)*(1 - b^2*x^2))/(b^3*Sqrt[Pi]) + x^3*Erfi[b*x])/3
```

Maple [A] time = 0.042, size = 50, normalized size = 0.9

$$\frac{1}{b^3}\left(\frac{b^3x^3\text{erfi}\,(bx)}{3} - \frac{2}{3\sqrt{\pi}}\left(\frac{b^2x^2\mathrm{e}^{b^2x^2}}{2} - \frac{\mathrm{e}^{b^2x^2}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfi(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*erfi(b*x)-2/3/Pi^(1/2)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp
(b^2*x^2)))
```

Maxima [A] time = 0.973776, size = 47, normalized size = 0.82

$$\frac{1}{3}\,x^3\,\mathrm{erfi}\left(bx\right) - \frac{\left(b^2x^2 - 1\right)e^{\left(b^2x^2\right)}}{3\,\sqrt{\pi}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*erfi(b*x) - 1/3*(b^2*x^2 - 1)*e^(b^2*x^2)/(sqrt(pi)*b^3)
```

Fricas [A] time = 2.3121, size = 100, normalized size = 1.75

$$\frac{\pi b^3x^3\,\mathrm{erfi}\left(bx\right) - \sqrt{\pi}\left(b^2x^2 - 1\right)e^{\left(b^2x^2\right)}}{3\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/3*(pi*b^3*x^3*erfi(b*x) - sqrt(pi)*(b^2*x^2 - 1)*e^(b^2*x^2))/(pi*b^3)
```

Sympy [A] time = 0.494234, size = 49, normalized size = 0.86

$$\begin{cases} \frac{x^3\,\mathrm{erfi}\left(bx\right)}{3} - \frac{x^2e^{b^2x^2}}{3\sqrt{\pi}b} + \frac{e^{b^2x^2}}{3\sqrt{\pi}b^3} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfi(b*x),x)
```

```
[Out] Piecewise((x**3*erfi(b*x)/3 - x**2*exp(b**2*x**2)/(3*sqrt(pi)*b) + exp(b**2
*x**2)/(3*sqrt(pi)*b**3), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi(b*x), x)
```

3.217 $\int \text{Erfi}(bx)\,dx$

Optimal. Leaf size=26

$$x\text{Erfi}(bx)-\frac{e^{b^2x^2}}{\sqrt{\pi}b}$$

```
[Out] -(E^(b^2*x^2)/(b*Sqrt[Pi])) + x*Erfi[b*x]
```

Rubi [A] time = 0.0047307, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6351}

$$x\text{Erfi}(bx)-\frac{e^{b^2x^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x],x]
```

```
[Out] -(E^(b^2*x^2)/(b*Sqrt[Pi])) + x*Erfi[b*x]
```

Rule 6351

```
Int[Erfi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x])/b
, x] - Simp[E^(a + b*x)^2/(b*Sqrt[Pi]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erfi}(bx)\,dx=-\frac{e^{b^2x^2}}{b\sqrt{\pi}}+x\text{erfi}(bx)$$

Mathematica [A] time = 0.0072138, size = 26, normalized size = 1.

$$x\text{Erfi}(bx)-\frac{e^{b^2x^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x],x]
```

```
[Out] -(E^(b^2*x^2)/(b*Sqrt[Pi])) + x*Erfi[b*x]
```

Maple [A] time = 0.04, size = 26, normalized size = 1.

$$\frac{1}{b}\left(bx\mathrm{erfi}\,(bx)-\frac{\mathrm{e}^{b^2x^2}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x),x)
```

```
[Out] 1/b*(b*x*erfi(b*x)-1/Pi^(1/2)*exp(b^2*x^2))
```

Maxima [A] time = 1.01803, size = 34, normalized size = 1.31

$$\frac{bx\,\mathrm{erfi}\,(bx)-\frac{e^{\left(b^2x^2\right)}}{\sqrt{\pi}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x),x, algorithm="maxima")
```

```
[Out] (b*x*erfi(b*x) - e^(b^2*x^2)/sqrt(pi))/b
```

Fricas [A] time = 2.26128, size = 68, normalized size = 2.62

$$\frac{\pi bx\,\mathrm{erfi}\,(bx)-\sqrt{\pi}e^{\left(b^2x^2\right)}}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x),x, algorithm="fricas")
```

```
[Out] (pi*b*x*erfi(b*x) - sqrt(pi)*e^(b^2*x^2))/(pi*b)
```

Sympy [A] time = 0.148698, size = 22, normalized size = 0.85

$$\begin{cases} x\,\mathrm{erfi}\,(bx) - \frac{e^{b^2x^2}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x),x)
```

```
[Out] Piecewise((x*erfi(b*x) - exp(b**2*x**2)/(sqrt(pi)*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{erfi}\,(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x), x)
```

3.218 $\int \frac{\mathbf{Erfi}(bx)}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{b\text{ExpIntegralEi}\left(b^2x^2\right)}{\sqrt{\pi}}-\frac{\text{Erfi}(bx)}{x}$$

[Out] -(Erfi[b*x]/x) + (b*ExpIntegralEi[b^2*x^2])/Sqrt[Pi]

Rubi [A] time = 0.0291507, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6363, 2210}

$$\frac{b\text{Ei}\left(b^2x^2\right)}{\sqrt{\pi}}-\frac{\text{Erfi}(bx)}{x}$$

Antiderivative was successfully verified.

[In] Int[Erfi[b*x]/x^2,x]

[Out] -(Erfi[b*x]/x) + (b*ExpIntegralEi[b^2*x^2])/Sqrt[Pi]

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\text{erfi}(bx)}{x^2}\,dx = -\frac{\text{erfi}(bx)}{x} + \frac{(2b)\int \frac{e^{b^2x^2}}{x}\,dx}{\sqrt{\pi}}$$
$$= -\frac{\text{erfi}(bx)}{x} + \frac{b\text{Ei}\left(b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0122441, size = 25, normalized size = 1.

$$\frac{b\text{ExpIntegralEi}\left(b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{Erfi}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x^2,x]
```

```
[Out] -(Erfi[b*x]/x) + (b*ExpIntegralEi[b^2*x^2])/Sqrt[Pi]
```

Maple [A] time = 0.043, size = 31, normalized size = 1.2

$$b\left(-\frac{\text{erfi}\,(bx)}{bx} - \frac{\text{Ei}\left(1, -b^2x^2\right)}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x^2,x)
```

```
[Out] b*(-1/b/x*erfi(b*x)-1/Pi^(1/2)*Ei(1,-b^2*x^2))
```

Maxima [A] time = 1.29618, size = 31, normalized size = 1.24

$$\frac{b\text{Ei}\left(b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erfi}\,(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] b*Ei(b^2*x^2)/sqrt(pi) - erfi(b*x)/x
```

Fricas [A] time = 2.35493, size = 68, normalized size = 2.72

$$\frac{\sqrt{\pi}bx\mathrm{Ei}\left(b^2x^2\right)-\pi\,\mathrm{erfi}\,(bx)}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*b*x*Ei(b^2*x^2) - pi*erfi(b*x))/(pi*x)
```

Sympy [C] time = 1.2032, size = 32, normalized size = 1.28

$$-\frac{b\,\mathrm{E}_1\left(b^2x^2e^{i\pi}\right)}{\sqrt{\pi}}-\frac{i\,\mathrm{erfc}\,(ibx)}{x}+\frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x**2,x)
```

```
[Out] -b*expint(1, b**2*x**2*exp_polar(I*pi))/sqrt(pi) - I*erfc(I*b*x)/x + I/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{erfi}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x^2, x)
```

3.219 $\int \frac{\text{Erfi}(bx)}{x^4} \, dx$

Optimal. Leaf size=54

$$\frac{b^3 \text{ExpIntegralEi}\left(b^2x^2\right)}{3\sqrt{\pi}} - \frac{be^{b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{Erfi}(bx)}{3x^3}$$

```
[Out] -(b*E^(b^2*x^2))/(3*Sqrt[Pi]*x^2) - Erfi[b*x]/(3*x^3) + (b^3*ExpIntegralEi[
b^2*x^2])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.050437, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2214, 2210}

$$\frac{b^3 \text{Ei}\left(b^2x^2\right)}{3\sqrt{\pi}} - \frac{be^{b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{Erfi}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/x^4,x]
```


```
[Out] -(b*E^(b^2*x^2))/(3*Sqrt[Pi]*x^2) - Erfi[b*x]/(3*x^3) + (b^3*ExpIntegralEi[
b^2*x^2])/(3*Sqrt[Pi])
```


Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{erfi}(bx)}{x^4}\,dx &= -\frac{\text{erfi}(bx)}{3x^3}+\frac{(2b)\int \frac{e^{b^2x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\ &= -\frac{be^{b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\text{erfi}(bx)}{3x^3}+\frac{\left(2b^3\right)\int \frac{e^{b^2x^2}}{x}\,dx}{3\sqrt{\pi}}\\ &= -\frac{be^{b^2x^2}}{3\sqrt{\pi}x^2}-\frac{\text{erfi}(bx)}{3x^3}+\frac{b^3\text{Ei}\left(b^2x^2\right)}{3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0214942, size = 50, normalized size = 0.93

$$-\frac{-\frac{b^3x^3\text{ExpIntegralEi}\left(b^2x^2\right)}{\sqrt{\pi}}+\frac{bxe^{b^2x^2}}{\sqrt{\pi}}+\text{Erfi}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x^4,x]
```

```
[Out] -((b*E^(b^2*x^2)*x)/Sqrt[Pi] + Erfi[b*x] - (b^3*x^3*ExpIntegralEi[b^2*x^2])
/Sqrt[Pi])/(3*x^3)
```

Maple [A] time = 0.043, size = 52, normalized size = 1.

$$b^3\left(-\frac{\text{erfi}\,(bx)}{3\,x^3b^3}+\frac{2}{3\,\sqrt{\pi}}\left(-\frac{e^{b^2x^2}}{2\,b^2x^2}-\frac{\text{Ei}\left(1,-b^2x^2\right)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x^4,x)
```

```
[Out] b^3*(-1/3/b^3/x^3*erfi(b*x)+2/3/Pi^(1/2)*(-1/2*exp(b^2*x^2)/b^2/x^2-1/2*Ei(
1,-b^2*x^2)))
```

Maxima [A] time = 1.28505, size = 38, normalized size = 0.7

$$\frac{b^3\Gamma\left(-1,-b^2x^2\right)}{3\sqrt{\pi}}-\frac{\operatorname{erfi}(bx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*gamma(-1, -b^2*x^2)/sqrt(pi) - 1/3*erfi(b*x)/x^3
```

Fricas [A] time = 2.28505, size = 109, normalized size = 2.02

$$-\frac{\pi\operatorname{erfi}(bx)-\sqrt{\pi}\left(b^3x^3\operatorname{Ei}\left(b^2x^2\right)-bxe^{\left(b^2x^2\right)}\right)}{3\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(pi*erfi(b*x) - sqrt(pi)*(b^3*x^3*Ei(b^2*x^2) - b*x*e^(b^2*x^2)))/(pi*
x^3)
```

Sympy [C] time = 2.35422, size = 63, normalized size = 1.17

$$-\frac{b^3E_1\left(b^2x^2e^{i\pi}\right)}{3\sqrt{\pi}}-\frac{be^{b^2x^2}}{3\sqrt{\pi}x^2}-\frac{i\operatorname{erfc}(ibx)}{3x^3}+\frac{i}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x**4,x)
```

```
[Out] -b**3*expint(1, b**2*x**2*exp_polar(I*pi))/(3*sqrt(pi)) - b*exp(b**2*x**2)/
(3*sqrt(pi)*x**2) - I*erfc(I*b*x)/(3*x**3) + I/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x^4, x)
```

3.220 $\int \frac{\textbf{Erfi}(bx)}{x^6}\,dx$

Optimal. Leaf size=78

$$\frac{b^5\text{ExpIntegralEi}\left(b^2x^2\right)}{10\sqrt{\pi}}-\frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2}-\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\text{Erfi}(bx)}{5x^5}$$

```
[Out] -(b*E^(b^2*x^2))/(10*Sqrt[Pi]*x^4) - (b^3*E^(b^2*x^2))/(10*Sqrt[Pi]*x^2) -
Erfi[b*x]/(5*x^5) + (b^5*ExpIntegralEi[b^2*x^2])/(10*Sqrt[Pi])
```

Rubi [A] time = 0.0701197, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6363, 2214, 2210}

$$\frac{b^5\text{Ei}\left(b^2x^2\right)}{10\sqrt{\pi}}-\frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2}-\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4}-\frac{\text{Erfi}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/x^6,x]
```

```
[Out] -(b*E^(b^2*x^2))/(10*Sqrt[Pi]*x^4) - (b^3*E^(b^2*x^2))/(10*Sqrt[Pi]*x^2) -
Erfi[b*x]/(5*x^5) + (b^5*ExpIntegralEi[b^2*x^2])/(10*Sqrt[Pi])
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erfi}(bx)}{x^6}\,dx &= -\frac{\operatorname{erfi}(bx)}{5x^5} + \frac{(2b)\int \frac{e^{b^2x^2}}{x^5}\,dx}{5\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4} - \frac{\operatorname{erfi}(bx)}{5x^5} + \frac{b^3\int \frac{e^{b^2x^2}}{x^3}\,dx}{5\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4} - \frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2} - \frac{\operatorname{erfi}(bx)}{5x^5} + \frac{b^5\int \frac{e^{b^2x^2}}{x}\,dx}{5\sqrt{\pi}} \\
&= -\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4} - \frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2} - \frac{\operatorname{erfi}(bx)}{5x^5} + \frac{b^5\operatorname{Ei}\left(b^2x^2\right)}{10\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.0273478, size = 61, normalized size = 0.78

$$\frac{b^5x^5\text{ExpIntegralEi}\left(b^2x^2\right) - bxe^{b^2x^2}\left(b^2x^2+1\right) - 2\sqrt{\pi}\text{Erfi}(bx)}{10\sqrt{\pi}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/x^6,x]
```

```
[Out] (-(b*E^(b^2*x^2)*x*(1 + b^2*x^2)) - 2*Sqrt[Pi]*Erfi[b*x] + b^5*x^5*ExpInteg
ralEi[b^2*x^2])/(10*Sqrt[Pi]*x^5)
```

Maple [A] time = 0.043, size = 68, normalized size = 0.9

$$b^5\left(-\frac{\operatorname{erfi}(bx)}{5\,b^5x^5} + \frac{2}{5\,\sqrt{\pi}}\left(-\frac{e^{b^2x^2}}{4\,x^4b^4} - \frac{e^{b^2x^2}}{4\,b^2x^2} - \frac{\operatorname{Ei}\left(1,-b^2x^2\right)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5/b^5/x^5*erfi(b*x)+2/5/Pi^(1/2)*(-1/4*exp(b^2*x^2)/b^4/x^4-1/4*exp
(b^2*x^2)/b^2/x^2-1/4*Ei(1,-b^2*x^2)))
```

Maxima [A] time = 1.12932, size = 38, normalized size = 0.49

$$-\frac{b^5\Gamma\left(-2,-b^2x^2\right)}{5\sqrt{\pi}}-\frac{\operatorname{erfi}(bx)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*b^5*gamma(-2, -b^2*x^2)/sqrt(pi) - 1/5*erfi(b*x)/x^5
```

Fricas [A] time = 2.39239, size = 130, normalized size = 1.67

$$-\frac{2\pi\operatorname{erfi}(bx)-\sqrt{\pi}\left(b^5x^5\operatorname{Ei}\left(b^2x^2\right)-\left(b^3x^3+bx\right)e^{\left(b^2x^2\right)}\right)}{10\pi x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(2*pi*erfi(b*x) - sqrt(pi)*(b^5*x^5*Ei(b^2*x^2) - (b^3*x^3 + b*x)*e^(
b^2*x^2)))/(pi*x^5)
```

Sympy [C] time = 5.01015, size = 85, normalized size = 1.09

$$-\frac{b^5\operatorname{E}_1\left(b^2x^2e^{i\pi}\right)}{10\sqrt{\pi}}-\frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2}-\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4}-\frac{i\operatorname{erfc}(ibx)}{5x^5}+\frac{i}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x**6,x)
```

```
[Out] -b**5*expint(1, b**2*x**2*exp_polar(I*pi))/(10*sqrt(pi)) - b**3*exp(b**2*x*
*2)/(10*sqrt(pi)*x**2) - b*exp(b**2*x**2)/(10*sqrt(pi)*x**4) - I*erfc(I*b*x
)/(5*x**5) + I/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)/x^6, x)
```

3.221 $\int (c+dx)^3 \mathbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=279

$$-\frac{d^2 e^{(a+bx)^2}(a+bx)^2(bc-ad)}{\sqrt{\pi}b^4}+\frac{d^2 e^{(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^4}-\frac{(bc-ad)^4\text{Erfi}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2\text{Erfi}(a+bx)}{4b^4}-\frac{e^{(a+bx)^2}(bc-ad}{\sqrt{\pi}b^4}$$

```
[Out] (d^2*(b*c - a*d)*E^(a + b*x)^2)/(b^4*Sqrt[Pi]) - ((b*c - a*d)^3*E^(a + b*x)
^2)/(b^4*Sqrt[Pi]) + (3*d^3*E^(a + b*x)^2*(a + b*x))/(8*b^4*Sqrt[Pi]) - (3*
d*(b*c - a*d)^2*E^(a + b*x)^2*(a + b*x))/(2*b^4*Sqrt[Pi]) - (d^2*(b*c - a*d
)*E^(a + b*x)^2*(a + b*x)^2)/(b^4*Sqrt[Pi]) - (d^3*E^(a + b*x)^2*(a + b*x)^
3)/(4*b^4*Sqrt[Pi]) - (3*d^3*Erfi[a + b*x])/(16*b^4) + (3*d*(b*c - a*d)^2*E
rfi[a + b*x])/(4*b^4) - ((b*c - a*d)^4*Erfi[a + b*x])/(4*b^4*d) + ((c + d*x
)^4*Erfi[a + b*x])/(4*d)
```

Rubi [A] time = 0.249907, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6363, 2226, 2204, 2209, 2212}

$$-\frac{d^2 e^{(a+bx)^2}(a+bx)^2(bc-ad)}{\sqrt{\pi}b^4}+\frac{d^2 e^{(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^4}-\frac{(bc-ad)^4\text{Erfi}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2\text{Erfi}(a+bx)}{4b^4}-\frac{e^{(a+bx)^2}(bc-ad}{\sqrt{\pi}b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Erfi[a + b*x],x]
```


```
[Out] (d^2*(b*c - a*d)*E^(a + b*x)^2)/(b^4*Sqrt[Pi]) - ((b*c - a*d)^3*E^(a + b*x)
^2)/(b^4*Sqrt[Pi]) + (3*d^3*E^(a + b*x)^2*(a + b*x))/(8*b^4*Sqrt[Pi]) - (3*
d*(b*c - a*d)^2*E^(a + b*x)^2*(a + b*x))/(2*b^4*Sqrt[Pi]) - (d^2*(b*c - a*d
)*E^(a + b*x)^2*(a + b*x)^2)/(b^4*Sqrt[Pi]) - (d^3*E^(a + b*x)^2*(a + b*x)^
3)/(4*b^4*Sqrt[Pi]) - (3*d^3*Erfi[a + b*x])/(16*b^4) + (3*d*(b*c - a*d)^2*E
rfi[a + b*x])/(4*b^4) - ((b*c - a*d)^4*Erfi[a + b*x])/(4*b^4*d) + ((c + d*x
)^4*Erfi[a + b*x])/(4*d)
```


Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^3\text{erfi}(a+bx)\,dx = \frac{(c+dx)^4\text{erfi}(a+bx)}{4d} - \frac{b\int e^{(a+bx)^2}(c+dx)^4\,dx}{2d\sqrt{\pi}}$$

$$= \frac{(c+dx)^4\text{erfi}(a+bx)}{4d} - \frac{b\int \left(\frac{(bc-ad)^4e^{(a+bx)^2}}{b^4} + \frac{4d(bc-ad)^3e^{(a+bx)^2}(a+bx)}{b^4} + \frac{6d^2(bc-ad)^2e^{(a+bx)^2}(a+bx)^2}{b^4} + \cdots\right.}{2d\sqrt{\pi}}$$

$$= \frac{(c+dx)^4\text{erfi}(a+bx)}{4d} - \frac{d^3\int e^{(a+bx)^2}(a+bx)^4\,dx}{2b^3\sqrt{\pi}} - \frac{\left(2d^2(bc-ad)\right)\int e^{(a+bx)^2}(a+bx)^3\,dx}{b^3\sqrt{\pi}} - \frac{(3\cdots}{\cdots}$$

$$= -\frac{(bc-ad)^3e^{(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{3d(bc-ad)^2e^{(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}} - \frac{d^2(bc-ad)e^{(a+bx)^2}(a+bx)^2}{b^4\sqrt{\pi}} - \frac{d^3e^{(a+bx)^2}\cdots}{4b^4\cdots}$$

$$= \frac{d^2(bc-ad)e^{(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{(bc-ad)^3e^{(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{3d^3e^{(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}} - \frac{3d(bc-ad)^2e^{(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}} \cdots$$

$$= \frac{d^2(bc-ad)e^{(a+bx)^2}}{b^4\sqrt{\pi}} - \frac{(bc-ad)^3e^{(a+bx)^2}}{b^4\sqrt{\pi}} + \frac{3d^3e^{(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}} - \frac{3d(bc-ad)^2e^{(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}} \cdots$$

Mathematica [A] time = 0.26782, size = 237, normalized size = 0.85

$$\frac{\sqrt{\pi}\text{Erfi}(a+bx)\left(12a^2d\left(d^2-2b^2c^2\right)+16a^3bcd^2-4a^4d^3+8a\left(2b^3c^3-3bcd^2\right)+4b^4x\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right)+12\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Erfi[a + b*x],x]
```

```
[Out] (-2*E^(a + b*x)^2*(a*(5 - 2*a^2)*d^3 + b*d^2*(8*(-1 + a^2)*c + (-3 + 2*a^2)
*d*x) - 2*a*b^2*d*(6*c^2 + 4*c*d*x + d^2*x^2) + 2*b^3*(4*c^3 + 6*c^2*d*x +
4*c*d^2*x^2 + d^3*x^3)) + Sqrt[Pi]*(12*b^2*c^2*d + 16*a^3*b*c*d^2 - 3*d^3 -
 4*a^4*d^3 + 12*a^2*d*(-2*b^2*c^2 + d^2) + 8*a*(2*b^3*c^3 - 3*b*c*d^2) + 4*
b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Erfi[a + b*x])/(16*b^4*S
qrt[Pi])
```

Maple [B] time = 0.049, size = 703, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*erfi(b*x+a),x)
```

```
[Out] 1/b*(1/4/b^3*d^3*erfi(b*x+a)*(b*x+a)^4-1/b^3*d^3*erfi(b*x+a)*(b*x+a)^3*a+1/
b^2*d^2*erfi(b*x+a)*(b*x+a)^3*c+3/2/b^3*d^3*erfi(b*x+a)*(b*x+a)^2*a^2-3/b^2
*d^2*erfi(b*x+a)*(b*x+a)^2*a*c+3/2/b*d*erfi(b*x+a)*(b*x+a)^2*c^2-1/b^3*d^3*
erfi(b*x+a)*(b*x+a)*a^3+3/b^2*d^2*erfi(b*x+a)*(b*x+a)*a^2*c-3/b*d*erfi(b*x+
a)*(b*x+a)*a*c^2+erfi(b*x+a)*(b*x+a)*c^3+1/4/b^3*d^3*erfi(b*x+a)*a^4-1/b^2*
d^2*erfi(b*x+a)*a^3*c+3/2/b*d*erfi(b*x+a)*a^2*c^2-erfi(b*x+a)*a*c^3+1/4*b/d
*erfi(b*x+a)*c^4-1/2/b^3/d/Pi^(1/2)*(d^4*(1/2*exp((b*x+a)^2)*(b*x+a)^3-3/4*
(b*x+a)*exp((b*x+a)^2)+3/8*Pi^(1/2)*erfi(b*x+a))+1/2*a^4*d^4*Pi^(1/2)*erfi(
b*x+a)+1/2*b^4*c^4*Pi^(1/2)*erfi(b*x+a)-2*a^3*d^4*exp((b*x+a)^2)+6*a^2*d^4*
(1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi(b*x+a))-4*a*d^4*(1/2*(b*x+a)^
2*exp((b*x+a)^2)-1/2*exp((b*x+a)^2))+2*b^3*c^3*d*exp((b*x+a)^2)+6*b^2*c^2*d
^2*(1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi(b*x+a))+4*b*c*d^3*(1/2*(b*
x+a)^2*exp((b*x+a)^2)-1/2*exp((b*x+a)^2))-2*a*b^3*c^3*d*Pi^(1/2)*erfi(b*x+a
)+3*a^2*b^2*c^2*d^2*Pi^(1/2)*erfi(b*x+a)-2*a^3*b*c*d^3*Pi^(1/2)*erfi(b*x+a)
-6*a*b^2*c^2*d^2*exp((b*x+a)^2)+6*a^2*b*c*d^3*exp((b*x+a)^2)-12*a*b*c*d^3*(
1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \operatorname{erfi}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3*erfi(b*x + a), x)
```

Fricas [A] time = 2.461, size = 581, normalized size = 2.08

$$-\frac{2\sqrt{\pi}\left(2\,b^3d^3x^3+8\,b^3c^3-12\,ab^2c^2d+8\left(a^2-1\right)bcd^2-\left(2\,a^3-5\,a\right)d^3+2\left(4\,b^3cd^2-ab^2d^3\right)x^2+\left(12\,b^3c^2d-8\,ab^2cd^2+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(pi)*(2*b^3*d^3*x^3 + 8*b^3*c^3 - 12*a*b^2*c^2*d + 8*(a^2 - 1)
*b*c*d^2 - (2*a^3 - 5*a)*d^3 + 2*(4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (12*b^3*c^
2*d - 8*a*b^2*c*d^2 + (2*a^2 - 3)*b*d^3)*x)*e^(b^2*x^2 + 2*a*b*x + a^2) - (
4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi*b^4*c^
3*x + pi*(16*a*b^3*c^3 - 12*(2*a^2 - 1)*b^2*c^2*d + 8*(2*a^3 - 3*a)*b*c*d^2
 - (4*a^4 - 12*a^2 + 3)*d^3))*erfi(b*x + a))/(pi*b^4)
```

Sympy [A] time = 18.2334, size = 746, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*erfi(b*x+a),x)
```

```
[Out] Piecewise((-a**4*d**3*erfi(a + b*x)/(4*b**4) + a**3*c*d**2*erfi(a + b*x)/b*
*3 + a**3*d**3*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(4*sqrt(pi)*b**4) - 3*
a**2*c**2*d*erfi(a + b*x)/(2*b**2) - a**2*c*d**2*exp(a**2)*exp(b**2*x**2)*e
xp(2*a*b*x)/(sqrt(pi)*b**3) - a**2*d**3*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*
b*x)/(4*sqrt(pi)*b**3) + 3*a**2*d**3*erfi(a + b*x)/(4*b**4) + a*c**3*erfi(a
 + b*x)/b + 3*a*c**2*d*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(pi)*b*
*2) + a*c*d**2*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b**2) + a*
d**3*x**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(4*sqrt(pi)*b**2) - 3*a*c*d
**2*erfi(a + b*x)/(2*b**3) - 5*a*d**3*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)
/(8*sqrt(pi)*b**4) + c**3*x*erfi(a + b*x) + 3*c**2*d*x**2*erfi(a + b*x)/2 +
 c*d**2*x**3*erfi(a + b*x) + d**3*x**4*erfi(a + b*x)/4 - c**3*exp(a**2)*exp
(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b) - 3*c**2*d*x*exp(a**2)*exp(b**2*x**2)
*exp(2*a*b*x)/(2*sqrt(pi)*b) - c*d**2*x**2*exp(a**2)*exp(b**2*x**2)*exp(2*a
*b*x)/(sqrt(pi)*b) - d**3*x**3*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(4*sqr
t(pi)*b) + 3*c**2*d*erfi(a + b*x)/(4*b**2) + c*d**2*exp(a**2)*exp(b**2*x**2
)*exp(2*a*b*x)/(sqrt(pi)*b**3) + 3*d**3*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*
b*x)/(8*sqrt(pi)*b**3) - 3*d**3*erfi(a + b*x)/(16*b**4), Ne(b, 0)), ((c**3*
x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*erfi(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \operatorname{erfi}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*erfi(b*x + a), x)
```

3.222 $\int (c+dx)^2 \mathbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=186

$$-\frac{(bc-ad)^3\text{Erfi}(a+bx)}{3b^3d}+\frac{d(bc-ad)\text{Erfi}(a+bx)}{2b^3}-\frac{e^{(a+bx)^2}(bc-ad)^2}{\sqrt{\pi}b^3}-\frac{de^{(a+bx)^2}(a+bx)(bc-ad)}{\sqrt{\pi}b^3}-\frac{d^2e^{(a+bx)^2}(a+bx)^2}{3\sqrt{\pi}b^3}+$$

```
[Out] (d^2*E^(a + b*x)^2)/(3*b^3*Sqrt[Pi]) - ((b*c - a*d)^2*E^(a + b*x)^2)/(b^3*S
qrt[Pi]) - (d*(b*c - a*d)*E^(a + b*x)^2*(a + b*x))/(b^3*Sqrt[Pi]) - (d^2*E^
(a + b*x)^2*(a + b*x)^2)/(3*b^3*Sqrt[Pi]) + (d*(b*c - a*d)*Erfi[a + b*x])/(
2*b^3) - ((b*c - a*d)^3*Erfi[a + b*x])/(3*b^3*d) + ((c + d*x)^3*Erfi[a + b*
x])/(3*d)
```

Rubi [A] time = 0.165187, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6363, 2226, 2204, 2209, 2212}

$$-\frac{(bc-ad)^3\text{Erfi}(a+bx)}{3b^3d}+\frac{d(bc-ad)\text{Erfi}(a+bx)}{2b^3}-\frac{e^{(a+bx)^2}(bc-ad)^2}{\sqrt{\pi}b^3}-\frac{de^{(a+bx)^2}(a+bx)(bc-ad)}{\sqrt{\pi}b^3}-\frac{d^2e^{(a+bx)^2}(a+bx)^2}{3\sqrt{\pi}b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Erfi[a + b*x],x]
```

```
[Out] (d^2*E^(a + b*x)^2)/(3*b^3*Sqrt[Pi]) - ((b*c - a*d)^2*E^(a + b*x)^2)/(b^3*S
qrt[Pi]) - (d*(b*c - a*d)*E^(a + b*x)^2*(a + b*x))/(b^3*Sqrt[Pi]) - (d^2*E^
(a + b*x)^2*(a + b*x)^2)/(3*b^3*Sqrt[Pi]) + (d*(b*c - a*d)*Erfi[a + b*x])/(
2*b^3) - ((b*c - a*d)^3*Erfi[a + b*x])/(3*b^3*d) + ((c + d*x)^3*Erfi[a + b*
x])/(3*d)
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
```

```
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\operatorname{erfi}(a+bx)\,dx &= \frac{(c+dx)^3\operatorname{erfi}(a+bx)}{3d} - \frac{(2b)\int e^{(a+bx)^2}(c+dx)^3\,dx}{3d\sqrt{\pi}} \\
&= \frac{(c+dx)^3\operatorname{erfi}(a+bx)}{3d} - \frac{(2b)\int \left(\frac{(bc-ad)^3e^{(a+bx)^2}}{b^3} + \frac{3d(bc-ad)^2e^{(a+bx)^2}(a+bx)}{b^3} + \frac{3d^2(bc-ad)e^{(a+bx)^2}(a+bx)^2}{b^3}\right.}{3d\sqrt{\pi}} \\
&= \frac{(c+dx)^3\operatorname{erfi}(a+bx)}{3d} - \frac{\left(2d^2\right)\int e^{(a+bx)^2}(a+bx)^3\,dx}{3b^2\sqrt{\pi}} - \frac{(2d(bc-ad))\int e^{(a+bx)^2}(a+bx)^2\,dx}{b^2\sqrt{\pi}} - \\
&= -\frac{(bc-ad)^2e^{(a+bx)^2}}{b^3\sqrt{\pi}} - \frac{d(bc-ad)e^{(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} - \frac{d^2e^{(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}} - \frac{(bc-ad)^3\operatorname{erfi}(a+b}{3b^3d} \\
&= \frac{d^2e^{(a+bx)^2}}{3b^3\sqrt{\pi}} - \frac{(bc-ad)^2e^{(a+bx)^2}}{b^3\sqrt{\pi}} - \frac{d(bc-ad)e^{(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} - \frac{d^2e^{(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}} + \frac{d(bc-a}{}
\end{aligned}$$

Mathematica [A] time = 0.174973, size = 142, normalized size = 0.76

$$\frac{\sqrt{\pi}\text{Erfi}(a+bx)\left(-6a^2bcd+2a^3d^2+a\left(6b^2c^2-3d^2\right)+2b^3x\left(3c^2+3cdx+d^2x^2\right)+3bcd\right)-2e^{(a+bx)^2}\left(\left(a^2-1\right)d^2-abd(3c\right.}{6\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Erfi[a + b*x],x]
```

```
[Out] (-2*E^(a + b*x)^2*((-1 + a^2)*d^2 - a*b*d*(3*c + d*x) + b^2*(3*c^2 + 3*c*d*
x + d^2*x^2)) + Sqrt[Pi]*(3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + a*(6*b^2*c^2
- 3*d^2) + 2*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Erfi[a + b*x])/(6*b^3*Sqrt[
Pi])
```

Maple [B] time = 0.049, size = 414, normalized size = 2.2

$$\frac{1}{b}\left(\frac{d^2\text{erfi}\left(bx+a\right)\left(bx+a\right)^3}{3\,b^2}-\frac{d^2\text{erfi}\left(bx+a\right)\left(bx+a\right)^2a}{b^2}+\frac{d\text{erfi}\left(bx+a\right)\left(bx+a\right)^2c}{b}+\frac{d^2\text{erfi}\left(bx+a\right)\left(bx+a\right)a^2}{b^2}-2\,\frac{d\text{erf}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*erfi(b*x+a),x)
```

```
[Out] 1/b*(1/3/b^2*d^2*erfi(b*x+a)*(b*x+a)^3-1/b^2*d^2*erfi(b*x+a)*(b*x+a)^2*a+1/
b*d*erfi(b*x+a)*(b*x+a)^2*c+1/b^2*d^2*erfi(b*x+a)*(b*x+a)*a^2-2/b*d*erfi(b*
x+a)*(b*x+a)*a*c+erfi(b*x+a)*(b*x+a)*c^2-1/3/b^2*d^2*erfi(b*x+a)*a^3+1/b*d*
erfi(b*x+a)*a^2*c-erfi(b*x+a)*a*c^2+1/3*b/d*erfi(b*x+a)*c^3-2/3/b^2/d/Pi^(1
/2)*(1/2*b^3*c^3*Pi^(1/2)*erfi(b*x+a)+d^3*(1/2*(b*x+a)^2*exp((b*x+a)^2)-1/2
*exp((b*x+a)^2))-1/2*a^3*d^3*Pi^(1/2)*erfi(b*x+a)+3/2*a^2*d^3*exp((b*x+a)^2
)-3*a*d^3*(1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi(b*x+a))+3/2*b^2*c^2
*d*exp((b*x+a)^2)+3*b*c*d^2*(1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi(b
*x+a))-3/2*a*b^2*c^2*d*Pi^(1/2)*erfi(b*x+a)+3/2*a^2*b*c*d^2*Pi^(1/2)*erfi(b
*x+a)-3*a*b*c*d^2*exp((b*x+a)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\,\text{erfi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*erfi(b*x + a), x)
```

Fricas [A] time = 2.31035, size = 362, normalized size = 1.95

$$-\frac{2\sqrt{\pi}\left(b^2d^2x^2+3\,b^2c^2-3\,abcd+\left(a^2-1\right)d^2+\left(3\,b^2cd-abd^2\right)x\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-\left(2\,\pi b^3d^2x^3+6\,\pi b^3cdx^2+6\,\pi b^3c^2x+\right.}{6\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(pi)*(b^2*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 - 1)*d^2 + (3*
b^2*c*d - a*b*d^2)*x)*e^(b^2*x^2 + 2*a*b*x + a^2) - (2*pi*b^3*d^2*x^3 + 6*p
i*b^3*c*d*x^2 + 6*pi*b^3*c^2*x + pi*(6*a*b^2*c^2 - 3*(2*a^2 - 1)*b*c*d + (2
*a^3 - 3*a)*d^2))*erfi(b*x + a))/(pi*b^3)
```

Sympy [A] time = 5.38124, size = 398, normalized size = 2.14

$$\begin{cases}\frac{a^3d^2\operatorname{erfi}(a+bx)}{3b^3}-\frac{a^2cd\operatorname{erfi}(a+bx)}{b^2}-\frac{a^2d^2e^{a^2}e^{b^2x^2}e^{2abx}}{3\sqrt{\pi}b^3}+\frac{ac^2\operatorname{erfi}(a+bx)}{b}+\frac{acde^{a^2}e^{b^2x^2}e^{2abx}}{\sqrt{\pi}b^2}+\frac{ad^2xe^{a^2}e^{b^2x^2}e^{2abx}}{3\sqrt{\pi}b^2}-\frac{ad^2\operatorname{erfi}(a+bx)}{2b^3}+c^2x\operatorname{erfi}(a \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\operatorname{erfi}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*erfi(b*x+a),x)
```

```
[Out] Piecewise((a**3*d**2*erfi(a + b*x)/(3*b**3) - a**2*c*d*erfi(a + b*x)/b**2 -
 a**2*d**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b**3) + a*c**2
*erfi(a + b*x)/b + a*c*d*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b*
*2) + a*d**2*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b**2) - a*
d**2*erfi(a + b*x)/(2*b**3) + c**2*x*erfi(a + b*x) + c*d*x**2*erfi(a + b*x)
 + d**2*x**3*erfi(a + b*x)/3 - c**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(
sqrt(pi)*b) - c*d*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b) - d*
*2*x**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b) + c*d*erfi(a +
 b*x)/(2*b**2) + d**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b**
3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*erfi(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \operatorname{erfi}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*erfi(b*x + a), x)
```

3.223 $\int (c+dx)\mathbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=115

$$-\frac{(bc-ad)^2\mathrm{Erfi}(a+bx)}{2b^2d}-\frac{e^{(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^2}+\frac{d\mathrm{Erfi}(a+bx)}{4b^2}-\frac{de^{(a+bx)^2}(a+bx)}{2\sqrt{\pi}b^2}+\frac{(c+dx)^2\mathrm{Erfi}(a+bx)}{2d}$$

```
[Out] -(((b*c - a*d)*E^(a + b*x)^2)/(b^2*Sqrt[Pi])) - (d*E^(a + b*x)^2*(a + b*x))
/(2*b^2*Sqrt[Pi]) + (d*Erfi[a + b*x])/(4*b^2) - ((b*c - a*d)^2*Erfi[a + b*x
])/(2*b^2*d) + ((c + d*x)^2*Erfi[a + b*x])/(2*d)
```

Rubi [A] time = 0.10527, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6363, 2226, 2204, 2209, 2212}

$$-\frac{(bc-ad)^2\mathrm{Erfi}(a+bx)}{2b^2d}-\frac{e^{(a+bx)^2}(bc-ad)}{\sqrt{\pi}b^2}+\frac{d\mathrm{Erfi}(a+bx)}{4b^2}-\frac{de^{(a+bx)^2}(a+bx)}{2\sqrt{\pi}b^2}+\frac{(c+dx)^2\mathrm{Erfi}(a+bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Erfi[a + b*x],x]
```

```
[Out] -(((b*c - a*d)*E^(a + b*x)^2)/(b^2*Sqrt[Pi])) - (d*E^(a + b*x)^2*(a + b*x))
/(2*b^2*Sqrt[Pi]) + (d*Erfi[a + b*x])/(4*b^2) - ((b*c - a*d)^2*Erfi[a + b*x
])/(2*b^2*d) + ((c + d*x)^2*Erfi[a + b*x])/(2*d)
```

Rule 6363

