

Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/7.5.2-Inverse-hyperbolic-secant-functions

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3.43	$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7} \, dx$	255
3.44	$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8} \, dx$	260
3.45	$\int e^{\operatorname{sech}^{-1}(ax^2)} x^7 \, dx$	265
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3.57	$\int e^{\operatorname{sech}^{-1}(ax^2)} x^m \, dx$	317
3.58	$\int e^{\operatorname{sech}^{-1}(ax)} x^m \, dx$	321
3.59	$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx$	325
3.60	$\int e^{\operatorname{sech}^{-1}(ax^p)} x^m \, dx$	329
3.61	$\int e^{\operatorname{sech}^{-1}(ax^p)} x \, dx$	333
3.62	$\int e^{\operatorname{sech}^{-1}(ax^p)} \, dx$	337
3.63	$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} \, dx$	341
3.64	$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2} \, dx$	346

3.65	$\int e^{2\operatorname{sech}^{-1}(ax)} x^4 dx$	350
3.66	$\int e^{2\operatorname{sech}^{-1}(ax)} x^3 dx$	355
3.67	$\int e^{2\operatorname{sech}^{-1}(ax)} x^2 dx$	360
3.68	$\int e^{2\operatorname{sech}^{-1}(ax)} x dx$	365
3.69	$\int e^{2\operatorname{sech}^{-1}(ax)} dx$	370
3.70	$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x} dx$	375
3.71	$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2} dx$	379
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3.73	$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^4} dx$	387
3.74	$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^5} dx$	391
3.75	$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6} dx$	395
3.76	$\int e^{-\operatorname{sech}^{-1}(ax)} x^4 dx$	399
3.77	$\int e^{-\operatorname{sech}^{-1}(ax)} x^3 dx$	404
3.78	$\int e^{-\operatorname{sech}^{-1}(ax)} x^2 dx$	409
3.79	$\int e^{-\operatorname{sech}^{-1}(ax)} x dx$	413
3.80	$\int e^{-\operatorname{sech}^{-1}(ax)} dx$	417
3.81	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x} dx$	422
3.82	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2} dx$	426
3.83	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3} dx$	430
3.84	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4} dx$	434
3.85	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5} dx$	438
3.86	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6} dx$	442
3.87	$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7} dx$	446
3.88	$\int \frac{e^{\operatorname{sech}^{-1}(cx)} (dx)^m}{1-c^2x^2} dx$	450
3.89	$\int \frac{e^{\operatorname{sech}^{-1}(cx)} x^4}{1-c^2x^2} dx$	454
3.90	$\int \frac{e^{\operatorname{sech}^{-1}(cx)} x^3}{1-c^2x^2} dx$	459
3.91	$\int \frac{e^{\operatorname{sech}^{-1}(cx)} x^2}{1-c^2x^2} dx$	464
3.92	$\int \frac{e^{\operatorname{sech}^{-1}(cx)} x}{1-c^2x^2} dx$	468
3.93	$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2} dx$	472

3.94	$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)} dx$	477
3.95	$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)} dx$	481
3.96	$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3(1-c^2x^2)} dx$	486
3.97	$\int \frac{x^{(-1+ae^{\operatorname{sech}^{-1}(ax)x})}}{1-a^2x^2} dx$	491
3.98	$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$	495
3.99	$\int x^3 \operatorname{sech}^{-1}(a+bx^4) dx$	500
3.100	$\int x^{-1+n} \operatorname{sech}^{-1}(a+bx^n) dx$	504
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [100]. This is test number [201].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (100)	% 0. (0)
Mathematica	% 98. (98)	% 2. (2)
Maple	% 74. (74)	% 26. (26)
Maxima	% 21. (21)	% 79. (79)
Fricas	% 69. (69)	% 31. (31)
Sympy	% 1. (1)	% 99. (99)
Giac	% 4. (4)	% 96. (96)

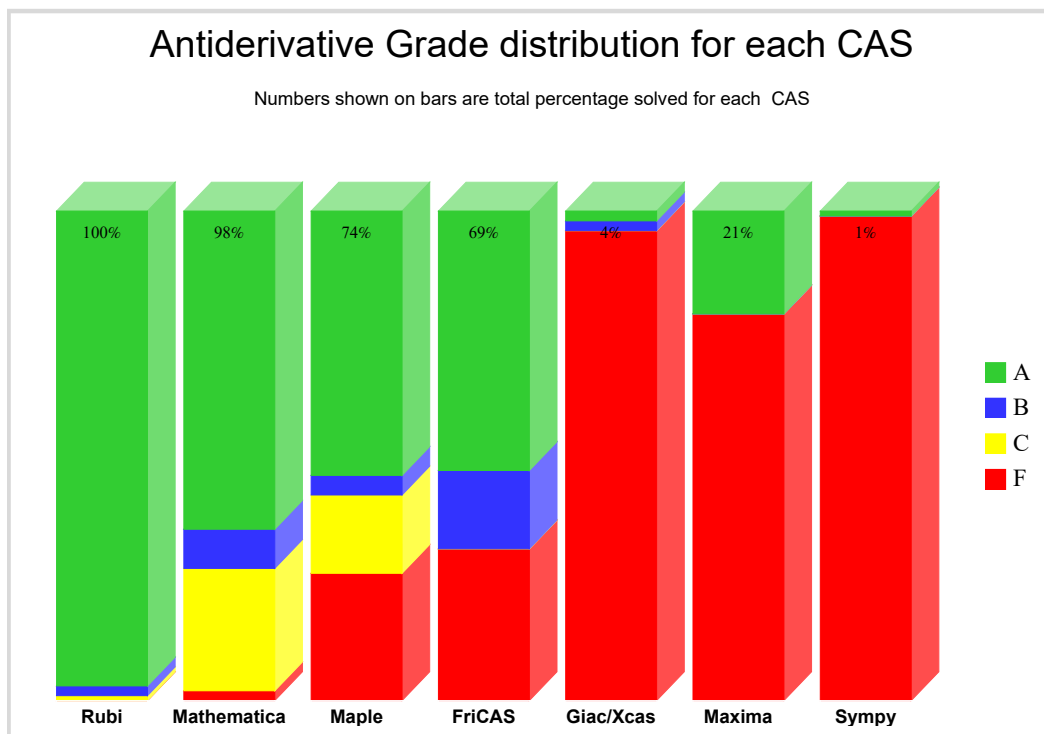
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

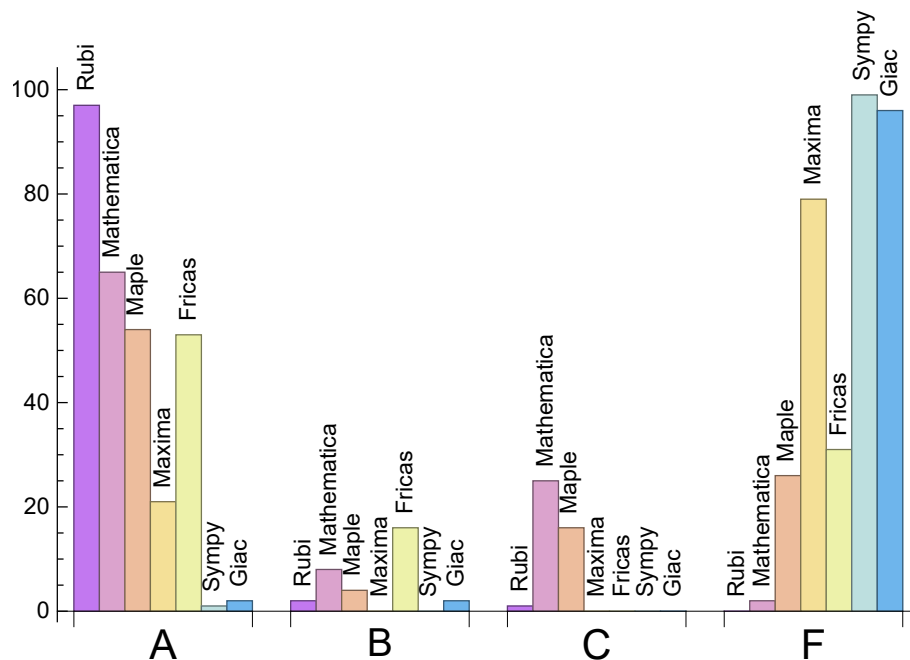
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	97.	2.	1.	0.
Mathematica	65.	8.	25.	2.
Maple	54.	4.	16.	26.
Maxima	21.	0.	0.	79.
Fricas	53.	16.	0.	31.
Sympy	1.	0.	0.	99.
Giac	2.	2.	0.	96.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	135.47	1.06	107.	1.
Mathematica	0.86	155.57	1.28	98.5	1.1
Maple	0.24	224.51	2.23	102.5	1.23
Maxima	1.02	67.24	0.78	62.	0.9
Fricas	2.08	313.65	3.61	197.	2.28
Sympy	15.49	48.	0.68	48.	0.68
Giac	1.17	208.25	2.82	183.	2.73

1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {32, 34, 36, 37, 38, 39, 47, 53, 63, 97}

Mathematica {4, 5, 9, 10, 11, 12, 15, 16, 17, 18, 24, 30, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

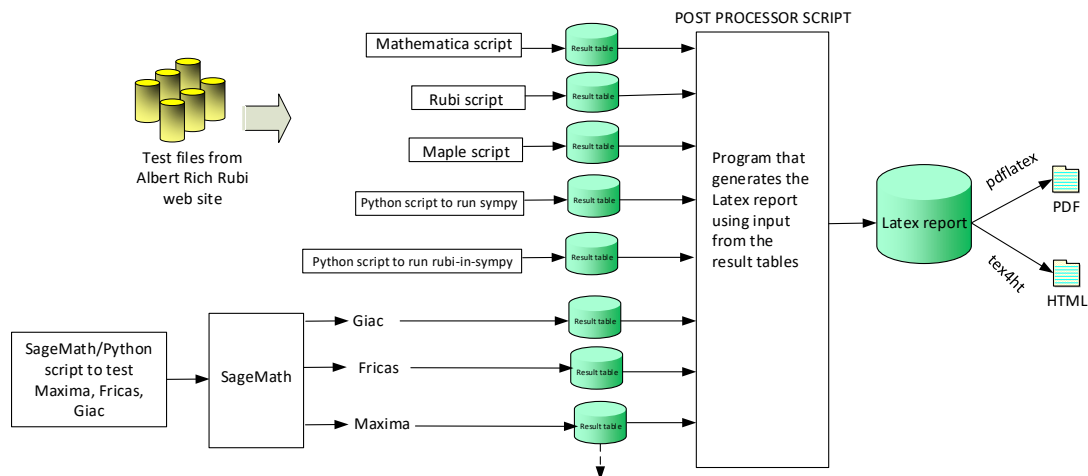
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100 }

B grade: { 38, 97 }

C grade: { 39 }

F grade: { }

2.1.2 Mathematica

A grade: { 8, 9, 10, 11, 12, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 34, 39, 40, 41, 42, 43, 44, 47, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 83, 84, 85, 86, 87, 89, 91, 93, 94, 95, 96, 98, 99, 100 }

B grade: { 4, 6, 7, 29, 31, 36, 38, 97 }

C grade: { 1, 2, 3, 5, 13, 14, 18, 33, 35, 37, 45, 46, 48, 49, 50, 52, 54, 63, 65, 67, 77, 79, 81, 90, 92 }

F grade: { 19, 88 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 9, 10, 11, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 66, 68, 70, 71, 72, 73, 74, 75, 89, 91, 93, 95, 97, 98, 99 }
}

B grade: { 6, 7, 8, 80 }

C grade: { 5, 33, 35, 37, 51, 55, 63, 65, 67, 69, 76, 78, 90, 92, 94, 96 }

F grade: { 12, 15, 16, 17, 18, 19, 30, 56, 57, 58, 59, 60, 61, 62, 64, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 100 }

2.1.4 Maxima

A grade: { 4, 20, 21, 22, 23, 25, 26, 27, 28, 32, 34, 39, 41, 43, 47, 66, 71, 73, 75, 99, 100 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 33, 35, 36, 37, 38, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 72, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

2.1.5 FriCAS

A grade: { 1, 20, 21, 22, 23, 25, 26, 27, 28, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 45, 47, 49, 55, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 91, 94, 96, 97 }

B grade: { 2, 3, 4, 6, 7, 8, 36, 38, 51, 53, 90, 92, 93, 95, 99, 100 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 46, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 88, 98 }

2.1.6 Sympy

A grade: { 93 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55,

56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100 }

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A grade: { 49, 71 }

B grade: { 45, 47 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	225	250	0	772	0	0
normalized size	1	1.	1.11	1.23	0.	3.8	0.	0.
time (sec)	N/A	0.155	0.494	0.292	0.	2.526	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	200	189	0	720	0	0
normalized size	1	1.	1.31	1.24	0.	4.71	0.	0.
time (sec)	N/A	0.099	0.258	0.244	0.	2.351	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	176	111	0	687	0	0
normalized size	1	1.	1.64	1.04	0.	6.42	0.	0.
time (sec)	N/A	0.062	0.161	0.234	0.	2.263	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	95	50	42	567	0	0
normalized size	1	1.	2.16	1.14	0.95	12.89	0.	0.
time (sec)	N/A	0.056	0.196	0.213	0.978	2.147	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	332	886	0	0	0	0
normalized size	1	1.	1.95	5.21	0.	0.	0.	0.
time (sec)	N/A	0.29	0.192	0.352	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	244	542	0	1462	0	0
normalized size	1	1.	3.49	7.74	0.	20.89	0.	0.
time (sec)	N/A	0.106	0.215	0.252	0.	2.199	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	315	1233	0	1912	0	0
normalized size	1	1.	2.37	9.27	0.	14.38	0.	0.
time (sec)	N/A	0.204	0.907	0.259	0.	2.481	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	368	1946	0	2163	0	0
normalized size	1	1.	1.87	9.88	0.	10.98	0.	0.
time (sec)	N/A	0.316	0.402	0.256	0.	2.668	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	305	655	0	0	0	0
normalized size	1	1.	1.09	2.35	0.	0.	0.	0.
time (sec)	N/A	0.242	1.915	0.483	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	172	396	0	0	0	0
normalized size	1	1.	1.15	2.66	0.	0.	0.	0.
time (sec)	N/A	0.138	0.418	0.392	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	105	209	0	0	0	0
normalized size	1	1.	1.31	2.61	0.	0.	0.	0.
time (sec)	N/A	0.068	0.21	0.285	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	280	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.463	0.295	0.392	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	678	367	0	0	0	0
normalized size	1	1.	3.03	1.64	0.	0.	0.	0.
time (sec)	N/A	0.389	2.625	0.357	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	1439	1026	0	0	0	0
normalized size	1	1.	2.68	1.91	0.	0.	0.	0.
time (sec)	N/A	0.75	7.763	0.563	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	0.552	0.559	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	153	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.14	0.353	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	384	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.511	0.319	0.373	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	1849	0	0	0	0	0
normalized size	1	1.	5.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.575	45.653	0.458	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	965	965	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.338	9.65	0.694	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	84	54	78	144	0	0
normalized size	1	1.	0.51	0.33	0.48	0.88	0.	0.
time (sec)	N/A	0.03	0.041	0.139	0.982	1.838	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	72	49	62	131	0	0
normalized size	1	1.	0.57	0.39	0.49	1.04	0.	0.
time (sec)	N/A	0.027	0.04	0.126	0.983	1.872	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	42	46	116	0	0
normalized size	1	1.	0.64	0.48	0.52	1.32	0.	0.
time (sec)	N/A	0.018	0.026	0.13	0.987	1.865	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	36	26	92	0	0
normalized size	1	1.	1.56	0.84	0.6	2.14	0.	0.
time (sec)	N/A	0.007	0.096	0.123	0.972	1.806	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	45	65	0	0	0	0
normalized size	1	1.	0.98	1.41	0.	0.	0.	0.
time (sec)	N/A	0.095	0.036	0.16	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	111	64	88	111	0	0
normalized size	1	1.	1.13	0.65	0.9	1.13	0.	0.
time (sec)	N/A	0.023	0.077	0.134	0.98	1.878	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	79	124	134	0	0
normalized size	1	1.	0.92	0.58	0.91	0.99	0.	0.
time (sec)	N/A	0.033	0.112	0.133	0.993	1.885	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	140	91	157	153	0	0
normalized size	1	1.	0.81	0.53	0.91	0.89	0.	0.
time (sec)	N/A	0.04	0.128	0.141	0.99	1.93	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	29	22	57	0	0
normalized size	1	1.	1.19	1.38	1.05	2.71	0.	0.
time (sec)	N/A	0.007	0.024	0.101	1.024	1.811	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	219	116	0	0	0	0
normalized size	1	1.	3.59	1.9	0.	0.	0.	0.
time (sec)	N/A	0.105	0.957	0.333	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	49	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.046	0.284	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	249	140	0	0	0	0
normalized size	1	1.	3.23	1.82	0.	0.	0.	0.
time (sec)	N/A	0.101	1.171	0.361	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	64	83	65	64	63	135	0	0
normalized size	1	1.3	1.02	1.	0.98	2.11	0.	0.
time (sec)	N/A	0.038	0.09	0.189	1.078	1.714	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	97	118	0	203	0	0
normalized size	1	1.	1.15	1.4	0.	2.42	0.	0.
time (sec)	N/A	0.032	0.127	0.179	0.	1.778	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	38	52	48	54	51	111	0	0
normalized size	1	1.37	1.26	1.42	1.34	2.92	0.	0.
time (sec)	N/A	0.024	0.059	0.173	1.07	1.821	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	75	92	0	171	0	0
normalized size	1	1.	1.42	1.74	0.	3.23	0.	0.
time (sec)	N/A	0.018	0.076	0.184	0.	1.808	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	24	39	79	79	0	258	0	0
normalized size	1	1.62	3.29	3.29	0.	10.75	0.	0.
time (sec)	N/A	0.137	0.04	0.184	0.	1.832	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	48	64	75	92	0	165	0	0
normalized size	1	1.33	1.56	1.92	0.	3.44	0.	0.
time (sec)	N/A	0.031	0.05	0.189	0.	2.024	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	A	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	35	99	93	90	0	278	0	0
normalized size	1	2.83	2.66	2.57	0.	7.94	0.	0.
time (sec)	N/A	0.041	0.06	0.204	0.	1.81	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	55	84	43	53	58	108	0	0
normalized size	1	1.53	0.78	0.96	1.05	1.96	0.	0.
time (sec)	N/A	0.037	0.043	0.178	1.045	1.835	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	110	110	0	298	0	0
normalized size	1	1.	0.83	0.83	0.	2.26	0.	0.
time (sec)	N/A	0.059	0.073	0.184	0.	1.809	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	60	63	69	128	0	0
normalized size	1	1.	0.52	0.55	0.6	1.11	0.	0.
time (sec)	N/A	0.052	0.067	0.179	1.04	1.922	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	129	132	0	324	0	0
normalized size	1	1.	0.79	0.81	0.	1.99	0.	0.
time (sec)	N/A	0.077	0.104	0.184	0.	2.194	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	76	71	81	151	0	0
normalized size	1	1.	0.52	0.49	0.55	1.03	0.	0.
time (sec)	N/A	0.071	0.085	0.191	1.037	2.129	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	145	152	0	348	0	0
normalized size	1	1.	0.75	0.78	0.	1.79	0.	0.
time (sec)	N/A	0.098	0.133	0.203	0.	2.133	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	111	137	0	251	0	360
normalized size	1	1.	1.	1.23	0.	2.26	0.	3.24
time (sec)	N/A	0.062	0.181	0.322	0.	2.125	0.	1.195

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	139	114	0	0	0	0
normalized size	1	1.	1.21	0.99	0.	0.	0.	0.
time (sec)	N/A	0.051	0.332	0.19	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	58	71	56	60	57	126	0	240
normalized size	1	1.22	0.97	1.03	0.98	2.17	0.	4.14
time (sec)	N/A	0.041	0.094	0.178	1.068	2.061	0.	1.182

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	140	136	0	0	0	0
normalized size	1	1.	1.25	1.21	0.	0.	0.	0.
time (sec)	N/A	0.067	0.435	0.181	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	92	112	0	225	0	107
normalized size	1	1.	1.46	1.78	0.	3.57	0.	1.7
time (sec)	N/A	0.037	0.104	0.33	0.	2.053	0.	1.163

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	116	102	0	0	0	0
normalized size	1	1.	1.73	1.52	0.	0.	0.	0.
time (sec)	N/A	0.028	0.189	0.204	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	100	127	0	298	0	0
normalized size	1	1.	1.47	1.87	0.	4.38	0.	0.
time (sec)	N/A	0.043	0.063	0.312	0.	2.099	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	135	132	0	0	0	0
normalized size	1	1.	0.92	0.9	0.	0.	0.	0.
time (sec)	N/A	0.068	0.282	0.187	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	80	93	113	103	0	230	0	0
normalized size	1	1.16	1.41	1.29	0.	2.88	0.	0.
time (sec)	N/A	0.053	0.144	0.284	0.	2.013	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	123	104	0	0	0	0
normalized size	1	1.	1.07	0.9	0.	0.	0.	0.
time (sec)	N/A	0.051	0.192	0.208	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	105	129	0	319	0	0
normalized size	1	1.	0.89	1.09	0.	2.7	0.	0.
time (sec)	N/A	0.061	0.256	0.299	0.	2.034	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	159	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	2.495	0.23	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	159	0	0	0	0	0
normalized size	1	1.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	2.303	0.194	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	145	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.302	0.174	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	139	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.825	0.643	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	186	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	4.953	1.224	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	159	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.297	1.102	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	139	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.386	1.154	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	87	106	96	145	0	197	0	0
normalized size	1	1.22	1.1	1.67	0.	2.26	0.	0.
time (sec)	N/A	0.069	0.137	0.326	0.	2.454	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	156	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.259	1.182	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	105	123	0	225	0	0
normalized size	1	1.	0.52	0.61	0.	1.11	0.	0.
time (sec)	N/A	0.7	0.141	0.206	0.	2.091	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	52	72	57	131	0	0
normalized size	1	1.	0.44	0.62	0.49	1.12	0.	0.
time (sec)	N/A	0.546	0.068	0.181	1.045	1.946	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	86	97	0	192	0	0
normalized size	1	1.	0.51	0.57	0.	1.14	0.	0.
time (sec)	N/A	0.465	0.07	0.182	0.	2.116	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	89	89	0	281	0	0
normalized size	1	1.	1.05	1.05	0.	3.31	0.	0.
time (sec)	N/A	0.434	0.06	0.208	0.	2.106	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	75	98	0	186	0	0
normalized size	1	1.	1.32	1.72	0.	3.26	0.	0.
time (sec)	N/A	0.174	0.095	0.209	0.	2.301	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	86	97	0	306	0	0
normalized size	1	1.	1.	1.13	0.	3.56	0.	0.
time (sec)	N/A	0.446	0.066	0.187	0.	2.149	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	73	62	130	0	126
normalized size	1	1.	0.91	1.28	1.09	2.28	0.	2.21
time (sec)	N/A	0.39	0.066	0.194	1.043	2.055	0.	1.152

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	121	131	0	316	0	0
normalized size	1	1.	0.82	0.89	0.	2.15	0.	0.
time (sec)	N/A	0.453	0.123	0.192	0.	2.123	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	69	84	76	150	0	0
normalized size	1	1.	0.38	0.46	0.42	0.82	0.	0.
time (sec)	N/A	0.505	0.079	0.196	1.046	2.166	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	137	153	0	344	0	0
normalized size	1	1.	0.51	0.57	0.	1.29	0.	0.
time (sec)	N/A	0.543	0.143	0.227	0.	2.116	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	301	301	85	92	88	174	0	0
normalized size	1	1.	0.28	0.31	0.29	0.58	0.	0.
time (sec)	N/A	0.576	0.099	0.24	1.078	2.082	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	65	531	0	135	0	0
normalized size	1	1.	0.44	3.61	0.	0.92	0.	0.
time (sec)	N/A	0.619	0.09	0.54	0.	2.037	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	97	0	0	203	0	0
normalized size	1	1.	0.6	0.	0.	1.25	0.	0.
time (sec)	N/A	0.569	0.113	180.	0.	2.07	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	48	269	0	111	0	0
normalized size	1	1.	0.64	3.59	0.	1.48	0.	0.
time (sec)	N/A	0.513	0.055	0.496	0.	1.91	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	173	0	0
normalized size	1	1.	0.8	0.	0.	1.84	0.	0.
time (sec)	N/A	0.309	0.068	180.	0.	2.024	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	72	2632	0	259	0	0
normalized size	1	1.	1.11	40.49	0.	3.98	0.	0.
time (sec)	N/A	0.166	0.031	0.475	0.	2.081	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	74	0	0	163	0	0
normalized size	1	1.	1.61	0.	0.	3.54	0.	0.
time (sec)	N/A	0.401	0.036	180.	0.	2.303	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	92	0	0	279	0	0
normalized size	1	1.	1.28	0.	0.	3.88	0.	0.
time (sec)	N/A	0.383	0.057	180.	0.	2.028	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	43	0	0	109	0	0
normalized size	1	1.	0.37	0.	0.	0.94	0.	0.
time (sec)	N/A	0.424	0.052	180.	0.	2.066	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	110	0	0	300	0	0
normalized size	1	1.	0.55	0.	0.	1.5	0.	0.
time (sec)	N/A	0.5	0.1	180.	0.	2.077	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	60	0	0	130	0	0
normalized size	1	1.	0.26	0.	0.	0.56	0.	0.
time (sec)	N/A	0.5	0.069	180.	0.	2.032	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	129	0	0	325	0	0
normalized size	1	1.	0.4	0.	0.	1.02	0.	0.
time (sec)	N/A	0.567	0.141	180.	0.	2.096	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	76	0	0	153	0	0
normalized size	1	1.	0.22	0.	0.	0.43	0.	0.
time (sec)	N/A	0.605	0.097	180.	0.	2.094	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	0.635	0.434	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	69	69	0	149	0	0
normalized size	1	1.	0.78	0.78	0.	1.69	0.	0.
time (sec)	N/A	0.181	0.189	0.211	0.	2.067	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	110	117	0	213	0	0
normalized size	1	1.	1.47	1.56	0.	2.84	0.	0.
time (sec)	N/A	0.165	0.154	0.213	0.	2.093	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	52	0	111	0	0
normalized size	1	1.	0.98	1.16	0.	2.47	0.	0.
time (sec)	N/A	0.125	0.09	0.2	0.	2.117	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	68	92	0	131	0	0
normalized size	1	1.	1.84	2.49	0.	3.54	0.	0.
time (sec)	N/A	0.114	0.044	0.227	0.	2.102	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	73	87	0	213	48	0
normalized size	1	1.	1.03	1.23	0.	3.	0.68	0.
time (sec)	N/A	0.119	0.04	0.204	0.	2.076	15.488	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	59	61	0	144	0	0
normalized size	1	1.	1.4	1.45	0.	3.43	0.	0.
time (sec)	N/A	0.143	0.164	0.21	0.	2.162	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	108	111	0	344	0	0
normalized size	1	1.	1.	1.03	0.	3.19	0.	0.
time (sec)	N/A	0.176	0.151	0.227	0.	2.12	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	90	86	0	197	0	0
normalized size	1	1.	1.06	1.01	0.	2.32	0.	0.
time (sec)	N/A	0.165	0.242	0.22	0.	2.206	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	A	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	12	26	28	36	0	69	0	0
normalized size	1	2.17	2.33	3.	0.	5.75	0.	0.
time (sec)	N/A	1.052	0.244	0.15	0.	2.042	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	52	104	0	0	0	0
normalized size	1	1.	0.85	1.7	0.	0.	0.	0.
time (sec)	N/A	0.098	0.055	0.283	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	84	62	51	613	0	0
normalized size	1	1.	1.47	1.09	0.89	10.75	0.	0.
time (sec)	N/A	0.115	0.202	0.289	0.994	2.473	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	84	0	54	1138	0	0
normalized size	1	1.	1.45	0.	0.93	19.62	0.	0.
time (sec)	N/A	0.113	0.248	0.401	0.999	2.608	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [15] had the largest ratio of [1.2]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.	10	0.7

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	7	6	1.	10	0.6
3	A	6	6	1.	8	0.75
4	A	4	4	1.	6	0.667
5	A	14	8	1.	10	0.8
6	A	5	5	1.	10	0.5
7	A	7	7	1.	10	0.7
8	A	8	8	1.	10	0.8
9	A	17	9	1.	12	0.75
10	A	11	8	1.	10	0.8
11	A	8	6	1.	8	0.75
12	A	17	9	1.	12	0.75
13	A	12	8	1.	12	0.667
14	A	23	11	1.	12	0.917
15	A	16	12	1.	10	1.2
16	A	10	7	1.	8	0.875
17	A	20	10	1.	12	0.833
18	A	14	9	1.	12	0.75
19	A	32	13	1.	12	1.083
20	A	4	3	1.	10	0.3
21	A	4	3	1.	10	0.3
22	A	4	3	1.	8	0.375
23	A	3	3	1.	6	0.5
24	A	7	6	1.	10	0.6
25	A	5	5	1.	10	0.5
26	A	6	5	1.	10	0.5
27	A	7	5	1.	10	0.5
28	A	3	3	1.	4	0.75
29	A	7	6	1.	10	0.6
30	A	7	6	1.	10	0.6
31	A	7	7	1.	10	0.7
32	A	5	5	1.3	10	0.5
33	A	5	5	1.	10	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	3	3	1.37	10	0.3
35	A	4	4	1.	8	0.5
36	A	3	3	1.62	6	0.5
37	A	5	5	1.33	10	0.5
38	B	6	6	2.83	10	0.6
39	C	5	5	1.53	10	0.5
40	A	8	6	1.	10	0.6
41	A	7	5	1.	10	0.5
42	A	10	6	1.	10	0.6
43	A	9	5	1.	10	0.5
44	A	12	6	1.	10	0.6
45	A	6	6	1.	12	0.5
46	A	5	5	1.	12	0.417
47	A	4	4	1.22	12	0.333
48	A	7	7	1.	12	0.583
49	A	5	5	1.	12	0.417
50	A	4	4	1.	12	0.333
51	A	6	6	1.	10	0.6
52	A	8	8	1.	8	1.
53	A	5	5	1.16	12	0.417
54	A	5	5	1.	12	0.417
55	A	7	7	1.	12	0.583
56	A	4	4	1.	12	0.333
57	A	4	4	1.	12	0.333
58	A	4	4	1.	10	0.4
59	A	5	5	1.	12	0.417
60	A	4	4	1.	12	0.333
61	A	4	4	1.	10	0.4
62	A	4	4	1.	8	0.5
63	A	6	6	1.22	12	0.5
64	A	4	4	1.	12	0.333
65	A	9	6	1.	12	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
66	A	8	6	1.	12	0.5
67	A	7	6	1.	12	0.5
68	A	8	5	1.	10	0.5
69	A	7	5	1.	8	0.625
70	A	5	3	1.	12	0.25
71	A	4	2	1.	12	0.167
72	A	5	3	1.	12	0.25
73	A	4	2	1.	12	0.167
74	A	5	3	1.	12	0.25
75	A	4	2	1.	12	0.167
76	A	8	5	1.	12	0.417
77	A	7	5	1.	12	0.417
78	A	6	5	1.	12	0.417
79	A	5	4	1.	10	0.4
80	A	6	4	1.	8	0.5
81	A	5	3	1.	12	0.25
82	A	5	3	1.	12	0.25
83	A	4	2	1.	12	0.167
84	A	5	3	1.	12	0.25
85	A	4	2	1.	12	0.167
86	A	5	3	1.	12	0.25
87	A	4	2	1.	12	0.167
88	A	5	4	1.	24	0.167
89	A	8	7	1.	22	0.318
90	A	7	7	1.	22	0.318
91	A	4	4	1.	22	0.182
92	A	5	5	1.	20	0.25
93	A	8	8	1.	19	0.421
94	A	5	5	1.	22	0.227
95	A	9	8	1.	22	0.364
96	A	8	7	1.	22	0.318
97	B	7	5	2.17	25	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	8	8	1.	19	0.421
99	A	5	5	1.	12	0.417
100	A	5	5	1.	14	0.357

Chapter 3

Listing of integrals

3.1 $\int x^3 \operatorname{sech}^{-1}(a + bx) dx$

Optimal. Leaf size=203

$$-\frac{(17a^2 + 2) \sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)}{12b^4} + \frac{(2a^2 + 1) a \tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{2b^4} - \frac{a^4 \operatorname{sech}^{-1}(a + bx)}{4b^4} - \frac{x^2 \sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)}{12b^2}$$

[Out] $-\left(\left(2 + 17a^2\right)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)\right)/(12b^4) - \left(x^2\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)\right)/(12b^2) + \left(a(a + bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)\right)/(3b^4) - \left(a^4\operatorname{ArcSech}[a + bx]\right)/(4b^4) + \left(x^4\operatorname{ArcSech}[a + bx]\right)/4 + \left(a(1 + 2a^2)\operatorname{ArcTan}\left[\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right]\right)/(2b^4)$

Rubi [A] time = 0.155323, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {6321, 5468, 3782, 4048, 3770, 3767, 8}

$$-\frac{(17a^2 + 2) \sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)}{12b^4} + \frac{(2a^2 + 1) a \tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{2b^4} - \frac{a^4 \operatorname{sech}^{-1}(a + bx)}{4b^4} - \frac{x^2 \sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)}{12b^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3 \operatorname{ArcSech}[a + bx], x]$

```
[Out] -((2 + 17*a^2)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(12*b^4) -
(x^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(12*b^2) + (a*(a + b*
x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(3*b^4) - (a^4*ArcSech[
a + b*x])/(4*b^4) + (x^4*ArcSech[a + b*x])/4 + (a*(1 + 2*a^2)*ArcTan[(Sqrt[
(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/(2*b^4)
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e
+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] := -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int x^3 \operatorname{sech}^{-1}(a + bx) dx &= -\frac{\operatorname{Subst}\left(\int x \operatorname{sech}(x)(-a + \operatorname{sech}(x))^3 \tanh(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^4} \\
 &= \frac{1}{4} x^4 \operatorname{sech}^{-1}(a + bx) - \frac{\operatorname{Subst}\left(\int (-a + \operatorname{sech}(x))^4 dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{4b^4} \\
 &= -\frac{x^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(a + bx) - \frac{\operatorname{Subst}\left(\int (-a + \operatorname{sech}(x))(-3a^3 + (2 + 9a^2)) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{12b^4} \\
 &= -\frac{x^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx) \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(a + bx) - \frac{\operatorname{Subst}\left(\int (-a + \operatorname{sech}(x))(-3a^3 + (2 + 9a^2)) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{12b^4} \\
 &= -\frac{x^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx) \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} - \frac{a^4 \operatorname{sech}^{-1}(a + bx)}{4b^4} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(a + bx) \\
 &= -\frac{x^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx) \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} - \frac{a^4 \operatorname{sech}^{-1}(a + bx)}{4b^4} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(a + bx) \\
 &= -\frac{(2 + 17a^2) \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^4} - \frac{x^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx) \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4}
 \end{aligned}$$

Mathematica [C] time = 0.493848, size = 225, normalized size = 1.11

$$\frac{\sqrt{\frac{a+bx-1}{a+bx+1}} \left((9a^2 - 4a + 2)bx + 13a^3 + 13a^2 + (1 - 3a)b^2x^2 + 2a + b^3x^3 + 2 \right) - 3a^4 \log(a + bx) + 3a^4 \log\left(a \sqrt{\frac{a+bx-1}{a+bx+1}}\right)}{12b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcSech[a + b*x], x]

[Out] -(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))])*(2 + 2*a + 13*a^2 + 13*a^3 + (2 - 4*a + 9*a^2)*b*x + (1 - 3*a)*b^2*x^2 + b^3*x^3) - 3*b^4*x^4*ArcSech[a + b*x]

$$- 3a^4 \text{Log}[a + bx] + 3a^4 \text{Log}[1 + \text{Sqrt}[-((-1 + a + bx)/(1 + a + bx))]] + a \text{Sqrt}[-((-1 + a + bx)/(1 + a + bx))] + bx \text{Sqrt}[-((-1 + a + bx)/(1 + a + bx))] + (6I) a (1 + 2a^2) \text{Log}[(-2I)(a + bx) + 2 \text{Sqrt}[-((-1 + a + bx)/(1 + a + bx))] (1 + a + bx)] / (12b^4)$$

Maple [A] time = 0.292, size = 250, normalized size = 1.2

$$\frac{1}{b^4} \left(\frac{(bx + a)^4 \operatorname{arcsech}(bx + a)}{4} - \operatorname{arcsech}(bx + a) (bx + a)^3 a + \frac{3 \operatorname{arcsech}(bx + a) (bx + a)^2 a^2}{2} - \operatorname{arcsech}(bx + a) (bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arcsech(b*x+a),x)

[Out] $\frac{1}{b^4} \left(\frac{1}{4} (bx+a)^4 \operatorname{arcsech}(bx+a) - \operatorname{arcsech}(bx+a) (bx+a)^3 a + \frac{3}{2} \operatorname{arcsech}(bx+a) (bx+a)^2 a^2 - \operatorname{arcsech}(bx+a) (bx+a) a^3 + \frac{1}{4} \operatorname{arcsech}(bx+a) a^4 - \frac{1}{12} \left(-\frac{bx+a-1}{bx+a} \right)^{1/2} (bx+a) \left(\frac{bx+a+1}{bx+a} \right)^{1/2} (3a^4 \operatorname{arctanh}\left(\frac{1}{(1-(bx+a)^2)^{1/2}}\right) + 12a^3 \arcsin(bx+a) + (1-(bx+a)^2)^{1/2} (bx+a)^2 - 6a (bx+a) (1-(bx+a)^2)^{1/2} + 18a^2 (1-(bx+a)^2)^{1/2} + 6a \arcsin(bx+a) + 2(1-(bx+a)^2)^{1/2} \right) / (1-(bx+a)^2)^{1/2} \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2b^4 x^4 \log(\sqrt{bx+a+1} \sqrt{-bx-a+1} bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a) - 2b^4 x^4 \log(bx+a) - b^2 x^2 + 6abx - a^4}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{8} (2b^4 x^4 \log(\sqrt{bx+a+1} \sqrt{-bx-a+1} bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a) + 1) \sqrt{-bx-a+1} a + bx + a - 2b^4 x^4 \log(bx+a) - b^2 x^2 + 6a b x - (a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1) - 2(b^4 x^4 - a^4) \log(bx+a) - (a^4 - 4a^3 + 6a^2 - 4a + 1) \log(-bx-a+1) / b^4 + \int \frac{1}{4} (b^2 x^5 + a b x^4) / (b^2 x^2 + 2a b x + a^2 + (b^2 x^2 + 2a b x + a^2 - 1) e^{1/2 \log(bx+a+1) + 1/2 \log(-bx-a+1)}) - 1, x)$

Fricas [A] time = 2.52586, size = 772, normalized size = 3.8

$$6 b^4 x^4 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2 x^2 + 2 abx + a^2 - 1}{b^2 x^2 + 2 abx + a^2}} + 1}{bx+a}\right) - 3 a^4 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2 x^2 + 2 abx + a^2 - 1}{b^2 x^2 + 2 abx + a^2}} + 1}{x}\right) + 3 a^4 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2 x^2 + 2 abx + a^2 - 1}{b^2 x^2 + 2 abx + a^2}} - 1}{x}\right) + 12 (2 a^3 + \dots)$$

$24 b^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(b*x+a),x, algorithm="fricas")

[Out] 1/24*(6*b^4*x^4*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - 3*a^4*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + 3*a^4*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 12*(2*a^3 + a)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b^3*x^3 - 3*a*b^2*x^2 + 13*a^3 + (9*a^2 + 2)*b*x + 2*a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{asech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*asech(b*x+a),x)

[Out] Integral(x**3*asech(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(b*x+a),x, algorithm="giac")

```
[Out] integrate(x^3*arcsech(b*x + a), x)
```

3.2 $\int x^2 \operatorname{sech}^{-1}(a + bx) dx$

Optimal. Leaf size=153

$$-\frac{(6a^2 + 1) \tan^{-1}\left(\frac{\sqrt{-a-bx+1}(a+bx+1)}{a+bx}\right)}{6b^3} + \frac{a^3 \operatorname{sech}^{-1}(a + bx)}{3b^3} + \frac{5a\sqrt{-a-bx+1}(a + bx + 1)}{6b^3} - \frac{x\sqrt{-a-bx+1}(a + bx + 1)}{6b^2} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a + bx)$$

[Out] (5*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^3) - (x*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^2) + (a^3*ArcSech[a + b*x])/(3*b^3) + (x^3*ArcSech[a + b*x])/3 - ((1 + 6*a^2)*ArcTan[(Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x])/(6*b^3)

Rubi [A] time = 0.0985505, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6321, 5468, 3782, 3770, 3767, 8}

$$-\frac{(6a^2 + 1) \tan^{-1}\left(\frac{\sqrt{-a-bx+1}(a+bx+1)}{a+bx}\right)}{6b^3} + \frac{a^3 \operatorname{sech}^{-1}(a + bx)}{3b^3} + \frac{5a\sqrt{-a-bx+1}(a + bx + 1)}{6b^3} - \frac{x\sqrt{-a-bx+1}(a + bx + 1)}{6b^2} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcSech[a + b*x], x]

[Out] (5*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^3) - (x*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^2) + (a^3*ArcSech[a + b*x])/(3*b^3) + (x^3*ArcSech[a + b*x])/3 - ((1 + 6*a^2)*ArcTan[(Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x])/(6*b^3)

Rule 6321

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

Int[((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_.)])^(n_.)*Tanh[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[((e

```

+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

```

Rule 3782

```

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]

```

Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

Rule 3767

```

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]

```

Rule 8

```

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

```

Rubi steps

$$\begin{aligned}
\int x^2 \operatorname{sech}^{-1}(a+bx) dx &= -\frac{\operatorname{Subst}\left(\int x \operatorname{sech}(x)(-a+\operatorname{sech}(x))^2 \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^3} \\
&= \frac{1}{3}x^3 \operatorname{sech}^{-1}(a+bx) - \frac{\operatorname{Subst}\left(\int (-a+\operatorname{sech}(x))^3 dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{3b^3} \\
&= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a+bx) - \frac{\operatorname{Subst}\left(\int (-2a^3+(1+6a^2)\operatorname{sech}(x)-5a\operatorname{sech}^2(x)) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{6b^3} \\
&= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3 \operatorname{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a+bx) + \frac{(5a)\operatorname{Subst}\left(\int \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{6b^3} \\
&= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3 \operatorname{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a+bx) - \frac{(1+6a^2)\tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{a}\right)}{6b^3} \\
&= \frac{5a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^3} - \frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3 \operatorname{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a+bx) - \frac{(1+6a^2)\tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{a}\right)}{6b^3}
\end{aligned}$$

Mathematica [C] time = 0.258412, size = 200, normalized size = 1.31

$$\frac{\sqrt{-\frac{a+bx-1}{a+bx+1}}(5a^2+a(4bx+5)-bx(bx+1))-2a^3\log(a+bx)+2a^3\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}}+bx\sqrt{-\frac{a+bx-1}{a+bx+1}}+\sqrt{-\frac{a+bx-1}{a+bx+1}}+1\right)}{6b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcSech[a + b*x], x]

[Out] (Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(5*a^2 - b*x*(1 + b*x) + a*(5 + 4*b*x)) + 2*b^3*x^3*ArcSech[a + b*x] - 2*a^3*Log[a + b*x] + 2*a^3*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]) + I*(1 + 6*a^2)*Log[(-2*I)*(a + b*x) + 2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)]/(6*b^3)

Maple [A] time = 0.244, size = 189, normalized size = 1.2

$$\frac{1}{b^3} \left(\frac{(bx+a)^3 \operatorname{arcsech}(bx+a)}{3} - \operatorname{arcsech}(bx+a)(bx+a)^2 a + \operatorname{arcsech}(bx+a)(bx+a)a^2 - \frac{\operatorname{arcsech}(bx+a)a^3}{3} + \frac{bx}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsech(b*x+a),x)`

[Out] $\frac{1}{b^3} \left(\frac{1}{3} (b*x+a)^3 \operatorname{arcsech}(b*x+a) - \operatorname{arcsech}(b*x+a) (b*x+a)^2 a + \operatorname{arcsech}(b*x+a) (b*x+a) a^2 - \frac{1}{3} \operatorname{arcsech}(b*x+a) a^3 + \frac{1}{6} \left(-\frac{(b*x+a-1)}{(b*x+a)} \right)^{1/2} (b*x+a) \left(\frac{(b*x+a+1)}{(b*x+a)} \right)^{1/2} (2a^3 \operatorname{arctanh}(1/(1-(b*x+a)^2))^{1/2}) + 6a^2 \operatorname{arcsin}(b*x+a) - (b*x+a) (1-(b*x+a)^2)^{1/2} + 6a (1-(b*x+a)^2)^{1/2} + \operatorname{arcsin}(b*x+a) \right) / (1-(b*x+a)^2)^{1/2} \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2b^3x^3 \log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a\right) - 2b^3x^3 \log(bx+a) - 2bx + (a^3 + 3a^2 + 3a + 1) \log(bx+a+1) - 2(b^3x^3 + a^3) \log(bx+a) + (a^3 - 3a^2 + 3a - 1) \log(-bx-a+1)}{6b^3} / b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(b*x+a),x, algorithm="maxima")`

[Out] $\frac{1}{6} \left(2b^3x^3 \log(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a) - 2b^3x^3 \log(bx+a) - 2bx + (a^3 + 3a^2 + 3a + 1) \log(bx+a+1) - 2(b^3x^3 + a^3) \log(bx+a) + (a^3 - 3a^2 + 3a - 1) \log(-bx-a+1) \right) / b^3 + \operatorname{integrate}(1/3(b^2x^4 + a^2bx^3) / (b^2x^2 + 2abx + a^2 + (b^2x^2 + 2abx + a^2 - 1)e^{1/2 \log(bx+a+1)} + 1/2 \log(-bx-a+1)) - 1), x)$

Fricas [B] time = 2.35117, size = 720, normalized size = 4.71

$$\frac{2b^3x^3 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right) + a^3 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right) - a^3 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right) - (6a^2 + 1) \operatorname{arctan}\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{x}\right)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(b*x+a),x, algorithm="fricas")`

```
[Out] 1/6*(2*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + a^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - (6*a^2 + 1)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (b^2*x^2 - 4*a*b*x - 5*a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(b*x+a), x)
```

```
[Out] Integral(x**2*asech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a), x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(b*x + a), x)
```

3.3 $\int x \operatorname{sech}^{-1}(a + bx) dx$

Optimal. Leaf size=107

$$-\frac{a^2 \operatorname{sech}^{-1}(a + bx)}{2b^2} - \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)}{2b^2} + \frac{a \tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a + bx)$$

[Out] $-(\operatorname{Sqrt}[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*b^2) - (a^2*\operatorname{ArcSech}[a + b*x])/(2*b^2) + (x^2*\operatorname{ArcSech}[a + b*x])/2 + (a*\operatorname{ArcTan}[(\operatorname{Sqrt}[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x]))/b^2$

Rubi [A] time = 0.0617878, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {6321, 5468, 3773, 3770, 3767, 8}

$$-\frac{a^2 \operatorname{sech}^{-1}(a + bx)}{2b^2} - \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx + 1)}{2b^2} + \frac{a \tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{ArcSech}[a + b*x], x]$

[Out] $-(\operatorname{Sqrt}[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*b^2) - (a^2*\operatorname{ArcSech}[a + b*x])/(2*b^2) + (x^2*\operatorname{ArcSech}[a + b*x])/2 + (a*\operatorname{ArcTan}[(\operatorname{Sqrt}[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x]))/b^2$

Rule 6321

$\operatorname{Int}[(a + \operatorname{ArcSech}(c + d*x))*(b + x)^p*((e + f*x)^m), x_Symbol] \rightarrow -\operatorname{Dist}[(d^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^p*\operatorname{Sech}[x]*\operatorname{Tanh}[x]*(d*e - c*f + f*\operatorname{Sech}[x])^m, x], x, \operatorname{ArcSech}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, x\}$ && $\operatorname{IGtQ}[p, 0]$ && $\operatorname{IntegerQ}[m]$

Rule 5468

$\operatorname{Int}[(e + f*x)^m*\operatorname{Sech}(c + d*x)*((a + b*\operatorname{Sech}(c + d*x))^n), x_Symbol] \rightarrow -\operatorname{Simp}[(e + f*x)^m*(a + b*\operatorname{Sech}[c + d*x])^{n+1}/(b*d*(n+1)), x] + \operatorname{Dist}[(f*m)/(b*d*(n+1)), \operatorname{Int}[(e + f*x)^{m-1}*(a + b*\operatorname{Sech}[c + d*x])^{n+1}, x], x] /;$

FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3773

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Simp[a^2*x, x] + (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int x \operatorname{sech}^{-1}(a + bx) dx &= -\frac{\operatorname{Subst}\left(\int x \operatorname{sech}(x)(-a + \operatorname{sech}(x)) \tanh(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^2} \\
 &= \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx) - \frac{\operatorname{Subst}\left(\int (-a + \operatorname{sech}(x))^2 dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{2b^2} \\
 &= -\frac{a^2 \operatorname{sech}^{-1}(a + bx)}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx) - \frac{\operatorname{Subst}\left(\int \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{2b^2} + \frac{a \operatorname{Subst}\left(\int 1 dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{2b} \\
 &= -\frac{a^2 \operatorname{sech}^{-1}(a + bx)}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx) + \frac{a \tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{b^2} - \frac{i \operatorname{Subst}\left(\int 1 dx, x, -i \operatorname{sech}^{-1}(a + bx)\right)}{2b} \\
 &= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2b^2} - \frac{a^2 \operatorname{sech}^{-1}(a + bx)}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx) + \frac{a \tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{b^2}
 \end{aligned}$$

Mathematica [C] time = 0.161123, size = 176, normalized size = 1.64

$$\frac{a^2 \log(a + bx) - a^2 \log\left(a\sqrt{\frac{a+bx-1}{a+bx+1}} + bx\sqrt{\frac{a+bx-1}{a+bx+1}} + \sqrt{\frac{a+bx-1}{a+bx+1}} + 1\right) + b^2 x^2 \operatorname{sech}^{-1}(a + bx) - \sqrt{\frac{a+bx-1}{a+bx+1}}(a + bx + 1) - 2i}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcSech[a + b*x], x]

[Out] $(-\operatorname{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + b^2*x^2*\operatorname{ArcSech}[a + b*x] + a^2*\operatorname{Log}[a + b*x] - a^2*\operatorname{Log}[1 + \operatorname{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] + a*\operatorname{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*\operatorname{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] - (2*I)*a*\operatorname{Log}[(-2*I)*(a + b*x) + 2*\operatorname{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]]*(1 + a + b*x)]/(2*b^2)$

Maple [A] time = 0.234, size = 111, normalized size = 1.

$$\frac{1}{b^2} \left(\frac{\operatorname{arcsech}(bx + a)(bx + a)^2}{2} - \operatorname{arcsech}(bx + a)a(bx + a) - \frac{bx + a}{2} \sqrt{\frac{bx + a - 1}{bx + a}} \sqrt{\frac{bx + a + 1}{bx + a}} \left(2a \arcsin(bx + a) + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arcsech(b*x+a), x)

[Out] $1/b^2*(1/2*\operatorname{arcsech}(b*x+a)*(b*x+a)^2-\operatorname{arcsech}(b*x+a)*a*(b*x+a)-1/2*(-(b*x+a-1)/(b*x+a))^{(1/2)}*(b*x+a)*((b*x+a+1)/(b*x+a))^{(1/2)}*(2*a*\arcsin(b*x+a)+(1-(b*x+a)^2)^{(1/2)})/(1-(b*x+a)^2)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2b^2x^2 \log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a\right) - 2b^2x^2 \log(bx + a) - (a^2 + 2a + 1) \log}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsech(b*x+a), x, algorithm="maxima")

```
[Out] 1/4*(2*b^2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^2*x^2*log(b*x + a) - (a^2 + 2*a + 1)*log(b*x + a + 1) - 2*(b^2*x^2 - a^2)*log(b*x + a) - (a^2 - 2*a + 1)*log(-b*x - a + 1))/b^2 + integrate(1/2*(b^2*x^3 + a*b*x^2)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1), x)
```

Fricas [B] time = 2.2629, size = 687, normalized size = 6.42

$$\frac{2b^2x^2 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right) - a^2 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right) + a^2 \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right) + 4a \arctan\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{x}\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - a^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + a^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*a*a*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{asech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a),x)
```

```
[Out] Integral(x*asech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a), x)
```

3.4 $\int \operatorname{sech}^{-1}(a + bx) dx$

Optimal. Leaf size=44

$$\frac{(a + bx)\operatorname{sech}^{-1}(a + bx)}{b} - \frac{2 \tan^{-1}\left(\sqrt{\frac{-a-bx+1}{a+bx+1}}\right)}{b}$$

[Out] ((a + b*x)*ArcSech[a + b*x])/b - (2*ArcTan[Sqrt[(1 - a - b*x)/(1 + a + b*x)]])/b

Rubi [A] time = 0.0556131, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6313, 1961, 12, 203}

$$\frac{(a + bx)\operatorname{sech}^{-1}(a + bx)}{b} - \frac{2 \tan^{-1}\left(\sqrt{\frac{-a-bx+1}{a+bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x], x]

[Out] ((a + b*x)*ArcSech[a + b*x])/b - (2*ArcTan[Sqrt[(1 - a - b*x)/(1 + a + b*x)]])/b

Rule 6313

Int[ArcSech[(c_) + (d_)*(x_)], x_Symbol] := Simp[((c + d*x)*ArcSech[c + d*x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; FreeQ[{c, d}, x]

Rule 1961

Int[(u_)^(r_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)*(u /. x -> (-a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n)]^r)/(b*e - d*x^q)^(1/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q), x] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegerQ[r]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^{-1}(a+bx) dx &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} + \int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx} dx \\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} - (4b) \operatorname{Subst}\left(\int \frac{1}{2b^2(1+x^2)} dx, x, \sqrt{\frac{1-a-bx}{1+a+bx}}\right) \\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b} \\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} - \frac{2 \tan^{-1}\left(\sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b} \end{aligned}$$

Mathematica [B] time = 0.195596, size = 95, normalized size = 2.16

$$x \operatorname{sech}^{-1}(a+bx) + \frac{2\sqrt{\frac{a+bx-1}{a+bx+1}} \left(a \tan^{-1}\left(\sqrt{\frac{a+bx-1}{a+bx+1}}\right) - \sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right) \right)}{b\sqrt{\frac{a+bx-1}{a+bx+1}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x], x]

[Out] x*ArcSech[a + b*x] + (2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(-ArcSinh[Sqrt[-1 + a + b*x]/Sqrt[2]] + a*ArcTan[Sqrt[(-1 + a + b*x)/(1 + a + b*x)]]))/(b*Sqrt[(-1 + a + b*x)/(1 + a + b*x)])