```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Erfi[a + b*x])/(d*(m + 1)), x] - Dist[(2*b)/(Sqrt[Pi]*d*
(m + 1)), Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)\text{erfi}(a+bx)\,dx &= \frac{(c+dx)^2\text{erfi}(a+bx)}{2d} - \frac{b\int e^{(a+bx)^2}(c+dx)^2\,dx}{d\sqrt{\pi}}\\
&= \frac{(c+dx)^2\text{erfi}(a+bx)}{2d} - \frac{b\int \left(\frac{(bc-ad)^2e^{(a+bx)^2}}{b^2} + \frac{2d(bc-ad)e^{(a+bx)^2}(a+bx)}{b^2} + \frac{d^2e^{(a+bx)^2}(a+bx)^2}{b^2}\right)dx}{d\sqrt{\pi}}\\
&= \frac{(c+dx)^2\text{erfi}(a+bx)}{2d} - \frac{d\int e^{(a+bx)^2}(a+bx)^2\,dx}{b\sqrt{\pi}} - \frac{(2(bc-ad))\int e^{(a+bx)^2}(a+bx)\,dx}{b\sqrt{\pi}} - \frac{(bc-ad}{}\\
&= -\frac{(bc-ad)e^{(a+bx)^2}}{b^2\sqrt{\pi}} - \frac{de^{(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}} - \frac{(bc-ad)^2\text{erfi}(a+bx)}{2b^2d} + \frac{(c+dx)^2\text{erfi}(a+bx)}{2d} + \frac{d\int}{}\\
&= -\frac{(bc-ad)e^{(a+bx)^2}}{b^2\sqrt{\pi}} - \frac{de^{(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}} + \frac{d\text{erfi}(a+bx)}{4b^2} - \frac{(bc-ad)^2\text{erfi}(a+bx)}{2b^2d} + \frac{(c+dx)^2\text{er}}{2d}\end{aligned}$$

Mathematica [A] time = 0.0706394, size = 78, normalized size = 0.68

$$\frac{\sqrt{\pi}\text{Erfi}(a+bx)\left(-2a^2d+4abc+4b^2cx+2b^2dx^2+d\right)-2e^{(a+bx)^2}(-ad+2bc+bdx)}{4\sqrt{\pi}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Erfi[a + b*x],x]
```

```
[Out] (-2*E^(a + b*x)^2*(2*b*c - a*d + b*d*x) + Sqrt[Pi]*(4*a*b*c + d - 2*a^2*d +
 4*b^2*c*x + 2*b^2*d*x^2)*Erfi[a + b*x])/(4*b^2*Sqrt[Pi])
```

Maple [A] time = 0.047, size = 117, normalized size = 1.

$$\frac{1}{b}\left(\frac{d\mathrm{erfi}\,(bx+a)\,(bx+a)^2}{2\,b}-\frac{\mathrm{erfi}\,(bx+a)\,(bx+a)\,ad}{b}+\mathrm{erfi}\,(bx+a)\,c\,(bx+a)-\frac{1}{\sqrt{\pi}b}\left(d\left(\frac{(bx+a)\,\mathrm{e}^{(bx+a)^2}}{2}-\frac{\sqrt{\pi}\mathrm{erfi}\,(bx}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*erfi(b*x+a),x)
```

```
[Out] 1/b*(1/2/b*erfi(b*x+a)*d*(b*x+a)^2-1/b*erfi(b*x+a)*(b*x+a)*a*d+erfi(b*x+a)*
c*(b*x+a)-1/Pi^(1/2)/b*(d*(1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi(b*x
+a))-a*d*exp((b*x+a)^2)+exp((b*x+a)^2)*b*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\,\mathrm{erfi}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*erfi(b*x + a), x)
```

Fricas [A] time = 2.32517, size = 212, normalized size = 1.84

$$-\frac{2\,\sqrt{\pi}(bdx+2\,bc-ad)e^{\left(b^2x^2+2\,abx+a^2\right)}-\left(2\,\pi b^2dx^2+4\,\pi b^2cx+\pi\left(4\,abc-\left(2\,a^2-1\right)d\right)\right)\mathrm{erfi}\,(bx+a)}{4\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(pi)*(b*d*x + 2*b*c - a*d)*e^(b^2*x^2 + 2*a*b*x + a^2) - (2*pi*
b^2*d*x^2 + 4*pi*b^2*c*x + pi*(4*a*b*c - (2*a^2 - 1)*d))*erfi(b*x + a))/(pi
*b^2)
```

Sympy [A] time = 1.50053, size = 178, normalized size = 1.55

$$\begin{cases} -\frac{a^2 d \operatorname{erfi}(a+bx)}{2b^2} + \frac{ac \operatorname{erfi}(a+bx)}{b} + \frac{ade^{a^2} e^{b^2x^2} e^{2abx}}{2\sqrt{\pi} b^2} + cx \operatorname{erfi}(a + bx) + \frac{dx^2 \operatorname{erfi}(a+bx)}{2} - \frac{ce^{a^2} e^{b^2x^2} e^{2abx}}{\sqrt{\pi} b} - \frac{dxe^{a^2} e^{b^2x^2} e^{2abx}}{2\sqrt{\pi} b} + \frac{d \operatorname{erfi}(a+bx)}{4b^2} & \text{fo} \\ \left(cx + \frac{dx^2}{2}\right) \operatorname{erfi}(a) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a),x)
```

```
[Out] Piecewise((-a**2*d*erfi(a + b*x)/(2*b**2) + a*c*erfi(a + b*x)/b + a*d*exp(a
**2)*exp(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(pi)*b**2) + c*x*erfi(a + b*x) + d*
x**2*erfi(a + b*x)/2 - c*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b)
 - d*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(pi)*b) + d*erfi(a + b*
x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*erfi(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \operatorname{erfi}(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*erfi(b*x + a), x)
```

3.224 $\int \textbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)\text{Erfi}(a+bx)}{b}-\frac{e^{(a+bx)^2}}{\sqrt{\pi}b}$$

```
[Out] -(E^(a + b*x)^2/(b*Sqrt[Pi])) + ((a + b*x)*Erfi[a + b*x])/b
```

Rubi [A] time = 0.0057444, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6351}

$$\frac{(a+bx)\text{Erfi}(a+bx)}{b}-\frac{e^{(a+bx)^2}}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[a + b*x],x]
```

```
[Out] -(E^(a + b*x)^2/(b*Sqrt[Pi])) + ((a + b*x)*Erfi[a + b*x])/b
```

Rule 6351

```
Int[Erfi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x])/b
, x] - Simp[E^(a + b*x)^2/(b*Sqrt[Pi]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erfi}(a+bx)\,dx=-\frac{e^{(a+bx)^2}}{b\sqrt{\pi}}+\frac{(a+bx)\text{erfi}(a+bx)}{b}$$

Mathematica [A] time = 0.0265767, size = 33, normalized size = 0.94

$$\frac{(a+bx)\text{Erfi}(a+bx)-\frac{e^{(a+bx)^2}}{\sqrt{\pi}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[a + b*x],x]
```

```
[Out] (-(E^(a + b*x)^2/Sqrt[Pi]) + (a + b*x)*Erfi[a + b*x])/b
```

Maple [A] time = 0.045, size = 31, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\,\mathrm{erfi}\,(bx+a)-\frac{\mathrm{e}^{(bx+a)^2}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*erfi(b*x+a)-1/Pi^(1/2)*exp((b*x+a)^2))
```

Maxima [A] time = 1.0806, size = 41, normalized size = 1.17

$$\frac{(bx+a)\,\mathrm{erfi}\,(bx+a)-\frac{e^{\left((bx+a)^2\right)}}{\sqrt{\pi}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*erfi(b*x + a) - e^((b*x + a)^2)/sqrt(pi))/b
```

Fricas [A] time = 2.43289, size = 107, normalized size = 3.06

$$\frac{(\pi bx+\pi a)\,\mathrm{erfi}\,(bx+a)-\sqrt{\pi}e^{\left(b^2x^2+2\,abx+a^2\right)}}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] ((pi*b*x + pi*a)*erfi(b*x + a) - sqrt(pi)*e^(b^2*x^2 + 2*a*b*x + a^2))/(pi*
b)
```

Sympy [A] time = 0.409487, size = 51, normalized size = 1.46

$$\begin{cases} \frac{a\,\mathrm{erfi}\,(a+bx)}{b} + x\,\mathrm{erfi}\,(a+bx) - \frac{e^{a^2}e^{b^2x^2}e^{2abx}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ x\,\mathrm{erfi}\,(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a),x)
```

```
[Out] Piecewise((a*erfi(a + b*x)/b + x*erfi(a + b*x) - exp(a**2)*exp(b**2*x**2)*e
xp(2*a*b*x)/(sqrt(pi)*b), Ne(b, 0)), (x*erfi(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{erfi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a), x)
```

3.225 $\int \frac{\text{Erfi}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{Erfi}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Erfi[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.013578, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Erfi[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfi}(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 1.04754, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Erfi[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.506, size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a)/(d*x+c),x)
```

```
[Out] int(erfi(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erfi}\,(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(erfi(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)/(d*x + c), x)
```

3.226 $\int \frac{\mathbf{Erfi}(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=49

$$\frac{2b\text{Unintegrable}\left(\frac{e^{(a+bx)^2}}{c+dx}, x\right)}{\sqrt{\pi}d} - \frac{\text{Erfi}(a+bx)}{d(c+dx)}$$

```
[Out] -(Erfi[a + b*x]/(d*(c + d*x))) + (2*b*Unintegrable[E^(a + b*x)^2/(c + d*x),
 x])/(d*Sqrt[Pi])
```

Rubi [A] time = 0.0358247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Erfi[a + b*x]/(d*(c + d*x))) + (2*b*Defer[Int][E^(a + b*x)^2/(c + d*x), x
])/(d*Sqrt[Pi])
```

Rubi steps

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^2} dx = -\frac{\text{erfi}(a+bx)}{d(c+dx)} + \frac{(2b)\int \frac{e^{(a+bx)^2}}{c+dx} dx}{d\sqrt{\pi}}$$

Mathematica [A] time = 0.298909, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Erfi[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx + a)}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(erfi(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx + a)}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx + a)}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(a + bx)}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(erfi(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx + a)}{(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)/(d*x + c)^2, x)
```

3.227 $\int \frac{\mathbf{Erfi}(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2b^2(bc-ad)\text{Unintegrable}\left(\frac{e^{(a+bx)^2}}{c+dx},x\right)}{\sqrt{\pi}d^3}+\frac{b^2\text{Erfi}(a+bx)}{d^3}-\frac{be^{(a+bx)^2}}{\sqrt{\pi}d^2(c+dx)}-\frac{\text{Erfi}(a+bx)}{2d(c+dx)^2}$$

```
[Out] -((b*E^(a + b*x)^2)/(d^2*Sqrt[Pi]*(c + d*x))) + (b^2*Erfi[a + b*x])/d^3 - E
rfi[a + b*x]/(2*d*(c + d*x)^2) - (2*b^2*(b*c - a*d)*Unintegrable[E^(a + b*x
)^2/(c + d*x), x])/(d^3*Sqrt[Pi])
```

Rubi [A] time = 0.0741336, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(a+bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -((b*E^(a + b*x)^2)/(d^2*Sqrt[Pi]*(c + d*x))) + (b^2*Erfi[a + b*x])/d^3 - E
rfi[a + b*x]/(2*d*(c + d*x)^2) - (2*b^2*(b*c - a*d)*Defer[Int][E^(a + b*x)^
2/(c + d*x), x])/(d^3*Sqrt[Pi])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{erfi}(a+bx)}{(c+dx)^3}\,dx &= -\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}+\frac{b\int \frac{e^{(a+bx)^2}}{(c+dx)^2}\,dx}{d\sqrt{\pi}}\\
&= -\frac{be^{(a+bx)^2}}{d^2\sqrt{\pi}(c+dx)}-\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}+\frac{\left(2b^3\right)\int e^{(a+bx)^2}\,dx}{d^3\sqrt{\pi}}-\frac{\left(2b^2(bc-ad)\right)\int \frac{e^{(a+bx)^2}}{c+dx}\,dx}{d^3\sqrt{\pi}}\\
&= -\frac{be^{(a+bx)^2}}{d^2\sqrt{\pi}(c+dx)}+\frac{b^2\text{erfi}(a+bx)}{d^3}-\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}-\frac{\left(2b^2(bc-ad)\right)\int \frac{e^{(a+bx)^2}}{c+dx}\,dx}{d^3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.522344, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(a+bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[a + b*x]/(c + d*x)^3,x]
```

```
[Out] Integrate[Erfi[a + b*x]/(c + d*x)^3, x]
```

Maple [A] time = 0.366, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a)/(d*x+c)^3,x)
```

```
[Out] int(erfi(b*x+a)/(d*x+c)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)/(d*x + c)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erfi}(bx+a)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)/(d*x + c)^3, x)
```

3.228 $\int x^5 \mathbf{Erfi}(bx)^2 \, dx$

Optimal. Leaf size=175

$$-\frac{x^5 e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}b}+\frac{5x^3e^{b^2x^2}\text{Erfi}(bx)}{6\sqrt{\pi}b^3}-\frac{5xe^{b^2x^2}\text{Erfi}(bx)}{4\sqrt{\pi}b^5}+\frac{5\text{Erfi}(bx)^2}{16b^6}+\frac{x^4e^{2b^2x^2}}{6\pi b^2}-\frac{7x^2e^{2b^2x^2}}{12\pi b^4}+\frac{11e^{2b^2x^2}}{12\pi b^6}+\frac{1}{6}x^6\text{Erfi}(bx)^2$$

```
[Out] (11*E^(2*b^2*x^2))/(12*b^6*Pi) - (7*E^(2*b^2*x^2)*x^2)/(12*b^4*Pi) + (E^(2*
b^2*x^2)*x^4)/(6*b^2*Pi) - (5*E^(b^2*x^2)*x*Erfi[b*x])/(4*b^5*Sqrt[Pi]) + (
5*E^(b^2*x^2)*x^3*Erfi[b*x])/(6*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^5*Erfi[b*x])
/(3*b*Sqrt[Pi]) + (5*Erfi[b*x]^2)/(16*b^6) + (x^6*Erfi[b*x]^2)/6
```

Rubi [A] time = 0.255586, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6366, 6387, 6375, 30, 2209, 2212}

$$-\frac{x^5 e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}b}+\frac{5x^3e^{b^2x^2}\text{Erfi}(bx)}{6\sqrt{\pi}b^3}-\frac{5xe^{b^2x^2}\text{Erfi}(bx)}{4\sqrt{\pi}b^5}+\frac{5\text{Erfi}(bx)^2}{16b^6}+\frac{x^4e^{2b^2x^2}}{6\pi b^2}-\frac{7x^2e^{2b^2x^2}}{12\pi b^4}+\frac{11e^{2b^2x^2}}{12\pi b^6}+\frac{1}{6}x^6\text{Erfi}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Erfi[b*x]^2,x]
```


```
[Out] (11*E^(2*b^2*x^2))/(12*b^6*Pi) - (7*E^(2*b^2*x^2)*x^2)/(12*b^4*Pi) + (E^(2*
b^2*x^2)*x^4)/(6*b^2*Pi) - (5*E^(b^2*x^2)*x*Erfi[b*x])/(4*b^5*Sqrt[Pi]) + (
5*E^(b^2*x^2)*x^3*Erfi[b*x])/(6*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^5*Erfi[b*x])
/(3*b*Sqrt[Pi]) + (5*Erfi[b*x]^2)/(16*b^6) + (x^6*Erfi[b*x]^2)/6
```


Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int x^5\text{erfi}(bx)^2\,dx = \frac{1}{6}x^6\text{erfi}(bx)^2 - \frac{(2b)\int e^{b^2x^2}x^6\text{erfi}(bx)\,dx}{3\sqrt{\pi}}$$
$$= -\frac{e^{b^2x^2}x^5\text{erfi}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx)^2 + \frac{2\int e^{2b^2x^2}x^5\,dx}{3\pi} + \frac{5\int e^{b^2x^2}x^4\text{erfi}(bx)\,dx}{3b\sqrt{\pi}}$$
$$= \frac{e^{2b^2x^2}x^4}{6b^2\pi} + \frac{5e^{b^2x^2}x^3\text{erfi}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5\text{erfi}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx)^2 - \frac{2\int e^{2b^2x^2}x^3\,dx}{3b^2\pi} - \frac{5\int e^{2b^2x^2}x^3\,dx}{3b^2\pi} - \frac{5}{}$$
$$= -\frac{7e^{2b^2x^2}x^2}{12b^4\pi} + \frac{e^{2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{b^2x^2}x\text{erfi}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3\text{erfi}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5\text{erfi}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx)^2 + \frac{\int e^{2b}}{3}$$
$$= \frac{11e^{2b^2x^2}}{12b^6\pi} - \frac{7e^{2b^2x^2}x^2}{12b^4\pi} + \frac{e^{2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{b^2x^2}x\text{erfi}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3\text{erfi}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5\text{erfi}(bx)}{3b\sqrt{\pi}} + \frac{1}{6}x^6\text{erfi}(bx$$
$$= \frac{11e^{2b^2x^2}}{12b^6\pi} - \frac{7e^{2b^2x^2}x^2}{12b^4\pi} + \frac{e^{2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{b^2x^2}x\text{erfi}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3\text{erfi}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5\text{erfi}(bx)}{3b\sqrt{\pi}} + \frac{5\text{erfi}(bx)^2}{16b^6}$$

Mathematica [A] time = 0.0386386, size = 99, normalized size = 0.57

$$\frac{\pi\left(8b^6x^6+15\right)\text{Erfi}(bx)^2 - 4\sqrt{\pi}bxe^{b^2x^2}\left(4b^4x^4-10b^2x^2+15\right)\text{Erfi}(bx) + 4e^{2b^2x^2}\left(2b^4x^4-7b^2x^2+11\right)}{48\pi b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Erfi[b*x]^2,x]
```

```
[Out] (4*E^(2*b^2*x^2)*(11 - 7*b^2*x^2 + 2*b^4*x^4) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*
(15 - 10*b^2*x^2 + 4*b^4*x^4)*Erfi[b*x] + Pi*(15 + 8*b^6*x^6)*Erfi[b*x]^2)/
(48*b^6*Pi)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int x^5\left(\text{erfi}\left(bx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erfi(b*x)^2,x)
```

```
[Out] int(x^5*erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfi(b*x)^2, x)
```

Fricas [A] time = 2.28581, size = 228, normalized size = 1.3

$$-\frac{4\sqrt{\pi}\left(4\,b^5x^5 - 10\,b^3x^3 + 15\,bx\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2\right)} - \left(15\,\pi + 8\,\pi b^6x^6\right)\operatorname{erfi}(bx)^2 - 4\left(2\,b^4x^4 - 7\,b^2x^2 + 11\right)e^{\left(2\,b^2x^2\right)}}{48\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(4*sqrt(pi)*(4*b^5*x^5 - 10*b^3*x^3 + 15*b*x)*erfi(b*x)*e^(b^2*x^2) -
 (15*pi + 8*pi*b^6*x^6)*erfi(b*x)^2 - 4*(2*b^4*x^4 - 7*b^2*x^2 + 11)*e^(2*b
^2*x^2))/(pi*b^6)
```

Sympy [A] time = 7.1129, size = 168, normalized size = 0.96

$$\begin{cases} \frac{x^6 \operatorname{erfi}^2(bx)}{6} - \frac{x^5 e^{b^2x^2} \operatorname{erfi}(bx)}{3\sqrt{\pi}b} + \frac{x^4 e^{2b^2x^2}}{6\pi b^2} + \frac{5x^3 e^{b^2x^2} \operatorname{erfi}(bx)}{6\sqrt{\pi}b^3} - \frac{7x^2 e^{2b^2x^2}}{12\pi b^4} - \frac{5x e^{b^2x^2} \operatorname{erfi}(bx)}{4\sqrt{\pi}b^5} + \frac{11 e^{2b^2x^2}}{12\pi b^6} + \frac{5 \operatorname{erfi}^2(bx)}{16b^6} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erfi(b*x)**2,x)
```

```
[Out] Piecewise((x**6*erfi(b*x)**2/6 - x**5*exp(b**2*x**2)*erfi(b*x)/(3*sqrt(pi)*
b) + x**4*exp(2*b**2*x**2)/(6*pi*b**2) + 5*x**3*exp(b**2*x**2)*erfi(b*x)/(6
*sqrt(pi)*b**3) - 7*x**2*exp(2*b**2*x**2)/(12*pi*b**4) - 5*x*exp(b**2*x**2)
*erfi(b*x)/(4*sqrt(pi)*b**5) + 11*exp(2*b**2*x**2)/(12*pi*b**6) + 5*erfi(b*
```

```
x)**2/(16*b**6), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*erfi(b*x)^2, x)
```

3.229 $\int x^3 \mathbf{Erfi}(bx)^2 \, dx$

Optimal. Leaf size=124

$$-\frac{x^3 e^{b^2x^2}\text{Erfi}(bx)}{2\sqrt{\pi}b}+\frac{3xe^{b^2x^2}\text{Erfi}(bx)}{4\sqrt{\pi}b^3}-\frac{3\text{Erfi}(bx)^2}{16b^4}+\frac{x^2e^{2b^2x^2}}{4\pi b^2}-\frac{e^{2b^2x^2}}{2\pi b^4}+\frac{1}{4}x^4\text{Erfi}(bx)^2$$

```
[Out] -E^(2*b^2*x^2)/(2*b^4*Pi) + (E^(2*b^2*x^2)*x^2)/(4*b^2*Pi) + (3*E^(b^2*x^2)
*x*Erfi[b*x])/(4*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^3*Erfi[b*x])/(2*b*Sqrt[Pi])
 - (3*Erfi[b*x]^2)/(16*b^4) + (x^4*Erfi[b*x]^2)/4
```

Rubi [A] time = 0.159084, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6366, 6387, 6375, 30, 2209, 2212}

$$-\frac{x^3 e^{b^2x^2}\text{Erfi}(bx)}{2\sqrt{\pi}b}+\frac{3xe^{b^2x^2}\text{Erfi}(bx)}{4\sqrt{\pi}b^3}-\frac{3\text{Erfi}(bx)^2}{16b^4}+\frac{x^2e^{2b^2x^2}}{4\pi b^2}-\frac{e^{2b^2x^2}}{2\pi b^4}+\frac{1}{4}x^4\text{Erfi}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Erfi[b*x]^2,x]
```

```
[Out] -E^(2*b^2*x^2)/(2*b^4*Pi) + (E^(2*b^2*x^2)*x^2)/(4*b^2*Pi) + (3*E^(b^2*x^2)
*x*Erfi[b*x])/(4*b^3*Sqrt[Pi]) - (E^(b^2*x^2)*x^3*Erfi[b*x])/(2*b*Sqrt[Pi])
 - (3*Erfi[b*x]^2)/(16*b^4) + (x^4*Erfi[b*x]^2)/4
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \text{erfi}(bx)^2\,dx &= \frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{b\int e^{b^2x^2}x^4\text{erfi}(bx)\,dx}{\sqrt{\pi}} \\
&= -\frac{e^{b^2x^2}x^3\text{erfi}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfi}(bx)^2 + \frac{\int e^{2b^2x^2}x^3\,dx}{\pi} + \frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)\,dx}{2b\sqrt{\pi}} \\
&= \frac{e^{2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{b^2x^2}x\text{erfi}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3\text{erfi}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{\int e^{2b^2x^2}x\,dx}{2b^2\pi} - \frac{3\int e^{2b^2x^2}x\,dx}{2b^2\pi} - \frac{3\int e^{b^2}}{} \\
&= -\frac{e^{2b^2x^2}}{2b^4\pi} + \frac{e^{2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{b^2x^2}x\text{erfi}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3\text{erfi}(bx)}{2b\sqrt{\pi}} + \frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{3\,\text{Subst}(\int x\,dx, x, \text{erfi}(bx))}{8b^4} \\
&= -\frac{e^{2b^2x^2}}{2b^4\pi} + \frac{e^{2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{b^2x^2}x\text{erfi}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3\text{erfi}(bx)}{2b\sqrt{\pi}} - \frac{3\text{erfi}(bx)^2}{16b^4} + \frac{1}{4}x^4\text{erfi}(bx)^2
\end{aligned}$$

Mathematica [A] time = 0.027615, size = 82, normalized size = 0.66

$$\frac{\pi\left(4b^4x^4-3\right)\operatorname{Erfi}(bx)^2-4\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2-3\right)\operatorname{Erfi}(bx)+4e^{2b^2x^2}\left(b^2x^2-2\right)}{16\pi b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Erfi[b*x]^2,x]
```

```
[Out] (4*E^(2*b^2*x^2)*(-2 + b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*b^2*x^
2)*Erfi[b*x] + Pi*(-3 + 4*b^4*x^4)*Erfi[b*x]^2)/(16*b^4*Pi)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x^3(\operatorname{erfi}(bx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erfi(b*x)^2,x)
```

```
[Out] int(x^3*erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{erfi}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfi(b*x)^2, x)
```

Fricas [A] time = 2.41216, size = 188, normalized size = 1.52

$$-\frac{4\sqrt{\pi}\left(2\,b^3x^3-3\,bx\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2\right)}+\left(3\,\pi-4\,\pi b^4x^4\right)\operatorname{erfi}(bx)^2-4\left(b^2x^2-2\right)e^{\left(2\,b^2x^2\right)}}{16\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(pi)*(2*b^3*x^3 - 3*b*x)*erfi(b*x)*e^(b^2*x^2) + (3*pi - 4*pi*
b^4*x^4)*erfi(b*x)^2 - 4*(b^2*x^2 - 2)*e^(2*b^2*x^2))/(pi*b^4)
```

Sympy [A] time = 2.10517, size = 116, normalized size = 0.94

$$\begin{cases} \frac{x^4 \operatorname{erfi}^2(bx)}{4} - \frac{x^3 e^{b^2x^2} \operatorname{erfi}(bx)}{2\sqrt{\pi}b} + \frac{x^2 e^{2b^2x^2}}{4\pi b^2} + \frac{3xe^{b^2x^2} \operatorname{erfi}(bx)}{4\sqrt{\pi}b^3} - \frac{e^{2b^2x^2}}{2\pi b^4} - \frac{3 \operatorname{erfi}^2(bx)}{16b^4} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erfi(b*x)**2,x)
```

```
[Out] Piecewise((x**4*erfi(b*x)**2/4 - x**3*exp(b**2*x**2)*erfi(b*x)/(2*sqrt(pi)*
b) + x**2*exp(2*b**2*x**2)/(4*pi*b**2) + 3*x*exp(b**2*x**2)*erfi(b*x)/(4*sq
rt(pi)*b**3) - exp(2*b**2*x**2)/(2*pi*b**4) - 3*erfi(b*x)**2/(16*b**4), Ne(
b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*erfi(b*x)^2, x)
```

3.230 $\int x\mathbf{Erfi}(bx)^2\,dx$

Optimal. Leaf size=71

$$-\frac{xe^{b^2x^2}\text{Erfi}(bx)}{\sqrt{\pi}b}+\frac{\text{Erfi}(bx)^2}{4b^2}+\frac{e^{2b^2x^2}}{2\pi b^2}+\frac{1}{2}x^2\text{Erfi}(bx)^2$$

```
[Out] E^(2*b^2*x^2)/(2*b^2*Pi) - (E^(b^2*x^2)*x*Erfi[b*x])/(b*Sqrt[Pi]) + Erfi[b*
x]^2/(4*b^2) + (x^2*Erfi[b*x]^2)/2
```

Rubi [A] time = 0.0766503, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6366, 6387, 6375, 30, 2209}

$$-\frac{xe^{b^2x^2}\text{Erfi}(bx)}{\sqrt{\pi}b}+\frac{\text{Erfi}(bx)^2}{4b^2}+\frac{e^{2b^2x^2}}{2\pi b^2}+\frac{1}{2}x^2\text{Erfi}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Erfi[b*x]^2,x]
```

```
[Out] E^(2*b^2*x^2)/(2*b^2*Pi) - (E^(b^2*x^2)*x*Erfi[b*x])/(b*Sqrt[Pi]) + Erfi[b*
x]^2/(4*b^2) + (x^2*Erfi[b*x]^2)/2
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x\text{erfi}(bx)^2\,dx &= \frac{1}{2}x^2\text{erfi}(bx)^2 - \frac{(2b)\int e^{b^2x^2}x^2\text{erfi}(bx)\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{b^2x^2}x\text{erfi}(bx)}{b\sqrt{\pi}} + \frac{1}{2}x^2\text{erfi}(bx)^2 + \frac{2\int e^{2b^2x^2}x\,dx}{\pi} + \frac{\int e^{b^2x^2}\text{erfi}(bx)\,dx}{b\sqrt{\pi}}\\ &= \frac{e^{2b^2x^2}}{2b^2\pi} - \frac{e^{b^2x^2}x\text{erfi}(bx)}{b\sqrt{\pi}} + \frac{1}{2}x^2\text{erfi}(bx)^2 + \frac{\text{Subst}\left(\int x\,dx, x, \text{erfi}(bx)\right)}{2b^2}\\ &= \frac{e^{2b^2x^2}}{2b^2\pi} - \frac{e^{b^2x^2}x\text{erfi}(bx)}{b\sqrt{\pi}} + \frac{\text{erfi}(bx)^2}{4b^2} + \frac{1}{2}x^2\text{erfi}(bx)^2\end{aligned}$$

Mathematica [A] time = 0.0160232, size = 63, normalized size = 0.89

$$\frac{\left(2\pi b^2x^2+\pi\right)\text{Erfi}(bx)^2 - 4\sqrt{\pi}bxe^{b^2x^2}\text{Erfi}(bx) + 2e^{2b^2x^2}}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erfi[b*x]^2,x]
```

```
[Out] (2*E^(2*b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*Erfi[b*x] + (Pi + 2*b^2*Pi*x^
2)*Erfi[b*x]^2)/(4*b^2*Pi)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int x\left(\operatorname{erfi}\left(bx\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfi(b*x)^2,x)
```

```
[Out] int(x*erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{erfi}\left(bx\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*erfi(b*x)^2, x)
```

Fricas [A] time = 2.30628, size = 143, normalized size = 2.01

$$-\frac{4\sqrt{\pi}bx\operatorname{erfi}\left(bx\right)e^{\left(b^{2}x^{2}\right)} - \left(\pi + 2\,\pi b^{2}x^{2}\right)\operatorname{erfi}\left(bx\right)^{2} - 2\,e^{\left(2\,b^{2}x^{2}\right)}}{4\,\pi b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(pi)*b*x*erfi(b*x)*e^(b^2*x^2) - (pi + 2*pi*b^2*x^2)*erfi(b*x)^
2 - 2*e^(2*b^2*x^2))/(pi*b^2)
```

Sympy [A] time = 0.52173, size = 63, normalized size = 0.89

$$\begin{cases} \frac{x^{2}\operatorname{erfi}^{2}\left(bx\right)}{2} - \frac{xe^{b^{2}x^{2}}\operatorname{erfi}\left(bx\right)}{\sqrt{\pi}b} + \frac{e^{2b^{2}x^{2}}}{2\pi b^{2}} + \frac{\operatorname{erfi}^{2}\left(bx\right)}{4b^{2}} & \text{for}\: b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)**2,x)
```

```
[Out] Piecewise((x**2*erfi(b*x)**2/2 - x*exp(b**2*x**2)*erfi(b*x)/(sqrt(pi)*b) +
exp(2*b**2*x**2)/(2*pi*b**2) + erfi(b*x)**2/(4*b**2), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*erfi(b*x)^2, x)
```

3.231 $\int \frac{\text{Erfi}(bx)^2}{x} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfi}(bx)^2}{x}, x\right)$$

```
[Out] Unintegrable[Erfi[b*x]^2/x, x]
```

Rubi [A] time = 0.0163806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[b*x]^2/x,x]
```

```
[Out] Defer[Int][Erfi[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\text{erfi}(bx)^2}{x} \, dx = \int \frac{\text{erfi}(bx)^2}{x} \, dx$$

Mathematica [A] time = 0.0263319, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[b*x]^2/x,x]
```

```
[Out] Integrate[Erfi[b*x]^2/x, x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfi}(bx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x,x)
```

```
[Out] int(erfi(b*x)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x,x)
```

```
[Out] Integral(erfi(b*x)**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x, x)
```

3.232 $\int \frac{\text{Erfi}(bx)^2}{x^3} dx$

Optimal. Leaf size=65

$$-\frac{2be^{b^2x^2}\text{Erfi}(bx)}{\sqrt{\pi}x}+b^2\text{Erfi}(bx)^2+\frac{2b^2\text{ExpIntegralEi}\left(2b^2x^2\right)}{\pi}-\frac{\text{Erfi}(bx)^2}{2x^2}$$

```
[Out] (-2*b*E^(b^2*x^2)*Erfi[b*x])/(Sqrt[Pi]*x) + b^2*Erfi[b*x]^2 - Erfi[b*x]^2/(
2*x^2) + (2*b^2*ExpIntegralEi[2*b^2*x^2])/Pi
```

Rubi [A] time = 0.0901547, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6366, 6393, 6375, 30, 2210}

$$-\frac{2be^{b^2x^2}\text{Erfi}(bx)}{\sqrt{\pi}x}+b^2\text{Erfi}(bx)^2+\frac{2b^2\text{Ei}\left(2b^2x^2\right)}{\pi}-\frac{\text{Erfi}(bx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]^2/x^3,x]
```

```
[Out] (-2*b*E^(b^2*x^2)*Erfi[b*x])/(Sqrt[Pi]*x) + b^2*Erfi[b*x]^2 - Erfi[b*x]^2/(
2*x^2) + (2*b^2*ExpIntegralEi[2*b^2*x^2])/Pi
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx &= -\frac{\operatorname{erfi}(bx)^2}{2x^2} + \frac{(2b)\int \frac{e^{b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx}{\sqrt{\pi}} \\
&= -\frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)^2}{2x^2} + \frac{\left(4b^2\right)\int \frac{e^{2b^2x^2}}{x}\,dx}{\pi} + \frac{\left(4b^3\right)\int e^{b^2x^2}\operatorname{erfi}(bx)\,dx}{\sqrt{\pi}} \\
&= -\frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)^2}{2x^2} + \frac{2b^2\operatorname{Ei}\left(2b^2x^2\right)}{\pi} + \left(2b^2\right)\operatorname{Subst}\left(\int x\,dx, x, \operatorname{erfi}(bx)\right) \\
&= -\frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{\sqrt{\pi}x} + b^2\operatorname{erfi}(bx)^2 - \frac{\operatorname{erfi}(bx)^2}{2x^2} + \frac{2b^2\operatorname{Ei}\left(2b^2x^2\right)}{\pi}
\end{aligned}$$

Mathematica [A] time = 0.0227362, size = 60, normalized size = 0.92

$$-\frac{2be^{b^2x^2}\operatorname{Erfi}(bx)}{\sqrt{\pi}x} + \left(b^2 - \frac{1}{2x^2}\right)\operatorname{Erfi}(bx)^2 + \frac{2b^2\operatorname{ExpIntegralEi}\left(2b^2x^2\right)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]^2/x^3,x]
```

```
[Out] (-2*b*E^(b^2*x^2)*Erfi[b*x])/(Sqrt[Pi]*x) + (b^2 - 1/(2*x^2))*Erfi[b*x]^2 +
 (2*b^2*ExpIntegralEi[2*b^2*x^2])/Pi
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfi}(bx))^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x^3,x)
```

```
[Out] int(erfi(b*x)^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x^3, x)
```

Fricas [A] time = 2.33436, size = 153, normalized size = 2.35

$$\frac{4\, b^2 x^2 \operatorname{Ei}\left(2\, b^2 x^2\right) - 4\, \sqrt{\pi} b x \operatorname{erfi}(bx)\, e^{\left(b^2 x^2\right)} - \left(\pi - 2\, \pi b^2 x^2\right) \operatorname{erfi}(bx)^2}{2\, \pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*Ei(2*b^2*x^2) - 4*sqrt(pi)*b*x*erfi(b*x)*e^(b^2*x^2) - (pi -
 2*pi*b^2*x^2)*erfi(b*x)^2)/(pi*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x**3,x)
```

```
[Out] Integral(erfi(b*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x^3, x)
```

3.233 $\int \frac{\mathbf{Erfi}(bx)^2}{x^5}\,dx$

Optimal. Leaf size=123

$$-\frac{2b^3e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}x}-\frac{be^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}x^3}+\frac{1}{3}b^4\text{Erfi}(bx)^2+\frac{4b^4\text{ExpIntegralEi}\left(2b^2x^2\right)}{3\pi}-\frac{b^2e^{2b^2x^2}}{3\pi x^2}-\frac{\text{Erfi}(bx)^2}{4x^4}$$

```
[Out] -(b^2*E^(2*b^2*x^2))/(3*Pi*x^2) - (b*E^(b^2*x^2)*Erfi[b*x])/(3*Sqrt[Pi]*x^3
) - (2*b^3*E^(b^2*x^2)*Erfi[b*x])/(3*Sqrt[Pi]*x) + (b^4*Erfi[b*x]^2)/3 - Er
fi[b*x]^2/(4*x^4) + (4*b^4*ExpIntegralEi[2*b^2*x^2])/(3*Pi)
```

Rubi [A] time = 0.165433, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6366, 6393, 6375, 30, 2210, 2214}

$$-\frac{2b^3e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}x}-\frac{be^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}x^3}+\frac{1}{3}b^4\text{Erfi}(bx)^2+\frac{4b^4\text{Ei}\left(2b^2x^2\right)}{3\pi}-\frac{b^2e^{2b^2x^2}}{3\pi x^2}-\frac{\text{Erfi}(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]^2/x^5,x]
```

```
[Out] -(b^2*E^(2*b^2*x^2))/(3*Pi*x^2) - (b*E^(b^2*x^2)*Erfi[b*x])/(3*Sqrt[Pi]*x^3
) - (2*b^3*E^(b^2*x^2)*Erfi[b*x])/(3*Sqrt[Pi]*x) + (b^4*Erfi[b*x]^2)/3 - Er
fi[b*x]^2/(4*x^4) + (4*b^4*ExpIntegralEi[2*b^2*x^2])/(3*Pi)
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\int \frac{\text{erfi}(bx)^2}{x^5}\,dx = -\frac{\text{erfi}(bx)^2}{4x^4}+\frac{b\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^4}\,dx}{\sqrt{\pi}}$$

$$= -\frac{be^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x^3}-\frac{\text{erfi}(bx)^2}{4x^4}+\frac{\left(2b^2\right)\int \frac{e^{2b^2x^2}}{x^3}\,dx}{3\pi}+\frac{\left(2b^3\right)\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}\,dx}{3\sqrt{\pi}}$$

$$= -\frac{b^2e^{2b^2x^2}}{3\pi x^2}-\frac{be^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x^3}-\frac{2b^3e^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x}-\frac{\text{erfi}(bx)^2}{4x^4}+2\frac{\left(4b^4\right)\int \frac{e^{2b^2x^2}}{x}\,dx}{3\pi}+\frac{\left(4b^5\right)\int e^{b^2x^2}\text{erfi}(bx)\,d}{3\sqrt{\pi}}$$

$$= -\frac{b^2e^{2b^2x^2}}{3\pi x^2}-\frac{be^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x^3}-\frac{2b^3e^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x}-\frac{\text{erfi}(bx)^2}{4x^4}+\frac{4b^4\text{Ei}\left(2b^2x^2\right)}{3\pi}+\frac{1}{3}\left(2b^4\right)\text{Subst}(\int x\,dx,x,$$

$$= -\frac{b^2e^{2b^2x^2}}{3\pi x^2}-\frac{be^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x^3}-\frac{2b^3e^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x}+\frac{1}{3}b^4\text{erfi}(bx)^2-\frac{\text{erfi}(bx)^2}{4x^4}+\frac{4b^4\text{Ei}\left(2b^2x^2\right)}{3\pi}$$

Mathematica [A] time = 0.0284174, size = 97, normalized size = 0.79

$$\frac{-4\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2+1\right)\operatorname{Erfi}(bx)+\pi\left(4b^4x^4-3\right)\operatorname{Erfi}(bx)^2-4b^2x^2\left(e^{2b^2x^2}-4b^2x^2\operatorname{ExpIntegralEi}\left(2b^2x^2\right)\right)}{12\pi x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]^2/x^5,x]
```

```
[Out] (-4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(1 + 2*b^2*x^2)*Erfi[b*x] + Pi*(-3 + 4*b^4*x^4
)*Erfi[b*x]^2 - 4*b^2*x^2*(E^(2*b^2*x^2) - 4*b^2*x^2*ExpIntegralEi[2*b^2*x^
2]))/(12*Pi*x^4)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{erfi}(bx)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x^5,x)
```

```
[Out] int(erfi(b*x)^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x^5, x)
```

Fricas [A] time = 2.52266, size = 212, normalized size = 1.72

$$\frac{16\,b^4x^4\operatorname{Ei}\left(2\,b^2x^2\right)-4\,b^2x^2e^{\left(2\,b^2x^2\right)}-4\,\sqrt{\pi}\left(2\,b^3x^3+bx\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2\right)}-\left(3\,\pi-4\,\pi b^4x^4\right)\operatorname{erfi}(bx)^2}{12\,\pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(16*b^4*x^4*Ei(2*b^2*x^2) - 4*b^2*x^2*e^(2*b^2*x^2) - 4*sqrt(pi)*(2*b^
3*x^3 + b*x)*erfi(b*x)*e^(b^2*x^2) - (3*pi - 4*pi*b^4*x^4)*erfi(b*x)^2)/(pi
*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x**5,x)
```

```
[Out] Integral(erfi(b*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x^5, x)
```

3.234 $\int \frac{\mathbf{Erfi}(bx)^2}{x^7}\,dx$

Optimal. Leaf size=174

$$-\frac{8b^5e^{b^2x^2}\mathrm{Erfi}(bx)}{45\sqrt{\pi}x}-\frac{4b^3e^{b^2x^2}\mathrm{Erfi}(bx)}{45\sqrt{\pi}x^3}-\frac{2be^{b^2x^2}\mathrm{Erfi}(bx)}{15\sqrt{\pi}x^5}+\frac{4}{45}b^6\mathrm{Erfi}(bx)^2+\frac{28b^6\mathrm{ExpIntegralEi}\left(2b^2x^2\right)}{45\pi}-\frac{2b^4e^{2b^2x^2}}{9\pi x^2}-$$

```
[Out] -(b^2*E^(2*b^2*x^2))/(15*Pi*x^4) - (2*b^4*E^(2*b^2*x^2))/(9*Pi*x^2) - (2*b*
E^(b^2*x^2)*Erfi[b*x])/(15*Sqrt[Pi]*x^5) - (4*b^3*E^(b^2*x^2)*Erfi[b*x])/(4
5*Sqrt[Pi]*x^3) - (8*b^5*E^(b^2*x^2)*Erfi[b*x])/(45*Sqrt[Pi]*x) + (4*b^6*Er
fi[b*x]^2)/45 - Erfi[b*x]^2/(6*x^6) + (28*b^6*ExpIntegralEi[2*b^2*x^2])/(45
*Pi)
```

Rubi [A] time = 0.278855, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6366, 6393, 6375, 30, 2210, 2214}

$$-\frac{8b^5e^{b^2x^2}\mathrm{Erfi}(bx)}{45\sqrt{\pi}x}-\frac{4b^3e^{b^2x^2}\mathrm{Erfi}(bx)}{45\sqrt{\pi}x^3}-\frac{2be^{b^2x^2}\mathrm{Erfi}(bx)}{15\sqrt{\pi}x^5}+\frac{4}{45}b^6\mathrm{Erfi}(bx)^2+\frac{28b^6\mathrm{Ei}\left(2b^2x^2\right)}{45\pi}-\frac{2b^4e^{2b^2x^2}}{9\pi x^2}-\frac{b^2e^{2b^2x^2}}{15\pi x^4}-\mathrm{Er}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]^2/x^7,x]
```


```
[Out] -(b^2*E^(2*b^2*x^2))/(15*Pi*x^4) - (2*b^4*E^(2*b^2*x^2))/(9*Pi*x^2) - (2*b*
E^(b^2*x^2)*Erfi[b*x])/(15*Sqrt[Pi]*x^5) - (4*b^3*E^(b^2*x^2)*Erfi[b*x])/(4
5*Sqrt[Pi]*x^3) - (8*b^5*E^(b^2*x^2)*Erfi[b*x])/(45*Sqrt[Pi]*x) + (4*b^6*Er
fi[b*x]^2)/45 - Erfi[b*x]^2/(6*x^6) + (28*b^6*ExpIntegralEi[2*b^2*x^2])/(45
*Pi)
```


Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
```

```
+ 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\int \frac{\operatorname{erfi}(bx)^2}{x^7}\,dx = -\frac{\operatorname{erfi}(bx)^2}{6x^6} + \frac{(2b)\int \frac{e^{b^2x^2}\operatorname{erfi}(bx)}{x^6}\,dx}{3\sqrt{\pi}}$$

$$= -\frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{15\sqrt{\pi}x^5} - \frac{\operatorname{erfi}(bx)^2}{6x^6} + \frac{\left(4b^2\right)\int \frac{e^{2b^2x^2}}{x^5}\,dx}{15\pi} + \frac{\left(4b^3\right)\int \frac{e^{b^2x^2}\operatorname{erfi}(bx)}{x^4}\,dx}{15\sqrt{\pi}}$$

$$= -\frac{b^2e^{2b^2x^2}}{15\pi x^4} - \frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x^3} - \frac{\operatorname{erfi}(bx)^2}{6x^6} + \frac{\left(8b^4\right)\int \frac{e^{2b^2x^2}}{x^3}\,dx}{45\pi} + \frac{\left(4b^4\right)\int \frac{e^{2b^2x^2}}{x^3}\,dx}{15\pi} + \frac{(8}{}$$

$$= -\frac{b^2e^{2b^2x^2}}{15\pi x^4} - \frac{2b^4e^{2b^2x^2}}{9\pi x^2} - \frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)^2}{6x^6} + 2\frac{\left(16b^6\right)\int}{45}$$

$$= -\frac{b^2e^{2b^2x^2}}{15\pi x^4} - \frac{2b^4e^{2b^2x^2}}{9\pi x^2} - \frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)^2}{6x^6} + \frac{28b^6\operatorname{Ei}\left(2\right.}{45\pi}$$

$$= -\frac{b^2e^{2b^2x^2}}{15\pi x^4} - \frac{2b^4e^{2b^2x^2}}{9\pi x^2} - \frac{2be^{b^2x^2}\operatorname{erfi}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5e^{b^2x^2}\operatorname{erfi}(bx)}{45\sqrt{\pi}x} + \frac{4}{45}b^6\operatorname{erfi}(bx)^2 - \frac{\operatorname{erfi}(}{6x}$$

Mathematica [A] time = 0.0372825, size = 114, normalized size = 0.66

$$\frac{-4\sqrt{\pi}bxe^{b^2x^2}\left(4b^4x^4+2b^2x^2+3\right)\operatorname{Erfi}(bx)+\pi\left(8b^6x^6-15\right)\operatorname{Erfi}(bx)^2+56b^6x^6\operatorname{ExpIntegralEi}\left(2b^2x^2\right)-2b^2x^2e^{2b^2x^2}\left(1\right.}{90\pi x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]^2/x^7,x]
```

```
[Out] (-2*b^2*E^(2*b^2*x^2)*x^2*(3 + 10*b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3
+ 2*b^2*x^2 + 4*b^4*x^4)*Erfi[b*x] + Pi*(-15 + 8*b^6*x^6)*Erfi[b*x]^2 + 56*
b^6*x^6*ExpIntegralEi[2*b^2*x^2])/(90*Pi*x^6)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{erfi}(bx)\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x^7,x)
```

```
[Out] int(erfi(b*x)^2/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x^7, x)
```

Fricas [A] time = 2.55369, size = 255, normalized size = 1.47

$$\frac{56\,b^6x^6\operatorname{Ei}\left(2\,b^2x^2\right) - 4\,\sqrt{\pi}\left(4\,b^5x^5 + 2\,b^3x^3 + 3\,bx\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2\right)} - \left(15\,\pi - 8\,\pi b^6x^6\right)\operatorname{erfi}(bx)^2 - 2\left(10\,b^4x^4 + 3\,b^2x^2\right)e^{\left(2\,b^2\right.}}{90\,\pi x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/90*(56*b^6*x^6*Ei(2*b^2*x^2) - 4*sqrt(pi)*(4*b^5*x^5 + 2*b^3*x^3 + 3*b*x)
*erfi(b*x)*e^(b^2*x^2) - (15*pi - 8*pi*b^6*x^6)*erfi(b*x)^2 - 2*(10*b^4*x^4
 + 3*b^2*x^2)*e^(2*b^2*x^2))/(pi*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x**7,x)
```

```
[Out] Integral(erfi(b*x)**2/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x^7, x)
```

3.235 $\int x^4 \mathbf{Erfi}(bx)^2 \, dx$

Optimal. Leaf size=162

$$-\frac{2x^4e^{b^2x^2}\text{Erfi}(bx)}{5\sqrt{\pi}b}+\frac{4x^2e^{b^2x^2}\text{Erfi}(bx)}{5\sqrt{\pi}b^3}-\frac{4e^{b^2x^2}\text{Erfi}(bx)}{5\sqrt{\pi}b^5}+\frac{43\text{Erfi}\left(\sqrt{2}bx\right)}{40\sqrt{2\pi}b^5}+\frac{x^3e^{2b^2x^2}}{5\pi b^2}-\frac{11xe^{2b^2x^2}}{20\pi b^4}+\frac{1}{5}x^5\text{Erfi}(bx)^2$$

```
[Out] (-11*E^(2*b^2*x^2)*x)/(20*b^4*Pi) + (E^(2*b^2*x^2)*x^3)/(5*b^2*Pi) - (4*E^(
b^2*x^2)*Erfi[b*x])/(5*b^5*Sqrt[Pi]) + (4*E^(b^2*x^2)*x^2*Erfi[b*x])/(5*b^3
*Sqrt[Pi]) - (2*E^(b^2*x^2)*x^4*Erfi[b*x])/(5*b*Sqrt[Pi]) + (x^5*Erfi[b*x]^
2)/5 + (43*Erfi[Sqrt[2]*b*x])/(40*b^5*Sqrt[2*Pi])
```

Rubi [A] time = 0.225014, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6366, 6387, 6384, 2204, 2212}

$$-\frac{2x^4e^{b^2x^2}\text{Erfi}(bx)}{5\sqrt{\pi}b}+\frac{4x^2e^{b^2x^2}\text{Erfi}(bx)}{5\sqrt{\pi}b^3}-\frac{4e^{b^2x^2}\text{Erfi}(bx)}{5\sqrt{\pi}b^5}+\frac{43\text{Erfi}\left(\sqrt{2}bx\right)}{40\sqrt{2\pi}b^5}+\frac{x^3e^{2b^2x^2}}{5\pi b^2}-\frac{11xe^{2b^2x^2}}{20\pi b^4}+\frac{1}{5}x^5\text{Erfi}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Erfi[b*x]^2,x]
```


```
[Out] (-11*E^(2*b^2*x^2)*x)/(20*b^4*Pi) + (E^(2*b^2*x^2)*x^3)/(5*b^2*Pi) - (4*E^(
b^2*x^2)*Erfi[b*x])/(5*b^5*Sqrt[Pi]) + (4*E^(b^2*x^2)*x^2*Erfi[b*x])/(5*b^3
*Sqrt[Pi]) - (2*E^(b^2*x^2)*x^4*Erfi[b*x])/(5*b*Sqrt[Pi]) + (x^5*Erfi[b*x]^
2)/5 + (43*Erfi[Sqrt[2]*b*x])/(40*b^5*Sqrt[2*Pi])
```


Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4\text{erfi}(bx)^2\,dx &= \frac{1}{5}x^5\text{erfi}(bx)^2 - \frac{(4b)\int e^{b^2x^2}x^5\text{erfi}(bx)\,dx}{5\sqrt{\pi}} \\
&= -\frac{2e^{b^2x^2}x^4\text{erfi}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfi}(bx)^2 + \frac{4\int e^{2b^2x^2}x^4\,dx}{5\pi} + \frac{8\int e^{b^2x^2}x^3\text{erfi}(bx)\,dx}{5b\sqrt{\pi}} \\
&= \frac{e^{2b^2x^2}x^3}{5b^2\pi} + \frac{4e^{b^2x^2}x^2\text{erfi}(bx)}{5b^3\sqrt{\pi}} - \frac{2e^{b^2x^2}x^4\text{erfi}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfi}(bx)^2 - \frac{3\int e^{2b^2x^2}x^2\,dx}{5b^2\pi} - \frac{8\int e^{2b^2x^2}x^2\,dx}{5b^2\pi} - \\
&= -\frac{11e^{2b^2x^2}x}{20b^4\pi} + \frac{e^{2b^2x^2}x^3}{5b^2\pi} - \frac{4e^{b^2x^2}\text{erfi}(bx)}{5b^5\sqrt{\pi}} + \frac{4e^{b^2x^2}x^2\text{erfi}(bx)}{5b^3\sqrt{\pi}} - \frac{2e^{b^2x^2}x^4\text{erfi}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfi}(bx)^2 + \frac{3\int e}{20} \\
&= -\frac{11e^{2b^2x^2}x}{20b^4\pi} + \frac{e^{2b^2x^2}x^3}{5b^2\pi} - \frac{4e^{b^2x^2}\text{erfi}(bx)}{5b^5\sqrt{\pi}} + \frac{4e^{b^2x^2}x^2\text{erfi}(bx)}{5b^3\sqrt{\pi}} - \frac{2e^{b^2x^2}x^4\text{erfi}(bx)}{5b\sqrt{\pi}} + \frac{1}{5}x^5\text{erfi}(bx)^2 + \frac{2\sqrt{\frac{2}{\pi}}}{}
\end{aligned}
$$

Mathematica [A] time = 0.0455509, size = 105, normalized size = 0.65

$$
\frac{16\pi b^5x^5\text{Erfi}(bx)^2 - 32\sqrt{\pi}e^{b^2x^2}\left(b^4x^4 - 2b^2x^2 + 2\right)\text{Erfi}(bx) + 4bxe^{2b^2x^2}\left(4b^2x^2 - 11\right) + 43\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}bx\right)}{80\pi b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Erfi[b*x]^2,x]
```

```
[Out] (4*b*E^(2*b^2*x^2)*x*(-11 + 4*b^2*x^2) - 32*E^(b^2*x^2)*Sqrt[Pi]*(2 - 2*b^2
*x^2 + b^4*x^4)*Erfi[b*x] + 16*b^5*Pi*x^5*Erfi[b*x]^2 + 43*Sqrt[2*Pi]*Erfi[
Sqrt[2]*b*x])/(80*b^5*Pi)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x^4 (\operatorname{erfi}(bx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erfi(b*x)^2,x)
```

```
[Out] int(x^4*erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfi(b*x)^2, x)
```

Fricas [A] time = 2.5078, size = 278, normalized size = 1.72

$$\frac{16\,\pi b^6 x^5 \operatorname{erfi}(bx)^2 - 32\,\sqrt{\pi}\left(b^5 x^4 - 2\,b^3 x^2 + 2\,b\right)\operatorname{erfi}(bx)\,e^{\left(b^2 x^2\right)} + 43\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{b^2}x\right) + 4\left(4\,b^4 x^3 - 11\,b^2 x\right)e^{\left(2\,b^2}{80\,\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*pi*b^6*x^5*erfi(b*x)^2 - 32*sqrt(pi)*(b^5*x^4 - 2*b^3*x^2 + 2*b)*e
rfi(b*x)*e^(b^2*x^2) + 43*sqrt(2)*sqrt(pi)*sqrt(b^2)*erfi(sqrt(2)*sqrt(b^2)
*x) + 4*(4*b^4*x^3 - 11*b^2*x)*e^(2*b^2*x^2))/(pi*b^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erfi(b*x)**2,x)
```

```
[Out] Integral(x**4*erfi(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*erfi(b*x)^2, x)
```

3.236 $\int x^2 \mathbf{Erfi}(bx)^2\, dx$

Optimal. Leaf size=111

$$-\frac{2x^2e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}b}+\frac{2e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}b^3}-\frac{5\text{Erfi}\left(\sqrt{2}bx\right)}{6\sqrt{2\pi}b^3}+\frac{xe^{2b^2x^2}}{3\pi b^2}+\frac{1}{3}x^3\text{Erfi}(bx)^2$$

```
[Out] (E^(2*b^2*x^2)*x)/(3*b^2*Pi) + (2*E^(b^2*x^2)*Erfi[b*x])/(3*b^3*Sqrt[Pi]) -
 (2*E^(b^2*x^2)*x^2*Erfi[b*x])/(3*b*Sqrt[Pi]) + (x^3*Erfi[b*x]^2)/3 - (5*Er
fi[Sqrt[2]*b*x])/(6*b^3*Sqrt[2*Pi])
```

Rubi [A] time = 0.119623, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6366, 6387, 6384, 2204, 2212}

$$-\frac{2x^2e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}b}+\frac{2e^{b^2x^2}\text{Erfi}(bx)}{3\sqrt{\pi}b^3}-\frac{5\text{Erfi}\left(\sqrt{2}bx\right)}{6\sqrt{2\pi}b^3}+\frac{xe^{2b^2x^2}}{3\pi b^2}+\frac{1}{3}x^3\text{Erfi}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erfi[b*x]^2,x]
```

```
[Out] (E^(2*b^2*x^2)*x)/(3*b^2*Pi) + (2*E^(b^2*x^2)*Erfi[b*x])/(3*b^3*Sqrt[Pi]) -
 (2*E^(b^2*x^2)*x^2*Erfi[b*x])/(3*b*Sqrt[Pi]) + (x^3*Erfi[b*x]^2)/3 - (5*Er
fi[Sqrt[2]*b*x])/(6*b^3*Sqrt[2*Pi])
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\operatorname{erfi}(bx)^2\,dx &= \frac{1}{3}x^3\operatorname{erfi}(bx)^2-\frac{(4b)\int e^{b^2x^2}x^3\operatorname{erfi}(bx)\,dx}{3\sqrt{\pi}}\\
&= -\frac{2e^{b^2x^2}x^2\operatorname{erfi}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\operatorname{erfi}(bx)^2+\frac{4\int e^{2b^2x^2}x^2\,dx}{3\pi}+\frac{4\int e^{b^2x^2}x\operatorname{erfi}(bx)\,dx}{3b\sqrt{\pi}}\\
&= \frac{e^{2b^2x^2}x}{3b^2\pi}+\frac{2e^{b^2x^2}\operatorname{erfi}(bx)}{3b^3\sqrt{\pi}}-\frac{2e^{b^2x^2}x^2\operatorname{erfi}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\operatorname{erfi}(bx)^2-\frac{\int e^{2b^2x^2}\,dx}{3b^2\pi}-\frac{4\int e^{2b^2x^2}\,dx}{3b^2\pi}\\
&= \frac{e^{2b^2x^2}x}{3b^2\pi}+\frac{2e^{b^2x^2}\operatorname{erfi}(bx)}{3b^3\sqrt{\pi}}-\frac{2e^{b^2x^2}x^2\operatorname{erfi}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\operatorname{erfi}(bx)^2-\frac{\sqrt{\frac{2}{\pi}}\operatorname{erfi}\left(\sqrt{2}bx\right)}{3b^3}-\frac{\operatorname{erfi}\left(\sqrt{2}bx\right)}{6b^3\sqrt{2\pi}}
\end{aligned}
$$

Mathematica [A] time = 0.0301299, size = 87, normalized size = 0.78

$$
\frac{4\pi b^3x^3\operatorname{Erfi}(bx)^2-8\sqrt{\pi}e^{b^2x^2}\left(b^2x^2-1\right)\operatorname{Erfi}(bx)+4bxe^{2b^2x^2}-5\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}bx\right)}{12\pi b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erfi[b*x]^2,x]
```

```
[Out] (4*b*E^(2*b^2*x^2)*x - 8*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erfi[b*x] + 4*
b^3*Pi*x^3*Erfi[b*x]^2 - 5*Sqrt[2*Pi]*Erfi[Sqrt[2]*b*x])/(12*b^3*Pi)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{erfi}(bx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfi(b*x)^2,x)
```

```
[Out] int(x^2*erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfi(b*x)^2, x)
```

Fricas [A] time = 2.47672, size = 232, normalized size = 2.09

$$\frac{4\,\pi b^4 x^3 \operatorname{erfi}(bx)^2 + 4\,b^2 x e^{\left(2\,b^2 x^2\right)} - 8\,\sqrt{\pi}\left(b^3 x^2 - b\right) \operatorname{erfi}(bx)\, e^{\left(b^2 x^2\right)} - 5\,\sqrt{2}\sqrt{\pi}\sqrt{b^2} \operatorname{erfi}\left(\sqrt{2}\sqrt{b^2} x\right)}{12\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi*b^4*x^3*erfi(b*x)^2 + 4*b^2*x*e^(2*b^2*x^2) - 8*sqrt(pi)*(b^3*x^
2 - b)*erfi(b*x)*e^(b^2*x^2) - 5*sqrt(2)*sqrt(pi)*sqrt(b^2)*erfi(sqrt(2)*sq
rt(b^2)*x))/(pi*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfi(b*x)**2,x)
```

```
[Out] Integral(x**2*erfi(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi(b*x)^2, x)
```

3.237 $\int \mathbf{Erfi}(bx)^2\,dx$

Optimal. Leaf size=54

$$-\frac{2e^{b^2x^2}\mathrm{Erfi}(bx)}{\sqrt{\pi}b}+x\mathrm{Erfi}(bx)^2+\frac{\sqrt{\frac{2}{\pi}}\mathrm{Erfi}\left(\sqrt{2}bx\right)}{b}$$

```
[Out] (-2*E^(b^2*x^2)*Erfi[b*x])/(b*Sqrt[Pi]) + x*Erfi[b*x]^2 + (Sqrt[2/Pi]*Erfi[
Sqrt[2]*b*x])/b
```

Rubi [A] time = 0.0445048, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6354, 12, 6384, 2204}

$$-\frac{2e^{b^2x^2}\mathrm{Erfi}(bx)}{\sqrt{\pi}b}+x\mathrm{Erfi}(bx)^2+\frac{\sqrt{\frac{2}{\pi}}\mathrm{Erfi}\left(\sqrt{2}bx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]^2,x]
```

```
[Out] (-2*E^(b^2*x^2)*Erfi[b*x])/(b*Sqrt[Pi]) + x*Erfi[b*x]^2 + (Sqrt[2/Pi]*Erfi[
Sqrt[2]*b*x])/b
```

Rule 6354

```
Int[Erfi[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x]^
2)/b, x] - Dist[4/Sqrt[Pi], Int[(a + b*x)*E^(a + b*x)^2*Erfi[a + b*x], x],
x] /; FreeQ[{a, b}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfi}(bx)^2\,dx &= x\operatorname{erfi}(bx)^2-\frac{4\int be^{b^2x^2}x\operatorname{erfi}(bx)\,dx}{\sqrt{\pi}}\\ &= x\operatorname{erfi}(bx)^2-\frac{(4b)\int e^{b^2x^2}x\operatorname{erfi}(bx)\,dx}{\sqrt{\pi}}\\ &= -\frac{2e^{b^2x^2}\operatorname{erfi}(bx)}{b\sqrt{\pi}}+x\operatorname{erfi}(bx)^2+\frac{4\int e^{2b^2x^2}\,dx}{\pi}\\ &= -\frac{2e^{b^2x^2}\operatorname{erfi}(bx)}{b\sqrt{\pi}}+x\operatorname{erfi}(bx)^2+\frac{\sqrt{\frac{2}{\pi}}\operatorname{erfi}\left(\sqrt{2}bx\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.011072, size = 54, normalized size = 1.

$$-\frac{2e^{b^2x^2}\operatorname{Erfi}(bx)}{\sqrt{\pi}b}+x\operatorname{Erfi}(bx)^2+\frac{\sqrt{\frac{2}{\pi}}\operatorname{Erfi}\left(\sqrt{2}bx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]^2,x]
```

```
[Out] (-2*E^(b^2*x^2)*Erfi[b*x])/(b*Sqrt[Pi]) + x*Erfi[b*x]^2 + (Sqrt[2/Pi]*Erfi[
Sqrt[2]*b*x])/b
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (\operatorname{erfi}(bx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2,x)
```

```
[Out] int(erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2, x)
```

Fricas [A] time = 2.43849, size = 169, normalized size = 3.13

$$\frac{\pi b^2 x \operatorname{erfi}(bx)^2 - 2\sqrt{\pi} b \operatorname{erfi}(bx) e^{\left(b^2 x^2\right)} + \sqrt{2}\sqrt{\pi}\sqrt{b^2} \operatorname{erfi}\left(\sqrt{2}\sqrt{b^2} x\right)}{\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] (pi*b^2*x*erfi(b*x)^2 - 2*sqrt(pi)*b*erfi(b*x)*e^(b^2*x^2) + sqrt(2)*sqrt(p
i)*sqrt(b^2)*erfi(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2,x)
```

```
[Out] Integral(erfi(b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2, x)
```

3.238 $\int \frac{\mathbf{Erfi}(bx)^2}{x^2}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfi}(bx)^2}{x^2},x\right)$$

[Out] Unintegrable[Erfi[b*x]^2/x^2, x]

Rubi [A] time = 0.0177361, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Erfi[b*x]^2/x^2,x]

[Out] Defer[Int][Erfi[b*x]^2/x^2, x]

Rubi steps

$$\int \frac{\text{erfi}(bx)^2}{x^2}\,dx = \int \frac{\text{erfi}(bx)^2}{x^2}\,dx$$

Mathematica [A] time = 0.0350481, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Erfi[b*x]^2/x^2,x]

[Out] Integrate[Erfi[b*x]^2/x^2, x]

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfi}(bx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x^2,x)
```

```
[Out] int(erfi(b*x)^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)^2/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x**2,x)
```

```
[Out] Integral(erfi(b*x)**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x^2, x)
```

3.239 $\int \frac{\mathbf{Erfi}(bx)^2}{x^4} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfi}(bx)^2}{x^4}, x\right)$$

[Out] Unintegrable[Erfi[b*x]^2/x^4, x]

Rubi [A] time = 0.0171603, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(bx)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[Erfi[b*x]^2/x^4,x]

[Out] Defer[Int][Erfi[b*x]^2/x^4, x]

Rubi steps

$$\int \frac{\text{erfi}(bx)^2}{x^4} dx = \int \frac{\text{erfi}(bx)^2}{x^4} dx$$

Mathematica [A] time = 0.0341987, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(bx)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[Erfi[b*x]^2/x^4,x]

[Out] Integrate[Erfi[b*x]^2/x^4, x]

Maple [A] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfi}(bx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x^4,x)
```

```
[Out] int(erfi(b*x)^2/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)^2/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x**4,x)
```

```
[Out] Integral(erfi(b*x)**2/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x^4, x)
```

3.240 $\int \frac{\mathbf{Erfi}(bx)^2}{x^6}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\text{Erfi}(bx)^2}{x^6},x\right)$$

```
[Out] Unintegrable[Erfi[b*x]^2/x^6, x]
```

Rubi [A] time = 0.016658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[b*x]^2/x^6,x]
```

```
[Out] Defer[Int][Erfi[b*x]^2/x^6, x]
```

Rubi steps

$$\int \frac{\text{erfi}(bx)^2}{x^6}\,dx = \int \frac{\text{erfi}(bx)^2}{x^6}\,dx$$

Mathematica [A] time = 0.034095, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[b*x]^2/x^6,x]
```

```
[Out] Integrate[Erfi[b*x]^2/x^6, x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfi}(bx))^2}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)^2/x^6,x)
```

```
[Out] int(erfi(b*x)^2/x^6,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2/x^6, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)^2/x^6, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(bx)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)**2/x**6,x)
```

```
[Out] Integral(erfi(b*x)**2/x**6, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2/x^6, x)
```

3.241 $\int (c+dx)^2 \mathbf{Erfi}(a+bx)^2 \, dx$

Optimal. Leaf size=366

$$\frac{d(a+bx)^2(bc-ad)\text{Erfi}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{Erfi}(a+bx)^2}{b^3}-\frac{2de^{(a+bx)^2}(a+bx)(bc-ad)\text{Erfi}(a+bx)}{\sqrt{\pi}b^3}+\frac{d(bc-a}{}$$

```
[Out] (d*(b*c - a*d)*E^(2*(a + b*x)^2))/(b^3*Pi) + (d^2*E^(2*(a + b*x)^2)*(a + b*
x))/(3*b^3*Pi) + (2*d^2*E^(a + b*x)^2*Erfi[a + b*x])/(3*b^3*Sqrt[Pi]) - (2*
(b*c - a*d)^2*E^(a + b*x)^2*Erfi[a + b*x])/(b^3*Sqrt[Pi]) - (2*d*(b*c - a*d
)*E^(a + b*x)^2*(a + b*x)*Erfi[a + b*x])/(b^3*Sqrt[Pi]) - (2*d^2*E^(a + b*x
)^2*(a + b*x)^2*Erfi[a + b*x])/(3*b^3*Sqrt[Pi]) + (d*(b*c - a*d)*Erfi[a + b
*x]^2)/(2*b^3) + ((b*c - a*d)^2*(a + b*x)*Erfi[a + b*x]^2)/b^3 + (d*(b*c -
a*d)*(a + b*x)^2*Erfi[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*Erfi[a + b*x]^2)/(
3*b^3) + ((b*c - a*d)^2*Sqrt[2/Pi]*Erfi[Sqrt[2]*(a + b*x)])/b^3 - (5*d^2*Er
fi[Sqrt[2]*(a + b*x)])/(6*b^3*Sqrt[2*Pi])
```

Rubi [A] time = 0.361178, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6369, 6354, 6384, 2204, 6366, 6387, 6375, 30, 2209, 2212}

$$\frac{d(a+bx)^2(bc-ad)\text{Erfi}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{Erfi}(a+bx)^2}{b^3}-\frac{2de^{(a+bx)^2}(a+bx)(bc-ad)\text{Erfi}(a+bx)}{\sqrt{\pi}b^3}+\frac{d(bc-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Erfi[a + b*x]^2,x]
```


```
[Out] (d*(b*c - a*d)*E^(2*(a + b*x)^2))/(b^3*Pi) + (d^2*E^(2*(a + b*x)^2)*(a + b*
x))/(3*b^3*Pi) + (2*d^2*E^(a + b*x)^2*Erfi[a + b*x])/(3*b^3*Sqrt[Pi]) - (2*
(b*c - a*d)^2*E^(a + b*x)^2*Erfi[a + b*x])/(b^3*Sqrt[Pi]) - (2*d*(b*c - a*d
)*E^(a + b*x)^2*(a + b*x)*Erfi[a + b*x])/(b^3*Sqrt[Pi]) - (2*d^2*E^(a + b*x
)^2*(a + b*x)^2*Erfi[a + b*x])/(3*b^3*Sqrt[Pi]) + (d*(b*c - a*d)*Erfi[a + b
*x]^2)/(2*b^3) + ((b*c - a*d)^2*(a + b*x)*Erfi[a + b*x]^2)/b^3 + (d*(b*c -
a*d)*(a + b*x)^2*Erfi[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*Erfi[a + b*x]^2)/(
3*b^3) + ((b*c - a*d)^2*Sqrt[2/Pi]*Erfi[Sqrt[2]*(a + b*x)])/b^3 - (5*d^2*Er
fi[Sqrt[2]*(a + b*x)])/(6*b^3*Sqrt[2*Pi])
```


Rule 6369

```
Int[Erfi[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Dist
[1/b^(m + 1), Subst[Int[ExpandIntegrand[Erfi[x]^2, (b*c - a*d + d*x)^m, x],
 x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6354

```
Int[Erfi[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x]^
2)/b, x] - Dist[4/Sqrt[Pi], Int[(a + b*x)*E^(a + b*x)^2*Erfi[a + b*x], x],
x] /; FreeQ[{a, b}, x]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int\left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\text{erfi}(x)^2+2bcd\left(1-\frac{ad}{bc}\right)x\text{erfi}(x)^2+d^2x^2\text{erfi}(x)^2\right)dx,x,a\right.}{b^3}$$

$$=\frac{d^2\,\text{Subst}\left(\int x^2\text{erfi}(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\,\text{Subst}\left(\int x\text{erfi}(x)^2\,dx,x,a+bx\right)}{b^3}+$$

$$=\frac{(bc-ad)^2(a+bx)\text{erfi}(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2\text{erfi}(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3\text{erfi}(a+}{3b^3}$$

$$=-\frac{2(bc-ad)^2e^{(a+bx)^2}\text{erfi}(a+bx)}{b^3\sqrt{\pi}}-\frac{2d(bc-ad)e^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)}{b^3\sqrt{\pi}}-\frac{2d^2e^{(a+bx)^2}(a}{3}$$

$$=\frac{d(bc-ad)e^{2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{2(a+bx)^2}(a+bx)}{3b^3\pi}+\frac{2d^2e^{(a+bx)^2}\text{erfi}(a+bx)}{3b^3\sqrt{\pi}}-\frac{2(bc-ad)^2e^{(a+bx)^2}\text{erf}}{b^3\sqrt{\pi}}$$

$$=\frac{d(bc-ad)e^{2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{2(a+bx)^2}(a+bx)}{3b^3\pi}+\frac{2d^2e^{(a+bx)^2}\text{erfi}(a+bx)}{3b^3\sqrt{\pi}}-\frac{2(bc-ad)^2e^{(a+bx)^2}\text{erf}}{b^3\sqrt{\pi}}$$

Mathematica [F] time = 0.736608, size = 0, normalized size = 0.

$$\int (c+dx)^2\text{Erfi}(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*Erfi[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^2*Erfi[a + b*x]^2, x]
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int (dx+c)^2 (\text{erfi}(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*erfi(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*erfi(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \,\text{erfi}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfi(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*erfi(b*x + a)^2, x)
```

Fricas [A] time = 2.55522, size = 645, normalized size = 1.76

$$\frac{\sqrt{2}\sqrt{\pi}\left(12\,b^2c^2-24\,abcd+\left(12\,a^2-5\right)d^2\right)\sqrt{b^2}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)-8\,\sqrt{\pi}\left(b^3d^2x^2+3\,b^3c^2-3\,ab^2cd+\left(a^2-1\right)bd^2+\left(3\,b^3c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2)*sqrt(pi)*(12*b^2*c^2 - 24*a*b*c*d + (12*a^2 - 5)*d^2)*sqrt(b^
2)*erfi(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 8*sqrt(pi)*(b^3*d^2*x^2 + 3*b^3*c^
2 - 3*a*b^2*c*d + (a^2 - 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x)*erfi(b*x + a
)*e^(b^2*x^2 + 2*a*b*x + a^2) + 2*(2*pi*b^4*d^2*x^3 + 6*pi*b^4*c*d*x^2 + 6*
pi*b^4*c^2*x + pi*(6*a*b^3*c^2 - 3*(2*a^2 - 1)*b^2*c*d + (2*a^3 - 3*a)*b*d^
2))*erfi(b*x + a)^2 + 4*(b^2*d^2*x + 3*b^2*c*d - 2*a*b*d^2)*e^(2*b^2*x^2 +
4*a*b*x + 2*a^2))/(pi*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{erfi}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*erfi(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*erfi(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{erfi}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*erfi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*erfi(b*x + a)^2, x)
```

3.242 $\int (c+dx)\,\mathbf{Erfi}(a+bx)^2\,dx$

Optimal. Leaf size=184

$$\frac{(a+bx)(bc-ad)\text{Erfi}(a+bx)^2}{b^2}-\frac{2e^{(a+bx)^2}(bc-ad)\text{Erfi}(a+bx)}{\sqrt{\pi}b^2}+\frac{\sqrt{\frac{2}{\pi}}(bc-ad)\text{Erfi}\left(\sqrt{2}(a+bx)\right)}{b^2}+\frac{d(a+bx)^2\text{Erfi}(a+bx}{2b^2}$$

```
[Out] (d*E^(2*(a + b*x)^2))/(2*b^2*Pi) - (2*(b*c - a*d)*E^(a + b*x)^2*Erfi[a + b*
x])/(b^2*Sqrt[Pi]) - (d*E^(a + b*x)^2*(a + b*x)*Erfi[a + b*x])/(b^2*Sqrt[Pi
]) + (d*Erfi[a + b*x]^2)/(4*b^2) + ((b*c - a*d)*(a + b*x)*Erfi[a + b*x]^2)/
b^2 + (d*(a + b*x)^2*Erfi[a + b*x]^2)/(2*b^2) + ((b*c - a*d)*Sqrt[2/Pi]*Erf
i[Sqrt[2]*(a + b*x)])/b^2
```

Rubi [A] time = 0.181994, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6369, 6354, 6384, 2204, 6366, 6387, 6375, 30, 2209}

$$\frac{(a+bx)(bc-ad)\text{Erfi}(a+bx)^2}{b^2}-\frac{2e^{(a+bx)^2}(bc-ad)\text{Erfi}(a+bx)}{\sqrt{\pi}b^2}+\frac{\sqrt{\frac{2}{\pi}}(bc-ad)\text{Erfi}\left(\sqrt{2}(a+bx)\right)}{b^2}+\frac{d(a+bx)^2\text{Erfi}(a+bx}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Erfi[a + b*x]^2,x]
```


```
[Out] (d*E^(2*(a + b*x)^2))/(2*b^2*Pi) - (2*(b*c - a*d)*E^(a + b*x)^2*Erfi[a + b*
x])/(b^2*Sqrt[Pi]) - (d*E^(a + b*x)^2*(a + b*x)*Erfi[a + b*x])/(b^2*Sqrt[Pi
]) + (d*Erfi[a + b*x]^2)/(4*b^2) + ((b*c - a*d)*(a + b*x)*Erfi[a + b*x]^2)/
b^2 + (d*(a + b*x)^2*Erfi[a + b*x]^2)/(2*b^2) + ((b*c - a*d)*Sqrt[2/Pi]*Erf
i[Sqrt[2]*(a + b*x)])/b^2
```


Rule 6369

```
Int[Erfi[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Dist
[1/b^(m + 1), Subst[Int[ExpandIntegrand[Erfi[x]^2, (b*c - a*d + d*x)^m, x],
 x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6354

```
Int[Erfi[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x]^
2)/b, x] - Dist[4/Sqrt[Pi], Int[(a + b*x)*E^(a + b*x)^2*Erfi[a + b*x], x],
```

```
x] /; FreeQ[{a, b}, x]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 6366

```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Erfi[b*x]^2
)/(m + 1), x] - Dist[(4*b)/(Sqrt[Pi]*(m + 1)), Int[x^(m + 1)*E^(b^2*x^2)*Er
fi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
```

```
[d*e - c*f, 0]
```

Rubi steps

$$\int (c+dx)\text{erfi}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)\text{erfi}(x)^2+dx\text{erfi}(x)^2\right)dx,x,a+bx\right)}{b^2}$$
$$= \frac{d\,\text{Subst}\left(\int x\text{erfi}(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int \text{erfi}(x)^2\,dx,x,a+bx\right)}{b^2}$$
$$= \frac{(bc-ad)(a+bx)\text{erfi}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{erfi}(a+bx)^2}{2b^2}-\frac{(2d)\,\text{Subst}\left(\int e^{x^2}x^2\text{erfi}(x)\,dx,x,\right.}{b^2\sqrt{\pi}}$$
$$= -\frac{2(bc-ad)e^{(a+bx)^2}\text{erfi}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)}{b^2\sqrt{\pi}}+\frac{(bc-ad)(a+bx)\text{erfi}(a+b}{b^2}$$
$$= \frac{de^{2(a+bx)^2}}{2b^2\pi}-\frac{2(bc-ad)e^{(a+bx)^2}\text{erfi}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)}{b^2\sqrt{\pi}}+\frac{(bc-ad)(a+bx}{b^2}$$
$$= \frac{de^{2(a+bx)^2}}{2b^2\pi}-\frac{2(bc-ad)e^{(a+bx)^2}\text{erfi}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)}{b^2\sqrt{\pi}}+\frac{d\text{erfi}(a+bx)^2}{4b^2}+$$

Mathematica [A] time = 0.143423, size = 128, normalized size = 0.7

$$\frac{\pi\text{Erfi}(a+bx)^2\left(-2a^2d+4abc+4b^2cx+2b^2dx^2+d\right)-4\sqrt{\pi}e^{(a+bx)^2}\text{Erfi}(a+bx)(-ad+2bc+bdx)+4\sqrt{2\pi}(bc-ad)\text{Erfi}\left(\right.}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Erfi[a + b*x]^2,x]
```

```
[Out] (2*d*E^(2*(a + b*x)^2) - 4*E^(a + b*x)^2*Sqrt[Pi]*(2*b*c - a*d + b*d*x)*Erf
i[a + b*x] + Pi*(4*a*b*c + d - 2*a^2*d + 4*b^2*c*x + 2*b^2*d*x^2)*Erfi[a +
b*x]^2 + 4*(b*c - a*d)*Sqrt[2*Pi]*Erfi[Sqrt[2]*(a + b*x)])/(4*b^2*Pi)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (dx+c)\left(\text{erfi}\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*erfi(b*x+a)^2,x)
```

```
[Out] int((d*x+c)*erfi(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\,\mathrm{erfi}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*erfi(b*x + a)^2, x)
```