Maple [A] time = 0.213, size = 50, normalized size = 1.1

$$x \operatorname{arcsech}(bx+a) + \frac{\operatorname{arcsech}(bx+a)a}{b} - \frac{1}{b} \arctan\left(\sqrt{(bx+a)^{-1}-1}\sqrt{(bx+a)^{-1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a),x)

[Out] x*arcsech(b*x+a)+1/b*arcsech(b*x+a)*a-1/b*arctan((1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))

Maxima [A] time = 0.9777, size = 42, normalized size = 0.95

$$\frac{(bx+a) \operatorname{arsec}(bx+a) - \arctan\left(\sqrt{\frac{1}{(bx+a)^2}-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a),x, algorithm="maxima")

[Out] ((b*x + a)*arcsech(b*x + a) - arctan(sqrt(1/(b*x + a)^2 - 1)))/b

Fricas [B] time = 2.1473, size = 567, normalized size = 12.89

$$\frac{2bx \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}+1}}{bx+a}\right) + a \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}+1}}{x}\right) - a \log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}-1}}{x}\right) - 2 \arctan\left(\frac{(b^2x^2+2abx+a^2-1)^{1/4}}{(b^2x^2+2abx+a^2)^{1/4}}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a),x, algorithm="fricas")

[Out] 1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*arctan((b^2*x^2 + 2*a*b*x + a^2 - 1)^(1/4)/(b^2*x^2 + 2*a*b*x + a^2)^(1/4))

```
*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a), x)
```

```
[Out] Integral(asech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a), x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a), x)
```


3.5 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x} dx$

Optimal. Leaf size=170

$$\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1 - \sqrt{1 - a^2}}\right) + \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1 - a^2} + 1}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a + bx) \log\left(1 - \frac{ae}{1}\right)$$

[Out] ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])] + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - PolyLog[2, -E^(2*ArcSech[a + b*x])]/2

Rubi [A] time = 0.289761, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$, Rules used = {6321, 5595, 5570, 3718, 2190, 2279, 2391, 5562}

$$\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1 - \sqrt{1 - a^2}}\right) + \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1 - a^2} + 1}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a + bx) \log\left(1 - \frac{ae}{1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]/x, x]

[Out] ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])] + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - PolyLog[2, -E^(2*ArcSech[a + b*x])]/2

Rule 6321

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5595

Int[(((e_.) + (f_.)*(x_.))^(m_.)*(F_)[(c_.) + (d_.)*(x_.)]^(n_.)*(G_)[(c_.) + (d_.)*(x_.)]^(p_.))/((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_.)]), x_Symbol] :> I

```
nt[((e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G] && IntegersQ[m, n, p]
```

Rule 5570

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Tanh[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sinh[c + d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/ (b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]
```

, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^{-1}(a+bx)}{x} dx &= -\operatorname{Subst}\left(\int \frac{x \operatorname{sech}(x) \tanh(x)}{-a + \operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
 &= -\operatorname{Subst}\left(\int \frac{x \tanh(x)}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
 &= -\left(a \operatorname{Subst}\left(\int \frac{x \sinh(x)}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - \operatorname{Subst}\left(\int x \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
 &= -\left(2 \operatorname{Subst}\left(\int \frac{e^{2x} x}{1+e^{2x}} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - a \operatorname{Subst}\left(\int \frac{e^x x}{1-\sqrt{1-a^2}-ae^x} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
 &= \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx) \\
 &= \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx) \\
 &= \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)
 \end{aligned}$$

Mathematica [C] time = 0.191604, size = 332, normalized size = 1.95

$$-\operatorname{PolyLog}\left(2, -\frac{(\sqrt{1-a^2}-1)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) - \operatorname{PolyLog}\left(2, \frac{(\sqrt{1-a^2}+1)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + \frac{1}{2} \operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(a+bx)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]/x, x]

[Out] $(-4*I)*\operatorname{ArcSin}\left[\frac{\sqrt{(-1+a)/a}}{\sqrt{2}}\right]*\operatorname{ArcTanh}\left[\frac{(1+a)*\operatorname{Tanh}[\operatorname{ArcSech}[a+b*x]/2]}{\sqrt{1-a^2}}\right] - \operatorname{ArcSech}[a+b*x]*\operatorname{Log}[1+E^{-2*\operatorname{ArcSech}[a+b*x]}]$
 $+ \operatorname{ArcSech}[a+b*x]*\operatorname{Log}\left[1+\frac{-1+\sqrt{1-a^2}}{aE^{\operatorname{ArcSech}[a+b*x]}}\right] + (2*I)*\operatorname{ArcSin}\left[\frac{\sqrt{(-1+a)/a}}{\sqrt{2}}\right]*\operatorname{Log}\left[1+\frac{-1+\sqrt{1-a^2}}{aE^{\operatorname{ArcSech}[a+b*x]}}\right] + \operatorname{ArcSech}[a+b*x]*\operatorname{Log}\left[1-\frac{1+\sqrt{1-a^2}}{aE^{\operatorname{ArcSech}[a+b*x]}}\right] - (2*I)*\operatorname{ArcSin}\left[\frac{\sqrt{(-1+a)/a}}{\sqrt{2}}\right]*\operatorname{Log}\left[1-\frac{1+\sqrt{1-a^2}}{aE^{\operatorname{ArcSech}[a+b*x]}}\right] + \operatorname{PolyLog}\left[2, -E^{-2*\operatorname{ArcSech}[a+b*x]}\right]/2 - \operatorname{PolyLog}\left[2, -\frac{-1+\sqrt{1-a^2}}{aE^{\operatorname{ArcSech}[a+b*x]}}\right] - \operatorname{PolyLog}\left[2, \frac{1+\sqrt{1-a^2}}{aE^{\operatorname{ArcSech}[a+b*x]}}\right]$

$\text{Sqrt}[1 - a^2]/(a * E^{\text{ArcSech}[a + b * x]})]$

Maple [C] time = 0.352, size = 886, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsech(b*x+a)/x,x)`

[Out] $\frac{1}{2} \text{arcsech}(b*x+a) * \ln\left(\frac{-a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} + 1} + \frac{1}{1 + (-a^2+1)^{1/2}}\right) + \frac{1}{2} \text{arcsech}(b*x+a) * \ln\left(\frac{a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} - 1} + \frac{1}{-1 + (-a^2+1)^{1/2}}\right) - \frac{1}{2} * \frac{(-a^2+1)^{1/2}}{a^2-1} * \text{arcsech}(b*x+a) * \ln\left(\frac{-a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} + 1} + \frac{1}{1 + (-a^2+1)^{1/2}}\right) + \frac{1}{2} * \frac{(-a^2+1)^{1/2}}{a^2-1} * \text{arcsech}(b*x+a) * \ln\left(\frac{a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} - 1} + \frac{1}{-1 + (-a^2+1)^{1/2}}\right) + \text{dilog}\left(\frac{-a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} + 1} + \frac{1}{1 + (-a^2+1)^{1/2}}\right) + \text{dilog}\left(\frac{a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} - 1} + \frac{1}{-1 + (-a^2+1)^{1/2}}\right) - \text{arcsech}(b*x+a) * \ln(1 + I * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}) - \text{arcsech}(b*x+a) * \ln(1 - I * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}) - \text{dilog}(1 + I * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}) - \text{dilog}(1 - I * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}) + \frac{1}{2} * \frac{a^2-1 - (-a^2+1)^{1/2}}{a^2} / \frac{(-a^2+1)^{1/2}}{a^2-1} * \text{arcsech}(b*x+a) * (\ln\left(\frac{a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} - 1} + \frac{1}{-1 + (-a^2+1)^{1/2}}\right) * a^2 + \ln\left(\frac{-a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} + 1} + \frac{1}{1 + (-a^2+1)^{1/2}}\right) * a^2 - 2 * \ln\left(\frac{-a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} - 1} + \frac{1}{-1 + (-a^2+1)^{1/2}}\right) + 2 * \ln\left(\frac{-a * (1/(b*x+a) + (1/(b*x+a) - 1)^{1/2}) * (1/(b*x+a) + 1)^{1/2}}{(-a^2+1)^{1/2} + 1} + \frac{1}{1 + (-a^2+1)^{1/2}}\right) * (-a^2+1)^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsech}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(b*x+a)/x,x, algorithm="maxima")`

[Out] integrate(arcsech(b*x + a)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsech}(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x,x, algorithm="fricas")

[Out] integral(arcsech(b*x + a)/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asech}(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)/x,x)

[Out] Integral(asech(a + b*x)/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsech}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)/x, x)

3.6 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=70

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{a+1} \tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}} - \frac{b\operatorname{sech}^{-1}(a+bx)}{a} - \frac{\operatorname{sech}^{-1}(a+bx)}{x}$$

[Out] -((b*ArcSech[a + b*x])/a) - ArcSech[a + b*x]/x + (2*b*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])

Rubi [A] time = 0.105885, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6321, 5468, 3783, 2659, 208}

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{a+1} \tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}} - \frac{b\operatorname{sech}^{-1}(a+bx)}{a} - \frac{\operatorname{sech}^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]/x^2,x]

[Out] -((b*ArcSech[a + b*x])/a) - ArcSech[a + b*x]/x + (2*b*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])

Rule 6321

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;

FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3783

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))⁽⁻¹⁾, x_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a² - b², 0]

Rule 2659

Int[((a_.) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])⁽⁻¹⁾, x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e²*x²), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a² - b², 0]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} dx &= -\left(b \operatorname{Subst}\left(\int \frac{x \operatorname{sech}(x) \tanh(x)}{(-a + \operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
 &= -\frac{\operatorname{sech}^{-1}(a+bx)}{x} + b \operatorname{Subst}\left(\int \frac{1}{-a + \operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
 &= -\frac{b \operatorname{sech}^{-1}(a+bx)}{a} - \frac{\operatorname{sech}^{-1}(a+bx)}{x} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a} \\
 &= -\frac{b \operatorname{sech}^{-1}(a+bx)}{a} - \frac{\operatorname{sech}^{-1}(a+bx)}{x} + \frac{(2b) \operatorname{Subst}\left(\int \frac{1}{1-a-(1+a)x^2} dx, x, \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(a+bx)\right)\right)}{a} \\
 &= -\frac{b \operatorname{sech}^{-1}(a+bx)}{a} - \frac{\operatorname{sech}^{-1}(a+bx)}{x} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{1+a} \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}
 \end{aligned}$$

Mathematica [B] time = 0.215134, size = 244, normalized size = 3.49

$$\frac{b\left(\sqrt{1-a^2} \log(a+bx) - \sqrt{1-a^2} \log\left(a\sqrt{\frac{a+bx-1}{a+bx+1}} + bx\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{-\frac{a+bx-1}{a+bx+1}} + 1\right) + \log\left(\sqrt{1-a^2} a\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{1-a}\right)\right)}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a + b*x]/x^2,x]

[Out] $-(\text{ArcSech}[a + b*x]/x) + (b*(-\text{Log}[x] + \text{Sqrt}[1 - a^2]*\text{Log}[a + b*x] - \text{Sqrt}[1 - a^2]*\text{Log}[1 + \text{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] + a*\text{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] + b*x*\text{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] + \text{Log}[1 - a^2 - a*b*x + \text{Sqrt}[1 - a^2]*\text{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] + a*\text{Sqrt}[1 - a^2]*\text{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]] + \text{Sqrt}[1 - a^2]*b*x*\text{Sqrt}[-((-1 + a + b*x)/(1 + a + b*x))]])/(a*\text{Sqrt}[1 - a^2])$

Maple [B] time = 0.252, size = 542, normalized size = 7.7

$$-\frac{\text{arcsech}(bx+a)}{x} - \frac{ab^2x}{(1+a)(a-1)} \sqrt{\frac{bx+a-1}{bx+a}} \sqrt{\frac{bx+a+1}{bx+a}} \text{Artanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right) \frac{1}{\sqrt{1-(bx+a)^2}} - \frac{a^2b}{(1+a)(a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)/x^2,x)

[Out] $-\text{arcsech}(b*x+a)/x - ((b*x+a-1)/(b*x+a))^{1/2} * ((b*x+a+1)/(b*x+a))^{1/2} / (1 - (b*x+a)^2)^{1/2} * a/(a-1)/(1+a) * \text{arctanh}(1/(1 - (b*x+a)^2)^{1/2}) * x * b^2 - b * ((b*x+a-1)/(b*x+a))^{1/2} * ((b*x+a+1)/(b*x+a))^{1/2} / (1 - (b*x+a)^2)^{1/2} * a^2/(a-1)/(1+a) * \text{arctanh}(1/(1 - (b*x+a)^2)^{1/2}) - ((b*x+a-1)/(b*x+a))^{1/2} * ((b*x+a+1)/(b*x+a))^{1/2} / (1 - (b*x+a)^2)^{1/2} / a/(a-1)/(1+a) * \ln(2 * ((-a^2+1)^{1/2} * (1 - (b*x+a)^2)^{1/2} - a * (b*x+a+1)/b/x) * (-a^2+1)^{1/2} * x * b^2 + ((b*x+a-1)/(b*x+a))^{1/2} * ((b*x+a+1)/(b*x+a))^{1/2} / (1 - (b*x+a)^2)^{1/2} / a/(a-1)/(1+a) * \text{arctanh}(1/(1 - (b*x+a)^2)^{1/2}) * x * b^2 - b * ((b*x+a-1)/(b*x+a))^{1/2} * ((b*x+a+1)/(b*x+a))^{1/2} / (1 - (b*x+a)^2)^{1/2} / (a-1)/(1+a) * \ln(2 * ((-a^2+1)^{1/2} * (1 - (b*x+a)^2)^{1/2} - a * (b*x+a+1)/b/x) * (-a^2+1)^{1/2} + b * ((b*x+a-1)/(b*x+a))^{1/2} * ((b*x+a+1)/(b*x+a))^{1/2} / (1 - (b*x+a)^2)^{1/2} / (a-1)/(1+a) * \text{arctanh}(1/(1 - (b*x+a)^2)^{1/2}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{b \log(x)}{a^3 - a} - \frac{(a^2b - ab)x \log(bx + a + 1) + (a^2b + ab)x \log(-bx - a + 1) + 2(a^3 - a) \log(\sqrt{bx + a + 1} \sqrt{-bx - a + 1} bx + \sqrt{bx + a + 1} \sqrt{-bx - a + 1})}{2(a^3 - a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="maxima")

[Out] $b \cdot \log(x) / (a^3 - a) - 1/2 \cdot ((a^2 \cdot b - a \cdot b) \cdot x \cdot \log(b \cdot x + a + 1) + (a^2 \cdot b + a \cdot b) \cdot x \cdot \log(-b \cdot x - a + 1) + 2 \cdot (a^3 - a) \cdot \log(\sqrt{b \cdot x + a + 1} \cdot \sqrt{-b \cdot x - a + 1}) \cdot b \cdot x + \sqrt{b \cdot x + a + 1} \cdot \sqrt{-b \cdot x - a + 1} \cdot a + b \cdot x + a) - 2 \cdot (a^3 + (a^2 \cdot b - b) \cdot x - a) \cdot \log(b \cdot x + a) - 2 \cdot (a^3 - a) \cdot \log(b \cdot x + a)) / ((a^3 - a) \cdot x) - \int \frac{(b^2 \cdot x + a \cdot b)}{(b^2 \cdot x^3 + 2 \cdot a \cdot b \cdot x^2 + (a^2 - 1) \cdot x + (b^2 \cdot x^3 + 2 \cdot a \cdot b \cdot x^2 + (a^2 - 1) \cdot x) \cdot e^{(1/2 \cdot \log(b \cdot x + a + 1) + 1/2 \cdot \log(-b \cdot x - a + 1))}} dx$

Fricas [B] time = 2.19925, size = 1462, normalized size = 20.89

$$\left[\frac{(a^2 - 1)bx \log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right) - (a^2 - 1)bx \log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right) + \sqrt{-a^2+1}bx \log\left(\frac{(2a^2-1)b^2x^2+2a^4+2a^2bx+a^2}{2(a^3-a)x}\right)}{2(a^3-a)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="fricas")

[Out] $[-1/2 \cdot ((a^2 - 1) \cdot b \cdot x \cdot \log(((b \cdot x + a) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) + 1)/x) - (a^2 - 1) \cdot b \cdot x \cdot \log(((b \cdot x + a) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) - 1)/x) + \sqrt{-a^2 + 1} \cdot b \cdot x \cdot \log(((2 \cdot a^2 - 1) \cdot b^2 \cdot x^2 + 2 \cdot a^4 + 4 \cdot (a^3 - a) \cdot b \cdot x - 4 \cdot a^2 - 2 \cdot (a \cdot b^2 \cdot x^2 + a^3 + (2 \cdot a^2 - 1) \cdot b \cdot x - a) \cdot \sqrt{-a^2 + 1}) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) + 2)/x^2) + 2 \cdot (a^3 - a) \cdot \log(((b \cdot x + a) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) + 1)/(b \cdot x + a)))/((a^3 - a) \cdot x), -1/2 \cdot ((a^2 - 1) \cdot b \cdot x \cdot \log(((b \cdot x + a) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) + 1)/x) - (a^2 - 1) \cdot b \cdot x \cdot \log(((b \cdot x + a) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) - 1)/x) + 2 \cdot \sqrt{a^2 - 1} \cdot b \cdot x \cdot \arctan((a \cdot b^2 \cdot x^2 + a^3 + (2 \cdot a^2 - 1) \cdot b \cdot x - a) \cdot \sqrt{a^2 - 1} \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) / ((a^2 - 1) \cdot b^2 \cdot x^2 + a^4 + 2 \cdot (a^3 - a) \cdot b \cdot x - 2 \cdot a^2 + 1)) + 2 \cdot (a^3 - a) \cdot \log(((b \cdot x + a) \cdot \sqrt{-(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2 - 1)/(b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + a^2)}) + 1)/(b \cdot x + a)))/((a^3 - a) \cdot x)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)/x**2,x)

[Out] Integral(asech(a + b*x)/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)/x^2, x)

3.7 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=133

$$\frac{b^2 \operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{(1-2a^2)b^2 \tanh^{-1}\left(\frac{\sqrt{a+1} \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2(1-a^2)^{3/2}} + \frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a(1-a^2)x} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2}$$

[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*a*(1 - a^2)*x) + (b^2*ArcSech[a + b*x])/(2*a^2) - ArcSech[a + b*x]/(2*x^2) - ((1 - 2*a^2)*b^2*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a^2*(1 - a^2)^(3/2))

Rubi [A] time = 0.203781, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {6321, 5468, 3785, 3919, 3831, 2659, 208}

$$\frac{b^2 \operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{(1-2a^2)b^2 \tanh^{-1}\left(\frac{\sqrt{a+1} \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2(1-a^2)^{3/2}} + \frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a(1-a^2)x} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]/x^3,x]

[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*a*(1 - a^2)*x) + (b^2*ArcSech[a + b*x])/(2*a^2) - ArcSech[a + b*x]/(2*x^2) - ((1 - 2*a^2)*b^2*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a^2*(1 - a^2)^(3/2))

Rule 6321

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)]^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e
+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Simp[(b^2*Cot[
c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} dx &= -\left(b^2 \operatorname{Subst}\left(\int \frac{x \operatorname{sech}(x) \tanh(x)}{(-a+\operatorname{sech}(x))^3} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2 \operatorname{Subst}\left(\int \frac{1}{(-a+\operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1-a^2-\operatorname{sech}(x)}{-a+\operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2a(1-a^2)} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)b^2) \operatorname{Subst}\left(\int \frac{\operatorname{sech}(x)}{-a+\operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2a^2(1-a^2)} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)b^2) \operatorname{Subst}\left(\int \frac{1}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2a^2(1-a^2)} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)b^2) \operatorname{Subst}\left(\int \frac{1}{1-a(1+\cosh(x))} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a^2(1-a^2)} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{(1-2a^2)b^2 \tanh^{-1}\left(\frac{\sqrt{1+a} \tanh\left(\frac{\operatorname{sech}^{-1}(a+bx)}{2}\right)}{\sqrt{1-a}}\right)}{a^2(1-a^2)^{3/2}}
\end{aligned}$$

Mathematica [B] time = 0.907066, size = 315, normalized size = 2.37

$$\frac{1}{2} \left(\frac{(2a^2-1)b^2 \log(x)}{a^2(1-a^2)^{3/2}} - \frac{b^2 \log(a+bx)}{a^2} + \frac{b^2 \log\left(a\sqrt{\frac{a+bx-1}{a+bx+1}} + bx\sqrt{\frac{a+bx-1}{a+bx+1}} + \sqrt{\frac{a+bx-1}{a+bx+1}} + 1\right)}{a^2} + \frac{(2a^2-1)b^2 \log\left(\sqrt{1-a}\right)}{a^2(1-a^2)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a + b*x]/x^3,x]

[Out] $(-((b\sqrt{-((-1+a+bx)/(1+a+bx))}*(1+a+bx))/((-1+a)*a*(1+a)*x)) - \operatorname{ArcSech}[a+bx]/x^2 - ((-1+2a^2)*b^2*\operatorname{Log}[x])/(a^2*(1-a^2)^{(3/2)}) - (b^2*\operatorname{Log}[a+bx])/a^2 + (b^2*\operatorname{Log}[1+\sqrt{-((-1+a+bx)/(1+a+bx))}] + a*\sqrt{-((-1+a+bx)/(1+a+bx))} + b*x*\sqrt{-((-1+a+bx)/(1+a+bx))})/a^2 + ((-1+2a^2)*b^2*\operatorname{Log}[1-a^2-a*bx+\sqrt{1-a^2}]*\sqrt{-((-1+a+bx)/(1+a+bx))} + a*\sqrt{1-a^2}*\sqrt{-((-1+a+bx)/(1+a+bx))} + \sqrt{1-a^2}*b*x*\sqrt{-((-1+a+bx)/(1+a+bx))})/(a^2*(1-a^2)^{(3/2)})/2$

Maple [B] time = 0.259, size = 1233, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsech(b*x+a)/x^3,x)`

[Out]
$$\begin{aligned} & -1/2*\text{arcsech}(b*x+a)/x^2+1/2*b^3*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}*x/(a^2-1)/(1+a)*a^2/(a-1)/(1-(b*x+a)^2)^{1/2}*\text{arctanh}(1/(1-(b*x+a)^2)^{1/2})+1/2*b^2*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/(a^2-1)/(1+a)*a^3/(a-1)/(1-(b*x+a)^2)^{1/2}*\text{arctanh}(1/(1-(b*x+a)^2)^{1/2})+b^3*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}*x/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^{1/2}*\ln(2*((-a^2+1)^{1/2}*(1-(b*x+a)^2)^{1/2}-a*(b*x+a)+1)/b/x)*(-a^2+1)^{1/2}-b^3*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}*x/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^{1/2}*\text{arctanh}(1/(1-(b*x+a)^2)^{1/2})+b^2*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/(a^2-1)/(1+a)*a/(a-1)/(1-(b*x+a)^2)^{1/2}*\ln(2*((-a^2+1)^{1/2}*(1-(b*x+a)^2)^{1/2}-a*(b*x+a)+1)/b/x)*(-a^2+1)^{1/2}-b^2*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/(a^2-1)/(1+a)*a/(a-1)/(1-(b*x+a)^2)^{1/2}*\text{arctanh}(1/(1-(b*x+a)^2)^{1/2})-1/2*b^3*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}*x/(a^2-1)/(1+a)/a^2/(a-1)/(1-(b*x+a)^2)^{1/2}*\ln(2*((-a^2+1)^{1/2}*(1-(b*x+a)^2)^{1/2}-a*(b*x+a)+1)/b/x)*(-a^2+1)^{1/2}-1/2*b^2*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/(a^2-1)/(1+a)*a/(a-1)+1/2*b^3*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}*x/(a^2-1)/(1+a)/a^2/(a-1)/(1-(b*x+a)^2)^{1/2}*\text{arctanh}(1/(1-(b*x+a)^2)^{1/2})-1/2*b^2*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/(a^2-1)/(1+a)/a/(a-1)/(1-(b*x+a)^2)^{1/2}*\ln(2*((-a^2+1)^{1/2}*(1-(b*x+a)^2)^{1/2}-a*(b*x+a)+1)/b/x)*(-a^2+1)^{1/2}-1/2*b*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/x/(a^2-1)/(1+a)*a^2/(a-1)+1/2*b^2*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/(a^2-1)/(1+a)/a/(a-1)+1/2*b*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2}/x/(a^2-1)/(1+a)/(a-1) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(3a^2b^2 - b^2) \log(x)}{2(a^6 - 2a^4 + a^2)} + \frac{(a^4b^2 - 2a^3b^2 + a^2b^2)x^2 \log(bx + a + 1) + (a^4b^2 + 2a^3b^2 + a^2b^2)x^2 \log(-bx - a + 1) - 2(a^3b - a^2b^2)}{2(a^6 - 2a^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2)*log(x)/(a^6 - 2*a^4 + a^2) + 1/4*((a^4*b^2 - 2*a^3*b
^2 + a^2*b^2)*x^2*log(b*x + a + 1) + (a^4*b^2 + 2*a^3*b^2 + a^2*b^2)*x^2*lo
g(-b*x - a + 1) - 2*(a^3*b - a*b)*x - 2*(a^6 - 2*a^4 + a^2)*log(sqrt(b*x +
a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*
x + a) + 2*(a^6 - 2*a^4 - (a^4*b^2 - 2*a^2*b^2 + b^2)*x^2 + a^2)*log(b*x +
a) + 2*(a^6 - 2*a^4 + a^2)*log(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2) - integr
ate(1/2*(b^2*x + a*b)/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a
*b*x^3 + (a^2 - 1)*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))),
x)
```

Fricas [B] time = 2.4814, size = 1912, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4
+ 4*(a^3 - a)*b*x - 4*a^2 + 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(
-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))) +
2)/x^2) - (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
+ a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))) + 1)/x) + (a^4 - 2*a^2 + 1)*b^2*x^2*
log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2
))) - 1)/x) + 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
+ a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))) + 1)/(b*x + a) + 2*((a^3 - a)*b^2*x^
2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x
+ a^2)))/((a^6 - 2*a^4 + a^2)*x^2), 1/4*(2*(2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x
^2*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(a^2 - 1)*sqrt(-(b^2*
x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^2 - 1)*b^2*x^2 + a^
4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a
)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))) + 1)/x) -
(a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1
)/(b^2*x^2 + 2*a*b*x + a^2))) - 1)/x) - 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a
)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))) + 1)/(b*x +
a) - 2*((a^3 - a)*b^2*x^2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a
^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^6 - 2*a^4 + a^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)/x**3,x)

[Out] Integral(asech(a + b*x)/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)/x^3, x)

3.8 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4} dx$

Optimal. Leaf size=197

$$\frac{(2-5a^2)b^2\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} + \frac{(6a^4-5a^2+2)b^3\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3(1-a^2)^{5/2}} + \frac{b\sqrt{\frac{-a}{a+1}}}{6}$$

[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a*(1 - a^2)*x^2) - ((2 - 5*a^2)*b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a^2*(1 - a^2)^2*x) - (b^3*ArcSech[a + b*x])/(3*a^3) - ArcSech[a + b*x]/(3*x^3) + ((2 - 5*a^2 + 6*a^4)*b^3*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])]/Sqrt[1 - a])/(3*a^3*(1 - a^2)^(5/2))

Rubi [A] time = 0.315907, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$, Rules used = {6321, 5468, 3785, 4060, 3919, 3831, 2659, 208}

$$\frac{(2-5a^2)b^2\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} + \frac{(6a^4-5a^2+2)b^3\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3(1-a^2)^{5/2}} + \frac{b\sqrt{\frac{-a}{a+1}}}{6}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]/x^4, x]

[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a*(1 - a^2)*x^2) - ((2 - 5*a^2)*b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a^2*(1 - a^2)^2*x) - (b^3*ArcSech[a + b*x])/(3*a^3) - ArcSech[a + b*x]/(3*x^3) + ((2 - 5*a^2 + 6*a^4)*b^3*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])]/Sqrt[1 - a])/(3*a^3*(1 - a^2)^(5/2))

Rule 6321

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)]^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e
+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[((A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
+ 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
+ (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)]^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

&& NeQ[a^2 - b^2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4} dx &= -\left(b^3 \operatorname{Subst}\left(\int \frac{x \operatorname{sech}(x) \tanh(x)}{(-a + \operatorname{sech}(x))^4} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
 &= -\frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3 \operatorname{Subst}\left(\int \frac{1}{(-a + \operatorname{sech}(x))^3} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} - \frac{b^3 \operatorname{Subst}\left(\int \frac{2(1-a^2)-2a\operatorname{sech}(x)-\operatorname{sech}^2(x)}{(-a+\operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{6a(1-a^2)} \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} + \frac{b^3 \operatorname{Subst}\left(\int \frac{2(1-a^2)-2a\operatorname{sech}(x)-\operatorname{sech}^2(x)}{(-a+\operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{6a(1-a^2)} \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} \\
 &= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3}
 \end{aligned}$$

Mathematica [A] time = 0.402399, size = 368, normalized size = 1.87

$$\frac{1}{6} \left(\frac{b\sqrt{-\frac{a+bx-1}{a+bx+1}}(a^2(5b^2x^2+5bx+1)+a^3(4bx-1)-a^4-abx+a-2bx(bx+1))}{(a-1)^2a^2(a+1)^2x^2} - \frac{(6a^4-5a^2+2)b^3\log(x)}{a^3(1-a^2)^{5/2}} + \frac{2b^3\log(x)}{a^3(1-a^2)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a + b*x]/x^4,x]

[Out]
$$\left((b\sqrt{-((-1+a+b*x)/(1+a+b*x))}) * (a - a^4 - a*b*x - 2*b*x*(1+b*x) + a^3*(-1+4*b*x) + a^2*(1+5*b*x+5*b^2*x^2)) / ((-1+a)^2*a^2*(1+a)^2*x^2) - (2*\text{ArcSech}[a+b*x])/x^3 - ((2-5*a^2+6*a^4)*b^3*\text{Log}[x]) / (a^3*(1-a^2)^{(5/2)}) + (2*b^3*\text{Log}[a+b*x])/a^3 - (2*b^3*\text{Log}[1+\sqrt{-((-1+a+b*x)/(1+a+b*x))}] + a*\sqrt{-((-1+a+b*x)/(1+a+b*x))} + b*x*\sqrt{-((-1+a+b*x)/(1+a+b*x))}) / a^3 + ((2-5*a^2+6*a^4)*b^3*\text{Log}[1-a^2-a*b*x+\sqrt{1-a^2}]*\sqrt{-((-1+a+b*x)/(1+a+b*x))} + a*\sqrt{1-a^2}*\sqrt{-((-1+a+b*x)/(1+a+b*x))} + \sqrt{1-a^2}*b*x*\sqrt{-((-1+a+b*x)/(1+a+b*x))}) / (a^3*(1-a^2)^{(5/2)}) \right) / 6$$

Maple [B] time = 0.256, size = 1946, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)/x^4,x)

[Out]
$$\begin{aligned} & -1/3*\text{arcsech}(b*x+a)/x^3 - 1/3*b^4*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a^3*\text{arctanh}(1/(1-(b*x+a)^2)^{(1/2)}) - b^4*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a*(-a^2+1)^{(1/2)}*\ln(2*((-a^2+1)^{(1/2)}*(1-(b*x+a)^2)^{(1/2)} - a*(b*x+a+1)/b/x) - 1/3*b^3*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a^4*\text{arctanh}(1/(1-(b*x+a)^2)^{(1/2)}) - b^3*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a^2*(-a^2+1)^{(1/2)}*\ln(2*((-a^2+1)^{(1/2)}*(1-(b*x+a)^2)^{(1/2)} - a*(b*x+a+1)/b/x) + b^4*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a*\text{arctanh}(1/(1-(b*x+a)^2)^{(1/2)}) + 5/6*b^4*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}/a*(-a^2+1)^{(1/2)}*\ln(2*((-a^2+1)^{(1/2)}*(1-(b*x+a)^2)^{(1/2)} - a*(b*x+a+1)/b/x) + b^3*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a^2*\text{arctanh}(1/(1-(b*x+a)^2)^{(1/2)}) + 5/6*b^3*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*a^2+5/6*b^3*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}*(-a^2+1)^{(1/2)}*\ln(2*((-a^2+1)^{(1/2)}*(1-(b*x+a)^2)^{(1/2)} - a*(b*x+a+1)/b/x) - b^4*(-(b*x+a-1)/(b*x+a))^{(1/2)}*((b*x+a+1)/(b*x+a))^{(1/2)}*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^{(1/2)}/a*\text{arctanh}(1/(1-(b*x+a)^2)^{(1/2)}) \end{aligned}$$

$$\begin{aligned}
&)) + 2/3 * b^2 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} / x / (a^2-1)^2 \\
&/ (1+a) / (a-1) * a^3 - 1/3 * b^4 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} \\
&/ 2 * x / (a^2-1)^2 / (1+a) / (a-1) / (1 - (b*x+a)^2)^{1/2} / a^3 * (-a^2+1)^{1/2} * \ln(2 * ((-a \\
&^2+1)^{1/2} * (1 - (b*x+a)^2)^{1/2} - a * (b*x+a) + 1) / b / x) - b^3 * (- (b*x+a-1) / (b*x+a))^{1/2} \\
&/ 2 * ((b*x+a+1) / (b*x+a))^{1/2} / (a^2-1)^2 / (1+a) / (a-1) / (1 - (b*x+a)^2)^{1/2} * a \\
&\operatorname{rctanh}(1 / (1 - (b*x+a)^2)^{1/2}) - 7/6 * b^3 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) \\
&/ (b*x+a))^{1/2} / (a^2-1)^2 / (1+a) / (a-1) - 1/6 * b * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b* \\
&x+a+1) / (b*x+a))^{1/2} / x^2 / (a^2-1)^2 / (1+a) / (a-1) * a^4 - 1/3 * b^3 * (- (b*x+a-1) / (b* \\
&x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} / (a^2-1)^2 / (1+a) / (a-1) / (1 - (b*x+a)^2)^{1/2} \\
&/ a^2 * (-a^2+1)^{1/2} * \ln(2 * ((-a^2+1)^{1/2} * (1 - (b*x+a)^2)^{1/2} - a * (b*x+a) + \\
&1) / b / x) + 1/3 * b^4 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} * x / (a^2 \\
&- 1)^2 / (1+a) / (a-1) / (1 - (b*x+a)^2)^{1/2} / a^3 * \operatorname{arctanh}(1 / (1 - (b*x+a)^2)^{1/2}) - 5/ \\
&6 * b^2 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} / x / (a^2-1)^2 / (1+a) \\
&/ (a-1) * a + 1/3 * b^3 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} / (a^2 \\
&- 1)^2 / (1+a) / (a-1) / (1 - (b*x+a)^2)^{1/2} / a^2 * \operatorname{arctanh}(1 / (1 - (b*x+a)^2)^{1/2}) + 1/ \\
&3 * b^3 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} / (a^2-1)^2 / (1+a) / \\
&(a-1) / a^2 + 1/3 * b * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+a))^{1/2} / x^2 / (a \\
&^2-1)^2 / (1+a) / (a-1) * a^2 + 1/6 * b^2 * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+a+1) / (b*x+ \\
&a))^{1/2} / x / (a^2-1)^2 / (1+a) / (a-1) / a - 1/6 * b * (- (b*x+a-1) / (b*x+a))^{1/2} * ((b*x+ \\
&a+1) / (b*x+a))^{1/2} / x^2 / (a^2-1)^2 / (1+a) / (a-1)
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(6a^4b^3 - 3a^2b^3 + b^3) \log(x)}{3(a^9 - 3a^7 + 3a^5 - a^3)} - \frac{(a^6b^3 - 3a^5b^3 + 3a^4b^3 - a^3b^3)x^3 \log(bx + a + 1) + (a^6b^3 + 3a^5b^3 + 3a^4b^3 + a^3b^3)x^3 \log(-bx - a + 1)}{3(a^9 - 3a^7 + 3a^5 - a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="maxima")

[Out] $\begin{aligned}
&1/3 * (6 * a^4 * b^3 - 3 * a^2 * b^3 + b^3) * \log(x) / (a^9 - 3 * a^7 + 3 * a^5 - a^3) - 1/6 * \\
&((a^6 * b^3 - 3 * a^5 * b^3 + 3 * a^4 * b^3 - a^3 * b^3) * x^3 * \log(b * x + a + 1) + (a^6 * b^3 \\
&+ 3 * a^5 * b^3 + 3 * a^4 * b^3 + a^3 * b^3) * x^3 * \log(-b * x - a + 1) - 2 * (3 * a^5 * b^2 - \\
&4 * a^3 * b^2 + a * b^2) * x^2 + (a^6 * b - 2 * a^4 * b + a^2 * b) * x + 2 * (a^9 - 3 * a^7 + 3 * \\
&a^5 - a^3) * \log(\operatorname{sqrt}(b * x + a + 1) * \operatorname{sqrt}(-b * x - a + 1) * b * x + \operatorname{sqrt}(b * x + a + 1) \\
&* \operatorname{sqrt}(-b * x - a + 1) * a + b * x + a) - 2 * (a^9 - 3 * a^7 + 3 * a^5 + (a^6 * b^3 - 3 * a^4 \\
&4 * b^3 + 3 * a^2 * b^3 - b^3) * x^3 - a^3) * \log(b * x + a) - 2 * (a^9 - 3 * a^7 + 3 * a^5 - \\
&a^3) * \log(b * x + a)) / ((a^9 - 3 * a^7 + 3 * a^5 - a^3) * x^3) - \operatorname{integrate}(1/3 * (b^2 * \\
&x + a * b) / (b^2 * x^5 + 2 * a * b * x^4 + (a^2 - 1) * x^3 + (b^2 * x^5 + 2 * a * b * x^4 + (a^2 \\
&- 1) * x^3) * e^{1/2 * \log(b * x + a + 1) + 1/2 * \log(-b * x - a + 1)}), x)
\end{aligned}$

Fricas [B] time = 2.66797, size = 2163, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/12*((6*a^4 - 5*a^2 + 2)*\sqrt{-a^2 + 1}*b^3*x^3*\log(((2*a^2 - 1)*b^2*x^2 \\ & + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - \\ & a)*\sqrt{-a^2 + 1}*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + \\ & a^2)) + 2)/x^2) + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*\log(((b*x + a)*\sqrt{ \\ & -(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - 2*(a^6 \\ & - 3*a^4 + 3*a^2 - 1)*b^3*x^3*\log(((b*x + a)*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 \\ & - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*(a^9 - 3*a^7 + 3*a^5 - a^3)*\log \\ & (((b*x + a)*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)} \\ & + 1)/(b*x + a)) - 2*((5*a^5 - 7*a^3 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)* \\ & b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2 \\ & *x^2 + 2*a*b*x + a^2)}}/(a^9 - 3*a^7 + 3*a^5 - a^3)*x^3, -1/6*((6*a^4 - 5 \\ & *a^2 + 2)*\sqrt{a^2 - 1}*b^3*x^3*\arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - \\ & a)*\sqrt{a^2 - 1}*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + \\ & a^2)}))/(a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^6 - 3*a \\ & ^4 + 3*a^2 - 1)*b^3*x^3*\log(((b*x + a)*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/ \\ & (b^2*x^2 + 2*a*b*x + a^2)} + 1)/x) - (a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*\log(\\ & ((b*x + a)*\sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)} - \\ & 1)/x) + 2*(a^9 - 3*a^7 + 3*a^5 - a^3)*\log(((b*x + a)*\sqrt{-(b^2*x^2 + 2*a* \\ & b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)} + 1)/(b*x + a)) - ((5*a^5 - 7*a^3 \\ & + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)*b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)* \\ & \sqrt{-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)}}/(a^9 - 3*a \\ & ^7 + 3*a^5 - a^3)*x^3] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}(a + bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)/x**4,x)

[Out] Integral(asech(a + b*x)/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{ar} \operatorname{sech}(bx + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)/x^4, x)

3.9 $\int x^2 \operatorname{sech}^{-1}(a + bx)^2 dx$

Optimal. Leaf size=279

$$\frac{2ia^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3} - \frac{2ia^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3} + \frac{i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{3b^3} - \frac{i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{3b^3}$$

```
[Out] -x/(3*b^2) + (2*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a
+ b*x])/b^3 - ((a + b*x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*A
rcSech[a + b*x])/(3*b^3) + (a^3*ArcSech[a + b*x]^2)/(3*b^3) + (x^3*ArcSech[
a + b*x]^2)/3 - (2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/(3*b^3) - (
4*a^2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/b^3 + (2*a*Log[a + b*x])
/b^3 + ((I/3)*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b^3 + ((2*I)*a^2*PolyLog
[2, (-I)*E^ArcSech[a + b*x]])/b^3 - ((I/3)*PolyLog[2, I*E^ArcSech[a + b*x]]
)/b^3 - ((2*I)*a^2*PolyLog[2, I*E^ArcSech[a + b*x]])/b^3
```

Rubi [A] time = 0.241625, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {6321, 5468, 4190, 4180, 2279, 2391, 4184, 3475, 4185}

$$\frac{2ia^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3} - \frac{2ia^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3} + \frac{i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{3b^3} - \frac{i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a + b*x]^2,x]
```

```
[Out] -x/(3*b^2) + (2*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a
+ b*x])/b^3 - ((a + b*x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*A
rcSech[a + b*x])/(3*b^3) + (a^3*ArcSech[a + b*x]^2)/(3*b^3) + (x^3*ArcSech[
a + b*x]^2)/3 - (2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/(3*b^3) - (
4*a^2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/b^3 + (2*a*Log[a + b*x])
/b^3 + ((I/3)*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b^3 + ((2*I)*a^2*PolyLog
[2, (-I)*E^ArcSech[a + b*x]])/b^3 - ((I/3)*PolyLog[2, I*E^ArcSech[a + b*x]]
)/b^3 - ((2*I)*a^2*PolyLog[2, I*E^ArcSech[a + b*x]])/b^3
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)])*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
```


b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 4190

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4180

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))]^(n_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int x^2 \operatorname{sech}^{-1}(a + bx)^2 dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}(x)(-a + \operatorname{sech}(x))^2 \tanh(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^3} \\
&= \frac{1}{3}x^3 \operatorname{sech}^{-1}(a + bx)^2 - \frac{2 \operatorname{Subst}\left(\int x(-a + \operatorname{sech}(x))^3 dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{3b^3} \\
&= \frac{1}{3}x^3 \operatorname{sech}^{-1}(a + bx)^2 - \frac{2 \operatorname{Subst}\left(\int (-a^3x + 3a^2x \operatorname{sech}(x) - 3ax \operatorname{sech}^2(x) + x \operatorname{sech}^3(x)) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{3b^3} \\
&= \frac{a^3 \operatorname{sech}^{-1}(a + bx)^2}{3b^3} + \frac{1}{3}x^3 \operatorname{sech}^{-1}(a + bx)^2 - \frac{2 \operatorname{Subst}\left(\int x \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{3b^3} + \dots \\
&= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{3b^3} \\
&= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{3b^3} \\
&= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{3b^3} \\
&= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{3b^3}
\end{aligned}$$

Mathematica [A] time = 1.91494, size = 305, normalized size = 1.09

$$-(6a^2 + 1) \left(2i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right) - 2i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right) - 2i \operatorname{sech}^{-1}(a+bx) \log\left(1 - ie^{\operatorname{sech}^{-1}(a+bx)}\right) + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*ArcSech[a + b*x]^2,x]

[Out] $-(2*(a + b*x)*\sqrt{-((-1 + a + b*x)/(1 + a + b*x))}*(1 + a + b*x)*\text{ArcSech}[a + b*x] + 6*a*(a + b*x)^2*\text{ArcSech}[a + b*x]^2 - 2*(a + b*x)^3*\text{ArcSech}[a + b*x]^2 + 2*(a + b*x - 6*a*\sqrt{-((-1 + a + b*x)/(1 + a + b*x))}*(1 + a + b*x)*\text{ArcSech}[a + b*x] - 3*a^2*(a + b*x)*\text{ArcSech}[a + b*x]^2 + 12*a*\text{Log}[(a + b*x)^{-1}] - (1 + 6*a^2)*(Pi*\text{Log}[1 - I*E^{\text{ArcSech}[a + b*x]}] - (2*I)*\text{ArcSech}[a + b*x]*\text{Log}[1 - I*E^{\text{ArcSech}[a + b*x]}] - Pi*\text{Log}[1 + I*E^{\text{ArcSech}[a + b*x]}] + (2*I)*\text{ArcSech}[a + b*x]*\text{Log}[1 + I*E^{\text{ArcSech}[a + b*x]}] - Pi*\text{Log}[\text{Cot}[(Pi + (2*I)*\text{ArcSech}[a + b*x])/4]]) + (2*I)*\text{PolyLog}[2, (-I)*E^{\text{ArcSech}[a + b*x]}] - (2*I)*\text{PolyLog}[2, I*E^{\text{ArcSech}[a + b*x]}]))/(6*b^3)$

Maple [A] time = 0.483, size = 655, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arcsech(b*x+a)^2,x)

[Out] $-1/3/b^3*a - 1/3*x/b^2 - 1/3/b*\text{arcsech}(b*x+a)*((b*x+a+1)/(b*x+a))^{1/2}*(-(b*x+a-1)/(b*x+a))^{1/2}*x^2 - 2*I/b^3*a^2*\text{arcsech}(b*x+a)*\ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) + 1/3*I/b^3*\text{arcsech}(b*x+a)*\ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) + 5/3/b^3*\text{arcsech}(b*x+a)*((b*x+a+1)/(b*x+a))^{1/2}*(-(b*x+a-1)/(b*x+a))^{1/2}*a^2 - 1/3*I/b^3*\text{arcsech}(b*x+a)*\ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) + 1/3*I/b^3*\text{dilog}(1+I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) - 2*I/b^3*a^2*\text{dilog}(1-I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) + 1/3*x^3*\text{arcsech}(b*x+a)^2 + 2*I/b^3*a^2*\text{arcsech}(b*x+a)*\ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) + 4/3/b^2*\text{arcsech}(b*x+a)*((b*x+a+1)/(b*x+a))^{1/2}*(-(b*x+a-1)/(b*x+a))^{1/2}*x*a + 1/3*a^3*\text{arcsech}(b*x+a)^2/b^3 - 2/b^3*a*\text{arcsech}(b*x+a) + 2*I/b^3*a^2*\text{dilog}(1+I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2})) - 2/b^3*a*\ln(1+(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2}))^2 + 4/b^3*a*\ln(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2}) - 1/3*I/b^3*\text{dilog}(1-I*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2}*(1/(b*x+a)+1)^{1/2}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3 \log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a\right)^2 - \int -\frac{2\left(6\left(b^3x^5 + 3ab^2x^4 + (3a^2b - b)\right.\right.}{\left.\left.\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-2/3*(6*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + 6*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 - (b^3*x^5 + 2*a*b^2*x^4 + (a^2*b - b)*x^3 + 6*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a) + (3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^5 + 4*a*b^2*x^4 + (2*a^2*b - b)*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \operatorname{arsech}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsech(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*asech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate(x^2*arcsech(b*x + a)^2, x)`

3.10 $\int x \operatorname{sech}^{-1}(a + bx)^2 dx$

Optimal. Leaf size=149

$$-\frac{2ia \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} + \frac{2ia \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} - \frac{a^2 \operatorname{sech}^{-1}(a + bx)^2}{2b^2} - \frac{\log(a + bx)}{b^2} - \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx)}{b^2}$$

[Out] $-\left(\frac{\sqrt{(1 - a - bx)/(1 + a + bx)}(1 + a + bx) \operatorname{ArcSech}[a + bx]}{b^2} - \frac{a^2 \operatorname{ArcSech}[a + bx]^2}{2b^2} + \frac{x^2 \operatorname{ArcSech}[a + bx]^2}{2} + \frac{4a \operatorname{ArcSech}[a + bx] \operatorname{ArcTan}[E^{\operatorname{ArcSech}[a + bx]}]}{b^2} - \frac{\log[a + bx]}{b^2} - \frac{(2I) a \operatorname{PolyLog}[2, (-I) E^{\operatorname{ArcSech}[a + bx]}]}{b^2} + \frac{(2I) a \operatorname{PolyLog}[2, I E^{\operatorname{ArcSech}[a + bx]}]}{b^2}\right)$

Rubi [A] time = 0.138342, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$, Rules used = {6321, 5468, 4190, 4180, 2279, 2391, 4184, 3475}

$$-\frac{2ia \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} + \frac{2ia \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} - \frac{a^2 \operatorname{sech}^{-1}(a + bx)^2}{2b^2} - \frac{\log(a + bx)}{b^2} - \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x \operatorname{ArcSech}[a + bx]^2, x]$

[Out] $-\left(\frac{\sqrt{(1 - a - bx)/(1 + a + bx)}(1 + a + bx) \operatorname{ArcSech}[a + bx]}{b^2} - \frac{a^2 \operatorname{ArcSech}[a + bx]^2}{2b^2} + \frac{x^2 \operatorname{ArcSech}[a + bx]^2}{2} + \frac{4a \operatorname{ArcSech}[a + bx] \operatorname{ArcTan}[E^{\operatorname{ArcSech}[a + bx]}]}{b^2} - \frac{\log[a + bx]}{b^2} - \frac{(2I) a \operatorname{PolyLog}[2, (-I) E^{\operatorname{ArcSech}[a + bx]}]}{b^2} + \frac{(2I) a \operatorname{PolyLog}[2, I E^{\operatorname{ArcSech}[a + bx]}]}{b^2}\right)$

Rule 6321

$\operatorname{Int}[\left((a_{.}) + \operatorname{ArcSech}[(c_{.}) + (d_{.})(x_{.})]\right)(b_{.})^{(p_{.})} \left((e_{.}) + (f_{.})(x_{.})\right)^{(m_{.})}, x_{\text{Symbol}}] \rightarrow -\operatorname{Dist}[(d^{(m+1)})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + bx)^p \operatorname{Sech}[x] \operatorname{Tanh}[x] (d e - c f + f \operatorname{Sech}[x])^m, x], x, \operatorname{ArcSech}[c + d x]], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

$\operatorname{Int}[\left((e_{.}) + (f_{.})(x_{.})\right)^{(m_{.})} \operatorname{Sech}[(c_{.}) + (d_{.})(x_{.})] \left((a_{.}) + (b_{.}) \operatorname{Sech}[(c_{.}) + (d_{.})(x_{.})]\right)^{(n_{.})} \operatorname{Tanh}[(c_{.}) + (d_{.})(x_{.})], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\left((e_{.}) + (f_{.})(x_{.})\right)^{(m_{.})} \operatorname{Sech}[(c_{.}) + (d_{.})(x_{.})] \left((a_{.}) + (b_{.}) \operatorname{Sech}[(c_{.}) + (d_{.})(x_{.})]\right)^{(n_{.})} \operatorname{Tanh}[(c_{.}) + (d_{.})(x_{.})], x_{\text{Symbol}}]$

+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 4190

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4180

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \operatorname{sech}^{-1}(a + bx)^2 dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}(x)(-a + \operatorname{sech}(x)) \tanh(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^2} \\
&= \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx)^2 - \frac{\operatorname{Subst}\left(\int x(-a + \operatorname{sech}(x))^2 dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^2} \\
&= \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx)^2 - \frac{\operatorname{Subst}\left(\int (a^2 x - 2ax \operatorname{sech}(x) + x \operatorname{sech}^2(x)) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^2} \\
&= -\frac{a^2 \operatorname{sech}^{-1}(a + bx)^2}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a + bx)^2 - \frac{\operatorname{Subst}\left(\int x \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(a + bx)\right)}{b^2} + \frac{(2a + bx) \operatorname{sech}^{-1}(a + bx)}{b^2} \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a+bx)^2 + \frac{4a \operatorname{sech}^{-1}(a+bx)}{b^2} \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a+bx)^2 + \frac{4a \operatorname{sech}^{-1}(a+bx)}{b^2} \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(a+bx)^2 + \frac{4a \operatorname{sech}^{-1}(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.417642, size = 172, normalized size = 1.15

$$-4ia \left(\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(a+bx)}\right) \right) + 2 \log\left(\frac{1}{a+bx}\right) + (a+bx)^2 \operatorname{sech}^{-1}(a+bx)^2 - 2a(a+bx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*ArcSech[a + b*x]^2, x]

[Out] $(-2\sqrt{-((-1 + a + b*x)/(1 + a + b*x))} * (1 + a + b*x) * \operatorname{ArcSech}[a + b*x] - 2*a*(a + b*x) * \operatorname{ArcSech}[a + b*x]^2 + (a + b*x)^2 * \operatorname{ArcSech}[a + b*x]^2 - (4*I)*a * \operatorname{ArcSech}[a + b*x] * (\operatorname{Log}[1 - I/E^{\operatorname{ArcSech}[a + b*x]}] - \operatorname{Log}[1 + I/E^{\operatorname{ArcSech}[a + b*x]}]) + 2*\operatorname{Log}[(a + b*x)^{-1}] - (4*I)*a * (\operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcSech}[a + b*x]}] - \operatorname{PolyLog}[2, I/E^{\operatorname{ArcSech}[a + b*x]}])) / (2*b^2)$

Maple [A] time = 0.392, size = 396, normalized size = 2.7

$$\frac{x^2 (\operatorname{arcsech}(bx + a))^2}{2} - \frac{x \operatorname{arcsech}(bx + a)}{b} \sqrt{\frac{bx + a - 1}{bx + a}} \sqrt{\frac{bx + a + 1}{bx + a}} - \frac{a^2 (\operatorname{arcsech}(bx + a))^2}{2b^2} - \frac{\operatorname{arcsech}(bx + a) a}{b^2} \sqrt{\frac{bx + a - 1}{bx + a}} \sqrt{\frac{bx + a + 1}{bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsech(b*x+a)^2,x)`

[Out] $\frac{1}{2}x^2 \operatorname{arcsech}(bx+a)^2 - \frac{1}{b} \operatorname{arcsech}(bx+a) \left(-\frac{bx+a-1}{bx+a} \right)^{\frac{1}{2}} \left(\frac{bx+a+1}{bx+a} \right)^{\frac{1}{2}} x - \frac{1}{2} a^2 \operatorname{arcsech}(bx+a)^2 / b^2 - \frac{1}{b^2} \operatorname{arcsech}(bx+a) \left(-\frac{bx+a-1}{bx+a} \right)^{\frac{1}{2}} \left(\frac{bx+a+1}{bx+a} \right)^{\frac{1}{2}} a + \frac{1}{b^2} \operatorname{arcsech}(bx+a) - \frac{2}{b^2} \ln \left(\frac{1}{bx+a} + \left(\frac{1}{bx+a} - 1 \right)^{\frac{1}{2}} \left(\frac{1}{bx+a} + 1 \right)^{\frac{1}{2}} \right) + \frac{1}{b^2} \ln \left(1 + \left(\frac{1}{bx+a} + \left(\frac{1}{bx+a} - 1 \right)^{\frac{1}{2}} \left(\frac{1}{bx+a} + 1 \right)^{\frac{1}{2}} \right)^2 \right) - 2 \frac{I}{b^2} a \operatorname{arcsech}(bx+a) \ln \left(1 + I \left(\frac{1}{bx+a} + \left(\frac{1}{bx+a} - 1 \right)^{\frac{1}{2}} \left(\frac{1}{bx+a} + 1 \right)^{\frac{1}{2}} \right) \right) + 2 \frac{I}{b^2} a \operatorname{arcsch}(bx+a) \ln \left(1 - I \left(\frac{1}{bx+a} + \left(\frac{1}{bx+a} - 1 \right)^{\frac{1}{2}} \left(\frac{1}{bx+a} + 1 \right)^{\frac{1}{2}} \right) \right) - 2 \frac{I}{b^2} a \operatorname{dilog} \left(1 + I \left(\frac{1}{bx+a} + \left(\frac{1}{bx+a} - 1 \right)^{\frac{1}{2}} \left(\frac{1}{bx+a} + 1 \right)^{\frac{1}{2}} \right) \right) + 2 \frac{I}{b^2} a \operatorname{dilog} \left(1 - I \left(\frac{1}{bx+a} + \left(\frac{1}{bx+a} - 1 \right)^{\frac{1}{2}} \left(\frac{1}{bx+a} + 1 \right)^{\frac{1}{2}} \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} x^2 \log \left(\sqrt{bx+a+1} \sqrt{-bx-a+1} bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a \right)^2 - \int -\frac{4(b^3 x^4 + 3ab^2 x^3 + (3a^2 b - b)x^2}{\dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(b*x+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{2}x^2 \log(\sqrt{bx+a+1} \sqrt{-bx-a+1} bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a)^2 - \int (-4(b^3 x^4 + 3ab^2 x^3 + (3a^2 b - b)x^2 + (a^3 - a)x) \sqrt{bx+a+1} \sqrt{-bx-a+1} \log(bx+a)^2 + 4(b^3 x^4 + 3ab^2 x^3 + (3a^2 b - b)x^2 + (a^3 - a)x) \log(bx+a)^2 - (b^3 x^4 + 2ab^2 x^3 + (a^2 b - b)x^2 + 4(b^3 x^4 + 3ab^2 x^3 + (3a^2 b - b)x^2 + (a^3 - a)x) \log(bx+a) + (2(b^3 x^4 + 3ab^2 x^3 + (3a^2 b - b)x^2 + (a^3 - a)x) \sqrt{bx+a+1} \log(bx+a) + (2b^3 x^4 + 4ab^2 x^3 + (2a^2 b - b)x^2 + 2(b^3 x^4 + 3ab^2 x^3 + (3a^2 b - b)x^2 + (a^3 - a)x) \log(bx+a)) \sqrt{bx+a+1}) \sqrt{-bx-a+1}) \log(\sqrt{bx+a+1} \sqrt{-bx-a+1} bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a)) / (b^3 x^3 + 3ab^2 x^2 + a^3 + (b^3 x^3 + 3ab^2 x^2 + a^3 + (3a^2 b - b)x - a) \sqrt{bx+a+1} \sqrt{-bx-a+1}) + (3a^2 b - b)x - a), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x \operatorname{arsh}(bx+a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{asech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a)**2,x)
```

```
[Out] Integral(x*asech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a)^2, x)
```