Fricas [A] time = 2.42785, size = 406, normalized size = 2.21

$$\frac{4\sqrt{2}\sqrt{\pi}\sqrt{b^2}(bc-ad)\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)-4\sqrt{\pi}\left(b^2dx+2\,b^2c-abd\right)\mathrm{erfi}\,(bx+a)\,e^{\left(b^2x^2+2\,abx+a^2\right)}+\left(2\,\pi b^3dx^2+4\,\pi b^3cx+\right.}{4\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*sqrt(pi)*sqrt(b^2)*(b*c - a*d)*erfi(sqrt(2)*sqrt(b^2)*(b*x +
 a)/b) - 4*sqrt(pi)*(b^2*d*x + 2*b^2*c - a*b*d)*erfi(b*x + a)*e^(b^2*x^2 +
2*a*b*x + a^2) + (2*pi*b^3*d*x^2 + 4*pi*b^3*c*x + pi*(4*a*b^2*c - (2*a^2 -
1)*b*d))*erfi(b*x + a)^2 + 2*b*d*e^(2*b^2*x^2 + 4*a*b*x + 2*a^2))/(pi*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\,\mathrm{erfi}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*erfi(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\,\mathrm{erfi}\,(bx + a)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*erfi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*erfi(b*x + a)^2, x)
```

3.243 $\int \mathbf{Erfi}(a+bx)^2\,dx$

Optimal. Leaf size=68

$$\frac{(a+bx)\text{Erfi}(a+bx)^2}{b}-\frac{2e^{(a+bx)^2}\text{Erfi}(a+bx)}{\sqrt{\pi}b}+\frac{\sqrt{\frac{2}{\pi}}\text{Erfi}\left(\sqrt{2}(a+bx)\right)}{b}$$

```
[Out] (-2*E^(a + b*x)^2*Erfi[a + b*x])/(b*Sqrt[Pi]) + ((a + b*x)*Erfi[a + b*x]^2)
/b + (Sqrt[2/Pi]*Erfi[Sqrt[2]*(a + b*x)])/b
```

Rubi [A] time = 0.134533, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6354, 6384, 2204}

$$\frac{(a+bx)\text{Erfi}(a+bx)^2}{b}-\frac{2e^{(a+bx)^2}\text{Erfi}(a+bx)}{\sqrt{\pi}b}+\frac{\sqrt{\frac{2}{\pi}}\text{Erfi}\left(\sqrt{2}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x)^2*Erfi[a + b*x])/(b*Sqrt[Pi]) + ((a + b*x)*Erfi[a + b*x]^2)
/b + (Sqrt[2/Pi]*Erfi[Sqrt[2]*(a + b*x)])/b
```

Rule 6354

```
Int[Erfi[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x]^
2)/b, x] - Dist[4/Sqrt[Pi], Int[(a + b*x)*E^(a + b*x)^2*Erfi[a + b*x], x],
x] /; FreeQ[{a, b}, x]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfi}(a+bx)^2\,dx &= \frac{(a+bx)\operatorname{erfi}(a+bx)^2}{b}-\frac{4\int e^{(a+bx)^2}(a+bx)\operatorname{erfi}(a+bx)\,dx}{\sqrt{\pi}}\\ &= \frac{(a+bx)\operatorname{erfi}(a+bx)^2}{b}-\frac{4\operatorname{Subst}\left(\int e^{x^2}x\operatorname{erfi}(x)\,dx,x,a+bx\right)}{b\sqrt{\pi}}\\ &= -\frac{2e^{(a+bx)^2}\operatorname{erfi}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erfi}(a+bx)^2}{b}+\frac{4\operatorname{Subst}\left(\int e^{2x^2}\,dx,x,a+bx\right)}{b\pi}\\ &= -\frac{2e^{(a+bx)^2}\operatorname{erfi}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erfi}(a+bx)^2}{b}+\frac{\sqrt{\frac{2}{\pi}}\operatorname{erfi}\left(\sqrt{2}(a+bx)\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0410586, size = 64, normalized size = 0.94

$$\frac{\sqrt{\pi}(a+bx)\operatorname{Erfi}(a+bx)^2-2e^{(a+bx)^2}\operatorname{Erfi}(a+bx)+\sqrt{2}\operatorname{Erfi}\left(\sqrt{2}(a+bx)\right)}{\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x)^2*Erfi[a + b*x] + Sqrt[Pi]*(a + b*x)*Erfi[a + b*x]^2 + Sqrt
[2]*Erfi[Sqrt[2]*(a + b*x)])/(b*Sqrt[Pi])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (\operatorname{erfi}(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a)^2,x)
```

```
[Out] int(erfi(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)^2, x)
```

Fricas [A] time = 2.47335, size = 231, normalized size = 3.4

$$-\frac{2\sqrt{\pi}b\operatorname{erfi}(bx+a)e^{\left(b^2x^2+2\,abx+a^2\right)}-\left(\pi b^2x+\pi ab\right)\operatorname{erfi}(bx+a)^2-\sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*sqrt(pi)*b*erfi(b*x + a)*e^(b^2*x^2 + 2*a*b*x + a^2) - (pi*b^2*x + pi*a
*b)*erfi(b*x + a)^2 - sqrt(2)*sqrt(pi)*sqrt(b^2)*erfi(sqrt(2)*sqrt(b^2)*(b*
x + a)/b))/(pi*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)**2,x)
```

```
[Out] Integral(erfi(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)^2, x)
```

3.244 $\int \frac{\text{Erfi}(a+bx)^2}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{Erfi}(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable[Erfi[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0226524, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{Erfi}(a+bx)^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Erfi[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{erfi}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{erfi}(a+bx)^2}{c+dx}\,dx$$

Mathematica [A] time = 0.0514338, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(a+bx)^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Erfi[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.33, size = 0, normalized size = 0.

$$\int \frac{\left(\operatorname{erfi}\left(bx+a\right)\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(erfi(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left(bx+a\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)^2/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}\left(bx+a\right)^{2}}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(erfi(a + b*x)**2/(c + d*x), x)
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)^2/(d*x + c), x)
```

3.245 $\int \frac{\text{Erfi}(a+bx)^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{Erfi}(a+bx)^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Erfi[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.021419, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{Erfi}(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Erfi[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Erfi[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx$$

Mathematica [A] time = 0.0985773, size = 0, normalized size = 0.

$$\int \frac{\text{Erfi}(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Erfi[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.323, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{erfi}(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(erfi(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)^2/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erfi}(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(erfi(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)^2/(d*x + c)^2, x)
```

3.246 $\int x^2 \mathbf{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=102

$$\frac{1}{3}x^3\text{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}x^3\left(cx^n\right)^{-3/n}e^{-\frac{3\left(4abd^2n+3\right)}{4b^2d^2n^2}}\text{Erfi}\left(\frac{2abd^2+2b^2d^2\log\left(cx^n\right)+\frac{3}{n}}{2bd}\right)$$

```
[Out] (x^3*Erfi[d*(a + b*Log[c*x^n])])/3 - (x^3*Erfi[(2*a*b*d^2 + 3/n + 2*b^2*d^2
*Log[c*x^n])/(2*b*d)])/(3*E^((3*(3 + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(c*x^n)
^(3/n))
```

Rubi [A] time = 0.219985, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6403, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{1}{3}x^3\text{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}x^3\left(cx^n\right)^{-3/n}e^{-\frac{3\left(4abd^2n+3\right)}{4b^2d^2n^2}}\text{Erfi}\left(\frac{2abd^2+2b^2d^2\log\left(cx^n\right)+\frac{3}{n}}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*Erfi[d*(a + b*Log[c*x^n])])/3 - (x^3*Erfi[(2*a*b*d^2 + 3/n + 2*b^2*d^2
*Log[c*x^n])/(2*b*d)])/(3*E^((3*(3 + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(c*x^n)
^(3/n))
```

Rule 6403

```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfi[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 - Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int x^2\text{erfi}\,(d\,(a+b\log{(cx^n)}))\;dx = \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{(2bdn)\int e^{d^2(a+b\log(cx^n))^2}x^2\,dx}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{(2bdn)\int \exp\left(a^2d^2+2abd^2\log{(cx^n)}+b^2d^2\log^2{(cx^n)}\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{(2bdn)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^2\,(cx^n)^{2abd^2}\;dx}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{\left(2bdnx^{-2abd^2n}\,(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^{2+2abd^2n}\ldots}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{\left(2bdx^3\,(cx^n)^{2abd^2-\frac{3+2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(a^2d^2+\frac{(3+2a\ldots}{\ldots}\right.\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{\left(2bde^{-\frac{3(3+4abd^2n)}{4b^2d^2n^2}}x^3\,(cx^n)^{2abd^2-\frac{3+2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(\left(\ldots\right.\right.\right.}{3\sqrt{\pi}}$$

$$= \frac{1}{3}x^3\text{erfi}\,(d\,(a+b\log{(cx^n)})) - \frac{1}{3}e^{-\frac{3(3+4abd^2n)}{4b^2d^2n^2}}x^3\,(cx^n)^{-3/n}\,\text{erfi}\left(\frac{2abd^2+\frac{3}{n}+2b^2d^2\log{(c\ldots}}{2bd}\right.$$

Mathematica [A] time = 0.307975, size = 90, normalized size = 0.88

$$\frac{1}{3}\left(x^3\text{Erfi}\,(d\,(a+b\log{(cx^n)})) - x^3\,(cx^n)^{-3/n}\,e^{-\frac{3(4abd^2n+3)}{4b^2d^2n^2}}\text{Erfi}\left(ad+bd\log{(cx^n)}+\frac{3}{2bdn}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*Erfi[d*(a + b*Log[c*x^n])] - (x^3*Erfi[a*d + 3/(2*b*d*n) + b*d*Log[c*x
^n]])/(E^((3*(3 + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(c*x^n)^(3/n)))/3
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int x^2\text{erfi}\,(d\,(a+b\ln{(cx^n)}))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*erfi(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfi((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 2.94648, size = 305, normalized size = 2.99

$$\frac{1}{3}x^3\operatorname{erfi}\left(bd\log\left(cx^n\right)+ad\right)-\frac{1}{3}\sqrt{b^2d^2n^2}\operatorname{erfi}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n+3\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{3\left(4\,b^2d^2n\log(c)+\right.}{4\,b^2d^2n}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*erfi(b*d*log(c*x^n) + a*d) - 1/3*sqrt(b^2*d^2*n^2)*erfi(1/2*(2*b^2*
d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n + 3)*sqrt(b^2*d^2*n^2)/(b
^2*d^2*n^2))*e^(-3/4*(4*b^2*d^2*n*log(c) + 4*a*b*d^2*n + 3)/(b^2*d^2*n^2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi((b*log(c*x^n) + a)*d), x)
```

3.247 $\int x\mathbf{Erfi}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}x^2\text{Erfi}(d(a+b\log(cx^n)))-\frac{1}{2}x^2(cx^n)^{-2/n}e^{-\frac{2abd^2n+1}{b^2d^2n^2}}\text{Erfi}\left(\frac{abd^2+b^2d^2\log(cx^n)+\frac{1}{n}}{bd}\right)$$

```
[Out] (x^2*Erfi[d*(a + b*Log[c*x^n])])/2 - (x^2*Erfi[(a*b*d^2 + n^(-1) + b^2*d^2*
Log[c*x^n])/(b*d)])/(2*E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(2/n))
```

Rubi [A] time = 0.16624, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6403, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{1}{2}x^2\text{Erfi}(d(a+b\log(cx^n)))-\frac{1}{2}x^2(cx^n)^{-2/n}e^{-\frac{2abd^2n+1}{b^2d^2n^2}}\text{Erfi}\left(\frac{abd^2+b^2d^2\log(cx^n)+\frac{1}{n}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*Erfi[d*(a + b*Log[c*x^n])])/2 - (x^2*Erfi[(a*b*d^2 + n^(-1) + b^2*d^2*
Log[c*x^n])/(b*d)])/(2*E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(2/n))
```

Rule 6403

```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfi[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 - Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{(bdn)\int e^{d^2(a+b\log(cx^n))^2}x\,dx}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{(bdn)\int \exp\left(a^2d^2+2abd^2\log(cx^n)+b^2d^2\log^2(cx^n)\right)x\,d}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{(bdn)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x(cx^n)^{2abd^2}\,dx}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{\left(bdnx^{-2abd^2n}(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^{1+2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{\left(bdx^2(cx^n)^{2abd^2-\frac{2+2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(a^2d^2+\frac{(2+2abd^2n)}{n}\right.\right.}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{\left(bde^{-\frac{1+2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{2abd^2-\frac{2+2abd^2n}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{\left(\frac{2+2abd^2n}{n}\right.}{4}\right.\right.}{\sqrt{\pi}}$$

$$= \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n))) - \frac{1}{2}e^{-\frac{1+2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{-2/n}\text{erfi}\left(\frac{abd^2+\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right)$$

Mathematica [A] time = 0.275637, size = 81, normalized size = 0.87

$$\frac{1}{2}\left(x^2\text{Erfi}(d(a+b\log(cx^n))) - x^2e^{-\frac{\frac{2abn+\frac{1}{d^2}}{b^2}+2n\log(cx^n)}{n^2}}\text{Erfi}\left(ad+bd\log(cx^n)+\frac{1}{bdn}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*Erfi[d*(a + b*Log[c*x^n])] - (x^2*Erfi[a*d + 1/(b*d*n) + b*d*Log[c*x^n
]])/E^(((d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n])/n^2))/2
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int x\text{erfi}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*erfi(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{erfi}\left((b\log\left(cx^{n}\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*erfi((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 3.15298, size = 286, normalized size = 3.08

$$\frac{1}{2}\,x^{2}\,\mathrm{erfi}\left(bd\log\left(cx^{n}\right)+ad\right)-\frac{1}{2}\,\sqrt{b^{2}d^{2}n^{2}}\,\mathrm{erfi}\left(\frac{\left(b^{2}d^{2}n^{2}\log\left(x\right)+b^{2}d^{2}n\log\left(c\right)+abd^{2}n+1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n}{b^{2}d^{2}n^{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*erfi(b*d*log(c*x^n) + a*d) - 1/2*sqrt(b^2*d^2*n^2)*erfi((b^2*d^2*n^
2*log(x) + b^2*d^2*n*log(c) + a*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2
))*e^(-(2*b^2*d^2*n*log(c) + 2*a*b*d^2*n + 1)/(b^2*d^2*n^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{erfi}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*erfi(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfi}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*erfi((b*log(c*x^n) + a)*d), x)
```

3.248 $\int \mathbf{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=91

$$x\mathrm{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-x\left(cx^n\right)^{-1/n}e^{-\frac{4abd^2n+1}{4b^2d^2n^2}}\mathrm{Erfi}\left(\frac{2abd^2+2b^2d^2\log\left(cx^n\right)+\frac{1}{n}}{2bd}\right)$$

```
[Out] x*Erfi[d*(a + b*Log[c*x^n])] - (x*Erfi[(2*a*b*d^2 + n^(-1) + 2*b^2*d^2*Log[
c*x^n])/(2*b*d)])/(E^((1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x^n)^n^(-1))
```

Rubi [A] time = 0.132879, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6399, 2277, 2274, 15, 2276, 2234, 2204}

$$x\mathrm{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)-x\left(cx^n\right)^{-1/n}e^{-\frac{4abd^2n+1}{4b^2d^2n^2}}\mathrm{Erfi}\left(\frac{2abd^2+2b^2d^2\log\left(cx^n\right)+\frac{1}{n}}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*Erfi[d*(a + b*Log[c*x^n])] - (x*Erfi[(2*a*b*d^2 + n^(-1) + 2*b^2*d^2*Log[
c*x^n])/(2*b*d)])/(E^((1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x^n)^n^(-1))
```

Rule 6399

```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*
Erfi[d*(a + b*Log[c*x^n])], x] - Dist[(2*b*d*n)/Sqrt[Pi], Int[E^(d*(a + b*L
og[c*x^n]))^2, x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = x\operatorname{erfi}(d(a+b\log(cx^n))) - \frac{(2bdn)\int e^{d^2(a+b\log(cx^n))^2}\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfi}(d(a+b\log(cx^n))) - \frac{(2bdn)\int \exp\left(a^2d^2+2abd^2\log(cx^n)+b^2d^2\log^2(cx^n)\right)\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfi}(d(a+b\log(cx^n))) - \frac{(2bdn)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}(cx^n)^{2abd^2}\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfi}(d(a+b\log(cx^n))) - \frac{\left(2bdnx^{-2abd^2n}(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^{2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= x\operatorname{erfi}(d(a+b\log(cx^n))) - \frac{\left(2bdx(cx^n)^{2abd^2-\frac{1+2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(a^2d^2+\frac{(1+2abd^2n)x}{n}\right.\right.}{\sqrt{\pi}}$$

$$= x\operatorname{erfi}(d(a+b\log(cx^n))) - \frac{\left(2bde^{-\frac{1+4abd^2n}{4b^2d^2n^2}}x(cx^n)^{2abd^2-\frac{1+2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{\left(\frac{1+2abd^2n}{n}+\right.}{4b^2d}\right.\right.}{\sqrt{\pi}}$$

$$= x\operatorname{erfi}(d(a+b\log(cx^n))) - e^{-\frac{1+4abd^2n}{4b^2d^2n^2}}x(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{2abd^2+\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)$$

Mathematica [A] time = 0.236883, size = 78, normalized size = 0.86

$$x\operatorname{Erfi}(d(a+b\log(cx^n))) - x\operatorname{Erfi}\left(ad+bd\log(cx^n)+\frac{1}{2bdn}\right)\exp\left(-\frac{\frac{4abn+\frac{1}{d^2}}{b^2}+4n\log(cx^n)}{4n^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*Erfi[d*(a + b*Log[c*x^n])] - (x*Erfi[a*d + 1/(2*b*d*n) + b*d*Log[c*x^n]])
/E^(((d^(-2) + 4*a*b*n)/b^2 + 4*n*Log[c*x^n])/(4*n^2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \operatorname{erfi}(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(erfi(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}\left((b\log\left(cx^{n}\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 3.38574, size = 293, normalized size = 3.22

$$-\sqrt{b^{2}d^{2}n^{2}}\operatorname{erfi}\left(\frac{\left(2\,b^{2}d^{2}n^{2}\log\left(x\right)+2\,b^{2}d^{2}n\log\left(c\right)+2\,abd^{2}n+1\right)\sqrt{b^{2}d^{2}n^{2}}}{2\,b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{4\,b^{2}d^{2}n\log(c)+4\,abd^{2}n+1}{4\,b^{2}d^{2}n^{2}}\right)}+x\operatorname{erfi}\left(bd\log\left(cx^{n}\right)+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -sqrt(b^2*d^2*n^2)*erfi(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*
a*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(-1/4*(4*b^2*d^2*n*log(c)
 + 4*a*b*d^2*n + 1)/(b^2*d^2*n^2)) + x*erfi(b*d*log(c*x^n) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(erfi(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}\left((b\log\left(cx^{n}\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d), x)
```

3.249 $\int \frac{\text{Erfi}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=64

$$\frac{(a+b\log(cx^n))\,\text{Erfi}(d(a+b\log(cx^n)))}{bn}-\frac{e^{(ad+bd\log(cx^n))^2}}{\sqrt{\pi}bdn}$$

```
[Out] -(E^(a*d + b*d*Log[c*x^n])^2/(b*d*n*Sqrt[Pi])) + (Erfi[d*(a + b*Log[c*x^n])
]*(a + b*Log[c*x^n]))/(b*n)
```

Rubi [A] time = 0.0379644, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6351}

$$\frac{(a+b\log(cx^n))\,\text{Erfi}(d(a+b\log(cx^n)))}{bn}-\frac{e^{(ad+bd\log(cx^n))^2}}{\sqrt{\pi}bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] -(E^(a*d + b*d*Log[c*x^n])^2/(b*d*n*Sqrt[Pi])) + (Erfi[d*(a + b*Log[c*x^n])
]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6351

```
Int[Erfi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erfi[a + b*x])/b
, x] - Simp[E^(a + b*x)^2/(b*Sqrt[Pi]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \text{erfi}(d(a+bx))\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int \text{erfi}(x)\,dx, x, ad+bd\log(cx^n)\right)}{bdn}\\ &= -\frac{e^{(ad+bd\log(cx^n))^2}}{bdn\sqrt{\pi}}+\frac{\text{erfi}(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [A] time = 0.0800035, size = 83, normalized size = 1.3

$$\frac{\sqrt{\pi}d\left(a+b\log\left(cx^{n}\right)\right)\operatorname{Erfi}\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)-\left(cx^{n}\right)^{2abd^{2}}e^{d^{2}\left(a^{2}+b^{2}\log^{2}\left(cx^{n}\right)\right)}}{\sqrt{\pi}bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (-(E^(d^2*(a^2 + b^2*Log[c*x^n]^2))*(c*x^n)^(2*a*b*d^2)) + d*Sqrt[Pi]*Erfi[
d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*d*n*Sqrt[Pi])
```

Maple [A] time = 0.106, size = 78, normalized size = 1.2

$$\frac{\ln\left(cx^{n}\right)\operatorname{erfi}\left(ad+bd\ln\left(cx^{n}\right)\right)}{n}+\frac{\operatorname{erfi}\left(ad+bd\ln\left(cx^{n}\right)\right)a}{bn}-\frac{e^{\left(ad+bd\ln\left(cx^{n}\right)\right)^{2}}}{bdn\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*erfi(a*d+b*d*ln(c*x^n))+1/n/b*erfi(a*d+b*d*ln(c*x^n))*a-exp((
a*d+b*d*ln(c*x^n))^2)/b/d/n/Pi^(1/2)
```

Maxima [A] time = 1.09948, size = 78, normalized size = 1.22

$$\frac{\left(b\log\left(cx^{n}\right)+a\right)d\operatorname{erfi}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)-\frac{e^{\left(\left(b\log\left(cx^{n}\right)+a\right)^{2}d^{2}\right)}}{\sqrt{\pi}}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*erfi((b*log(c*x^n) + a)*d) - e^((b*log(c*x^n) + a)^2*
d^2)/sqrt(pi))/(b*d*n)
```

Fricas [A] time = 3.32816, size = 284, normalized size = 4.44

$$\frac{(\pi bdn\log(x)+\pi bd\log(c)+\pi ad)\,\mathrm{erfi}\,(bd\log(cx^n)+ad)-\sqrt{\pi}e^{\left(b^2d^2n^2\log(x)^2+b^2d^2\log(c)^2+2\,abd^2\log(c)+a^2d^2+2\,\left(b^2d^2n\log(c)+abd^2n\right)\right.}}{\pi bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] ((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*erfi(b*d*log(c*x^n) + a*d) - sq
rt(pi)*e^(b^2*d^2*n^2*log(x)^2 + b^2*d^2*log(c)^2 + 2*a*b*d^2*log(c) + a^2*
d^2 + 2*(b^2*d^2*n*log(c) + a*b*d^2*n)*log(x)))/(pi*b*d*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(ad+bd\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(erfi(a*d + b*d*log(c*x**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,((b\log(cx^n)+a)d)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d)/x, x)
```

3.250 $\int \frac{\text{Erfi}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=94

$$\frac{(cx^n)^{\frac{1}{n}}\,e^{\frac{a}{bn}-\frac{1}{4b^2d^2n^2}}\,\text{Erfi}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)}{x}-\frac{\text{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

```
[Out] -(Erfi[d*(a + b*Log[c*x^n])]/x) + (E^(-1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)
^n^(-1)*Erfi[(2*a*b*d^2 - n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x
```

Rubi [A] time = 0.217014, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6403, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{(cx^n)^{\frac{1}{n}}\,e^{\frac{a}{bn}-\frac{1}{4b^2d^2n^2}}\,\text{Erfi}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)}{x}-\frac{\text{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -(Erfi[d*(a + b*Log[c*x^n])]/x) + (E^(-1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)
^n^(-1)*Erfi[(2*a*b*d^2 - n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x
```

Rule 6403

```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfi[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 - Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x^2}\,dx = -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{(2bdn)\int \frac{e^{d^2\left(a+b\log\left(cx^n\right)\right)^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{(2bdn)\int \frac{\exp\left(a^2d^2+2abd^2\log(cx^n)+b^2d^2\log^2(cx^n)\right)}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{(2bdn)\int \frac{e^{a^2d^2+b^2d^2\log^2(cx^n)}(cx^n)^{2abd^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{\left(2bdnx^{-2abd^2n}\,(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^{-2+2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{\left(2bd\,(cx^n)^{2abd^2-\frac{-1+2abd^2n}{n}}\right)\mathrm{Subst}\left(\int \exp\left(a^2d^2+\frac{(-1+2abd^2n)}{n}\right.\right.}{\sqrt{\pi}x}$$

$$= -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{\left(2bde^{-\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}\,(cx^n)^{2abd^2-\frac{-1+2abd^2n}{n}}\right)\mathrm{Subst}\left(\int \exp\left(\frac{\left(\frac{-1+2ab}{n}\right.}{}\right.\right.}{\sqrt{\pi}x}$$

$$= -\frac{\mathrm{erfi}\,(d\,(a+b\log{(cx^n)}))}{x} + \frac{e^{-\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}\,(cx^n)^{\frac{1}{n}}\,\mathrm{erfi}\left(\frac{2abd^2-\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)}{x}$$

Mathematica [A] time = 0.259407, size = 82, normalized size = 0.87

$$\frac{(cx^n)^{\frac{1}{n}}\,e^{\frac{4abd^2n-1}{4b^2d^2n^2}}\,\mathrm{Erfi}\left(ad+bd\log{(cx^n)}-\frac{1}{2bdn}\right)-\mathrm{Erfi}\,(d\,(a+b\log{(cx^n)}))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (-Erfi[d*(a + b*Log[c*x^n])] + E^((-1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x^
n)^n^(-1)*Erfi[a*d - 1/(2*b*d*n) + b*d*Log[c*x^n]])/x
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(d\,(a+b\ln{(cx^n)}))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(erfi(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [A] time = 3.30408, size = 296, normalized size = 3.15

$$\frac{\sqrt{b^{2}d^{2}n^{2}}x\operatorname{erfi}\left(\frac{\left(2\,b^{2}d^{2}n^{2}\log(x)+2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n-1\right)\sqrt{b^{2}d^{2}n^{2}}}{2\,b^{2}d^{2}n^{2}}\right)e^{\left(\frac{4\,b^{2}d^{2}n\log(c)+4\,abd^{2}n-1}{4\,b^{2}d^{2}n^{2}}\right)}-\operatorname{erfi}\left(bd\log\left(cx^{n}\right)+ad\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(b^2*d^2*n^2)*x*erfi(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) +
2*a*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*n*log(c
) + 4*a*b*d^2*n - 1)/(b^2*d^2*n^2)) - erfi(b*d*log(c*x^n) + a*d))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(erfi(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d)/x^2, x)
```

3.251 $\int \frac{\textbf{Erfi}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=95

$$\frac{(cx^n)^{2/n}e^{-\frac{1-2abd^2n}{b^2d^2n^2}}\operatorname{Erfi}\left(\frac{abd^2+b^2d^2\log(cx^n)-\frac{1}{n}}{bd}\right)}{2x^2}-\frac{\operatorname{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

```
[Out] -Erfi[d*(a + b*Log[c*x^n])]/(2*x^2) + ((c*x^n)^(2/n)*Erfi[(a*b*d^2 - n^(-1)
 + b^2*d^2*Log[c*x^n])/(b*d)])/(2*E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^2)
```

Rubi [A] time = 0.205497, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6403, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{(cx^n)^{2/n}e^{-\frac{1-2abd^2n}{b^2d^2n^2}}\operatorname{Erfi}\left(\frac{abd^2+b^2d^2\log(cx^n)-\frac{1}{n}}{bd}\right)}{2x^2}-\frac{\operatorname{Erfi}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -Erfi[d*(a + b*Log[c*x^n])]/(2*x^2) + ((c*x^n)^(2/n)*Erfi[(a*b*d^2 - n^(-1)
 + b^2*d^2*Log[c*x^n])/(b*d)])/(2*E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^2)
```

Rule 6403

```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfi[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 - Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{(bdn)\int \frac{e^{d^2(a+b\log(cx^n))^2}}{x^3}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{(bdn)\int \frac{\exp\left(a^2d^2+2abd^2\log(cx^n)+b^2d^2\log^2(cx^n)\right)}{x^3}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{(bdn)\int \frac{e^{a^2d^2+b^2d^2\log^2(cx^n)}(cx^n)^{2abd^2}}{x^3}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bdnx^{-2abd^2n}(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^{-3+2abd^2n}\,dx}{\sqrt{\pi}}$$

$$= -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bd(cx^n)^{2abd^2-\frac{-2+2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(a^2d^2+\frac{(-2+2abd^2n)x}{n}+\right.\right.}{\sqrt{\pi}x^2}$$

$$= -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bde^{-\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2abd^2-\frac{-2+2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{\left(\frac{-2+2abd^2n}{n}+2b\right.}{4b^2d^2}\right.\right.}{\sqrt{\pi}x^2}$$

$$= -\frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{e^{-\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\operatorname{erfi}\left(\frac{abd^2-\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right)}{2x^2}$$

Mathematica [A] time = 0.265812, size = 80, normalized size = 0.84

$$\frac{e^{\frac{\frac{2abn-\frac{1}{d^2}}{b^2}+2n\log(cx^n)}{n^2}}\operatorname{Erfi}\left(ad+bd\log(cx^n)-\frac{1}{bdn}\right)-\operatorname{Erfi}(d(a+b\log(cx^n)))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (-Erfi[d*(a + b*Log[c*x^n])] + E^((((-d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n]
)/n^2)*Erfi[a*d - 1/(b*d*n) + b*d*Log[c*x^n]])/(2*x^2)
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(erfi(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [A] time = 3.00626, size = 288, normalized size = 3.03

$$\frac{\sqrt{b^{2}d^{2}n^{2}}x^{2}\operatorname{erfi}\left(\frac{\left(b^{2}d^{2}n^{2}\log(x)+b^{2}d^{2}n\log(c)+abd^{2}n-1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(\frac{2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n-1}{b^{2}d^{2}n^{2}}\right)}-\operatorname{erfi}\left(bd\log\left(cx^{n}\right)+ad\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2*d^2*n^2)*x^2*erfi((b^2*d^2*n^2*log(x) + b^2*d^2*n*log(c) + a*
b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^((2*b^2*d^2*n*log(c) + 2*a*
b*d^2*n - 1)/(b^2*d^2*n^2)) - erfi(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left((b \log\left(cx^{n}\right) + a)d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi((b*log(c*x^n) + a)*d)/x^3, x)
```

3.252 $\int (ex)^m \mathbf{Erfi}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=126

$$\frac{(ex)^{m+1}\text{Erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(-\frac{(m+1)\left(4abd^2n+m+1\right)}{4b^2d^2n^2}\right)\text{Erfi}\left(\frac{2abd^2n+2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}$$

```
[Out] ((e*x)^(1 + m)*Erfi[d*(a + b*Log[c*x^n])])/(e*(1 + m)) - (x*(e*x)^m*Erfi[(1
 + m + 2*a*b*d^2*n + 2*b^2*d^2*n*Log[c*x^n])/(2*b*d*n)])/(E^(((1 + m)*(1 +
m + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(1 + m)*(c*x^n)^((1 + m)/n))
```

Rubi [A] time = 0.322351, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6403, 2278, 2274, 15, 20, 2276, 2234, 2204}

$$\frac{(ex)^{m+1}\text{Erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(-\frac{(m+1)\left(4abd^2n+m+1\right)}{4b^2d^2n^2}\right)\text{Erfi}\left(\frac{2abd^2n+2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^(1 + m)*Erfi[d*(a + b*Log[c*x^n])])/(e*(1 + m)) - (x*(e*x)^m*Erfi[(1
 + m + 2*a*b*d^2*n + 2*b^2*d^2*n*Log[c*x^n])/(2*b*d*n)])/(E^(((1 + m)*(1 +
m + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(1 + m)*(c*x^n)^((1 + m)/n))
```

Rule 6403

```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((e*x)^(m + 1)*Erfi[d*(a + b*Log[c*x^n])])/(e*(m + 1)), x]
 - Dist[(2*b*d*n)/(Sqrt[Pi]*(m + 1)), Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))^
2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int (ex)^m \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(2bdn)\int e^{d^2(a+b\log(cx^n))^2}(ex)^m\,dx}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(2bdn)\int \exp\left(a^2d^2+2abd^2\log(cx^n)+b^2d^2\log\right.}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(2bdn)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}(ex)^m(cx^n)^{2abd^2}\,dx}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bdnx^{-2abd^2n}(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log^2(cx^n)}x^{2a}}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bdnx^{-m-2abd^2n}(ex)^m(cx^n)^{2abd^2}\right)\int e^{a^2d^2+b^2d^2\log}}{(1+m)\sqrt{\pi}}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bdx(ex)^m(cx^n)^{2abd^2-\frac{1+m+2abd^2n}{n}}\right)\operatorname{Subst}\left(\int \exp\right.}{(1+}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(2bd\exp\left(-\frac{(1+m)(1+m+4abd^2n)}{4b^2d^2n^2}\right)x(ex)^m(cx^n)^{2abd^2}\right.}{}$$

$$= \frac{(ex)^{1+m}\operatorname{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\exp\left(-\frac{(1+m)(1+m+4abd^2n)}{4b^2d^2n^2}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\right.}{1+m}$$

Mathematica [A] time = 0.441594, size = 126, normalized size = 1.

$$\frac{(ex)^m\left(x\operatorname{Erfi}(d(a+b\log(cx^n)))-x^{-m}\operatorname{Erfi}\left(\frac{2abd^2n+m+1}{2bdn}+bd\log(cx^n)\right)\exp\left(-\frac{(m+1)(4abd^2n+4b^2d^2n\log(cx^n)-4b^2d^2n^2\log(x)+m+1}{4b^2d^2n^2}\right.\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Erfi[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(x*Erfi[d*(a + b*Log[c*x^n])] - Erfi[(1 + m + 2*a*b*d^2*n)/(2*b*d*
n) + b*d*Log[c*x^n]]/(E^(((1 + m)*(1 + m + 4*a*b*d^2*n - 4*b^2*d^2*n^2*Log[
x] + 4*b^2*d^2*n*Log[c*x^n]))/(4*b^2*d^2*n^2))*x^m)))/(1 + m)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfi}\left(d\left(a+b\ln\left(cx^n\right)\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*erfi(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*erfi(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfi}\left((b\log\left(cx^n\right)+a)d\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*erfi((b*log(c*x^n) + a)*d), x)
```

Fricas [A] time = 3.10303, size = 429, normalized size = 3.4

$$\frac{x \operatorname{erfi}\left(bd\log\left(cx^n\right)+ad\right) e^{(m\log(e)+m\log(x))} - \sqrt{b^2d^2n^2}\operatorname{erfi}\left(\frac{\left(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n+m+1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(\frac{4\,b^2d^2mn^2\log(e)-4\,(b^2d^2}{}\right.}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] (x*erfi(b*d*log(c*x^n) + a*d)*e^(m*log(e) + m*log(x)) - sqrt(b^2*d^2*n^2)*e
rfi(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n + m + 1)*s
qrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*m*n^2*log(e) - 4*(b^2*d^2
*m + b^2*d^2)*n*log(c) - m^2 - 4*(a*b*d^2*m + a*b*d^2)*n - 2*m - 1)/(b^2*d^
2*n^2)))/(m + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfi}(ad + bd \log(cx^n)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*erfi(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*erfi(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{erfi}((b \log(cx^n) + a)d) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*erfi((b*log(c*x^n) + a)*d), x)
```

3.253 $\int e^{c+b^2x^2}\textbf{Erfi}(bx)^2\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^3}{6b}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^3)/(6*b)
```

Rubi [A] time = 0.0274825, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6375, 30}

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*Erfi[b*x]^2,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^3)/(6*b)
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}\text{erfi}(bx)^2\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x^2\,dx,x,\text{erfi}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\text{erfi}(bx)^3}{6b}\end{aligned}$$

Mathematica [A] time = 0.0082775, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*Erfi[b*x]^2,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^3)/(6*b)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}\left(\text{erfi}\,(bx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)^2,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfi}\,(bx)^2\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^2*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.68126, size = 43, normalized size = 2.05

$$\frac{\sqrt{\pi}\,\text{erfi}\,(bx)^3\,e^c}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(pi)*erfi(b*x)^3*e^c/b
```

Sympy [A] time = 2.71562, size = 19, normalized size = 0.9

$$\begin{cases} \frac{\sqrt{\pi}e^c \operatorname{erfi}^3(bx)}{6b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)**2,x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)*erfi(b*x)**3/(6*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)^2 e^{\left(b^2x^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^2*e^(b^2*x^2 + c), x)
```

3.254 $\int e^{c+b^2x^2}\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{4b}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Rubi [A] time = 0.0178662, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6375, 30}

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}\text{erfi}(bx)\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx, x, \text{erfi}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\,\text{erfi}(bx)^2}{4b}\end{aligned}$$

Mathematica [A] time = 0.0049837, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}\text{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfi}(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.93153, size = 43, normalized size = 2.05

$$\frac{\sqrt{\pi}\,\text{erfi}(bx)^2\,e^c}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*erfi(b*x)^2*e^c/b
```

Sympy [A] time = 0.727783, size = 19, normalized size = 0.9

$$\begin{cases} \frac{\sqrt{\pi}e^c \operatorname{erfi}^2(bx)}{4b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x),x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)*erfi(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx) e^{\left(b^2x^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.255 $\int \frac{e^{c+b^2x^2}}{\mathbf{Erfi}(bx)}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{\pi}e^c\log(\mathbf{Erfi}(bx))}{2b}$$

```
[Out] (E^c*Sqrt[Pi]*Log[Erfi[b*x]])/(2*b)
```

Rubi [A] time = 0.0287121, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6375, 29}

$$\frac{\sqrt{\pi}e^c\log(\mathbf{Erfi}(bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)/Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Log[Erfi[b*x]])/(2*b)
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \operatorname{erfi}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\log(\operatorname{erfi}(bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.0105771, size = 20, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\log(\text{Erfi}(bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)/Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Log[Erfi[b*x]])/(2*b)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}}{\text{erfi}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)/erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)/erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(b^2x^2+c\right)}}{\text{erfi}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(b^2*x^2 + c)/erfi(b*x), x)
```

Fricas [A] time = 2.94376, size = 47, normalized size = 2.35

$$\frac{\sqrt{\pi}e^c\log\left(\text{erfi}\,(bx)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*e^c*log(erfi(b*x))/b
```

Sympy [A] time = 0.422322, size = 24, normalized size = 1.2

$$\begin{cases} \frac{\sqrt{\pi}e^{c}\log\left(\operatorname{erfi}\left(bx\right)\right)}{2b} & \text{for}\ b \neq 0 \\ \tilde{\infty}xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)/erfi(b*x),x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)*log(erfi(b*x))/(2*b), Ne(b, 0)), (zoo*x*exp(c),
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(b^{2}x^{2}+c\right)}}{\operatorname{erfi}\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(b^2*x^2 + c)/erfi(b*x), x)
```