3.11 $\int \operatorname{sech}^{-1}(a + bx)^2 dx$

Optimal. Leaf size=80

$$\frac{2i\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} - \frac{2i\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}$$

[Out] $((a + b*x)*\operatorname{ArcSech}[a + b*x]^2)/b - (4*\operatorname{ArcSech}[a + b*x]*\operatorname{ArcTan}[E^{\operatorname{ArcSech}[a + b*x]}])/b + ((2*I)*\operatorname{PolyLog}[2, (-I)*E^{\operatorname{ArcSech}[a + b*x]}])/b - ((2*I)*\operatorname{PolyLog}[2, I*E^{\operatorname{ArcSech}[a + b*x]}])/b$

Rubi [A] time = 0.067922, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {6315, 6279, 5418, 4180, 2279, 2391}

$$\frac{2i\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} - \frac{2i\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSech}[a + b*x]^2, x]$

[Out] $((a + b*x)*\operatorname{ArcSech}[a + b*x]^2)/b - (4*\operatorname{ArcSech}[a + b*x]*\operatorname{ArcTan}[E^{\operatorname{ArcSech}[a + b*x]}])/b + ((2*I)*\operatorname{PolyLog}[2, (-I)*E^{\operatorname{ArcSech}[a + b*x]}])/b - ((2*I)*\operatorname{PolyLog}[2, I*E^{\operatorname{ArcSech}[a + b*x]}])/b$

Rule 6315

$\operatorname{Int}[(a + \operatorname{ArcSech}[c + (d*x)]*(b))^p, x_Symbol] \rightarrow \operatorname{Dist}[1/d, \operatorname{Subst}[\operatorname{Int}[(a + b*\operatorname{ArcSech}[x])^p, x], x, c + d*x], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{IGtQ}[p, 0]$

Rule 6279

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*(x)]*(b))^n, x_Symbol] \rightarrow -\operatorname{Dist}[c^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c, n\}, x\} \ \&\& \ \operatorname{IGtQ}[n, 0]$

Rule 5418

```
Int[(x_)^(m_)*Sech[(a_) + (b_)*(x_)^(n_)]^(p_)*Tanh[(a_) + (b_)*(x_)
^(n_)]^(q_), x_Symbol] := -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4180

```
Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_)
)^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^{-1}(a+bx)^2 dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{-1}(x)^2 dx, x, a+bx\right)}{b} \\
&= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{2 \operatorname{Subst}\left(\int x \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(2i) \operatorname{Subst}\left(\int \log(1-ie^x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(2i) \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{2i \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} - \frac{2i \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.21023, size = 105, normalized size = 1.31

$$\frac{i\left(2\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - 2\operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a+bx)\left(-i(a+bx)\operatorname{sech}^{-1}(a+bx) + 2\log(1 - ie^{-\operatorname{sech}^{-1}(a+bx)})\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]^2, x]

[Out] (I*(ArcSech[a + b*x]*((-I)*(a + b*x)*ArcSech[a + b*x] + 2*Log[1 - I/E^ArcSech[a + b*x]] - 2*Log[1 + I/E^ArcSech[a + b*x]])) + 2*PolyLog[2, (-I)/E^ArcSech[a + b*x]] - 2*PolyLog[2, I/E^ArcSech[a + b*x]]))/b

Maple [A] time = 0.285, size = 209, normalized size = 2.6

$$x(\operatorname{arcsech}(bx+a))^2 + \frac{(\operatorname{arcsech}(bx+a))^2 a}{b} + \frac{2i \operatorname{arcsech}(bx+a)}{b} \ln\left(1 + i\left((bx+a)^{-1} + \sqrt{(bx+a)^{-1} - 1}\sqrt{(bx+a)^{-1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)^2, x)

```
[Out] x*arcsech(b*x+a)^2+1/b*arcsech(b*x+a)^2*a+2*I/b*arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))-2*I/b*arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))-2*I/b*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))+2*I/b*dilog(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x \log \left(\sqrt{bx+a+1} \sqrt{-bx-a+1} bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a \right)^2 - \int -\frac{2(2(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b))}{$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{arsech}(bx+a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)**2,x)

[Out] Integral(asech(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^2,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)^2, x)

$$3.12 \quad \int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} dx$$

Optimal. Leaf size=274

$$2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + 2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 2\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$

```
[Out] ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]^2*Log[1 + E^(2*ArcSech[a + b*x])] + 2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]*PolyLog[2, -E^(2*ArcSech[a + b*x])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + PolyLog[3, -E^(2*ArcSech[a + b*x])]/2
```

Rubi [A] time = 0.463482, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {6321, 5595, 5570, 3718, 2190, 2531, 2282, 6589, 5562}

$$2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + 2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 2\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x, x]
```

```
[Out] ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]^2*Log[1 + E^(2*ArcSech[a + b*x])] + 2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]*PolyLog[2, -E^(2*ArcSech[a + b*x])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + PolyLog[3, -E^(2*ArcSech[a + b*x])]/2
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)])*(b_.)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
```


$\text{nh}[x]*(d*e - c*f + f*\text{Sech}[x])^m, x, \text{ArcSech}[c + d*x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{IGtQ}[p, 0] \ \&\& \text{IntegerQ}[m]$

Rule 5595

$\text{Int}[(((e_.) + (f_.)*(x_.))^{(m_.)}*(F_.)[(c_.) + (d_.)*(x_.)]^{(n_.)}*(G_.)[(c_.) + (d_.)*(x_.)]^{(p_.)})/((a_.) + (b_.)*\text{Sech}[(c_.) + (d_.)*(x_.)]), x_Symbol] \text{:>} \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]*F[c + d*x]^n*G[c + d*x]^p/(b + a*\text{Cosh}[c + d*x]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{HyperbolicQ}[F] \ \&\& \text{HyperbolicQ}[G] \ \&\& \text{IntegersQ}[m, n, p]$

Rule 5570

$\text{Int}[(((e_.) + (f_.)*(x_.))^{(m_.)}*\text{Tanh}[(c_.) + (d_.)*(x_.)]^{(n_.)})/(\text{Cosh}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \text{:>} \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Tanh}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Sinh}[c + d*x]*\text{Tanh}[c + d*x]^{(n - 1)}/(a + b*\text{Cosh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{IGtQ}[m, 0] \ \&\& \text{IGtQ}[n, 0]$

Rule 3718

$\text{Int}[(((c_.) + (d_.)*(x_.))^{(m_.)}*\text{tan}[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]), x_Symbol] \text{:>} -\text{Simp}[(I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*(-(I*e) + f*fz*x))}/(1 + \text{E}^{(2*(-(I*e) + f*fz*x))}), x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))})^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))})^{(n_.)}), x_Symbol] \text{:>} \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \text{IGtQ}[m, 0]$

Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_.)^{((c_.)*((a_.) + (b_.)*(x_.)))})^{(n_.)}]*((f_.) + (g_.)*(x_.))^{(m_.)}, x_Symbol] \text{:>} -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x \ \&\& \text{GtQ}[m, 0]$

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} dx &= -\operatorname{Subst}\left(\int \frac{x^2 \operatorname{sech}(x) \tanh(x)}{-a + \operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\operatorname{Subst}\left(\int \frac{x^2 \tanh(x)}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\left(a \operatorname{Subst}\left(\int \frac{x^2 \sinh(x)}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - \operatorname{Subst}\left(\int x^2 \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{e^{2x} x^2}{1+e^{2x}} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - a \operatorname{Subst}\left(\int \frac{e^x x^2}{1-\sqrt{1-a^2}-ae^x} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx) \\
&= \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx) \\
&= \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx) \\
&= \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)
\end{aligned}$$

Mathematica [A] time = 0.294566, size = 280, normalized size = 1.02

$$2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, -\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right) + 2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 2\operatorname{PolyLog}\left(3, -\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]^2/x, x]

[Out] $(-2*\operatorname{ArcSech}[a + b*x]^3)/3 - \operatorname{ArcSech}[a + b*x]^2*\log[1 + E^{(-2*\operatorname{ArcSech}[a + b*x])}] + \operatorname{ArcSech}[a + b*x]^2*\log[1 + (a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2])] + \operatorname{ArcSech}[a + b*x]^2*\log[1 - (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])] + \operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[2, -E^{(-2*\operatorname{ArcSech}[a + b*x])}] + 2*\operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[2, -((a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2]))] + 2*\operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[2, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])] + \operatorname{PolyLog}[3, -E^{(-2*\operatorname{ArcSech}[a + b*x])}]/2 - 2*\operatorname{PolyLog}[3, -((a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2]))] - 2*\operatorname{PolyLog}[3, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])]$

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arcsech}(bx + a))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)^2/x,x)

[Out] int(arcsech(b*x+a)^2/x,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="maxima")

[Out] integrate(arcsech(b*x + a)^2/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsech}(bx + a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="fricas")

[Out] integral(arcsech(b*x + a)^2/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^2(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x,x)
```

```
[Out] Integral(asech(a + b*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{ar} \operatorname{sech}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2/x, x)
```

3.13 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=224

$$\frac{2b\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}}$$

[Out] $-\left(\frac{b\operatorname{ArcSech}[a + b*x]^2}{a}\right) - \operatorname{ArcSech}[a + b*x]^2/x + (2*b*\operatorname{ArcSech}[a + b*x]*\operatorname{Log}[1 - (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 - \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2]) - (2*b*\operatorname{ArcSech}[a + b*x]*\operatorname{Log}[1 - (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2]) + (2*b*\operatorname{PolyLog}[2, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 - \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2]) - (2*b*\operatorname{PolyLog}[2, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2])$

Rubi [A] time = 0.388906, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6321, 5468, 4191, 3320, 2264, 2190, 2279, 2391}

$$\frac{2b\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSech}[a + b*x]^2/x^2, x]$

[Out] $-\left(\frac{b\operatorname{ArcSech}[a + b*x]^2}{a}\right) - \operatorname{ArcSech}[a + b*x]^2/x + (2*b*\operatorname{ArcSech}[a + b*x]*\operatorname{Log}[1 - (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 - \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2]) - (2*b*\operatorname{ArcSech}[a + b*x]*\operatorname{Log}[1 - (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2]) + (2*b*\operatorname{PolyLog}[2, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 - \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2]) - (2*b*\operatorname{PolyLog}[2, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])])/(a*\operatorname{Sqrt}[1 - a^2])$

Rule 6321

$\operatorname{Int}[(a_. + \operatorname{ArcSech}[(c_. + (d_.)*(x_.)]*(b_.))^{(p_.)*((e_. + (f_.)*(x_.))^{(m_.)}, x_Symbol] := -\operatorname{Dist}[(d^{(m+1)})^{-1}], \operatorname{Subst}[\operatorname{Int}[(a + b*x)^p*\operatorname{Sech}[x]*\operatorname{Tanh}[x]*(d*e - c*f + f*\operatorname{Sech}[x])^m, x], x, \operatorname{ArcSech}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{IntegerQ}[m]$

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e
+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^((n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2} dx &= -\left(b \operatorname{Subst}\left(\int \frac{x^2 \operatorname{sech}(x) \tanh(x)}{(-a + \operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + (2b) \operatorname{Subst}\left(\int \frac{x}{-a + \operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + (2b) \operatorname{Subst}\left(\int \left(-\frac{x}{a} + \frac{x}{a(1-a \cosh(x))}\right) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{(2b) \operatorname{Subst}\left(\int \frac{x}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{(4b) \operatorname{Subst}\left(\int \frac{e^x x}{-a+2e^x-ae^{2x}} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} - \frac{(4b) \operatorname{Subst}\left(\int \frac{e^x x}{2-2\sqrt{1-a^2}-2ae^x} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a^2}} + \dots \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b \operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [C] time = 2.62484, size = 678, normalized size = 3.03

$$-\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b \left(i \left(\operatorname{PolyLog}\left(2, \frac{(-1-i\sqrt{a^2-1})(-i\sqrt{a^2-1} \tanh(\frac{1}{2}\operatorname{sech}^{-1}(a+bx))+a-1)}{a(i\sqrt{a^2-1} \tanh(\frac{1}{2}\operatorname{sech}^{-1}(a+bx))+a-1)}\right) - \operatorname{PolyLog}\left(2, \frac{(\sqrt{a^2-1}+i)(-i\sqrt{a^2-1} \tanh(\frac{1}{2}\operatorname{sech}^{-1}(a+bx))+a-1)}{a(\sqrt{a^2-1} \tanh(\frac{1}{2}\operatorname{sech}^{-1}(a+bx))-i(a-1))}\right) \right) + 2\operatorname{sech}^{-1}(a+bx) \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) - \frac{2b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}} \right)}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a + b*x]^2/x^2,x]

[Out]
$$\begin{aligned} & -\left(\frac{(a + b*x)*\text{ArcSech}[a + b*x]^2}{x} + (2*b*(2*\text{ArcSech}[a + b*x]*\text{ArcTan}[\frac{(-1 + a)*\text{Coth}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}] - (2*I)*\text{ArcCos}[a^{(-1)}]*\text{ArcTan}[\frac{((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}] + (\text{ArcCos}[a^{(-1)}] + 2*(\text{ArcTan}[\frac{(-1 + a)*\text{Coth}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}] + \text{ArcTan}[\frac{((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}])*\text{Log}[\frac{\sqrt{-1 + a^2}}{\sqrt{2}*\sqrt{a}*E^{(\text{ArcSech}[a + b*x]/2)*\sqrt{-((b*x)/(a + b*x))}}] + (\text{ArcCos}[a^{(-1)}] - 2*(\text{ArcTan}[\frac{(-1 + a)*\text{Coth}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}] + \text{ArcTan}[\frac{((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}])*\text{Log}[(\sqrt{-1 + a^2})*E^{(\text{ArcSech}[a + b*x]/2)}]/(\sqrt{2}*\sqrt{a}*E^{(\text{ArcSech}[a + b*x]/2)}) - (\text{ArcCos}[a^{(-1)}] + 2*\text{ArcTan}[\frac{((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}])*\text{Log}[-\frac{((-1 + a)*(1 + a - I*\sqrt{-1 + a^2})*(-1 + \text{Tanh}[\text{ArcSech}[a + b*x]/2]))}{(a*(-1 + a + I*\sqrt{-1 + a^2})*\text{Tanh}[\text{ArcSech}[a + b*x]/2]))}] - (\text{ArcCos}[a^{(-1)}] - 2*\text{ArcTan}[\frac{((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2]}{\sqrt{-1 + a^2}}])*\text{Log}[\frac{((-1 + a)*(1 + a + I*\sqrt{-1 + a^2})*(1 + \text{Tanh}[\text{ArcSech}[a + b*x]/2]))}{(a*(-1 + a + I*\sqrt{-1 + a^2})*\text{Tanh}[\text{ArcSech}[a + b*x]/2]))}] + I*(\text{PolyLog}[2, \frac{((-1 - I*\sqrt{-1 + a^2})*(-1 + a - I*\sqrt{-1 + a^2})*\text{Tanh}[\text{ArcSech}[a + b*x]/2])}{(a*(-1 + a + I*\sqrt{-1 + a^2})*\text{Tanh}[\text{ArcSech}[a + b*x]/2])}] - \text{PolyLog}[2, \frac{(I + \sqrt{-1 + a^2})*(-1 + a - I*\sqrt{-1 + a^2})*\text{Tanh}[\text{ArcSech}[a + b*x]/2])}{(a*(-I)*(-1 + a) + \sqrt{-1 + a^2}*\text{Tanh}[\text{ArcSech}[a + b*x]/2])}])\right)/\sqrt{-1 + a^2}/a \end{aligned}$$

Maple [A] time = 0.357, size = 367, normalized size = 1.6

$$\frac{b(\text{arcsech}(bx+a))^2}{a} - \frac{(\text{arcsech}(bx+a))^2}{x} + 2\frac{b\sqrt{-a^2+1}\text{arcsech}(bx+a)}{a(a^2-1)} \ln\left(\frac{-a\left((bx+a)^{-1} + \sqrt{(bx+a)^{-1}-1}\sqrt{b}\right)}{1+\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)^2/x^2,x)

[Out]
$$\begin{aligned} & -b*\text{arcsech}(b*x+a)^2/a - \text{arcsech}(b*x+a)^2/x + 2*b*(-a^2+1)^{(1/2)}/a/(a^2-1)*\text{arcsech}(b*x+a)*\ln\left(\frac{-a*(1/(b*x+a)+(1/(b*x+a)-1)^{(1/2})*(1/(b*x+a)+1)^{(1/2}))+(-a^2+1)^{(1/2}+1)/(1+(-a^2+1)^{(1/2}))}{-2*b*(-a^2+1)^{(1/2)}/a/(a^2-1)*\text{arcsech}(b*x+a)*\ln\left(\frac{a*(1/(b*x+a)+(1/(b*x+a)-1)^{(1/2})*(1/(b*x+a)+1)^{(1/2}))+(-a^2+1)^{(1/2}-1)/(-1+(-a^2+1)^{(1/2}))}{+2*b*(-a^2+1)^{(1/2)}/a/(a^2-1)*\text{dilog}\left(\frac{-a*(1/(b*x+a)+(1/(b*x+a)-1)^{(1/2})*(1/(b*x+a)+1)^{(1/2}))+(-a^2+1)^{(1/2}+1)/(1+(-a^2+1)^{(1/2}))}{-2*b*(-a^2+1)^{(1/2)}/a/(a^2-1)*\text{dilog}\left(\frac{a*(1/(b*x+a)+(1/(b*x+a)-1)^{(1/2})*(1/(b*x+a)+1)^{(1/2}))+(-a^2+1)^{(1/2}-1)/(-1+(-a^2+1)^{(1/2}))}{\right)}\right)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a\right)^2}{x} - \int \frac{2\left(2\left(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="maxima")

[Out] -log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2/x - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 + (b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsech}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] integral(arcsech(b*x + a)^2/x^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asech}^2(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asech(b*x+a)**2/x**2,x)`

[Out] `Integral(asech(a + b*x)**2/x**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{ar} \operatorname{sech}(bx + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(b*x+a)^2/x^2,x, algorithm="giac")`

[Out] `integrate(arcsech(b*x + a)^2/x^2, x)`

3.14 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=537

$$-\frac{2b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2 \sqrt{1-a^2}} + \frac{b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2 (1-a^2)^{3/2}} + \frac{2b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2 \sqrt{1-a^2}} - \frac{b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2 (1-a^2)^{3/2}}$$

```
[Out] (b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/(a*(
1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^2)/(2*a^2) -
ArcSech[a + b*x]^2/(2*x^2) + (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a +
b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (2*b^2*ArcSech[a + b*x
]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
- (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])
)/(a^2*(1 - a^2)^(3/2)) + (2*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a +
b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (b^2*Log[x/(a + b*x)])/(a
^2*(1 - a^2)) + (b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])
)/(a^2*(1 - a^2)^(3/2)) - (2*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqr
t[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(
1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (2*b^2*PolyLog[2, (a*E^ArcSec
h[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
```

Rubi [A] time = 0.749842, antiderivative size = 537, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6321, 5468, 4191, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{2b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2 \sqrt{1-a^2}} + \frac{b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2 (1-a^2)^{3/2}} + \frac{2b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2 \sqrt{1-a^2}} - \frac{b^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2 (1-a^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x^3, x]
```

```
[Out] (b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/(a*(
1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^2)/(2*a^2) -
ArcSech[a + b*x]^2/(2*x^2) + (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a +
b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (2*b^2*ArcSech[a + b*x
]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
- (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])
```

$$\begin{aligned} &)/(a^2*(1 - a^2)^{(3/2)}) + (2*b^2*ArcSech[a + b*x]*Log[1 - (a*E^{ArcSech[a + b*x]})/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (b^2*Log[x/(a + b*x)])/(a^2*(1 - a^2)) + (b^2*PolyLog[2, (a*E^{ArcSech[a + b*x]})/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^{(3/2)}) - (2*b^2*PolyLog[2, (a*E^{ArcSech[a + b*x]})/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (b^2*PolyLog[2, (a*E^{ArcSech[a + b*x]})/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^{(3/2)}) + (2*b^2*PolyLog[2, (a*E^{ArcSech[a + b*x]})/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) \end{aligned}$$

Rule 6321

$$\text{Int}[(a_. + \text{ArcSech}[c_. + (d_.)(x_.)]*(b_.))^{\text{p}_.}*((e_.) + (f_.)(x_.))^{\text{m}_.}, x_Symbol] \text{ :> } -\text{Dist}[(d^{\text{m} + 1})^{-1}, \text{Subst}[\text{Int}[(a + b*x)^p*\text{Sech}[x]*\text{Tanh}[x]*(d*e - c*f + f*\text{Sech}[x])^m, x], x, \text{ArcSech}[c + d*x]], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IntegerQ}[m]$$

Rule 5468

$$\text{Int}[(e_. + (f_.)(x_.))^{\text{m}_.}*\text{Sech}[c_. + (d_.)(x_.)]*((a_.) + (b_.)*\text{Sech}[c_. + (d_.)(x_.)])^{\text{n}_.}*\text{Tanh}[c_. + (d_.)(x_.)], x_Symbol] \text{ :> } -\text{Simp}[(e + f*x)^m*(a + b*\text{Sech}[c + d*x])^{n+1}/(b*d*(n+1)), x] + \text{Dist}[(f*m)/(b*d*(n+1)), \text{Int}[(e + f*x)^{m-1}*(a + b*\text{Sech}[c + d*x])^{n+1}, x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[n, -1]$$

Rule 4191

$$\text{Int}[(\text{csc}[e_. + (f_.)(x_.)]*(b_.) + (a_.))^{\text{n}_.}*((c_.) + (d_.)(x_.))^{\text{m}_.}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, 1/(\text{Sin}[e + f*x]^n/(b + a*\text{Sin}[e + f*x])^n)], x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0]$$

Rule 3324

$$\text{Int}[(c_. + (d_.)(x_.))^{\text{m}_.}/((a_.) + (b_.)*\text{sin}[e_. + (f_.)(x_.)])^2, x_Symbol] \text{ :> } \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x])/(f*(a^2 - b^2)*(a + b*\text{Sin}[e + f*x])), x] + (\text{Dist}[a/(a^2 - b^2), \text{Int}[(c + d*x)^m/(a + b*\text{Sin}[e + f*x]), x], x] - \text{Dist}[(b*d*m)/(f*(a^2 - b^2)), \text{Int}[(c + d*x)^{m-1}*\text{Cos}[e + f*x]/(a + b*\text{Sin}[e + f*x]), x], x]) \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m, 0]$$

Rule 3320

$$\text{Int}[(c_. + (d_.)(x_.))^{\text{m}_.}/((a_.) + (b_.)*\text{sin}[e_. + \text{Pi}*(k_.) + (\text{Complex}[0, fz_])*(f_.)(x_.)]), x_Symbol] \text{ :> } \text{Dist}[2, \text{Int}[(c + d*x)^m*E^{-\text{I}*e} + f*fz*x)/(E^{\text{I}*\text{Pi}*(k - 1/2)}*(b + (2*a*E^{-\text{I}*e} + f*fz*x))/E^{\text{I}*\text{Pi}*(k - 1/2)}) - (b*E^{2*(-\text{I}*e} + f*fz*x)})/E^{2*\text{I}*k*\text{Pi}})], x], x] \text{ /; } \text{FreeQ}[\{a, b, c,$$

d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*(F_)^(e_.)*((c_.) + (d_.)*(x_))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3} dx &= -\left(b^2 \operatorname{Subst}\left(\int \frac{x^2 \operatorname{sech}(x) \tanh(x)}{(-a+\operatorname{sech}(x))^3} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + b^2 \operatorname{Subst}\left(\int \frac{x}{(-a+\operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + b^2 \operatorname{Subst}\left(\int \left(\frac{x}{a^2} + \frac{x}{a^2(-1+a \cosh(x))^2} + \frac{2x}{a^2(-1+a \cosh(x))}\right) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{x}{(-1+a \cosh(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a^2} + \dots \\
&= \frac{b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{(4b^2) \operatorname{Subst}\left(\int \dots\right)}{a^2} \\
&= \frac{b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \operatorname{Subst}\left(\int \dots\right)}{a^2} \\
&= \frac{b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} - \frac{2b^2 \operatorname{sech}^{-1}(a+bx)}{a^2} \\
&= \frac{b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{a^2} \\
&= \frac{b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{a^2} \\
&= \frac{b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)}{a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)}{a^2}
\end{aligned}$$

Mathematica [C] time = 7.76266, size = 1439, normalized size = 2.68

result too large to display

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a + b*x]^2/x^3,x]

```

[Out] -((a + b*x)^2*ArcSech[a + b*x]^2)/(2*a^2*x^2) + (b*ArcSech[a + b*x]*(-(a*Sqr
rt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + (-1 + a^2)*(a + b*x)*A
rcSech[a + b*x]))/((-1 + a)*a^2*(1 + a)*x) + (b^2*Log[(b*x)/(a + b*x)])/(a^
2 - a^4) - (2*b^2*(2*ArcSech[a + b*x]*ArcTan[((-1 + a)*Coth[ArcSech[a + b*x
]/2)])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*ArcTan[((1 + a)*Tanh[ArcSech[a
 + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] + 2*(ArcTan[((-1 + a)*Coth[Ar
cSech[a + b*x]/2)])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/
2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^(ArcSech[a + b*x
]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)] - 2*(ArcTan[((-1 + a)*Co
th[ArcSech[a + b*x]/2)])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[a +
b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*E^(ArcSech[a + b*x]/2))/(Sqr
t[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (ArcCos[a^(-1)] + 2*ArcTan[((1 +
a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[-(((1 + a)*(1 + a - I*Sqr
t[-1 + a^2]))*(-1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2
]*Tanh[ArcSech[a + b*x]/2]))] - (ArcCos[a^(-1)] - 2*ArcTan[((1 + a)*Tanh[A
rcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[(((1 + a)*(1 + a + I*Sqrt[-1 + a^2
])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSe
ch[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*Sqrt[-1 + a^2]))*(-1 + a - I*Sqrt
[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[Arc
Sech[a + b*x]/2]))] - PolyLog[2, ((I + Sqrt[-1 + a^2]))*(-1 + a - I*Sqrt[-1
 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-I)*(-1 + a) + Sqrt[-1 + a^2]*Tanh[
ArcSech[a + b*x]/2])))])))/(-1 + a^2)^(3/2) + (b^2*(2*ArcSech[a + b*x]*ArcTa
n[((-1 + a)*Coth[ArcSech[a + b*x]/2)])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)
]*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1
)] + 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2)])/Sqrt[-1 + a^2]] + ArcTan
[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(S
qrt[2]*Sqrt[a]*E^(ArcSech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[
a^(-1)] - 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2)])/Sqrt[-1 + a^2]] + A
rcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a
^2]*E^(ArcSech[a + b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (
ArcCos[a^(-1)] + 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2
]])*Log[-(((1 + a)*(1 + a - I*Sqrt[-1 + a^2]))*(-1 + Tanh[ArcSech[a + b*x]/2
]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - (ArcCos[a^
(-1)] - 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[(((
-1 + a)*(1 + a + I*Sqrt[-1 + a^2]))*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 +
a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*S
qrt[-1 + a^2]))*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-
1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + Sqr
t[-1 + a^2]))*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-I)
*(-1 + a) + Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))])))/(-1 + a^2)^(3/2))

```


Maple [A] time = 0.563, size = 1026, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsech(b*x+a)^2/x^3,x)`

[Out]
$$\begin{aligned} & -b^2 \operatorname{arcsech}(b*x+a)/a/(a^2-1)*(-(b*x+a-1)/(b*x+a))^{1/2}*((b*x+a+1)/(b*x+a))^{1/2} \\ & + 1/2*b^2 \operatorname{arcsech}(b*x+a)^2/(a^2-1) - b \operatorname{arcsech}(b*x+a)/x/(a^2-1)*(-(b*x+a-1)/(b*x+a))^{1/2} \\ & *((b*x+a+1)/(b*x+a))^{1/2} - 1/2*b^2 \operatorname{arcsech}(b*x+a)^2/a^2/(a^2-1) - 1/2*\operatorname{arcsech}(b*x+a)^2*a^2/x^2/(a^2-1) \\ & - b^2 \operatorname{arcsech}(b*x+a)/a^2/(a^2-1) + 1/2*\operatorname{arcsech}(b*x+a)^2/x^2/(a^2-1) - b^2/a^2/(a^2-1)*\ln(a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})^2 + a-2/(b*x+a)-2*(1/(b*x+a)-1)^{1/2}*(1/(b*x+a)+1)^{1/2}) \\ & + 2*b^2/a^2/(a^2-1)*\ln(1/(b*x+a)+(1/(b*x+a)-1)^{1/2}*(1/(b*x+a)+1)^{1/2}) \\ & + b^2*(-a^2+1)^{1/2}/a^2/(a^2-1)^2*\operatorname{arcsech}(b*x+a)*\ln((-a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}+1)/(1+(-a^2+1)^{1/2})) \\ & - b^2*(-a^2+1)^{1/2}/a^2/(a^2-1)^2*\operatorname{arcsech}(b*x+a)*\ln((a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}-1)/(-1+(-a^2+1)^{1/2})) \\ & + b^2*(-a^2+1)^{1/2}/a^2/(a^2-1)^2*\operatorname{dilog}((-a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}+1)/(1+(-a^2+1)^{1/2})) \\ & - b^2*(-a^2+1)^{1/2}/a^2/(a^2-1)^2*\operatorname{dilog}((a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}-1)/(-1+(-a^2+1)^{1/2})) \\ & - 2*b^2*(-a^2+1)^{1/2}/(a^2-1)^2*\operatorname{arcsech}(b*x+a)*\ln((-a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}+1)/(1+(-a^2+1)^{1/2})) \\ & + 2*b^2*(-a^2+1)^{1/2}/(a^2-1)^2*\operatorname{arcsech}(b*x+a)*\ln((a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}-1)/(-1+(-a^2+1)^{1/2})) \\ & + 2*b^2*(-a^2+1)^{1/2}/(a^2-1)^2*\operatorname{dilog}((-a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}+1)/(1+(-a^2+1)^{1/2})) \\ & + 2*b^2*(-a^2+1)^{1/2}/(a^2-1)^2*\operatorname{dilog}((a*(1/(b*x+a)+(1/(b*x+a)-1))^{1/2} \\ & *(1/(b*x+a)+1)^{1/2})+(-a^2+1)^{1/2}-1)/(-1+(-a^2+1)^{1/2})) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a\right)^2}{2x^2} - \int \frac{4(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x)}{2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(b*x+a)^2/x^3,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/2*\log(\sqrt{b*x + a + 1}*\sqrt{-b*x - a + 1})*b*x + \sqrt{b*x + a + 1}*\sqrt{-b*x - a + 1} \\ & *a + b*x + a)^2/x^2 - \operatorname{integrate}(-(4*(b^3*x^3 + 3*a*b^2*x^2 + a \end{aligned}$$

$$\begin{aligned} &^3 + (3a^2b - b)x - a) \sqrt{bx + a + 1} \sqrt{-bx - a + 1} \log(bx + a) \\ &^2 + 4(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x - a) \log(bx + a)^2 + \\ &(b^3x^3 + 2ab^2x^2 + (a^2b - b)x - 4(b^3x^3 + 3ab^2x^2 + a^3 + \\ &(3a^2b - b)x - a) \log(bx + a) - (2(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x - a) \sqrt{bx + a + 1} \log(bx + a) - (2b^3x^3 + 4ab^2x^2 + \\ &(2a^2b - b)x - 2(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x - a) \log(bx + a)) \sqrt{bx + a + 1}) \sqrt{-bx - a + 1}) \log(\sqrt{bx + a + 1} \sqrt{-bx - a + 1} * b * x + \sqrt{bx + a + 1} \sqrt{-bx - a + 1} * a + bx + a)) / (\\ &b^3x^6 + 3ab^2x^5 + (3a^2b - b)x^4 + (a^3 - a)x^3 + (b^3x^6 + 3ab^2x^5 + (3a^2b - b)x^4 + (a^3 - a)x^3) \sqrt{bx + a + 1} \sqrt{-bx - a + 1}), x) \end{aligned}$$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsech}(bx + a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="fricas")

[Out] integral(arcsech(b*x + a)^2/x^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^2(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)**2/x**3,x)

[Out] Integral(asech(a + b*x)**2/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2/x^3, x)
```

3.15 $\int x \operatorname{sech}^{-1}(a + bx)^3 dx$

Optimal. Leaf size=260

$$\frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} + \frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2b^2}$$

[Out] $(-3 \operatorname{ArcSech}[a + b*x]^2)/(2*b^2) - (3 \operatorname{Sqrt}[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x) \operatorname{ArcSech}[a + b*x]^2)/(2*b^2) - (a^2 \operatorname{ArcSech}[a + b*x]^3)/(2*b^2) + (x^2 \operatorname{ArcSech}[a + b*x]^3)/2 + (6*a \operatorname{ArcSech}[a + b*x]^2 \operatorname{ArcTan}[E^{\operatorname{ArcSech}[a + b*x]}])/b^2 + (3 \operatorname{ArcSech}[a + b*x] \operatorname{Log}[1 + E^{(2 \operatorname{ArcSech}[a + b*x])}])/b^2 - ((6*I) * a \operatorname{ArcSech}[a + b*x] \operatorname{PolyLog}[2, (-I) * E^{\operatorname{ArcSech}[a + b*x]}])/b^2 + ((6*I) * a \operatorname{ArcSech}[a + b*x] \operatorname{PolyLog}[2, I * E^{\operatorname{ArcSech}[a + b*x]}])/b^2 + (3 \operatorname{PolyLog}[2, -E^{(2 \operatorname{ArcSech}[a + b*x])}])/b^2 + ((6*I) * a \operatorname{PolyLog}[3, (-I) * E^{\operatorname{ArcSech}[a + b*x]}])/b^2 - ((6*I) * a \operatorname{PolyLog}[3, I * E^{\operatorname{ArcSech}[a + b*x]}])/b^2$

Rubi [A] time = 0.257207, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.2$, Rules used = {6321, 5468, 4190, 4180, 2531, 2282, 6589, 4184, 3718, 2190, 2279, 2391}

$$\frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} + \frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x \operatorname{ArcSech}[a + b*x]^3, x]$

[Out] $(-3 \operatorname{ArcSech}[a + b*x]^2)/(2*b^2) - (3 \operatorname{Sqrt}[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x) \operatorname{ArcSech}[a + b*x]^2)/(2*b^2) - (a^2 \operatorname{ArcSech}[a + b*x]^3)/(2*b^2) + (x^2 \operatorname{ArcSech}[a + b*x]^3)/2 + (6*a \operatorname{ArcSech}[a + b*x]^2 \operatorname{ArcTan}[E^{\operatorname{ArcSech}[a + b*x]}])/b^2 + (3 \operatorname{ArcSech}[a + b*x] \operatorname{Log}[1 + E^{(2 \operatorname{ArcSech}[a + b*x])}])/b^2 - ((6*I) * a \operatorname{ArcSech}[a + b*x] \operatorname{PolyLog}[2, (-I) * E^{\operatorname{ArcSech}[a + b*x]}])/b^2 + ((6*I) * a \operatorname{ArcSech}[a + b*x] \operatorname{PolyLog}[2, I * E^{\operatorname{ArcSech}[a + b*x]}])/b^2 + (3 \operatorname{PolyLog}[2, -E^{(2 \operatorname{ArcSech}[a + b*x])}])/b^2 + ((6*I) * a \operatorname{PolyLog}[3, (-I) * E^{\operatorname{ArcSech}[a + b*x]}])/b^2 - ((6*I) * a \operatorname{PolyLog}[3, I * E^{\operatorname{ArcSech}[a + b*x]}])/b^2$

Rule 6321

$\operatorname{Int}[(a + \operatorname{ArcSech}[c + (d \cdot x)]) \cdot (b \cdot x)^p \cdot (e + (f \cdot x))^m, x_Symbol] \rightarrow -\operatorname{Dist}[(d^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^p \operatorname{Sech}[x] \operatorname{Tanh}[x] \cdot (d \cdot e - c \cdot f + f \cdot \operatorname{Sech}[x])^m, x], x, \operatorname{ArcSech}[c + d*x]], x] /;$ FreeQ[{a,

b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 4190

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4180

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d

, e, n, p}, x] && EqQ[b*d, a*e]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int x \operatorname{sech}^{-1}(a+bx)^3 dx &= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}(x)(-a+\operatorname{sech}(x)) \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^2} \\
&= \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 - \frac{3 \operatorname{Subst}\left(\int x^2(-a+\operatorname{sech}(x))^2 dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2b^2} \\
&= \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 - \frac{3 \operatorname{Subst}\left(\int (a^2x^2 - 2ax^2 \operatorname{sech}(x) + x^2 \operatorname{sech}^2(x)) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2b^2} \\
&= -\frac{a^2 \operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 - \frac{3 \operatorname{Subst}\left(\int x^2 \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2b^2} + \dots \\
&= -\frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 + \dots \\
&= -\frac{3 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 \\
&= -\frac{3 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 \\
&= -\frac{3 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3 \\
&= -\frac{3 \operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2 \operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \operatorname{sech}^{-1}(a+bx)^3
\end{aligned}$$

Mathematica [A] time = 0.552299, size = 254, normalized size = 0.98

$$-3 \operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(a+bx)}\right) + 6ia \left(-2 \operatorname{sech}^{-1}(a+bx) \left(\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*ArcSech[a + b*x]^3,x]

[Out] $(-3 \sqrt{-((-1+a+bx)/(1+a+bx))} (1+a+bx) \operatorname{ArcSech}[a+bx]^2 - 2a(a+bx) \operatorname{ArcSech}[a+bx]^3 + (a+bx)^2 \operatorname{ArcSech}[a+bx]^3 + 3 \operatorname{ArcSech}[a+bx] (\operatorname{ArcSech}[a+bx] + 2 \operatorname{Log}[1 + E^{-2 \operatorname{ArcSech}[a+bx]})]) - 3 \operatorname{PolyLog}[2, -E^{-2 \operatorname{ArcSech}[a+bx]}] + (6I)a (-\operatorname{ArcSech}[a+bx]^2 (\operatorname{Log}[1 - I/E^{\operatorname{ArcSech}[a+bx]}] - \operatorname{Log}[1 + I/E^{\operatorname{ArcSech}[a+bx]}])) - 2 \operatorname{ArcSech}[a+bx] (\operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcSech}[a+bx]}] - \operatorname{PolyLog}[2, I/E^{\operatorname{ArcSech}[a+bx]}]) - 2 \operatorname{PolyLog}[3, (-I)/E^{\operatorname{ArcSech}[a+bx]}] + 2 \operatorname{PolyLog}[3, I/E^{\operatorname{ArcSech}[a+bx]}]))/(2b^2)$

Maple [F] time = 0.559, size = 0, normalized size = 0.

$$\int x (\operatorname{arcsech}(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arcsech(b*x+a)^3,x)

[Out] int(x*arcsech(b*x+a)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 \log(\sqrt{bx+a+1} \sqrt{-bx-a+1}) bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a)^3 - \int (1/2(16(b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \sqrt{bx+a+1} \sqrt{-bx-a+1} \log(bx+a)^3 + 16(b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \log(bx+a)^3 + 3(b^3x^4 + 2ab^2x^3 + (a^2b - b)x^2 + 4(b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \log(bx+a) + (2(b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \sqrt{bx+a+1} \log(bx+a) + (2b^3x^4 + 4ab^2x^3 + (2a^2b - b)x^2 + 2(b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \log(bx+a)) \sqrt{bx+a+1}) \sqrt{-bx-a+1}) \log(\sqrt{bx+a+1} \sqrt{-bx-a+1}) bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a)^2 - 24((b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \sqrt{bx+a+1} \sqrt{-bx-a+1} \log(bx+a)^2 + (b^3x^4 + 3ab^2x^3 + (3a^2b - b)x^2 + (a^3 - a)x) \log(bx+a)^2) \log(\sqrt{bx+a+1} \sqrt{-bx-a+1}) bx + \sqrt{bx+a+1} \sqrt{-bx-a+1} a + bx + a) / (b^3x^3 + 3ab^2x^2 + a^3 + (b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x - a) \sqrt{bx+a+1} \sqrt{-bx-a+1} + (3a^2b - b)x - a), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x \operatorname{ar} \operatorname{sech}(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(b*x + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{asech}^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a)**3,x)
```

```
[Out] Integral(x*asech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a)^3, x)
```

3.16 $\int \operatorname{sech}^{-1}(a + bx)^3 dx$

Optimal. Leaf size=136

$$\frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a + bx)}\right)}{b} - \frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a + bx)}\right)}{b} - \frac{6i \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(a + bx)}\right)}{b}$$

[Out] ((a + b*x)*ArcSech[a + b*x]^3)/b - (6*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a + b*x]])/b + ((6*I)*ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b - ((6*I)*ArcSech[a + b*x]*PolyLog[2, I*E^ArcSech[a + b*x]])/b - ((6*I)*PolyLog[3, (-I)*E^ArcSech[a + b*x]])/b + ((6*I)*PolyLog[3, I*E^ArcSech[a + b*x]])/b

Rubi [A] time = 0.104783, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6315, 6279, 5418, 4180, 2531, 2282, 6589}

$$\frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(a + bx)}\right)}{b} - \frac{6i \operatorname{sech}^{-1}(a + bx) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(a + bx)}\right)}{b} - \frac{6i \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(a + bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]^3, x]

[Out] ((a + b*x)*ArcSech[a + b*x]^3)/b - (6*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a + b*x]])/b + ((6*I)*ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b - ((6*I)*ArcSech[a + b*x]*PolyLog[2, I*E^ArcSech[a + b*x]])/b - ((6*I)*PolyLog[3, (-I)*E^ArcSech[a + b*x]])/b + ((6*I)*PolyLog[3, I*E^ArcSech[a + b*x]])/b

Rule 6315

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Dist[1/d, Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

Rule 6279

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := -Dist[c^(-1), Subst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,

b, c, n}, x] && IGtQ[n, 0]

Rule 5418

```
Int[(x_)^(m_)*Sech[(a_) + (b_)*(x_)^(n_)]^(p_)*Tanh[(a_) + (b_)*(x_)
^(n_)]^(q_), x_Symbol] := -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4180

```
Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_
))^m_, x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^{-1}(a+bx)^3 dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{-1}(x)^3 dx, x, a+bx\right)}{b} \\
&= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{3 \operatorname{Subst}\left(\int x^2 \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(6i) \operatorname{Subst}\left(\int x \log(1-ie^x) dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.140201, size = 153, normalized size = 1.12

$$\frac{-(a+bx)\operatorname{sech}^{-1}(a+bx)^3 + 3i\left(-2\operatorname{sech}^{-1}(a+bx)\left(\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right) - 2\left(\operatorname{PolyLog}\left(3, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{PolyLog}\left(3, ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]^3, x]

[Out] $-\left(-\left(\left(a + b*x\right) * \operatorname{ArcSech}\left[a + b*x\right]^3\right) + \left(3*I\right) * \left(-\left(\operatorname{ArcSech}\left[a + b*x\right]^2 * \left(\operatorname{Log}\left[1 - I / E^{\operatorname{ArcSech}\left[a + b*x\right]}\right] - \operatorname{Log}\left[1 + I / E^{\operatorname{ArcSech}\left[a + b*x\right]}\right]\right)\right) - 2 * \operatorname{ArcSech}\left[a + b*x\right] * \left(\operatorname{PolyLog}\left[2, \left(-I\right) / E^{\operatorname{ArcSech}\left[a + b*x\right]}\right] - \operatorname{PolyLog}\left[2, I / E^{\operatorname{ArcSech}\left[a + b*x\right]}\right]\right) - 2 * \left(\operatorname{PolyLog}\left[3, \left(-I\right) / E^{\operatorname{ArcSech}\left[a + b*x\right]}\right] - \operatorname{PolyLog}\left[3, I / E^{\operatorname{ArcSech}\left[a + b*x\right]}\right]\right)\right) / b$

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int (\operatorname{arcsech}(bx+a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)^3,x)

[Out] int(arcsech(b*x+a)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x \log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a\right)^3 - \int \frac{8(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x - a)}{\dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3,x, algorithm="maxima")

[Out] x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^3 - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 + 8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 + 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 12*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{arsech}(bx+a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3,x, algorithm="fricas")

[Out] `integral(arcsech(b*x + a)^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asech}^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asech(b*x+a)**3,x)`

[Out] `Integral(asech(a + b*x)**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(b*x+a)^3,x, algorithm="giac")`

[Out] `integrate(arcsech(b*x + a)^3, x)`

$$3.17 \quad \int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} dx$$

Optimal. Leaf size=378

$$3\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + 3\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 6\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) - 6\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) + 6\operatorname{PolyLog}\left(4, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + 6\operatorname{PolyLog}\left(4, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 3\operatorname{PolyLog}\left(4, -E^{(2*\operatorname{ArcSech}[a+bx])}\right)/4$$

```
[Out] ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]^3*Log[1 + E^(2*ArcSech[a + b*x])] + 3*ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 3*ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (3*ArcSech[a + b*x]^2*PolyLog[2, -E^(2*ArcSech[a + b*x])])/2 - 6*ArcSech[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - 6*ArcSech[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*ArcSech[a + b*x]*PolyLog[3, -E^(2*ArcSech[a + b*x])])/2 + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (3*PolyLog[4, -E^(2*ArcSech[a + b*x])])/4
```

Rubi [A] time = 0.51077, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6321, 5595, 5570, 3718, 2190, 2531, 6609, 2282, 6589, 5562}

$$3\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + 3\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 6\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) - 6\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) + 6\operatorname{PolyLog}\left(4, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + 6\operatorname{PolyLog}\left(4, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 3\operatorname{PolyLog}\left(4, -E^{(2*\operatorname{ArcSech}[a+bx])}\right)/4$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x, x]
```

```
[Out] ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]^3*Log[1 + E^(2*ArcSech[a + b*x])] + 3*ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 3*ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (3*ArcSech[a + b*x]^2*PolyLog[2, -E^(2*ArcSech[a + b*x])])/2 - 6*ArcSech[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - 6*ArcSech[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*ArcSech[a + b*x]*PolyLog[3, -E^(2*ArcSech[a + b*x])])/2 + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (3*PolyLog[4, -E^(2*ArcSech[a + b*x])])/4
```

$3 \text{PolyLog}[4, -E^{(2 \text{ArcSech}[a + b x])}]/4$

Rule 6321

$\text{Int}[(a + \text{ArcSech}[c + (d \cdot x)] \cdot (b))^{(p)} \cdot ((e) + (f) \cdot (x))^{(m)}, x_Symbol] \rightarrow -\text{Dist}[(d^{(m+1)})^{(-1)}, \text{Subst}[\text{Int}[(a + b x)^p \text{Sech}[x] \text{Tanh}[x] \cdot (d e - c f + f \text{Sech}[x])^m, x], x, \text{ArcSech}[c + d x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 5595

$\text{Int}[(e + (f \cdot x))^{(m)} \cdot (F)[(c) + (d \cdot x)]^{(n)} \cdot (G)[(c) + (d \cdot x)]^{(p)} / ((a) + (b \cdot x) \cdot \text{Sech}[c + (d \cdot x)]), x_Symbol] \rightarrow \text{Int}[(e + f x)^m \text{Cosh}[c + d x] \cdot F[c + d x]^n \cdot G[c + d x]^p / (b + a \text{Cosh}[c + d x]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{HyperbolicQ}[F] \ \&\& \ \text{HyperbolicQ}[G] \ \&\& \ \text{IntegersQ}[m, n, p]$

Rule 5570

$\text{Int}[(e + (f \cdot x))^{(m)} \cdot \text{Tanh}[(c) + (d \cdot x)]^{(n)} / (\text{Cosh}[(c) + (d \cdot x)] \cdot (b) + (a)), x_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f x)^m \text{Tanh}[c + d x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f x)^m \text{Sinh}[c + d x] \cdot \text{Tanh}[c + d x]^{(n-1)} / (a + b \text{Cosh}[c + d x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 3718

$\text{Int}[(c + (d \cdot x))^{(m)} \cdot \tan[(e) + (\text{Complex}[0, fz]) \cdot (f \cdot x)], x_Symbol] \rightarrow -\text{Simp}[(I \cdot (c + d x)^{(m+1)}) / (d \cdot (m+1)), x] + \text{Dist}[2 \cdot I, \text{Int}[(c + d x)^m \cdot E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))} / (1 + E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))}), x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F)^{(g \cdot (e + (f \cdot x)))^{(n)} \cdot ((c) + (d \cdot x))^{(m)} / ((a) + (b \cdot x) \cdot (F)^{(g \cdot (e + (f \cdot x)))^{(n)}}), x_Symbol] \rightarrow \text{Simp}[(c + d x)^m \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f x)))^n}) / a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d x)^{(m-1)} \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f x)))^n}) / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2531

$\text{Int}[\text{Log}[1 + (e) \cdot (F)^{(c \cdot (a) + (b \cdot x))}]^{(n)} \cdot ((f) + (g \cdot x))^{(m)}, x_Symbol] \rightarrow -\text{Simp}[(f + g x)^m \cdot \text{PolyLog}[2, -(e \cdot (F^{(c \cdot (a + b x))})^n)] / (b \cdot c \cdot n \cdot \text{Log}[F]), x] + \text{Dist}[(g \cdot m) / (b \cdot c \cdot n \cdot \text{Log}[F]), \text{Int}[(f + g x)^{(m-1)}], x]$


```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} dx &= -\operatorname{Subst}\left(\int \frac{x^3 \operatorname{sech}(x) \tanh(x)}{-a + \operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\operatorname{Subst}\left(\int \frac{x^3 \tanh(x)}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\left(a \operatorname{Subst}\left(\int \frac{x^3 \sinh(x)}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - \operatorname{Subst}\left(\int x^3 \tanh(x) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{e^{2x} x^3}{1+e^{2x}} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - a \operatorname{Subst}\left(\int \frac{e^x x^3}{1-\sqrt{1-a^2}-ae^x} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)^3 \\
&= \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)^3 \\
&= \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)^3 \\
&= \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)^3 \\
&= \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)^3
\end{aligned}$$

Mathematica [A] time = 0.318892, size = 384, normalized size = 1.02

$$3 \operatorname{sech}^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2, -\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right) + 3 \operatorname{sech}^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right) - 6 \operatorname{sech}^{-1}(a+bx) \operatorname{PolyLog}\left(3, -\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right) - 6 \operatorname{sech}^{-1}(a+bx) \operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]^3/x, x]

[Out] $-\operatorname{ArcSech}[a + b*x]^4/2 - \operatorname{ArcSech}[a + b*x]^3 \operatorname{Log}[1 + E^{(-2*\operatorname{ArcSech}[a + b*x])}] + \operatorname{ArcSech}[a + b*x]^3 \operatorname{Log}[1 + (a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2])] + \operatorname{ArcSech}[a + b*x]^3 \operatorname{Log}[1 - (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])] + (3*\operatorname{ArcSech}[a + b*x]^2*\operatorname{PolyLog}[2, -E^{(-2*\operatorname{ArcSech}[a + b*x])}])/2 + 3*\operatorname{ArcSech}[a + b*x]^2*\operatorname{PolyLog}[2, -((a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2]))] + 3*\operatorname{ArcSech}[a + b*x]^2*\operatorname{PolyLog}[2, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])] + (3*\operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[3, -E^{(-2*\operatorname{ArcSech}[a + b*x])}])/2 - 6*\operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[3, -((a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2]))] - 6*\operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[3, (a*E^{\operatorname{ArcSech}[a + b*x]})/(1 + \operatorname{Sqrt}[1 - a^2])] - 6*\operatorname{ArcSech}[a + b*x]*\operatorname{PolyLog}[3, (a*E^{\operatorname{ArcSech}[a + b*x]})/(-1 + \operatorname{Sqrt}[1 - a^2])]$

```
[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*PolyLog[4, -E^(-2*ArcSech[a + b*x])])/4 + 6*PolyLog[4, -((a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2]))] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
```