3.256 $\int \frac{e^{c+b^2x^2}}{\mathbf{Erfi}(bx)^2}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{\pi}e^c}{2b\text{Erfi}(bx)}$$

```
[Out] -(E^c*Sqrt[Pi])/(2*b*Erfi[b*x])
```

Rubi [A] time = 0.0261904, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6375, 30}

$$-\frac{\sqrt{\pi}e^c}{2b\text{Erfi}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)/Erfi[b*x]^2,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(2*b*Erfi[b*x])
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^2}\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \text{erfi}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}}{2b\text{erfi}(bx)}\end{aligned}$$

Mathematica [A] time = 0.0062358, size = 21, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c}{2b\text{Erfi}(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)/Erfi[b*x]^2,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(2*b*Erfi[b*x])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}}{(\text{erfi}\,(bx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)/erfi(b*x)^2,x)
```

```
[Out] int(exp(b^2*x^2+c)/erfi(b*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(b^2x^2+c\right)}}{\text{erfi}\,(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(b^2*x^2 + c)/erfi(b*x)^2, x)
```

Fricas [A] time = 2.98027, size = 45, normalized size = 2.14

$$-\frac{\sqrt{\pi}e^c}{2\,b\,\text{erfi}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(pi)*e^c/(b*erfi(b*x))
```

Sympy [A] time = 1.39146, size = 24, normalized size = 1.14

$$\begin{cases} -\frac{\sqrt{\pi}e^{c}}{2b\operatorname{erfi}(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)/erfi(b*x)**2,x)
```

```
[Out] Piecewise((-sqrt(pi)*exp(c)/(2*b*erfi(b*x)), Ne(b, 0)), (zoo*x*exp(c), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(b^{2}x^{2}+c\right)}}{\operatorname{erfi}(bx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(b^2*x^2 + c)/erfi(b*x)^2, x)
```

3.257 $\int \frac{e^{c+b^2x^2}}{\mathbf{Erfi}(bx)^3}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{\pi}e^c}{4b\text{Erfi}(bx)^2}$$

```
[Out] -(E^c*Sqrt[Pi])/(4*b*Erfi[b*x]^2)
```

Rubi [A] time = 0.0272756, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6375, 30}

$$-\frac{\sqrt{\pi}e^c}{4b\text{Erfi}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)/Erfi[b*x]^3,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(4*b*Erfi[b*x]^2)
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^3}\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, \text{erfi}(bx)\right)}{2b}\\ &= -\frac{e^c\sqrt{\pi}}{4b\text{erfi}(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0062796, size = 21, normalized size = 1.

$$-\frac{\sqrt{\pi}e^c}{4b\text{Erfi}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)/Erfi[b*x]^3,x]
```

```
[Out] -(E^c*Sqrt[Pi])/(4*b*Erfi[b*x]^2)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}}{(\operatorname{erfi}(bx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)/erfi(b*x)^3,x)
```

```
[Out] int(exp(b^2*x^2+c)/erfi(b*x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(b^2x^2+c\right)}}{\operatorname{erfi}(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(b^2*x^2 + c)/erfi(b*x)^3, x)
```

Fricas [A] time = 2.60472, size = 47, normalized size = 2.24

$$-\frac{\sqrt{\pi}e^c}{4\,b\operatorname{erfi}(bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*e^c/(b*erfi(b*x)^2)
```

Sympy [A] time = 3.67687, size = 26, normalized size = 1.24

$$\begin{cases} -\frac{\sqrt{\pi}e^{c}}{4b\operatorname{erfi}^{2}(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}xe^{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)/erfi(b*x)**3,x)
```

```
[Out] Piecewise((-sqrt(pi)*exp(c)/(4*b*erfi(b*x)**2), Ne(b, 0)), (zoo*x*exp(c), T
rue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(b^{2}x^{2}+c\right)}}{\operatorname{erfi}(bx)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)/erfi(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(b^2*x^2 + c)/erfi(b*x)^3, x)
```

3.258 $\int e^{c+b^2x^2}\mathbf{Erfi}(bx)^n\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{\pi}e^c\mathrm{Erfi}(bx)^{n+1}}{2b(n+1)}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^(1 + n))/(2*b*(1 + n))
```

Rubi [A] time = 0.0318396, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6375, 30}

$$\frac{\sqrt{\pi}e^c\mathrm{Erfi}(bx)^{n+1}}{2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*Erfi[b*x]^n,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^(1 + n))/(2*b*(1 + n))
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}\mathrm{erfi}(bx)^n\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\mathrm{Subst}\left(\int x^n\,dx,x,\mathrm{erfi}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\mathrm{erfi}(bx)^{1+n}}{2b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0105179, size = 28, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^{n+1}}{2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*Erfi[b*x]^n,x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^(1 + n))/(2*b*(1 + n))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}\left(\text{erfi}\left(bx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)^n,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \text{erfi}\left(bx\right)^n e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)^n*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.66277, size = 68, normalized size = 2.43

$$\frac{\sqrt{\pi}\,\text{erfi}\left(bx\right)^n\,\text{erfi}\left(bx\right)e^c}{2\left(bn+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)^n,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(b*x)^n*erfi(b*x)*e^c/(b*n + b)
```

Sympy [A] time = 10.9657, size = 63, normalized size = 2.25

$$\begin{cases} \tilde{\infty} x e^{c} & \text{for}\: b = 0 \wedge n = -1 \\ 0^{n} x e^{c} & \text{for}\: b = 0 \\ \frac{\sqrt{\pi} e^{c} \log{\left(\operatorname{erfi}{\left(b x \right)} \right)}}{2 b} & \text{for}\: n = -1 \\ \frac{\sqrt{\pi} e^{c} \operatorname{erfi}{\left(b x \right)} \operatorname{erfi}^{n}{\left(b x \right)}}{2 b n + 2 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)**n,x)
```

```
[Out] Piecewise((zoo*x*exp(c), Eq(b, 0) & Eq(n, -1)), (0**n*x*exp(c), Eq(b, 0)),
(sqrt(pi)*exp(c)*log(erfi(b*x))/(2*b), Eq(n, -1)), (sqrt(pi)*exp(c)*erfi(b*
x)*erfi(b*x)**n/(2*b*n + 2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)^{n} e^{\left(b^{2}x^{2}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)^n*e^(b^2*x^2 + c), x)
```

3.259 $\int e^{c+dx^2}x^5\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=257

$$-\frac{be^c\mathrm{Erfi}\left(x\sqrt{b^2+d}\right)}{d^3\sqrt{b^2+d}}-\frac{be^c\mathrm{Erfi}\left(x\sqrt{b^2+d}\right)}{2d^2\left(b^2+d\right)^{3/2}}+\frac{bxe^{x^2\left(b^2+d\right)+c}}{\sqrt{\pi}d^2\left(b^2+d\right)}-\frac{3be^c\mathrm{Erfi}\left(x\sqrt{b^2+d}\right)}{8d\left(b^2+d\right)^{5/2}}-\frac{bx^3e^{x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}+\frac{3bxe^{x^2\left(b^2+d\right)+c}}{4\sqrt{\pi}d\left(b^2+d\right)^2}$$

```
[Out] (3*b*E^(c + (b^2 + d)*x^2)*x)/(4*d*(b^2 + d)^2*Sqrt[Pi]) + (b*E^(c + (b^2 +
 d)*x^2)*x)/(d^2*(b^2 + d)*Sqrt[Pi]) - (b*E^(c + (b^2 + d)*x^2)*x^3)/(2*d*(
b^2 + d)*Sqrt[Pi]) + (E^(c + d*x^2)*Erfi[b*x])/d^3 - (E^(c + d*x^2)*x^2*Erf
i[b*x])/d^2 + (E^(c + d*x^2)*x^4*Erfi[b*x])/(2*d) - (3*b*E^c*Erfi[Sqrt[b^2
+ d]*x])/(8*d*(b^2 + d)^(5/2)) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d^2*(b^2
+ d)^(3/2)) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(d^3*Sqrt[b^2 + d])
```

Rubi [A] time = 0.408909, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6387, 6384, 2204, 2212}

$$-\frac{be^c\mathrm{Erfi}\left(x\sqrt{b^2+d}\right)}{d^3\sqrt{b^2+d}}-\frac{be^c\mathrm{Erfi}\left(x\sqrt{b^2+d}\right)}{2d^2\left(b^2+d\right)^{3/2}}+\frac{bxe^{x^2\left(b^2+d\right)+c}}{\sqrt{\pi}d^2\left(b^2+d\right)}-\frac{3be^c\mathrm{Erfi}\left(x\sqrt{b^2+d}\right)}{8d\left(b^2+d\right)^{5/2}}-\frac{bx^3e^{x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}+\frac{3bxe^{x^2\left(b^2+d\right)+c}}{4\sqrt{\pi}d\left(b^2+d\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^5*Erfi[b*x],x]
```

```
[Out] (3*b*E^(c + (b^2 + d)*x^2)*x)/(4*d*(b^2 + d)^2*Sqrt[Pi]) + (b*E^(c + (b^2 +
 d)*x^2)*x)/(d^2*(b^2 + d)*Sqrt[Pi]) - (b*E^(c + (b^2 + d)*x^2)*x^3)/(2*d*(
b^2 + d)*Sqrt[Pi]) + (E^(c + d*x^2)*Erfi[b*x])/d^3 - (E^(c + d*x^2)*x^2*Erf
i[b*x])/d^2 + (E^(c + d*x^2)*x^4*Erfi[b*x])/(2*d) - (3*b*E^c*Erfi[Sqrt[b^2
+ d]*x])/(8*d*(b^2 + d)^(5/2)) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d^2*(b^2
+ d)^(3/2)) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(d^3*Sqrt[b^2 + d])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx = \frac{e^{c+dx^2}x^4\text{erfi}(bx)}{2d} - \frac{2\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx}{d} - \frac{b\int e^{c+\left(b^2+d\right)x^2}x^4\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{c+\left(b^2+d\right)x^2}x^3}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{e^{c+dx^2}x^2\text{erfi}(bx)}{d^2} + \frac{e^{c+dx^2}x^4\text{erfi}(bx)}{2d} + \frac{2\int e^{c+dx^2}x\text{erfi}(bx)\,dx}{d^2} + \frac{(2b)\int e^{c+\left(b^2\right.}}{d^2\sqrt{}}$$

$$= \frac{3be^{c+\left(b^2+d\right)x^2}x}{4d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{be^{c+\left(b^2+d\right)x^2}x}{d^2\left(b^2+d\right)\sqrt{\pi}} - \frac{be^{c+\left(b^2+d\right)x^2}x^3}{2d\left(b^2+d\right)\sqrt{\pi}} + \frac{e^{c+dx^2}\text{erfi}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\text{erfi}(bx)}{d^2} + \frac{e^{c+}}{}$$

$$= \frac{3be^{c+\left(b^2+d\right)x^2}x}{4d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{be^{c+\left(b^2+d\right)x^2}x}{d^2\left(b^2+d\right)\sqrt{\pi}} - \frac{be^{c+\left(b^2+d\right)x^2}x^3}{2d\left(b^2+d\right)\sqrt{\pi}} + \frac{e^{c+dx^2}\text{erfi}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\text{erfi}(bx)}{d^2} + \frac{e^{c+}}{}$$

Mathematica [A] time = 0.278333, size = 131, normalized size = 0.51

$$\frac{e^c\left(-\frac{b\left(20b^2d+8b^4+15d^2\right)\text{Erfi}\left(x\sqrt{b^2+d}\right)}{\left(b^2+d\right)^{5/2}} - \frac{2bdxe^{x^2\left(b^2+d\right)}\left(2b^2\left(dx^2-2\right)+d\left(2dx^2-7\right)\right)}{\sqrt{\pi}\left(b^2+d\right)^2} + 4e^{dx^2}\left(d^2x^4-2dx^2+2\right)\text{Erfi}(bx)\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^5*Erfi[b*x],x]
```

```
[Out] (E^c*((-2*b*d*E^((b^2 + d)*x^2)*x*(2*b^2*(-2 + d*x^2) + d*(-7 + 2*d*x^2)))/
((b^2 + d)^2*Sqrt[Pi]) + 4*E^(d*x^2)*(2 - 2*d*x^2 + d^2*x^4)*Erfi[b*x] - (b
*(8*b^4 + 20*b^2*d + 15*d^2)*Erfi[Sqrt[b^2 + d]*x])/(b^2 + d)^(5/2)))/(8*d^
3)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^5\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^5*erfi(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^5*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\operatorname{erfi}(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfi(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.80757, size = 547, normalized size = 2.13

$$\frac{\pi\left(8\,b^5+20\,b^3d+15\,bd^2\right)\sqrt{-b^2-d}\operatorname{erf}\left(\sqrt{-b^2-d}x\right)e^c+4\left(\pi\left(b^6d^2+3\,b^4d^3+3\,b^2d^4+d^5\right)x^4-2\,\pi\left(b^6d+3\,b^4d^2+3\,b^2d^3\right.\right.}{8\,\pi\left(b^6d^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*(8*b^5 + 20*b^3*d + 15*b*d^2)*sqrt(-b^2 - d)*erf(sqrt(-b^2 - d)*x)*
e^c + 4*(pi*(b^6*d^2 + 3*b^4*d^3 + 3*b^2*d^4 + d^5)*x^4 - 2*pi*(b^6*d + 3*b
^4*d^2 + 3*b^2*d^3 + d^4)*x^2 + 2*pi*(b^6 + 3*b^4*d + 3*b^2*d^2 + d^3))*erf
i(b*x)*e^(d*x^2 + c) - 2*sqrt(pi)*(2*(b^5*d^2 + 2*b^3*d^3 + b*d^4)*x^3 - (4
*b^5*d + 11*b^3*d^2 + 7*b*d^3)*x)*e^(b^2*x^2 + d*x^2 + c))/(pi*(b^6*d^3 + 3
*b^4*d^4 + 3*b^2*d^5 + d^6))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**5*erfi(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^5*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfi(b*x)*e^(d*x^2 + c), x)
```

3.260 $\int e^{c+dx^2}x^3\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=142

$$\frac{be^c\text{Erfi}\left(x\sqrt{b^2+d}\right)}{2d^2\sqrt{b^2+d}}+\frac{be^c\text{Erfi}\left(x\sqrt{b^2+d}\right)}{4d\left(b^2+d\right)^{3/2}}-\frac{bxe^{x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}-\frac{\text{Erfi}(bx)e^{c+dx^2}}{2d^2}+\frac{x^2\text{Erfi}(bx)e^{c+dx^2}}{2d}$$

```
[Out] -(b*E^(c + (b^2 + d)*x^2)*x)/(2*d*(b^2 + d)*Sqrt[Pi]) - (E^(c + d*x^2)*Erfi
[b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfi[b*x])/(2*d) + (b*E^c*Erfi[Sqrt[b^2
 + d]*x])/(4*d*(b^2 + d)^(3/2)) + (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d^2*Sqrt
[b^2 + d])
```

Rubi [A] time = 0.151724, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6387, 6384, 2204, 2212}

$$\frac{be^c\text{Erfi}\left(x\sqrt{b^2+d}\right)}{2d^2\sqrt{b^2+d}}+\frac{be^c\text{Erfi}\left(x\sqrt{b^2+d}\right)}{4d\left(b^2+d\right)^{3/2}}-\frac{bxe^{x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}-\frac{\text{Erfi}(bx)e^{c+dx^2}}{2d^2}+\frac{x^2\text{Erfi}(bx)e^{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^3*Erfi[b*x],x]
```

```
[Out] -(b*E^(c + (b^2 + d)*x^2)*x)/(2*d*(b^2 + d)*Sqrt[Pi]) - (E^(c + d*x^2)*Erfi
[b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfi[b*x])/(2*d) + (b*E^c*Erfi[Sqrt[b^2
 + d]*x])/(4*d*(b^2 + d)^(3/2)) + (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d^2*Sqrt
[b^2 + d])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
```

```
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx &= \frac{e^{c+dx^2}x^2\text{erfi}(bx)}{2d}-\frac{\int e^{c+dx^2}x\text{erfi}(bx)\,dx}{d}-\frac{b\int e^{c+\left(b^2+d\right)x^2}x^2\,dx}{d\sqrt{\pi}}\\ &= -\frac{be^{c+\left(b^2+d\right)x^2}x}{2d\left(b^2+d\right)\sqrt{\pi}}-\frac{e^{c+dx^2}\text{erfi}(bx)}{2d^2}+\frac{e^{c+dx^2}x^2\text{erfi}(bx)}{2d}+\frac{b\int e^{c+\left(b^2+d\right)x^2}\,dx}{d^2\sqrt{\pi}}+\frac{b\int e^{c+\left(b^2+d\right)x^2}\,dx}{2d\left(b^2+d\right)\sqrt{\pi}}\\ &= -\frac{be^{c+\left(b^2+d\right)x^2}x}{2d\left(b^2+d\right)\sqrt{\pi}}-\frac{e^{c+dx^2}\text{erfi}(bx)}{2d^2}+\frac{e^{c+dx^2}x^2\text{erfi}(bx)}{2d}+\frac{be^c\text{erfi}\left(\sqrt{b^2+d}x\right)}{4d\left(b^2+d\right)^{3/2}}+\frac{be^c\text{erfi}\left(\sqrt{b^2+d}x\right.}{2d^2\sqrt{b^2+d}}\end{aligned}$$

Mathematica [A] time = 0.173665, size = 91, normalized size = 0.64

$$\frac{e^c\left(\frac{\left(2b^3+3bd\right)\text{Erfi}\left(x\sqrt{b^2+d}\right)}{\left(b^2+d\right)^{3/2}}-\frac{2bdxe^{x^2\left(b^2+d\right)}}{\sqrt{\pi}\left(b^2+d\right)}+2e^{dx^2}\left(dx^2-1\right)\text{Erfi}(bx)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^3*Erfi[b*x],x]
```

```
[Out] (E^c*((-2*b*d*E^((b^2 + d)*x^2)*x)/((b^2 + d)*Sqrt[Pi]) + 2*E^(d*x^2)*(-1 +
 d*x^2)*Erfi[b*x] + ((2*b^3 + 3*b*d)*Erfi[Sqrt[b^2 + d]*x])/(b^2 + d)^(3/2)
```

```
))/(4*d^2)
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^3\mathrm{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^3*erfi(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^3*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\mathrm{erfi}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfi(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.72853, size = 332, normalized size = 2.34

$$-\frac{\pi\left(2\,b^3+3\,bd\right)\sqrt{-b^2-d}\,\mathrm{erf}\left(\sqrt{-b^2-d}x\right)e^c+2\,\sqrt{\pi}\left(b^3d+bd^2\right)xe^{\left(b^2x^2+dx^2+c\right)}-2\left(\pi\left(b^4d+2\,b^2d^2+d^3\right)x^2-\pi\left(b^4+2\,b^2d\right.\right.}{4\,\pi\left(b^4d^2+2\,b^2d^3+d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfi(b*x),x, algorithm="fricas")
```

```
[Out] -1/4*(pi*(2*b^3 + 3*b*d)*sqrt(-b^2 - d)*erf(sqrt(-b^2 - d)*x)*e^c + 2*sqrt(
pi)*(b^3*d + b*d^2)*x*e^(b^2*x^2 + d*x^2 + c) - 2*(pi*(b^4*d + 2*b^2*d^2 +
d^3)*x^2 - pi*(b^4 + 2*b^2*d + d^2))*erfi(b*x)*e^(d*x^2 + c))/(pi*(b^4*d^2
```

```
+ 2*b^2*d^3 + d^4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**3*erfi(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfi(b*x)*e^(d*x^2 + c), x)
```

3.261 $\int e^{c+dx^2} x \mathbf{Erfi}(bx) \, dx$

Optimal. Leaf size=53

$$\frac{\text{Erfi}(bx)e^{c+dx^2}}{2d} - \frac{be^c \text{Erfi}\left(x\sqrt{b^2+d}\right)}{2d\sqrt{b^2+d}}$$

```
[Out] (E^(c + d*x^2)*Erfi[b*x])/(2*d) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d*Sqrt[b
^2 + d])
```

Rubi [A] time = 0.0391289, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6384, 2204}

$$\frac{\text{Erfi}(bx)e^{c+dx^2}}{2d} - \frac{be^c \text{Erfi}\left(x\sqrt{b^2+d}\right)}{2d\sqrt{b^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x*Erfi[b*x],x]
```

```
[Out] (E^(c + d*x^2)*Erfi[b*x])/(2*d) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d*Sqrt[b
^2 + d])
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{c+dx^2}x\text{erfi}(bx)\,dx = \frac{e^{c+dx^2}\text{erfi}(bx)}{2d} - \frac{b\int e^{c+(b^2+d)x^2}\,dx}{d\sqrt{\pi}}$$
$$= \frac{e^{c+dx^2}\text{erfi}(bx)}{2d} - \frac{be^c\text{erfi}\left(\sqrt{b^2+d}\,x\right)}{2d\sqrt{b^2+d}}$$

Mathematica [A] time = 0.017985, size = 47, normalized size = 0.89

$$\frac{e^c\left(e^{dx^2}\text{Erfi}(bx) - \frac{b\text{Erfi}\left(x\sqrt{b^2+d}\right)}{\sqrt{b^2+d}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x*Erfi[b*x],x]
```

```
[Out] (E^c*(E^(d*x^2)*Erfi[b*x] - (b*Erfi[Sqrt[b^2 + d]*x])/Sqrt[b^2 + d]))/(2*d)
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x\text{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x*erfi(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*erfi(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.70942, size = 135, normalized size = 2.55

$$\frac{\sqrt{-b^2-d}\,b\operatorname{erf}\left(\sqrt{-b^2-d}\,x\right)e^c+\left(b^2+d\right)\operatorname{erfi}(bx)\,e^{\left(dx^2+c\right)}}{2\left(b^2d+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-b^2 - d)*b*erf(sqrt(-b^2 - d)*x)*e^c + (b^2 + d)*erfi(b*x)*e^(d*
x^2 + c))/(b^2*d + d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c\int xe^{dx^2}\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x*erfi(b*x),x)
```

```
[Out] exp(c)*Integral(x*exp(d*x**2)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{erfi}(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*erfi(b*x)*e^(d*x^2 + c), x)
```

3.262 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(bx)}{x}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{\text{Erfi}(bx)e^{c+dx^2}}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + d*x^2)*Erfi[b*x])/x, x]
```

Rubi [A] time = 0.0386478, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + d*x^2)*Erfi[b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx$$

Mathematica [A] time = 0.131031, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x, x]
```

Maple [A] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{d x^2+c} \operatorname{erfi}(b x)}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x)/x,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(b x) e^{\left(d x^2+c\right)}}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erfi}(b x) e^{\left(d x^2+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(d*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int \frac{e^{d x^{2}} \operatorname{erfi}(b x)}{x}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x)/x,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x, x)
```

3.263 $\int \frac{e^{c+dx^2}\textbf{Erfi}(bx)}{x^3}\,dx$

Optimal. Leaf size=92

$$d\text{Unintegrable}\left(\frac{\text{Erfi}(bx)e^{c+dx^2}}{x},x\right)+be^c\sqrt{b^2+d}\,\text{Erfi}\left(x\sqrt{b^2+d}\right)-\frac{be^{x^2\left(b^2+d\right)+c}}{\sqrt{\pi}x}-\frac{\text{Erfi}(bx)e^{c+dx^2}}{2x^2}$$

```
[Out] -((b*E^(c + (b^2 + d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)*Erfi[b*x])/(2*x^
2) + b*Sqrt[b^2 + d]*E^c*Erfi[Sqrt[b^2 + d]*x] + d*Unintegrable[(E^(c + d*x
^2)*Erfi[b*x])/x, x]
```

Rubi [A] time = 0.14796, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[b*x])/x^3,x]
```

```
[Out] -((b*E^(c + (b^2 + d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)*Erfi[b*x])/(2*x^
2) + b*Sqrt[b^2 + d]*E^c*Erfi[Sqrt[b^2 + d]*x] + d*Defer[Int][(E^(c + d*x^2
)*Erfi[b*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx &= -\frac{e^{c+dx^2}\text{erfi}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx+\frac{b\int \frac{e^{c+\left(b^2+d\right)x^2}}{x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{c+\left(b^2+d\right)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\text{erfi}(bx)}{2x^2}+d\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx+\frac{\left(2b\left(b^2+d\right)\right)\int e^{c+\left(b^2+d\right)x^2}\,dx}{\sqrt{\pi}}\\
&= -\frac{be^{c+\left(b^2+d\right)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\text{erfi}(bx)}{2x^2}+b\sqrt{b^2+d}e^c\text{erfi}\left(\sqrt{b^2+d}x\right)+d\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 0.183742, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^3, x]
```

Maple [A] time = 0.274, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfi}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x)/x^3,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(d*x^2 + c)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int\frac{e^{dx^{2}}\operatorname{erfi}(bx)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x)/x**3,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erfi}(bx)\,e^{\left(dx^{2}+c\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^3, x)
```

3.264 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(bx)}{x^5}\,dx$

Optimal. Leaf size=211

$$\frac{1}{2}d^2\text{Unintegrable}\left(\frac{\text{Erfi}(bx)e^{c+dx^2}}{x},x\right)+\frac{1}{2}be^cd\sqrt{b^2+d}\text{Erfi}\left(x\sqrt{b^2+d}\right)+\frac{1}{3}be^c\left(b^2+d\right)^{3/2}\text{Erfi}\left(x\sqrt{b^2+d}\right)-\frac{bde^{x^2\left(b^2+d\right)}}{2\sqrt{\pi}x}$$

```
[Out] -(b*E^(c + (b^2 + d)*x^2))/(6*Sqrt[Pi]*x^3) - (b*d*E^(c + (b^2 + d)*x^2))/(
2*Sqrt[Pi]*x) - (b*(b^2 + d)*E^(c + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x) - (E^(c
+ d*x^2)*Erfi[b*x])/(4*x^4) - (d*E^(c + d*x^2)*Erfi[b*x])/(4*x^2) + (b*d*Sq
rt[b^2 + d]*E^c*Erfi[Sqrt[b^2 + d]*x])/2 + (b*(b^2 + d)^(3/2)*E^c*Erfi[Sqrt
[b^2 + d]*x])/3 + (d^2*Unintegrable[(E^(c + d*x^2)*Erfi[b*x])/x, x])/2
```

Rubi [A] time = 0.320188, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[b*x])/x^5,x]
```

```
[Out] -(b*E^(c + (b^2 + d)*x^2))/(6*Sqrt[Pi]*x^3) - (b*d*E^(c + (b^2 + d)*x^2))/(
2*Sqrt[Pi]*x) - (b*(b^2 + d)*E^(c + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x) - (E^(c
+ d*x^2)*Erfi[b*x])/(4*x^4) - (d*E^(c + d*x^2)*Erfi[b*x])/(4*x^2) + (b*d*Sq
rt[b^2 + d]*E^c*Erfi[Sqrt[b^2 + d]*x])/2 + (b*(b^2 + d)^(3/2)*E^c*Erfi[Sqrt
[b^2 + d]*x])/3 + (d^2*Defer[Int][(E^(c + d*x^2)*Erfi[b*x])/x, x])/2
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = -\frac{e^{c+dx^2}\text{erfi}(bx)}{4x^4} + \frac{1}{2}d\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx + \frac{b\int \frac{e^{c+\left(b^2+d\right)x^2}}{x^4}\,dx}{2\sqrt{\pi}}$$

$$= -\frac{be^{c+\left(b^2+d\right)x^2}}{6\sqrt{\pi}x^3} - \frac{e^{c+dx^2}\text{erfi}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erfi}(bx)}{4x^2} + \frac{1}{2}d^2\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx + \frac{(bd)\int \frac{e^{c+\left(b^2+d\right)x^2}}{x^2}\,dx}{2\sqrt{\pi}} +$$

$$= -\frac{be^{c+\left(b^2+d\right)x^2}}{6\sqrt{\pi}x^3} - \frac{bde^{c+\left(b^2+d\right)x^2}}{2\sqrt{\pi}x} - \frac{b\left(b^2+d\right)e^{c+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erfi}(bx)}{4x^2} + \frac{1}{2}d^2\int \frac{e^{c}}{}$$

$$= -\frac{be^{c+\left(b^2+d\right)x^2}}{6\sqrt{\pi}x^3} - \frac{bde^{c+\left(b^2+d\right)x^2}}{2\sqrt{\pi}x} - \frac{b\left(b^2+d\right)e^{c+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erfi}(bx)}{4x^2} + \frac{1}{2}bd\sqrt{b^2}$$

Mathematica [A] time = 0.233364, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^5,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^5, x]
```

Maple [A] time = 0.297, size = 0, normalized size = 0.

$$\int \frac{\text{e}^{dx^2+c}\text{erfi}\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x)/x^5,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\left(bx\right)e^{\left(dx^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erfi}\left(bx\right)e^{\left(dx^2+c\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(d*x^2 + c)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\left(bx\right)e^{\left(dx^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^5, x)
```

3.265 $\int e^{c+dx^2} x^4 \mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=170

$$\frac{3\text{Unintegrable}\left(\text{Erfi}(bx)e^{c+dx^2},x\right)}{4d^2}+\frac{3be^{x^2(b^2+d)+c}}{4\sqrt{\pi}d^2\left(b^2+d\right)}-\frac{bx^2e^{x^2(b^2+d)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}+\frac{be^{x^2(b^2+d)+c}}{2\sqrt{\pi}d\left(b^2+d\right)^2}-\frac{3x\text{Erfi}(bx)e^{c+dx^2}}{4d^2}+\frac{x^3\text{Erfi}(b}{2}$$

```
[Out] (b*E^(c + (b^2 + d)*x^2))/(2*d*(b^2 + d)^2*Sqrt[Pi]) + (3*b*E^(c + (b^2 + d
)*x^2))/(4*d^2*(b^2 + d)*Sqrt[Pi]) - (b*E^(c + (b^2 + d)*x^2)*x^2)/(2*d*(b^
2 + d)*Sqrt[Pi]) - (3*E^(c + d*x^2)*x*Erfi[b*x])/(4*d^2) + (E^(c + d*x^2)*x
^3*Erfi[b*x])/(2*d) + (3*Unintegrable[E^(c + d*x^2)*Erfi[b*x], x])/(4*d^2)
```

Rubi [A] time = 0.234193, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^4\text{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^4*Erfi[b*x],x]
```

```
[Out] (b*E^(c + (b^2 + d)*x^2))/(2*d*(b^2 + d)^2*Sqrt[Pi]) + (3*b*E^(c + (b^2 + d
)*x^2))/(4*d^2*(b^2 + d)*Sqrt[Pi]) - (b*E^(c + (b^2 + d)*x^2)*x^2)/(2*d*(b^
2 + d)*Sqrt[Pi]) - (3*E^(c + d*x^2)*x*Erfi[b*x])/(4*d^2) + (E^(c + d*x^2)*x
^3*Erfi[b*x])/(2*d) + (3*Defer[Int][E^(c + d*x^2)*Erfi[b*x], x])/(4*d^2)
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx &= \frac{e^{c+dx^2}x^3\text{erfi}(bx)}{2d}-\frac{3\int e^{c+dx^2}x^2\text{erfi}(bx)\,dx}{2d}-\frac{b\int e^{c+\left(b^2+d\right)x^2}x^3\,dx}{d\sqrt{\pi}}\\ &= -\frac{be^{c+\left(b^2+d\right)x^2}x^2}{2d\left(b^2+d\right)\sqrt{\pi}}-\frac{3e^{c+dx^2}x\text{erfi}(bx)}{4d^2}+\frac{e^{c+dx^2}x^3\text{erfi}(bx)}{2d}+\frac{3\int e^{c+dx^2}\text{erfi}(bx)\,dx}{4d^2}+\frac{(3b)\int e^{c+\left(b^2+d\right)}}{2d^2\sqrt{\pi}}\\ &= \frac{be^{c+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}}+\frac{3be^{c+\left(b^2+d\right)x^2}}{4d^2\left(b^2+d\right)\sqrt{\pi}}-\frac{be^{c+\left(b^2+d\right)x^2}x^2}{2d\left(b^2+d\right)\sqrt{\pi}}-\frac{3e^{c+dx^2}x\text{erfi}(bx)}{4d^2}+\frac{e^{c+dx^2}x^3\text{erfi}(bx)}{2d}+\frac{3}{}\end{aligned}$$

Mathematica [A] time = 0.268197, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^4\text{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^4*Erfi[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^4*Erfi[b*x], x]
```

Maple [A] time = 0.11, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^4\text{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^4*erfi(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^4*erfi(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfi(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\,\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*erfi(b*x)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**4*erfi(b*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx) e^{\left(dx^2+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfi(b*x)*e^(d*x^2 + c), x)
```

3.266 $\int e^{c+dx^2}x^2\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=78

$$-\frac{\text{Unintegrable}\left(\text{Erfi}(bx)e^{c+dx^2},x\right)}{2d}-\frac{be^{x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}+\frac{x\text{Erfi}(bx)e^{c+dx^2}}{2d}$$

```
[Out] -(b*E^(c + (b^2 + d)*x^2))/(2*d*(b^2 + d)*Sqrt[Pi]) + (E^(c + d*x^2)*x*Erfi
[b*x])/(2*d) - Unintegrable[E^(c + d*x^2)*Erfi[b*x], x]/(2*d)
```

Rubi [A] time = 0.0945238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^2\text{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^2*Erfi[b*x],x]
```

```
[Out] -(b*E^(c + (b^2 + d)*x^2))/(2*d*(b^2 + d)*Sqrt[Pi]) + (E^(c + d*x^2)*x*Erfi
[b*x])/(2*d) - Defer[Int][E^(c + d*x^2)*Erfi[b*x], x]/(2*d)
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2}x^2\text{erfi}(bx)\,dx&=\frac{e^{c+dx^2}x\text{erfi}(bx)}{2d}-\frac{\int e^{c+dx^2}\text{erfi}(bx)\,dx}{2d}-\frac{b\int e^{c+\left(b^2+d\right)x^2}x\,dx}{d\sqrt{\pi}}\\&=-\frac{be^{c+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)\sqrt{\pi}}+\frac{e^{c+dx^2}x\text{erfi}(bx)}{2d}-\frac{\int e^{c+dx^2}\text{erfi}(bx)\,dx}{2d}\end{aligned}$$

Mathematica [A] time = 0.193643, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^2\text{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^2*Erfi[b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^2*Erfi[b*x], x]
```

Maple [A] time = 0.214, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^2\mathrm{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^2*erfi(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*x^2*erfi(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfi(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\,\mathrm{erfi}\,(bx)\,e^{(dx^2+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*erfi(b*x)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^c \int x^2 e^{dx^2} \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**2*erfi(b*x),x)
```

```
[Out] exp(c)*Integral(x**2*exp(d*x**2)*erfi(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi(b*x)*e^(d*x^2 + c), x)
```

3.267 $\int e^{c+dx^2}\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\text{Erfi}(bx)e^{c+dx^2},x\right)$$

```
[Out] Unintegrable[E^(c + d*x^2)*Erfi[b*x], x]
```

Rubi [A] time = 0.0148574, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}\text{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*Erfi[b*x],x]

[Out] Defer[Int][E^(c + d*x^2)*Erfi[b*x], x]
```

Rubi steps

$$\int e^{c+dx^2}\text{erfi}(bx)\,dx = \int e^{c+dx^2}\text{erfi}(bx)\,dx$$

Mathematica [A] time = 0.0241942, size = 0, normalized size = 0.

$$\int e^{c+dx^2}\text{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*Erfi[b*x],x]

[Out] Integrate[E^(c + d*x^2)*Erfi[b*x], x]
```

Maple [A] time = 0.1, size = 0, normalized size = 0.

$$\int \mathrm{e}^{d x^{2}+c} \operatorname{erfi}\left(b x\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x),x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}\left(bx\right)e^{\left(dx^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{erfi}\left(bx\right)e^{\left(dx^{2}+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int e^{dx^{2}}\operatorname{erfi}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x),x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c), x)
```

3.268 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(bx)}{x^2}\,dx$

Optimal. Leaf size=58

$$2d\text{Unintegrable}\left(\text{Erfi}(bx)e^{c+dx^2},x\right)+\frac{be^c\text{ExpIntegralEi}\left(x^2\left(b^2+d\right)\right)}{\sqrt{\pi}}-\frac{\text{Erfi}(bx)e^{c+dx^2}}{x}$$

```
[Out] -((E^(c + d*x^2)*Erfi[b*x])/x) + (b*E^c*ExpIntegralEi[(b^2 + d)*x^2])/Sqrt[
Pi] + 2*d*Unintegrable[E^(c + d*x^2)*Erfi[b*x], x]
```

Rubi [A] time = 0.113821, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[b*x])/x^2,x]
```

```
[Out] -((E^(c + d*x^2)*Erfi[b*x])/x) + (b*E^c*ExpIntegralEi[(b^2 + d)*x^2])/Sqrt[
Pi] + 2*d*Defer[Int][E^(c + d*x^2)*Erfi[b*x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^2}\,dx &= -\frac{e^{c+dx^2}\text{erfi}(bx)}{x}+(2d)\int e^{c+dx^2}\text{erfi}(bx)\,dx+\frac{(2b)\int \frac{e^{c+\left(b^2+d\right)x^2}}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+dx^2}\text{erfi}(bx)}{x}+\frac{be^c\text{Ei}\left(\left(b^2+d\right)x^2\right)}{\sqrt{\pi}}+(2d)\int e^{c+dx^2}\text{erfi}(bx)\,dx\end{aligned}$$

Mathematica [A] time = 0.188564, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^2,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^2, x]
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{dx^2+c}\mathrm{erfi}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x)/x^2,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(d*x^2 + c)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int \frac{e^{dx^{2}}\operatorname{erfi}(bx)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\,e^{\left(dx^{2}+c\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^2, x)
```

3.269 $\int \frac{e^{c+dx^2}\textbf{Erfi}(bx)}{x^4}\,dx$

Optimal. Leaf size=143

$$\frac{4}{3}d^2\text{Unintegrable}\left(\text{Erfi}(bx)e^{c+dx^2},x\right)+\frac{2be^cd\text{ExpIntegralEi}\left(x^2\left(b^2+d\right)\right)}{3\sqrt{\pi}}+\frac{be^c\left(b^2+d\right)\text{ExpIntegralEi}\left(x^2\left(b^2+d\right)\right)}{3\sqrt{\pi}}$$

```
[Out] -(b*E^(c + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + d*x^2)*Erfi[b*x])/(3*
x^3) - (2*d*E^(c + d*x^2)*Erfi[b*x])/(3*x) + (2*b*d*E^c*ExpIntegralEi[(b^2
+ d)*x^2])/(3*Sqrt[Pi]) + (b*(b^2 + d)*E^c*ExpIntegralEi[(b^2 + d)*x^2])/(3
*Sqrt[Pi]) + (4*d^2*Unintegrable[E^(c + d*x^2)*Erfi[b*x], x])/3
```

Rubi [A] time = 0.272621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[b*x])/x^4,x]
```

```
[Out] -(b*E^(c + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + d*x^2)*Erfi[b*x])/(3*
x^3) - (2*d*E^(c + d*x^2)*Erfi[b*x])/(3*x) + (2*b*d*E^c*ExpIntegralEi[(b^2
+ d)*x^2])/(3*Sqrt[Pi]) + (b*(b^2 + d)*E^c*ExpIntegralEi[(b^2 + d)*x^2])/(3
*Sqrt[Pi]) + (4*d^2*Defer[Int][E^(c + d*x^2)*Erfi[b*x], x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^4}\,dx &= -\frac{e^{c+dx^2}\text{erfi}(bx)}{3x^3}+\frac{1}{3}(2d)\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^2}\,dx+\frac{(2b)\int \frac{e^{c+\left(b^2+d\right)x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{c+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\text{erfi}(bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erfi}(bx)}{3x}+\frac{1}{3}\left(4d^2\right)\int e^{c+dx^2}\text{erfi}(bx)\,dx+\frac{(4bd)\int \frac{e^{c+\left(b^2+d\right)x^2}}{x}}{3\sqrt{\pi}}\\
&= -\frac{be^{c+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\text{erfi}(bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erfi}(bx)}{3x}+\frac{2bde^c\text{Ei}\left(\left(b^2+d\right)x^2\right)}{3\sqrt{\pi}}+\frac{b\left(b^2+d\right)e^c\text{Ei}\left(\left(b^2+\right.\right.}{3\sqrt{\pi}}
\end{aligned}$$

Mathematica [A] time = 0.291054, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^4,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^4, x]
```