Maple [F] time = 0.373, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arcsech}(bx + a))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3/x,x)
```

```
[Out] int(arcsech(b*x+a)^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(b*x + a)^3/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsech}(bx + a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^3(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)**3/x,x)

[Out] Integral(asech(a + b*x)**3/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)^3/x, x)

$$3.18 \quad \int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2} dx$$

Optimal. Leaf size=330

$$\frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{6b\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

[Out] -((b*ArcSech[a + b*x]^3)/a) - ArcSech[a + b*x]^3/x + (3*b*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (3*b*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])

Rubi [A] time = 0.575216, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {6321, 5468, 4191, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{6b\operatorname{PolyLog}\left(3, \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]^3/x^2,x]

[Out] -((b*ArcSech[a + b*x]^3)/a) - ArcSech[a + b*x]^3/x + (3*b*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (3*b*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])

Rule 6321

```
Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.), x_Symbol] := -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*Sech[
(c_.) + (d_.)*(x_.)])^(n_.)*Tanh[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[((e
+ f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_.)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/
((a_.) + (b_.)*(F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_)^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2} dx &= -\left(b \operatorname{Subst}\left(\int \frac{x^3 \operatorname{sech}(x) \tanh(x)}{(-a + \operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + (3b) \operatorname{Subst}\left(\int \frac{x^2}{-a + \operatorname{sech}(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + (3b) \operatorname{Subst}\left(\int \left(-\frac{x^2}{a} + \frac{x^2}{a(1-a \cosh(x))}\right) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{(3b) \operatorname{Subst}\left(\int \frac{x^2}{1-a \cosh(x)} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{(6b) \operatorname{Subst}\left(\int \frac{e^x x^2}{-a+2e^x-ae^{2x}} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} - \frac{(6b) \operatorname{Subst}\left(\int \frac{e^x x^2}{2-2\sqrt{1-a^2}-2ae^x} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a^2}} + \dots \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}} \\
&= -\frac{b \operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b \operatorname{sech}^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3b \operatorname{sech}^{-1}(a+bx)}{a\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [C] time = 45.6533, size = 1849, normalized size = 5.6

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]^3/x^2,x]

[Out] -((-3*b*x*ArcCos[-a^(-1)]*ArcSech[a + b*x]^2 + a*Sqrt[-1 + a^2]*ArcSech[a + b*x]^3 + Sqrt[-1 + a^2]*b*x*ArcSech[a + b*x]^3 + 12*b*x*ArcCos[-a^(-1)]*Ar

$$\begin{aligned}
& c\text{Sech}[a + b*x]*\text{ArcTanh}[\text{Tanh}[\text{ArcSech}[a + b*x]/2]] - 12*b*x*\text{ArcCos}[-a^{(-1)}]*\text{ArcTanh}[\text{Tanh}[\text{ArcSech}[a + b*x]/2]]^2 - 6*b*x*\text{ArcCos}[-a^{(-1)}]*\text{ArcSech}[a + b*x] \\
& * \text{Log}[\text{Sqrt}[-1 + a^2]/(\text{Sqrt}[2]*\text{Sqrt}[a]*E^{(I*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})}]/\text{Sqrt}[-1 + a^2]]*\text{Sqrt}[1 + a*\text{Cos}[2*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})]] + (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Coth}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[\text{Sqrt}[-1 + a^2]/(\text{Sqrt}[2]*\text{Sqrt}[a]*E^{(I*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})}]/\text{Sqrt}[-1 + a^2]]*\text{Sqrt}[1 + a*\text{Cos}[2*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})]] - (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Tanh}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[\text{Sqrt}[-1 + a^2]/(\text{Sqrt}[2]*\text{Sqrt}[a]*E^{(I*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})}]/\text{Sqrt}[-1 + a^2]]*\text{Sqrt}[1 + a*\text{Cos}[2*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})]] - 6*b*x*\text{ArcCos}[-a^{(-1)}]*\text{ArcSech}[a + b*x]*\text{Log}[(\text{Sqrt}[-1 + a^2]*E^{(I*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})}]/(\text{Sqrt}[2]*\text{Sqrt}[a]*\text{Sqrt}[1 + a*\text{Cos}[2*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})]]) - (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Coth}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[(\text{Sqrt}[-1 + a^2]*E^{(I*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})}]/(\text{Sqrt}[2]*\text{Sqrt}[a]*\text{Sqrt}[1 + a*\text{Cos}[2*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})]]) + (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Tanh}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[(\text{Sqrt}[-1 + a^2]*E^{(I*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})}]/(\text{Sqrt}[2]*\text{Sqrt}[a]*\text{Sqrt}[1 + a*\text{Cos}[2*\text{ArcTan}[\frac{(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2)]}{\text{Sqrt}[-1 + a^2]})]]) + 6*b*x*\text{ArcCos}[-a^{(-1)}]*\text{ArcSech}[a + b*x]*\text{Log}[(\text{Sqrt}[-1 + a^2] + I*(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])/(2*\text{Sqrt}[a]*\text{Sqrt}[-(((-1 + a^2)*(a + b*x))/(b*x))])* \text{Sqrt}[-((b*x)/((-1 + a)*(1 + a + b*x))])] + (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Coth}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[(\text{Sqrt}[-1 + a^2] + I*(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])/(2*\text{Sqrt}[a]*\text{Sqrt}[-(((-1 + a^2)*(a + b*x))/(b*x))])* \text{Sqrt}[-((b*x)/((-1 + a)*(1 + a + b*x))])] - (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Tanh}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[(\text{Sqrt}[-1 + a^2] + I*(1+a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])/(2*\text{Sqrt}[a]*\text{Sqrt}[-(((-1 + a^2)*(a + b*x))/(b*x))])* \text{Sqrt}[-((b*x)/((-1 + a)*(1 + a + b*x))])] + 6*b*x*\text{ArcCos}[-a^{(-1)}]*\text{ArcSech}[a + b*x]*\text{Log}[(I*(-1 + a^2)*\text{Sqrt}[-((b*x)/((-1 + a)*(1 + a + b*x))])]/(\text{Sqrt}[a]*\text{Sqrt}[-(((-1 + a^2)*(a + b*x))/(b*x))])*((-I)*\text{Sqrt}[-1 + a^2] + (1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])) - (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Coth}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[(I*(-1 + a^2)*\text{Sqrt}[-((b*x)/((-1 + a)*(1 + a + b*x))])]/(\text{Sqrt}[a]*\text{Sqrt}[-(((-1 + a^2)*(a + b*x))/(b*x))])*((-I)*\text{Sqrt}[-1 + a^2] + (1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])) + (12*I)*b*x*\text{ArcSech}[a + b*x]*\text{ArcTanh}[\text{Tanh}[\text{ArcSech}[a + b*x]/2]]*\text{Log}[(I*(-1 + a^2)*\text{Sqrt}[-((b*x)/((-1 + a)*(1 + a + b*x))])]/(\text{Sqrt}[a]*\text{Sqrt}[-(((-1 + a^2)*(a + b*x))/(b*x))])*((-I)*\text{Sqrt}[-1 + a^2] + (1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])) - (3*I)*b*x*\text{ArcSech}[a + b*x]^2*\text{Log}[(I - I*a + \text{Sqrt}[-1 + a^2])*(-I + ((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])/\text{Sqrt}[-1 + a^2])]/(a*(1 + \text{Tanh}[\text{ArcSech}[a + b*x]/2])) + (3*I)*b*x*\text{ArcSech}[a + b*x]^2*\text{Log}[(I - I*a + \text{Sqrt}[-1 + a^2])*(I + ((1 + a)*\text{Tanh}[\text{ArcSech}[a + b*x]/2])/\text{Sqrt}[-1 + a^2])]/(a*(1 + \text{Tanh}[\text{ArcSech}[a + b*x]/2])) - (6*I)*b*x*\text{ArcSech}[a + b*x]*\text{PolyLog}[2, ((1 - I*\text{Sqrt}[-1 + a^2])*(1 - \text{Sqrt}[-(((-1 + a + b*x)/(1 + a + b*x))])*(1 + a + b*x)))/(a*(a + b*x))] + (6*I)*b*x*\text{ArcSech}[a + b*x]*\text{PolyLog}[2, ((1 + I*\text{Sqrt}[-1 + a^2])*(1 - \text{Sqrt}[-(((-1 + a + b*x)/(1 + a + b*x))])
\end{aligned}$$

$$\frac{(1 + a + bx)}{a(a + bx)} - (6I)bx \operatorname{PolyLog}[3, ((1 - I\sqrt{-1 + a^2}) * (1 - \sqrt{-((-1 + a + bx)/(1 + a + bx))}) * (1 + a + bx)) / (a(a + bx))] + (6I)bx \operatorname{PolyLog}[3, ((1 + I\sqrt{-1 + a^2}) * (1 - \sqrt{-((-1 + a + bx)/(1 + a + bx))}) * (1 + a + bx)) / (a(a + bx))] / (a\sqrt{-1 + a^2} * x)$$

Maple [F] time = 0.458, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arcsech}(bx + a))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsech(b*x+a)^3/x^2,x)`

[Out] `int(arcsech(b*x+a)^3/x^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(\sqrt{bx + a + 1}\sqrt{-bx - a + 1}bx + \sqrt{bx + a + 1}\sqrt{-bx - a + 1}a + bx + a)^3}{x} - \int \frac{8(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x - a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(b*x+a)^3/x^2,x, algorithm="maxima")`

[Out] `-log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^3/x - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 + 8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 - 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 12*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)`

$*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*\sqrt{b*x + a + 1}*\sqrt{-b*x - a + 1}), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsech}(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="fricas")

[Out] integral(arcsech(b*x + a)^3/x^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^3(a+bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)**3/x**2,x)

[Out] Integral(asech(a + b*x)**3/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx+a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)^3/x^2, x)

$$3.19 \quad \int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3} dx$$

Optimal. Leaf size=965

result too large to display

```
[Out] (-3*b^2*ArcSech[a + b*x]^2)/(2*a^2*(1 - a^2)) + (3*b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x]^2)/(2*a*(1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^3)/(2*a^2) - ArcSech[a + b*x]^3/(2*x^2) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(2*a^2*(1 - a^2)^(3/2)) - (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)) - (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(2*a^2*(1 - a^2)^(3/2)) + (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (3*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (6*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (3*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)) - (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (6*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (3*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (6*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (3*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (6*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
```

Rubi [A] time = 1.33793, antiderivative size = 965, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6321, 5468, 4191, 3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} - \frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} - \frac{3b^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \operatorname{sech}^{-1}(a+bx)^2}{a^2 \sqrt{1-a^2}} + \frac{3b^2 \log\left(1 - \frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \operatorname{sech}^{-1}(a+bx)^2}{a^2 \sqrt{1-a^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]^3/x^3,x]

[Out]
$$\begin{aligned} & (-3*b^2*ArcSech[a + b*x]^2)/(2*a^2*(1 - a^2)) + (3*b^2*sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x]^2)/(2*a*(1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^3)/(2*a^2) - ArcSech[a + b*x]^3/(2*x^2) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(2*a^2*(1 - a^2)^(3/2)) - (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*sqrt[1 - a^2]) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*(1 - a^2)) - (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(2*a^2*(1 - a^2)^(3/2)) + (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*sqrt[1 - a^2]) + (3*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*(1 - a^2)^(3/2)) - (6*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*sqrt[1 - a^2]) + (3*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*(1 - a^2)) - (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*(1 - a^2)^(3/2)) + (6*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*sqrt[1 - a^2]) - (3*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*(1 - a^2)^(3/2)) + (6*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])]/(1 - Sqrt[1 - a^2]))/(a^2*sqrt[1 - a^2]) + (3*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*(1 - a^2)^(3/2)) - (6*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])]/(1 + Sqrt[1 - a^2]))/(a^2*sqrt[1 - a^2]) \end{aligned}$$

Rule 6321

Int[((a_.) + ArcSech[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Tanh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]

Rule 5468

Int[((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_.)])^(n_.)*Tanh[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_.)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.))))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

, g, n}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3} dx &= -\left(b^2 \operatorname{Subst}\left(\int \frac{x^3 \operatorname{sech}(x) \tanh(x)}{(-a+\operatorname{sech}(x))^3} dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{1}{2}(3b^2) \operatorname{Subst}\left(\int \frac{x^2}{(-a+\operatorname{sech}(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{1}{2}(3b^2) \operatorname{Subst}\left(\int \left(\frac{x^2}{a^2} + \frac{x^2}{a^2(-1+a \cosh(x))^2} + \frac{2x^2}{a^2(-1+a \cosh(x))}\right) dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{(3b^2) \operatorname{Subst}\left(\int \frac{x^2}{(-1+a \cosh(x))^2} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2a^2} + \\
&= \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{(6b^2) \operatorname{Subst}\left(\int \frac{x^2}{(-1+a \cosh(x))} dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2a^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
&= -\frac{3b^2 \operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)} + \frac{3b^2 \sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx) \operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2 \operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2}
\end{aligned}$$

Mathematica [F] time = 9.65002, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSech[a + b*x]^3/x^3,x]

[Out] Integrate[ArcSech[a + b*x]^3/x^3, x]

Maple [F] time = 0.694, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arcsech}(bx + a))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)^3/x^3,x)

[Out] int(arcsech(b*x+a)^3/x^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a)^3}{2x^2} - \int \frac{16(b^3x^3 + 3ab^2x^2 + a^3 + (3a^2b - b)x)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="maxima")

[Out] $-1/2*\log(\sqrt{bx+a+1}*\sqrt{-bx-a+1})*bx + \sqrt{bx+a+1}*\sqrt{-bx-a+1}*a + bx + a)^3/x^2 - \operatorname{integrate}(1/2*(16*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*\sqrt{bx+a+1}*\sqrt{-bx-a+1}*\log(b*x + a)^3 + 16*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*\log(b*x + a)^3 - 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*\log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*\sqrt{bx+a+1}*\log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*\log(b*x + a))*\sqrt{bx+a+1}*\sqrt{-bx-a+1})*\log(\sqrt{bx+a+1}*\sqrt{-bx-a+1})*bx + \sqrt{bx+a+1}*\sqrt{-bx-a+1}*a + bx + a)^2 - 24*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*\sqrt{bx+a+1}*\sqrt{-bx-a+1}*\log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 +$

$(3a^2b - b)x - a) \log(bx + a)^2 \log(\sqrt{bx + a + 1} \sqrt{-bx - a + 1} bx + \sqrt{bx + a + 1} \sqrt{-bx - a + 1} a + bx + a) / (b^3x^6 + 3ab^2x^5 + (3a^2b - b)x^4 + (a^3 - a)x^3 + (b^3x^6 + 3ab^2x^5 + (3a^2b - b)x^4 + (a^3 - a)x^3) \sqrt{bx + a + 1} \sqrt{-bx - a + 1}), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsech}(bx + a)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="fricas")

[Out] integral(arcsech(b*x + a)^3/x^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^3(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)**3/x**3,x)

[Out] Integral(asech(a + b*x)**3/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="giac")

[Out] integrate(arcsech(b*x + a)^3/x^3, x)

3.20 $\int x^3 \operatorname{sech}^{-1}(\sqrt{x}) dx$

Optimal. Leaf size=164

$$\frac{1}{4}x^4 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{(1-x)^4}{28\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{3(1-x)^3}{20\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} + \frac{(1-x)^2}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{1-x}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

[Out] $-(1-x)/(4*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (1-x)^2/(4*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) - (3*(1-x)^3)/(20*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (1-x)^4/(28*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (x^4*\operatorname{ArcSech}[\operatorname{Sqrt}[x]])/4$

Rubi [A] time = 0.0304525, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6345, 12, 43}

$$\frac{1}{4}x^4 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{(1-x)^4}{28\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{3(1-x)^3}{20\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} + \frac{(1-x)^2}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{1-x}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{ArcSech}[\operatorname{Sqrt}[x]], x]$

[Out] $-(1-x)/(4*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (1-x)^2/(4*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) - (3*(1-x)^3)/(20*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (1-x)^4/(28*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (x^4*\operatorname{ArcSech}[\operatorname{Sqrt}[x]])/4$

Rule 6345

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[u_]*(b_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*(a + b*\operatorname{ArcSech}[u])]/(d*(m+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1-u^2])]/(d*(m+1)*u*\operatorname{Sqrt}[-1+1/u]*\operatorname{Sqrt}[1+1/u]), \operatorname{Int}[\operatorname{SimplifyIntegrand}[(c + d*x)^{(m+1)}*D[u, x]]/(u*\operatorname{Sqrt}[1-u^2]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \operatorname{NeQ}[m, -1] \ \&\& \operatorname{InverseFunctionFreeQ}[u, x] \ \&\& \operatorname{!FunctionOfQ}[(c + d*x)^{(m+1)}, u, x] \ \&\& \operatorname{!FunctionOfExponentialQ}[u, x]$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
 \int x^3 \operatorname{sech}^{-1}(\sqrt{x}) \, dx &= \frac{1}{4} x^4 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{x^3}{2\sqrt{1-x}} \, dx}{4\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
 &= \frac{1}{4} x^4 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{x^3}{\sqrt{1-x}} \, dx}{8\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
 &= \frac{1}{4} x^4 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \left(\frac{1}{\sqrt{1-x}} - 3\sqrt{1-x} + 3(1-x)^{3/2} - (1-x)^{5/2} \right) \, dx}{8\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
 &= -\frac{1-x}{4\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{4\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{3(1-x)^3}{20\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^4}{28\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}}
 \end{aligned}$$

Mathematica [A] time = 0.04074, size = 84, normalized size = 0.51

$$\frac{1}{4} x^4 \operatorname{sech}^{-1}(\sqrt{x}) - \frac{1}{140} \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} (5x^{7/2} + 5x^3 + 6x^{5/2} + 6x^2 + 8x^{3/2} + 8x + 16\sqrt{x} + 16)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[Sqrt[x]], x]
```

```
[Out] -(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(16 + 16*Sqrt[x] + 8*x + 8*x^(3/2) + 6*x^2 + 6*x^(5/2) + 5*x^3 + 5*x^(7/2)))/140 + (x^4*ArcSech[Sqrt[x]])/4
```

Maple [A] time = 0.139, size = 54, normalized size = 0.3

$$\frac{x^4}{4} \operatorname{arcsech}(\sqrt{x}) - \frac{5x^3 + 6x^2 + 8x + 16}{140} \sqrt{-(-1 + \sqrt{x})} \frac{1}{\sqrt{x}} \sqrt{x} \sqrt{(1 + \sqrt{x})} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arcsech(x^(1/2)),x)`

[Out] $\frac{1}{4}x^4 \operatorname{arcsech}(x^{(1/2)}) - \frac{1}{140}(-(-1+x^{(1/2)})/x^{(1/2)})^{(1/2)}x^{(1/2)}((1+x^{(1/2)})/x^{(1/2)})^{(1/2)}(5x^3+6x^2+8x+16)$

Maxima [A] time = 0.982, size = 78, normalized size = 0.48

$$\frac{1}{28}x^{\frac{7}{2}}\left(\frac{1}{x}-1\right)^{\frac{7}{2}} - \frac{3}{20}x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}} + \frac{1}{4}x^4 \operatorname{ar} \operatorname{sech}(\sqrt{x}) + \frac{1}{4}x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}} - \frac{1}{4}\sqrt{x}\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsech(x^(1/2)),x, algorithm="maxima")`

[Out] $\frac{1}{28}x^{(7/2)}*(1/x - 1)^{(7/2)} - \frac{3}{20}x^{(5/2)}*(1/x - 1)^{(5/2)} + \frac{1}{4}x^4 \operatorname{arc} \operatorname{sech}(\sqrt{x}) + \frac{1}{4}x^{(3/2)}*(1/x - 1)^{(3/2)} - \frac{1}{4}\sqrt{x}*\sqrt{1/x - 1}$

Fricas [A] time = 1.83839, size = 144, normalized size = 0.88

$$\frac{1}{4}x^4 \log\left(\frac{x\sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right) - \frac{1}{140}(5x^3 + 6x^2 + 8x + 16)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsech(x^(1/2)),x, algorithm="fricas")`

[Out] $\frac{1}{4}x^4 \log((x*\sqrt{-(x-1)/x} + \sqrt{x})/x) - \frac{1}{140}(5x^3 + 6x^2 + 8x + 16)*\sqrt{x}*\sqrt{-(x-1)/x}$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*asech(x**(1/2)),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="giac")

[Out] integrate(x^3*arcsech(sqrt(x)), x)

3.21 $\int x^2 \operatorname{sech}^{-1}(\sqrt{x}) dx$

Optimal. Leaf size=126

$$\frac{1}{3}x^3 \operatorname{sech}^{-1}(\sqrt{x}) - \frac{(1-x)^3}{15\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} + \frac{2(1-x)^2}{9\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{1-x}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

[Out] $-(1-x)/(3*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (2*(1-x)^2)/(9*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) - (1-x)^3/(15*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (x^3*\operatorname{ArcSech}[\operatorname{Sqrt}[x]])/3$

Rubi [A] time = 0.0270286, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6345, 12, 43}

$$\frac{1}{3}x^3 \operatorname{sech}^{-1}(\sqrt{x}) - \frac{(1-x)^3}{15\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} + \frac{2(1-x)^2}{9\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{1-x}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{ArcSech}[\operatorname{Sqrt}[x]], x]$

[Out] $-(1-x)/(3*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (2*(1-x)^2)/(9*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) - (1-x)^3/(15*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (x^3*\operatorname{ArcSech}[\operatorname{Sqrt}[x]])/3$

Rule 6345

$\operatorname{Int}[(a + \operatorname{ArcSech}[u]*(b.))*((c.) + (d.)*(x.))^{(m.)}, x_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*(a + b*\operatorname{ArcSech}[u])/(d*(m+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1-u^2])/(d*(m+1)*u*\operatorname{Sqrt}[-1+1/u]*\operatorname{Sqrt}[1+1/u]), \operatorname{Int}[\operatorname{SimplifyIntegrand}[(c + d*x)^{(m+1)}*D[u, x]]/(u*\operatorname{Sqrt}[1-u^2]), x], x] /;$ FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^{(m+1)}, u, x] && !FunctionOfExponentialQ[u, x]

Rule 12

$\operatorname{Int}[(a.)*(u.), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b.)*(v.) /; FreeQ[b, x]]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \operatorname{sech}^{-1}(\sqrt{x}) \, dx &= \frac{1}{3} x^3 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{x^2}{2\sqrt{1-x}} \, dx}{3\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1}{3} x^3 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{x^2}{\sqrt{1-x}} \, dx}{6\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1}{3} x^3 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \left(\frac{1}{\sqrt{1-x}} - 2\sqrt{1-x} + (1-x)^{3/2} \right) \, dx}{6\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= -\frac{1-x}{3\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{2(1-x)^2}{9\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{(1-x)^3}{15\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{3} x^3 \operatorname{sech}^{-1}
\end{aligned}$$

Mathematica [A] time = 0.0395421, size = 72, normalized size = 0.57

$$\frac{1}{3} x^3 \operatorname{sech}^{-1}(\sqrt{x}) - \frac{1}{45} \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} (3x^{5/2} + 3x^2 + 4x^{3/2} + 4x + 8\sqrt{x} + 8)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcSech[Sqrt[x]], x]

[Out] -(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 4*x + 4*x^(3/2) + 3*x^2 + 3*x^(5/2)))/45 + (x^3*ArcSech[Sqrt[x]])/3

Maple [A] time = 0.126, size = 49, normalized size = 0.4

$$\frac{x^3}{3} \operatorname{arcsech}(\sqrt{x}) - \frac{3x^2 + 4x + 8}{45} \sqrt{-(-1 + \sqrt{x})} \frac{1}{\sqrt{x}} \sqrt{x} \sqrt{(1 + \sqrt{x})} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsech(x^(1/2)),x)`

[Out] `1/3*x^3*arcsech(x^(1/2))-1/45*(-(-1+x^(1/2))/x^(1/2))^(1/2)*x^(1/2)*((1+x^(1/2))/x^(1/2))^(1/2)*(3*x^2+4*x+8)`

Maxima [A] time = 0.982506, size = 62, normalized size = 0.49

$$-\frac{1}{15} x^{\frac{5}{2}} \left(\frac{1}{x} - 1\right)^{\frac{5}{2}} + \frac{1}{3} x^3 \operatorname{arsech}(\sqrt{x}) + \frac{2}{9} x^{\frac{3}{2}} \left(\frac{1}{x} - 1\right)^{\frac{3}{2}} - \frac{1}{3} \sqrt{x} \sqrt{\frac{1}{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(x^(1/2)),x, algorithm="maxima")`

[Out] `-1/15*x^(5/2)*(1/x - 1)^(5/2) + 1/3*x^3*arcsech(sqrt(x)) + 2/9*x^(3/2)*(1/x - 1)^(3/2) - 1/3*sqrt(x)*sqrt(1/x - 1)`

Fricas [A] time = 1.87177, size = 131, normalized size = 1.04

$$\frac{1}{3} x^3 \log\left(\frac{x \sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right) - \frac{1}{45} (3x^2 + 4x + 8) \sqrt{x} \sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(x^(1/2)),x, algorithm="fricas")`

[Out] `1/3*x^3*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/45*(3*x^2 + 4*x + 8)*sqrt(x)*sqrt(-(x - 1)/x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*asech(x**(1/2)),x)

[Out] Integral(x**2*asech(sqrt(x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="giac")

[Out] integrate(x^2*arcsech(sqrt(x)), x)

3.22 $\int x \operatorname{sech}^{-1}(\sqrt{x}) dx$

Optimal. Leaf size=88

$$\frac{1}{2}x^2 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{(1-x)^2}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

[Out] $-(1-x)/(2*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (1-x)^2/(6*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (x^2*\operatorname{ArcSech}[\operatorname{Sqrt}[x]])/2$

Rubi [A] time = 0.0183515, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6345, 12, 43}

$$\frac{1}{2}x^2 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{(1-x)^2}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{ArcSech}[\operatorname{Sqrt}[x]], x]$

[Out] $-(1-x)/(2*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (1-x)^2/(6*\operatorname{Sqrt}[-1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[1+1/\operatorname{Sqrt}[x]]*\operatorname{Sqrt}[x]) + (x^2*\operatorname{ArcSech}[\operatorname{Sqrt}[x]])/2$

Rule 6345

$\operatorname{Int}[(a + \operatorname{ArcSech}[u]*(b + (c + d*x)^m)), x_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1}*(a + b*\operatorname{ArcSech}[u])/(d*(m+1)), x] + \operatorname{Dist}[b*\operatorname{Sqrt}[1-u^2], \operatorname{Int}[\operatorname{SimplifyIntegrand}[(c + d*x)^{m+1}*D[u, x]]/(u*\operatorname{Sqrt}[1-u^2]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \operatorname{NeQ}[m, -1] \ \&\& \ \operatorname{InverseFunctionFreeQ}[u, x] \ \&\& \ \operatorname{!FunctionOfQ}[(c + d*x)^{m+1}, u, x] \ \&\& \ \operatorname{!FunctionOfExponentialQ}[u, x]$

Rule 12

$\operatorname{Int}[(a)*(u), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ $\operatorname{FreeQ}[a, x] \ \&\& \ \operatorname{!MatchQ}[u, (b)*(v)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x \operatorname{sech}^{-1}(\sqrt{x}) dx &= \frac{1}{2} x^2 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{x}{2\sqrt{1-x}} dx}{2\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1}{2} x^2 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{x}{\sqrt{1-x}} dx}{4\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1}{2} x^2 \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right) dx}{4\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= -\frac{1-x}{2\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{6\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.0263739, size = 56, normalized size = 0.64

$$\frac{1}{2} x^2 \operatorname{sech}^{-1}(\sqrt{x}) - \frac{1}{6} \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} (x^{3/2} + x + 2\sqrt{x} + 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[Sqrt[x]], x]
```

```
[Out] -(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + x + x^(3/2)))/6 + (x^2
*ArcSech[Sqrt[x]])/2
```

Maple [A] time = 0.13, size = 42, normalized size = 0.5

$$\frac{x^2}{2} \operatorname{arcsech}(\sqrt{x}) - \frac{x+2}{6} \sqrt{-(-1+\sqrt{x})} \frac{1}{\sqrt{x}} \sqrt{x} \sqrt{(1+\sqrt{x})} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsech(x^(1/2)),x)`

[Out] $\frac{1}{2}x^2 \operatorname{arcsech}(x^{1/2}) - \frac{1}{6} * (-(-1+x^{1/2})/x^{1/2})^{1/2} * x^{1/2} * ((1+x^{1/2})/x^{1/2})^{1/2} * (x+2)$

Maxima [A] time = 0.987311, size = 46, normalized size = 0.52

$$\frac{1}{6} x^2 \left(\frac{1}{x} - 1\right)^{\frac{3}{2}} + \frac{1}{2} x^2 \operatorname{ar} \operatorname{sech}(\sqrt{x}) - \frac{1}{2} \sqrt{x} \sqrt{\frac{1}{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(x^(1/2)),x, algorithm="maxima")`

[Out] $\frac{1}{6}x^{3/2} * (1/x - 1)^{3/2} + \frac{1}{2}x^2 * \operatorname{arcsech}(\sqrt{x}) - \frac{1}{2} * \sqrt{x} * \sqrt{1/x - 1}$

Fricas [A] time = 1.86514, size = 116, normalized size = 1.32

$$\frac{1}{2} x^2 \log\left(\frac{x \sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right) - \frac{1}{6} (x+2) \sqrt{x} \sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(x^(1/2)),x, algorithm="fricas")`

[Out] $\frac{1}{2}x^2 * \log((x * \sqrt{-(x-1)/x} + \sqrt{x})/x) - \frac{1}{6} * (x+2) * \sqrt{x} * \sqrt{-(x-1)/x}$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{asech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asech(x**(1/2)),x)`

[Out] `Integral(x*asech(sqrt(x)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(x^(1/2)),x, algorithm="giac")`

[Out] `integrate(x*arcsech(sqrt(x)), x)`

3.23 $\int \operatorname{sech}^{-1}(\sqrt{x}) dx$

Optimal. Leaf size=43

$$x \operatorname{sech}^{-1}(\sqrt{x}) - \frac{1-x}{\sqrt{\frac{1}{\sqrt{x}}-1} \sqrt{\frac{1}{\sqrt{x}}+1} \sqrt{x}}$$

[Out] -((1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])) + x*ArcSech[Sqrt[x]]

Rubi [A] time = 0.0072149, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6343, 12, 32}

$$x \operatorname{sech}^{-1}(\sqrt{x}) - \frac{1-x}{\sqrt{\frac{1}{\sqrt{x}}-1} \sqrt{\frac{1}{\sqrt{x}}+1} \sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[Sqrt[x]], x]

[Out] -((1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])) + x*ArcSech[Sqrt[x]]

Rule 6343

Int[ArcSech[u_], x_Symbol] := Simp[x*ArcSech[u], x] + Dist[Sqrt[1 - u^2]/(u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(x*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^{-1}(\sqrt{x}) dx &= x \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{1}{2\sqrt{1-x}} dx}{\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} \\
&= x \operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-x}} dx}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} \\
&= -\frac{1-x}{\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} + x \operatorname{sech}^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.0958415, size = 67, normalized size = 1.56

$$x \operatorname{sech}^{-1}(\sqrt{x}) - \frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} \sqrt{\sqrt{x}+1} \sqrt{1-x}}{\sqrt{1-\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[Sqrt[x]], x]

[Out] -((Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[1 + Sqrt[x]]*Sqrt[1 - x])/Sqrt[1 - Sqrt[x]]) + x*ArcSech[Sqrt[x]]

Maple [A] time = 0.123, size = 36, normalized size = 0.8

$$\operatorname{arcsech}(\sqrt{x}) - \sqrt{(1 + \sqrt{x}) \frac{1}{\sqrt{x}} \sqrt{-(-1 + \sqrt{x}) \frac{1}{\sqrt{x}} \sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(x^(1/2)), x)

[Out] x*arcsech(x^(1/2))-((1+x^(1/2))/x^(1/2))^(1/2)*((-(-1+x^(1/2))/x^(1/2))^(1/2))*x^(1/2)

Maxima [A] time = 0.971505, size = 26, normalized size = 0.6

$$x \operatorname{arsech}(\sqrt{x}) - \sqrt{x} \sqrt{\frac{1}{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2)),x, algorithm="maxima")

[Out] x*arcsech(sqrt(x)) - sqrt(x)*sqrt(1/x - 1)

Fricas [A] time = 1.80622, size = 92, normalized size = 2.14

$$x \log \left(\frac{x \sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x} \right) - \sqrt{x} \sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2)),x, algorithm="fricas")

[Out] x*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - sqrt(x)*sqrt(-(x - 1)/x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(x**(1/2)),x)

[Out] Integral(asech(sqrt(x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x)), x)
```

$$3.24 \quad \int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} dx$$

Optimal. Leaf size=46

$$-\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right) + \operatorname{sech}^{-1}(\sqrt{x})^2 - 2\operatorname{sech}^{-1}(\sqrt{x}) \log\left(e^{2\operatorname{sech}^{-1}(\sqrt{x})} + 1\right)$$

[Out] ArcSech[Sqrt[x]]^2 - 2*ArcSech[Sqrt[x]]*Log[1 + E^(2*ArcSech[Sqrt[x]])] - PolyLog[2, -E^(2*ArcSech[Sqrt[x]])]

Rubi [A] time = 0.0952382, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right) + \operatorname{sech}^{-1}(\sqrt{x})^2 - 2\operatorname{sech}^{-1}(\sqrt{x}) \log\left(e^{2\operatorname{sech}^{-1}(\sqrt{x})} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSech[Sqrt[x]]/x, x]

[Out] ArcSech[Sqrt[x]]^2 - 2*ArcSech[Sqrt[x]]*Log[1 + E^(2*ArcSech[Sqrt[x]])] - PolyLog[2, -E^(2*ArcSech[Sqrt[x]])]

Rule 6281

Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := -Subst[Int[(a + b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]

Rule 5660

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m * E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} dx &= 2 \operatorname{Subst} \left(\int \frac{\operatorname{sech}^{-1}(x)}{x} dx, x, \sqrt{x} \right) \\
&= - \left(2 \operatorname{Subst} \left(\int \frac{\cosh^{-1}(x)}{x} dx, x, \frac{1}{\sqrt{x}} \right) \right) \\
&= - \left(2 \operatorname{Subst} \left(\int x \tanh(x) dx, x, \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right) \right) \right) \\
&= \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)^2 - 4 \operatorname{Subst} \left(\int \frac{e^{2x} x}{1 + e^{2x}} dx, x, \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right) \right) \\
&= \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)^2 - 2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right) \log \left(1 + e^{2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)} \right) + 2 \operatorname{Subst} \left(\int \log(1 + e^{2x}) dx, x, \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right) \right) \\
&= \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)^2 - 2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right) \log \left(1 + e^{2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)} \right) + \operatorname{Subst} \left(\int \frac{\log(1 + x)}{x} dx, x, e^{2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)} \right) \\
&= \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)^2 - 2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right) \log \left(1 + e^{2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)} \right) - \operatorname{Li}_2 \left(-e^{2 \cosh^{-1} \left(\frac{1}{\sqrt{x}} \right)} \right)
\end{aligned}$$

Mathematica [A] time = 0.0359203, size = 45, normalized size = 0.98

$$\operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(\sqrt{x})} \right) - \operatorname{sech}^{-1}(\sqrt{x}) \left(\operatorname{sech}^{-1}(\sqrt{x}) + 2 \log \left(e^{-2 \operatorname{sech}^{-1}(\sqrt{x})} + 1 \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[Sqrt[x]]/x,x]

[Out] -(ArcSech[Sqrt[x]]*(ArcSech[Sqrt[x]] + 2*Log[1 + E^(-2*ArcSech[Sqrt[x]])])) + PolyLog[2, -E^(-2*ArcSech[Sqrt[x]])]

Maple [A] time = 0.16, size = 65, normalized size = 1.4

$$\left(\operatorname{arcsech}(\sqrt{x})\right)^2 - 2 \operatorname{arcsech}(\sqrt{x}) \ln \left(1 + \left(\frac{1}{\sqrt{x}} + \sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \right)^2 \right) - \operatorname{polylog} \left(2, - \left(\frac{1}{\sqrt{x}} + \sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(x^(1/2))/x,x)

[Out] arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2))*(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2))*(1+1/x^(1/2))^(1/2))^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4} \log(x)^2 + \log(x) \log\left(\sqrt{\sqrt{x}+1}\sqrt{-\sqrt{x}+1}+1\right) - \log(\sqrt{x}+1) \log(\sqrt{x}) - \log(\sqrt{x}) \log(-\sqrt{x}+1) - \operatorname{Li}_2(-\sqrt{x}) - \operatorname{Li}_2(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2))/x,x, algorithm="maxima")

[Out] -1/4*log(x)^2 + log(x)*log(sqrt(sqrt(x) + 1)*sqrt(-sqrt(x) + 1) + 1) - log(sqrt(x) + 1)*log(sqrt(x)) - log(sqrt(x))*log(-sqrt(x) + 1) - dilog(-sqrt(x)) - dilog(sqrt(x)) + integrate(1/2*log(x)/((x - 1)*e^(1/2*log(sqrt(x) + 1) + 1/2*log(-sqrt(x) + 1)) + x - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsech}(\sqrt{x})}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2))/x,x, algorithm="fricas")

[Out] integral(arcsech(sqrt(x))/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asech}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(x**(1/2))/x,x)

[Out] Integral(asech(sqrt(x))/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsech}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2))/x,x, algorithm="giac")

[Out] integrate(arcsech(sqrt(x))/x, x)

$$3.25 \quad \int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2} dx$$

Optimal. Leaf size=98

$$\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}x^{3/2}} + \frac{\sqrt{1-x}\tanh^{-1}(\sqrt{1-x})}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

[Out] (1 - x)/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/x + (Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])

Rubi [A] time = 0.0233795, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6345, 12, 51, 63, 206}

$$\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}x^{3/2}} + \frac{\sqrt{1-x}\tanh^{-1}(\sqrt{1-x})}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[Sqrt[x]]/x^2,x]

[Out] (1 - x)/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/x + (Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])

Rule 6345

Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] :> Simp[(((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 - u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*D[u, x]]/(u*Sqrt[1 - u^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u, x]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2} dx &= -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} - \frac{\sqrt{1-x} \int \frac{1}{2\sqrt{1-xx^2}} dx}{\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} \\
&= -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} - \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-xx^2}} dx}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} \\
&= \frac{1-x}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} - \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-xx}} dx}{4\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} \\
&= \frac{1-x}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} + \frac{\sqrt{1-x} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x}\right)}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}} \\
&= \frac{1-x}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} + \frac{\sqrt{1-x} \tanh^{-1}(\sqrt{1-x})}{2\sqrt{-1 + \frac{1}{\sqrt{x}}} \sqrt{1 + \frac{1}{\sqrt{x}}} \sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0771281, size = 111, normalized size = 1.13

$$\frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x+1}}}(\sqrt{x}+1) + x \log\left(\sqrt{x}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x+1}}} + \sqrt{\frac{1-\sqrt{x}}{\sqrt{x+1}}} + 1\right) - \frac{1}{2}x \log(x) - 2\operatorname{sech}^{-1}(\sqrt{x})}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[Sqrt[x]]/x^2, x]

[Out] (Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(1 + Sqrt[x]) - 2*ArcSech[Sqrt[x]] + x*Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])])*Sqrt[x]] - (x*Log[x])/2)/(2*x)

Maple [A] time = 0.134, size = 64, normalized size = 0.7

$$-\frac{1}{x} \operatorname{arcsech}(\sqrt{x}) + \frac{1}{2} \sqrt{-(-1 + \sqrt{x})} \frac{1}{\sqrt{x}} \sqrt{(1 + \sqrt{x})} \frac{1}{\sqrt{x}} \left(\operatorname{Artanh}\left(\frac{1}{\sqrt{1-x}}\right) x + \sqrt{1-x} \right) \frac{1}{\sqrt{x}} \frac{1}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsech(x^(1/2))/x^2,x)`

[Out] $-\text{arcsech}(x^{1/2})/x + 1/2 * (-(-1+x^{1/2})/x^{1/2})^{1/2} / x^{1/2} * ((1+x^{1/2})/x^{1/2})^{1/2} * (\text{arctanh}(1/(1-x)^{1/2}) * x + (1-x)^{1/2}) / (1-x)^{1/2}$

Maxima [A] time = 0.980277, size = 88, normalized size = 0.9

$$\frac{\sqrt{x}\sqrt{\frac{1}{x}-1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)} - \frac{\text{arsech}(\sqrt{x})}{x} + \frac{1}{4} \log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right) - \frac{1}{4} \log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(x^(1/2))/x^2,x, algorithm="maxima")`

[Out] $-1/2 * \text{sqrt}(x) * \text{sqrt}(1/x - 1) / (x * (1/x - 1) - 1) - \text{arcsech}(\text{sqrt}(x)) / x + 1/4 * \log(\text{sqrt}(x) * \text{sqrt}(1/x - 1) + 1) - 1/4 * \log(\text{sqrt}(x) * \text{sqrt}(1/x - 1) - 1)$

Fricas [A] time = 1.87779, size = 111, normalized size = 1.13

$$\frac{(x-2) \log\left(\frac{x\sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right) + \sqrt{x}\sqrt{-\frac{x-1}{x}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(x^(1/2))/x^2,x, algorithm="fricas")`

[Out] $1/2 * ((x - 2) * \log((x * \text{sqrt}(-(x - 1)/x) + \text{sqrt}(x))/x) + \text{sqrt}(x) * \text{sqrt}(-(x - 1)/x)) / x$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asech}(\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**2,x)
```

```
[Out] Integral(asech(sqrt(x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{ar} \operatorname{sech}(\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^2, x)
```

$$3.26 \quad \int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3} dx$$

Optimal. Leaf size=136

$$\frac{3(1-x)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}x^{3/2}} + \frac{1-x}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}x^{5/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} + \frac{3\sqrt{1-x}\tanh^{-1}(\sqrt{1-x})}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

[Out] (1 - x)/(8*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (3*(1 - x))/(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(2*x^2) + (3*Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])

Rubi [A] time = 0.0332407, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6345, 12, 51, 63, 206}

$$\frac{3(1-x)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}x^{3/2}} + \frac{1-x}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}x^{5/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} + \frac{3\sqrt{1-x}\tanh^{-1}(\sqrt{1-x})}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[Sqrt[x]]/x^3,x]

[Out] (1 - x)/(8*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (3*(1 - x))/(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(2*x^2) + (3*Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3} dx &= \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{\sqrt{1-x} \int \frac{1}{2\sqrt{1-xx^3}} dx}{2\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-xx^3}} dx}{4\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{8\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{5/2}}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{(3\sqrt{1-x}) \int \frac{1}{\sqrt{1-xx^2}} dx}{16\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{8\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{5/2}}} + \frac{3(1-x)}{16\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{3/2}}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{(3\sqrt{1-x}) \int \frac{1}{\sqrt{1-xx}} dx}{32\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{8\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{5/2}}} + \frac{3(1-x)}{16\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{3/2}}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} + \frac{(3\sqrt{1-x}) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx\right)}{16\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{8\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{5/2}}} + \frac{3(1-x)}{16\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}x^{3/2}}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} + \frac{3\sqrt{1-x} \tanh^{-1}(\sqrt{1-x})}{16\sqrt{-1 + \frac{1}{\sqrt{x}}}\sqrt{1 + \frac{1}{\sqrt{x}}}\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.112036, size = 125, normalized size = 0.92

$$\frac{1}{16} \left(\frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} (3x^{3/2} + 3x + 2\sqrt{x} + 2)}{x^2} - \frac{8\operatorname{sech}^{-1}(\sqrt{x})}{x^2} + 3 \log \left(\sqrt{x} \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} + \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} + 1 \right) - \frac{3 \log(x)}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[Sqrt[x]]/x^3,x]

[Out] ((Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + 3*x + 3*x^(3/2)))/x^2 - (8*ArcSech[Sqrt[x]])/x^2 + 3*Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (3*Log[x])/2)/16

Maple [A] time = 0.133, size = 79, normalized size = 0.6

$$-\frac{1}{2x^2} \operatorname{arcsech}(\sqrt{x}) + \frac{1}{16} \sqrt{-(-1 + \sqrt{x})} \frac{1}{\sqrt{x}} \sqrt{(1 + \sqrt{x})} \frac{1}{\sqrt{x}} \left(3 \operatorname{Arctanh}\left(\frac{1}{\sqrt{1-x}}\right) x^2 + 3 \sqrt{1-x} x + 2 \sqrt{1-x} \right) x^{-\frac{3}{2}} \frac{1}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(x^(1/2))/x^3,x)

[Out] $-\frac{1}{2} \operatorname{arcsech}(x^{1/2}) / x^2 + \frac{1}{16} * (-(-1 + x^{1/2}) / x^{1/2})^{1/2} / x^{3/2} * ((1 + x^{1/2}) / x^{1/2})^{1/2} * (3 * \operatorname{arctanh}(1 / (1-x)^{1/2})) * x^2 + 3 * (1-x)^{1/2} * x + 2 * (1-x)^{1/2} / (1-x)^{1/2}$

Maxima [A] time = 0.993, size = 124, normalized size = 0.91

$$\frac{3x^{\frac{3}{2}} \left(\frac{1}{x} - 1\right)^{\frac{3}{2}} - 5\sqrt{x} \sqrt{\frac{1}{x} - 1}}{16 \left(x^2 \left(\frac{1}{x} - 1\right)^2 - 2x \left(\frac{1}{x} - 1\right) + 1\right)} - \frac{\operatorname{arosech}(\sqrt{x})}{2x^2} + \frac{3}{32} \log\left(\sqrt{x} \sqrt{\frac{1}{x} - 1} + 1\right) - \frac{3}{32} \log\left(\sqrt{x} \sqrt{\frac{1}{x} - 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="maxima")

[Out] $-\frac{1}{16} * (3 * x^{3/2} * (1/x - 1)^{3/2} - 5 * \sqrt{x} * \sqrt{1/x - 1}) / (x^2 * (1/x - 1)^2 - 2 * x * (1/x - 1) + 1) - \frac{1}{2} * \operatorname{arcsech}(\sqrt{x}) / x^2 + \frac{3}{32} * \log(\sqrt{x} * \sqrt{1/x - 1} + 1) - \frac{3}{32} * \log(\sqrt{x} * \sqrt{1/x - 1} - 1)$

Fricas [A] time = 1.88476, size = 134, normalized size = 0.99

$$\frac{(3x + 2)\sqrt{x} \sqrt{-\frac{x-1}{x}} + (3x^2 - 8) \log\left(\frac{x \sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right)}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="fricas")

[Out] $\frac{1}{16} \left((3x + 2) \sqrt{x} \sqrt{-(x - 1)/x} + (3x^2 - 8) \log\left(\frac{x \sqrt{-(x - 1)}}{x} + \sqrt{x}\right) \right) / x^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}(\sqrt{x})}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asech(x**(1/2))/x**3,x)`

[Out] `Integral(asech(sqrt(x))/x**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(\sqrt{x})}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(x^(1/2))/x^3,x, algorithm="giac")`

[Out] `integrate(arcsech(sqrt(x))/x^3, x)`

$$3.27 \quad \int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4} dx$$

Optimal. Leaf size=172

$$\frac{5(1-x)}{48\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{3/2}} + \frac{5(1-x)}{72\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{5/2}} + \frac{1-x}{18\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{7/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} + \frac{5\sqrt{1-x}\tanh^{-1}(\sqrt{1-x})}{48\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{3/2}}$$

[Out] (1 - x)/(18*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*x^(7/2)) + (5*(1 - x))/(72*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (5*(1 - x))/(48*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(3*x^3) + (5*sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(48*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*sqrt[x])

Rubi [A] time = 0.039765, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6345, 12, 51, 63, 206}

$$\frac{5(1-x)}{48\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{3/2}} + \frac{5(1-x)}{72\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{5/2}} + \frac{1-x}{18\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{7/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} + \frac{5\sqrt{1-x}\tanh^{-1}(\sqrt{1-x})}{48\sqrt{\frac{1}{\sqrt{x}}}-1\sqrt{\frac{1}{\sqrt{x}}}+1x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[Sqrt[x]]/x^4, x]

[Out] (1 - x)/(18*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*x^(7/2)) + (5*(1 - x))/(72*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (5*(1 - x))/(48*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(3*x^3) + (5*sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(48*sqrt[-1 + 1/Sqrt[x]]*sqrt[1 + 1/Sqrt[x]]*sqrt[x])

Rule 6345

Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*sqrt[1 - u^2])/(d*(m + 1)*u*sqrt[-1 + 1/u]*sqrt[1 + 1/u]), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*D[u, x]]/(u*sqrt[1 - u^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u, x]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4} dx &= -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{\sqrt{1-x} \int \frac{1}{2\sqrt{1-xx^4}} dx}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-xx^4}} dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{7/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{(5\sqrt{1-x}) \int \frac{1}{\sqrt{1-xx^3}} dx}{36\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{5/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{(5\sqrt{1-x}) \int \frac{1}{\sqrt{1-xx^2}} dx}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} \\
&= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} \\
&= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.128421, size = 140, normalized size = 0.81

$$\frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} (15x^{5/2} + 15x^2 + 10x^{3/2} + 10x + 8\sqrt{x} + 8) + 15x^3 \log\left(\sqrt{x}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} + \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} + 1\right) - \frac{15}{2}x^3 \log(x) - 48\operatorname{sech}^{-1}(\sqrt{x})}{144x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[Sqrt[x]]/x^4, x]

[Out] (Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])])*(8 + 8*Sqrt[x] + 10*x + 10*x^(3/2) + 15*x^2 + 15*x^(5/2)) - 48*ArcSech[Sqrt[x]] + 15*x^3*Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x] - (15*x^3*Log[

$x])/2)/(144*x^3)$

Maple [A] time = 0.141, size = 91, normalized size = 0.5

$$-\frac{1}{3x^3}\operatorname{arcsech}(\sqrt{x}) + \frac{1}{144}\sqrt{-(-1+\sqrt{x})}\frac{1}{\sqrt{x}}\sqrt{(1+\sqrt{x})}\frac{1}{\sqrt{x}}\left(15\operatorname{Artanh}\left(\frac{1}{\sqrt{1-x}}\right)x^3 + 15\sqrt{1-xx^2} + 10\sqrt{1-xx} + 8\sqrt{1-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsech(x^(1/2))/x^4,x)`

[Out] $-1/3*\operatorname{arcsech}(x^{(1/2)})/x^3+1/144*(-(-1+x^{(1/2)})/x^{(1/2)})^{(1/2)}/x^{(5/2)}*((1+x^{(1/2)})/x^{(1/2)})^{(1/2)}*(15*\operatorname{arctanh}(1/(1-x)^{(1/2)})*x^3+15*(1-x)^{(1/2)}*x^2+10*(1-x)^{(1/2)}*x+8*(1-x)^{(1/2)})/(1-x)^{(1/2)}$

Maxima [A] time = 0.989615, size = 157, normalized size = 0.91

$$\frac{15x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}-40x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}+33\sqrt{x}\sqrt{\frac{1}{x}-1}}{144\left(x^3\left(\frac{1}{x}-1\right)^3-3x^2\left(\frac{1}{x}-1\right)^2+3x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arosech}(\sqrt{x})}{3x^3}+\frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right)-\frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(x^(1/2))/x^4,x, algorithm="maxima")`

[Out] $-1/144*(15*x^{(5/2)}*(1/x-1)^{(5/2)}-40*x^{(3/2)}*(1/x-1)^{(3/2)}+33*\sqrt{x}*\sqrt{1/x-1})/(x^3*(1/x-1)^3-3*x^2*(1/x-1)^2+3*x*(1/x-1)-1)-1/3*\operatorname{arcsech}(\sqrt{x})/x^3+5/96*\log(\sqrt{x}*\sqrt{1/x-1}+1)-5/96*\log(\sqrt{x}*\sqrt{1/x-1}-1)$

Fricas [A] time = 1.92981, size = 153, normalized size = 0.89

$$\frac{(15x^2+10x+8)\sqrt{x}\sqrt{-\frac{x-1}{x}}+3(5x^3-16)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{144x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/144*((15*x^2 + 10*x + 8)*sqrt(x)*sqrt(-(x - 1)/x) + 3*(5*x^3 - 16)*log((x
*sqrt(-(x - 1)/x) + sqrt(x))/x))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{ar} \operatorname{sech}(\sqrt{x})}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^4, x)
```

3.28 $\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=21

$$x \cosh^{-1}(x) - \sqrt{x-1}\sqrt{x+1}$$

[Out] -(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]

Rubi [A] time = 0.0069606, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {6327, 5654, 74}

$$x \cosh^{-1}(x) - \sqrt{x-1}\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[x^(-1)],x]

[Out] -(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]

Rule 6327

Int[ArcSech[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[u*ArcCosh[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]

Rule 5654

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\begin{aligned}
 \int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx &= \int \cosh^{-1}(x) dx \\
 &= x \cosh^{-1}(x) - \int \frac{x}{\sqrt{-1+x}\sqrt{1+x}} dx \\
 &= -\sqrt{-1+x}\sqrt{1+x} + x \cosh^{-1}(x)
 \end{aligned}$$

Mathematica [A] time = 0.0235574, size = 25, normalized size = 1.19

$$x \operatorname{sech}^{-1}\left(\frac{1}{x}\right) - \frac{x-1}{\sqrt{\frac{x-1}{x+1}}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[x^(-1)],x]

[Out] -((-1 + x)/Sqrt[(-1 + x)/(1 + x)]) + x*ArcSech[x^(-1)]

Maple [A] time = 0.101, size = 29, normalized size = 1.4

$$x \operatorname{arcsech}(x^{-1}) - \sqrt{-(x^{-1}-1)x} \sqrt{(x^{-1}+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(1/x),x)

[Out] x*arcsech(1/x)-(-(1/x-1)*x)^(1/2)*((1/x+1)*x)^(1/2)

Maxima [A] time = 1.02406, size = 22, normalized size = 1.05

$$x \operatorname{arsech}\left(\frac{1}{x}\right) - \sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(1/x),x, algorithm="maxima")

[Out] $x \cdot \operatorname{arcsech}(1/x) - \sqrt{x^2 - 1}$

Fricas [A] time = 1.81089, size = 57, normalized size = 2.71

$$x \log\left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(1/x),x, algorithm="fricas")`

[Out] $x \cdot \log(x + \sqrt{x^2 - 1}) - \sqrt{x^2 - 1}$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asech}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asech(1/x),x)`

[Out] `Integral(asech(1/x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(1/x),x, algorithm="giac")`

[Out] `integrate(arcsech(1/x), x)`

$$3.29 \quad \int \frac{\operatorname{sech}^{-1}(ax^n)}{x} dx$$

Optimal. Leaf size=61

$$-\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n} + \frac{\operatorname{sech}^{-1}(ax^n)^2}{2n} - \frac{\operatorname{sech}^{-1}(ax^n) \log\left(e^{2\operatorname{sech}^{-1}(ax^n)} + 1\right)}{n}$$

[Out] ArcSech[a*x^n]^2/(2*n) - (ArcSech[a*x^n]*Log[1 + E^(2*ArcSech[a*x^n])])/n - PolyLog[2, -E^(2*ArcSech[a*x^n])]/(2*n)

Rubi [A] time = 0.105367, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n} + \frac{\operatorname{sech}^{-1}(ax^n)^2}{2n} - \frac{\operatorname{sech}^{-1}(ax^n) \log\left(e^{2\operatorname{sech}^{-1}(ax^n)} + 1\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a*x^n]/x, x]

[Out] ArcSech[a*x^n]^2/(2*n) - (ArcSech[a*x^n]*Log[1 + E^(2*ArcSech[a*x^n])])/n - PolyLog[2, -E^(2*ArcSech[a*x^n])]/(2*n)

Rule 6281

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] :> -Subst[Int[(a + b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]

Rule 5660

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] :> Subst[Int[(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m * E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;

FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax^n)}{x} dx &= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}^{-1}(ax)}{x} dx, x, x^n\right)}{n} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)}{x} dx, x, x^{-n}\right)}{n} \\
&= -\frac{\operatorname{Subst}\left(\int x \tanh(x) dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{2 \operatorname{Subst}\left(\int \frac{e^{2x}}{1+e^{2x}} dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \log(1 + e^{2x}) dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2 \cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n} \\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} - \frac{\operatorname{Li}_2\left(-e^{2 \cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}
\end{aligned}$$

Mathematica [B] time = 0.956552, size = 219, normalized size = 3.59

$$\frac{\sqrt{\frac{1-ax^n}{ax^n+1}} \left(\sqrt{1-a^2x^{2n}} \left(-4 \operatorname{PolyLog}\left(2, \frac{1}{2} - \frac{1}{2} \sqrt{1-a^2x^{2n}}\right) + \log^2(a^2x^{2n}) + 2 \log^2\left(\frac{1}{2} \left(\sqrt{1-a^2x^{2n}} + 1 \right)\right) - 4 \log\left(\frac{1}{2} \left(\sqrt{1-a^2x^{2n}} + 1 \right)\right) \right) \right)}{8(n-ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a*x^n]/x,x]

[Out] ArcSech[a*x^n]*Log[x] + (Sqrt[(1 - a*x^n)/(1 + a*x^n)]*(4*Sqrt[-1 + a^2*x^(2*n)]]*ArcTan[Sqrt[-1 + a^2*x^(2*n)]]*(2*n*Log[x] - Log[a^2*x^(2*n)]) + Sqrt[1 - a^2*x^(2*n)]*(Log[a^2*x^(2*n)]^2 - 4*Log[a^2*x^(2*n)]*Log[(1 + Sqrt[1 - a^2*x^(2*n)])]/2] + 2*Log[(1 + Sqrt[1 - a^2*x^(2*n)])]/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - a^2*x^(2*n)]]/2)))/(8*(n - a*n*x^n))

Maple [A] time = 0.333, size = 116, normalized size = 1.9

$$\frac{(\operatorname{arcsech}(ax^n))^2}{2n} - \frac{\operatorname{arcsech}(ax^n)}{n} \ln \left(1 + \left(\frac{1}{ax^n} + \sqrt{\frac{1}{ax^n} - 1} \sqrt{\frac{1}{ax^n} + 1} \right)^2 \right) - \frac{1}{2n} \operatorname{polylog} \left(2, - \left(\frac{1}{ax^n} + \sqrt{\frac{1}{ax^n} - 1} \sqrt{\frac{1}{ax^n} + 1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x^n)/x,x)

[Out] 1/2*arcsech(a*x^n)^2/n-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/2))*(1/a/(x^n)+1)^(1/2))^2)/n-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(1/a/(x^n)+1)^(1/2))^2)/n

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 n \int \frac{x^{2n} \log(x)}{a^2 x x^{2n} + (a^2 x x^{2n} - x) \sqrt{ax^n + 1} \sqrt{-ax^n + 1} - x} dx + n \int \frac{\log(x)}{2(axx^n + x)} dx - n \int \frac{\log(x)}{2(axx^n - x)} dx + \log(\sqrt{ax^n + 1} \sqrt{-ax^n + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x^n)/x,x, algorithm="maxima")

[Out] a^2*n*integrate(x^(2*n)*log(x)/(a^2*x*x^(2*n) + (a^2*x*x^(2*n) - x)*sqrt(a*x^n + 1)*sqrt(-a*x^n + 1) - x), x) + n*integrate(1/2*log(x)/(a*x*x^n + x), x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1)*log(x) - log(a)*log(x) - log(x)*log(x^n))

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x^n)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x**n)/x,x)

[Out] Integral(asech(a*x**n)/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x^n)/x,x, algorithm="giac")

[Out] integrate(arcsech(a*x^n)/x, x)

$$3.30 \quad \int \frac{\operatorname{sech}^{-1}(ax^5)}{x} dx$$

Optimal. Leaf size=54

$$-\frac{1}{10}\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax^5)}\right) + \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{sech}^{-1}(ax^5)\log\left(e^{2\operatorname{sech}^{-1}(ax^5)} + 1\right)$$

[Out] ArcSech[a*x^5]^2/10 - (ArcSech[a*x^5]*Log[1 + E^(2*ArcSech[a*x^5])])/5 - PolyLog[2, -E^(2*ArcSech[a*x^5])]/10

Rubi [A] time = 0.107238, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{1}{10}\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax^5)}\right) + \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{sech}^{-1}(ax^5)\log\left(e^{2\operatorname{sech}^{-1}(ax^5)} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a*x^5]/x,x]

[Out] ArcSech[a*x^5]^2/10 - (ArcSech[a*x^5]*Log[1 + E^(2*ArcSech[a*x^5])])/5 - PolyLog[2, -E^(2*ArcSech[a*x^5])]/10

Rule 6281

Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]

Rule 5660

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*(c_) + (d_)*(x_)))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax^5)}{x} dx &= \frac{1}{5} \operatorname{Subst} \left(\int \frac{\operatorname{sech}^{-1}(ax)}{x} dx, x, x^5 \right) \\
&= - \left(\frac{1}{5} \operatorname{Subst} \left(\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)}{x} dx, x, \frac{1}{x^5} \right) \right) \\
&= - \left(\frac{1}{5} \operatorname{Subst} \left(\int x \tanh(x) dx, x, \operatorname{sech}^{-1}(ax^5) \right) \right) \\
&= \frac{1}{10} \operatorname{sech}^{-1}(ax^5)^2 - \frac{2}{5} \operatorname{Subst} \left(\int \frac{e^{2x}}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(ax^5) \right) \\
&= \frac{1}{10} \operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5} \operatorname{sech}^{-1}(ax^5) \log \left(1 + e^{2\operatorname{sech}^{-1}(ax^5)} \right) + \frac{1}{5} \operatorname{Subst} \left(\int \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(ax^5) \right) \\
&= \frac{1}{10} \operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5} \operatorname{sech}^{-1}(ax^5) \log \left(1 + e^{2\operatorname{sech}^{-1}(ax^5)} \right) + \frac{1}{10} \operatorname{Subst} \left(\int \frac{\log(1 + x)}{x} dx, x, e^{2\operatorname{sech}^{-1}(ax^5)} \right) \\
&= \frac{1}{10} \operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5} \operatorname{sech}^{-1}(ax^5) \log \left(1 + e^{2\operatorname{sech}^{-1}(ax^5)} \right) - \frac{1}{10} \operatorname{Li}_2 \left(-e^{2\operatorname{sech}^{-1}(ax^5)} \right)
\end{aligned}$$

Mathematica [A] time = 0.0460431, size = 49, normalized size = 0.91

$$\frac{1}{10} \left(\operatorname{PolyLog} \left(2, -e^{-2\operatorname{sech}^{-1}(ax^5)} \right) - \operatorname{sech}^{-1}(ax^5) \left(\operatorname{sech}^{-1}(ax^5) + 2 \log \left(e^{-2\operatorname{sech}^{-1}(ax^5)} + 1 \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a*x^5]/x,x]

[Out] $(-\text{ArcSech}[a*x^5]*(\text{ArcSech}[a*x^5] + 2*\text{Log}[1 + E^{(-2*\text{ArcSech}[a*x^5])}])) + \text{PolyLog}[2, -E^{(-2*\text{ArcSech}[a*x^5])}])/10$

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int \frac{\text{arcsech}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x^5)/x,x)

[Out] int(arcsech(a*x^5)/x,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsech}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x^5)/x,x, algorithm="maxima")

[Out] integrate(arcsech(a*x^5)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsech}(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(a*x^5)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x**5)/x,x)
```

```
[Out] Integral(asech(a*x**5)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x^5)/x, x)
```

3.31 $\int \operatorname{sech}^{-1}(ce^{a+bx}) dx$

Optimal. Leaf size=77

$$-\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ce^{a+bx})}\right)}{2b} + \frac{\operatorname{sech}^{-1}(ce^{a+bx})^2}{2b} - \frac{\operatorname{sech}^{-1}(ce^{a+bx}) \log\left(e^{2\operatorname{sech}^{-1}(ce^{a+bx})} + 1\right)}{b}$$

[Out] ArcSech[c*E^(a + b*x)]^2/(2*b) - (ArcSech[c*E^(a + b*x)]*Log[1 + E^(2*ArcSech[c*E^(a + b*x)])])/b - PolyLog[2, -E^(2*ArcSech[c*E^(a + b*x)])]/(2*b)

Rubi [A] time = 0.101172, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {2282, 6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ce^{a+bx})}\right)}{2b} + \frac{\operatorname{sech}^{-1}(ce^{a+bx})^2}{2b} - \frac{\operatorname{sech}^{-1}(ce^{a+bx}) \log\left(e^{2\operatorname{sech}^{-1}(ce^{a+bx})} + 1\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[c*E^(a + b*x)], x]

[Out] ArcSech[c*E^(a + b*x)]^2/(2*b) - (ArcSech[c*E^(a + b*x)]*Log[1 + E^(2*ArcSech[c*E^(a + b*x)])])/b - PolyLog[2, -E^(2*ArcSech[c*E^(a + b*x)])]/(2*b)

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := -Subst[Int[(a + b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

0]

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^{-1}(ce^{a+bx}) dx &= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{c}\right)}{x} dx, x, e^{-a-bx}\right)}{b} \\
&= -\frac{\operatorname{Subst}\left(\int x \tanh(x) dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{2 \operatorname{Subst}\left(\int \frac{e^{2x}}{1+e^{2x}} dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\operatorname{Subst}\left(\int \log(1 + e^{2x}) dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2 \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b} \\
&= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{\operatorname{Li}_2\left(-e^{2 \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b}
\end{aligned}$$

Mathematica [B] time = 1.17138, size = 249, normalized size = 3.23

$$x \operatorname{sech}^{-1}(ce^{a+bx}) - \frac{\sqrt{\frac{1-ce^{a+bx}}{ce^{a+bx}+1}} \sqrt{ce^{a+bx}+1} \left(4 \operatorname{PolyLog}\left(2, \frac{1}{2} \left(1 - \sqrt{1 - c^2 e^{2(a+bx)}}\right)\right) - \log^2(c^2 e^{2(a+bx)}) - 2 \log^2\left(\frac{1}{2} \left(\sqrt{1 - c^2 e^{2(a+bx)}}\right)\right)\right)}{8}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[c*E^(a + b*x)], x]

[Out] x*ArcSech[c*E^(a + b*x)] - (Sqrt[(1 - c*E^(a + b*x))/(1 + c*E^(a + b*x))]*Sqrt[1 + c*E^(a + b*x)]*(ArcTanh[Sqrt[1 - c^2*E^(2*(a + b*x))]]*(8*b*x - 4*Log[c^2*E^(2*(a + b*x))]) - Log[c^2*E^(2*(a + b*x))]^2 + 4*Log[c^2*E^(2*(a + b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] - 2*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2]^2 + 4*PolyLog[2, (1 - Sqrt[1 - c^2*E^(2*(a + b*x))])/2]))/(8*b*Sqrt[1 - c*E^(a + b*x)])

Maple [A] time = 0.361, size = 140, normalized size = 1.8

$$\frac{(\operatorname{arcsech}(ce^{bx+a}))^2}{2b} - \frac{\operatorname{arcsech}(ce^{bx+a})}{b} \ln \left(1 + \left(\frac{1}{ce^{bx+a}} + \sqrt{\frac{1}{ce^{bx+a}} - 1} \sqrt{\frac{1}{ce^{bx+a}} + 1} \right)^2 \right) - \frac{1}{2b} \operatorname{polylog} \left(2, - \left(\frac{1}{ce^{bx+a}} + \sqrt{\frac{1}{ce^{bx+a}} - 1} \sqrt{\frac{1}{ce^{bx+a}} + 1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(c*exp(b*x+a)),x)

[Out] 1/2*arcsech(c*exp(b*x+a))^2/b-arcsech(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b-1/2*polylog(2,-(1/c/exp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$bc^2 \int \frac{xe^{(2bx+2a)}}{c^2e^{(2bx+2a)} + (c^2e^{(2bx+2a)} - 1)e^{\left(\frac{1}{2} \log(ce^{(bx+a)}+1) + \frac{1}{2} \log(-ce^{(bx+a)}+1)\right)} - 1} dx - \frac{1}{2}bx^2 - (a + \log(c))x + x \log \left(\sqrt{ce^{(bx+a)}} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="maxima")

[Out] b*c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(-c*e^(b*x + a) + 1)) - 1), x) - 1/2*b*x^2 - (a + log(c))*x + x*log(sqrt(c*e^(b*x + a) + 1)*sqrt(-c*e^(b*x + a) + 1) + 1) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a) + 1))/b - 1/2*(b*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asech}(ce^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(c*exp(b*x+a)), x)

[Out] Integral(asech(c*exp(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}(ce^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(c*exp(b*x+a)), x, algorithm="giac")

[Out] integrate(arcsech(c*e^(b*x + a)), x)

3.32 $\int e^{\operatorname{sech}^{-1}(ax)} x^4 dx$

Optimal. Leaf size=64

$$\frac{x^2}{15a^3} - \frac{x^3 e^{\operatorname{sech}^{-1}(ax)}}{15a^2} - \frac{2x e^{\operatorname{sech}^{-1}(ax)}}{15a^4} + \frac{x^4}{20a} + \frac{1}{5} x^5 e^{\operatorname{sech}^{-1}(ax)}$$

[Out] $(-2 * E^{\operatorname{ArcSech}[a * x] * x}) / (15 * a^4) + x^2 / (15 * a^3) - (E^{\operatorname{ArcSech}[a * x] * x^3}) / (15 * a^2) + x^4 / (20 * a) + (E^{\operatorname{ArcSech}[a * x] * x^5}) / 5$

Rubi [A] time = 0.0384958, antiderivative size = 83, normalized size of antiderivative = 1.3, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 30, 100, 12, 74}

$$-\frac{x^2 \sqrt{1-ax}}{15a^3 \sqrt{\frac{1}{ax+1}}} - \frac{2\sqrt{1-ax}}{15a^5 \sqrt{\frac{1}{ax+1}}} + \frac{x^4}{20a} + \frac{1}{5} x^5 e^{\operatorname{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

[In] Int[E^ArcSech[a*x]*x^4, x]

[Out] $x^4 / (20 * a) + (E^{\operatorname{ArcSech}[a * x] * x^5}) / 5 - (2 * \operatorname{Sqrt}[1 - a * x]) / (15 * a^5 * \operatorname{Sqrt}[(1 + a * x)^{-1}]) - (x^2 * \operatorname{Sqrt}[1 - a * x]) / (15 * a^3 * \operatorname{Sqrt}[(1 + a * x)^{-1}])$

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 100

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)

$\int (a + b*x)^{(m-2)} * (c + d*x)^n * (e + f*x)^p * \text{Simp}[a^2 * d*f*(m+n+p+1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m+n+p) - b*(d*e*(m+n) + c*f*(m+p))]*x, x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{NeQ}[m+n+p+1, 0] \&\& \text{IntegerQ}[m]$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 74

$\text{Int}[(a_*) + (b_*)*(x_*)]*((c_*) + (d_*)*(x_*)^{(n_*)})*((e_*) + (f_*)*(x_*)^{(p_*)}), x_Symbol] \rightarrow \text{Simp}[(b*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/(d*f*(n+p+2)), x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{NeQ}[n+p+2, 0] \&\& \text{EqQ}[a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1)), 0]$

Rubi steps

$$\begin{aligned} \int e^{\text{sech}^{-1}(ax)} x^4 dx &= \frac{1}{5} e^{\text{sech}^{-1}(ax)} x^5 + \frac{\int x^3 dx}{5a} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x^3}{\sqrt{1-ax}\sqrt{1+ax}} dx}{5a} \\ &= \frac{x^4}{20a} + \frac{1}{5} e^{\text{sech}^{-1}(ax)} x^5 - \frac{x^2 \sqrt{1-ax}}{15a^3 \sqrt{\frac{1}{1+ax}}} - \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int -\frac{2x}{\sqrt{1-ax}\sqrt{1+ax}} dx}{15a^3} \\ &= \frac{x^4}{20a} + \frac{1}{5} e^{\text{sech}^{-1}(ax)} x^5 - \frac{x^2 \sqrt{1-ax}}{15a^3 \sqrt{\frac{1}{1+ax}}} + \frac{\left(2\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}} dx}{15a^3} \\ &= \frac{x^4}{20a} + \frac{1}{5} e^{\text{sech}^{-1}(ax)} x^5 - \frac{2\sqrt{1-ax}}{15a^5 \sqrt{\frac{1}{1+ax}}} - \frac{x^2 \sqrt{1-ax}}{15a^3 \sqrt{\frac{1}{1+ax}}} \end{aligned}$$

Mathematica [A] time = 0.0899093, size = 65, normalized size = 1.02

$$\frac{15a^4 x^4 + 4\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2(3a^3 x^3 - 3a^2 x^2 + 2ax - 2)}{60a^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]*x^4,x]

[Out] $(15a^4x^4 + 4\sqrt{(1-ax)/(1+ax)})(1+ax)^2(-2+2ax-3a^2x^2+3a^3x^3)/(60a^5)$

Maple [A] time = 0.189, size = 64, normalized size = 1.

$$\frac{x(a^2x^2-1)(3a^2x^2+2)}{15a^4}\sqrt{\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}+\frac{x^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x^4,x)

[Out] $1/15*(-(ax-1)/a/x)^(1/2)*x*((ax+1)/a/x)^(1/2)*(a^2x^2-1)*(3a^2x^2+2)/a^4+1/4*x^4/a$

Maxima [A] time = 1.07833, size = 63, normalized size = 0.98

$$\frac{x^4}{4a} + \frac{(3a^4x^4 - a^2x^2 - 2)\sqrt{ax+1}\sqrt{-ax+1}}{15a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x^4,x, algorithm="maxima")

[Out] $1/4*x^4/a + 1/15*(3*a^4*x^4 - a^2*x^2 - 2)*sqrt(ax + 1)*sqrt(-ax + 1)/a^5$

Fricas [A] time = 1.71408, size = 135, normalized size = 2.11

$$\frac{15a^3x^4 + 4(3a^4x^5 - a^2x^3 - 2x)\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{ax-1}{ax}}}{60a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(15*a^3*x^4 + 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqrt(-
(a*x - 1)/(a*x)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x^3 dx + \int ax^4 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**4,x)
```

```
[Out] (Integral(x**3, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)),
x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="giac")
```

```
[Out] integrate(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.33 $\int e^{\operatorname{sech}^{-1}(ax)} x^3 dx$

Optimal. Leaf size=84

$$-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)}{8a^4} + \frac{x^3}{12a} + \frac{1}{4}x^4e^{\operatorname{sech}^{-1}(ax)}$$

[Out] $x^3/(12*a) + (E^{\operatorname{ArcSech}[a*x]}*x^4)/4 - (x*\operatorname{Sqrt}[1 - a*x])/(8*a^3*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcSin}[a*x])/(8*a^4)$

Rubi [A] time = 0.0319382, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 30, 90, 41, 216}

$$-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)}{8a^4} + \frac{x^3}{12a} + \frac{1}{4}x^4e^{\operatorname{sech}^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x]*x^3,x]

[Out] $x^3/(12*a) + (E^{\operatorname{ArcSech}[a*x]}*x^4)/4 - (x*\operatorname{Sqrt}[1 - a*x])/(8*a^3*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcSin}[a*x])/(8*a^4)$

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 90

```
Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)(n + 1)*(e + f*x)(p + 1)]/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)n*(e + f*x)p*Simp[a2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))(m_.)*((c_) + (d_.)*(x_))(m_.), x_Symbol] := Int[(a*c + b*d*x2)m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax)} x^3 dx &= \frac{1}{4} e^{\operatorname{sech}^{-1}(ax)} x^4 + \frac{\int x^2 dx}{4a} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x^2}{\sqrt{1-ax}\sqrt{1+ax}} dx}{4a} \\ &= \frac{x^3}{12a} + \frac{1}{4} e^{\operatorname{sech}^{-1}(ax)} x^4 - \frac{x\sqrt{1-ax}}{8a^3 \sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}} dx}{8a^3} \\ &= \frac{x^3}{12a} + \frac{1}{4} e^{\operatorname{sech}^{-1}(ax)} x^4 - \frac{x\sqrt{1-ax}}{8a^3 \sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{1}{\sqrt{1-a^2x^2}} dx}{8a^3} \\ &= \frac{x^3}{12a} + \frac{1}{4} e^{\operatorname{sech}^{-1}(ax)} x^4 - \frac{x\sqrt{1-ax}}{8a^3 \sqrt{\frac{1}{1+ax}}} + \frac{\sqrt{\frac{1}{1+ax}} \sqrt{1+ax} \sin^{-1}(ax)}{8a^4} \end{aligned}$$

Mathematica [C] time = 0.126645, size = 97, normalized size = 1.15

$$\frac{8a^3x^3 - 3a\sqrt{\frac{1-ax}{ax+1}}(-2a^3x^4 - 2a^2x^3 + ax^2 + x) + 3i \log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]*x^3,x]

[Out] $(8a^3x^3 - 3a\sqrt{(1-ax)/(1+ax)})(x + ax^2 - 2a^2x^3 - 2a^3x^4) + (3I)\text{Log}[(-2I)ax + 2\sqrt{(1-ax)/(1+ax)}(1+ax)]/(24a^4)$

Maple [C] time = 0.179, size = 118, normalized size = 1.4

$$\frac{x \operatorname{csgn}(a)}{8a^3} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(2 \operatorname{csgn}(a) x^3 a^3 \sqrt{-a^2x^2+1} - x \sqrt{-a^2x^2+1} \operatorname{csgn}(a) a + \arctan \left(x \operatorname{csgn}(a) a \frac{1}{\sqrt{-a^2x^2+1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x^3,x)

[Out] $1/8 * (- (ax-1)/ax)^{1/2} * x * ((ax+1)/ax)^{1/2} * (2 * \operatorname{csgn}(a) * x^3 * a^3 * (-a^2x^2+1)^{1/2} - x * (-a^2x^2+1)^{1/2} * \operatorname{csgn}(a) * a + \arctan(\operatorname{csgn}(a) * a * x / (-a^2x^2+1)^{1/2})) * \operatorname{csgn}(a) / (-a^2x^2+1)^{1/2} / a^3 + 1/3 * x^3/a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3a} + \frac{-\frac{(-a^2x^2+1)^{3/2}x}{4a^2} + \frac{\sqrt{-a^2x^2+1}x}{8a^2} + \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x^3,x, algorithm="maxima")

[Out] $1/3 * x^3/a + \text{integrate}(\text{sqrt}(ax + 1) * \text{sqrt}(-ax + 1) * x^2, x)/a$

Fricas [A] time = 1.77774, size = 203, normalized size = 2.42

$$\frac{8a^3x^3 + 3(2a^4x^4 - a^2x^2)\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{ax-1}{ax}} - 3\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{ax-1}{ax}}\right)}{24a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="fricas")

[Out] 1/24*(8*a^3*x^3 + 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x^2 dx + \int ax^3 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**3,x)

[Out] (Integral(x**2, x) + Integral(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="giac")

[Out] integrate(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

3.34 $\int e^{\operatorname{sech}^{-1}(ax)} x^2 dx$

Optimal. Leaf size=38

$$-\frac{x e^{\operatorname{sech}^{-1}(ax)}}{3a^2} + \frac{x^2}{6a} + \frac{1}{3} x^3 e^{\operatorname{sech}^{-1}(ax)}$$

[Out] $-(E^{\operatorname{ArcSech}[a*x]*x})/(3*a^2) + x^2/(6*a) + (E^{\operatorname{ArcSech}[a*x]*x^3})/3$

Rubi [A] time = 0.024187, antiderivative size = 52, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6335, 30, 74}

$$-\frac{\sqrt{1-ax}}{3a^3 \sqrt{\frac{1}{ax+1}}} + \frac{x^2}{6a} + \frac{1}{3} x^3 e^{\operatorname{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

[In] Int[E^ArcSech[a*x]*x^2,x]

[Out] $x^2/(6*a) + (E^{\operatorname{ArcSech}[a*x]*x^3})/3 - \operatorname{Sqrt}[1 - a*x]/(3*a^3*\operatorname{Sqrt}[(1 + a*x)^{-1}])$

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax)} x^2 dx &= \frac{1}{3} e^{\operatorname{sech}^{-1}(ax)} x^3 + \frac{\int x dx}{3a} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}} dx}{3a} \\ &= \frac{x^2}{6a} + \frac{1}{3} e^{\operatorname{sech}^{-1}(ax)} x^3 - \frac{\sqrt{1-ax}}{3a^3 \sqrt{\frac{1}{1+ax}}}\end{aligned}$$

Mathematica [A] time = 0.0587718, size = 48, normalized size = 1.26

$$\frac{3a^2x^2 + 2(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2}{6a^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]*x^2,x]

[Out] (3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)

Maple [A] time = 0.173, size = 54, normalized size = 1.4

$$\frac{x(a^2x^2-1)}{3a^2} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x)

[Out] 1/3*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)/a^2+1/2*x^2/a

Maxima [A] time = 1.06955, size = 51, normalized size = 1.34

$$\frac{x^2}{2a} + \frac{(a^2x^2-1)\sqrt{ax+1}\sqrt{-ax+1}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^3
```

Fricas [A] time = 1.82112, size = 111, normalized size = 2.92

$$\frac{3ax^2 + 2(a^2x^3 - x)\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{-ax-1}{ax}}}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*x^2 + 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x dx + \int ax^2 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**2,x)
```

```
[Out] (Integral(x, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.35 $\int e^{\operatorname{sech}^{-1}(ax)} x dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\frac{1}{ax+1}} \sqrt{ax+1} \sin^{-1}(ax)}{2a^2} + \frac{1}{2} x^2 e^{\operatorname{sech}^{-1}(ax)} + \frac{x}{2a}$$

[Out] $x/(2*a) + (E^{\operatorname{ArcSech}[a*x]*x^2})/2 + (\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcSin}[a*x])/(2*a^2)$

Rubi [A] time = 0.0181567, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 8, 41, 216}

$$\frac{\sqrt{\frac{1}{ax+1}} \sqrt{ax+1} \sin^{-1}(ax)}{2a^2} + \frac{1}{2} x^2 e^{\operatorname{sech}^{-1}(ax)} + \frac{x}{2a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x]*x}, x]$

[Out] $x/(2*a) + (E^{\operatorname{ArcSech}[a*x]*x^2})/2 + (\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcSin}[a*x])/(2*a^2)$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_)]*(x_)^{(p_)}]*(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1/(1 + a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1 - a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p\}, x \ \&\& \operatorname{NeQ}[m, -1]$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 41

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(m_)}), x_Symbol] \rightarrow \operatorname{Int}[(a*c + b*d*x^2)^m, x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x \ \&\& \operatorname{EqQ}[b*c + a*d, 0] \ \&\& ($

IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax)} x \, dx &= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax)} x^2 + \frac{\int 1 \, dx}{2a} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}} \, dx}{2a} \\ &= \frac{x}{2a} + \frac{1}{2} e^{\operatorname{sech}^{-1}(ax)} x^2 + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{1}{\sqrt{1-a^2x^2}} \, dx}{2a} \\ &= \frac{x}{2a} + \frac{1}{2} e^{\operatorname{sech}^{-1}(ax)} x^2 + \frac{\sqrt{\frac{1}{1+ax}} \sqrt{1+ax} \sin^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [C] time = 0.0759349, size = 75, normalized size = 1.42

$$\frac{2ax + ax\sqrt{\frac{1-ax}{ax+1}}(ax+1) + i \log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]*x, x]

[Out] (2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(2*a^2)

Maple [C] time = 0.184, size = 92, normalized size = 1.7

$$\frac{\operatorname{xcsgn}(a)}{2a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(x\sqrt{-a^2x^2+1} \operatorname{csgn}(a) a + \arctan\left(\operatorname{xcsgn}(a) a \frac{1}{\sqrt{-a^2x^2+1}}\right) \right) \frac{1}{\sqrt{-a^2x^2+1}} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x, x)

[Out] $\frac{1}{2} * \left(\frac{-a*x-1}{a/x} \right)^{(1/2)} * x * \left(\frac{a*x+1}{a/x} \right)^{(1/2)} * \left(x * \left(-a^2*x^2+1 \right)^{(1/2)} * \text{csgn}(a) * a + \arctan(\text{csgn}(a) * a * x / \left(-a^2*x^2+1 \right)^{(1/2)}) \right) / \left(-a^2*x^2+1 \right)^{(1/2)} * \text{csgn}(a) / a + x / a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{a} + \frac{\frac{1}{2} \sqrt{-a^2 x^2 + 1} x + \frac{\arcsin\left(\frac{a^2 x}{\sqrt{a^2}}\right)}{2 \sqrt{a^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x,x, algorithm="maxima")`

[Out] `x/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a`

Fricas [A] time = 1.80814, size = 171, normalized size = 3.23

$$\frac{a^2 x^2 \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 2ax - \arctan\left(\sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x,x, algorithm="fricas")`

[Out] `1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*a*x - arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int 1 dx + \int ax \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))*x,x)`

[Out] (Integral(1, x) + Integral(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x,x, algorithm="giac")

[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

3.36 $\int e^{\operatorname{sech}^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\frac{\log(x)}{a} + xe^{\operatorname{sech}^{-1}(ax)} - \frac{\operatorname{sech}^{-1}(ax)}{a}$$

[Out] $E^{\operatorname{ArcSech}[a*x]}*x - \operatorname{ArcSech}[a*x]/a + \operatorname{Log}[x]/a$

Rubi [A] time = 0.136978, antiderivative size = 39, normalized size of antiderivative = 1.62, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6329, 1962, 208}

$$\frac{\log(x)}{a} - \frac{2 \tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a} + xe^{\operatorname{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x]}, x]$

[Out] $E^{\operatorname{ArcSech}[a*x]}*x - (2*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/a + \operatorname{Log}[x]/a$

Rule 6329

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_.)*(x_)]}, x_Symbol] \rightarrow \operatorname{Simp}[x*E^{\operatorname{ArcSech}[a*x]}, x] + (\operatorname{Dist}[1/a, \operatorname{Int}[(1*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])/(x*(1 - a*x)), x], x] + \operatorname{Simp}[\operatorname{Log}[x]/a, x]) /;$ $\operatorname{FreeQ}[a, x]$

Rule 1962

$\operatorname{Int}[(u_)^{(r_.)}*(x_)^{(m_.)}*((e_.)*((a_.) + (b_.)*(x_)^{(n_.)}))/((c_) + (d_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[p]\}, \operatorname{Dist}[(q*e*(b*c - a*d))/n, \operatorname{Subst}[\operatorname{Int}[\operatorname{SimplifyIntegrand}[(x^{(q*(p + 1) - 1)*(-a*e) + c*x^q)^{((m + 1)/n - 1)*(u /. x \rightarrow (-a*e) + c*x^q)^{(1/n)/(b*e - d*x^q)^{(1/n))}^r)/(b*e - d*x^q)^{((m + 1)/n + 1)}, x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^{(1/q)}], x]] /;$ $\operatorname{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \operatorname{PolynomialQ}[u, x] \ \&\& \operatorname{FractionQ}[p] \ \&\& \operatorname{IntegerQ}[1/n] \ \&\& \operatorname{IntegersQ}[m, r]$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax)} dx &= e^{\operatorname{sech}^{-1}(ax)} x + \frac{\log(x)}{a} + \frac{\int \frac{\sqrt{\frac{1-ax}{1+ax}}}{x(1-ax)} dx}{a} \\ &= e^{\operatorname{sech}^{-1}(ax)} x + \frac{\log(x)}{a} - 4 \operatorname{Subst} \left(\int \frac{1}{2a - 2ax^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\ &= e^{\operatorname{sech}^{-1}(ax)} x - \frac{2 \tanh^{-1} \left(\sqrt{\frac{1-ax}{1+ax}} \right)}{a} + \frac{\log(x)}{a} \end{aligned}$$

Mathematica [B] time = 0.0402629, size = 79, normalized size = 3.29

$$\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1) + 2 \log(ax) - \log \left(ax \sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x], x]

[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*Log[a*x] - Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]] + a*x*Sqrt[(1 - a*x)/(1 + a*x)])/a

Maple [A] time = 0.184, size = 79, normalized size = 3.3

$$\frac{\ln(x)}{a} + x \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(\sqrt{-a^2x^2+1} - \operatorname{Arctanh} \left(\frac{1}{\sqrt{-a^2x^2+1}} \right) \right) \frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2), x)

[Out] ln(x)/a+(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)/(-a^2*x^2+1)^(1/2)*((-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)

Fricas [B] time = 1.83216, size = 258, normalized size = 10.75

$$\frac{2ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1\right) + \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 1\right) + 2\log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="fricas")

[Out] 1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*log(x))/a

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x} dx + \int a\sqrt{-1 + \frac{1}{ax}}\sqrt{1 + \frac{1}{ax}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2),x)

[Out] (Integral(1/x, x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

$$3.37 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} dx$$

Optimal. Leaf size=48

$$2 \tan^{-1} \left(\sqrt{\frac{1-ax}{ax+1}} \right) - \frac{2}{1 - \sqrt{\frac{1-ax}{ax+1}}}$$

[Out] -2/(1 - Sqrt[(1 - a*x)/(1 + a*x)]) + 2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]

Rubi [A] time = 0.0308311, antiderivative size = 64, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6334, 97, 12, 41, 216}

$$-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{ax+1}}} - \frac{1}{ax} - \sqrt{\frac{1}{ax+1}} \sqrt{ax+1} \sin^{-1}(ax)$$

Warning: Unable to verify antiderivative.

[In] Int[E^ArcSech[a*x]/x,x]

[Out] -(1/(a*x)) - Sqrt[1 - a*x]/(a*x*Sqrt[(1 + a*x)^(-1)]) - Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcSin[a*x]

Rule 6334

Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] := -Simp[(a*p*x^p)^(-1), x] + Dist[(Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/a, Int[(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p])/x^(p + 1), x], x] /; FreeQ[{a, p}, x]

Rule 97

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_) + (d_.)*(x_)^(m_.), x_Symbol] := Int[(a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
 \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} dx &= -\frac{1}{ax} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{\sqrt{1-ax}\sqrt{1+ax}}{x^2} dx}{a} \\
 &= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{1+ax}} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{a^2}{\sqrt{1-ax}\sqrt{1+ax}} dx}{a} \\
 &= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{1+ax}} - \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}} dx \\
 &= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{1+ax}} - \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{\sqrt{1-a^2x^2}} dx \\
 &= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{1+ax}} - \sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \sin^{-1}(ax)
 \end{aligned}$$

Mathematica [C] time = 0.0497251, size = 75, normalized size = 1.56

$$\sqrt{\frac{1-ax}{ax+1}} \left(-\frac{1}{ax} - 1\right) - \frac{1}{ax} - i \log \left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x, x]
```

[Out] $-(1/(a*x)) + (-1 - 1/(a*x))*\text{Sqrt}[(1 - a*x)/(1 + a*x)] - I*\text{Log}[(-2*I)*a*x + 2*\text{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)]$

Maple [C] time = 0.189, size = 92, normalized size = 1.9

$$-\text{csgn}(a) \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(\arctan \left(\text{csgn}(a) ax \frac{1}{\sqrt{-a^2x^2+1}} \right) xa + \text{csgn}(a) \sqrt{-a^2x^2+1} \right) \frac{1}{\sqrt{-a^2x^2+1}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((1/a/x+(1/a/x-1)^{(1/2)}*(1+1/a/x)^{(1/2}))/x,x)$

[Out] $-(-(a*x-1)/a/x)^{(1/2)}*((a*x+1)/a/x)^{(1/2)}*(\arctan(\text{csgn}(a)*a*x/(-a^2*x^2+1))^{(1/2)})*x*a+\text{csgn}(a)*(-a^2*x^2+1)^{(1/2)}*\text{csgn}(a)/(-a^2*x^2+1)^{(1/2)}-1/a/x$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{a^2 \arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - \frac{\sqrt{-a^2x^2+1}}{x}}{a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((1/a/x+(1/a/x-1)^{(1/2)}*(1+1/a/x)^{(1/2}))/x,x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1)/x^2, x)/a - 1/(a*x)$

Fricas [A] time = 2.02397, size = 165, normalized size = 3.44

$$\frac{ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - ax \arctan \left(\sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} \right) + 1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((1/a/x+(1/a/x-1)^{(1/2)}*(1+1/a/x)^{(1/2}))/x,x, \text{algorithm}=\text{"fricas"})$

[Out] $-(a*x*\sqrt{(a*x + 1)/(a*x)})*\sqrt{-(a*x - 1)/(a*x)} - a*x*\arctan(\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)}) + 1/(a*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))/x,x)`

[Out] `(Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x, x))/a`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x,x, algorithm="giac")`

[Out] `integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x, x)`

$$3.38 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2} dx$$

Optimal. Leaf size=35

$$a \tanh^{-1} \left(\sqrt{\frac{1-ax}{ax+1}} \right) - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x}$$

[Out] $-E^{\operatorname{ArcSech}[a*x]/(2*x)} + a*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]$

Rubi [B] time = 0.0412326, antiderivative size = 99, normalized size of antiderivative = 2.83, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{ax+1}}} + \frac{1}{2ax^2} + \frac{1}{2}a\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}$$

Warning: Unable to verify antiderivative.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x]/x^2}, x]$

[Out] $1/(2*a*x^2) - E^{\operatorname{ArcSech}[a*x]/x} + \operatorname{Sqrt}[1 - a*x]/(2*a*x^2*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (a*\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - a*x]*\operatorname{Sqrt}[1 + a*x]])/2$

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p]/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 103

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^ (p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 92

```

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{x} - \frac{\int \frac{1}{x^3} dx}{a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx}{a} \\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{a^2}{x\sqrt{1-ax}\sqrt{1+ax}} dx}{2a} \\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{2} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{2} \left(a^2\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \operatorname{Subst} \left(\int \frac{1}{a-ax^2} dx, x, \sqrt{1-ax}\sqrt{1+ax} \right) \\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{2} a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \tanh^{-1} \left(\sqrt{1-ax}\sqrt{1+ax} \right)
\end{aligned}$$

Mathematica [B] time = 0.0598851, size = 93, normalized size = 2.66

$$\frac{1}{2} \left(-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax^2} - \frac{1}{ax^2} - a \log(x) + a \log \left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]/x^2,x]

[Out] $\left(-\frac{1}{(a*x^2)} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{(a*x^2)} - a \operatorname{Log}[x] + a \operatorname{Log}\left[1 + \sqrt{\frac{1-ax}{1+ax}} + a*x\sqrt{\frac{1-ax}{1+ax}}\right] \right) / 2$

Maple [A] time = 0.204, size = 90, normalized size = 2.6

$$-\frac{1}{2x} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(-a^2x^2 \operatorname{Artanh} \left(\frac{1}{\sqrt{-a^2x^2+1}} \right) + \sqrt{-a^2x^2+1} \right) \frac{1}{\sqrt{-a^2x^2+1}} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)

[Out] $-1/2*(-(a*x-1)/a/x)^{(1/2)}/x*((a*x+1)/a/x)^{(1/2)}*(-a^2*x^2*\operatorname{arctanh}(1/(-a^2*x^2+1))^{(1/2)}+(-a^2*x^2+1)^{(1/2)})/(-a^2*x^2+1)^{(1/2)}-1/2/a/x^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{2} a^2 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{1}{2} \sqrt{-a^2x^2+1} a^2 - \frac{(-a^2x^2+1)^{\frac{3}{2}}}{2x^2}}{a} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a - 1/2/(a*x^2)`

Fricas [B] time = 1.81015, size = 278, normalized size = 7.94

$$\frac{a^2x^2 \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{ax-1}{ax}} + 1\right) - a^2x^2 \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{ax-1}{ax}} - 1\right) - 2ax\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{ax-1}{ax}} - 2}{4ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^2,x, algorithm="fricas")`

[Out] $1/4*(a^2*x^2*\log(a*x*\sqrt{(a*x + 1)/(a*x)})*\sqrt{-(a*x - 1)/(a*x)} + 1) - a^2*x^2*\log(a*x*\sqrt{(a*x + 1)/(a*x)})*\sqrt{-(a*x - 1)/(a*x)} - 1) - 2*a*x*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} - 2)/(a*x^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^3} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^2} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

```
[Out] (Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
2, x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^2, x)
```

$$3.39 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3} dx$$

Optimal. Leaf size=55

$$-\frac{8a^2 \left(\frac{1-ax}{ax+1}\right)^{3/2}}{3 \left(1 - \frac{1-ax}{ax+1}\right)^3} - \frac{1}{3ax^3}$$

[Out] $-1/(3*a*x^3) - (8*a^2*((1 - a*x)/(1 + a*x))^(3/2))/(3*(1 - (1 - a*x)/(1 + a*x))^3)$

Rubi [C] time = 0.0366551, antiderivative size = 84, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 30, 103, 12, 95}

$$\frac{\sqrt{1-ax}}{6ax^3 \sqrt{\frac{1}{ax+1}}} + \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{a\sqrt{1-ax}}{3x\sqrt{\frac{1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

[In] Int[E^ArcSech[a*x]/x^3, x]

[Out] $1/(6*a*x^3) - E^{\operatorname{ArcSech}[a*x]}/(2*x^2) + \operatorname{Sqrt}[1 - a*x]/(6*a*x^3*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (a*\operatorname{Sqrt}[1 - a*x])/(3*x*\operatorname{Sqrt}[(1 + a*x)^{-1}])$

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 103

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_., x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 95

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_., x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3} dx &= \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} - \frac{\int \frac{1}{x^4} dx}{2a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^4\sqrt{1-ax}\sqrt{1+ax}} dx}{2a} \\
&= \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{1+ax}} dx}{6a} \\
&= \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{3} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^2\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{3x\sqrt{\frac{1}{1+ax}}}
\end{aligned}$$

Mathematica [A] time = 0.0431778, size = 43, normalized size = 0.78

$$\frac{(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2-1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcSech[a*x]/x^3,x]

[Out] $(-1 + (-1 + a*x)*\text{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a*x^3)$

Maple [A] time = 0.178, size = 53, normalized size = 1.

$$\frac{a^2x^2 - 1}{3x^2} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{1}{3x^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)

[Out] $1/3*(-(a*x-1)/a/x)^{(1/2)}/x^2*((a*x+1)/a/x)^{(1/2)}*(a^2*x^2-1)-1/3/x^3/a$

Maxima [A] time = 1.04497, size = 58, normalized size = 1.05

$$\frac{(a^2x^3 - x)\sqrt{ax+1}\sqrt{-ax+1}}{3ax^4} - \frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxima")

[Out] $1/3*(a^2*x^3 - x)*\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1)/(a*x^4) - 1/3/(a*x^3)$

Fricas [A] time = 1.83451, size = 108, normalized size = 1.96

$$\frac{(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1)/(a*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^4} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^3} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

```
[Out] (Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**3, x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^3, x)
```

$$3.40 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4} dx$$

Optimal. Leaf size=132

$$\frac{1}{8}a^3\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{ax+1}}} + \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3}$$

[Out] 1/(12*a*x^4) - E^ArcSech[a*x]/(3*x^3) + Sqrt[1 - a*x]/(12*a*x^4*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(8*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/8

Rubi [A] time = 0.058869, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{1}{8}a^3\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{ax+1}}} + \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x]/x^4, x]

[Out] 1/(12*a*x^4) - E^ArcSech[a*x]/(3*x^3) + Sqrt[1 - a*x]/(12*a*x^4*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(8*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/8

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} - \frac{\int \frac{1}{x^5} dx}{3a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^5\sqrt{1-ax}\sqrt{1+ax}} dx}{3a} \\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx}{12a} \\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{4} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{8} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{a^2}{x\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{8} \left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8} \left(a^4\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \operatorname{Subst}\left(\int \frac{1}{a-ax^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8} a^3 \sqrt{\frac{1}{1+ax}} \sqrt{1+ax} \tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)
\end{aligned}$$

Mathematica [A] time = 0.0733636, size = 110, normalized size = 0.83

$$\frac{\sqrt{\frac{1-ax}{ax+1}} \left(a^3x^3 + a^2x^2 - 2ax - 2\right) - a^4x^4 \log(x) + a^4x^4 \log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) - 2}{8ax^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]/x^4, x]

[Out] (-2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*x^4*Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(8*a*x^4)

Maple [A] time = 0.184, size = 110, normalized size = 0.8

$$\frac{1}{8x^3} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(\operatorname{Arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) x^4 a^4 + a^2x^2\sqrt{-a^2x^2+1} - 2\sqrt{-a^2x^2+1}\right) \frac{1}{\sqrt{-a^2x^2+1}} - \frac{1}{4x^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^4,x)`

[Out] $\frac{1}{8} * (-a*x-1)/a/x)^{(1/2)}/x^3 * ((a*x+1)/a/x)^{(1/2)} * (\operatorname{arctanh}(1/(-a^2*x^2+1)^{(1/2)})) * x^4 * a^4 + a^2 * x^2 * (-a^2*x^2+1)^{(1/2)} - 2 * (-a^2*x^2+1)^{(1/2)}) / (-a^2*x^2+1)^{(1/2)} - 1/4/x^4/a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{8} a^4 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{1}{8} \sqrt{-a^2x^2+1} a^4 - \frac{(-a^2x^2+1)^{\frac{3}{2}} a^2}{8x^2} - \frac{(-a^2x^2+1)^{\frac{3}{2}}}{4x^4}}{a} - \frac{1}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a - 1/4/(a*x^4)`

Fricas [A] time = 1.80857, size = 298, normalized size = 2.26

$$\frac{a^4 x^4 \log\left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 1\right) - a^4 x^4 \log\left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 1\right) + 2(a^3 x^3 - 2ax) \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 4}{16ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^4,x, algorithm="fricas")`

[Out] $\frac{1}{16} * (a^4 * x^4 * \log(a*x * \sqrt{(a*x + 1)/(a*x)} * \sqrt{-(a*x - 1)/(a*x)} + 1) - a^4 * x^4 * \log(a*x * \sqrt{(a*x + 1)/(a*x)} * \sqrt{-(a*x - 1)/(a*x)} - 1) + 2 * (a^3 * x^3 - 2 * a * x) * \sqrt{(a*x + 1)/(a*x)} * \sqrt{-(a*x - 1)/(a*x)} - 4) / (a*x^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^5} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^4} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))/x**4,x)

[Out] (Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**4, x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^4, x)

3.41

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^5} dx$$

Optimal. Leaf size=115

$$\frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{ax+1}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{ax+1}}} + \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4}$$

[Out] 1/(20*a*x^5) - E^ArcSech[a*x]/(4*x^4) + Sqrt[1 - a*x]/(20*a*x^5*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(15*x^3*Sqrt[(1 + a*x)^(-1)]) + (2*a^3*Sqrt[1 - a*x])/(15*x*Sqrt[(1 + a*x)^(-1)])

Rubi [A] time = 0.051863, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 30, 103, 12, 95}

$$\frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{ax+1}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{ax+1}}} + \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x]/x^5, x]

[Out] 1/(20*a*x^5) - E^ArcSech[a*x]/(4*x^4) + Sqrt[1 - a*x]/(20*a*x^5*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(15*x^3*Sqrt[(1 + a*x)^(-1)]) + (2*a^3*Sqrt[1 - a*x])/(15*x*Sqrt[(1 + a*x)^(-1)])

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x)) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^5} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4} - \frac{\int \frac{1}{x^6} dx}{4a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^6\sqrt{1-ax}\sqrt{1+ax}} dx}{4a} \\
&= \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{4a^2}{x^4\sqrt{1-ax}\sqrt{1+ax}} dx}{20a} \\
&= \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} - \frac{1}{5} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x^4\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{1}{15} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int -\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{15} \left(2a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x^2\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{20ax^5} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{1+ax}}}
\end{aligned}$$

Mathematica [A] time = 0.0669814, size = 60, normalized size = 0.52

$$\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2(2a^3x^3-2a^2x^2+3ax-3)-3}{15ax^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]/x^5,x]

[Out] (-3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 + 2*a^3*x^3))/(15*a*x^5)

Maple [A] time = 0.179, size = 63, normalized size = 0.6

$$\frac{(a^2x^2-1)(2a^2x^2+3)}{15x^4} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{1}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)`

[Out] $1/15*(-(a*x-1)/a/x)^(1/2)/x^4*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)*(2*a^2*x^2+3)-1/5/a/x^5$

Maxima [A] time = 1.03999, size = 69, normalized size = 0.6

$$\frac{(2a^4x^5 + a^2x^3 - 3x)\sqrt{ax+1}\sqrt{-ax+1}}{15ax^6} - \frac{1}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxima")`

[Out] $1/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1)/(a*x^6) - 1/5/(a*x^5)$

Fricas [A] time = 1.92158, size = 128, normalized size = 1.11

$$\frac{(2a^5x^5 + a^3x^3 - 3ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 3}{15ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="fricas")`

[Out] $1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*\text{sqrt}((a*x + 1)/(a*x))*\text{sqrt}(-(a*x - 1)/(a*x)) - 3)/(a*x^5)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^6} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^5} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)

[Out] (Integral(x**(-6), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**5, x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^5, x)

$$3.42 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6} dx$$

Optimal. Leaf size=163

$$\frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{ax+1}}} + \frac{1}{16}a^5\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{ax+1}}} + \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5}$$

[Out] 1/(30*a*x^6) - E^ArcSech[a*x]/(5*x^5) + Sqrt[1 - a*x]/(30*a*x^6*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(24*x^4*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[1 - a*x])/(16*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^5*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/16

Rubi [A] time = 0.0769087, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{ax+1}}} + \frac{1}{16}a^5\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{ax+1}}} + \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x]/x^6,x]

[Out] 1/(30*a*x^6) - E^ArcSech[a*x]/(5*x^5) + Sqrt[1 - a*x]/(30*a*x^6*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(24*x^4*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[1 - a*x])/(16*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^5*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/16

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} - \frac{\int \frac{1}{x^7} dx}{5a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^7\sqrt{1-ax}\sqrt{1+ax}} dx}{5a} \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{5a^2}{x^5\sqrt{1-ax}\sqrt{1+ax}} dx}{30a} \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} - \frac{1}{6} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x^5\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{1}{24} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int -\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{8} \left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{16} \left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{16} \left(a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16} \left(a^6\sqrt{\frac{1}{1+ax}}\sqrt{1+ax} \right) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}} dx \right) \\
&= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16} a^5 \sqrt{\frac{1}{1+ax}} \sqrt{1+ax} \tanh^{-1} \left(\sqrt{\frac{1-ax}{1+ax}} \right)
\end{aligned}$$

Mathematica [A] time = 0.104171, size = 129, normalized size = 0.79

$$\frac{\sqrt{\frac{1-ax}{ax+1}} \left(3a^5x^5 + 3a^4x^4 + 2a^3x^3 + 2a^2x^2 - 8ax - 8 \right) - 3a^6x^6 \log(x) + 3a^6x^6 \log \left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right) - 8}{48ax^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]/x^6,x]

[Out] (-8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 + 3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]])/(48*a*x^6)

+ a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)])/(48*a*x^6)

Maple [A] time = 0.184, size = 132, normalized size = 0.8

$$\frac{1}{48x^5} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(3 \operatorname{Arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) x^6 a^6 + 3 \sqrt{-a^2x^2+1} x^4 a^4 + 2 a^2 x^2 \sqrt{-a^2x^2+1} - 8 \sqrt{-a^2x^2+1} \right) \sqrt{-\frac{ax-1}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^6,x)

[Out] 1/48*(-(a*x-1)/a/x)^(1/2)/x^5*((a*x+1)/a/x)^(1/2)*(3*arctanh(1/(-a^2*x^2+1)^(1/2))*x^6*a^6+3*(-a^2*x^2+1)^(1/2)*x^4*a^4+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-8*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/6/x^6/a

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{16} a^6 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{1}{16} \sqrt{-a^2x^2+1} a^6 - \frac{(-a^2x^2+1)^{\frac{3}{2}} a^4}{16x^2} - \frac{(-a^2x^2+1)^{\frac{3}{2}} a^2}{8x^4} - \frac{(-a^2x^2+1)^{\frac{3}{2}}}{6x^6}}{a} - \frac{1}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a - 1/6/(a*x^6)

Fricas [A] time = 2.19449, size = 324, normalized size = 1.99

$$\frac{3 a^6 x^6 \log\left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 1\right) - 3 a^6 x^6 \log\left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 1\right) + 2(3 a^5 x^5 + 2 a^3 x^3 - 8 ax) \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 16}{96 a x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^6,x, algorithm="fricas")

```
[Out] 1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(3
*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
- 16)/(a*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^7} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^6} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

```
[Out] (Integral(x**(-7), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
6, x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^6, x)
```

$$3.43 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7} dx$$

Optimal. Leaf size=146

$$\frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{ax+1}}} + \frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{ax+1}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{ax+1}}} + \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6}$$

[Out] 1/(42*a*x^7) - E^ArcSech[a*x]/(6*x^6) + Sqrt[1 - a*x]/(42*a*x^7*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(35*x^5*Sqrt[(1 + a*x)^(-1)]) + (4*a^3*Sqrt[1 - a*x])/(105*x^3*Sqrt[(1 + a*x)^(-1)]) + (8*a^5*Sqrt[1 - a*x])/(105*x*Sqrt[(1 + a*x)^(-1)])