Maple [A] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfi}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x)/x^4,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erfi}\,(bx)\,e^{\left(dx^2+c\right)}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(d*x^2 + c)/x^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int\frac{e^{dx^{2}}\operatorname{erfi}(bx)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x)/x**4,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erfi}(bx)\,e^{\left(dx^{2}+c\right)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(d*x^2 + c)/x^4, x)
```

3.270 $\int e^{-b^2x^2}x^5\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=107

$$-\frac{x^4e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{x^2e^{-b^2x^2}\text{Erfi}(bx)}{b^4}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{b^6}+\frac{2x^3}{3\sqrt{\pi}b^3}+\frac{2x}{\sqrt{\pi}b^5}+\frac{x^5}{5\sqrt{\pi}b}$$

```
[Out] (2*x)/(b^5*Sqrt[Pi]) + (2*x^3)/(3*b^3*Sqrt[Pi]) + x^5/(5*b*Sqrt[Pi]) - Erfi
[b*x]/(b^6*E^(b^2*x^2)) - (x^2*Erfi[b*x])/(b^4*E^(b^2*x^2)) - (x^4*Erfi[b*x
])/(2*b^2*E^(b^2*x^2))
```

Rubi [A] time = 0.115105, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6387, 6384, 8, 30}

$$-\frac{x^4e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{x^2e^{-b^2x^2}\text{Erfi}(bx)}{b^4}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{b^6}+\frac{2x^3}{3\sqrt{\pi}b^3}+\frac{2x}{\sqrt{\pi}b^5}+\frac{x^5}{5\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] (2*x)/(b^5*Sqrt[Pi]) + (2*x^3)/(3*b^3*Sqrt[Pi]) + x^5/(5*b*Sqrt[Pi]) - Erfi
[b*x]/(b^6*E^(b^2*x^2)) - (x^2*Erfi[b*x])/(b^4*E^(b^2*x^2)) - (x^4*Erfi[b*x
])/(2*b^2*E^(b^2*x^2))
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^5\text{erfi}(bx)\,dx &= -\frac{e^{-b^2x^2}x^4\text{erfi}(bx)}{2b^2}+\frac{2\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx}{b^2}+\frac{\int x^4\,dx}{b\sqrt{\pi}}\\ &= \frac{x^5}{5b\sqrt{\pi}}-\frac{e^{-b^2x^2}x^2\text{erfi}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfi}(bx)}{2b^2}+\frac{2\int e^{-b^2x^2}x\text{erfi}(bx)\,dx}{b^4}+\frac{2\int x^2\,dx}{b^3\sqrt{\pi}}\\ &= \frac{2x^3}{3b^3\sqrt{\pi}}+\frac{x^5}{5b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\text{erfi}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfi}(bx)}{2b^2}+\frac{2\int 1\,dx}{b^5\sqrt{\pi}}\\ &= \frac{2x}{b^5\sqrt{\pi}}+\frac{2x^3}{3b^3\sqrt{\pi}}+\frac{x^5}{5b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\text{erfi}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfi}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0436194, size = 68, normalized size = 0.64

$$\frac{\frac{6b^5x^5+20b^3x^3+60bx}{\sqrt{\pi}}-15e^{-b^2x^2}\left(b^4x^4+2b^2x^2+2\right)\text{Erfi}(bx)}{30b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] ((60*b*x + 20*b^3*x^3 + 6*b^5*x^5)/Sqrt[Pi] - (15*(2 + 2*b^2*x^2 + b^4*x^4)
*Erfi[b*x])/E^(b^2*x^2))/(30*b^6)
```

Maple [A] time = 0.303, size = 103, normalized size = 1.

$$\frac{6\,x^5e^{b^2x^2}b^5-15\,\text{erfi}\,(bx)\,x^4b^4\sqrt{\pi}+20\,e^{b^2x^2}b^3x^3-30\,\sqrt{\pi}\text{erfi}\,(bx)\,b^2x^2+60\,e^{b^2x^2}bx-30\,\sqrt{\pi}\text{erfi}\,(bx)}{30\,b^6\sqrt{\pi}e^{b^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*erfi(b*x)/exp(b^2*x^2),x)
```

```
[Out] 1/30*(6*x^5*exp(b^2*x^2)*b^5-15*erfi(b*x)*x^4*b^4*Pi^(1/2)+20*exp(b^2*x^2)*
b^3*x^3-30*Pi^(1/2)*erfi(b*x)*b^2*x^2+60*exp(b^2*x^2)*b*x-30*Pi^(1/2)*erfi(
b*x))/b^6/Pi^(1/2)/exp(b^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx) e^{(-b^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.58599, size = 185, normalized size = 1.73

$$\frac{\left(2\sqrt{\pi}\left(3b^5x^5 + 10b^3x^3 + 30bx\right)e^{\left(b^2x^2\right)} - 15\left(2\pi + \pi b^4x^4 + 2\pi b^2x^2\right)\operatorname{erfi}(bx)\right)e^{\left(-b^2x^2\right)}}{30\pi b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/30*(2*sqrt(pi)*(3*b^5*x^5 + 10*b^3*x^3 + 30*b*x)*e^(b^2*x^2) - 15*(2*pi +
 pi*b^4*x^4 + 2*pi*b^2*x^2)*erfi(b*x))*e^(-b^2*x^2)/(pi*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx)e^{(-b^2x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfi(b*x)*e^(-b^2*x^2), x)
```

3.271 $\int e^{-b^2x^2}x^3\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=71

$$-\frac{x^2e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2b^4}+\frac{x}{\sqrt{\pi}b^3}+\frac{x^3}{3\sqrt{\pi}b}$$

```
[Out] x/(b^3*Sqrt[Pi]) + x^3/(3*b*Sqrt[Pi]) - Erfi[b*x]/(2*b^4*E^(b^2*x^2)) - (x^
2*Erfi[b*x])/(2*b^2*E^(b^2*x^2))
```

Rubi [A] time = 0.0685207, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6387, 6384, 8, 30}

$$-\frac{x^2e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2b^4}+\frac{x}{\sqrt{\pi}b^3}+\frac{x^3}{3\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] x/(b^3*Sqrt[Pi]) + x^3/(3*b*Sqrt[Pi]) - Erfi[b*x]/(2*b^4*E^(b^2*x^2)) - (x^
2*Erfi[b*x])/(2*b^2*E^(b^2*x^2))
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^3\operatorname{erfi}(bx)\,dx &= -\frac{e^{-b^2x^2}x^2\operatorname{erfi}(bx)}{2b^2}+\frac{\int e^{-b^2x^2}x\operatorname{erfi}(bx)\,dx}{b^2}+\frac{\int x^2\,dx}{b\sqrt{\pi}}\\ &= \frac{x^3}{3b\sqrt{\pi}}-\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\operatorname{erfi}(bx)}{2b^2}+\frac{\int 1\,dx}{b^3\sqrt{\pi}}\\ &= \frac{x}{b^3\sqrt{\pi}}+\frac{x^3}{3b\sqrt{\pi}}-\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\operatorname{erfi}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.026179, size = 51, normalized size = 0.72

$$\frac{\frac{2bx\left(b^2x^2+3\right)}{\sqrt{\pi}}-3e^{-b^2x^2}\left(b^2x^2+1\right)\operatorname{Erfi}(bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] ((2*b*x*(3 + b^2*x^2))/Sqrt[Pi] - (3*(1 + b^2*x^2)*Erfi[b*x])/E^(b^2*x^2))/
(6*b^4)
```

Maple [A] time = 0.115, size = 72, normalized size = 1.

$$\frac{2\,e^{b^2x^2}b^3x^3-3\,\sqrt{\pi}\operatorname{erfi}(bx)\,b^2x^2+6\,e^{b^2x^2}bx-3\,\sqrt{\pi}\operatorname{erfi}(bx)}{6\,\sqrt{\pi}b^4e^{b^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*erfi(b*x)/exp(b^2*x^2),x)
```

```
[Out] 1/6*(2*exp(b^2*x^2)*b^3*x^3-3*Pi^(1/2)*erfi(b*x)*b^2*x^2+6*exp(b^2*x^2)*b*x
-3*Pi^(1/2)*erfi(b*x))/Pi^(1/2)/b^4/exp(b^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx) e^{(-b^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.57583, size = 138, normalized size = 1.94

$$\frac{\left(2\sqrt{\pi}\left(b^3x^3 + 3\,bx\right)e^{\left(b^2x^2\right)} - 3\left(\pi + \pi b^2x^2\right)\operatorname{erfi}(bx)\right)e^{\left(-b^2x^2\right)}}{6\,\pi b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(pi)*(b^3*x^3 + 3*b*x)*e^(b^2*x^2) - 3*(pi + pi*b^2*x^2)*erfi(b*
x))*e^(-b^2*x^2)/(pi*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx) e^{(-b^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfi(b*x)*e^(-b^2*x^2), x)
```

3.272 $\int e^{-b^2x^2}x\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=32

$$\frac{x}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}$$

```
[Out] x/(b*Sqrt[Pi]) - Erfi[b*x]/(2*b^2*E^(b^2*x^2))
```

Rubi [A] time = 0.027782, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6384, 8}

$$\frac{x}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] x/(b*Sqrt[Pi]) - Erfi[b*x]/(2*b^2*E^(b^2*x^2))
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x\text{erfi}(bx)\,dx&=-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{\int 1\,dx}{b\sqrt{\pi}}\\&=\frac{x}{b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0140244, size = 32, normalized size = 1.

$$\frac{x}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] x/(b*Sqrt[Pi]) - Erfi[b*x]/(2*b^2*E^(b^2*x^2))
```

Maple [A] time = 0.063, size = 41, normalized size = 1.3

$$\frac{2\,e^{b^2x^2}bx-\sqrt{\pi}\text{erfi}\,(bx)}{2\,b^2\sqrt{\pi}e^{b^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*erfi(b*x)/exp(b^2*x^2),x)
```

```
[Out] 1/2*(2*exp(b^2*x^2)*b*x-Pi^(1/2)*erfi(b*x))/Pi^(1/2)/b^2/exp(b^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [A] time = 2.43269, size = 96, normalized size = 3.

$$\frac{\left(2\,\sqrt{\pi}bxe^{\left(b^2x^2\right)}-\pi\,\text{erfi}\,(bx)\right)e^{\left(-b^2x^2\right)}}{2\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(pi)*b*x*e^(b^2*x^2) - pi*erfi(b*x))*e^(-b^2*x^2)/(pi*b^2)
```

Sympy [A] time = 32.3085, size = 27, normalized size = 0.84

$$\begin{cases} \frac{x}{\sqrt{\pi}b} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{2b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Piecewise((x/(sqrt(pi)*b) - exp(-b**2*x**2)*erfi(b*x)/(2*b**2), Ne(b, 0)),
(0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfi}(bx) e^{\left(-b^2x^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*erfi(b*x)*e^(-b^2*x^2), x)
```

3.273 $\int \frac{e^{-b^2x^2}\mathbf{Erfi}(bx)}{x}\,dx$

Optimal. Leaf size=30

$$\frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0364644, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6390}

$$\frac{2bx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x),x]
```

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rule 6390

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*
b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0153554, size = 30, normalized size = 1.

$$\frac{2bx\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x),x]
```

```
[Out] (2*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{\mathrm{e}^{b^2x^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x,x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)e^{(-b^2x^2)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x, x)
```

3.274 $\int \frac{e^{-b^2x^2}\mathbf{Erfi}(bx)}{x^3}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^3x\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

```
[Out] -(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2) - (2*b^3*x*Hypergeometric
PFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0748705, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6393, 6390, 30}

$$-\frac{2b^3x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x^3),x]
```

```
[Out] -(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2) - (2*b^3*x*Hypergeometric
PFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6390

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*
b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = -\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{2x^2} - b^2\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x}\,dx + \frac{b\int \frac{1}{x^2}\,dx}{\sqrt{\pi}}$$
$$= -\frac{b}{\sqrt{\pi}x} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{2x^2} - \frac{2b^3x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0146599, size = 32, normalized size = 0.49

$$-\frac{2b\,\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},1\right\},\left\{\frac{1}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^3),x]
```

```
[Out] (-2*b*HypergeometricPFQ[{-1/2, 1}, {1/2, 3/2}, -(b^2*x^2)])/(Sqrt[Pi]*x)
```

Maple [F] time = 0.189, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2x^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x^3,x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

3.275 $\int \frac{e^{-b^2x^2}\mathbf{Erfi}(bx)}{x^5}\,dx$

Optimal. Leaf size=105

$$\frac{b^5x\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-b^2x^2\right)}{\sqrt{\pi}}+\frac{b^2e^{-b^2x^2}\text{Erfi}(bx)}{4x^2}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{4x^4}+\frac{b^3}{2\sqrt{\pi}x}-\frac{b}{6\sqrt{\pi}x^3}$$

```
[Out] -b/(6*Sqrt[Pi]*x^3) + b^3/(2*Sqrt[Pi]*x) - Erfi[b*x]/(4*E^(b^2*x^2)*x^4) +
(b^2*Erfi[b*x])/(4*E^(b^2*x^2)*x^2) + (b^5*x*HypergeometricPFQ[{1/2, 1}, {3
/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.113068, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6393, 6390, 30}

$$\frac{b^5x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}+\frac{b^2e^{-b^2x^2}\text{Erfi}(bx)}{4x^2}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{4x^4}+\frac{b^3}{2\sqrt{\pi}x}-\frac{b}{6\sqrt{\pi}x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x^5),x]
```

```
[Out] -b/(6*Sqrt[Pi]*x^3) + b^3/(2*Sqrt[Pi]*x) - Erfi[b*x]/(4*E^(b^2*x^2)*x^4) +
(b^2*Erfi[b*x])/(4*E^(b^2*x^2)*x^2) + (b^5*x*HypergeometricPFQ[{1/2, 1}, {3
/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6390

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*
b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = -\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^4} - \frac{1}{2}b^2\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx + \frac{b\int \frac{1}{x^4}\,dx}{2\sqrt{\pi}}$$
$$= -\frac{b}{6\sqrt{\pi}x^3} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x}\,dx - \frac{b^3\int \frac{1}{x^2}\,dx}{2\sqrt{\pi}}$$
$$= -\frac{b}{6\sqrt{\pi}x^3} + \frac{b^3}{2\sqrt{\pi}x} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^2} + \frac{b^5x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0164657, size = 34, normalized size = 0.32

$$-\frac{2b\text{HypergeometricPFQ}\left(\left\{-\frac{3}{2},1\right\},\left\{-\frac{1}{2},\frac{3}{2}\right\},-b^2x^2\right)}{3\sqrt{\pi}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^5),x]
```

```
[Out] (-2*b*HypergeometricPFQ[{-3/2, 1}, {-1/2, 3/2}, -(b^2*x^2)])/(3*Sqrt[Pi]*x^
3)
```

Maple [F] time = 0.639, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2x^2}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x^5,x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2)/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx) e^{(-b^2x^2)}}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx) e^{(-b^2x^2)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx) e^{(-b^2x^2)}}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^5, x)
```

3.276 $\int e^{-b^2x^2}x^6\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=148

$$\frac{15x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{8\sqrt{\pi}b^5}-\frac{x^5e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{5x^3e^{-b^2x^2}\text{Erfi}(bx)}{4b^4}-\frac{15xe^{-b^2x^2}\text{Erfi}(bx)}{8b^6}+\frac{5x^4}{8\sqrt{\pi}b^3}$$

```
[Out] (15*x^2)/(8*b^5*Sqrt[Pi]) + (5*x^4)/(8*b^3*Sqrt[Pi]) + x^6/(6*b*Sqrt[Pi]) -
 (15*x*Erfi[b*x])/(8*b^6*E^(b^2*x^2)) - (5*x^3*Erfi[b*x])/(4*b^4*E^(b^2*x^2
)) - (x^5*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (15*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(b^2*x^2)])/(8*b^5*Sqrt[Pi])
```

Rubi [A] time = 0.141605, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6387, 6378, 30}

$$\frac{15x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{8\sqrt{\pi}b^5}-\frac{x^5e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{5x^3e^{-b^2x^2}\text{Erfi}(bx)}{4b^4}-\frac{15xe^{-b^2x^2}\text{Erfi}(bx)}{8b^6}+\frac{5x^4}{8\sqrt{\pi}b^3}+\frac{15x^2}{8\sqrt{\pi}b^5}+\frac{x^6}{6\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] (15*x^2)/(8*b^5*Sqrt[Pi]) + (5*x^4)/(8*b^3*Sqrt[Pi]) + x^6/(6*b*Sqrt[Pi]) -
 (15*x*Erfi[b*x])/(8*b^6*E^(b^2*x^2)) - (5*x^3*Erfi[b*x])/(4*b^4*E^(b^2*x^2
)) - (x^5*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (15*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(b^2*x^2)])/(8*b^5*Sqrt[Pi])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
```

```
c, d}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx &= -\frac{e^{-b^2x^2}x^5\text{erfi}(bx)}{2b^2}+\frac{5\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx}{2b^2}+\frac{\int x^5\,dx}{b\sqrt{\pi}}\\
&= \frac{x^6}{6b\sqrt{\pi}}-\frac{5e^{-b^2x^2}x^3\text{erfi}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^5\text{erfi}(bx)}{2b^2}+\frac{15\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx}{4b^4}+\frac{5\int x^3\,dx}{2b^3\sqrt{\pi}}\\
&= \frac{5x^4}{8b^3\sqrt{\pi}}+\frac{x^6}{6b\sqrt{\pi}}-\frac{15e^{-b^2x^2}x\text{erfi}(bx)}{8b^6}-\frac{5e^{-b^2x^2}x^3\text{erfi}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^5\text{erfi}(bx)}{2b^2}+\frac{15\int e^{-b^2x^2}\text{erfi}(b\cdots}{8b^6}\\
&= \frac{15x^2}{8b^5\sqrt{\pi}}+\frac{5x^4}{8b^3\sqrt{\pi}}+\frac{x^6}{6b\sqrt{\pi}}-\frac{15e^{-b^2x^2}x\text{erfi}(bx)}{8b^6}-\frac{5e^{-b^2x^2}x^3\text{erfi}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^5\text{erfi}(bx)}{2b^2}+\frac{15x^2\,{}_2\cdots}{}
\end{aligned}$$

Mathematica [A] time = 0.0234395, size = 52, normalized size = 0.35

$$\frac{x^2\left(-9\text{HypergeometricPFQ}\left(\{1,1\},\left\{-\frac{3}{2},2\right\},-b^2x^2\right)+4b^4x^4+3b^2x^2+9\right)}{24\sqrt{\pi}b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] (x^2*(9 + 3*b^2*x^2 + 4*b^4*x^4 - 9*HypergeometricPFQ[{1, 1}, {-3/2, 2}, -(
b^2*x^2)]))/(24*b^5*Sqrt[Pi])
```

Maple [F] time = 0.43, size = 0, normalized size = 0.

$$\int \frac{x^6\text{erfi}\,(bx)}{e^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*erfi(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(x^6*erfi(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \operatorname{erfi}(bx) e^{\left(-b^2x^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^6 \operatorname{erfi}(bx) e^{\left(-b^2x^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^6*erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \operatorname{erfi}(bx) e^{\left(-b^2x^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^6*erfi(b*x)*e^(-b^2*x^2), x)
```

3.277 $\int e^{-b^2x^2}x^4\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=109

$$\frac{3x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{4\sqrt{\pi}b^3}-\frac{x^3e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{3xe^{-b^2x^2}\text{Erfi}(bx)}{4b^4}+\frac{3x^2}{4\sqrt{\pi}b^3}+\frac{x^4}{4\sqrt{\pi}b}$$

```
[Out] (3*x^2)/(4*b^3*Sqrt[Pi]) + x^4/(4*b*Sqrt[Pi]) - (3*x*Erfi[b*x])/(4*b^4*E^(b
^2*x^2)) - (x^3*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (3*x^2*HypergeometricPFQ[{
1, 1}, {3/2, 2}, -(b^2*x^2)])/(4*b^3*Sqrt[Pi])
```

Rubi [A] time = 0.0987593, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6387, 6378, 30}

$$\frac{3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{4\sqrt{\pi}b^3}-\frac{x^3e^{-b^2x^2}\text{Erfi}(bx)}{2b^2}-\frac{3xe^{-b^2x^2}\text{Erfi}(bx)}{4b^4}+\frac{3x^2}{4\sqrt{\pi}b^3}+\frac{x^4}{4\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] (3*x^2)/(4*b^3*Sqrt[Pi]) + x^4/(4*b*Sqrt[Pi]) - (3*x*Erfi[b*x])/(4*b^4*E^(b
^2*x^2)) - (x^3*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (3*x^2*HypergeometricPFQ[{
1, 1}, {3/2, 2}, -(b^2*x^2)])/(4*b^3*Sqrt[Pi])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-b^2x^2}x^4\operatorname{erfi}(bx)\,dx &= -\frac{e^{-b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}+\frac{3\int e^{-b^2x^2}x^2\operatorname{erfi}(bx)\,dx}{2b^2}+\frac{\int x^3\,dx}{b\sqrt{\pi}}\\ &= \frac{x^4}{4b\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\operatorname{erfi}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}+\frac{3\int e^{-b^2x^2}\operatorname{erfi}(bx)\,dx}{4b^4}+\frac{3\int x\,dx}{2b^3\sqrt{\pi}}\\ &= \frac{3x^2}{4b^3\sqrt{\pi}}+\frac{x^4}{4b\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\operatorname{erfi}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}+\frac{3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{4b^3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0196826, size = 43, normalized size = 0.39

$$\frac{x^2\left(-\text{HypergeometricPFQ}\left(\{1,1\},\left\{-\frac{1}{2},2\right\},-b^2x^2\right)+b^2x^2+1\right)}{4\sqrt{\pi}b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] (x^2*(1 + b^2*x^2 - HypergeometricPFQ[{1, 1}, {-1/2, 2}, -(b^2*x^2)]))/(4*b
^3*Sqrt[Pi])
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{x^4\operatorname{erfi}(bx)}{e^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*erfi(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(x^4*erfi(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx) e^{(-b^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \operatorname{erfi}(bx) e^{(-b^2x^2)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^4*erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx) e^{(-b^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfi(b*x)*e^(-b^2*x^2), x)
```

3.278 $\int e^{-b^2x^2}x^2\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=70

$$\frac{x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{Erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}$$

```
[Out] x^2/(2*b*Sqrt[Pi]) - (x*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/(2*b*Sqrt[Pi])
```

Rubi [A] time = 0.0563927, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6387, 6378, 30}

$$\frac{x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{Erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Erfi[b*x])/E^(b^2*x^2),x]
```

```
[Out] x^2/(2*b*Sqrt[Pi]) - (x*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/(2*b*Sqrt[Pi])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int e^{-b^2x^2}x^2\operatorname{erfi}(bx)\,dx = -\frac{e^{-b^2x^2}x\operatorname{erfi}(bx)}{2b^2} + \frac{\int e^{-b^2x^2}\operatorname{erfi}(bx)\,dx}{2b^2} + \frac{\int x\,dx}{b\sqrt{\pi}}$$
$$= \frac{x^2}{2b\sqrt{\pi}} - \frac{e^{-b^2x^2}x\operatorname{erfi}(bx)}{2b^2} + \frac{x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2b\sqrt{\pi}}$$

Mathematica [A] time = 0.0140755, size = 36, normalized size = 0.51

$$\frac{x^2\left(1-\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{1}{2},2\right\},-b^2x^2\right)\right)}{2\sqrt{\pi}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Erfi[b*x])/E^(b^2*x^2),x]

[Out] (x^2*(1 - HypergeometricPFQ[{1, 1}, {1/2, 2}, -(b^2*x^2)]))/(2*b*Sqrt[Pi])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{x^2\operatorname{erfi}(bx)}{e^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*erfi(b*x)/exp(b^2*x^2),x)

[Out] int(x^2*erfi(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \operatorname{erfi}(bx)\, e^{\left(-b^2x^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)\, e^{\left(-b^2x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi(b*x)*e^(-b^2*x^2), x)
```

3.279 $\int e^{-b^2x^2}\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=27

$$\frac{bx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

```
[Out] (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0160768, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6378}

$$\frac{bx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/E^(b^2*x^2),x]
```

```
[Out] (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx=\frac{bx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.0084946, size = 27, normalized size = 1.

$$\frac{bx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/E^(b^2*x^2),x]
```

```
[Out] (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{\mathrm{e}^{b^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2),x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)e^{\left(-b^2x^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2), x)
```

3.280 $\int \frac{e^{-b^2x^2}\mathbf{Erfi}(bx)}{x^2}\,dx$

Optimal. Leaf size=60

$$-\frac{2b^3x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}$$

```
[Out] -(Erfi[b*x]/(E^(b^2*x^2)*x)) - (2*b^3*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, -(b^2*x^2)])/Sqrt[Pi] + (2*b*Log[x])/Sqrt[Pi]
```

Rubi [A] time = 0.0563698, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6393, 6378, 29}

$$-\frac{2b^3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x^2),x]
```

```
[Out] -(Erfi[b*x]/(E^(b^2*x^2)*x)) - (2*b^3*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, -(b^2*x^2)])/Sqrt[Pi] + (2*b*Log[x])/Sqrt[Pi]
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}\,dx &= -\frac{e^{-b^2x^2}\text{erfi}(bx)}{x} - \left(2b^2\right)\int e^{-b^2x^2}\text{erfi}(bx)\,dx + \frac{(2b)\int \frac{1}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{-b^2x^2}\text{erfi}(bx)}{x} - \frac{2b^3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}} + \frac{2b\log(x)}{\sqrt{\pi}}\end{aligned}$$

Mathematica [C] time = 0.0163195, size = 26, normalized size = 0.43

$$-\frac{1}{2}bG_{2,3}^{2,1}\left(b^2x^2|\begin{array}{c}0,1\\0,0,-\frac{1}{2}\end{array}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^2),x]
```

```
[Out] -(b*MeijerG[{{0}, {1}}, {{0, 0}, {-1/2}}, b^2*x^2])/2
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)}{e^{b^2x^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x^2,x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2)/x^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^2, x)
```

3.281 $\int \frac{e^{-b^2x^2}\textbf{Erfi}(bx)}{x^4}\,dx$

Optimal. Leaf size=105

$$\frac{4b^5x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{3\sqrt{\pi}}+\frac{2b^2e^{-b^2x^2}\text{Erfi}(bx)}{3x}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{3x^3}-\frac{4b^3\log(x)}{3\sqrt{\pi}}-\frac{b}{3\sqrt{\pi}x^2}$$

```
[Out] -b/(3*Sqrt[Pi]*x^2) - Erfi[b*x]/(3*E^(b^2*x^2)*x^3) + (2*b^2*Erfi[b*x])/(3*
E^(b^2*x^2)*x) + (4*b^5*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]
)/(3*Sqrt[Pi]) - (4*b^3*Log[x])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.0976992, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6393, 6378, 29, 30}

$$\frac{4b^5x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{3\sqrt{\pi}}+\frac{2b^2e^{-b^2x^2}\text{Erfi}(bx)}{3x}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{3x^3}-\frac{4b^3\log(x)}{3\sqrt{\pi}}-\frac{b}{3\sqrt{\pi}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x^4),x]
```

```
[Out] -b/(3*Sqrt[Pi]*x^2) - Erfi[b*x]/(3*E^(b^2*x^2)*x^3) + (2*b^2*Erfi[b*x])/(3*
E^(b^2*x^2)*x) + (4*b^5*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]
)/(3*Sqrt[Pi]) - (4*b^3*Log[x])/(3*Sqrt[Pi])
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^4}\,dx &= -\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{3x^3}-\frac{1}{3}\left(2b^2\right)\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx+\frac{(2b)\int \frac{1}{x^3}\,dx}{3\sqrt{\pi}}\\ &= -\frac{b}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{-b^2x^2}\operatorname{erfi}(bx)\,dx-\frac{\left(4b^3\right)\int \frac{1}{x}\,dx}{3\sqrt{\pi}}\\ &= -\frac{b}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{3x}+\frac{4b^5x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{3\sqrt{\pi}}-\frac{4b^3\log(x)}{3\sqrt{\pi}}\end{aligned}$$

Mathematica [C] time = 0.016869, size = 29, normalized size = 0.28

$$-\frac{bG_{2,3}^{2,1}\left(b^2x^2|\begin{array}{c}0,2\\0,1,-\frac{1}{2}\end{array}\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^4),x]
```

```
[Out] -(b*MeijerG[{{0}, {2}}, {{0, 1}, {-1/2}}, b^2*x^2])/(2*x^2)
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2x^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x^4,x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\, e^{\left(-b^2x^2\right)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erfi}(bx)\, e^{\left(-b^2x^2\right)}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\, e^{\left(-b^2x^2\right)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^4, x)
```

3.282 $\int \frac{e^{-b^2x^2}\mathbf{Erfi}(bx)}{x^6}\,dx$

Optimal. Leaf size=144

$$-\frac{8b^7x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{15\sqrt{\pi}}-\frac{4b^4e^{-b^2x^2}\text{Erfi}(bx)}{15x}+\frac{2b^2e^{-b^2x^2}\text{Erfi}(bx)}{15x^3}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{5x^5}+\frac{2b^3}{15\sqrt{\pi}}$$

```
[Out] -b/(10*Sqrt[Pi]*x^4) + (2*b^3)/(15*Sqrt[Pi]*x^2) - Erfi[b*x]/(5*E^(b^2*x^2)
*x^5) + (2*b^2*Erfi[b*x])/(15*E^(b^2*x^2)*x^3) - (4*b^4*Erfi[b*x])/(15*E^(b
^2*x^2)*x) - (8*b^7*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/(1
5*Sqrt[Pi]) + (8*b^5*Log[x])/(15*Sqrt[Pi])
```

Rubi [A] time = 0.136732, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6393, 6378, 29, 30}

$$-\frac{8b^7x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{15\sqrt{\pi}}-\frac{4b^4e^{-b^2x^2}\text{Erfi}(bx)}{15x}+\frac{2b^2e^{-b^2x^2}\text{Erfi}(bx)}{15x^3}-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{5x^5}+\frac{2b^3}{15\sqrt{\pi}x^2}+\frac{8b^5\log(x)}{15\sqrt{\pi}}-\frac{b}{10\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x^6),x]
```

```
[Out] -b/(10*Sqrt[Pi]*x^4) + (2*b^3)/(15*Sqrt[Pi]*x^2) - Erfi[b*x]/(5*E^(b^2*x^2)
*x^5) + (2*b^2*Erfi[b*x])/(15*E^(b^2*x^2)*x^3) - (4*b^4*Erfi[b*x])/(15*E^(b
^2*x^2)*x) - (8*b^7*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/(1
5*Sqrt[Pi]) + (8*b^5*Log[x])/(15*Sqrt[Pi])
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
```

```
c, d}, x] && EqQ[d, -b^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^6}\,dx = -\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{5x^5} - \frac{1}{5}\left(2b^2\right)\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^4}\,dx + \frac{(2b)\int \frac{1}{x^5}\,dx}{5\sqrt{\pi}}$$

$$= -\frac{b}{10\sqrt{\pi}x^4} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{5x^5} + \frac{2b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{15x^3} + \frac{1}{15}\left(4b^4\right)\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx - \frac{\left(4b^3\right)\int \frac{1}{x^3}\,dx}{15\sqrt{\pi}}$$

$$= -\frac{b}{10\sqrt{\pi}x^4} + \frac{2b^3}{15\sqrt{\pi}x^2} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{5x^5} + \frac{2b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{15x^3} - \frac{4b^4e^{-b^2x^2}\operatorname{erfi}(bx)}{15x} - \frac{1}{15}\left(8b^6\right)\int e^{-b^2x^2}$$

$$= -\frac{b}{10\sqrt{\pi}x^4} + \frac{2b^3}{15\sqrt{\pi}x^2} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{5x^5} + \frac{2b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{15x^3} - \frac{4b^4e^{-b^2x^2}\operatorname{erfi}(bx)}{15x} - \frac{8b^7x^2\,{}_2F_2\left(1,1;\frac{3}{2},\right.}{15\sqrt{\pi}}$$

Mathematica [C] time = 0.0182751, size = 29, normalized size = 0.2

$$-\frac{bG_{2,3}^{2,1}\left(b^2x^2\left|\begin{array}{c}0,3\\0,2,-\frac{1}{2}\end{array}\right.\right)}{2x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^6),x]
```

```
[Out] -(b*MeijerG[{{0}, {3}}, {{0, 2}, {-1/2}}, b^2*x^2])/(2*x^4)
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2x^2}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x^6,x)
```

```
[Out] int(erfi(b*x)/exp(b^2*x^2)/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)e^{\left(-b^2x^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{erfi}(bx)e^{\left(-b^2x^2\right)}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(-b^2*x^2)/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(-b^2*x^2)/x^6, x)
```

3.283 $\int e^{c+b^2x^2}x^5\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=144

$$\frac{x^4e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{x^2e^{b^2x^2+c}\text{Erfi}(bx)}{b^4}+\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{b^6}-\frac{43e^c\text{Erfi}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{x^3e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}+\frac{11xe^{2b^2x^2+c}}{16\sqrt{\pi}b^5}$$

```
[Out] (11*E^(c + 2*b^2*x^2)*x)/(16*b^5*Sqrt[Pi]) - (E^(c + 2*b^2*x^2)*x^3)/(4*b^3
*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfi[b*x])/b^6 - (E^(c + b^2*x^2)*x^2*Erfi[b*
x])/b^4 + (E^(c + b^2*x^2)*x^4*Erfi[b*x])/(2*b^2) - (43*E^c*Erfi[Sqrt[2]*b*
x])/(32*Sqrt[2]*b^6)
```

Rubi [A] time = 0.228873, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6387, 6384, 2204, 2212}

$$\frac{x^4e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{x^2e^{b^2x^2+c}\text{Erfi}(bx)}{b^4}+\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{b^6}-\frac{43e^c\text{Erfi}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{x^3e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}+\frac{11xe^{2b^2x^2+c}}{16\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^5*Erfi[b*x],x]
```

```
[Out] (11*E^(c + 2*b^2*x^2)*x)/(16*b^5*Sqrt[Pi]) - (E^(c + 2*b^2*x^2)*x^3)/(4*b^3
*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfi[b*x])/b^6 - (E^(c + b^2*x^2)*x^2*Erfi[b*
x])/b^4 + (E^(c + b^2*x^2)*x^4*Erfi[b*x])/(2*b^2) - (43*E^c*Erfi[Sqrt[2]*b*
x])/(32*Sqrt[2]*b^6)
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
```

```
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx &= \frac{e^{c+b^2x^2}x^4\text{erfi}(bx)}{2b^2}-\frac{2\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx}{b^2}-\frac{\int e^{c+2b^2x^2}x^4\,dx}{b\sqrt{\pi}}\\ &=-\frac{e^{c+2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfi}(bx)}{2b^2}+\frac{2\int e^{c+b^2x^2}x\text{erfi}(bx)\,dx}{b^4}+\frac{3\int e^{c+2b^2x^2}x^2\,dx}{4b^3\sqrt{\pi}}\\ &=\frac{11e^{c+2b^2x^2}x}{16b^5\sqrt{\pi}}-\frac{e^{c+2b^2x^2}x^3}{4b^3\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfi}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfi}(bx)}{2b^2}-\frac{3\int e^{c+2b^2x^2}}{16b^5\sqrt{\pi}}\\ &=\frac{11e^{c+2b^2x^2}x}{16b^5\sqrt{\pi}}-\frac{e^{c+2b^2x^2}x^3}{4b^3\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfi}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfi}(bx)}{2b^2}-\frac{43e^c\text{erfi}\left(\sqrt{2}\right.}{32\sqrt{2}b^6}\end{aligned}$$

Mathematica [A] time = 0.0630072, size = 95, normalized size = 0.66

$$\frac{e^c\left(32\sqrt{\pi}e^{b^2x^2}\left(b^4x^4-2b^2x^2+2\right)\text{Erfi}(bx)-4bxe^{2b^2x^2}\left(4b^2x^2-11\right)-43\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}bx\right)\right)}{64\sqrt{\pi}b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^5*Erfi[b*x],x]
```

```
[Out] (E^c*(-4*b*E^(2*b^2*x^2)*x*(-11 + 4*b^2*x^2) + 32*E^(b^2*x^2)*Sqrt[Pi]*(2 -
 2*b^2*x^2 + b^4*x^4)*Erfi[b*x] - 43*Sqrt[2*Pi]*Erfi[Sqrt[2]*b*x]))/(64*b^6
```

```
*Sqrt[Pi])
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x^5\mathrm{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^5*erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^5*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\,\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.70616, size = 262, normalized size = 1.82

$$-\frac{43\,\sqrt{2}\pi\sqrt{b^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{b^2}x\right)e^c-32\left(\pi b^5x^4-2\,\pi b^3x^2+2\,\pi b\right)\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}+4\,\sqrt{\pi}\left(4\,b^4x^3-11\,b^2x\right)e^{\left(2\,b^2x^2+c\right)}}{64\,\pi b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erfi(b*x),x, algorithm="fricas")
```

```
[Out] -1/64*(43*sqrt(2)*pi*sqrt(b^2)*erfi(sqrt(2)*sqrt(b^2)*x)*e^c - 32*(pi*b^5*x
^4 - 2*pi*b^3*x^2 + 2*pi*b)*erfi(b*x)*e^(b^2*x^2 + c) + 4*sqrt(pi)*(4*b^4*x
^3 - 11*b^2*x)*e^(2*b^2*x^2 + c))/(pi*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**5*erfi(b*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{erfi}(bx) e^{(b^2x^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^5*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.284 $\int e^{c+b^2x^2}x^3\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=97

$$\frac{x^2e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{2b^4}+\frac{5e^c\text{Erfi}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{xe^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

```
[Out] -(E^(c + 2*b^2*x^2)*x)/(4*b^3*Sqrt[Pi]) - (E^(c + b^2*x^2)*Erfi[b*x])/(2*b^
4) + (E^(c + b^2*x^2)*x^2*Erfi[b*x])/(2*b^2) + (5*E^c*Erfi[Sqrt[2]*b*x])/(8
*Sqrt[2]*b^4)
```

Rubi [A] time = 0.114576, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6387, 6384, 2204, 2212}

$$\frac{x^2e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{2b^4}+\frac{5e^c\text{Erfi}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{xe^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^3*Erfi[b*x],x]
```

```
[Out] -(E^(c + 2*b^2*x^2)*x)/(4*b^3*Sqrt[Pi]) - (E^(c + b^2*x^2)*Erfi[b*x])/(2*b^
4) + (E^(c + b^2*x^2)*x^2*Erfi[b*x])/(2*b^2) + (5*E^c*Erfi[Sqrt[2]*b*x])/(8
*Sqrt[2]*b^4)
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx &= \frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}x\text{erfi}(bx)\,dx}{b^2}-\frac{\int e^{c+2b^2x^2}x^2\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{c+2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{2b^2}+\frac{\int e^{c+2b^2x^2}\,dx}{4b^3\sqrt{\pi}}+\frac{\int e^{c+2b^2x^2}\,dx}{b^3\sqrt{\pi}}\\ &= -\frac{e^{c+2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{2b^2}+\frac{5e^c\text{erfi}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}\end{aligned}$$

Mathematica [A] time = 0.0379454, size = 77, normalized size = 0.79

$$\frac{e^c\left(8\sqrt{\pi}e^{b^2x^2}\left(b^2x^2-1\right)\text{Erfi}(bx)-4bxe^{2b^2x^2}+5\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}bx\right)\right)}{16\sqrt{\pi}b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^3*Erfi[b*x],x]
```

```
[Out] (E^c*(-4*b*E^(2*b^2*x^2)*x + 8*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erfi[b*x
] + 5*Sqrt[2*Pi]*Erfi[Sqrt[2]*b*x]))/(16*b^4*Sqrt[Pi])
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x^3\text{erfi}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^3*erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^3*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx) e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.66692, size = 213, normalized size = 2.2

$$-\frac{4\sqrt{\pi}b^2xe^{\left(2\,b^2x^2+c\right)} - 5\sqrt{2}\pi\sqrt{b^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{b^2}x\right)e^c - 8\left(\pi b^3x^2 - \pi b\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2+c\right)}}{16\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erfi(b*x),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(pi)*b^2*x*e^(2*b^2*x^2 + c) - 5*sqrt(2)*pi*sqrt(b^2)*erfi(sqr
t(2)*sqrt(b^2)*x)*e^c - 8*(pi*b^3*x^2 - pi*b)*erfi(b*x)*e^(b^2*x^2 + c))/(p
i*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c\int x^3e^{b^2x^2}\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**3*erfi(b*x),x)
```

```
[Out] exp(c)*Integral(x**3*exp(b**2*x**2)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx) e^{(b^2x^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^3*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.285 $\int e^{c+b^2x^2} x \textbf{Erfi}(bx)\, dx$

Optimal. Leaf size=47

$$\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2} - \frac{e^c\text{Erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