Rubi [A] time = 0.0711514, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 30, 103, 12, 95}

$$\frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{ax+1}}} + \frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{ax+1}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{ax+1}}} + \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x]/x^7, x]

[Out] 1/(42*a*x^7) - E^ArcSech[a*x]/(6*x^6) + Sqrt[1 - a*x]/(42*a*x^7*Sqrt[(1 + a*x)^(-1)]) + (a*Sqrt[1 - a*x])/(35*x^5*Sqrt[(1 + a*x)^(-1)]) + (4*a^3*Sqrt[1 - a*x])/(105*x^3*Sqrt[(1 + a*x)^(-1)]) + (8*a^5*Sqrt[1 - a*x])/(105*x*Sqrt[(1 + a*x)^(-1)])

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} - \frac{\int \frac{1}{x^8} dx}{6a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^8\sqrt{1-ax}\sqrt{1+ax}} dx}{6a} \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{6a^2}{x^6\sqrt{1-ax}\sqrt{1+ax}} dx}{42a} \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} - \frac{1}{7} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^6\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{1}{35} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{4a^2}{x^4\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} - \frac{1}{35} \left(4a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^4\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{1}{105} \left(4a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{105} \left(8a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{1+ax}}}
\end{aligned}$$

Mathematica [A] time = 0.085222, size = 76, normalized size = 0.52

$$\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2(8a^5x^5 - 8a^4x^4 + 12a^3x^3 - 12a^2x^2 + 15ax - 15) - 15}{105ax^7}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]/x^7, x]

[Out] (-15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a*x^7)

Maple [A] time = 0.191, size = 71, normalized size = 0.5

$$\frac{(a^2x^2 - 1)(8x^4a^4 + 12a^2x^2 + 15)}{105x^6} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{1}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)

[Out] 1/105*(-(a*x-1)/a/x)^(1/2)/x^6*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+12*a^2*x^2+15)-1/7/a/x^7

Maxima [A] time = 1.03706, size = 81, normalized size = 0.55

$$\frac{(8a^6x^7 + 4a^4x^5 + 3a^2x^3 - 15x)\sqrt{ax+1}\sqrt{-ax+1}}{105ax^8} - \frac{1}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxima")

[Out] 1/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^8) - 1/7/(a*x^7)

Fricas [A] time = 2.12879, size = 151, normalized size = 1.03

$$\frac{(8a^7x^7 + 4a^5x^5 + 3a^3x^3 - 15ax)\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{-ax-1}{ax}} - 15}{105ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="fricas")

[Out] 1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 15)/(a*x^7)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^8} dx + \int \frac{a \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}}{x^7} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)

[Out] (Integral(x**(-8), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**7, x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^7, x)

3.44 $\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8} dx$

Optimal. Leaf size=194

$$\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{ax+1}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{ax+1}}} + \frac{5}{128}a^7\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{ax+1}}} + \frac{1}{56ax^8}$$

[Out] $1/(56*a*x^8) - E^{\operatorname{ArcSech}[a*x]}/(7*x^7) + \operatorname{Sqrt}[1 - a*x]/(56*a*x^8*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (a*\operatorname{Sqrt}[1 - a*x])/(48*x^6*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (5*a^3*\operatorname{Sqrt}[1 - a*x])/(192*x^4*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (5*a^5*\operatorname{Sqrt}[1 - a*x])/(128*x^2*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (5*a^7*\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - a*x]*\operatorname{Sqrt}[1 + a*x]])/128$

Rubi [A] time = 0.0984763, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{ax+1}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{ax+1}}} + \frac{5}{128}a^7\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{ax+1}}} + \frac{1}{56ax^8}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x]}/x^8, x]$

[Out] $1/(56*a*x^8) - E^{\operatorname{ArcSech}[a*x]}/(7*x^7) + \operatorname{Sqrt}[1 - a*x]/(56*a*x^8*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (a*\operatorname{Sqrt}[1 - a*x])/(48*x^6*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (5*a^3*\operatorname{Sqrt}[1 - a*x])/(192*x^4*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (5*a^5*\operatorname{Sqrt}[1 - a*x])/(128*x^2*\operatorname{Sqrt}[(1 + a*x)^{-1}]) + (5*a^7*\operatorname{Sqrt}[(1 + a*x)^{-1}]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - a*x]*\operatorname{Sqrt}[1 + a*x]])/128$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_.)*(x_)^{(p_.)}]}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1/(1 + a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1 - a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p, x\} \&\& \operatorname{NeQ}[m, -1]$

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} - \frac{\int \frac{1}{x^9} dx}{7a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^9\sqrt{1-ax}\sqrt{1+ax}} dx}{7a} \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{7a^2}{x^7\sqrt{1-ax}\sqrt{1+ax}} dx}{56a} \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} - \frac{1}{8} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^7\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{1}{48} \left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{5a^2}{x^5\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} - \frac{1}{48} \left(5a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^5\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{1}{192} \left(5a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int -\frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{64} \left(5a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{128} \left(5a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{128} \left(5a^7\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{128} \left(5a^8\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right) \int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} + \frac{5}{128} a^7 \sqrt{\frac{1}{1+ax}} \sqrt{1+ax}
\end{aligned}$$

Mathematica [A] time = 0.132619, size = 145, normalized size = 0.75

$$\frac{\sqrt{\frac{1-ax}{ax+1}} (15a^7x^7 + 15a^6x^6 + 10a^5x^5 + 10a^4x^4 + 8a^3x^3 + 8a^2x^2 - 48ax - 48) - 15a^8x^8 \log(x) + 15a^8x^8 \log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}}\right)}{384ax^8}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]/x^8,x]

[Out] $(-48 + \text{Sqrt}[(1 - a*x)/(1 + a*x)])*(-48 - 48*a*x + 8*a^2*x^2 + 8*a^3*x^3 + 10*a^4*x^4 + 10*a^5*x^5 + 15*a^6*x^6 + 15*a^7*x^7) - 15*a^8*x^8*\text{Log}[x] + 15*a^8*x^8*\text{Log}[1 + \text{Sqrt}[(1 - a*x)/(1 + a*x)]] + a*x*\text{Sqrt}[(1 - a*x)/(1 + a*x)]]/(384*a*x^8)$

Maple [A] time = 0.203, size = 152, normalized size = 0.8

$$\frac{1}{384 x^7} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(15 \text{Arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) x^8 a^8 + 15 \sqrt{-a^2x^2+1} x^6 a^6 + 10 \sqrt{-a^2x^2+1} x^4 a^4 + 8 a^2 x^2 \sqrt{-a^2x^2+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x)

[Out] $1/384*(-(a*x-1)/a/x)^(1/2)/x^7*((a*x+1)/a/x)^(1/2)*(15*\text{arctanh}(1/(-a^2*x^2+1)^(1/2))*x^8*a^8+15*(-a^2*x^2+1)^(1/2)*x^6*a^6+10*(-a^2*x^2+1)^(1/2)*x^4*a^4+8*a^2*x^2*(-a^2*x^2+1)^(1/2)-48*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/8/a/x^8$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{5}{128} a^8 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{5}{128} \sqrt{-a^2x^2+1} a^8 - \frac{5(-a^2x^2+1)^{\frac{3}{2}} a^6}{128 x^2} - \frac{5(-a^2x^2+1)^{\frac{3}{2}} a^4}{64 x^4} - \frac{5(-a^2x^2+1)^{\frac{3}{2}} a^2}{48 x^6} - \frac{(-a^2x^2+1)^{\frac{3}{2}}}{8 x^8}}{a} - \frac{1}{8 a x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^9, x)/a - 1/8/(a*x^8)

Fricas [A] time = 2.13341, size = 348, normalized size = 1.79

$$\frac{15 a^8 x^8 \log\left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 1\right) - 15 a^8 x^8 \log\left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 1\right) + 2\left(15 a^7 x^7 + 10 a^5 x^5 + 8 a^3 x^3 - 48 ax\right) \sqrt{\frac{ax+1}{ax}}}{768 a x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="fricas")
```

```
[Out] 1/768*(15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
- 15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2
*(15*a^7*x^7 + 10*a^5*x^5 + 8*a^3*x^3 - 48*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(
-(a*x - 1)/(a*x)) - 96)/(a*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^9} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^8} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**8,x)
```

```
[Out] (Integral(x**(-9), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
8, x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^8, x)
```


3.45 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^7 dx$

Optimal. Leaf size=111

$$-\frac{x^2 \sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sqrt{1-a^2x^4}}{16a^3} + \frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sin^{-1}(ax^2)}{16a^4} + \frac{x^6}{24a} + \frac{1}{8} x^8 e^{\operatorname{sech}^{-1}(ax^2)}$$

[Out] $x^6/(24*a) + (E^{\operatorname{ArcSech}[a*x^2]}*x^8)/8 - (x^2*\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{Sqrt}[1 - a^2*x^4])/(16*a^3) + (\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{ArcSin}[a*x^2])/(16*a^4)$

Rubi [A] time = 0.0616554, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6335, 30, 259, 275, 321, 216}

$$-\frac{x^2 \sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sqrt{1-a^2x^4}}{16a^3} + \frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sin^{-1}(ax^2)}{16a^4} + \frac{x^6}{24a} + \frac{1}{8} x^8 e^{\operatorname{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^2]}*x^7, x]$

[Out] $x^6/(24*a) + (E^{\operatorname{ArcSech}[a*x^2]}*x^8)/8 - (x^2*\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{Sqrt}[1 - a^2*x^4])/(16*a^3) + (\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{ArcSin}[a*x^2])/(16*a^4)$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^p]}*(x^m), x] := \operatorname{Simp}[(x^{m+1})*E^{\operatorname{ArcSech}[a*x^p]}/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{m-p}], x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1/(1 + a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{m-p}]/(\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1 - a*x^p]), x], x) /; \operatorname{FreeQ}[a, m, p], x \&\& \operatorname{NeQ}[m, -1]$

Rule 30

$\operatorname{Int}[x^m, x] := \operatorname{Simp}[x^{m+1}/(m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 259

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int e^{\operatorname{sech}^{-1}(ax^2)} x^7 dx &= \frac{1}{8} e^{\operatorname{sech}^{-1}(ax^2)} x^8 + \frac{\int x^5 dx}{4a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^5}{\sqrt{1-ax^2} \sqrt{1+ax^2}} dx}{4a} \\
 &= \frac{x^6}{24a} + \frac{1}{8} e^{\operatorname{sech}^{-1}(ax^2)} x^8 + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^5}{\sqrt{1-a^2x^4}} dx}{4a} \\
 &= \frac{x^6}{24a} + \frac{1}{8} e^{\operatorname{sech}^{-1}(ax^2)} x^8 + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}} dx, x, x^2\right)}{8a} \\
 &= \frac{x^6}{24a} + \frac{1}{8} e^{\operatorname{sech}^{-1}(ax^2)} x^8 - \frac{x^2 \sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{16a^3} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}} dx, x, x^2\right)}{16a^3} \\
 &= \frac{x^6}{24a} + \frac{1}{8} e^{\operatorname{sech}^{-1}(ax^2)} x^8 - \frac{x^2 \sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{16a^3} + \frac{\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sin^{-1}(ax^2)}{16a^4}
 \end{aligned}$$

Mathematica [C] time = 0.181418, size = 111, normalized size = 1.

$$\frac{8a^3x^6 - 3a\sqrt{\frac{1-ax^2}{ax^2+1}}(-2a^3x^8 - 2a^2x^6 + ax^4 + x^2) + 3i \log\left(2\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2 + 1) - 2iax^2\right)}{48a^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x^7,x]

[Out] (8*a^3*x^6 - 3*a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4 - 2*a^2*x^6 - 2*a^3*x^8) + (3*I)*Log[(-2*I)*a*x^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)])/(48*a^4)

Maple [A] time = 0.322, size = 137, normalized size = 1.2

$$\frac{x^2}{16a^4} \sqrt{-\frac{ax^2-1}{ax^2}} \sqrt{\frac{ax^2+1}{ax^2}} \left(2x^6 \sqrt{-\frac{a^2x^4-1}{a^2}} a^4 - x^2 \sqrt{-\frac{a^2x^4-1}{a^2}} a^2 + \arctan\left(x^2 \frac{1}{\sqrt{-\frac{a^2x^4-1}{a^2}}}\right) \right) \frac{1}{\sqrt{-\frac{a^2x^4-1}{a^2}}} + \frac{x^6}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^7,x)

[Out] 1/16*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(2*x^6*(-(a^2*x^4-1)/a^2)^(1/2)*a^4-x^2*(-(a^2*x^4-1)/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4-1)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^(1/2)/a^4+1/6*x^6/a

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^6}{6a} + \frac{-\frac{(-a^2x^4+1)^3x^2}{8a^2} + \frac{\sqrt{-a^2x^4+1}x^2}{16a^2} + \frac{\arcsin\left(\frac{a^2x^2}{\sqrt{a^2}}\right)}{16\sqrt{a^2a^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="maxima")

[Out] $1/6*x^6/a + \text{integrate}(\text{sqrt}(a*x^2 + 1)*\text{sqrt}(-a*x^2 + 1)*x^5, x)/a$

Fricas [A] time = 2.12503, size = 251, normalized size = 2.26

$$\frac{8a^3x^6 + 3(2a^4x^8 - a^2x^4)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} - 6 \arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1}{ax^2}\right)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="fricas")`

[Out] $1/48*(8*a^3*x^6 + 3*(2*a^4*x^8 - a^2*x^4)*\text{sqrt}((a*x^2 + 1)/(a*x^2))*\text{sqrt}(-(a*x^2 - 1)/(a*x^2)) - 6*\arctan((a*x^2*\text{sqrt}((a*x^2 + 1)/(a*x^2))*\text{sqrt}(-(a*x^2 - 1)/(a*x^2)) - 1)/(a*x^2)))/a^4$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x**2+(1/a/x**2-1)**(1/2))*(1/a/x**2+1)**(1/2))*x**7,x)`

[Out] Timed out

Giac [B] time = 1.19495, size = 360, normalized size = 3.24

$$8a^4x^6 - \frac{30a^4 \arcsin\left(\frac{\sqrt{2}\sqrt{-a^2x^2+a}}{2\sqrt{a}}\right) - 12\left(\sqrt{a^2x^2+a}\sqrt{-a^2x^2+aa^2x^2} - 2a^2 \arcsin\left(\frac{\sqrt{2}\sqrt{-a^2x^2+a}}{2\sqrt{a}}\right)\right)a^2 - \sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}(15a^3 + (a^2x^2 - a)(2(3a^2x^2 + 14a)(a^2x^2 - a)))}{48a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="giac")`

```
[Out] 1/48*(8*a^4*x^6 - (30*a^4*arcsin(1/2*sqrt(2)*sqrt(-a^2*x^2 + a)/sqrt(a)) -
12*(sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*a^2*x^2 - 2*a^2*arcsin(1/2*sqrt(2)
*sqrt(-a^2*x^2 + a)/sqrt(a)))*a^2 - sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*(1
5*a^3 + (a^2*x^2 - a)*(2*(3*a^2*x^2 + 14*a)*(a^2*x^2 - a) + 59*a^2)) - 8*(6
*a^3*arcsin(1/2*sqrt(2)*sqrt(-a^2*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2 + a)*sqr
t(-a^2*x^2 + a)*((2*a^2*x^2 + 5*a)*(a^2*x^2 - a) + 3*a^2))*a)/a^3)/a^5
```

3.46 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^6 dx$

Optimal. Leaf size=115

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\operatorname{EllipticF}(\sin^{-1}(\sqrt{ax}), -1)}{21a^{7/2}} - \frac{2x\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{21a^3} + \frac{2x^5}{35a} + \frac{1}{7}x^7e^{\operatorname{sech}^{-1}(ax^2)}$$

[Out] (2*x^5)/(35*a) + (E^ArcSech[a*x^2]*x^7)/7 - (2*x*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(21*a^3) + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(21*a^(7/2))

Rubi [A] time = 0.051338, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6335, 30, 259, 321, 221}

$$-\frac{2x\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{21a^3} + \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F(\sin^{-1}(\sqrt{ax})|-1)}{21a^{7/2}} + \frac{2x^5}{35a} + \frac{1}{7}x^7e^{\operatorname{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^2]*x^6,x]

[Out] (2*x^5)/(35*a) + (E^ArcSech[a*x^2]*x^7)/7 - (2*x*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(21*a^3) + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(21*a^(7/2))

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 259

```
Int[((c_.)*(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_) + (b2_.)*(x_)
^(n_.))^(p_.), x_Symbol] := Int[(c*x)^(m*(a1*a2 + b1*b2*x^(2*n))^(p), x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 321

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^2)} x^6 dx &= \frac{1}{7} e^{\operatorname{sech}^{-1}(ax^2)} x^7 + \frac{2 \int x^4 dx}{7a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^4}{\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{7a} \\ &= \frac{2x^5}{35a} + \frac{1}{7} e^{\operatorname{sech}^{-1}(ax^2)} x^7 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^4}{\sqrt{1-a^2x^4}} dx}{7a} \\ &= \frac{2x^5}{35a} + \frac{1}{7} e^{\operatorname{sech}^{-1}(ax^2)} x^7 - \frac{2x\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{21a^3} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{\sqrt{1-a^2x^4}} dx}{21a^3} \\ &= \frac{2x^5}{35a} + \frac{1}{7} e^{\operatorname{sech}^{-1}(ax^2)} x^7 - \frac{2x\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{21a^3} + \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} F\left(\sin^{-1}(\sqrt{ax}) \mid -1\right)}{21a^{7/2}} \end{aligned}$$

Mathematica [C] time = 0.332462, size = 139, normalized size = 1.21

$$-\frac{2i\sqrt{\frac{1-ax^2}{ax^2+1}} \sqrt{1-a^2x^4} \operatorname{EllipticF}\left(i \sinh^{-1}(\sqrt{-ax}), -1\right)}{21(-a)^{7/2}(ax^2-1)} + \frac{x\sqrt{\frac{1-ax^2}{ax^2+1}}(3a^3x^6+3a^2x^4-2ax^2-2)}{21a^3} + \frac{x^5}{5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^6,x]
```

[Out] $x^5/(5*a) + (x*\text{Sqrt}[(1 - a*x^2)/(1 + a*x^2)]*(-2 - 2*a*x^2 + 3*a^2*x^4 + 3*a^3*x^6))/(21*a^3) - (((2*I)/21)*\text{Sqrt}[(1 - a*x^2)/(1 + a*x^2)]*\text{Sqrt}[1 - a^2*x^4]*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-a]*x], -1])/((-a)^(7/2)*(-1 + a*x^2))$

Maple [A] time = 0.19, size = 114, normalized size = 1.

$$\frac{x^2}{21 a^2 x^4 - 21} \sqrt{-\frac{ax^2 - 1}{ax^2}} \sqrt{\frac{ax^2 + 1}{ax^2}} \left(3 x^9 a^{9/2} - 5 x^5 a^{5/2} - 2 \text{EllipticF}(x\sqrt{a}, i) \sqrt{-ax^2 + 1} \sqrt{ax^2 + 1} + 2 x \sqrt{a} \right) a^{-5/2} + \frac{x^5}{5 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x)`

[Out] $1/21*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(3*x^9*a^(9/2)-5*x^5*a^(5/2)-2*\text{EllipticF}(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)+2*x*a^(1/2))/a^(5/2)/(a^2*x^4-1)+1/5*x^5/a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^5}{5a} + \frac{\int \sqrt{ax^2 + 1} \sqrt{-ax^2 + 1} x^4 dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="maxima")`

[Out] $1/5*x^5/a + \text{integrate}(\text{sqrt}(a*x^2 + 1)*\text{sqrt}(-a*x^2 + 1)*x^4, x)/a$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{ax^6 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} + x^4}{a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^6*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + x^4)
/a, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \left(\sqrt{\frac{1}{ax^2} + 1} \sqrt{\frac{1}{ax^2} - 1} + \frac{1}{ax^2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="g
iac")
```

```
[Out] integrate(x^6*(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2)), x)
```

3.47 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^5 dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{1-ax^2}}{6a^3\sqrt{\frac{1}{ax^2+1}}} + \frac{x^4}{12a} + \frac{1}{6}x^6e^{\operatorname{sech}^{-1}(ax^2)}$$

[Out] $x^4/(12*a) + (E^{\operatorname{ArcSech}[a*x^2]}*x^6)/6 - \operatorname{Sqrt}[1 - a*x^2]/(6*a^3*\operatorname{Sqrt}[(1 + a*x^2)^{-1}])$

Rubi [A] time = 0.0412168, antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6335, 30, 259, 261}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{6a^3} + \frac{x^4}{12a} + \frac{1}{6}x^6e^{\operatorname{sech}^{-1}(ax^2)}$$

Warning: Unable to verify antiderivative.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^2]}*x^5, x]$

[Out] $x^4/(12*a) + (E^{\operatorname{ArcSech}[a*x^2]}*x^6)/6 - (\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{Sqrt}[1 - a^2*x^4])/(6*a^3)$

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
```

$Q[\{a1, b1, a2, b2, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[a1, 0] \ \&\& \ \text{GtQ}[a2, 0]))$

Rule 261

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \ :> \ \text{Simp}[(a + b*x^n)^{(p + 1)/(b*n*(p + 1))}, x] \ /; \ \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{NeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int e^{\text{sech}^{-1}(ax^2)} x^5 dx &= \frac{1}{6} e^{\text{sech}^{-1}(ax^2)} x^6 + \frac{\int x^3 dx}{3a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^3}{\sqrt{1-ax^2} \sqrt{1+ax^2}} dx}{3a} \\ &= \frac{x^4}{12a} + \frac{1}{6} e^{\text{sech}^{-1}(ax^2)} x^6 + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^3}{\sqrt{1-a^2x^4}} dx}{3a} \\ &= \frac{x^4}{12a} + \frac{1}{6} e^{\text{sech}^{-1}(ax^2)} x^6 - \frac{\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{6a^3} \end{aligned}$$

Mathematica [A] time = 0.0941505, size = 56, normalized size = 0.97

$$\frac{(ax^2 - 1) \sqrt{\frac{1-ax^2}{ax^2+1}} (ax^2 + 1)^2}{6a^3} + \frac{x^4}{4a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x^5,x]

[Out] $x^4/(4*a) + ((-1 + a*x^2)*\text{Sqrt}[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)^2)/(6*a^3)$

Maple [A] time = 0.178, size = 60, normalized size = 1.

$$\frac{x^2 (a^2 x^4 - 1)}{6 a^2} \sqrt{-\frac{ax^2 - 1}{ax^2}} \sqrt{\frac{ax^2 + 1}{ax^2}} + \frac{x^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x)`

[Out] $1/6*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^2*x^4-1)/a^2+1/4*x^4/a$

Maxima [A] time = 1.06829, size = 57, normalized size = 0.98

$$\frac{x^4}{4a} + \frac{(a^2x^4 - 1)\sqrt{ax^2 + 1}\sqrt{-ax^2 + 1}}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="maxima")`

[Out] $1/4*x^4/a + 1/6*(a^2*x^4 - 1)*\text{sqrt}(a*x^2 + 1)*\text{sqrt}(-a*x^2 + 1)/a^3$

Fricas [A] time = 2.06129, size = 126, normalized size = 2.17

$$\frac{3ax^4 + 2(a^2x^6 - x^2)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}}{12a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="fricas")`

[Out] $1/12*(3*a*x^4 + 2*(a^2*x^6 - x^2)*\text{sqrt}((a*x^2 + 1)/(a*x^2))*\text{sqrt}(-(a*x^2 - 1)/(a*x^2)))/a^2$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**5,x)

[Out] Timed out

Giac [B] time = 1.18225, size = 240, normalized size = 4.14

$$3(a^2x^2 + a)^2 - 6(a^2x^2 + a)a - \frac{6a^3 \arcsin\left(\frac{\sqrt{2}\sqrt{-a^2x^2+a}}{2\sqrt{a}}\right) - \sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}((2a^2x^2+5a)(a^2x^2-a)+3a^2) + 3\left(\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}a^2x^2 - 2a^2 \arcsin\left(\frac{\sqrt{2}\sqrt{-a^2x^2+a}}{2\sqrt{a}}\right)\right)}{12a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="giac")

[Out] 1/12*(3*(a^2*x^2 + a)^2 - 6*(a^2*x^2 + a)*a - (6*a^3*arcsin(1/2*sqrt(2)*sqrt(-a^2*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((2*a^2*x^2 + 5*a)*(a^2*x^2 - a) + 3*a^2) + 3*(sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*a^2*x^2 - 2*a^2*arcsin(1/2*sqrt(2)*sqrt(-a^2*x^2 + a)/sqrt(a))))*a/a^5

3.48 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^4 dx$

Optimal. Leaf size=112

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\operatorname{EllipticF}(\sin^{-1}(\sqrt{ax}), -1)}{5a^{5/2}} + \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E(\sin^{-1}(\sqrt{ax})|-1)}{5a^{5/2}} + \frac{2x^3}{15a} + \frac{1}{5}x^5e^{\operatorname{sech}^{-1}(ax^2)}$$

[Out] (2*x^3)/(15*a) + (E^ArcSech[a*x^2]*x^5)/5 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticE[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2)) - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2))

Rubi [A] time = 0.0665785, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6335, 30, 259, 307, 221, 1199, 424}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F(\sin^{-1}(\sqrt{ax})|-1)}{5a^{5/2}} + \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E(\sin^{-1}(\sqrt{ax})|-1)}{5a^{5/2}} + \frac{2x^3}{15a} + \frac{1}{5}x^5e^{\operatorname{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^2]*x^4,x]

[Out] (2*x^3)/(15*a) + (E^ArcSech[a*x^2]*x^5)/5 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticE[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2)) - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2))

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 259

```
Int[((c_.)*(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax^2)} x^4 dx &= \frac{1}{5} e^{\operatorname{sech}^{-1}(ax^2)} x^5 + \frac{2 \int x^2 dx}{5a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^2}{\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{5a} \\
&= \frac{2x^3}{15a} + \frac{1}{5} e^{\operatorname{sech}^{-1}(ax^2)} x^5 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^2}{\sqrt{1-a^2x^4}} dx}{5a} \\
&= \frac{2x^3}{15a} + \frac{1}{5} e^{\operatorname{sech}^{-1}(ax^2)} x^5 - \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{\sqrt{1-a^2x^4}} dx}{5a^2} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1+ax^2}{\sqrt{1-a^2x^4}} dx}{5a^2} \\
&= \frac{2x^3}{15a} + \frac{1}{5} e^{\operatorname{sech}^{-1}(ax^2)} x^5 - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} F(\sin^{-1}(\sqrt{ax})|-1)}{5a^{5/2}} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{\sqrt{1+ax^2}}{\sqrt{1-ax^2}} dx}{5a^2} \\
&= \frac{2x^3}{15a} + \frac{1}{5} e^{\operatorname{sech}^{-1}(ax^2)} x^5 + \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} E(\sin^{-1}(\sqrt{ax})|-1)}{5a^{5/2}} - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} F(\sin^{-1}(\sqrt{ax})|-1)}{5a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.435428, size = 140, normalized size = 1.25

$$\frac{1}{15} \left(\frac{6i\sqrt{\frac{1-ax^2}{ax^2+1}} \sqrt{1-a^2x^4} (E(i\sinh^{-1}(\sqrt{-ax})|-1) - \operatorname{EllipticF}(i\sinh^{-1}(\sqrt{-ax}), -1))}{(-a)^{5/2}(ax^2-1)} + \frac{5x^3}{a} + \frac{3\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^5+x^3)}{a} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x^4,x]

[Out] ((5*x^3)/a + (3*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^3 + a*x^5))/a + ((6*I)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a]*x], -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/((-a)^(5/2)*(-1 + a*x^2))/15

Maple [A] time = 0.181, size = 136, normalized size = 1.2

$$\frac{x^2}{5a^2x^4-5}\sqrt{-\frac{ax^2-1}{ax^2}}\sqrt{\frac{ax^2+1}{ax^2}}\left(a^{\frac{7}{2}}x^7-x^3a^{\frac{3}{2}}+2\operatorname{EllipticF}(x\sqrt{a},i)\sqrt{-ax^2+1}\sqrt{ax^2+1}-2\sqrt{-ax^2+1}\sqrt{ax^2+1}\operatorname{EllipticF}(x\sqrt{a},i)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^4,x)

[Out] $\frac{1}{5} * \left(-\frac{a*x^2-1}{a/x^2} \right)^{1/2} * x^2 * \left(\frac{a*x^2+1}{a/x^2} \right)^{1/2} * \left(a^{7/2} * x^7 - x^3 * a^{3/2} + 2 * \text{EllipticF}(x*a^{1/2}, I) * (-a*x^2+1)^{1/2} * (a*x^2+1)^{1/2} - 2 * (-a*x^2+1)^{1/2} * (a*x^2+1)^{1/2} * \text{EllipticE}(x*a^{1/2}, I) \right) / \left(a^2 * x^4 - 1 \right) / a^{3/2} + \frac{1}{3} * x^3 / a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3a} + \frac{\int \sqrt{ax^2+1} \sqrt{-ax^2+1} x^2 dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="maxima")`

[Out] $\frac{1}{3} * x^3 / a + \text{integrate}(\text{sqrt}(a*x^2 + 1) * \text{sqrt}(-a*x^2 + 1) * x^2, x) / a$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{ax^4 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{\frac{-ax^2-1}{ax^2}} + x^2}{a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="fricas")`

[Out] `integral((a*x^4*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + x^2)/a, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x^2 dx + \int ax^4 \sqrt{-1 + \frac{1}{ax^2}} \sqrt{1 + \frac{1}{ax^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**4,x)

[Out] (Integral(x**2, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(\sqrt{\frac{1}{ax^2} + 1} \sqrt{\frac{1}{ax^2} - 1} + \frac{1}{ax^2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="giac")

[Out] integrate(x^4*(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2)), x)

3.49 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^3 dx$

Optimal. Leaf size=63

$$\frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sin^{-1}(ax^2)}{4a^2} + \frac{x^2}{4a} + \frac{1}{4} x^4 e^{\operatorname{sech}^{-1}(ax^2)}$$

[Out] $x^2/(4*a) + (E^{\operatorname{ArcSech}[a*x^2]}*x^4)/4 + (\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{ArcSin}[a*x^2])/(4*a^2)$

Rubi [A] time = 0.0368681, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6335, 30, 259, 275, 216}

$$\frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sin^{-1}(ax^2)}{4a^2} + \frac{x^2}{4a} + \frac{1}{4} x^4 e^{\operatorname{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^2]}*x^3, x]$

[Out] $x^2/(4*a) + (E^{\operatorname{ArcSech}[a*x^2]}*x^4)/4 + (\operatorname{Sqrt}[(1 + a*x^2)^{-1}]*\operatorname{Sqrt}[1 + a*x^2]*\operatorname{ArcSin}[a*x^2])/(4*a^2)$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_)]*(x_)^{(p_)}]*(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1/(1 + a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1 - a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p\}, x \&\& \operatorname{NeQ}[m, -1]$

Rule 30

$\operatorname{Int}[(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)}/(m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 259

$\operatorname{Int}[((c_)*(x_))^{(m_)}*((a1_)+(b1_)*(x_)^{(n_)})^{(p_)}*((a2_)+(b2_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Int}[(c*x)^m*(a1*a2 + b1*b2*x^{(2*n)})^p, x] /; \operatorname{FreeQ}[a1, a2, b1, b2, c, m, n, p], x]$

$Q[\{a1, b1, a2, b2, c, m, n, p\}, x] \&\& \text{EqQ}[a2*b1 + a1*b2, 0] \&\& (\text{IntegerQ}[p] \mid\mid (\text{GtQ}[a1, 0] \&\& \text{GtQ}[a2, 0]))$

Rule 275

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)*(a + b*x^{(n/k)})^p}, x], x, x^{k}], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \text{ :> Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

Rubi steps

$$\begin{aligned} \int e^{\text{sech}^{-1}(ax^2)} x^3 dx &= \frac{1}{4} e^{\text{sech}^{-1}(ax^2)} x^4 + \frac{\int x dx}{2a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x}{\sqrt{1-ax^2} \sqrt{1+ax^2}} dx}{2a} \\ &= \frac{x^2}{4a} + \frac{1}{4} e^{\text{sech}^{-1}(ax^2)} x^4 + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x}{\sqrt{1-a^2x^4}} dx}{2a} \\ &= \frac{x^2}{4a} + \frac{1}{4} e^{\text{sech}^{-1}(ax^2)} x^4 + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}} dx, x, x^2\right)}{4a} \\ &= \frac{x^2}{4a} + \frac{1}{4} e^{\text{sech}^{-1}(ax^2)} x^4 + \frac{\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sin^{-1}(ax^2)}{4a^2} \end{aligned}$$

Mathematica [C] time = 0.103778, size = 92, normalized size = 1.46

$$\frac{2ax^2 + a\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^4 + x^2) + i \log\left(2\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2 + 1) - 2iax^2\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x^3,x]

[Out] (2*a*x^2 + a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4) + I*Log[(-2*I)*a*x^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)])/(4*a^2)

Maple [A] time = 0.33, size = 112, normalized size = 1.8

$$\frac{x^2}{4a^2} \sqrt{-\frac{ax^2-1}{ax^2}} \sqrt{\frac{ax^2+1}{ax^2}} \left(x^2 \sqrt{-\frac{a^2x^4-1}{a^2}} a^2 + \arctan \left(x^2 \frac{1}{\sqrt{-\frac{a^2x^4-1}{a^2}}} \right) \right) \frac{1}{\sqrt{-\frac{a^2x^4-1}{a^2}}} + \frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x)`

[Out] $\frac{1}{4} * (- (a * x^2 - 1) / a / x^2)^{(1/2)} * x^2 * ((a * x^2 + 1) / a / x^2)^{(1/2)} * (x^2 * (- (a^2 * x^4 - 1) / a^2)^{(1/2)} * a^2 + \arctan(x^2 / (- (a^2 * x^4 - 1) / a^2)^{(1/2)})) / (- (a^2 * x^4 - 1) / a^2)^{(1/2)} / a^2 + 1/2 * x^2 / a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2a} + \frac{\frac{1}{4} \sqrt{-a^2x^4 + 1} x^2 + \frac{\arcsin\left(\frac{a^2x^2}{\sqrt{a^2}}\right)}{4\sqrt{a^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="maxima")`

[Out] $1/2 * x^2 / a + \text{integrate}(\text{sqrt}(a * x^2 + 1) * \text{sqrt}(-a * x^2 + 1) * x, x) / a$

Fricas [A] time = 2.05342, size = 225, normalized size = 3.57

$$\frac{a^2x^4 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} + 2ax^2 - 2 \arctan \left(\frac{ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} - 1}{ax^2} \right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="fricas")`

[Out] $\frac{1}{4}*(a^2*x^4*\sqrt{(a*x^2 + 1)/(a*x^2)}*\sqrt{-(a*x^2 - 1)/(a*x^2)} + 2*a*x^2 - 2*\arctan((a*x^2*\sqrt{(a*x^2 + 1)/(a*x^2)}*\sqrt{-(a*x^2 - 1)/(a*x^2)} - 1)/(a*x^2)))/a^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x dx + \int ax^3 \sqrt{-1 + \frac{1}{ax^2}} \sqrt{1 + \frac{1}{ax^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x**2+(1/a/x**2-1)**(1/2))*(1/a/x**2+1)**(1/2))*x**3,x)`

[Out] `(Integral(x, x) + Integral(a*x**3*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)), x))/a`

Giac [A] time = 1.16296, size = 107, normalized size = 1.7

$$\frac{2(a^2x^2 + a)a^2 + \left(\sqrt{a^2x^2 + a} \sqrt{-a^2x^2 + a} a^2x^2 - 2a^2 \arcsin\left(\frac{\sqrt{2}\sqrt{-a^2x^2 + a}}{2\sqrt{a}}\right) \right) a}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="giac")`

[Out] $\frac{1}{4}*(2*(a^2*x^2 + a)*a^2 + (\sqrt{a^2*x^2 + a}*\sqrt{-a^2*x^2 + a}*a^2*x^2 - 2*a^2*\arcsin(1/2*\sqrt{2}*\sqrt{-a^2*x^2 + a}/\sqrt{a}))*a)/a^5$

3.50 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^2 dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\operatorname{EllipticF}\left(\sin^{-1}(\sqrt{ax}), -1\right)}{3a^{3/2}} + \frac{1}{3}x^3e^{\operatorname{sech}^{-1}(ax^2)} + \frac{2x}{3a}$$

[Out] (2*x)/(3*a) + (E^ArcSech[a*x^2]*x^3)/3 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(3*a^(3/2))

Rubi [A] time = 0.0279784, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6335, 8, 248, 221}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\sin^{-1}(\sqrt{ax})\middle| -1\right)}{3a^{3/2}} + \frac{1}{3}x^3e^{\operatorname{sech}^{-1}(ax^2)} + \frac{2x}{3a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^2]*x^2,x]

[Out] (2*x)/(3*a) + (E^ArcSech[a*x^2]*x^3)/3 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(3*a^(3/2))

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 248

Int[((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0])

))

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^2)} x^2 dx &= \frac{1}{3} e^{\operatorname{sech}^{-1}(ax^2)} x^3 + \frac{2 \int 1 dx}{3a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{3a} \\ &= \frac{2x}{3a} + \frac{1}{3} e^{\operatorname{sech}^{-1}(ax^2)} x^3 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{\sqrt{1-a^2x^4}} dx}{3a} \\ &= \frac{2x}{3a} + \frac{1}{3} e^{\operatorname{sech}^{-1}(ax^2)} x^3 + \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} F\left(\sin^{-1}(\sqrt{ax}) \middle| -1\right)}{3a^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.189033, size = 116, normalized size = 1.73

$$-\frac{2i\sqrt{\frac{1-ax^2}{ax^2+1}}\sqrt{1-a^2x^4}\operatorname{EllipticF}\left(i\sinh^{-1}(\sqrt{-ax}), -1\right)}{3(-a)^{3/2}(ax^2-1)} + \frac{\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^3+x)}{3a} + \frac{x}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x^2, x]

[Out] x/a + (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x + a*x^3))/(3*a) - (((2*I)/3)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*EllipticF[I*ArcSinh[Sqrt[-a]*x], -1])/((-a)^(3/2)*(-1 + a*x^2))

Maple [A] time = 0.204, size = 102, normalized size = 1.5

$$\frac{x^2}{3a^2x^4-3}\sqrt{\frac{ax^2-1}{ax^2}}\sqrt{\frac{ax^2+1}{ax^2}}\left(x^5a^{\frac{5}{2}}-2\operatorname{EllipticF}\left(x\sqrt{a}, i\right)\sqrt{-ax^2+1}\sqrt{ax^2+1}-x\sqrt{a}\right)\frac{1}{\sqrt{a}}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x)`

[Out] `1/3*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(x^5*a^(5/2)-2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-x*a^(1/2))/(a^2*x^4-1)/a^(1/2)+x/a`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{a} + \frac{\int \sqrt{ax^2 + 1} \sqrt{-ax^2 + 1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="maxima")`

[Out] `x/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), x)/a`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{\frac{-ax^2-1}{ax^2}} + 1}{a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="fricas")`

[Out] `integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/a, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int 1 dx + \int ax^2 \sqrt{-1 + \frac{1}{ax^2}} \sqrt{1 + \frac{1}{ax^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**2,x)

[Out] (Integral(1, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\sqrt{\frac{1}{ax^2} + 1} \sqrt{\frac{1}{ax^2} - 1} + \frac{1}{ax^2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="giac")

[Out] integrate(x^2*(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2)), x)

3.51 $\int e^{\operatorname{sech}^{-1}(ax^2)} x dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a} + \frac{1}{2}x^2e^{\operatorname{sech}^{-1}(ax^2)} + \frac{\log(x)}{a}$$

[Out] (E^ArcSech[a*x^2]*x^2)/2 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcTanh[Sqrt[1 - a^2*x^4]])/(2*a) + Log[x]/a

Rubi [A] time = 0.0425703, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6335, 29, 259, 266, 63, 208}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a} + \frac{1}{2}x^2e^{\operatorname{sech}^{-1}(ax^2)} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^2]*x, x]

[Out] (E^ArcSech[a*x^2]*x^2)/2 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcTanh[Sqrt[1 - a^2*x^4]])/(2*a) + Log[x]/a

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^p)*((a2_.) + (b2_.)*(x_)^(n_.))^p, x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
```

$Q[\{a1, b1, a2, b2, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[a1, 0] \ \&\& \ \text{GtQ}[a2, 0]))$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \ :> \ \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] \ /; \ \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 63

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \ :> \ \text{With}[\{p = \text{Denominator}[m]\}, \ \text{Dist}[p/b, \ \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x}], x, (a + b*x)^{(1/p)}], x]] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \ \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned}
 \int e^{\text{sech}^{-1}(ax^2)} x \, dx &= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\int \frac{1}{x} dx}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{x\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{a} \\
 &= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{x\sqrt{1-a^2x^4}} dx}{a} \\
 &= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^4\right)}{4a} \\
 &= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} - \frac{\left(\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \text{Subst}\left(\int \frac{1}{\frac{1}{a^2} - \frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^4}\right)}{2a^3} \\
 &= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 - \frac{\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a} + \frac{\log(x)}{a}
 \end{aligned}$$

Mathematica [A] time = 0.0633233, size = 100, normalized size = 1.47

$$\frac{\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2+1) + 2\log(ax^2) - \log\left(ax^2\sqrt{\frac{1-ax^2}{ax^2+1}} + \sqrt{\frac{1-ax^2}{ax^2+1}} + 1\right)}{2a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x,x]

[Out] (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2) + 2*Log[a*x^2] - Log[1 + Sqrt[(1 - a*x^2)/(1 + a*x^2)]] + a*x^2*Sqrt[(1 - a*x^2)/(1 + a*x^2)])/(2*a)

Maple [C] time = 0.312, size = 127, normalized size = 1.9

$$\frac{x^2 \operatorname{csgn}(a^{-1}) \sqrt{-\frac{ax^2-1}{ax^2}} \sqrt{\frac{ax^2+1}{ax^2}} \left(\operatorname{csgn}(a^{-1}) a \sqrt{-\frac{a^2x^4-1}{a^2}} - \ln \left(2 \frac{1}{a^2x^2} \left(\operatorname{csgn}(a^{-1}) a \sqrt{-\frac{a^2x^4-1}{a^2}} + 1 \right) \right) \right)}{\sqrt{-\frac{a^2x^4-1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x,x)

[Out] 1/2*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(csgn(1/a)*a*(-(a^2*x^4-1)/a^2)^(1/2)-ln(2*(csgn(1/a)*a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2))*csgn(1/a)/a/(-(a^2*x^4-1)/a^2)^(1/2)+ln(x)/a

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{2} \sqrt{-a^2x^4+1} - \frac{1}{2} \log\left(\frac{2\sqrt{-a^2x^4+1}}{x^2} + \frac{2}{x^2}\right)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x,x, algorithm="maxima")

[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x, x)/a + log(x)/a

Fricas [B] time = 2.09933, size = 298, normalized size = 4.38

$$\frac{2ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{\frac{-ax^2-1}{ax^2}} - \log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{\frac{-ax^2-1}{ax^2}} + 1\right) + \log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{\frac{-ax^2-1}{ax^2}} - 1\right) + 4\log(x)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x,x, algorithm="fricas")

[Out] 1/4*(2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1) + log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1) + 4*log(x))/a

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x} dx + \int ax\sqrt{-1 + \frac{1}{ax^2}}\sqrt{1 + \frac{1}{ax^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2))*(1/a/x**2+1)**(1/2))*x,x)

[Out] (Integral(1/x, x) + Integral(a*x*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)), x))/a

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))*x,x, algorithm="giac")

[Out] Exception raised: TypeError

3.52 $\int e^{\operatorname{sech}^{-1}(ax^2)} dx$

Optimal. Leaf size=147

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\operatorname{EllipticF}\left(\sin^{-1}(\sqrt{ax}), -1\right)}{\sqrt{a}} - \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{ax} + xe^{\operatorname{sech}^{-1}(ax^2)} - \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E\left(\sin^{-1}(\sqrt{ax})\right)}{\sqrt{a}}$$

```
[Out] -2/(a*x) + E^ArcSech[a*x^2]*x - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(a*x) - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```

Rubi [A] time = 0.0680163, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6330, 30, 259, 325, 307, 221, 1199, 424}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{ax} + xe^{\operatorname{sech}^{-1}(ax^2)} + \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\sin^{-1}(\sqrt{ax})\right)}{\sqrt{a}} - \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E\left(\sin^{-1}(\sqrt{ax})\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2], x]
```

```
[Out] -2/(a*x) + E^ArcSech[a*x^2]*x - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(a*x) - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```