```
[Out] (E^(c + b^2*x^2)*Erfi[b*x])/(2*b^2) - (E^c*Erfi[Sqrt[2]*b*x])/(2*Sqrt[2]*b^
2)
```

Rubi [A] time = 0.0380216, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6384, 2204}

$$\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2} - \frac{e^c\text{Erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x*Erfi[b*x],x]
```

```
[Out] (E^(c + b^2*x^2)*Erfi[b*x])/(2*b^2) - (E^c*Erfi[Sqrt[2]*b*x])/(2*Sqrt[2]*b^
2)
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{c+b^2x^2}x\text{erfi}(bx)\,dx = \frac{e^{c+b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\int e^{c+2b^2x^2}\,dx}{b\sqrt{\pi}}$$
$$= \frac{e^{c+b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{e^c\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

Mathematica [A] time = 0.009964, size = 42, normalized size = 0.89

$$\frac{e^c\left(2e^{b^2x^2}\text{Erfi}(bx) - \sqrt{2}\text{Erfi}\left(\sqrt{2}bx\right)\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x*Erfi[b*x],x]

[Out] (E^c*(2*E^(b^2*x^2)*Erfi[b*x] - Sqrt[2]*Erfi[Sqrt[2]*b*x]))/(4*b^2)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x\text{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x*erfi(b*x),x)

[Out] int(exp(b^2*x^2+c)*x*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.69679, size = 123, normalized size = 2.62

$$\frac{2\,b\,\mathrm{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)}-\sqrt{2}\sqrt{b^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{b^2}x\right)e^{c}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*erfi(b*x)*e^(b^2*x^2 + c) - sqrt(2)*sqrt(b^2)*erfi(sqrt(2)*sqrt(b^
2)*x)*e^c)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int xe^{b^2x^2}\,\mathrm{erfi}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x*erfi(b*x),x)
```

```
[Out] exp(c)*Integral(x*exp(b**2*x**2)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.286 $\int \frac{e^{c+b^2x^2}\mathbf{Erfi}(bx)}{x}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + b^2*x^2)*Erfi[b*x])/x, x]
```

Rubi [A] time = 0.039339, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+b^2x^2}\text{Erfi}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + b^2*x^2)*Erfi[b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + b^2*x^2)*Erfi[b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx=\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx$$

Mathematica [A] time = 0.0928287, size = 0, normalized size = 0.

$$\int \frac{e^{c+b^2x^2}\text{Erfi}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x,x]
```

```
[Out] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x, x]
```

Maple [A] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\mathrm{erfi}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(b^2*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int \frac{e^{b^2x^2}\,\mathrm{erfi}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)/x,x)
```

```
[Out] exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\, e^{\left(b^2x^2+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x, x)
```

3.287 $\int \frac{e^{c+b^2x^2}\textbf{Erfi}(bx)}{x^3}\,dx$

Optimal. Leaf size=92

$$b^2\text{Unintegrable}\left(\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{x},x\right)-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{2x^2}+\sqrt{2}b^2e^c\text{Erfi}\left(\sqrt{2}bx\right)-\frac{be^{2b^2x^2+c}}{\sqrt{\pi}x}$$

```
[Out] -((b*E^(c + 2*b^2*x^2))/(Sqrt[Pi]*x)) - (E^(c + b^2*x^2)*Erfi[b*x])/(2*x^2)
 + Sqrt[2]*b^2*E^c*Erfi[Sqrt[2]*b*x] + b^2*Unintegrable[(E^(c + b^2*x^2)*Er
fi[b*x])/x, x]
```

Rubi [A] time = 0.111064, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+b^2x^2}\text{Erfi}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^3,x]
```

```
[Out] -((b*E^(c + 2*b^2*x^2))/(Sqrt[Pi]*x)) - (E^(c + b^2*x^2)*Erfi[b*x])/(2*x^2)
 + Sqrt[2]*b^2*E^c*Erfi[Sqrt[2]*b*x] + b^2*Defer[Int][(E^(c + b^2*x^2)*Erfi
[b*x])/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^3}\,dx &= -\frac{e^{c+b^2x^2}\text{erfi}(bx)}{2x^2}+b^2\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx+\frac{b\int \frac{e^{c+2b^2x^2}}{x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{be^{c+2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{2x^2}+b^2\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx+\frac{\left(4b^3\right)\int e^{c+2b^2x^2}\,dx}{\sqrt{\pi}}\\ &= -\frac{be^{c+2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{2x^2}+\sqrt{2}b^2e^c\text{erfi}\left(\sqrt{2}bx\right)+b^2\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 0.146748, size = 0, normalized size = 0.

$$\int \frac{e^{c+b^2x^2}\mathrm{Erfi}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^3, x]
```

Maple [A] time = 0.263, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\mathrm{erfi}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)/x^3,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int\frac{e^{b^{2}x^{2}}\operatorname{erfi}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)/x**3,x)
```

```
[Out] exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{erfi}\left(bx\right)e^{\left(b^{2}x^{2}+c\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

3.288 $\int \frac{e^{c+b^2x^2}\mathbf{Erfi}(bx)}{x^5}\,dx$

Optimal. Leaf size=173

$$\frac{1}{2}b^4\text{Unintegrable}\left(\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{x},x\right)-\frac{b^2e^{b^2x^2+c}\text{Erfi}(bx)}{4x^2}-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{4x^4}+\frac{2}{3}\sqrt{2}b^4e^c\text{Erfi}\left(\sqrt{2}bx\right)+\frac{b^4e^c\text{Erfi}\left(\sqrt{2}bx\right)}{\sqrt{2}}-$$

```
[Out] -(b*E^(c + 2*b^2*x^2))/(6*Sqrt[Pi]*x^3) - (7*b^3*E^(c + 2*b^2*x^2))/(6*Sqrt
[Pi]*x) - (E^(c + b^2*x^2)*Erfi[b*x])/(4*x^4) - (b^2*E^(c + b^2*x^2)*Erfi[b
*x])/(4*x^2) + (b^4*E^c*Erfi[Sqrt[2]*b*x])/Sqrt[2] + (2*Sqrt[2]*b^4*E^c*Erf
i[Sqrt[2]*b*x])/3 + (b^4*Unintegrable[(E^(c + b^2*x^2)*Erfi[b*x])/x, x])/2
```

Rubi [A] time = 0.218594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+b^2x^2}\text{Erfi}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^5,x]
```

```
[Out] -(b*E^(c + 2*b^2*x^2))/(6*Sqrt[Pi]*x^3) - (7*b^3*E^(c + 2*b^2*x^2))/(6*Sqrt
[Pi]*x) - (E^(c + b^2*x^2)*Erfi[b*x])/(4*x^4) - (b^2*E^(c + b^2*x^2)*Erfi[b
*x])/(4*x^2) + (b^4*E^c*Erfi[Sqrt[2]*b*x])/Sqrt[2] + (2*Sqrt[2]*b^4*E^c*Erf
i[Sqrt[2]*b*x])/3 + (b^4*Defer[Int][(E^(c + b^2*x^2)*Erfi[b*x])/x, x])/2
```

Rubi steps

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^5}\,dx = -\frac{e^{c+b^2x^2}\text{erfi}(bx)}{4x^4} + \frac{1}{2}b^2\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^3}\,dx + \frac{b\int \frac{e^{c+2b^2x^2}}{x^4}\,dx}{2\sqrt{\pi}}$$

$$= -\frac{be^{c+2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{e^{c+b^2x^2}\text{erfi}(bx)}{4x^4} - \frac{b^2e^{c+b^2x^2}\text{erfi}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx + \frac{b^3\int \frac{e^{c+2b^2x^2}}{x^2}\,dx}{2\sqrt{\pi}} +$$

$$= -\frac{be^{c+2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{7b^3e^{c+2b^2x^2}}{6\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\text{erfi}(bx)}{4x^4} - \frac{b^2e^{c+b^2x^2}\text{erfi}(bx)}{4x^2} + \frac{1}{2}b^4\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx + \frac{(2b^5}{}$$

$$= -\frac{be^{c+2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{7b^3e^{c+2b^2x^2}}{6\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\text{erfi}(bx)}{4x^4} - \frac{b^2e^{c+b^2x^2}\text{erfi}(bx)}{4x^2} + \frac{b^4e^c\text{erfi}\left(\sqrt{2}bx\right)}{\sqrt{2}} + \frac{2}{3}\sqrt{2}b^4e^c\text{erfi}$$

Mathematica [A] time = 0.192628, size = 0, normalized size = 0.

$$\int \frac{e^{c+b^2x^2}\text{Erfi}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^5,x]
```

```
[Out] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^5, x]
```

Maple [A] time = 0.283, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}\text{erfi}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)/x^5,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)/x^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^c\int\frac{e^{b^2x^2}\,\text{erfi}\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)/x**5,x)
```

```
[Out] exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**5, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

3.289 $\int e^{c+b^2x^2} x^4 \textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=121

$$\frac{x^3 e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{3xe^{b^2x^2+c}\text{Erfi}(bx)}{4b^4}+\frac{3\sqrt{\pi}e^c\text{Erfi}(bx)^2}{16b^5}-\frac{x^2e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}+\frac{e^{2b^2x^2+c}}{2\sqrt{\pi}b^5}$$

```
[Out] E^(c + 2*b^2*x^2)/(2*b^5*Sqrt[Pi]) - (E^(c + 2*b^2*x^2)*x^2)/(4*b^3*Sqrt[Pi
]) - (3*E^(c + b^2*x^2)*x*Erfi[b*x])/(4*b^4) + (E^(c + b^2*x^2)*x^3*Erfi[b*
x])/(2*b^2) + (3*E^c*Sqrt[Pi]*Erfi[b*x]^2)/(16*b^5)
```

Rubi [A] time = 0.162773, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6387, 6375, 30, 2209, 2212}

$$\frac{x^3 e^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{3xe^{b^2x^2+c}\text{Erfi}(bx)}{4b^4}+\frac{3\sqrt{\pi}e^c\text{Erfi}(bx)^2}{16b^5}-\frac{x^2e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}+\frac{e^{2b^2x^2+c}}{2\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^4*Erfi[b*x],x]
```

```
[Out] E^(c + 2*b^2*x^2)/(2*b^5*Sqrt[Pi]) - (E^(c + 2*b^2*x^2)*x^2)/(4*b^3*Sqrt[Pi
]) - (3*E^(c + b^2*x^2)*x*Erfi[b*x])/(4*b^4) + (E^(c + b^2*x^2)*x^3*Erfi[b*
x])/(2*b^2) + (3*E^c*Sqrt[Pi]*Erfi[b*x]^2)/(16*b^5)
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^4\operatorname{erfi}(bx)\,dx &= \frac{e^{c+b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}-\frac{3\int e^{c+b^2x^2}x^2\operatorname{erfi}(bx)\,dx}{2b^2}-\frac{\int e^{c+2b^2x^2}x^3\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{c+2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\operatorname{erfi}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}+\frac{3\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx}{4b^4}+\frac{\int e^{c+2b^2x^2}x\,dx}{2b^3\sqrt{\pi}}+\\ &= \frac{e^{c+2b^2x^2}}{2b^5\sqrt{\pi}}-\frac{e^{c+2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\operatorname{erfi}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}+\frac{\left(3e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(b\right.}{8b^5}\\ &= \frac{e^{c+2b^2x^2}}{2b^5\sqrt{\pi}}-\frac{e^{c+2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\operatorname{erfi}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\operatorname{erfi}(bx)}{2b^2}+\frac{3e^c\sqrt{\pi}\operatorname{erfi}(bx)^2}{16b^5}\end{aligned}$$

Mathematica [A] time = 0.0364366, size = 78, normalized size = 0.64

$$\frac{e^c\left(4\sqrt{\pi}bxe^{b^2x^2}\left(2b^2x^2-3\right)\operatorname{Erfi}(bx)-4e^{2b^2x^2}\left(b^2x^2-2\right)+3\pi\operatorname{Erfi}(bx)^2\right)}{16\sqrt{\pi}b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^4*Erfi[b*x],x]
```

```
[Out] (E^c*(-4*E^(2*b^2*x^2)*(-2 + b^2*x^2) + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*
b^2*x^2)*Erfi[b*x] + 3*Pi*Erfi[b*x]^2))/(16*b^5*Sqrt[Pi])
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x^4\mathrm{erfi}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^4*erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^4*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.53352, size = 180, normalized size = 1.49

$$\frac{\left(4\left(2\,\pi b^3x^3-3\,\pi bx\right)\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2\right)}+\sqrt{\pi}\left(3\,\pi\,\mathrm{erfi}\,(bx)^2-4\left(b^2x^2-2\right)e^{\left(2\,b^2x^2\right)}\right)\right)e^c}{16\,\pi b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(2*pi*b^3*x^3 - 3*pi*b*x)*erfi(b*x)*e^(b^2*x^2) + sqrt(pi)*(3*pi*er
fi(b*x)^2 - 4*(b^2*x^2 - 2)*e^(2*b^2*x^2)))*e^c/(pi*b^5)
```

Sympy [A] time = 32.1354, size = 124, normalized size = 1.02

$$\begin{cases} \frac{x^3e^ce^{b^2x^2}\,\mathrm{erfi}\,(bx)}{2b^2} - \frac{x^2e^ce^{2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{3xe^ce^{b^2x^2}\,\mathrm{erfi}\,(bx)}{4b^4} + \frac{e^ce^{2b^2x^2}}{2\sqrt{\pi}b^5} + \frac{3\sqrt{\pi}e^c\,\mathrm{erfi}^2\,(bx)}{16b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**4*erfi(b*x),x)
```

```
[Out] Piecewise((x**3*exp(c)*exp(b**2*x**2)*erfi(b*x)/(2*b**2) - x**2*exp(c)*exp(
2*b**2*x**2)/(4*sqrt(pi)*b**3) - 3*x*exp(c)*exp(b**2*x**2)*erfi(b*x)/(4*b**
4) + exp(c)*exp(2*b**2*x**2)/(2*sqrt(pi)*b**5) + 3*sqrt(pi)*exp(c)*erfi(b*x
)**2/(16*b**5), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^4*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.290 $\int e^{c+b^2x^2}x^2\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=69

$$\frac{xe^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{8b^3}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

```
[Out] -E^(c + 2*b^2*x^2)/(4*b^3*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfi[b*x])/(2*b^2)
 - (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b^3)
```

Rubi [A] time = 0.0757135, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6387, 6375, 30, 2209}

$$\frac{xe^{b^2x^2+c}\text{Erfi}(bx)}{2b^2}-\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{8b^3}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*x^2*Erfi[b*x],x]
```

```
[Out] -E^(c + 2*b^2*x^2)/(4*b^3*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfi[b*x])/(2*b^2)
 - (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b^3)
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx &= \frac{e^{c+b^2x^2}x\text{erfi}(bx)}{2b^2}-\frac{\int e^{c+b^2x^2}\text{erfi}(bx)\,dx}{2b^2}-\frac{\int e^{c+2b^2x^2}x\,dx}{b\sqrt{\pi}}\\ &= -\frac{e^{c+2b^2x^2}}{4b^3\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\text{erfi}(bx)}{2b^2}-\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx,x,\text{erfi}(bx)\right)}{4b^3}\\ &= -\frac{e^{c+2b^2x^2}}{4b^3\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\text{erfi}(bx)}{2b^2}-\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.0160547, size = 58, normalized size = 0.84

$$-\frac{e^c\left(-4\sqrt{\pi}bxe^{b^2x^2}\text{Erfi}(bx)+2e^{2b^2x^2}+\pi\text{Erfi}(bx)^2\right)}{8\sqrt{\pi}b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*x^2*Erfi[b*x],x]
```

```
[Out] -(E^c*(2*E^(2*b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*Erfi[b*x] + Pi*Erfi[b*x
]^2))/(8*b^3*Sqrt[Pi])
```

Maple [F] time = 0.225, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}x^2\text{erfi}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*x^2*erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*x^2*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)\, e^{\left(b^2x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.61336, size = 131, normalized size = 1.9

$$\frac{\left(4\,\pi bx \operatorname{erfi}(bx)\, e^{\left(b^2x^2\right)} - \sqrt{\pi}\left(\pi \operatorname{erfi}(bx)^2 + 2\, e^{\left(2\,b^2x^2\right)}\right)\right)e^c}{8\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*pi*b*x*erfi(b*x)*e^(b^2*x^2) - sqrt(pi)*(pi*erfi(b*x)^2 + 2*e^(2*b^2
*x^2)))*e^c/(pi*b^3)
```

Sympy [A] time = 5.38369, size = 68, normalized size = 0.99

$$\begin{cases} \frac{xe^{c}e^{b^2x^2}\operatorname{erfi}(bx)}{2b^2} - \frac{e^{c}e^{2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{\sqrt{\pi}e^{c}\operatorname{erfi}^2(bx)}{8b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*x**2*erfi(b*x),x)
```

```
[Out] Piecewise((x*exp(c)*exp(b**2*x**2)*erfi(b*x)/(2*b**2) - exp(c)*exp(2*b**2*x
**2)/(4*sqrt(pi)*b**3) - sqrt(pi)*exp(c)*erfi(b*x)**2/(8*b**3), Ne(b, 0)),
(0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx)e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*x^2*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.291 $\int e^{c+b^2x^2}\textbf{Erfi}(bx)\,dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{4b}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Rubi [A] time = 0.01751, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6375, 30}

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + b^2*x^2)*Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{c+b^2x^2}\text{erfi}(bx)\,dx &= \frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx, x, \text{erfi}(bx)\right)}{2b}\\ &= \frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{4b}\end{aligned}$$

Mathematica [A] time = 0.004696, size = 21, normalized size = 1.

$$\frac{\sqrt{\pi}e^c\mathrm{Erfi}(bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + b^2*x^2)*Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int e^{b^2x^2+c}\mathrm{erfi}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x),x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

Fricas [A] time = 2.58174, size = 43, normalized size = 2.05

$$\frac{\sqrt{\pi}\,\mathrm{erfi}\,(bx)^2\,e^c}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*erfi(b*x)^2*e^c/b
```

Sympy [A] time = 0.759273, size = 19, normalized size = 0.9

$$\begin{cases} \frac{\sqrt{\pi}e^{c}\operatorname{erfi}^{2}(bx)}{4b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x),x)
```

```
[Out] Piecewise((sqrt(pi)*exp(c)*erfi(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)\,e^{\left(b^{2}x^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

3.292 $\int \frac{e^{c+b^2x^2}\textbf{Erfi}(bx)}{x^2}\,dx$

Optimal. Leaf size=59

$$-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{x}+\frac{be^c\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}be^c\text{Erfi}(bx)^2$$

```
[Out] -((E^(c + b^2*x^2)*Erfi[b*x])/x) + (b*E^c*Sqrt[Pi]*Erfi[b*x]^2)/2 + (b*E^c*
ExpIntegralEi[2*b^2*x^2])/Sqrt[Pi]
```

Rubi [A] time = 0.0816846, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6393, 6375, 30, 2210}

$$-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{x}+\frac{be^c\text{Ei}\left(2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}be^c\text{Erfi}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^2,x]
```

```
[Out] -((E^(c + b^2*x^2)*Erfi[b*x])/x) + (b*E^c*Sqrt[Pi]*Erfi[b*x]^2)/2 + (b*E^c*
ExpIntegralEi[2*b^2*x^2])/Sqrt[Pi]
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx &= -\frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x}+\left(2b^2\right)\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx+\frac{(2b)\int\frac{e^{c+2b^2x^2}}{x}\,dx}{\sqrt{\pi}}\\ &= -\frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x}+\frac{be^c\operatorname{Ei}\left(2b^2x^2\right)}{\sqrt{\pi}}+\left(be^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(bx)\right)\\ &= -\frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x}+\frac{1}{2}be^c\sqrt{\pi}\operatorname{erfi}(bx)^2+\frac{be^c\operatorname{Ei}\left(2b^2x^2\right)}{\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0210227, size = 56, normalized size = 0.95

$$\frac{1}{2}e^c\left(-\frac{2e^{b^2x^2}\operatorname{Erfi}(bx)}{x}+\frac{2b\operatorname{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}+\sqrt{\pi}b\operatorname{Erfi}(bx)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^2,x]
```

```
[Out] (E^c*((-2*E^(b^2*x^2)*Erfi[b*x])/x + b*Sqrt[Pi]*Erfi[b*x]^2 + (2*b*ExpInteg
ralEi[2*b^2*x^2])/Sqrt[Pi]))/2
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{e^{b^2x^2+c}\operatorname{erfi}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)/x^2,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\, e^{\left(b^2x^2+c\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Fricas [A] time = 2.69375, size = 135, normalized size = 2.29

$$-\frac{\left(2\,\pi \operatorname{erfi}(bx)\, e^{\left(b^2x^2\right)} - \sqrt{\pi}\left(\pi bx \operatorname{erfi}(bx)^2 + 2\,bx\operatorname{Ei}\left(2\,b^2x^2\right)\right)\right)e^c}{2\,\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*pi*erfi(b*x)*e^(b^2*x^2) - sqrt(pi)*(pi*b*x*erfi(b*x)^2 + 2*b*x*Ei(
2*b^2*x^2)))*e^c/(pi*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^c \int \frac{e^{b^2x^2} \operatorname{erfi}(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\, e^{\left(b^2x^2+c\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

3.293 $\int \frac{e^{c+b^2x^2}\mathbf{Erfi}(bx)}{x^4}\,dx$

Optimal. Leaf size=118

$$-\frac{2b^2e^{b^2x^2+c}\text{Erfi}(bx)}{3x}-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{3x^3}+\frac{1}{3}\sqrt{\pi}b^3e^c\text{Erfi}(bx)^2+\frac{4b^3e^c\text{ExpIntegralEi}\left(2b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{2b^2x^2+c}}{3\sqrt{\pi}x^2}$$

```
[Out] -(b*E^(c + 2*b^2*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erfi[b*x])/(3*x^
3) - (2*b^2*E^(c + b^2*x^2)*Erfi[b*x])/(3*x) + (b^3*E^c*Sqrt[Pi]*Erfi[b*x]^
2)/3 + (4*b^3*E^c*ExpIntegralEi[2*b^2*x^2])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.172764, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6393, 6375, 30, 2210, 2214}

$$-\frac{2b^2e^{b^2x^2+c}\text{Erfi}(bx)}{3x}-\frac{e^{b^2x^2+c}\text{Erfi}(bx)}{3x^3}+\frac{1}{3}\sqrt{\pi}b^3e^c\text{Erfi}(bx)^2+\frac{4b^3e^c\text{Ei}\left(2b^2x^2\right)}{3\sqrt{\pi}}-\frac{be^{2b^2x^2+c}}{3\sqrt{\pi}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^4,x]
```

```
[Out] -(b*E^(c + 2*b^2*x^2))/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erfi[b*x])/(3*x^
3) - (2*b^2*E^(c + b^2*x^2)*Erfi[b*x])/(3*x) + (b^3*E^c*Sqrt[Pi]*Erfi[b*x]^
2)/3 + (4*b^3*E^c*ExpIntegralEi[2*b^2*x^2])/(3*Sqrt[Pi])
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1))
, x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^
n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-
4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0
] && LeQ[-n, m + 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx &= -\frac{e^{c+b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{1}{3}\left(2b^2\right)\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx+\frac{(2b)\int \frac{e^{c+2b^2x^2}}{x^3}\,dx}{3\sqrt{\pi}}\\
&= -\frac{be^{c+2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\text{erfi}(bx)}{3x}+\frac{1}{3}\left(4b^4\right)\int e^{c+b^2x^2}\text{erfi}(bx)\,dx+2\frac{\left(4b^3\right)\int \frac{e^{c+2b^2x^2}}{x}}{3\sqrt{\pi}}\\
&= -\frac{be^{c+2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\text{erfi}(bx)}{3x}+\frac{4b^3e^c\text{Ei}\left(2b^2x^2\right)}{3\sqrt{\pi}}+\frac{1}{3}\left(2b^3e^c\sqrt{\pi}\right)\text{Subst}(\int x\,d\\
&= -\frac{be^{c+2b^2x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{2b^2e^{c+b^2x^2}\text{erfi}(bx)}{3x}+\frac{1}{3}b^3e^c\sqrt{\pi}\text{erfi}(bx)^2+\frac{4b^3e^c\text{Ei}\left(2b^2x^2\right)}{3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.0357399, size = 91, normalized size = 0.77

$$-\frac{e^c\left(\pi\left(-b^3\right)x^3\text{Erfi}(bx)^2+\sqrt{\pi}e^{b^2x^2}\left(2b^2x^2+1\right)\text{Erfi}(bx)+bx\left(e^{2b^2x^2}-4b^2x^2\text{ExpIntegralEi}\left(2b^2x^2\right)\right)\right)}{3\sqrt{\pi}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^4,x]
```

```
[Out] -(E^c*(E^(b^2*x^2)*Sqrt[Pi]*(1 + 2*b^2*x^2)*Erfi[b*x] - b^3*Pi*x^3*Erfi[b*x
]^2 + b*x*(E^(2*b^2*x^2) - 4*b^2*x^2*ExpIntegralEi[2*b^2*x^2])))/(3*Sqrt[Pi
]*x^3)
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{b^2x^2+c}\operatorname{erfi}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^2*x^2+c)*erfi(b*x)/x^4,x)
```

```
[Out] int(exp(b^2*x^2+c)*erfi(b*x)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx)\,e^{\left(b^2x^2+c\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Fricas [A] time = 2.65977, size = 196, normalized size = 1.66

$$-\frac{\left(\left(\pi+2\,\pi b^2x^2\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2\right)}-\sqrt{\pi}\left(\pi b^3x^3\operatorname{erfi}(bx)^2+4\,b^3x^3\operatorname{Ei}\left(2\,b^2x^2\right)-bxe^{\left(2\,b^2x^2\right)}\right)\right)e^c}{3\,\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*((pi + 2*pi*b^2*x^2)*erfi(b*x)*e^(b^2*x^2) - sqrt(pi)*(pi*b^3*x^3*erfi
(b*x)^2 + 4*b^3*x^3*Ei(2*b^2*x^2) - b*x*e^(2*b^2*x^2)))*e^c/(pi*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{c}\int \frac{e^{b^{2}x^{2}}\operatorname{erfi}\left(bx\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b**2*x**2+c)*erfi(b*x)/x**4,x)
```

```
[Out] exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left(bx\right)e^{\left(b^{2}x^{2}+c\right)}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^2*x^2+c)*erfi(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

3.294 $\int e^{c+dx^2}x^3\mathbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{be^{\frac{a^2d}{b^2+d}+c}\operatorname{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2d^2\sqrt{b^2+d}}-\frac{a^2b^3e^{\frac{a^2d}{b^2+d}+c}\operatorname{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2d\left(b^2+d\right)^{5/2}}+\frac{be^{\frac{a^2d}{b^2+d}+c}\operatorname{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4d\left(b^2+d\right)^{3/2}}+\frac{ab^2e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)^2}-\frac{bxe^{a^2+2ab}}{2\sqrt{\pi}d}$$

```
[Out] (a*b^2*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)^2*Sqrt[Pi]) -
(b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x)/(2*d*(b^2 + d)*Sqrt[Pi]) - (E^(
c + d*x^2)*Erfi[a + b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfi[a + b*x])/(2*d)
 - (a^2*b^3*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d
]])/(2*d*(b^2 + d)^(5/2)) + (b*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 +
 d)*x)/Sqrt[b^2 + d]])/(4*d*(b^2 + d)^(3/2)) + (b*E^(c + (a^2*d)/(b^2 + d))
*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d^2*Sqrt[b^2 + d])
```

Rubi [A] time = 0.468279, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6387, 6384, 2234, 2204, 2241, 2240}

$$\frac{be^{\frac{a^2d}{b^2+d}+c}\operatorname{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2d^2\sqrt{b^2+d}}-\frac{a^2b^3e^{\frac{a^2d}{b^2+d}+c}\operatorname{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2d\left(b^2+d\right)^{5/2}}+\frac{be^{\frac{a^2d}{b^2+d}+c}\operatorname{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4d\left(b^2+d\right)^{3/2}}+\frac{ab^2e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)^2}-\frac{bxe^{a^2+2ab}}{2\sqrt{\pi}d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x^3*Erfi[a + b*x],x]
```

```
[Out] (a*b^2*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)^2*Sqrt[Pi]) -
(b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x)/(2*d*(b^2 + d)*Sqrt[Pi]) - (E^(
c + d*x^2)*Erfi[a + b*x])/(2*d^2) + (E^(c + d*x^2)*x^2*Erfi[a + b*x])/(2*d)
 - (a^2*b^3*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d
]])/(2*d*(b^2 + d)^(5/2)) + (b*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 +
 d)*x)/Sqrt[b^2 + d]])/(4*d*(b^2 + d)^(3/2)) + (b*E^(c + (a^2*d)/(b^2 + d))
*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d^2*Sqrt[b^2 + d])
```

Rule 6387

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m - 1)*E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] + (-Dist[(m - 1)/
```

```
(2*d), Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[b/(d*Sqrt[P
i]), Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2241

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] +
 (-Dist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x],
 x] - Dist[((m - 1)*e^2)/(2*c*Log[F]), Int[(d + e*x)^(m - 2)*F^(a + b*x + c
*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && Gt
Q[m, 1]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(e*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] - Dist[(b*e - 2*c*d)/(2*
c), Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[
b*e - 2*c*d, 0]
```

Rubi steps

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx = \frac{e^{c+dx^2}x^2\text{erfi}(a+bx)}{2d} - \frac{\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx}{d} - \frac{b\int e^{a^2+c+2abx+(b^2+d)x^2}x^2\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+(b^2+d)x^2}x}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfi}(a+bx)}{2d} + \frac{b\int e^{a^2+c+2abx+(b^2+d)x^2}\ d}{d^2\sqrt{\pi}}$$

$$= \frac{ab^2e^{a^2+c+2abx+(b^2+d)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} - \frac{be^{a^2+c+2abx+(b^2+d)x^2}x}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfi}(a+bx)}{2d} -$$

$$= \frac{ab^2e^{a^2+c+2abx+(b^2+d)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} - \frac{be^{a^2+c+2abx+(b^2+d)x^2}x}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfi}(a+bx)}{2d} +$$

$$= \frac{ab^2e^{a^2+c+2abx+(b^2+d)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} - \frac{be^{a^2+c+2abx+(b^2+d)x^2}x}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfi}(a+bx)}{2d} -$$

Mathematica [A] time = 2.11667, size = 206, normalized size = 0.68

$$\frac{e^c\left(\frac{2be^{\frac{a^2d}{b^2+d}}\text{Erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{\sqrt{b^2+d}} - \frac{bde^{\frac{a^2d}{b^2+d}}\left(\sqrt{\pi}\sqrt{b^2+d}\left((2a^2-1)b^2-d\right)\text{Erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)+2(b^2+d)e^{\frac{(ab+x(b^2+d))^2}{b^2+d}}\left(x(b^2+d)-ab\right)\right)}{\sqrt{\pi}(b^2+d)^3} + 2e^{dx^2}\left(dx^2-1\right)\text{Erfi}(a\right.}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x^3*Erfi[a + b*x],x]
```

```
[Out] (E^c*(2*E^(d*x^2)*(-1 + d*x^2)*Erfi[a + b*x] + (2*b*E^((a^2*d)/(b^2 + d))*E
rfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/Sqrt[b^2 + d] - (b*d*E^((a^2*d)/(b^
2 + d))*(2*(b^2 + d)*E^((a*b + (b^2 + d)*x)^2/(b^2 + d))*(-(a*b) + (b^2 + d
)*x) + ((-1 + 2*a^2)*b^2 - d)*Sqrt[b^2 + d]*Sqrt[Pi]*Erfi[(a*b + (b^2 + d)*
x)/Sqrt[b^2 + d]]))/((b^2 + d)^3*Sqrt[Pi])))/(4*d^2)
```

Maple [F] time = 0.383, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^3\text{erfi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^3*erfi(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^3*erfi(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\text{erfi}\,(bx+a)\,e^{\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.71188, size = 567, normalized size = 1.87

$$-\frac{\pi\left(2\,b^5-\left(2\,a^2-5\right)b^3d+3\,bd^2\right)\sqrt{-b^2-d}\,\text{erf}\left(\frac{\left(ab+\left(b^2+d\right)x\right)\sqrt{-b^2-d}}{b^2+d}\right)e^{\left(\frac{b^2c+\left(a^2+c\right)d}{b^2+d}\right)}-2\left(\pi\left(b^6d+3\,b^4d^2+3\,b^2d^3+d^4\right)x^2-\pi\left(b\right.\right.}{4\,\pi\left(b^6d^2+3\,b^4d^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(pi*(2*b^5 - (2*a^2 - 5)*b^3*d + 3*b*d^2)*sqrt(-b^2 - d)*erf((a*b + (b
^2 + d)*x)*sqrt(-b^2 - d)/(b^2 + d))*e^((b^2*c + (a^2 + c)*d)/(b^2 + d)) -
2*(pi*(b^6*d + 3*b^4*d^2 + 3*b^2*d^3 + d^4)*x^2 - pi*(b^6 + 3*b^4*d + 3*b^2
*d^2 + d^3))*erfi(b*x + a)*e^(d*x^2 + c) - 2*sqrt(pi)*(a*b^4*d + a*b^2*d^2
- (b^5*d + 2*b^3*d^2 + b*d^3)*x)*e^(b^2*x^2 + 2*a*b*x + d*x^2 + a^2 + c))/(
pi*(b^6*d^2 + 3*b^4*d^3 + 3*b^2*d^4 + d^5))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**3*erfi(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{erfi}(bx + a) e^{(dx^2+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^3*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*erfi(b*x + a)*e^(d*x^2 + c), x)
```

3.295 $\int e^{c+dx^2} x \mathbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=78

$$\frac{e^{c+dx^2}\text{Erfi}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2+d}+c}\text{Erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{2d\sqrt{b^2+d}}$$

```
[Out] (E^(c + d*x^2)*Erfi[a + b*x])/(2*d) - (b*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*
b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d*Sqrt[b^2 + d])
```

Rubi [A] time = 0.0532274, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6384, 2234, 2204}

$$\frac{e^{c+dx^2}\text{Erfi}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2+d}+c}\text{Erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{2d\sqrt{b^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x^2)*x*Erfi[a + b*x],x]
```

```
[Out] (E^(c + d*x^2)*Erfi[a + b*x])/(2*d) - (b*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*
b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d*Sqrt[b^2 + d])
```

Rule 6384

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[(E^(c + d*x^2)*Erfi[a + b*x])/(2*d), x] - Dist[b/(d*Sqrt[Pi]), Int[E^(a^
2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx &= \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{b\int e^{a^2+c+2abx+(b^2+d)x^2}\,dx}{d\sqrt{\pi}}\\ &= \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{\left(be^{c+\frac{a^2d}{b^2+d}}\right)\int e^{\frac{(2ab+2(b^2+d)x)^2}{4(b^2+d)}}\,dx}{d\sqrt{\pi}}\\ &= \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{be^{c+\frac{a^2d}{b^2+d}}\text{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{2d\sqrt{b^2+d}}\end{aligned}$$

Mathematica [A] time = 0.0707114, size = 73, normalized size = 0.94

$$\frac{e^c\left(e^{dx^2}\text{Erfi}(a+bx)-\frac{be^{\frac{a^2d}{b^2+d}}\text{Erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{\sqrt{b^2+d}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x^2)*x*Erfi[a + b*x],x]
```

```
[Out] (E^c*(E^(d*x^2)*Erfi[a + b*x] - (b*E^((a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 +
 d)*x)/Sqrt[b^2 + d]])/Sqrt[b^2 + d]))/(2*d)
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x\text{erfi}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x*erfi(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x*erfi(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 2.67521, size = 221, normalized size = 2.83

$$\frac{\sqrt{-b^2-d}\, b \operatorname{erf}\left(\frac{\left(ab+\left(b^2+d\right)x\right)\sqrt{-b^2-d}}{b^2+d}\right) e^{\left(\frac{b^2c+\left(a^2+c\right)d}{b^2+d}\right)} + \left(b^2+d\right) \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}}{2\left(b^2d+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-b^2 - d)*b*erf((a*b + (b^2 + d)*x)*sqrt(-b^2 - d)/(b^2 + d))*e^(
(b^2*c + (a^2 + c)*d)/(b^2 + d)) + (b^2 + d)*erfi(b*x + a)*e^(d*x^2 + c))/(
b^2*d + d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x*erfi(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*erfi(b*x + a)*e^(d*x^2 + c), x)
```

3.296 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable[(E^(c + d*x^2)*Erfi[a + b*x])/x, x]
```

Rubi [A] time = 0.037334, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[a + b*x])/x,x]
```

```
[Out] Defer[Int][(E^(c + d*x^2)*Erfi[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.169817, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x, x]
```

Maple [A] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{d x^{2}+c} \operatorname{erfi}\left(b x+a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left(b x+a\right) e^{\left(d x^{2}+c\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erfi}\left(b x+a\right) e^{\left(d x^{2}+c\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)*e^(d*x^2 + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int \frac{e^{d x^{2}} \operatorname{erfi}{\left(a + b x \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x+a)/x,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)\,e^{\left(dx^2+c\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x, x)
```

3.297 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=165

$$\frac{2ab^2\text{Unintegrable}\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{x},x\right)}{\sqrt{\pi}}+d\text{Unintegrable}\left(\frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x},x\right)+b\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{Erfi}\left(\frac{ab+x\left(b^2+\right.}{\sqrt{b^2+d}}\right.$$

```
[Out] -((b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)*
Erfi[a + b*x])/(2*x^2) + b*Sqrt[b^2 + d]*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*
b + (b^2 + d)*x)/Sqrt[b^2 + d]] + (2*a*b^2*Unintegrable[E^(a^2 + c + 2*a*b*
x + (b^2 + d)*x^2)/x, x])/Sqrt[Pi] + d*Unintegrable[(E^(c + d*x^2)*Erfi[a +
 b*x])/x, x]
```

Rubi [A] time = 0.386632, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[a + b*x])/x^3,x]
```

```
[Out] -((b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(Sqrt[Pi]*x)) - (E^(c + d*x^2)*
Erfi[a + b*x])/(2*x^2) + b*Sqrt[b^2 + d]*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*
b + (b^2 + d)*x)/Sqrt[b^2 + d]] + (2*a*b^2*Defer[Int][E^(a^2 + c + 2*a*b*x
+ (b^2 + d)*x^2)/x, x])/Sqrt[Pi] + d*Defer[Int][(E^(c + d*x^2)*Erfi[a + b*x
])/x, x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^3}\,dx = -\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx + \frac{b\int \frac{e^{a^2+c+2abx+(b^2+d)x^2}}{x^2}\,dx}{\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+(b^2+d)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx + \frac{(2ab^2)\int \frac{e^{a^2+c+2abx+(b^2+\cdots}}{x}}{\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+(b^2+d)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2} + d\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx + \frac{(2ab^2)\int \frac{e^{a^2+c+2abx+(b^2+\cdots}}{x}}{\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+(b^2+d)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2} + b\sqrt{b^2+d}e^{c+\frac{a^2d}{b^2+d}}\text{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right) + d\int \frac{e^{c+dx\cdots}}{}$$

Mathematica [A] time = 0.305966, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^3,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^3, x]
```