Rule 6330

```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] := Simp[x*E^ArcSech[a*x^p], x] + (Dist[p/a, Int[1/x^p, x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])]/a, Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x) /; FreeQ[{a, p}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax^2)} dx &= e^{\operatorname{sech}^{-1}(ax^2)} x + \frac{2 \int \frac{1}{x^2} dx}{a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{x^2 \sqrt{1-ax^2} \sqrt{1+ax^2}} dx}{a} \\
&= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)} x + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{x^2 \sqrt{1-a^2x^4}} dx}{a} \\
&= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)} x - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{ax} - \left(2a\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^2}{\sqrt{1-a^2x^4}} dx \\
&= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)} x - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{ax} + \left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{1}{\sqrt{1-a^2x^4}} dx - \left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^2}{\sqrt{1-a^2x^4}} dx \\
&= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)} x - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{ax} + \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} F(\sin^{-1}(\sqrt{ax})| -1)}{\sqrt{a}} - \left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^2}{\sqrt{1-a^2x^4}} dx \\
&= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)} x - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{ax} - \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} E(\sin^{-1}(\sqrt{ax})| -1)}{\sqrt{a}} + \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}}{\sqrt{a}}
\end{aligned}$$

Mathematica [C] time = 0.281666, size = 135, normalized size = 0.92

$$\frac{2i\sqrt{\frac{1-ax^2}{ax^2+1}} \sqrt{1-a^2x^4} \left(E\left(i \sinh^{-1}(\sqrt{-ax})\right) - 1\right) - \operatorname{EllipticF}\left(i \sinh^{-1}(\sqrt{-ax}), -1\right)}{\sqrt{-a}(ax^2 - 1)} + \sqrt{\frac{1-ax^2}{ax^2+1}} \left(-\frac{1}{ax} - x\right) - \frac{1}{ax}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2], x]

[Out] $-(1/(a*x)) + (-1/(a*x)) - x) * \operatorname{Sqrt}[(1 - a*x^2)/(1 + a*x^2)] - ((2*I) * \operatorname{Sqrt}[(1 - a*x^2)/(1 + a*x^2)] * \operatorname{Sqrt}[1 - a^2*x^4] * (\operatorname{EllipticE}[I * \operatorname{ArcSinh}[\operatorname{Sqrt}[-a]*x], -1] - \operatorname{EllipticF}[I * \operatorname{ArcSinh}[\operatorname{Sqrt}[-a]*x], -1])) / (\operatorname{Sqrt}[-a] * (-1 + a*x^2))$

Maple [A] time = 0.187, size = 132, normalized size = 0.9

$$-\frac{1}{ax} - \frac{x}{a^2x^4 - 1} \sqrt{-\frac{ax^2 - 1}{ax^2}} \sqrt{\frac{ax^2 + 1}{ax^2}} \left(a^2x^4 + 2\sqrt{-ax^2 + 1} \sqrt{ax^2 + 1} x \operatorname{EllipticF}(x\sqrt{a}, i) \sqrt{a} - 2\sqrt{-ax^2 + 1} \sqrt{ax^2 + 1} x \operatorname{EllipticE}(x\sqrt{a}, i)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x)`

[Out]
$$-1/a/x - (-a*x^2-1)/a/x^2)^{(1/2)} * x * ((a*x^2+1)/a/x^2)^{(1/2)} * (a^2*x^4+2*(-a*x^2+1)^{(1/2)}*(a*x^2+1)^{(1/2)} * x * \text{EllipticF}(x*a^{(1/2)}, I) * a^{(1/2)} - 2*(-a*x^2+1)^{(1/2)} * (a*x^2+1)^{(1/2)} * x * \text{EllipticE}(x*a^{(1/2)}, I) * a^{(1/2)} - 1) / (a^2*x^4-1)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{ax^2+1}\sqrt{-ax^2+1}}{x^2} dx}{a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^2, x)/a - 1/(a*x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1}{ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(a*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2} dx + \int a\sqrt{-1 + \frac{1}{ax^2}}\sqrt{1 + \frac{1}{ax^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2),x)`

[Out] `(Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)), x))/a`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1}{ax^2} + 1} \sqrt{\frac{1}{ax^2} - 1} + \frac{1}{ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2), x)`

$$3.53 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} dx$$

Optimal. Leaf size=80

$$-\frac{\sqrt{1-ax^2}}{2ax^2\sqrt{\frac{1}{ax^2+1}}} - \frac{1}{2ax^2} - \frac{1}{2}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sin^{-1}(ax^2)$$

[Out] -1/(2*a*x^2) - Sqrt[1 - a*x^2]/(2*a*x^2*Sqrt[(1 + a*x^2)^(-1)]) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcSin[a*x^2])/2

Rubi [A] time = 0.0530153, antiderivative size = 93, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6334, 259, 275, 277, 216}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{2ax^2} - \frac{1}{2ax^2} - \frac{1}{2}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sin^{-1}(ax^2)$$

Warning: Unable to verify antiderivative.

[In] Int[E^ArcSech[a*x^2]/x,x]

[Out] -1/(2*a*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(2*a*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcSin[a*x^2])/2

Rule 6334

Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] := -Simp[(a*p*x^p)^(-1), x] + Dist[(Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])]/a, Int[(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p])/x^(p + 1), x], x] /; FreeQ[{a, p}, x]

Rule 259

Int[((c_.)*(x_))^(m_.)*((a1_.) + (b1_.)*(x_)^(n_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^(n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} dx &= -\frac{1}{2ax^2} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{\sqrt{1-ax^2}\sqrt{1+ax^2}}{x^3} dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{\sqrt{1-a^2x^4}}{x^3} dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1-a^2x^2}}{x^2} dx, x, x^2\right)}{2a} \\
&= -\frac{1}{2ax^2} - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{2ax^2} - \frac{1}{2} \left(a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}} dx, x, x^2\right) \\
&= -\frac{1}{2ax^2} - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{2ax^2} - \frac{1}{2} \sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2} \sin^{-1}(ax^2)
\end{aligned}$$

Mathematica [A] time = 0.144055, size = 113, normalized size = 1.41

$$\frac{\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2+1)\tanh^{-1}\left(\frac{ax^2}{\sqrt{a^2x^4-1}}\right)}{2\sqrt{a^2x^4-1}} + \sqrt{\frac{1-ax^2}{ax^2+1}}\left(-\frac{1}{2ax^2} - \frac{1}{2}\right) - \frac{1}{2ax^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]/x,x]

[Out] $-\frac{1}{2\sqrt{2ax^2}} + \left(-\frac{1}{2} - \frac{1}{2\sqrt{2ax^2}}\right) \sqrt{\frac{1-ax^2}{1+ax^2}} + \left(\sqrt{\frac{1-ax^2}{1+ax^2}} * (1+ax^2) * \text{ArcTanh}\left[\frac{ax^2}{\sqrt{-1+a^2x^4}}\right]\right) / (2 * \sqrt{-1+a^2x^4})$

Maple [A] time = 0.284, size = 103, normalized size = 1.3

$$-\frac{1}{2} \sqrt{\frac{ax^2-1}{ax^2}} \sqrt{\frac{ax^2+1}{ax^2}} \left(\arctan\left(x^2 \frac{1}{\sqrt{-\frac{a^2x^4-1}{a^2}}}\right) x^2 + \sqrt{\frac{a^2x^4-1}{a^2}} \right) \frac{1}{\sqrt{-\frac{a^2x^4-1}{a^2}}} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x)

[Out] $-\frac{1}{2} * \left(-\frac{ax^2-1}{a/x^2}\right)^{(1/2)} * \left(\frac{ax^2+1}{a/x^2}\right)^{(1/2)} * \left(\arctan\left(\frac{x^2}{\left(-\frac{a^2x^4-1}{a^2}\right)^{(1/2)}}\right) * x^2 + \left(-\frac{a^2x^4-1}{a^2}\right)^{(1/2)}\right) / \left(-\frac{a^2x^4-1}{a^2}\right)^{(1/2)} - \frac{1}{2} / a/x^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{a^2 \arcsin\left(\frac{a^2x^2}{\sqrt{a^2}}\right)}{2\sqrt{a^2}} - \frac{\sqrt{-a^2x^4+1}}{2x^2}}{a} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="maxima")

[Out] integrate(sqrt(ax^2 + 1)*sqrt(-ax^2 + 1)/x^3, x)/a - 1/2/(ax^2)

Fricas [B] time = 2.0125, size = 230, normalized size = 2.88

$$\frac{ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{\frac{-ax^2-1}{ax^2}} - 2ax^2 \arctan\left(\frac{ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{\frac{-ax^2-1}{ax^2}} - 1}{ax^2}\right) + 1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="fricas")

[Out] -1/2*(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2*a*x^2*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1)/(a*x^2)) + 1)/(a*x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^3} dx + \int \frac{a \sqrt{-1 + \frac{1}{ax^2}} \sqrt{1 + \frac{1}{ax^2}}}{x} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x,x)

[Out] (Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2))/x, x))/a

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.54 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2} dx$$

Optimal. Leaf size=115

$$-\frac{2}{3}\sqrt{a}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\operatorname{EllipticF}\left(\sin^{-1}(\sqrt{ax}), -1\right) + \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{3ax^3} + \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}$$

[Out] 2/(3*a*x^3) - E^ArcSech[a*x^2]/x + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(3*a*x^3) - (2*Sqrt[a]*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/3

Rubi [A] time = 0.0510283, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6335, 30, 259, 325, 221}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{3ax^3} + \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} - \frac{2}{3}\sqrt{a}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\operatorname{F}\left(\sin^{-1}(\sqrt{ax})\middle| -1\right)$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^2]/x^2,x]

[Out] 2/(3*a*x^3) - E^ArcSech[a*x^2]/x + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(3*a*x^3) - (2*Sqrt[a]*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/3

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 259

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] := Int[(c*x)^(m*(a1*a2 + b1*b2*x^(2*n)))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} - \frac{2 \int \frac{1}{x^4} dx}{a} - \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{1}{x^4\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{a} \\ &= \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} - \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{1}{x^4\sqrt{1-a^2x^4}} dx}{a} \\ &= \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3} - \frac{1}{3} \left(2a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{1}{\sqrt{1-a^2x^4}} dx \\ &= \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3} - \frac{2}{3}\sqrt{a}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}F(\sin^{-1}(\sqrt{ax})|-1) \end{aligned}$$

Mathematica [C] time = 0.191668, size = 123, normalized size = 1.07

$$\frac{2i\sqrt{-a}\sqrt{\frac{1-ax^2}{ax^2+1}}\sqrt{1-a^2x^4}\operatorname{EllipticF}\left(i\sinh^{-1}(\sqrt{-ax}), -1\right)}{3ax^2-3} - \frac{\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2+1)}{3ax^3} - \frac{1}{3ax^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]/x^2,x]

[Out] $-\frac{1}{3ax^3} - \frac{\sqrt{(1-ax^2)/(1+ax^2)}(1+ax^2)}{3ax^3} + \frac{(2I)\sqrt{-a}\sqrt{(1-ax^2)/(1+ax^2)}\sqrt{1-a^2x^4}\text{EllipticF}[I\text{ArcSinh}[\sqrt{-a}x], -1]}{(-3+3ax^2)}$

Maple [A] time = 0.208, size = 104, normalized size = 0.9

$$\frac{1}{3x(a^2x^4-1)}\sqrt{\frac{ax^2-1}{ax^2}}\sqrt{\frac{ax^2+1}{ax^2}}\left(2\sqrt{-ax^2+1}\sqrt{ax^2+1}\text{EllipticF}(x\sqrt{a},i)x^3a^{3/2}-a^2x^4+1\right)-\frac{1}{3x^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x)

[Out] $\frac{1}{3}\left(-\frac{ax^2-1}{ax^2}\right)^{1/2}/x\left(\frac{ax^2+1}{ax^2}\right)^{1/2}\left(2\left(-\frac{ax^2+1}{ax^2}\right)^{1/2}\left(\frac{ax^2+1}{ax^2}\right)^{1/2}\text{EllipticF}\left(x\sqrt{a},I\right)+x^3a^{3/2}-a^2x^4+1\right)-\frac{1}{3x^3/a}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{ax^2+1}\sqrt{-ax^2+1}}{x^4} dx}{a} - \frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(ax^2 + 1)*sqrt(-ax^2 + 1)/x^4, x)/a - 1/3/(ax^3)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{\frac{ax^2-1}{ax^2}}+1}{ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="fricas")

[Out] integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(a*x^4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^4} dx + \int \frac{a\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}}{x^2} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**2,x)

[Out] (Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)))/x**2, x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax^2} + 1}\sqrt{\frac{1}{ax^2} - 1} + \frac{1}{ax^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2))/x^2, x)

$$3.55 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3} dx$$

Optimal. Leaf size=118

$$\frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sqrt{1-a^2x^4}}{4ax^4} + \frac{1}{4} a \sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \tanh^{-1}\left(\sqrt{1-a^2x^4}\right) + \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2}$$

[Out] $1/(4*a*x^4) - E^{\operatorname{ArcSech}[a*x^2]/(2*x^2)} + (\operatorname{Sqrt}[(1+a*x^2)^{-1}]*\operatorname{Sqrt}[1+a*x^2]*\operatorname{Sqrt}[1-a^2*x^4])/(4*a*x^4) + (a*\operatorname{Sqrt}[(1+a*x^2)^{-1}]*\operatorname{Sqrt}[1+a*x^2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-a^2*x^4]])/4$

Rubi [A] time = 0.0611727, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6335, 30, 259, 266, 51, 63, 208}

$$\frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sqrt{1-a^2x^4}}{4ax^4} + \frac{1}{4} a \sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \tanh^{-1}\left(\sqrt{1-a^2x^4}\right) + \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^2]}/x^3, x]$

[Out] $1/(4*a*x^4) - E^{\operatorname{ArcSech}[a*x^2]/(2*x^2)} + (\operatorname{Sqrt}[(1+a*x^2)^{-1}]*\operatorname{Sqrt}[1+a*x^2]*\operatorname{Sqrt}[1-a^2*x^4])/(4*a*x^4) + (a*\operatorname{Sqrt}[(1+a*x^2)^{-1}]*\operatorname{Sqrt}[1+a*x^2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-a^2*x^4]])/4$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_.)*(x_)^{(p_.)}]}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1/(1+a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1-a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p\}, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)}/(m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} - \frac{\int \frac{1}{x^5} dx}{a} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{1}{x^5\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{a} \\
&= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \int \frac{1}{x^5\sqrt{1-a^2x^4}} dx}{a} \\
&= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}} dx, x, x^4\right)}{4a} \\
&= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} - \frac{1}{8} \left(a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^4\right) \\
&= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2} - \frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^4}\right)}{4a} \\
&= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{1}{4} a \sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \tanh^{-1}\left(\sqrt{1-a^2x^4}\right)
\end{aligned}$$

Mathematica [A] time = 0.255935, size = 105, normalized size = 0.89

$$-\frac{a^2\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2+1)\tan^{-1}\left(\sqrt{a^2x^4-1}\right) + \frac{\sqrt{\frac{1-ax^2}{ax^2+1}}(ax^2+1)}{x^4} + \frac{1}{x^4}}{4a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]/x^3,x]

[Out] $-(x^{(-4)} + (\operatorname{Sqrt}[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2))/x^4 - (a^2*\operatorname{Sqrt}[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)*\operatorname{ArcTan}[\operatorname{Sqrt}[-1 + a^2*x^4]])/\operatorname{Sqrt}[-1 + a^2*x^4])/(4*a)$

Maple [C] time = 0.299, size = 129, normalized size = 1.1

$$\frac{\operatorname{csgn}(a^{-1})\sqrt{-\frac{ax^2-1}{ax^2}}\sqrt{\frac{ax^2+1}{ax^2}}\left(\ln\left(2\frac{1}{a^2x^2}\left(\operatorname{csgn}(a^{-1})a\sqrt{-\frac{a^2x^4-1}{a^2}}+1\right)\right)\right)x^4a - \sqrt{-\frac{a^2x^4-1}{a^2}}\operatorname{csgn}(a^{-1})}{\sqrt{-\frac{a^2x^4-1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))/x^3,x)

[Out] 1/4*(-(a*x^2-1)/a/x^2)^(1/2)/x^2*((a*x^2+1)/a/x^2)^(1/2)*(ln(2*(csgn(1/a))*
*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2)*x^4*a-(-(a^2*x^4-1)/a^2)^(1/2)*csgn(1
/a))*csgn(1/a)/(-(a^2*x^4-1)/a^2)^(1/2)-1/4/x^4/a

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{4} a^2 \log\left(\frac{2\sqrt{-a^2x^4+1}}{x^2} + \frac{2}{x^2}\right) - \frac{1}{4} \sqrt{-a^2x^4+1} a^2 - \frac{(-a^2x^4+1)^{\frac{3}{2}}}{4x^4}}{a} - \frac{1}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^5, x)/a - 1/4/(a*x^4)

Fricas [A] time = 2.03432, size = 319, normalized size = 2.7

$$\frac{a^2x^4 \log\left(ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} + 1\right) - a^2x^4 \log\left(ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} - 1\right) - 2ax^2 \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} - 2}{8ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2))*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/8*(a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2))
+ 1) - a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2
2)) - 1) - 2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2
)/(a*x^4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^5} dx + \int \frac{a \sqrt{-1 + \frac{1}{ax^2}} \sqrt{1 + \frac{1}{ax^2}}}{x^3} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**3,x)

[Out] (Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2))/x**3, x))/a

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError

3.56 $\int e^{\operatorname{sech}^{-1}(ax^3)} x^m dx$

Optimal. Leaf size=109

$$\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}x^{m-2}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m-2}{6}, \frac{m+4}{6}, a^2x^6\right)}{a(-m^2+m+2)} - \frac{3x^{m-2}}{a(-m^2+m+2)} + \frac{x^{m+1}e^{\operatorname{sech}^{-1}(ax^3)}}{m+1}$$

[Out] $(-3*x^{(-2+m)})/(a*(2+m-m^2)) + (E^{\operatorname{ArcSech}[a*x^3]}*x^{(1+m)})/(1+m) - (3*x^{(-2+m)}*\operatorname{Sqrt}[(1+a*x^3)^{-1}]*\operatorname{Sqrt}[1+a*x^3]*\operatorname{Hypergeometric2F1}[1/2, (-2+m)/6, (4+m)/6, a^2*x^6])/(a*(2+m-m^2))$

Rubi [A] time = 0.0677183, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6335, 30, 259, 364}

$$\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}x^{m-2}{}_2F_1\left(\frac{1}{2}, \frac{m-2}{6}; \frac{m+4}{6}; a^2x^6\right)}{a(-m^2+m+2)} - \frac{3x^{m-2}}{a(-m^2+m+2)} + \frac{x^{m+1}e^{\operatorname{sech}^{-1}(ax^3)}}{m+1}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^3]}*x^m, x]$

[Out] $(-3*x^{(-2+m)})/(a*(2+m-m^2)) + (E^{\operatorname{ArcSech}[a*x^3]}*x^{(1+m)})/(1+m) - (3*x^{(-2+m)}*\operatorname{Sqrt}[(1+a*x^3)^{-1}]*\operatorname{Sqrt}[1+a*x^3]*\operatorname{Hypergeometric2F1}[1/2, (-2+m)/6, (4+m)/6, a^2*x^6])/(a*(2+m-m^2))$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[a_*](x_*)^{(p_*)}]}*(x_*)^{(m_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1/(1+a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1-a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p\}, x \&\& \operatorname{NeQ}[m, -1]$

Rule 30

$\operatorname{Int}[(x_*)^{(m_*)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)}/(m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
]])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^3)} x^m dx &= \frac{e^{\operatorname{sech}^{-1}(ax^3)} x^{1+m}}{1+m} + \frac{3 \int x^{-3+m} dx}{a(1+m)} + \frac{\left(3\sqrt{\frac{1}{1+ax^3}} \sqrt{1+ax^3}\right) \int \frac{x^{-3+m}}{\sqrt{1-ax^3}\sqrt{1+ax^3}} dx}{a(1+m)} \\ &= -\frac{3x^{-2+m}}{a(2+m-m^2)} + \frac{e^{\operatorname{sech}^{-1}(ax^3)} x^{1+m}}{1+m} + \frac{\left(3\sqrt{\frac{1}{1+ax^3}} \sqrt{1+ax^3}\right) \int \frac{x^{-3+m}}{\sqrt{1-a^2x^6}} dx}{a(1+m)} \\ &= -\frac{3x^{-2+m}}{a(2+m-m^2)} + \frac{e^{\operatorname{sech}^{-1}(ax^3)} x^{1+m}}{1+m} - \frac{3x^{-2+m} \sqrt{\frac{1}{1+ax^3}} \sqrt{1+ax^3} {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-2+m); \frac{4+m}{6}; a^2x^6\right)}{a(2+m-m^2)} \end{aligned}$$

Mathematica [A] time = 2.49469, size = 159, normalized size = 1.46

$$\frac{2^{\frac{m+1}{3}} x^{m+1} (ax^3)^{\frac{1}{3}(-m-1)} e^{\operatorname{sech}^{-1}(ax^3)} \left(\frac{e^{\operatorname{sech}^{-1}(ax^3)}}{e^{2\operatorname{sech}^{-1}(ax^3)}+1}\right)^{\frac{m+1}{3}} \left((m+10)\operatorname{Hypergeometric2F1}\left(1, \frac{2-m}{6}, \frac{m+10}{6}, -e^{2\operatorname{sech}^{-1}(ax^3)}\right) - (m+4)(m+10)\right)}{(m+4)(m+10)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^3]*x^m,x]

[Out] (2^((1 + m)/3)*E^ArcSech[a*x^3]*(E^ArcSech[a*x^3]/(1 + E^(2*ArcSech[a*x^3])))^((1 + m)/3)*x^(1 + m)*(a*x^3)^((-1 - m)/3)*((10 + m)*Hypergeometric2F1[1, (2 - m)/6, (10 + m)/6, -E^(2*ArcSech[a*x^3])] - E^(2*ArcSech[a*x^3])*(4 + m)*Hypergeometric2F1[1, (8 - m)/6, (16 + m)/6, -E^(2*ArcSech[a*x^3])])/(

$4 + m) * (10 + m)$

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \left(\frac{1}{x^{3a}} + \sqrt{\frac{1}{x^{3a}} - 1} \sqrt{\frac{1}{x^{3a}} + 1} \right) x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^3/a+(1/x^3/a-1)^(1/2)*(1/x^3/a+1)^(1/2))*x^m,x)

[Out] int((1/x^3/a+(1/x^3/a-1)^(1/2)*(1/x^3/a+1)^(1/2))*x^m,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{ax^3 x^m \sqrt{\frac{ax^3+1}{ax^3}} \sqrt{-\frac{ax^3-1}{ax^3}} + x^m}{ax^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="fricas")

[Out] integral((a*x^3*x^m*sqrt((a*x^3 + 1)/(a*x^3))*sqrt(-(a*x^3 - 1)/(a*x^3)) + x^m)/(a*x^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**3+(1/a/x**3-1)**(1/2)*(1/a/x**3+1)**(1/2))*x**m,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\sqrt{\frac{1}{ax^3} + 1} \sqrt{\frac{1}{ax^3} - 1} + \frac{1}{ax^3} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="giac")

[Out] integrate(x^m*(sqrt(1/(a*x^3) + 1)*sqrt(1/(a*x^3) - 1) + 1/(a*x^3)), x)

3.57 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^m dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}x^{m-1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m-1}{4}, \frac{m+3}{4}, a^2x^4\right)}{a(1-m^2)} - \frac{2x^{m-1}}{a(1-m^2)} + \frac{x^{m+1}e^{\operatorname{sech}^{-1}(ax^2)}}{m+1}$$

[Out] $(-2*x^{(-1+m)})/(a*(1-m^2)) + (E^{\operatorname{ArcSech}[a*x^2]}*x^{(1+m)})/(1+m) - (2*x^{(-1+m)}*\sqrt{(1+a*x^2)^{-1}}*\sqrt{1+a*x^2}*\operatorname{Hypergeometric2F1}[1/2, (-1+m)/4, (3+m)/4, a^2*x^4])/(a*(1-m^2))$

Rubi [A] time = 0.0557968, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6335, 30, 259, 364}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}x^{m-1}{}_2F_1\left(\frac{1}{2}, \frac{m-1}{4}; \frac{m+3}{4}; a^2x^4\right)}{a(1-m^2)} - \frac{2x^{m-1}}{a(1-m^2)} + \frac{x^{m+1}e^{\operatorname{sech}^{-1}(ax^2)}}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^2]*x^m,x]

[Out] $(-2*x^{(-1+m)})/(a*(1-m^2)) + (E^{\operatorname{ArcSech}[a*x^2]}*x^{(1+m)})/(1+m) - (2*x^{(-1+m)}*\sqrt{(1+a*x^2)^{-1}}*\sqrt{1+a*x^2}*\operatorname{Hypergeometric2F1}[1/2, (-1+m)/4, (3+m)/4, a^2*x^4])/(a*(1-m^2))$

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m+1)*E^ArcSech[a*x^p])/(m+1), x] + (Dist[p/(a*(m+1)), Int[x^(m-p), x], x] + Dist[(p*Sqrt[1+a*x^p]*Sqrt[1/(1+a*x^p)])/(a*(m+1)), Int[x^(m-p)/(Sqrt[1+a*x^p]*Sqrt[1-a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 259

Int[((c_)*(x_))^(m_)*((a1_)+(b1_)*(x_)^(n_))^(p_)*((a2_)+(b2_)*(x_)^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2+b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1+a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 364

Int[((c_)*(x_))^(m_)*((a_)+(b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^2)} x^m dx &= \frac{e^{\operatorname{sech}^{-1}(ax^2)} x^{1+m}}{1+m} + \frac{2 \int x^{-2+m} dx}{a(1+m)} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^{-2+m}}{\sqrt{1-ax^2}\sqrt{1+ax^2}} dx}{a(1+m)} \\ &= -\frac{2x^{-1+m}}{a(1-m^2)} + \frac{e^{\operatorname{sech}^{-1}(ax^2)} x^{1+m}}{1+m} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2}\right) \int \frac{x^{-2+m}}{\sqrt{1-a^2x^4}} dx}{a(1+m)} \\ &= -\frac{2x^{-1+m}}{a(1-m^2)} + \frac{e^{\operatorname{sech}^{-1}(ax^2)} x^{1+m}}{1+m} - \frac{2x^{-1+m} \sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-1+m); \frac{3+m}{4}; a^2x^4\right)}{a(1-m^2)} \end{aligned}$$

Mathematica [A] time = 2.30329, size = 159, normalized size = 1.49

$$\frac{2^{\frac{m+1}{2}} x^{m+1} (ax^2)^{\frac{1}{2}(-m-1)} e^{\operatorname{sech}^{-1}(ax^2)} \left(\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{e^{2\operatorname{sech}^{-1}(ax^2)}+1}\right)^{\frac{m+1}{2}} \left((m+7)\operatorname{Hypergeometric2F1}\left(1, \frac{1-m}{4}, \frac{m+7}{4}, -e^{2\operatorname{sech}^{-1}(ax^2)}\right) - (m+3)\right)}{(m+3)(m+7)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^2]*x^m, x]

[Out] (2^((1+m)/2)*E^ArcSech[a*x^2]*(E^ArcSech[a*x^2]/(1+E^(2*ArcSech[a*x^2])))^((1+m)/2)*x^(1+m)*(a*x^2)^((-1-m)/2)*((7+m)*Hypergeometric2F1[1, (1-m)/4, (7+m)/4, -E^(2*ArcSech[a*x^2])]) - E^(2*ArcSech[a*x^2])*(3+m)*Hypergeometric2F1[1, (5-m)/4, (11+m)/4, -E^(2*ArcSech[a*x^2])])/(3

+ m)*(7 + m))

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \left(\frac{1}{ax^2} + \sqrt{\frac{1}{ax^2} - 1} \sqrt{\frac{1}{ax^2} + 1} \right) x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)

[Out] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{ax^2 x^m \sqrt{\frac{ax^2+1}{ax^2}} \sqrt{-\frac{ax^2-1}{ax^2}} + x^m}{ax^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="fricas")

[Out] integral((a*x^2*x^m*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + x^m)/(a*x^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**m,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\sqrt{\frac{1}{ax^2} + 1} \sqrt{\frac{1}{ax^2} - 1} + \frac{1}{ax^2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="giac")

[Out] integrate(x^m*(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2)), x)

3.58 $\int e^{\operatorname{sech}^{-1}(ax)} x^m dx$

Optimal. Leaf size=91

$$\frac{\sqrt{\frac{1}{ax+1}} \sqrt{ax+1} x^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{m+2}{2}, a^2 x^2\right)}{am(m+1)} + \frac{x^m}{am(m+1)} + \frac{x^{m+1} e^{\operatorname{sech}^{-1}(ax)}}{m+1}$$

[Out] $x^m/(a*m*(1+m)) + (E^{\operatorname{ArcSech}[a*x]}*x^{(1+m)})/(1+m) + (x^m*\operatorname{Sqrt}[(1+a*x)^{-1}]*\operatorname{Sqrt}[1+a*x]*\operatorname{Hypergeometric2F1}[1/2, m/2, (2+m)/2, a^2*x^2])/(a*m*(1+m))$

Rubi [A] time = 0.0395333, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6335, 30, 125, 364}

$$\frac{\sqrt{\frac{1}{ax+1}} \sqrt{ax+1} x^m {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{m+2}{2}; a^2 x^2\right)}{am(m+1)} + \frac{x^m}{am(m+1)} + \frac{x^{m+1} e^{\operatorname{sech}^{-1}(ax)}}{m+1}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x]}*x^m, x]$

[Out] $x^m/(a*m*(1+m)) + (E^{\operatorname{ArcSech}[a*x]}*x^{(1+m)})/(1+m) + (x^m*\operatorname{Sqrt}[(1+a*x)^{-1}]*\operatorname{Sqrt}[1+a*x]*\operatorname{Hypergeometric2F1}[1/2, m/2, (2+m)/2, a^2*x^2])/(a*m*(1+m))$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_)]*(x_)^{(p_)}]*(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1/(1+a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1-a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p\}, x \&\& \operatorname{NeQ}[m, -1]$

Rule 30

$\operatorname{Int}[(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)}/(m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 125

```
Int[((f_)*(x_))^(p_)*((a_)+(b_)*(x_))^(m_)*((c_)+(d_)*(x_))^(n_
), x_Symbol] := Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_)*(x_))^(m_)*((a_)+(b_)*(x_))^(n_)*((p_)), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax)} x^m dx &= \frac{e^{\operatorname{sech}^{-1}(ax)} x^{1+m}}{1+m} + \frac{\int x^{-1+m} dx}{a(1+m)} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x^{-1+m}}{\sqrt{1-ax}\sqrt{1+ax}} dx}{a(1+m)} \\ &= \frac{x^m}{am(1+m)} + \frac{e^{\operatorname{sech}^{-1}(ax)} x^{1+m}}{1+m} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x^{-1+m}}{\sqrt{1-a^2x^2}} dx}{a(1+m)} \\ &= \frac{x^m}{am(1+m)} + \frac{e^{\operatorname{sech}^{-1}(ax)} x^{1+m}}{1+m} + \frac{x^m \sqrt{\frac{1}{1+ax}} \sqrt{1+ax} {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{2+m}{2}; a^2x^2\right)}{am(1+m)} \end{aligned}$$

Mathematica [A] time = 0.302242, size = 145, normalized size = 1.59

$$\frac{2^{m+1} x^m (ax)^{-m} e^{2 \operatorname{sech}^{-1}(ax)} \left(\frac{e^{\operatorname{sech}^{-1}(ax)}}{e^{2 \operatorname{sech}^{-1}(ax)} + 1}\right)^m \left(e^{2 \operatorname{sech}^{-1}(ax)} + 1\right)^m \left((m+2) e^{2 \operatorname{sech}^{-1}(ax)} \operatorname{Hypergeometric2F1}\left(\frac{m}{2} + 2, m + 2, \frac{m}{2} + 2, -E^{2 \operatorname{sech}^{-1}(ax)}\right) + E^{2 \operatorname{sech}^{-1}(ax)} (2 + m) \operatorname{Hypergeometric2F1}\left[2 + \frac{m}{2}, 2 + m, 3 + \frac{m}{2}, -E^{2 \operatorname{sech}^{-1}(ax)}\right]\right)}{a(m+2)(m+4)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x]*x^m, x]

[Out] -((2^(1 + m)*E^(2*ArcSech[a*x])*(E^ArcSech[a*x]/(1 + E^(2*ArcSech[a*x])))^m*(1 + E^(2*ArcSech[a*x]))^m*x^m*(-((4 + m)*Hypergeometric2F1[1 + m/2, 2 + m, 2 + m/2, -E^(2*ArcSech[a*x])]) + E^(2*ArcSech[a*x])*(2 + m)*Hypergeometric2F1[2 + m/2, 2 + m, 3 + m/2, -E^(2*ArcSech[a*x])])))/(a*(2 + m)*(4 + m)*(a*x)^m)

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right) x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)

[Out] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{axx^m \sqrt{\frac{ax+1}{ax}} \sqrt{\frac{-ax-1}{ax}} + x^m}{ax}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="fricas")

[Out] integral((a*x*x^m*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + x^m)/(a*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^m}{x} dx + \int ax^m \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**m,x)

[Out] (Integral(x**m/x, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="giac")

[Out] integrate(x^m*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

3.59 $\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m dx$

Optimal. Leaf size=109

$$\frac{\sqrt{\frac{1}{\frac{a}{x}+1}} \sqrt{\frac{a}{x}+1} x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-m-2), -\frac{m}{2}, \frac{a^2}{x^2}\right)}{a(m^2+3m+2)} - \frac{x^{m+2}}{a(m^2+3m+2)} + \frac{x^{m+1} e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

[Out] (E^ArcSech[a/x]*x^(1+m))/(1+m) - x^(2+m)/(a*(2+3*m+m^2)) - (Sqrt[(1+a/x)^(-1)]*Sqrt[1+a/x]*x^(2+m)*Hypergeometric2F1[1/2, (-2-m)/2, -m/2, a^2/x^2])/(a*(2+3*m+m^2))

Rubi [A] time = 0.0751257, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6335, 30, 259, 339, 364}

$$\frac{\sqrt{\frac{1}{\frac{a}{x}+1}} \sqrt{\frac{a}{x}+1} x^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-m-2); -\frac{m}{2}; \frac{a^2}{x^2}\right)}{a(m^2+3m+2)} - \frac{x^{m+2}}{a(m^2+3m+2)} + \frac{x^{m+1} e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a/x]*x^m, x]

[Out] (E^ArcSech[a/x]*x^(1+m))/(1+m) - x^(2+m)/(a*(2+3*m+m^2)) - (Sqrt[(1+a/x)^(-1)]*Sqrt[1+a/x]*x^(2+m)*Hypergeometric2F1[1/2, (-2-m)/2, -m/2, a^2/x^2])/(a*(2+3*m+m^2))

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m+1)*E^ArcSech[a*x^p])/(m+1), x] + (Dist[p/(a*(m+1)), Int[x^(m-p), x], x] + Dist[(p*Sqrt[1+a*x^p]*Sqrt[1/(1+a*x^p)])/(a*(m+1)), Int[x^(m-p)/(Sqrt[1+a*x^p]*Sqrt[1-a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 259

Int[((c_)*(x_))^(m_)*((a1_) + (b1_)*(x_)^(n_))^(p_)*((a2_) + (b2_)*(x_)^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 339

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c*x)^(m+1)*(1/x)^(m+1))/c, Subst[Int[(a + b/x^n)^p/x^(m+2), x], x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/ (c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
 \int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m dx &= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{\int x^{1+m} dx}{a(1+m)} - \frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\right) \int \frac{x^{1+m}}{\sqrt{1-\frac{a}{x}}\sqrt{1+\frac{a}{x}}} dx}{a(1+m)} \\
 &= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a(2+3m+m^2)} - \frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\right) \int \frac{x^{1+m}}{\sqrt{1-\frac{a^2}{x^2}}} dx}{a(1+m)} \\
 &= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a(2+3m+m^2)} + \frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\left(\frac{1}{x}\right)^m x^m\right) \operatorname{Subst}\left(\int \frac{x^{-3-m}}{\sqrt{1-a^2x^2}} dx, x, \frac{1}{x}\right)}{a(1+m)} \\
 &= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a(2+3m+m^2)} - \frac{\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}} x^{2+m} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-2-m); -\frac{m}{2}; \frac{a^2}{x^2}\right)}{a(2+3m+m^2)}
 \end{aligned}$$

Mathematica [A] time = 0.825083, size = 139, normalized size = 1.28

$$\frac{a 2^{-m-1} x^m \left(\frac{a}{x}\right)^m e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} \left(\frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}}{e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}+1}\right)^{-m-1} \left(m e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} \operatorname{Hypergeometric2F1}\left(1, \frac{m}{2}+2, 2-\frac{m}{2}, -e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}\right) - (m-2)\right)}{(m-2)m}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a/x]*x^m,x]

[Out] $-\left(\left(2^{-1-m} a E^{\operatorname{ArcSech}[a/x]} \left(E^{\operatorname{ArcSech}[a/x]} / \left(1 + E^{2 \operatorname{ArcSech}[a/x]}\right)\right)\right)^{-1-m} (a/x)^m x^m \left(-\left(-2+m\right) \operatorname{Hypergeometric2F1}\left[1, 1+m/2, 1-m/2, -E^{2 \operatorname{ArcSech}[a/x]}\right]\right) + E^{2 \operatorname{ArcSech}[a/x]} m \operatorname{Hypergeometric2F1}\left[1, 2+m/2, 2-m/2, -E^{2 \operatorname{ArcSech}[a/x]}\right]\right) / \left(-2+m\right) m$

Maple [F] time = 0.643, size = 0, normalized size = 0.

$$\int \left(\frac{x}{a} + \sqrt{-1 + \frac{x}{a}} \sqrt{1 + \frac{x}{a}} \right) x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x)

[Out] int((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2 x^m}{a(m+2)} + \frac{\int \sqrt{a+x} \sqrt{-a+x} x^m dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="maxima")

[Out] x^2*x^m/(a*(m+2)) + integrate(sqrt(a+x)*sqrt(-a+x)*x^m,x)/a

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{a x^m \sqrt{\frac{a+x}{a}} \sqrt{\frac{-a-x}{a}} + x x^m}{a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="fricas")

[Out] integral((a*x^m*sqrt((a + x)/a)*sqrt(-(a - x)/a) + x*x^m)/a, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int xx^m dx + \int ax^m \sqrt{-1 + \frac{x}{a}} \sqrt{1 + \frac{x}{a}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/a+(-1+x/a)**(1/2)*(1+x/a)**(1/2))*x**m,x)

[Out] (Integral(x*x**m, x) + Integral(a*x**m*sqrt(-1 + x/a)*sqrt(1 + x/a), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\sqrt{\frac{x}{a} + 1} \sqrt{\frac{x}{a} - 1 + \frac{x}{a}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="giac")

[Out] integrate(x^m*(sqrt(x/a + 1)*sqrt(x/a - 1) + x/a), x)

3.60 $\int e^{\operatorname{sech}^{-1}(ax^p)} x^m dx$

Optimal. Leaf size=133

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}x^{m-p+1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m-p+1}{2p}, \frac{m+p+1}{2p}, a^2x^{2p}\right)}{a(m+1)(m-p+1)} + \frac{px^{m-p+1}}{a(m+1)(m-p+1)} + \frac{x^{m+1}e^{\operatorname{sech}^{-1}(ax^p)}}{m+1}$$

[Out] (E^ArcSech[a*x^p]*x^(1+m))/(1+m) + (p*x^(1+m-p))/(a*(1+m)*(1+m-p)) + (p*x^(1+m-p)*Sqrt[(1+a*x^p)^(-1)]*Sqrt[1+a*x^p]*Hypergeometric2F1[1/2, (1+m-p)/(2*p), (1+m+p)/(2*p), a^2*x^(2*p)])/(a*(1+m)*(1+m-p))

Rubi [A] time = 0.0834416, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6335, 30, 259, 364}

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}x^{m-p+1}{}_2F_1\left(\frac{1}{2}, \frac{m-p+1}{2p}; \frac{m+p+1}{2p}; a^2x^{2p}\right)}{a(m+1)(m-p+1)} + \frac{px^{m-p+1}}{a(m+1)(m-p+1)} + \frac{x^{m+1}e^{\operatorname{sech}^{-1}(ax^p)}}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^p]*x^m, x]

[Out] (E^ArcSech[a*x^p]*x^(1+m))/(1+m) + (p*x^(1+m-p))/(a*(1+m)*(1+m-p)) + (p*x^(1+m-p)*Sqrt[(1+a*x^p)^(-1)]*Sqrt[1+a*x^p]*Hypergeometric2F1[1/2, (1+m-p)/(2*p), (1+m+p)/(2*p), a^2*x^(2*p)])/(a*(1+m)*(1+m-p))

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m+1)*E^ArcSech[a*x^p])/(m+1), x] + (Dist[p/(a*(m+1)), Int[x^(m-p), x], x] + Dist[(p*Sqrt[1+a*x^p]*Sqrt[1/(1+a*x^p)])/(a*(m+1)), Int[x^(m-p)/(Sqrt[1+a*x^p]*Sqrt[1-a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^p)} x^m dx &= \frac{e^{\operatorname{sech}^{-1}(ax^p)} x^{1+m}}{1+m} + \frac{p \int x^{m-p} dx}{a(1+m)} + \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{m-p}}{\sqrt{1-ax^p} \sqrt{1+ax^p}} dx}{a(1+m)} \\ &= \frac{e^{\operatorname{sech}^{-1}(ax^p)} x^{1+m}}{1+m} + \frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{m-p}}{\sqrt{1-a^2x^{2p}}} dx}{a(1+m)} \\ &= \frac{e^{\operatorname{sech}^{-1}(ax^p)} x^{1+m}}{1+m} + \frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{px^{1+m-p} \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p} {}_2F_1\left(\frac{1}{2}, \frac{1+m-p}{2p}; \frac{1+m+p}{2p}; a^2x^{2p}\right)}{a(1+m)(1+m-p)} \end{aligned}$$

Mathematica [A] time = 4.95324, size = 186, normalized size = 1.4

$$\frac{2^{\frac{m+1}{p}} x^{m+1} (ax^p)^{-\frac{m+1}{p}} e^{\operatorname{sech}^{-1}(ax^p)} \left(\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{e^{2\operatorname{sech}^{-1}(ax^p)} + 1}\right)^{\frac{m+1}{p}} \left((m+3p+1) \operatorname{Hypergeometric2F1}\left(1, 1 - \frac{m+p+1}{2p}, \frac{m+3p+1}{2p}, -e^{2\operatorname{sech}^{-1}(ax^p)}\right) + (m+p+1)(m+3p+1)\right)}{(m+p+1)(m+3p+1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^p]*x^m,x]

[Out] (2^((1+m)/p)*E^ArcSech[a*x^p]*(E^ArcSech[a*x^p]/(1+E^(2*ArcSech[a*x^p])))^((1+m)/p)*x^(1+m)*(-(E^(2*ArcSech[a*x^p]))*(1+m+p)*Hypergeometric2F1[1, -(1+m-3*p)/(2*p), (1+m+5*p)/(2*p), -E^(2*ArcSech[a*x^p])]) + (1+m+3*p)*Hypergeometric2F1[1, 1-(1+m+p)/(2*p), (1+m+3*p)/(2*p), -E^(2*ArcSech[a*x^p])])/(1+m+p)*(1+m+3*p)*(a*x^p)^((1+m)/p)

))

Maple [F] time = 1.224, size = 0, normalized size = 0.

$$\int \left(\frac{1}{ax^p} + \sqrt{\frac{1}{ax^p} - 1} \sqrt{\frac{1}{ax^p} + 1} \right) x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)

[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x^m x^{-p} dx + \int ax^m \sqrt{-1 + \frac{x^{-p}}{a}} \sqrt{1 + \frac{x^{-p}}{a}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x**m,x)

[Out] (Integral(x**m*x**(-p), x) + Integral(a*x**m*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p)/a), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\sqrt{\frac{1}{ax^p} + 1} \sqrt{\frac{1}{ax^p} - 1} + \frac{1}{ax^p} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algorithm="giac")

[Out] integrate(x^m*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)

3.61 $\int e^{\operatorname{sech}^{-1}(ax^p)} x dx$

Optimal. Leaf size=119

$$\frac{px^{2-p} \sqrt{\frac{1}{ax^p+1}} \sqrt{ax^p+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(\frac{2}{p}-1\right), \frac{1}{2}\left(\frac{2}{p}+1\right), a^2x^{2p}\right)}{2a(2-p)} + \frac{px^{2-p}}{2a(2-p)} + \frac{1}{2}x^2 e^{\operatorname{sech}^{-1}(ax^p)}$$

[Out] (E^ArcSech[a*x^p]*x^2)/2 + (p*x^(2 - p))/(2*a*(2 - p)) + (p*x^(2 - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + 2/p)/2, (1 + 2/p)/2, a^2*x^(2*p)])/(2*a*(2 - p))

Rubi [A] time = 0.0528343, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6335, 30, 259, 364}

$$\frac{px^{2-p} \sqrt{\frac{1}{ax^p+1}} \sqrt{ax^p+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}\left(\frac{2}{p}-1\right); \frac{1}{2}\left(1 + \frac{2}{p}\right); a^2x^{2p}\right)}{2a(2-p)} + \frac{px^{2-p}}{2a(2-p)} + \frac{1}{2}x^2 e^{\operatorname{sech}^{-1}(ax^p)}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^p]*x, x]

[Out] (E^ArcSech[a*x^p]*x^2)/2 + (p*x^(2 - p))/(2*a*(2 - p)) + (p*x^(2 - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + 2/p)/2, (1 + 2/p)/2, a^2*x^(2*p)])/(2*a*(2 - p))

Rule 6335

Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*E^ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 259

```
Int[((c_)*(x_))^(m_)*((a1_)+(b1_)*(x_)^(n_))^(p_)*((a2_)+(b2_)*(x_)^(n_))^(p_), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_)*(x_))^(m_)*((a_)+(b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^p)} x \, dx &= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^p)} x^2 + \frac{p \int x^{1-p} \, dx}{2a} + \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{1-p}}{\sqrt{1-ax^p} \sqrt{1+ax^p}} \, dx}{2a} \\ &= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^p)} x^2 + \frac{px^{2-p}}{2a(2-p)} + \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{1-p}}{\sqrt{1-a^2x^{2p}}} \, dx}{2a} \\ &= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^p)} x^2 + \frac{px^{2-p}}{2a(2-p)} + \frac{px^{2-p} \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} \left(-1 + \frac{2}{p}\right); \frac{1}{2} \left(1 + \frac{2}{p}\right); a^2x^{2p}\right)}{2a(2-p)} \end{aligned}$$

Mathematica [A] time = 0.29688, size = 159, normalized size = 1.34

$$\frac{x^{2-p} \left(\frac{a^2 p x^{2p} \sqrt{\frac{1-ax^p}{ax^p+1}} \sqrt{1-a^2x^{2p}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{p} + \frac{1}{2}, \frac{1}{p} + \frac{3}{2}, a^2x^{2p}\right)}{(p+2)(ax^p-1)} - ax^p \sqrt{\frac{1-ax^p}{ax^p+1}} - \sqrt{\frac{1-ax^p}{ax^p+1}} - 1 \right)}{a(p-2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^p]*x,x]

[Out] (x^(2-p)*(-1 - Sqrt[(1 - a*x^p)/(1 + a*x^p)] - a*x^p*Sqrt[(1 - a*x^p)/(1 + a*x^p)] + (a^2*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p)])*Hypergeometric2F1[1/2, 1/2 + p^(-1), 3/2 + p^(-1), a^2*x^(2*p)])/((2 + p)*(-1 + a*x^p)))/(a*(-2 + p))

Maple [F] time = 1.102, size = 0, normalized size = 0.

$$\int \left(\frac{1}{ax^p} + \sqrt{\frac{1}{ax^p} - 1} \sqrt{\frac{1}{ax^p} + 1} \right) x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)

[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int xx^{-p} dx + \int ax\sqrt{-1 + \frac{x^{-p}}{a}}\sqrt{1 + \frac{x^{-p}}{a}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x,x)

[Out] (Integral(x*x**(-p), x) + Integral(a*x*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p)/a), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\sqrt{\frac{1}{ax^p} + 1} \sqrt{\frac{1}{ax^p} - 1} + \frac{1}{ax^p} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorithm="giac")

[Out] integrate(x*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)

3.62 $\int e^{\operatorname{sech}^{-1}(ax^p)} dx$

Optimal. Leaf size=105

$$\frac{px^{1-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(\frac{1}{p}-1\right), \frac{p+1}{2p}, a^2x^{2p}\right)}{a(1-p)} + \frac{px^{1-p}}{a(1-p)} + xe^{\operatorname{sech}^{-1}(ax^p)}$$

[Out] E^ArcSech[a*x^p]*x + (p*x^(1 - p))/(a*(1 - p)) + (p*x^(1 - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + p^(-1))/2, (1 + p)/(2*p), a^2*x^(2*p)])/(a*(1 - p))

Rubi [A] time = 0.0468155, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6330, 30, 259, 364}

$$\frac{px^{1-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}{}_2F_1\left(\frac{1}{2}, \frac{1}{2}\left(\frac{1}{p}-1\right); \frac{p+1}{2p}; a^2x^{2p}\right)}{a(1-p)} + \frac{px^{1-p}}{a(1-p)} + xe^{\operatorname{sech}^{-1}(ax^p)}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[a*x^p], x]

[Out] E^ArcSech[a*x^p]*x + (p*x^(1 - p))/(a*(1 - p)) + (p*x^(1 - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + p^(-1))/2, (1 + p)/(2*p), a^2*x^(2*p)])/(a*(1 - p))

Rule 6330

Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] := Simp[x*E^ArcSech[a*x^p], x] + (Dist[p/a, Int[1/x^p, x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])]/a, Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x) /; FreeQ[{a, p}, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 259

```
Int[((c_)*(x_)^(m_)*((a1_) + (b1_)*(x_)^(n_))^(p_)*((a2_) + (b2_)*(x_)
^(n_))^(p_), x_Symbol] := Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
|| (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
]]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int e^{\operatorname{sech}^{-1}(ax^p)} dx &= e^{\operatorname{sech}^{-1}(ax^p)} x + \frac{p \int x^{-p} dx}{a} + \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{-p}}{\sqrt{1-ax^p} \sqrt{1+ax^p}} dx}{a} \\ &= e^{\operatorname{sech}^{-1}(ax^p)} x + \frac{px^{1-p}}{a(1-p)} + \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{-p}}{\sqrt{1-a^2x^{2p}}} dx}{a} \\ &= e^{\operatorname{sech}^{-1}(ax^p)} x + \frac{px^{1-p}}{a(1-p)} + \frac{px^{1-p} \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} \left(-1 + \frac{1}{p}\right); \frac{1+p}{2p}; a^2x^{2p}\right)}{a(1-p)} \end{aligned}$$

Mathematica [A] time = 0.385518, size = 139, normalized size = 1.32

$$\frac{x \left(-\frac{a^2 p x^p \sqrt{\frac{1-ax^p}{ax^p+1}} \sqrt{1-a^2x^{2p}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{p+1}{2p}, \frac{1}{2}\left(\frac{1}{p}+3\right), a^2x^{2p}\right)}{(p+1)(ax^p-1)} + (a+x^{-p}) \sqrt{\frac{1-ax^p}{ax^p+1}} + x^{-p} \right)}{a-ap}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^p], x]

[Out] (x*(x^(-p) + (a + x^(-p))*Sqrt[(1 - a*x^p)/(1 + a*x^p)] - (a^2*p*x^p*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p)]*Hypergeometric2F1[1/2, (1 + p)/(2*p), (3 + p^(-1))/2, a^2*x^(2*p)])/((1 + p)*(-1 + a*x^p))))/(a - a*p)

Maple [F] time = 1.154, size = 0, normalized size = 0.

$$\int \frac{1}{ax^p} + \sqrt{\frac{1}{ax^p} - 1} \sqrt{\frac{1}{ax^p} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)`

[Out] `int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x^{-p} dx + \int a \sqrt{-1 + \frac{x^{-p}}{a}} \sqrt{1 + \frac{x^{-p}}{a}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2),x)

[Out] (Integral(x**(-p), x) + Integral(a*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p)/a), x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1}{ax^p} + 1} \sqrt{\frac{1}{ax^p} - 1} + \frac{1}{ax^p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p), x)

$$3.63 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} dx$$

Optimal. Leaf size=87

$$\frac{x^{-p}\sqrt{1-ax^p}}{ap\sqrt{\frac{1}{ax^p+1}}} - \frac{x^{-p}}{ap} - \frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\sin^{-1}(ax^p)}{p}$$

[Out] $-(1/(a*p*x^p)) - \operatorname{Sqrt}[1 - a*x^p]/(a*p*x^p*\operatorname{Sqrt}[(1 + a*x^p)^{-1}]) - (\operatorname{Sqrt}[(1 + a*x^p)^{-1}]*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{ArcSin}[a*x^p])/p$

Rubi [A] time = 0.0689316, antiderivative size = 106, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6334, 259, 345, 242, 277, 216}

$$-\frac{x^{-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\sqrt{1-a^2x^{2p}}}{ap} - \frac{x^{-p}}{ap} - \frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\operatorname{csc}^{-1}\left(\frac{x^{-p}}{a}\right)}{p}$$

Warning: Unable to verify antiderivative.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^p]}/x, x]$

[Out] $-(1/(a*p*x^p)) - (\operatorname{Sqrt}[(1 + a*x^p)^{-1}]*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1 - a^2*x^{(2*p)}])/ (a*p*x^p) - (\operatorname{Sqrt}[(1 + a*x^p)^{-1}]*\operatorname{Sqrt}[1 + a*x^p]*\operatorname{ArcCsc}[1/(a*x^p)]) /p$

Rule 6334

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_)*(x_)^{(p_)}]}/(x_), x_Symbol] \rightarrow -\operatorname{Simp}[(a*p*x^p)^{-1}, x] + \operatorname{Dist}[(\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1/(1 + a*x^p)])]/a, \operatorname{Int}[(\operatorname{Sqrt}[1 + a*x^p]*\operatorname{Sqrt}[1 - a*x^p])/x^{(p+1)}, x], x] /;$ FreeQ[{a, p}, x]

Rule 259

$\operatorname{Int}[(c_)*(x_)^{(m_)*((a1_)+(b1_)*(x_)^{(n_))}^{(p_*)}*((a2_)+(b2_)*(x_)^{(n_*)}^{(p_*)})^{(p_*)}, x_Symbol] \rightarrow \operatorname{Int}[(c*x)^m*(a1*a2 + b1*b2*x^{(2*n)})^p, x] /;$ FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} dx &= -\frac{x^{-p}}{ap} + \frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right) \int x^{-1-p}\sqrt{1-ax^p}\sqrt{1+ax^p} dx}{a} \\
&= -\frac{x^{-p}}{ap} + \frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right) \int x^{-1-p}\sqrt{1-a^2x^{2p}} dx}{a} \\
&= -\frac{x^{-p}}{ap} - \frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right) \operatorname{Subst}\left(\int \sqrt{1-\frac{a^2}{x^2}} dx, x, x^{-p}\right)}{ap} \\
&= -\frac{x^{-p}}{ap} + \frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1-a^2x^2}}{x^2} dx, x, x^p\right)}{ap} \\
&= -\frac{x^{-p}}{ap} - \frac{x^{-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\sqrt{1-a^2x^{2p}}}{ap} - \frac{\left(a\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}} dx, x, x^p\right)}{p} \\
&= -\frac{x^{-p}}{ap} - \frac{x^{-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\sqrt{1-a^2x^{2p}}}{ap} - \frac{\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p} \sin^{-1}(ax^p)}{p}
\end{aligned}$$

Mathematica [C] time = 0.137093, size = 96, normalized size = 1.1

$$\frac{i\left(-i(a+x^{-p})\sqrt{\frac{1-ax^p}{ax^p+1}} + a \log\left(2\sqrt{\frac{1-ax^p}{ax^p+1}}(ax^p+1) - 2iax^p\right) - ix^{-p}\right)}{ap}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^p]/x, x]

[Out] $((-I)*((-I)/x^p - I*(a + x^{(-p)})*Sqrt[(1 - a*x^p)/(1 + a*x^p)]) + a*Log[(-2*I)*a*x^p + 2*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p)])/(a*p)$

Maple [C] time = 0.326, size = 145, normalized size = 1.7

$$-\frac{\operatorname{csgn}(a)x^p a}{p} \sqrt{-\frac{ax^p-1}{ax^p}} \sqrt{\frac{1+ax^p}{ax^p}} \arctan\left(\operatorname{csgn}(a)x^p a \frac{1}{\sqrt{-(x^p)^2 a^2 + 1}}\right) \frac{1}{\sqrt{-(x^p)^2 a^2 + 1}} - \frac{(\operatorname{csgn}(a))^2}{p} \sqrt{-\frac{ax^p-1}{ax^p}} \sqrt{\frac{1+ax^p}{ax^p}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x)`

[Out] $-1/p*(-(a*x^{p-1})/a/(x^p))^{(1/2)*((1+a*x^p)/a/(x^p))^{(1/2)*csgn(a)/(-(x^p)^2*a^{2+1})^{(1/2)*arctan(csgn(a)*a*x^p/(-(x^p)^2*a^{2+1})^{(1/2)*x^p*a-1/p*(-(a*x^{p-1})/a/(x^p))^{(1/2)*((1+a*x^p)/a/(x^p))^{(1/2)*csgn(a)^2-1/a/p/(x^p)}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 2.45426, size = 197, normalized size = 2.26

$$\frac{ax^p \sqrt{\frac{ax^p+1}{ax^p}} \sqrt{-\frac{ax^p-1}{ax^p}} - ax^p \arctan\left(\sqrt{\frac{ax^p+1}{ax^p}} \sqrt{-\frac{ax^p-1}{ax^p}}\right) + 1}{apx^p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorithm="fricas")`

[Out] $-(a*x^p*\sqrt{(a*x^p + 1)/(a*x^p)})*\sqrt{-(a*x^p - 1)/(a*x^p)} - a*x^p*\arctan(\sqrt{(a*x^p + 1)/(a*x^p)}*\sqrt{-(a*x^p - 1)/(a*x^p)}) + 1/(a*p*x^p)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{-p}}{x} dx + \int \frac{a\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}}{x} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x,x)

[Out] (Integral(x**(-p)/x, x) + Integral(a*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p)/a)/x, x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax^p} + 1} \sqrt{\frac{1}{ax^p} - 1} + \frac{1}{ax^p}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorithm m="giac")

[Out] integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x, x)

$$3.64 \quad \int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2} dx$$

Optimal. Leaf size=107

$$\frac{px^{-p-1} \sqrt{\frac{1}{ax^p+1}} \sqrt{ax^p+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{p+1}{2p}, -\frac{1-p}{2p}, a^2x^{2p}\right)}{a(p+1)} + \frac{px^{-p-1}}{a(p+1)} - \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}$$

[Out] $-(E^{\operatorname{ArcSech}[a*x^p]}/x) + (p*x^{(-1-p)})/(a*(1+p)) + (p*x^{(-1-p)}*\operatorname{Sqrt}[(1+a*x^p)^{-1}]*\operatorname{Sqrt}[1+a*x^p]*\operatorname{Hypergeometric2F1}[1/2, -(1+p)/(2*p), -(1-p)/(2*p), a^2*x^{(2*p)}])/(a*(1+p))$

Rubi [A] time = 0.0604132, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6335, 30, 259, 364}

$$\frac{px^{-p-1} \sqrt{\frac{1}{ax^p+1}} \sqrt{ax^p+1} {}_2F_1\left(\frac{1}{2}, -\frac{p+1}{2p}; -\frac{1-p}{2p}; a^2x^{2p}\right)}{a(p+1)} + \frac{px^{-p-1}}{a(p+1)} - \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[a*x^p]}/x^2, x]$

[Out] $-(E^{\operatorname{ArcSech}[a*x^p]}/x) + (p*x^{(-1-p)})/(a*(1+p)) + (p*x^{(-1-p)}*\operatorname{Sqrt}[(1+a*x^p)^{-1}]*\operatorname{Sqrt}[1+a*x^p]*\operatorname{Hypergeometric2F1}[1/2, -(1+p)/(2*p), -(1-p)/(2*p), a^2*x^{(2*p)}])/(a*(1+p))$

Rule 6335

$\operatorname{Int}[E^{\operatorname{ArcSech}[(a_.)*(x_)^{(p_.)}]}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*E^{\operatorname{ArcSech}[a*x^p]})/(m+1), x] + (\operatorname{Dist}[p/(a*(m+1)), \operatorname{Int}[x^{(m-p)}, x], x] + \operatorname{Dist}[(p*\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1/(1+a*x^p)])/(a*(m+1)), \operatorname{Int}[x^{(m-p)}/(\operatorname{Sqrt}[1+a*x^p]*\operatorname{Sqrt}[1-a*x^p]), x], x]) /; \operatorname{FreeQ}\{a, m, p\}, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)}/(m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 259

Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[(c*x)^(m*(a1*a2 + b1*b2*x^(2*n)))^p, x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2} dx &= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} - \frac{p \int x^{-2-p} dx}{a} - \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{-2-p}}{\sqrt{1-ax^p} \sqrt{1+ax^p}} dx}{a} \\ &= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} - \frac{\left(p \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p}\right) \int \frac{x^{-2-p}}{\sqrt{1-a^2x^{2p}}} dx}{a} \\ &= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} + \frac{px^{-1-p} \sqrt{\frac{1}{1+ax^p}} \sqrt{1+ax^p} {}_2F_1\left(\frac{1}{2}, -\frac{1+p}{2p}; -\frac{1-p}{2p}; a^2x^{2p}\right)}{a(1+p)} \end{aligned}$$

Mathematica [A] time = 0.258708, size = 156, normalized size = 1.46

$$x^{-p-1} \left(\frac{apx^{2p} \sqrt{\frac{1-ax^p}{ax^p+1}} \sqrt{1-a^2x^{2p}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{p-1}{2p}, \frac{3}{2} - \frac{1}{2p}, a^2x^{2p}\right)}{(p-1)(p+1)(ax^p-1)} - \frac{\sqrt{\frac{1-ax^p}{ax^p+1}} (ax^p+1)}{a(p+1)} - \frac{1}{ap+a} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[a*x^p]/x^2,x]

[Out] x^(-1 - p)*(-(a + a*p)^(-1) - (Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p)))/(a*(1 + p)) + (a*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p)])*Hypergeometric2F1[1/2, (-1 + p)/(2*p), 3/2 - 1/(2*p), a^2*x^(2*p)]/((-1 + p)*(1 + p)*(-1 + a*x^p))

Maple [F] time = 1.182, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(\frac{1}{ax^p} + \sqrt{\frac{1}{ax^p} - 1} \sqrt{\frac{1}{ax^p} + 1} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)

[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^{-p}}{x^2} dx + \int \frac{a \sqrt{-1 + \frac{x^{-p}}{a}} \sqrt{1 + \frac{x^{-p}}{a}}}{x^2} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x**2,x)

[Out] (Integral(x**(-p)/x**2, x) + Integral(a*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p)/a)/x**2, x))/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{ax^p} + 1} \sqrt{\frac{1}{ax^p} - 1 + \frac{1}{ax^p}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x^2, x)

3.65 $\int e^{2\operatorname{sech}^{-1}(ax)} x^4 dx$

Optimal. Leaf size=203

$$\frac{(1-ax)(ax+1)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{ax+1}} \left(5 - 6\sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)^4}{10a^5} - \frac{\left(45\sqrt{\frac{1-ax}{ax+1}} + 4\right) (ax+1)^3}{30a^5} + \frac{5\sqrt{\frac{1-ax}{ax+1}} (ax+1)^2}{4a^5} + \frac{\left(4 - \sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)}{4a^5}$$

[Out] (5*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(4*a^5) + ((1 - a*x)*(1 + a*x)^4)/(5*a^5) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4*(5 - 6*Sqrt[(1 - a*x)/(1 + a*x)]))/(10*a^5) + ((1 + a*x)*(4 - Sqrt[(1 - a*x)/(1 + a*x)]))/(4*a^5) - ((1 + a*x)^3*(4 + 45*Sqrt[(1 - a*x)/(1 + a*x)]))/(30*a^5) - ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]/(2*a^5)

Rubi [A] time = 0.699971, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6337, 1804, 1811, 1814, 639, 203}

$$\frac{(1-ax)(ax+1)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{ax+1}} \left(5 - 6\sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)^4}{10a^5} - \frac{\left(45\sqrt{\frac{1-ax}{ax+1}} + 4\right) (ax+1)^3}{30a^5} + \frac{5\sqrt{\frac{1-ax}{ax+1}} (ax+1)^2}{4a^5} + \frac{\left(4 - \sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)}{4a^5}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcSech[a*x])*x^4,x]