Maple [A] time = 0.314, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfi}\,(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x+a)/x^3,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x+a)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)e^{(dx^2+c)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx+a)e^{(dx^2+c)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x+a)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)e^{(dx^2+c)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

3.298 $\int e^{c+dx^2} x^4 \textbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=467

$$\frac{3\text{Unintegrable}\left(e^{c+dx^2}\text{Erfi}(a+bx),x\right)}{4d^2}-\frac{3ab^2e^{\frac{a^2d}{b^2+d}+c}\text{Erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{4d^2\left(b^2+d\right)^{3/2}}+\frac{3be^{a^2+2abx+x^2(b^2+d)+c}}{4\sqrt{\pi}d^2\left(b^2+d\right)}+\frac{a^3b^4e^{\frac{a^2d}{b^2+d}+c}\text{Erfi}\left(\frac{ab+x(b^2}{\sqrt{b^2+}}\right.}{2d\left(b^2+d\right)^{7/2}}$$

```
[Out] -(a^2*b^3*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)^3*Sqrt[Pi])
 + (b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)^2*Sqrt[Pi]) + (
3*b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(4*d^2*(b^2 + d)*Sqrt[Pi]) + (a*
b^2*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x)/(2*d*(b^2 + d)^2*Sqrt[Pi]) - (
b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x^2)/(2*d*(b^2 + d)*Sqrt[Pi]) - (3*
E^(c + d*x^2)*x*Erfi[a + b*x])/(4*d^2) + (E^(c + d*x^2)*x^3*Erfi[a + b*x])/
(2*d) + (a^3*b^4*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^
2 + d]])/(2*d*(b^2 + d)^(7/2)) - (3*a*b^2*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a
*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(4*d*(b^2 + d)^(5/2)) - (3*a*b^2*E^(c + (
a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(4*d^2*(b^2 + d)
^(3/2)) + (3*Unintegrable[E^(c + d*x^2)*Erfi[a + b*x], x])/(4*d^2)
```

Rubi [A] time = 0.892451, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^4\text{Erfi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^4*Erfi[a + b*x],x]
```


```
[Out] -(a^2*b^3*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)^3*Sqrt[Pi])
 + (b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)^2*Sqrt[Pi]) + (
3*b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(4*d^2*(b^2 + d)*Sqrt[Pi]) + (a*
b^2*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x)/(2*d*(b^2 + d)^2*Sqrt[Pi]) - (
b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x^2)/(2*d*(b^2 + d)*Sqrt[Pi]) - (3*
E^(c + d*x^2)*x*Erfi[a + b*x])/(4*d^2) + (E^(c + d*x^2)*x^3*Erfi[a + b*x])/
(2*d) + (a^3*b^4*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^
2 + d]])/(2*d*(b^2 + d)^(7/2)) - (3*a*b^2*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a
*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(4*d*(b^2 + d)^(5/2)) - (3*a*b^2*E^(c + (
a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(4*d^2*(b^2 + d)
```

```
^(3/2)) + (3*Defer[Int][E^(c + d*x^2)*Erfi[a + b*x], x])/(4*d^2)
```

Rubi steps

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx = \frac{e^{c+dx^2}x^3\text{erfi}(a+bx)}{2d} - \frac{3\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx}{2d} - \frac{b\int e^{a^2+c+2abx+\left(b^2+d\right)x^2}x^3\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}x^2}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{3e^{c+dx^2}x\text{erfi}(a+bx)}{4d^2} + \frac{e^{c+dx^2}x^3\text{erfi}(a+bx)}{2d} + \frac{3\int e^{c+dx^2}\text{erfi}(a+bx}{4d^2}$$

$$= \frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{3be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{4d^2\left(b^2+d\right)\sqrt{\pi}} + \frac{ab^2e^{a^2+c+2abx+\left(b^2+d\right)x^2}x}{2d\left(b^2+d\right)^2\sqrt{\pi}} - \frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}x^2}{2d\left(b^2+d\right)\sqrt{\pi}}$$

$$= -\frac{a^2b^3e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^3\sqrt{\pi}} + \frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{3be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{4d^2\left(b^2+d\right)\sqrt{\pi}} + \frac{ab^2e^{a^2+c+2abx+\left(b^2+d\right)x}}{2d\left(b^2+d\right)^2\sqrt{\pi}}$$

$$= -\frac{a^2b^3e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^3\sqrt{\pi}} + \frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{3be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{4d^2\left(b^2+d\right)\sqrt{\pi}} + \frac{ab^2e^{a^2+c+2abx+\left(b^2+d\right)x}}{2d\left(b^2+d\right)^2\sqrt{\pi}}$$

$$= -\frac{a^2b^3e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^3\sqrt{\pi}} + \frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{3be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{4d^2\left(b^2+d\right)\sqrt{\pi}} + \frac{ab^2e^{a^2+c+2abx+\left(b^2+d\right)x}}{2d\left(b^2+d\right)^2\sqrt{\pi}}$$

Mathematica [A] time = 0.485338, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^4\text{Erfi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^4*Erfi[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^4*Erfi[a + b*x], x]
```

Maple [A] time = 0.115, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^4\text{erfi}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^4*erfi(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^4*erfi(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx + a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^4 \operatorname{erfi}(bx + a)\, e^{\left(dx^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^4*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**4*erfi(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^4*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*erfi(b*x + a)*e^(d*x^2 + c), x)
```

3.299 $\int e^{c+dx^2}x^2\,\mathbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{\text{Unintegrable}\left(e^{c+dx^2}\text{Erfi}(a+bx),x\right)}{2d}+\frac{ab^2e^{\frac{a^2d}{b^2+d}+c}\text{Erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2d\left(b^2+d\right)^{3/2}}-\frac{be^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\sqrt{\pi}d\left(b^2+d\right)}+\frac{xe^{c+dx^2}\text{Erfi}(a+bx)}{2d}$$

```
[Out] -(b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)*Sqrt[Pi]) + (E^(c
 + d*x^2)*x*Erfi[a + b*x])/(2*d) + (a*b^2*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a
*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d*(b^2 + d)^(3/2)) - Unintegrable[E^(c
 + d*x^2)*Erfi[a + b*x], x]/(2*d)
```

Rubi [A] time = 0.174312, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}x^2\text{Erfi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*x^2*Erfi[a + b*x],x]
```

```
[Out] -(b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(2*d*(b^2 + d)*Sqrt[Pi]) + (E^(c
 + d*x^2)*x*Erfi[a + b*x])/(2*d) + (a*b^2*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a
*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d*(b^2 + d)^(3/2)) - Defer[Int][E^(c +
 d*x^2)*Erfi[a + b*x], x]/(2*d)
```

Rubi steps

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx = \frac{e^{c+dx^2}x\text{erfi}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfi}(a+bx)\,dx}{2d} - \frac{b\int e^{a^2+c+2abx+\left(b^2+d\right)x^2}x\,dx}{d\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erfi}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfi}(a+bx)\,dx}{2d} + \frac{\left(ab^2\right)\int e^{a^2+c+2abx+\left(b^2\right.}}{d\left(b^2+d\right)\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erfi}(a+bx)}{2d} - \frac{\int e^{c+dx^2}\text{erfi}(a+bx)\,dx}{2d} + \frac{\left(ab^2e^{c+\frac{a^2d}{b^2+d}}\right)\int e^{\frac{(2ab+2(}{4(b}}}{d\left(b^2+d\right)\sqrt{}}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{2d\left(b^2+d\right)\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erfi}(a+bx)}{2d} + \frac{ab^2e^{c+\frac{a^2d}{b^2+d}}\text{erfi}\left(\frac{ab+\left(b^2+d\right)x}{\sqrt{b^2+d}}\right)}{2d\left(b^2+d\right)^{3/2}} - \frac{\int e^{c+dx^2}\text{erfi}(a+b}{2d}$$

Mathematica [A] time = 0.351461, size = 0, normalized size = 0.

$$\int e^{c+dx^2}x^2\text{Erfi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*x^2*Erfi[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*x^2*Erfi[a + b*x], x]
```

Maple [A] time = 0.224, size = 0, normalized size = 0.

$$\int e^{dx^2+c}x^2\text{erfi}\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*x^2*erfi(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*x^2*erfi(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^2 \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*x**2*erfi(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*x^2*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*erfi(b*x + a)*e^(d*x^2 + c), x)
```

3.300 $\int e^{c+dx^2}\textbf{Erfi}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(e^{c+dx^2}\text{Erfi}(a+bx),x\right)$$

```
[Out] Unintegrable[E^(c + d*x^2)*Erfi[a + b*x], x]
```

Rubi [A] time = 0.0142708, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{c+dx^2}\text{Erfi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^(c + d*x^2)*Erfi[a + b*x],x]
```

```
[Out] Defer[Int][E^(c + d*x^2)*Erfi[a + b*x], x]
```

Rubi steps

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\int e^{c+dx^2}\text{erfi}(a+bx)\,dx$$

Mathematica [A] time = 0.0331666, size = 0, normalized size = 0.

$$\int e^{c+dx^2}\text{Erfi}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + d*x^2)*Erfi[a + b*x],x]
```

```
[Out] Integrate[E^(c + d*x^2)*Erfi[a + b*x], x]
```

Maple [A] time = 0.107, size = 0, normalized size = 0.

$$\int \mathrm{e}^{d x^2+c} \operatorname{erfi}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x+a),x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c} \int e^{dx^{2}} \operatorname{erfi}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x+a),x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx+a)\, e^{\left(dx^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c), x)
```

3.301 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=77

$$\frac{2b\text{Unintegrable}\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{x},x\right)}{\sqrt{\pi}}+2d\text{Unintegrable}\left(e^{c+dx^2}\text{Erfi}(a+bx),x\right)-\frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x}$$

```
[Out] -((E^(c + d*x^2)*Erfi[a + b*x])/x) + (2*b*Unintegrable[E^(a^2 + c + 2*a*b*x
 + (b^2 + d)*x^2)/x, x])/Sqrt[Pi] + 2*d*Unintegrable[E^(c + d*x^2)*Erfi[a +
 b*x], x]
```

Rubi [A] time = 0.203134, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[a + b*x])/x^2,x]
```

```
[Out] -((E^(c + d*x^2)*Erfi[a + b*x])/x) + (2*b*Defer[Int][E^(a^2 + c + 2*a*b*x +
 (b^2 + d)*x^2)/x, x])/Sqrt[Pi] + 2*d*Defer[Int][E^(c + d*x^2)*Erfi[a + b*x
], x]
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx=-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}+(2d)\int e^{c+dx^2}\text{erfi}(a+bx)\,dx+\frac{(2b)\int \frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x}\,dx}{\sqrt{\pi}}$$

Mathematica [A] time = 0.341624, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^2,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^2, x]
```

Maple [A] time = 0.131, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\operatorname{erfi}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x+a)/x^2,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x+a)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)\,e^{(dx^2+c)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{erfi}(bx+a)\,e^{(dx^2+c)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$e^{c}\int \frac{e^{dx^{2}}\operatorname{erfi}\left(a+bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x+a)/x**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x**2)*erfi(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}\left(bx+a\right)e^{\left(dx^{2}+c\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

3.302 $\int \frac{e^{c+dx^2}\mathbf{Erfi}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=319

$$\frac{4a^2b^3\text{Unintegrable}\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{x},x\right)}{3\sqrt{\pi}}+\frac{4bd\text{Unintegrable}\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{x},x\right)}{3\sqrt{\pi}}+\frac{2b\left(b^2+d\right)\text{Unintegrable}\left(\frac{e^{a^2+2ab}}{}\right.}{3\sqrt{\pi}}$$

```
[Out] -(b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x^2) - (2*a*b^2*E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x) - (E^(c + d*x^2)*Erfi[a +
 b*x])/(3*x^3) - (2*d*E^(c + d*x^2)*Erfi[a + b*x])/(3*x) + (2*a*b^2*Sqrt[b^
2 + d]*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/3
 + (4*a^2*b^3*Unintegrable[E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)/x, x])/(3*
Sqrt[Pi]) + (4*b*d*Unintegrable[E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)/x, x]
)/(3*Sqrt[Pi]) + (2*b*(b^2 + d)*Unintegrable[E^(a^2 + c + 2*a*b*x + (b^2 +
d)*x^2)/x, x])/(3*Sqrt[Pi]) + (4*d^2*Unintegrable[E^(c + d*x^2)*Erfi[a + b*
x], x])/3
```

Rubi [A] time = 0.853859, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(c + d*x^2)*Erfi[a + b*x])/x^4,x]
```

```
[Out] -(b*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x^2) - (2*a*b^2*E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2))/(3*Sqrt[Pi]*x) - (E^(c + d*x^2)*Erfi[a +
 b*x])/(3*x^3) - (2*d*E^(c + d*x^2)*Erfi[a + b*x])/(3*x) + (2*a*b^2*Sqrt[b^
2 + d]*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/3
 + (4*a^2*b^3*Defer[Int][E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)/x, x])/(3*Sq
rt[Pi]) + (4*b*d*Defer[Int][E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)/x, x])/(3
*Sqrt[Pi]) + (2*b*(b^2 + d)*Defer[Int][E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2
)/x, x])/(3*Sqrt[Pi]) + (4*d^2*Defer[Int][E^(c + d*x^2)*Erfi[a + b*x], x])/
3
```

Rubi steps

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx = -\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3} + \frac{1}{3}(2d)\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx + \frac{(2b)\int \frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x^3}\,dx}{3\sqrt{\pi}}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x^2} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erfi}(a+bx)}{3x} + \frac{1}{3}\left(4d^2\right)\int e^{c+dx^2}\text{erfi}(a+bx$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x^2} - \frac{2ab^2e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erfi}(a+bx)}{3x} + \frac{1}{3}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x^2} - \frac{2ab^2e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erfi}(a+bx)}{3x} + \frac{1}{3}$$

$$= -\frac{be^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x^2} - \frac{2ab^2e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3} - \frac{2de^{c+dx^2}\text{erfi}(a+bx)}{3x} + \frac{2}{3}$$

Mathematica [A] time = 0.484685, size = 0, normalized size = 0.

$$\int \frac{e^{c+dx^2}\text{Erfi}(a+bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^4,x]
```

```
[Out] Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^4, x]
```

Maple [A] time = 0.327, size = 0, normalized size = 0.

$$\int \frac{e^{dx^2+c}\text{erfi}\,(bx+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x^2+c)*erfi(b*x+a)/x^4,x)
```

```
[Out] int(exp(d*x^2+c)*erfi(b*x+a)/x^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{erfi}(bx+a)\,e^{(dx^2+c)}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x**2+c)*erfi(b*x+a)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{erfi}(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x^2+c)*erfi(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

3.303 $\int \left(\frac{e^{-b^2x^2}\textbf{Erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\textbf{Erfi}(bx)}{x}\right)dx$

Optimal. Leaf size=33

$$-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

```
[Out] -(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2)
```

Rubi [A] time = 0.11913, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6393, 6390, 30}

$$-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfi[b*x])/(E^(b^2*x^2)*x),x]
```

```
[Out] -(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2)
```

Rule 6393

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[(x^(m + 1)*E^(c + d*x^2)*Erfi[a + b*x])/(m + 1), x] + (-Dist[(2*d)/
(m + 1), Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Dist[(2*b)/((m
 + 1)*Sqrt[Pi]), Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Rule 6390

```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[(2*
b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)dx = b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx+\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}\,dx$$
$$= -\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}+\frac{2b^3x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}-b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx+\frac{b\int}{\sqrt{}}$$
$$= -\frac{b}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}$$

Mathematica [A] time = 0.0577235, size = 33, normalized size = 1.

$$-\frac{e^{-b^2x^2}\text{Erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfi[b*x])/(E^(b^2*x^2)*x),x]
```

```
[Out] -(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2)
```

Maple [A] time = 0.201, size = 41, normalized size = 1.2

$$\frac{-2\,e^{b^2x^2}bx-\sqrt{\pi}\text{erfi}\,(bx)}{2\,\sqrt{\pi}x^2e^{b^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x)
```

```
[Out] 1/2*(-2*exp(b^2*x^2)*b*x-Pi^(1/2)*erfi(b*x))/Pi^(1/2)/x^2/exp(b^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^2\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}+\frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x, algori
thm="maxima")
```

```
[Out] integrate(b^2*erfi(b*x)*e^(-b^2*x^2)/x + erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Fricas [A] time = 2.63857, size = 97, normalized size = 2.94

$$-\frac{\left(2\sqrt{\pi}bxe^{\left(b^2x^2\right)} + \pi\,\mathrm{erfi}\left(bx\right)\right)e^{\left(-b^2x^2\right)}}{2\,\pi x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x, algori
thm="fricas")
```

```
[Out] -1/2*(2*sqrt(pi)*b*x*e^(b^2*x^2) + pi*erfi(b*x))*e^(-b^2*x^2)/(pi*x^2)
```

Sympy [A] time = 137.103, size = 53, normalized size = 1.61

$$\frac{2b^3x_2F_2\left(\begin{matrix}\frac{1}{2},1\\\frac{3}{2},\frac{3}{2}\end{matrix}\middle|-b^2x^2\right)}{\sqrt{\pi}} - \frac{2b_2F_2\left(\begin{matrix}-\frac{1}{2},1\\\frac{1}{2},\frac{3}{2}\end{matrix}\middle|-b^2x^2\right)}{\sqrt{\pi}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b**2*x**2)/x**3+b**2*erfi(b*x)/exp(b**2*x**2)/x,x)
```

```
[Out] 2*b**3*x*hyper((1/2, 1), (3/2, 3/2), -b**2*x**2)/sqrt(pi) - 2*b*hyper((-1/2
, 1), (1/2, 3/2), -b**2*x**2)/(sqrt(pi)*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^2\,\mathrm{erfi}\left(bx\right)e^{\left(-b^2x^2\right)}}{x} + \frac{\mathrm{erfi}\left(bx\right)e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x, algori
thm="giac")
```

```
[Out] integrate(b^2*erfi(b*x)*e^(-b^2*x^2)/x + erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

3.304 $\int \textbf{Erfi}(bx) \sin\left(c + ib^2x^2\right) dx$

Optimal. Leaf size=67

$$\frac{i\sqrt{\pi}e^{-ic}\text{Erfi}(bx)^2}{8b} - \frac{ibe^{ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}$$

```
[Out] ((I/8)*Sqrt[Pi]*Erfi[b*x]^2)/(b*E^(I*c)) - ((I/2)*b*E^(I*c)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] time = 0.0557588, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6406, 6375, 30, 6378}

$$\frac{i\sqrt{\pi}e^{-ic}\text{Erfi}(bx)^2}{8b} - \frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]*Sin[c + I*b^2*x^2],x]
```

```
[Out] ((I/8)*Sqrt[Pi]*Erfi[b*x]^2)/(b*E^(I*c)) - ((I/2)*b*E^(I*c)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rule 6406

```
Int[Erfi[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[I/2, Int[
E^(-(I*c) - I*d*x^2)*Erfi[b*x], x], x] - Dist[I/2, Int[E^(I*c + I*d*x^2)*Er
fi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\begin{aligned}\int \text{erfi}(bx)\sin\left(c+ib^2x^2\right)dx &= -\left(\frac{1}{2}i\int e^{ic-b^2x^2}\text{erfi}(bx)\,dx\right)+\frac{1}{2}i\int e^{-ic+b^2x^2}\text{erfi}(bx)\,dx\\ &= -\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(ie^{-ic}\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx,x,\text{erfi}(bx)\right)}{4b}\\ &= \frac{ie^{-ic}\sqrt{\pi}\,\text{erfi}(bx)^2}{8b}-\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 0.560063, size = 0, normalized size = 0.

$$\int \text{Erfi}(bx)\sin\left(c+ib^2x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[b*x]*Sin[c + I*b^2*x^2],x]

[Out] Integrate[Erfi[b*x]*Sin[c + I*b^2*x^2], x]
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \text{erfi}\,(bx)\sin\left(c+ib^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)*sin(c+I*b^2*x^2),x)

[Out] int(erfi(b*x)*sin(c+I*b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{i\sqrt{\pi}\cos\left(c\right)\operatorname{erfi}\left(bx\right)^{2}}{8\,b}+\frac{\sqrt{\pi}\operatorname{erfi}\left(bx\right)^{2}\sin\left(c\right)}{8\,b}-\frac{1}{2}i\,\cos\left(c\right)\int\operatorname{erfi}\left(bx\right)e^{\left(-b^{2}x^{2}\right)}\,dx+\frac{1}{2}\int\operatorname{erfi}\left(bx\right)e^{\left(-b^{2}x^{2}\right)}\,dx\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sin(c+I*b^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*I*sqrt(pi)*cos(c)*erfi(b*x)^2/b + 1/8*sqrt(pi)*erfi(b*x)^2*sin(c)/b - 1
/2*I*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) + 1/2*integrate(erfi(b*x)*
e^(-b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2}\left(-i\,\operatorname{erfi}\left(bx\right)e^{\left(-2\,b^{2}x^{2}+2i\,c\right)}+i\,\operatorname{erfi}\left(bx\right)\right)e^{\left(b^{2}x^{2}-i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sin(c+I*b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/2*(-I*erfi(b*x)*e^(-2*b^2*x^2 + 2*I*c) + I*erfi(b*x))*e^(b^2*x^2
 - I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(ib^{2}x^{2}+c\right)\operatorname{erfi}\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sin(c+I*b**2*x**2),x)
```

```
[Out] Integral(sin(I*b**2*x**2 + c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{erfi}\left(bx\right)\sin\left(i\,b^{2}x^{2}+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sin(c+I*b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*sin(I*b^2*x^2 + c), x)
```

3.305 $\int \textbf{Erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx$

Optimal. Leaf size=67

$$\frac{ibe^{-ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{ic}\text{Erfi}(bx)^2}{8b}$$

```
[Out] ((-I/8)*E^(I*c)*Sqrt[Pi]*Erfi[b*x]^2)/b + ((I/2)*b*x^2*HypergeometricPFQ[{1
, 1}, {3/2, 2}, -(b^2*x^2)])/(E^(I*c)*Sqrt[Pi])
```

Rubi [A] time = 0.0553932, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6406, 6378, 6375, 30}

$$\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{ic}\text{Erfi}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]*Sin[c - I*b^2*x^2],x]
```

```
[Out] ((-I/8)*E^(I*c)*Sqrt[Pi]*Erfi[b*x]^2)/b + ((I/2)*b*x^2*HypergeometricPFQ[{1
, 1}, {3/2, 2}, -(b^2*x^2)])/(E^(I*c)*Sqrt[Pi])
```

Rule 6406

```
Int[Erfi[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[I/2, Int[
E^(-(I*c) - I*d*x^2)*Erfi[b*x], x], x] - Dist[I/2, Int[E^(I*c + I*d*x^2)*Er
fi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx &= \frac{1}{2}i\int e^{-ic-b^2x^2}\operatorname{erfi}(bx)\,dx-\frac{1}{2}i\int e^{ic+b^2x^2}\operatorname{erfi}(bx)\,dx\\ &= \frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}-\frac{\left(ie^{ic}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(bx)\right)}{4b}\\ &= -\frac{ie^{ic}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 0.532506, size = 0, normalized size = 0.

$$\int \operatorname{Erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Erfi[b*x]*Sin[c - I*b^2*x^2],x]
```

```
[Out] Integrate[Erfi[b*x]*Sin[c - I*b^2*x^2], x]
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int -\operatorname{erfi}(bx)\sin\left(-c+ib^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-erfi(b*x)*sin(-c+I*b^2*x^2),x)
```

```
[Out] int(-erfi(b*x)*sin(-c+I*b^2*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i\sqrt{\pi}\cos(c)\,\mathrm{erfi}(bx)^2}{8\,b}+\frac{\sqrt{\pi}\,\mathrm{erfi}(bx)^2\sin(c)}{8\,b}+\frac{1}{2}i\,\cos(c)\int \mathrm{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx+\frac{1}{2}\int \mathrm{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sin(-c+I*b^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/8*I*sqrt(pi)*cos(c)*erfi(b*x)^2/b + 1/8*sqrt(pi)*erfi(b*x)^2*sin(c)/b +
1/2*I*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) + 1/2*integrate(erfi(b*x)
*e^(-b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{2}\left(i\,\mathrm{erfi}(bx)\,e^{\left(-2\,b^2x^2-2i\,c\right)}-i\,\mathrm{erfi}(bx)\right)e^{\left(b^2x^2+i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sin(-c+I*b^2*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/2*(I*erfi(b*x)*e^(-2*b^2*x^2 - 2*I*c) - I*erfi(b*x))*e^(b^2*x^2
+ I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \sin\left(ib^2x^2-c\right)\mathrm{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sin(-c+I*b**2*x**2),x)
```

```
[Out] -Integral(sin(I*b**2*x**2 - c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{erfi}(bx)\sin\left(i\,b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sin(-c+I*b^2*x^2),x, algorithm="giac")
```

```
[Out] integrate(-erfi(b*x)*sin(I*b^2*x^2 - c), x)
```

3.306 $\int \cos\left(c + ib^2x^2\right) \mathbf{Erfi}(bx)\, dx$

Optimal. Leaf size=63

$$\frac{be^{ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-ic}\text{Erfi}(bx)^2}{8b}$$

```
[Out] (Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^(I*c)) + (b*E^(I*c)*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0549715, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6409, 6375, 30, 6378}

$$\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-ic}\text{Erfi}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + I*b^2*x^2]*Erfi[b*x],x]
```

```
[Out] (Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^(I*c)) + (b*E^(I*c)*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Rule 6409

```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[
E^(-(I*c) - I*d*x^2)*Erfi[b*x], x], x] + Dist[1/2, Int[E^(I*c + I*d*x^2)*Er
fi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\begin{aligned}\int \cos\left(c+ib^2x^2\right)\operatorname{erfi}(bx)\,dx &= \frac{1}{2}\int e^{ic-b^2x^2}\operatorname{erfi}(bx)\,dx+\frac{1}{2}\int e^{-ic+b^2x^2}\operatorname{erfi}(bx)\,dx\\ &= \frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^{-ic}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(bx)\right)}{4b}\\ &= \frac{e^{-ic}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 0.533993, size = 0, normalized size = 0.

$$\int \cos\left(c+ib^2x^2\right)\operatorname{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + I*b^2*x^2]*Erfi[b*x],x]
```

```
[Out] Integrate[Cos[c + I*b^2*x^2]*Erfi[b*x], x]
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \cos\left(c+ib^2x^2\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+I*b^2*x^2)*erfi(b*x),x)
```

```
[Out] int(cos(c+I*b^2*x^2)*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\cos(c)\operatorname{erfi}(bx)^2}{8\,b} - \frac{i\,\sqrt{\pi}\operatorname{erfi}(bx)^2\sin(c)}{8\,b} + \frac{1}{2}\cos(c)\int \operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx + \frac{1}{2}i\int \operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*cos(c)*erfi(b*x)^2/b - 1/8*I*sqrt(pi)*erfi(b*x)^2*sin(c)/b + 1
/2*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) + 1/2*I*integrate(erfi(b*x)*
e^(-b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{2}\left(\operatorname{erfi}(bx)\,e^{\left(-2\,b^2x^2+2i\,c\right)} + \operatorname{erfi}(bx)\right)e^{\left(b^2x^2-i\,c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(erfi(b*x)*e^(-2*b^2*x^2 + 2*I*c) + erfi(b*x))*e^(b^2*x^2 - I*
c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(ib^2x^2 + c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b**2*x**2)*erfi(b*x),x)
```

```
[Out] Integral(cos(I*b**2*x**2 + c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(i\,b^2x^2 + c\right)\operatorname{erfi}(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+I*b^2*x^2)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(I*b^2*x^2 + c)*erfi(b*x), x)
```

3.307 $\int \cos\left(c - ib^2x^2\right) \mathbf{Erfi}(bx)\, dx$

Optimal. Leaf size=63

$$\frac{be^{-ic}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{ic}\text{Erfi}(bx)^2}{8b}$$

```
[Out] (E^(I*c)*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, -(b^2*x^2)])/(2*E^(I*c)*Sqrt[Pi])
```

Rubi [A] time = 0.0534083, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6409, 6378, 6375, 30}

$$\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{ic}\text{Erfi}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c - I*b^2*x^2]*Erfi[b*x],x]
```

```
[Out] (E^(I*c)*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, -(b^2*x^2)])/(2*E^(I*c)*Sqrt[Pi])
```

Rule 6409

```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int[
E^(-(I*c) - I*d*x^2)*Erfi[b*x], x], x] + Dist[1/2, Int[E^(I*c + I*d*x^2)*Er
fi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos\left(c-ib^2x^2\right)\operatorname{erfi}(bx)\,dx &= \frac{1}{2}\int e^{-ic-b^2x^2}\operatorname{erfi}(bx)\,dx+\frac{1}{2}\int e^{ic+b^2x^2}\operatorname{erfi}(bx)\,dx\\ &= \frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^{ic}\sqrt{\pi}\right)\operatorname{Subst}(\int x\,dx,x,\operatorname{erfi}(bx))}{4b}\\ &= \frac{e^{ic}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [F] time = 0.563999, size = 0, normalized size = 0.

$$\int \cos\left(c-ib^2x^2\right)\operatorname{Erfi}(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c - I*b^2*x^2]*Erfi[b*x],x]
```

```
[Out] Integrate[Cos[c - I*b^2*x^2]*Erfi[b*x], x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \cos\left(-c+ib^2x^2\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(-c+I*b^2*x^2)*erfi(b*x),x)
```

```
[Out] int(cos(-c+I*b^2*x^2)*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\pi}\cos(c)\operatorname{erfi}(bx)^2}{8b}+\frac{i\sqrt{\pi}\operatorname{erfi}(bx)^2\sin(c)}{8b}+\frac{1}{2}\cos(c)\int\operatorname{erfi}(bx)e^{\left(-b^2x^2\right)}\,dx-\frac{1}{2}i\int\operatorname{erfi}(bx)e^{\left(-b^2x^2\right)}\,dx\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*cos(c)*erfi(b*x)^2/b + 1/8*I*sqrt(pi)*erfi(b*x)^2*sin(c)/b + 1
/2*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) - 1/2*I*integrate(erfi(b*x)*
e^(-b^2*x^2), x)*sin(c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{2}\left(\operatorname{erfi}(bx)e^{\left(-2b^2x^2-2ic\right)}+\operatorname{erfi}(bx)\right)e^{\left(b^2x^2+ic\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(erfi(b*x)*e^(-2*b^2*x^2 - 2*I*c) + erfi(b*x))*e^(b^2*x^2 + I*
c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(ib^2x^2-c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b**2*x**2)*erfi(b*x),x)
```

```
[Out] Integral(cos(I*b**2*x**2 - c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(ib^2x^2 - c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-c+I*b^2*x^2)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(I*b^2*x^2 - c)*erfi(b*x), x)
```

3.308 $\int \mathbf{Erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{8b}-\frac{be^{-c}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, -(b^2*x^2)])/(2*E^c*Sqrt[Pi])
```

Rubi [A] time = 0.0508234, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6412, 6375, 30, 6378}

$$\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]*Sinh[c + b^2*x^2],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, -(b^2*x^2)])/(2*E^c*Sqrt[Pi])
```

Rule 6412

```
Int[Erfi[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfi[b*x], x], x] - Dist[1/2, Int[E^(-c - d*x^2)*Erfi[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\begin{aligned}\int \text{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx &= -\left(\frac{1}{2}\int e^{-c-b^2x^2}\text{erfi}(bx)\,dx\right)+\frac{1}{2}\int e^{c+b^2x^2}\text{erfi}(bx)\,dx\\ &= -\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^c\sqrt{\pi}\right)\text{Subst}\left(\int x\,dx,x,\text{erfi}(bx)\right)}{4b}\\ &= \frac{e^c\sqrt{\pi}\,\text{erfi}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 1.22833, size = 74, normalized size = 1.3

$$\frac{4b^2x^2(\cosh(c)-\sinh(c))\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+\pi\text{Erfi}(bx)(\text{Erfi}(bx)(\sinh(c)+\cosh(c))-2\text{Erf}(bx}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]*Sinh[c + b^2*x^2],x]
```

```
[Out] (4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] - Sinh[c])
 + Pi*Erfi[b*x]*(-2*Erf[b*x]*(Cosh[c] - Sinh[c]) + Erfi[b*x]*(Cosh[c] + Sin
h[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \text{erfi}\,(bx)\sinh\left(b^2x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erfi(b*x)*sinh(b^2*x^2+c),x)
```

```
[Out] int(erfi(b*x)*sinh(b^2*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sinh(b^2*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(erfi(b*x)*sinh(b^2*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{erfi}(bx)\sinh\left(b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sinh(b^2*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(erfi(b*x)*sinh(b^2*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(b^2x^2+c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sinh(b**2*x**2+c),x)
```

```
[Out] Integral(sinh(b**2*x**2 + c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{erfi}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erfi(b*x)*sinh(b^2*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(erfi(b*x)*sinh(b^2*x^2 + c), x)
```

3.309 $\int \mathbf{Erfi}(bx) \sinh\left(c - b^2x^2\right) dx$

Optimal. Leaf size=57

$$\frac{be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)^2}{8b}$$

```
[Out] -(Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0514296, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6412, 6378, 6375, 30}

$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Erfi[b*x]*Sinh[c - b^2*x^2],x]
```

```
[Out] -(Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Rule 6412

```
Int[Erfi[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfi[b*x], x], x] - Dist[1/2, Int[E^(-c - d*x^2)*Erfi[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx &= \frac{1}{2}\int e^{c-b^2x^2}\operatorname{erfi}(bx)\,dx-\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erfi}(bx)\,dx\\ &= \frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}-\frac{\left(e^{-c}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(bx)\right)}{4b}\\ &= -\frac{e^{-c}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.767608, size = 72, normalized size = 1.26

$$\frac{\pi\operatorname{Erfi}(bx)(2\operatorname{Erf}(bx)(\sinh(c)+\cosh(c))+\operatorname{Erfi}(bx)(\sinh(c)-\cosh(c)))-4b^2x^2(\sinh(c)+\cosh(c))\text{HypergeometricPFQ}}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erfi[b*x]*Sinh[c - b^2*x^2],x]
```

```
[Out] (-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c]
) + Pi*Erfi[b*x]*(Erfi[b*x]*(-Cosh[c] + Sinh[c]) + 2*Erf[b*x]*(Cosh[c] + Si
nh[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int -\operatorname{erfi}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-erfi(b*x)*sinh(b^2*x^2-c),x)
```

```
[Out] int(-erfi(b*x)*sinh(b^2*x^2-c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \operatorname{erfi}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sinh(b^2*x^2-c),x, algorithm="maxima")
```

```
[Out] -integrate(erfi(b*x)*sinh(b^2*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\operatorname{erfi}(bx)\sinh\left(b^2x^2-c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sinh(b^2*x^2-c),x, algorithm="fricas")
```

```
[Out] integral(-erfi(b*x)*sinh(b^2*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \sinh\left(b^2x^2-c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sinh(b**2*x**2-c),x)
```

```
[Out] -Integral(sinh(b**2*x**2 - c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{erfi}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-erfi(b*x)*sinh(b^2*x^2-c),x, algorithm="giac")
```

```
[Out] integrate(-erfi(b*x)*sinh(b^2*x^2 - c), x)
```

3.310 $\int \cosh\left(c+b^2x^2\right) \mathbf{Erfi}(bx) \, dx$

Optimal. Leaf size=57

$$\frac{be^{-c}x^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{8b}$$

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, -(b^2*x^2)])/(2*E^c*Sqrt[Pi])
```

Rubi [A] time = 0.0490827, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6415, 6375, 30, 6378}

$$\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^c\text{Erfi}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + b^2*x^2]*Erfi[b*x],x]
```

```
[Out] (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, -(b^2*x^2)])/(2*E^c*Sqrt[Pi])
```

Rule 6415

```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfi[b*x], x], x] + Dist[1/2, Int[E^(-c - d*x^2)*Erfi[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rubi steps

$$\begin{aligned}\int \cosh\left(c+b^2x^2\right)\operatorname{erfi}(bx)\,dx &= \frac{1}{2}\int e^{-c-b^2x^2}\operatorname{erfi}(bx)\,dx+\frac{1}{2}\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx\\ &= \frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^c\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(bx)\right)}{4b}\\ &= \frac{e^c\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 10.6839, size = 74, normalized size = 1.3

$$\frac{4b^2x^2(\sinh(c)-\cosh(c))\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},b^2x^2\right)+\pi\text{Erfi}(bx)(2\text{Erf}(bx)(\cosh(c)-\sinh(c))+\text{Erfi}(bx}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + b^2*x^2]*Erfi[b*x],x]
```

```
[Out] (4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(-Cosh[c] + Sinh[c]
) + Pi*Erfi[b*x]*(2*Erf[b*x]*(Cosh[c] - Sinh[c]) + Erfi[b*x]*(Cosh[c] + Sin
h[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2+c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b^2*x^2+c)*erfi(b*x),x)
```

```
[Out] int(cosh(b^2*x^2+c)*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2 + c\right) \operatorname{erfi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b^2*x^2 + c)*erfi(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\cosh \left(b^2x^2 + c\right) \operatorname{erfi}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(b^2*x^2 + c)*erfi(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2 + c\right) \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b**2*x**2+c)*erfi(b*x),x)
```

```
[Out] Integral(cosh(b**2*x**2 + c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh \left(b^2x^2 + c\right) \operatorname{erfi}(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2+c)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(b^2*x^2 + c)*erfi(b*x), x)
```

3.311 $\int \cosh\left(c-b^2x^2\right)\mathbf{Erfi}(bx)\,dx$

Optimal. Leaf size=57

$$\frac{be^cx^2\text{HypergeometricPFQ}\left(\{1,1\},\left\{\frac{3}{2},2\right\},-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)^2}{8b}$$

```
[Out] (Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.0508761, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6415, 6378, 6375, 30}

$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{Erfi}(bx)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c - b^2*x^2]*Erfi[b*x],x]
```

```
[Out] (Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Rule 6415

```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Dist[1/2, Int
[E^(c + d*x^2)*Erfi[b*x], x], x] + Dist[1/2, Int[E^(-c - d*x^2)*Erfi[b*x],
x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh\left(c-b^2x^2\right)\operatorname{erfi}(bx)\,dx &= \frac{1}{2}\int e^{c-b^2x^2}\operatorname{erfi}(bx)\,dx+\frac{1}{2}\int e^{-c+b^2x^2}\operatorname{erfi}(bx)\,dx\\ &= \frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\left(e^{-c}\sqrt{\pi}\right)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}(bx)\right)}{4b}\\ &= \frac{e^{-c}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 1.58836, size = 72, normalized size = 1.26

$$\frac{\pi\operatorname{Erfi}(bx)(2\operatorname{Erf}(bx)(\sinh(c)+\cosh(c))+\operatorname{Erfi}(bx)(\cosh(c)-\sinh(c)))-4b^2x^2(\sinh(c)+\cosh(c))\text{HypergeometricPFQ}}{8\sqrt{\pi}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c - b^2*x^2]*Erfi[b*x],x]
```

```
[Out] (-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c]
) + Pi*Erfi[b*x]*(Erfi[b*x]*(Cosh[c] - Sinh[c]) + 2*Erf[b*x]*(Cosh[c] + Sin
h[c])))/(8*b*Sqrt[Pi])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2-c\right)\operatorname{erfi}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b^2*x^2-c)*erfi(b*x),x)
```

```
[Out] int(cosh(b^2*x^2-c)*erfi(b*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2 - c\right) \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erfi(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b^2*x^2 - c)*erfi(b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(b^2x^2 - c\right) \operatorname{erfi}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erfi(b*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(b^2*x^2 - c)*erfi(b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2 - c\right) \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b**2*x**2-c)*erfi(b*x),x)
```

```
[Out] Integral(cosh(b**2*x**2 - c)*erfi(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(b^2x^2 - c\right) \operatorname{erfi}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b^2*x^2-c)*erfi(b*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(b^2*x^2 - c)*erfi(b*x), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
    ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```