[Out] (5*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(4*a^5) + ((1 - a*x)*(1 + a*x)^4)/(5*a^5) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4*(5 - 6*Sqrt[(1 - a*x)/(1 + a*x)]))/(10*a^5) + ((1 + a*x)*(4 - Sqrt[(1 - a*x)/(1 + a*x)]))/(4*a^5) - ((1 + a*x)^3*(4 + 45*Sqrt[(1 - a*x)/(1 + a*x)]))/(30*a^5) - ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]/(2*a^5)

Rule 6337

Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] := Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
  1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_)*(u_)] /; IntegerQ[m]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient
[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
  0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e
- c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{sech}^{-1}(ax)} x^4 dx &= \int x^4 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax} \right)^2 dx \\
&= -\frac{4 \operatorname{Subst} \left(\int \frac{(-1+x)^2 x (1+x)^6}{(1+x^2)^6} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^5} \\
&= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{2 \operatorname{Subst} \left(\int \frac{-42x-40x^2+130x^3+80x^4-30x^5-40x^6-10x^7}{(1+x^2)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{5a^5} \\
&= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{2 \operatorname{Subst} \left(\int \frac{x(-42-40x+130x^2+80x^3-30x^4-40x^5-10x^6)}{(1+x^2)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{5a^5} \\
&= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4 \left(5 - 6\sqrt{\frac{1-ax}{1+ax}} \right)}{10a^5} - \frac{\operatorname{Subst} \left(\int \frac{160-48x-960x^2+160x^3+320x^4+80x^5}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} \\
&= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4 \left(5 - 6\sqrt{\frac{1-ax}{1+ax}} \right)}{10a^5} - \frac{(1+ax)^3 \left(4 + 45\sqrt{\frac{1-ax}{1+ax}} \right)}{30a^5} + \frac{\operatorname{Subst} \left(\int \frac{480-480x}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{30a^5} \\
&= \frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5} + \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4 \left(5 - 6\sqrt{\frac{1-ax}{1+ax}} \right)}{10a^5} - \frac{(1+ax)^3 \left(4 + 45\sqrt{\frac{1-ax}{1+ax}} \right)}{30a^5} \\
&= \frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5} + \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4 \left(5 - 6\sqrt{\frac{1-ax}{1+ax}} \right)}{10a^5} + \frac{(1+ax) \left(4 - \sqrt{\frac{1-ax}{1+ax}} \right)}{4a^5} - \frac{(1+ax)^3 \left(4 + 45\sqrt{\frac{1-ax}{1+ax}} \right)}{30a^5} \\
&= \frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5} + \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4 \left(5 - 6\sqrt{\frac{1-ax}{1+ax}} \right)}{10a^5} + \frac{(1+ax) \left(4 - \sqrt{\frac{1-ax}{1+ax}} \right)}{4a^5} - \frac{(1+ax)^3 \left(4 + 45\sqrt{\frac{1-ax}{1+ax}} \right)}{30a^5}
\end{aligned}$$

Mathematica [C] time = 0.141372, size = 105, normalized size = 0.52

$$\frac{-12a^5x^5 + 40a^3x^3 - 15a\sqrt{\frac{1-ax}{ax+1}} \left(-2a^3x^4 - 2a^2x^3 + ax^2 + x \right) + 15i \log \left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax \right)}{60a^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])*x^4,x]

[Out] (40*a^3*x^3 - 12*a^5*x^5 - 15*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*x^4) + (15*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1

+ a*x)]/(60*a^5)

Maple [C] time = 0.206, size = 123, normalized size = 0.6

$$-\frac{x^5}{5} + \frac{2x^3}{3a^2} + \frac{x \operatorname{csgn}(a)}{4a^4} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(2 \operatorname{csgn}(a) x^3 a^3 \sqrt{-a^2x^2+1} - x \sqrt{-a^2x^2+1} \operatorname{csgn}(a) a + \arctan \left(x \operatorname{csgn}(a) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2*x^4,x)

[Out] -1/5*x^5+2/3*x^3/a^2+1/4/a^4*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(2*csgn(a)*x^3*a^3*(-a^2*x^2+1)^(1/2)-x*(-a^2*x^2+1)^(1/2)*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2x^3}{3a^2} + \frac{2 \left(-\frac{(-a^2x^2+1)^{\frac{3}{2}}x}{4a^2} + \frac{\sqrt{-a^2x^2+1}x}{8a^2} + \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{8\sqrt{a^2}a^2} \right)}{a^2} - \int x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="maxima")

[Out] 2/3*x^3/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a^2 - integrate(x^4, x)

Fricas [A] time = 2.09114, size = 225, normalized size = 1.11

$$\frac{12a^5x^5 - 40a^3x^3 - 15(2a^4x^4 - a^2x^2)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 15\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{60a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^5*x^5 - 40*a^3*x^3 - 15*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 15*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="giac")
```

```
[Out] integrate(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

3.66 $\int e^{2\operatorname{sech}^{-1}(ax)} x^3 dx$

Optimal. Leaf size=117

$$\frac{(1-ax)(ax+1)^3}{4a^4} + \frac{\sqrt{\frac{1-ax}{ax+1}} \left(4 - 3\sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)^3}{6a^4} + \frac{\left(3 - 8\sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)^2}{6a^4} - \frac{x}{a^3}$$

[Out] $-(x/a^3) + ((1 - a*x)*(1 + a*x)^3)/(4*a^4) + ((1 + a*x)^2*(3 - 8*sqrt[(1 - a*x)/(1 + a*x])))/(6*a^4) + (sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3*(4 - 3*sqrt[(1 - a*x)/(1 + a*x])))/(6*a^4)$

Rubi [A] time = 0.546433, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6337, 1804, 1811, 1814, 12, 261}

$$\frac{(1-ax)(ax+1)^3}{4a^4} + \frac{\sqrt{\frac{1-ax}{ax+1}} \left(4 - 3\sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)^3}{6a^4} + \frac{\left(3 - 8\sqrt{\frac{1-ax}{ax+1}}\right) (ax+1)^2}{6a^4} - \frac{x}{a^3}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcSech[a*x])*x^3,x]

[Out] $-(x/a^3) + ((1 - a*x)*(1 + a*x)^3)/(4*a^4) + ((1 + a*x)^2*(3 - 8*sqrt[(1 - a*x)/(1 + a*x])))/(6*a^4) + (sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3*(4 - 3*sqrt[(1 - a*x)/(1 + a*x])))/(6*a^4)$

Rule 6337

Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)]/u)^n, x] /; FreeQ[m, x] && Integer Q[n]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a,

b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 1811

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[x*PolynomialQuotient[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_.)*(u_.)] /; IntegerQ[m]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*(a + b*x^2)^(p + 1)/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{sech}^{-1}(ax)} x^3 dx &= \int x^3 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax} \right)^2 dx \\
&= \frac{4 \operatorname{Subst} \left(\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^4} \\
&= \frac{(1-ax)(1+ax)^3}{4a^4} - \frac{\operatorname{Subst} \left(\int \frac{24x+32x^2-32x^3-32x^4-8x^5}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{2a^4} \\
&= \frac{(1-ax)(1+ax)^3}{4a^4} - \frac{\operatorname{Subst} \left(\int \frac{x(24+32x-32x^2-32x^3-8x^4)}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{2a^4} \\
&= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3 \left(4 - 3\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4} + \frac{\operatorname{Subst} \left(\int \frac{-64-48x+192x^2+48x^3}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{12a^4} \\
&= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2 \left(3 - 8\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3 \left(4 - 3\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4} - \frac{\operatorname{Subst} \left(\int -\frac{192x}{(1+x^2)^2} \right)}{48a^4} \\
&= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2 \left(3 - 8\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3 \left(4 - 3\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4} + \frac{4 \operatorname{Subst} \left(\int \frac{x}{(1+x^2)^2} \right)}{a^4} \\
&= -\frac{x}{a^3} + \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2 \left(3 - 8\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3 \left(4 - 3\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^4}
\end{aligned}$$

Mathematica [A] time = 0.0679191, size = 52, normalized size = 0.44

$$\frac{x^2}{a^2} + \frac{2(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2}{3a^4} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcSech[a*x])*x^3,x]

[Out] x^2/a^2 - x^4/4 + (2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a^4)

Maple [A] time = 0.181, size = 72, normalized size = 0.6

$$\frac{1}{a^2} \left(-\frac{a^2 x^4}{4} + \frac{x^2}{2} \right) + \frac{2x(a^2 x^2 - 1)}{3a^3} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{x^2}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x)

[Out] 1/a^2*(-1/4*a^2*x^4+1/2*x^2)+2/3/a^3*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)+1/2*x^2/a^2

Maxima [A] time = 1.04463, size = 57, normalized size = 0.49

$$-\frac{1}{4}x^4 + \frac{x^2}{a^2} + \frac{2(a^2x^2 - 1)\sqrt{ax+1}\sqrt{-ax+1}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="maxima")

[Out] -1/4*x^4 + x^2/a^2 + 2/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^4

Fricas [A] time = 1.9465, size = 131, normalized size = 1.12

$$\frac{3a^3x^4 - 12ax^2 - 8(a^2x^3 - x)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{12a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="fricas")

[Out] -1/12*(3*a^3*x^4 - 12*a*x^2 - 8*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)))/a^3

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int 2x dx + \int -a^2 x^3 dx + \int 2ax^2 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))**2*x**3,x)

[Out] (Integral(2*x, x) + Integral(-a**2*x**3, x) + Integral(2*a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="giac")

[Out] integrate(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)

3.67 $\int e^{2\operatorname{sech}^{-1}(ax)} x^2 dx$

Optimal. Leaf size=169

$$\frac{(ax+1)^3 \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^4}{12a^3} - \frac{\sqrt{\frac{1-ax}{ax+1}} (ax+1)^2 \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3}{6a^3} + \frac{(ax+1) \left(1 - \sqrt{\frac{1-ax}{ax+1}}\right) \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)}{2a^3} - \frac{2 \tan^{-1} \left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^3}$$

[Out] ((1 + a*x)*(1 - Sqrt[(1 - a*x)/(1 + a*x)])*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) / (2*a^3) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))^3 / (6*a^3) + ((1 + a*x)^3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) / (12*a^3) - (2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]) / a^3

Rubi [A] time = 0.46505, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6337, 821, 12, 729, 723, 203}

$$\frac{(ax+1)^3 \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^4}{12a^3} - \frac{\sqrt{\frac{1-ax}{ax+1}} (ax+1)^2 \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3}{6a^3} + \frac{(ax+1) \left(1 - \sqrt{\frac{1-ax}{ax+1}}\right) \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)}{2a^3} - \frac{2 \tan^{-1} \left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcSech[a*x])*x^2, x]

[Out] ((1 + a*x)*(1 - Sqrt[(1 - a*x)/(1 + a*x)])*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) / (2*a^3) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))^3 / (6*a^3) + ((1 + a*x)^3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) / (12*a^3) - (2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]) / a^3

Rule 6337

Int[E^(ArcSech[u]*(n_.))*(x_)^(m_.), x_Symbol] := Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]

Rule 821

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x)) / (2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x]


```

/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 729

```

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=> -Simp
[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2
*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /
; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3,
0] && LtQ[p, -1]

```

Rule 723

```

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
+ c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]

```

Rule 203

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{sech}^{-1}(ax)} x^2 dx &= \int x^2 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax} \right)^2 dx \\
&= \frac{4 \operatorname{Subst} \left(\int \frac{x(1+x)^4}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^3} \\
&= \frac{(1+ax)^3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2 \operatorname{Subst} \left(\int \frac{4(1+x)^3}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{3a^3} \\
&= \frac{(1+ax)^3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{8 \operatorname{Subst} \left(\int \frac{(1+x)^3}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{3a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2 \operatorname{Subst} \left(\int \frac{(1+x)^2}{(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^3} \\
&= \frac{(1+ax) \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right) \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - 2 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\
&= \frac{(1+ax) \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right) \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - 2 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)
\end{aligned}$$

Mathematica [C] time = 0.0703845, size = 86, normalized size = 0.51

$$\sqrt{\frac{1-ax}{ax+1}} \left(\frac{x}{a^2} + \frac{x^2}{a} \right) + \frac{2x}{a^2} + \frac{i \log \left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax \right)}{a^3} - \frac{x^3}{3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])*x^2,x]

[Out] (2*x)/a^2 - x^3/3 + Sqrt[(1 - a*x)/(1 + a*x)]*(x/a^2 + x^2/a) + (I*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^3

Maple [C] time = 0.182, size = 97, normalized size = 0.6

$$-\frac{x^3}{3} + 2\frac{x}{a^2} + \frac{x \operatorname{sgn}(a)}{a^2} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(x\sqrt{-a^2x^2+1} \operatorname{sgn}(a) a + \arctan \left(x \operatorname{sgn}(a) a \frac{1}{\sqrt{-a^2x^2+1}} \right) \right) \frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x)`

[Out]
$$-1/3*x^3+2*x/a^2+1/a^2*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(x*(-a^2*x^2+1)^(1/2)*\text{csgn}(a)*a+\arctan(\text{csgn}(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)*\text{csgn}(a)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2x}{a^2} + \frac{2 \left(\frac{1}{2} \sqrt{-a^2 x^2 + 1} x + \frac{\arcsin\left(\frac{a^2 x}{\sqrt{a^2}}\right)}{2 \sqrt{a^2}} \right)}{a^2} - \int x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="maxima")`

[Out]
$$2*x/a^2 + 2*\text{integrate}(\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1), x)/a^2 - \text{integrate}(x^2, x)$$

Fricas [A] time = 2.11647, size = 192, normalized size = 1.14

$$\frac{a^3 x^3 - 3 a^2 x^2 \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 6 ax + 3 \arctan\left(\sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}}\right)}{3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="fricas")`

[Out]
$$-1/3*(a^3*x^3 - 3*a^2*x^2*\text{sqrt}((a*x + 1)/(a*x))*\text{sqrt}(-(a*x - 1)/(a*x)) - 6*a*x + 3*\arctan(\text{sqrt}((a*x + 1)/(a*x))*\text{sqrt}(-(a*x - 1)/(a*x))))/a^3$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="giac")

[Out] integrate(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)

3.68 $\int e^{2\operatorname{sech}^{-1}(ax)} x dx$

Optimal. Leaf size=85

$$-\frac{(ax+1)^2}{2a^2} + \frac{\left(2\sqrt{\frac{1-ax}{ax+1}}+1\right)(ax+1)}{a^2} + \frac{2\log(ax+1)}{a^2} + \frac{4\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

[Out] $-(1 + a*x)^2/(2*a^2) + ((1 + a*x)*(1 + 2*sqrt[(1 - a*x)/(1 + a*x)]))/a^2 + (2*Log[1 + a*x])/a^2 + (4*Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]])/a^2$

Rubi [A] time = 0.434226, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6337, 1647, 1593, 801, 260}

$$-\frac{(ax+1)^2}{2a^2} + \frac{\left(2\sqrt{\frac{1-ax}{ax+1}}+1\right)(ax+1)}{a^2} + \frac{2\log(ax+1)}{a^2} + \frac{4\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcSech[a*x])*x,x]

[Out] $-(1 + a*x)^2/(2*a^2) + ((1 + a*x)*(1 + 2*sqrt[(1 - a*x)/(1 + a*x)]))/a^2 + (2*Log[1 + a*x])/a^2 + (4*Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]])/a^2$

Rule 6337

Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q]/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{sech}^{-1}(ax)} x dx &= \int x \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax} \right) dx \\
&= \frac{4 \operatorname{Subst} \left(\int \frac{x(1+x)^3}{(-1+x)(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} \\
&= -\frac{(1+ax)^2}{2a^2} - \frac{\operatorname{Subst} \left(\int \frac{-12x-4x^2}{(-1+x)(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} \\
&= -\frac{(1+ax)^2}{2a^2} - \frac{\operatorname{Subst} \left(\int \frac{(-12-4x)x}{(-1+x)(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} \\
&= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax) \left(1 + 2\sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} + \frac{\operatorname{Subst} \left(\int \frac{8+8x}{(-1+x)(1+x^2)} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{2a^2} \\
&= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax) \left(1 + 2\sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} + \frac{\operatorname{Subst} \left(\int \left(\frac{8}{-1+x} - \frac{8x}{1+x^2} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{2a^2} \\
&= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax) \left(1 + 2\sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} + \frac{4 \log \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} - \frac{4 \operatorname{Subst} \left(\int \frac{x}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} \\
&= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax) \left(1 + 2\sqrt{\frac{1-ax}{1+ax}} \right)}{a^2} + \frac{2 \log(1+ax)}{a^2} + \frac{4 \log \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0600083, size = 89, normalized size = 1.05

$$\frac{-a^2 x^2 + 4 \sqrt{\frac{1-ax}{ax+1}} (ax+1) - 4 \log \left(ax \sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right) + 8 \log(x)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])*x,x]

[Out] $(-(a^2 x^2) + 4 \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 8 \operatorname{Log}[x] - 4 \operatorname{Log}[1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)] + a*x \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/(2*a^2)$

Maple [A] time = 0.208, size = 89, normalized size = 1.1

$$-\frac{x^2}{2} + 2 \frac{\ln(x)}{a^2} + 2 \frac{x \left(\sqrt{-a^2 x^2 + 1} - \operatorname{Artanh} \left(\frac{1}{\sqrt{-a^2 x^2 + 1}} \right) \right)}{a \sqrt{-a^2 x^2 + 1}} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x)

[Out] -1/2*x^2+2/a^2*ln(x)+2/a*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)/(-a^2*x^2+1)^(1/2)*((-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="maxima")

[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)

Fricas [A] time = 2.10643, size = 281, normalized size = 3.31

$$\frac{a^2 x^2 - 4 a x \sqrt{\frac{ax+1}{ax}} \sqrt{\frac{ax-1}{ax}} + 2 \log \left(a x \sqrt{\frac{ax+1}{ax}} \sqrt{\frac{ax-1}{ax}} + 1 \right) - 2 \log \left(a x \sqrt{\frac{ax+1}{ax}} \sqrt{\frac{ax-1}{ax}} - 1 \right) - 4 \log(x)}{2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="fricas")

[Out] -1/2*(a^2*x^2 - 4*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - 2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 4*log(x))/a^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="giac")

[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)

3.69 $\int e^{2\operatorname{sech}^{-1}(ax)} dx$

Optimal. Leaf size=57

$$-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{4 \tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a} - x$$

[Out] $-x - 4/(a*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) + (4*\operatorname{ArcTan}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/a$

Rubi [A] time = 0.173586, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6332, 1647, 12, 801, 203}

$$-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{4 \tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a} - x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])}, x]$

[Out] $-x - 4/(a*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) + (4*\operatorname{ArcTan}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/a$

Rule 6332

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*n_)}, x_Symbol] \rightarrow \operatorname{Int}[(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /; \operatorname{IntegerQ}[n]$

Rule 1647

$\operatorname{Int}[(Pq_)*((d_) + (e_)*(x_))^{(m_)*((a_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] > \operatorname{With}\{Q = \operatorname{PolynomialQuotient}[(d + e*x)^m*Pq, a + c*x^2, x], f = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]\}, \operatorname{Simp}[(a*g - c*f*x)*(a + c*x^2)^{(p+1)}/(2*a*c*(p+1)), x] + \operatorname{Dist}[1/(2*a*c*(p+1)), \operatorname{Int}[(d + e*x)^m*(a + c*x^2)^{(p+1)}*\operatorname{ExpandToSum}[(2*a*c*(p+1)*Q)/(d + e*x)^m + (c*f*(2*p+3))/(d + e*x)^m, x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x\} \&\& \operatorname{PolyQ}[Pq, x] \&$

& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 801

Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{sech}^{-1}(ax)} dx &= \int \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax} \right)^2 dx \\
&= -\frac{4 \operatorname{Subst} \left(\int \frac{x(1+x)^2}{(-1+x)^2(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a} \\
&= -x + \frac{2 \operatorname{Subst} \left(\int -\frac{4x}{(-1+x)^2(1+x^2)} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a} \\
&= -x - \frac{8 \operatorname{Subst} \left(\int \frac{x}{(-1+x)^2(1+x^2)} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a} \\
&= -x - \frac{8 \operatorname{Subst} \left(\int \left(\frac{1}{2(-1+x)^2} - \frac{1}{2(1+x^2)} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a} \\
&= -x - \frac{4}{a \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)} + \frac{4 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a} \\
&= -x - \frac{4}{a \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)} + \frac{4 \tan^{-1} \left(\sqrt{\frac{1-ax}{1+ax}} \right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0948087, size = 75, normalized size = 1.32

$$\frac{a^2 x^2 + 2\sqrt{\frac{1-ax}{ax+1}}(ax+1) + 2ax \tan^{-1} \left(\frac{ax}{\sqrt{\frac{1-ax}{ax+1}}(ax+1)} \right) + 2}{a^2 x}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x]),x]

[Out] -((2 + a^2*x^2 + 2*sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*a*x*ArcTan[(a*x)/(sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))])/(a^2*x))

Maple [C] time = 0.209, size = 98, normalized size = 1.7

$$-x - 2 \frac{1}{a^2 x} - 2 \frac{\operatorname{csgn}(a)}{a \sqrt{-a^2 x^2 + 1}} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(\arctan \left(\frac{\operatorname{csgn}(a) ax}{\sqrt{-a^2 x^2 + 1}} \right) xa + \operatorname{csgn}(a) \sqrt{-a^2 x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2,x`

[Out] `-x-2/a^2/x-2/a*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*(arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2))*x*a+csgn(a)*(-a^2*x^2+1)^(1/2))*csgn(a)/(-a^2*x^2+1)^(1/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x + \frac{2 \left(-\frac{a^2 \arcsin\left(\frac{a^2 x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - \frac{\sqrt{-a^2 x^2 + 1}}{x} \right)}{a^2} + \frac{-\frac{1}{x}}{a^2} - \frac{1}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2,x, algorithm="maxima")`

[Out] `-x + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a^2 + integrate(x^(-2), x)/a^2 - 1/(a^2*x)`

Fricas [A] time = 2.3013, size = 186, normalized size = 3.26

$$\frac{a^2 x^2 + 2 a x \sqrt{\frac{ax+1}{ax}} \sqrt{\frac{ax-1}{ax}} - 2 a x \arctan \left(\sqrt{\frac{ax+1}{ax}} \sqrt{\frac{ax-1}{ax}} \right) + 2}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2,x, algorithm="fricas")`

[Out] `-(a^2*x^2 + 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 2)/(a^2*x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int -a^2 dx + \int \frac{2}{x^2} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))**2,x)

[Out] (Integral(-a**2, x) + Integral(2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x, x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)

$$3.70 \quad \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x} dx$$

Optimal. Leaf size=86

$$\frac{2}{1 - \sqrt{\frac{1-ax}{ax+1}}} - \frac{2}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \log(ax+1) - 2 \log\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)$$

[Out] $-2/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 + 2/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]) - \operatorname{Log}[1 + a*x] - 2*\operatorname{Log}[1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]$

Rubi [A] time = 0.445547, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 1629, 260}

$$\frac{2}{1 - \sqrt{\frac{1-ax}{ax+1}}} - \frac{2}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \log(ax+1) - 2 \log\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])}/x, x]$

[Out] $-2/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 + 2/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]) - \operatorname{Log}[1 + a*x] - 2*\operatorname{Log}[1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*n_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /;$ FreeQ[m, x] && Integer Q[n]

Rule 1629

$\operatorname{Int}[(Pq_)*((d_) + (e_)*(x_))^{(m_)}*((a_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x)^m*(a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x} dx &= \int \frac{\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x} dx \\
 &= 4 \operatorname{Subst}\left(\int \frac{x(1+x)}{(-1+x)^3(1+x^2)} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= 4 \operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^3} + \frac{1}{2(-1+x)^2} - \frac{1}{2(-1+x)} + \frac{x}{2(1+x^2)}\right) dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= -\frac{2}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{2}{1 - \sqrt{\frac{1-ax}{1+ax}}} - 2 \log\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right) + 2 \operatorname{Subst}\left(\int \frac{x}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= -\frac{2}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{2}{1 - \sqrt{\frac{1-ax}{1+ax}}} - \log(1+ax) - 2 \log\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.0658266, size = 86, normalized size = 1.

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{a^2x^2} - \frac{1}{a^2x^2} + \log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) - 2\log(x)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])/x,x]

[Out] -(1/(a^2*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a^2*x^2) - 2*Log[x] + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]]

Maple [A] time = 0.187, size = 97, normalized size = 1.1

$$-\ln(x) - \frac{1}{a^2x^2} - \frac{1}{ax} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(-a^2x^2 \operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \sqrt{-a^2x^2+1}\right) \frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x,x)`

[Out] `-ln(x)-1/a^2/x^2-1/a*(-(a*x-1)/a/x)^(1/2)/x*((a*x+1)/a/x)^(1/2)*(-a^2*x^2*arctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \left(\frac{1}{2} a^2 \log \left(\frac{2 \sqrt{-a^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) - \frac{1}{2} \sqrt{-a^2 x^2 + 1} a^2 - \frac{(-a^2 x^2 + 1)^{\frac{3}{2}}}{2 x^2} \right)}{a^2} - \frac{1}{a^2 x^2} - \int \frac{1}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x,x, algorithm="maxima")`

[Out] `2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a^2 - 1/(a^2*x^2) - integrate(1/x, x)`

Fricas [A] time = 2.14903, size = 306, normalized size = 3.56

$$\frac{a^2 x^2 \log \left(a x \sqrt{\frac{a x + 1}{a x}} \sqrt{-\frac{a x - 1}{a x}} + 1 \right) - a^2 x^2 \log \left(a x \sqrt{\frac{a x + 1}{a x}} \sqrt{-\frac{a x - 1}{a x}} - 1 \right) - 2 a^2 x^2 \log(x) - 2 a x \sqrt{\frac{a x + 1}{a x}} \sqrt{-\frac{a x - 1}{a x}} - 2}{2 a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x,x, algorithm="fricas")`

[Out] `1/2*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a^2*x^2*log(x) - 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a^2*x^2)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2}{x^3} dx + \int -\frac{a^2}{x} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^2} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))**2/x,x)

[Out] (Integral(2/x**3, x) + Integral(-a**2/x, x) + Integral(2*a*sqrt(-1 + 1/(a*x)))*sqrt(1 + 1/(a*x))/x**2, x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x, x)

$$3.71 \quad \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2} dx$$

Optimal. Leaf size=57

$$\frac{2a}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{4a}{3\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3}$$

[Out] $(-4*a)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (2*a)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2$

Rubi [A] time = 0.390062, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 43}

$$\frac{2a}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{4a}{3\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])}/x^2, x]$

[Out] $(-4*a)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (2*a)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*n_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)])/u)^n, x] /;$ FreeQ[m, x] && IntegerQ[n]

Rule 43

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2} dx &= \int \frac{\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^2} dx \\
&= -\left(4a\right) \operatorname{Subst}\left(\int \frac{x}{(-1+x)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
&= -\left(4a\right) \operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^4} + \frac{1}{(-1+x)^3}\right) dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
&= -\frac{4a}{3\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{2a}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0662674, size = 52, normalized size = 0.91

$$\frac{3a^2x^2 + 2(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2 - 2}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcSech[a*x])/x^2,x]

[Out] (-2 + 3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a^2*x^3)

Maple [A] time = 0.194, size = 73, normalized size = 1.3

$$\frac{1}{a^2} \left(\frac{a^2}{x} - \frac{1}{3x^3} \right) + \frac{2a^2x^2 - 2}{3ax^2} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{1}{3x^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x)

[Out] 1/a^2*(a^2/x-1/3/x^3)+2/3/a*(-(a*x-1)/a/x)^(1/2)/x^2*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)-1/3/a^2/x^3

Maxima [A] time = 1.04345, size = 62, normalized size = 1.09

$$\frac{1}{x} + \frac{2(a^2x^3 - x)\sqrt{ax+1}\sqrt{-ax+1}}{3a^2x^4} - \frac{2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="maxima")

[Out] 1/x + 2/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^4) - 2/3/(a^2*x^3)

Fricas [A] time = 2.05545, size = 130, normalized size = 2.28

$$\frac{3a^2x^2 + 2(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="fricas")

[Out] 1/3*(3*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a^2*x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2}{x^4} dx + \int -\frac{a^2}{x^2} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^3} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**2,x)

[Out] (Integral(2/x**4, x) + Integral(-a**2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**3, x))/a**2

Giac [A] time = 1.15194, size = 126, normalized size = 2.21

$$\frac{2\left(a^2 + \frac{a}{x}\right)^{\frac{3}{2}}\left(a^2 - \frac{a}{x}\right)\sqrt{-a^2 + \frac{a}{x}}a^4 - \left(a^4 + \frac{a}{x^3}\right)a^6 + \left(3\left(a^2 + \frac{a}{x}\right)^2a^2 - \left(a^2 + \frac{a}{x}\right)^3\right)a^4}{3a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="giac")

[Out] 1/3*(2*(a^2 + a/x)^(3/2)*(a^2 - a/x)*sqrt(-a^2 + a/x)*a^4 - (a^4 + a/x^3)*a^6 + (3*(a^2 + a/x)^2*a^2 - (a^2 + a/x)^3)*a^4)/a^9

$$3.72 \quad \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3} dx$$

Optimal. Leaf size=147

$$\frac{a^2}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{3a^2}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{2a^2}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{a^2}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} + \frac{1}{2}a^2 \tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

[Out] $-(a^2/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + (2*a^2)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3 - (3*a^2)/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) + a^2/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) + (a^2*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/2$

Rubi [A] time = 0.45291, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 1612, 207}

$$\frac{a^2}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{3a^2}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{2a^2}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{a^2}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} + \frac{1}{2}a^2 \tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])/x^3}, x]$

[Out] $-(a^2/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + (2*a^2)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3 - (3*a^2)/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) + a^2/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) + (a^2*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/2$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*(n_.))*(x_)^{(m_.)}, x_Symbol] := \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /;$ FreeQ[m, x] && Integer Q[n]

Rule 1612

$\operatorname{Int}[(P_x)*((a_.) + (b_.)*(x_))^{(m_.)*((c_.) + (d_.)*(x_))^{(n_.)*((e_.) + (f_.)*(x_))^{(p_.)}, x_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[P_x*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P_x, x] && IntegersQ[m, n]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3} dx &= \int \frac{\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^3} dx \\
 &= (4a^2) \operatorname{Subst}\left(\int \frac{x(1+x^2)}{(-1+x)^5(1+x)} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= (4a^2) \operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5} + \frac{3}{2(-1+x)^4} + \frac{3}{4(-1+x)^3} + \frac{1}{8(-1+x)^2} - \frac{1}{8(-1+x^2)}\right) dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= -\frac{a^2}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{2a^2}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{3a^2}{2\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^2}{2\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{1}{2}a^2 \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= -\frac{a^2}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{2a^2}{\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{3a^2}{2\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^2}{2\left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} + \frac{1}{2}a^2 \tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.122758, size = 121, normalized size = 0.82

$$\frac{(ax+1)\left(a^2x^2\sqrt{\frac{1-ax}{ax+1}}+2ax-2\sqrt{\frac{1-ax}{ax+1}}-2\right)}{x^4} + a^4(-\log(x)) + a^4\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right)$$

$$4a^2$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])/x^3, x]

[Out] (((1 + a*x)*(-2 + 2*a*x - 2*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a*x)/(1 + a*x)]))/x^4 - a^4*Log[x] + a^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(4*a^2)

Maple [A] time = 0.192, size = 131, normalized size = 0.9

$$\frac{1}{a^2} \left(-\frac{1}{4x^4} + \frac{a^2}{2x^2} \right) + \frac{1}{4x^3a} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(\operatorname{Artanh} \left(\frac{1}{\sqrt{-a^2x^2+1}} \right) x^4 a^4 + a^2 x^2 \sqrt{-a^2x^2+1} - 2 \sqrt{-a^2x^2+1} \right) \frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^3,x)

[Out] $\frac{1}{a^2} \left(-\frac{1}{4x^4} + \frac{1}{2} a^2/x^2 \right) + \frac{1}{4a} \left(-\frac{ax-1}{a/x} \right)^{1/2} / x^3 \left(\frac{ax+1}{a/x} \right)^{1/2} \left(\operatorname{arctanh} \left(\frac{1}{\sqrt{-a^2x^2+1}} \right) x^4 a^4 + a^2 x^2 \sqrt{-a^2x^2+1} - 2 \sqrt{-a^2x^2+1} \right) / \left(-a^2x^2+1 \right)^{1/2} - \frac{1}{4a^2} / x^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \left(\frac{1}{8} a^4 \log \left(\frac{2 \sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|} \right) - \frac{1}{8} \sqrt{-a^2x^2+1} a^4 - \frac{(-a^2x^2+1)^{\frac{3}{2}} a^2}{8x^2} - \frac{(-a^2x^2+1)^{\frac{3}{2}}}{4x^4} \right)}{a^2} - \frac{1}{2a^2x^4} - \int \frac{1}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="maxima")

[Out] $2 * \operatorname{integrate}(\operatorname{sqrt}(ax+1) * \operatorname{sqrt}(-ax+1) / x^5, x) / a^2 - 1/2 / (a^2 * x^4) - \operatorname{integrate}(x^{-3}, x)$

Fricas [A] time = 2.12267, size = 316, normalized size = 2.15

$$\frac{a^4 x^4 \log \left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 1 \right) - a^4 x^4 \log \left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 1 \right) + 4 a^2 x^2 + 2 (a^3 x^3 - 2 ax) \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 4}{8 a^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="fricas")

[Out] $\frac{1}{8}(a^4x^4\log(ax\sqrt{(ax+1)/(ax)})\sqrt{-(ax-1)/(ax)}+1) - a^4x^4\log(ax\sqrt{(ax+1)/(ax)})\sqrt{-(ax-1)/(ax)} - 1 + 4a^2x^2 + 2(a^3x^3 - 2ax)\sqrt{(ax+1)/(ax)}\sqrt{-(ax-1)/(ax)} - 4)/(a^2x^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2}{x^5} dx + \int -\frac{a^2}{x^3} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^4} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))**2/x**3,x)

[Out] (Integral(2/x**5, x) + Integral(-a**2/x**3, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**4, x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1+\frac{1}{ax}}\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^3, x)

$$3.73 \quad \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^4} dx$$

Optimal. Leaf size=183

$$\frac{a^3}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{a^3}{4\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{3a^3}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{7a^3}{3\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} + \frac{2a^3}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} - \frac{4a^3}{5\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^5}$$

[Out] $(-4*a^3)/(5*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + (2*a^3)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 - (7*a^3)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (3*a^3)/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - a^3/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^3/(4*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rubi [A] time = 0.504922, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$\frac{a^3}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{a^3}{4\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{3a^3}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{7a^3}{3\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} + \frac{2a^3}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} - \frac{4a^3}{5\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^5}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])}/x^4, x]$

[Out] $(-4*a^3)/(5*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + (2*a^3)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 - (7*a^3)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (3*a^3)/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - a^3/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^3/(4*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u]*(n_.))}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1-u)/(1+u)] + (1*\operatorname{Sqrt}[(1-u)/(1+u)])/u)^n, x] /;$ FreeQ[m, x] && Integer Q[n]

Rule 1612

$\operatorname{Int}[(Px_)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}*((e_.) + (f_.)*(x_))^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,

x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned}
 \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^4} dx &= \int \frac{\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^4} dx \\
 &= -\left(4a\right) \operatorname{Subst}\left(\int \frac{x(a+ax^2)^2}{(-1+x)^6(1+x)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= -\left(4a\right) \operatorname{Subst}\left(\int \left(\frac{a^2}{(-1+x)^6} + \frac{2a^2}{(-1+x)^5} + \frac{7a^2}{4(-1+x)^4} + \frac{3a^2}{4(-1+x)^3} + \frac{a^2}{16(-1+x)^2} - \frac{a^2}{16(1+x)^2}\right) dx, \sqrt{\frac{1-ax}{1+ax}}\right) \\
 &= -\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5} + \frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} - \frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^3}{4\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}
 \end{aligned}$$

Mathematica [A] time = 0.0786524, size = 69, normalized size = 0.38

$$\frac{5a^2x^2 + 2\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2(2a^3x^3 - 2a^2x^2 + 3ax - 3) - 6}{15a^2x^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])/x^4, x]

[Out] (-6 + 5*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 + 2*a^3*x^3))/(15*a^2*x^5)

Maple [A] time = 0.196, size = 84, normalized size = 0.5

$$\frac{1}{a^2} \left(-\frac{1}{5x^5} + \frac{a^2}{3x^3} \right) + \frac{(2a^2x^2 - 2)(2a^2x^2 + 3)}{15x^4a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{1}{5x^5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^4, x)

[Out] $1/a^2*(-1/5/x^5+1/3*a^2/x^3)+2/15/a*(-(a*x-1)/a/x)^{(1/2)}/x^4*((a*x+1)/a/x)^{(1/2)}*(a^2*x^2-1)*(2*a^2*x^2+3)-1/5/a^2/x^5$

Maxima [A] time = 1.04593, size = 76, normalized size = 0.42

$$\frac{1}{3x^3} + \frac{2(2a^4x^5 + a^2x^3 - 3x)\sqrt{ax+1}\sqrt{-ax+1}}{15a^2x^6} - \frac{2}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="maxima")`

[Out] $1/3/x^3 + 2/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1)/(a^2*x^6) - 2/5/(a^2*x^5)$

Fricas [A] time = 2.16558, size = 150, normalized size = 0.82

$$\frac{5a^2x^2 + 2(2a^5x^5 + a^3x^3 - 3ax)\sqrt{\frac{ax+1}{ax}}\sqrt{\frac{-ax-1}{ax}} - 6}{15a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="fricas")`

[Out] $1/15*(5*a^2*x^2 + 2*(2*a^5*x^5 + a^3*x^3 - 3*a*x)*\text{sqrt}((a*x + 1)/(a*x))*\text{sqrt}(-(a*x - 1)/(a*x)) - 6)/(a^2*x^5)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2}{x^6} dx + \int -\frac{a^2}{x^4} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^5} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**4,x)

[Out] (Integral(2/x**6, x) + Integral(-a**2/x**4, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**5, x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}\right)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^4, x)

$$3.74 \quad \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^5} dx$$

Optimal. Leaf size=267

$$\frac{3a^4}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} - \frac{11a^4}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} + \frac{8a^4}{3\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{3a^4}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} + \frac{2a^4}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)}$$

[Out] $(-2*a^4)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6) + (2*a^4)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5 - (3*a^4)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 + (8*a^4)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) - (11*a^4)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^2 + (3*a^4)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^4/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^2 + a^4/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) + (a^4*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/4$

Rubi [A] time = 0.542506, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 1612, 207}

$$\frac{3a^4}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} - \frac{11a^4}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} + \frac{8a^4}{3\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{3a^4}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} + \frac{2a^4}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])/x^5}, x]$

[Out] $(-2*a^4)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6) + (2*a^4)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5 - (3*a^4)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 + (8*a^4)/(3*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) - (11*a^4)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^2 + (3*a^4)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^4/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^2 + a^4/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) + (a^4*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/4$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u]*(n_.))*(x_)^m}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 1612

```
Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^5} dx &= \int \frac{\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^5} dx \\ &= (4a) \operatorname{Subst}\left(\int \frac{x(a+ax^2)^3}{(-1+x)^7(1+x)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\ &= (4a) \operatorname{Subst}\left(\int \left(\frac{a^3}{(-1+x)^7} + \frac{5a^3}{2(-1+x)^6} + \frac{3a^3}{(-1+x)^5} + \frac{2a^3}{(-1+x)^4} + \frac{11a^3}{16(-1+x)^3} + \frac{3a^3}{32(-1+x)^2} + \frac{3a^3}{16(-1+x)}\right) dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\ &= -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6} + \frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5} - \frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} \\ &= -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6} + \frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5} - \frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} \end{aligned}$$

Mathematica [A] time = 0.142864, size = 137, normalized size = 0.51

$$\frac{6a^2x^2 + \sqrt{\frac{1-ax}{ax+1}}(3a^5x^5 + 3a^4x^4 + 2a^3x^3 + 2a^2x^2 - 8ax - 8) - 3a^6x^6 \log(x) + 3a^6x^6 \log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) - 8}{24a^2x^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])/x^5, x]

[Out] $(-8 + 6a^2x^2 + \text{Sqrt}[(1 - ax)/(1 + ax)]) * (-8 - 8ax + 2a^2x^2 + 2a^3x^3 + 3a^4x^4 + 3a^5x^5) - 3a^6x^6 * \text{Log}[x] + 3a^6x^6 * \text{Log}[1 + \text{Sqrt}[(1 - ax)/(1 + ax)]] + ax * \text{Sqrt}[(1 - ax)/(1 + ax)] / (24a^2x^6)$

Maple [A] time = 0.227, size = 153, normalized size = 0.6

$$\frac{1}{a^2} \left(\frac{a^2}{4x^4} - \frac{1}{6x^6} \right) + \frac{1}{24ax^5} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \left(3 \text{Artanh} \left(\frac{1}{\sqrt{-a^2x^2+1}} \right) x^6 a^6 + 3 \sqrt{-a^2x^2+1} x^4 a^4 + 2 a^2 x^2 \sqrt{-a^2x^2+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((1/a/x + (1/a/x - 1)^{(1/2)}) * (1 + 1/a/x)^{(1/2)})^2 / x^5, x)$

[Out] $1/a^2 * (1/4 * a^2/x^4 - 1/6/x^6) + 1/24/a * (- (a*x-1)/a/x)^{(1/2)} / x^5 * ((a*x+1)/a/x)^{(1/2)} * (3 * \text{arctanh}(1/(-a^2*x^2+1)^{(1/2)}) * x^6 * a^6 + 3 * (-a^2*x^2+1)^{(1/2)} * x^4 * a^4 + 2 * a^2 * x^2 * (-a^2*x^2+1)^{(1/2)} - 8 * (-a^2*x^2+1)^{(1/2)}) / (-a^2*x^2+1)^{(1/2)} - 1/6/a^2/x^6$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \left(\frac{1}{16} a^6 \log \left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|} \right) - \frac{1}{16} \sqrt{-a^2x^2+1} a^6 - \frac{(-a^2x^2+1)^{\frac{3}{2}} a^4}{16x^2} - \frac{(-a^2x^2+1)^{\frac{3}{2}} a^2}{8x^4} - \frac{(-a^2x^2+1)^{\frac{3}{2}}}{6x^6} \right)}{a^2} - \frac{1}{3a^2x^6} - \int \frac{1}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((1/a/x + (1/a/x - 1)^{(1/2)}) * (1 + 1/a/x)^{(1/2)})^2 / x^5, x, \text{algorithm}="maxima")$

[Out] $2 * \text{integrate}(\text{sqrt}(a*x + 1) * \text{sqrt}(-a*x + 1) / x^7, x) / a^2 - 1/3 / (a^2 * x^6) - \text{integrate}(x^{(-5)}, x)$

Fricas [A] time = 2.11585, size = 344, normalized size = 1.29

$$\frac{3 a^6 x^6 \log \left(a x \sqrt{\frac{a x+1}{a x}} \sqrt{-\frac{a x-1}{a x}} + 1 \right) - 3 a^6 x^6 \log \left(a x \sqrt{\frac{a x+1}{a x}} \sqrt{-\frac{a x-1}{a x}} - 1 \right) + 12 a^2 x^2 + 2 \left(3 a^5 x^5 + 2 a^3 x^3 - 8 a x \right) \sqrt{\frac{a x+1}{a x}} \sqrt{-\frac{a x-1}{a x}}}{48 a^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="fricas")

[Out] 1/48*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 12*a^2*x^2 + 2*(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 16)/(a^2*x^6)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2}{x^7} dx + \int -\frac{a^2}{x^5} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^6} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**5,x)

[Out] (Integral(2/x**7, x) + Integral(-a**2/x**5, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**6, x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}\right)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^5, x)

$$3.75 \quad \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6} dx$$

Optimal. Leaf size=301

$$\frac{a^5}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{a^5}{4\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{11a^5}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} - \frac{35a^5}{12\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} + \frac{4a^5}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} - \frac{4a^5}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^4}$$

[Out] $(-4*a^5)/(7*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^7) + (2*a^5)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6 - (18*a^5)/(5*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + (4*a^5)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 - (35*a^5)/(12*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (11*a^5)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^5/(12*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^5/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rubi [A] time = 0.575837, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$\frac{a^5}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{a^5}{4\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{11a^5}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} - \frac{35a^5}{12\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} + \frac{4a^5}{\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4} - \frac{4a^5}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{(2*\operatorname{ArcSech}[a*x])/x^6}, x]$

[Out] $(-4*a^5)/(7*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^7) + (2*a^5)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6 - (18*a^5)/(5*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + (4*a^5)/(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 - (35*a^5)/(12*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (11*a^5)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^5/(12*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^5/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u]*(n_..))*(x_..)^{(m_..)}, x_Symbol] := \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{Integer}$

Q[n]

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6} dx &= \int \frac{\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^6} dx \\ &= -\left(4a\right) \operatorname{Subst}\left(\int \frac{x(a+ax^2)^4}{(-1+x)^8(1+x)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right) \\ &= -\left(4a\right) \operatorname{Subst}\left(\int \left(\frac{a^4}{(-1+x)^8} + \frac{3a^4}{(-1+x)^7} + \frac{9a^4}{2(-1+x)^6} + \frac{4a^4}{(-1+x)^5} + \frac{35a^4}{16(-1+x)^4} + \frac{11a^4}{16(-1+x)^3} + \frac{11a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^7} + \frac{2a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6} - \frac{18a^5}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5} + \frac{4a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} - \frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{11a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}\right) \end{aligned}$$

Mathematica [A] time = 0.0991938, size = 85, normalized size = 0.28

$$\frac{21a^2x^2 + 2\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2(8a^5x^5 - 8a^4x^4 + 12a^3x^3 - 12a^2x^2 + 15ax - 15) - 30}{105a^2x^7}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(2*ArcSech[a*x])/x^6, x]

[Out] (-30 + 21*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a^2*x^7)

Maple [A] time = 0.24, size = 92, normalized size = 0.3

$$\frac{1}{a^2} \left(-\frac{1}{7x^7} + \frac{a^2}{5x^5} \right) + \frac{(2a^2x^2 - 2)(8x^4a^4 + 12a^2x^2 + 15)}{105x^6a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{1}{7a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x)`

[Out] $1/a^2*(-1/7/x^7+1/5*a^2/x^5)+2/105/a*(-(a*x-1)/a/x)^(1/2)/x^6*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+12*a^2*x^2+15)-1/7/a^2/x^7$

Maxima [A] time = 1.07831, size = 88, normalized size = 0.29

$$\frac{1}{5x^5} + \frac{2(8a^6x^7 + 4a^4x^5 + 3a^2x^3 - 15x)\sqrt{ax+1}\sqrt{-ax+1}}{105a^2x^8} - \frac{2}{7a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="maxima")`

[Out] $1/5/x^5 + 2/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1)/(a^2*x^8) - 2/7/(a^2*x^7)$

Fricas [A] time = 2.08195, size = 174, normalized size = 0.58

$$\frac{21a^2x^2 + 2(8a^7x^7 + 4a^5x^5 + 3a^3x^3 - 15ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 30}{105a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="fricas")`

[Out] $1/105*(21*a^2*x^2 + 2*(8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*\text{sqrt}((a*x + 1)/(a*x))*\text{sqrt}(-(a*x - 1)/(a*x)) - 30)/(a^2*x^7)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2}{x^8} dx + \int -\frac{a^2}{x^6} dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^7} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**6,x)

[Out] (Integral(2/x**8, x) + Integral(-a**2/x**6, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**7, x))/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}\right)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="giac")

[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^6, x)

3.76 $\int e^{-\operatorname{sech}^{-1}(ax)} x^4 dx$

Optimal. Leaf size=147

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^5}{5a^5} + \frac{\left(16\sqrt{\frac{1-ax}{ax+1}}+5\right)(ax+1)^4}{20a^5} - \frac{\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)(ax+1)^3}{15a^5} + \frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+9\right)(ax+1)^2}{6a^5} - \frac{x}{a^4}$$

[Out] $-(x/a^4) - (\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^5)/(5*a^5) + ((1 + a*x)^2*(9 + 4*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(6*a^5) + ((1 + a*x)^4*(5 + 16*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(20*a^5) - ((1 + a*x)^3*(15 + 17*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(15*a^5)$

Rubi [A] time = 0.619208, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6337, 1804, 1814, 12, 261}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^5}{5a^5} + \frac{\left(16\sqrt{\frac{1-ax}{ax+1}}+5\right)(ax+1)^4}{20a^5} - \frac{\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)(ax+1)^3}{15a^5} + \frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+9\right)(ax+1)^2}{6a^5} - \frac{x}{a^4}$$

Antiderivative was successfully verified.

[In] `Int[x^4/E^ArcSech[a*x], x]`

[Out] $-(x/a^4) - (\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^5)/(5*a^5) + ((1 + a*x)^2*(9 + 4*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(6*a^5) + ((1 + a*x)^4*(5 + 16*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(20*a^5) - ((1 + a*x)^3*(15 + 17*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(15*a^5)$

Rule 6337

`Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] := Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)]/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]`

Rule 1804

`Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]`

```
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)} x^4 dx &= \int \frac{x^4}{\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}} dx \\
&= \frac{4 \operatorname{Subst} \left(\int \frac{(-1+x)^5 x (1+x)^3}{(1+x^2)^6} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{a^5} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^5}{5a^5} - \frac{2 \operatorname{Subst} \left(\int \frac{-16+10x+140x^2-30x^3-80x^4+30x^5+20x^6-10x^7}{(1+x^2)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{5a^5} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^5}{5a^5} + \frac{(1+ax)^4 \left(5 + 16\sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} + \frac{\operatorname{Subst} \left(\int \frac{-128+560x+800x^2-320x^3-160x^4+80x^5}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^5}{5a^5} + \frac{(1+ax)^4 \left(5 + 16\sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} - \frac{(1+ax)^3 \left(15 + 17\sqrt{\frac{1-ax}{1+ax}} \right)}{15a^5} - \frac{\operatorname{Subst} \left(\int \frac{-320+2400x+1600x^2-320x^3-160x^4+80x^5}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right)}{15a^5} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^5}{5a^5} + \frac{(1+ax)^2 \left(9 + 4\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^5} + \frac{(1+ax)^4 \left(5 + 16\sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} - \frac{(1+ax)^3 \left(15 + 17\sqrt{\frac{1-ax}{1+ax}} \right)}{15a^5} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^5}{5a^5} + \frac{(1+ax)^2 \left(9 + 4\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^5} + \frac{(1+ax)^4 \left(5 + 16\sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} - \frac{(1+ax)^3 \left(15 + 17\sqrt{\frac{1-ax}{1+ax}} \right)}{15a^5} \\
&= -\frac{x}{a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)^5}{5a^5} + \frac{(1+ax)^2 \left(9 + 4\sqrt{\frac{1-ax}{1+ax}} \right)}{6a^5} + \frac{(1+ax)^4 \left(5 + 16\sqrt{\frac{1-ax}{1+ax}} \right)}{20a^5} - \frac{(1+ax)^3 \left(15 + 17\sqrt{\frac{1-ax}{1+ax}} \right)}{15a^5}
\end{aligned}$$

Mathematica [A] time = 0.0900951, size = 65, normalized size = 0.44

$$\frac{15a^4x^4 - 4\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2(3a^3x^3 - 3a^2x^2 + 2ax - 2)}{60a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/E^ArcSech[a*x], x]

[Out] (15*a^4*x^4 - 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*x^2 + 3*a^3*x^3))/(60*a^5)

Maple [C] time = 0.54, size = 531, normalized size = 3.6

$$-\frac{ax-1}{60x^7a^{12}} \left(15a^{10}x^{10} \left(\frac{ax+1}{ax} \right)^{7/2} \left(-\frac{ax-1}{ax} \right)^{5/2} + 30a^8x^8 \left(\frac{ax+1}{ax} \right)^{7/2} \left(-\frac{ax-1}{ax} \right)^{5/2} + 30 \left(\frac{ax+1}{ax} \right)^{7/2} \left(-\frac{ax-1}{ax} \right)^{3/2} x^8a^8 + 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)

[Out] -1/60*(a*x-1)/x^7*(15*a^10*x^10*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(5/2)+30*a^8*x^8*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(5/2)+30*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(3/2)*x^8*a^8+30*x^6*ln(a^2*x^2)*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(5/2)*a^6-30*a^7*x^7*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(3/2)+60*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(3/2)*ln(a^2*x^2)*x^6*a^6+12*x^11*a^11-60*x^5*ln(a^2*x^2)*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(3/2)*a^5+30*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(1/2)*ln(a^2*x^2)*x^6*a^6+12*x^10*a^10-60*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(1/2)*ln(a^2*x^2)*x^5*a^5-40*x^9*a^9+30*x^4*ln(a^2*x^2)*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(1/2)*a^4-40*x^8*a^8+40*a^7*x^7+40*x^6*a^6-20*x^3*a^3-20*a^2*x^2+8*a*x+8)/a^12/((a*x+1)/a/x)^(7/2)/(-(a*x-1)/a/x)^(7/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")

[Out] integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

Fricas [A] time = 2.03716, size = 135, normalized size = 0.92

$$\frac{15a^3x^4 - 4(3a^4x^5 - a^2x^3 - 2x)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{60a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a^3*x^4 - 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqrt(-
(a*x - 1)/(a*x)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{x^5}{ax\sqrt{-1 + \frac{1}{ax}}\sqrt{1 + \frac{1}{ax}} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**5/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.77 $\int e^{-\operatorname{sech}^{-1}(ax)} x^3 dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^4}{4a^4} + \frac{\left(9\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{12a^4} - \frac{\left(5\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)^2}{8a^4} + \frac{\left(\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)}{8a^4} + \frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{4a^4}$$

[Out] $-(\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4)/(4*a^4) + ((1 + a*x)*(8 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(8*a^4) - ((1 + a*x)^2*(8 + 5*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(8*a^4) + ((1 + a*x)^3*(4 + 9*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(12*a^4) + \operatorname{ArcTan}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]/(4*a^4)$

Rubi [A] time = 0.56946, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6337, 1804, 1814, 639, 203}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^4}{4a^4} + \frac{\left(9\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{12a^4} - \frac{\left(5\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)^2}{8a^4} + \frac{\left(\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)}{8a^4} + \frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{4a^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/E^{\operatorname{ArcSech}[a*x]}, x]$

[Out] $-(\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4)/(4*a^4) + ((1 + a*x)*(8 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(8*a^4) - ((1 + a*x)^2*(8 + 5*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(8*a^4) + ((1 + a*x)^3*(4 + 9*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(12*a^4) + \operatorname{ArcTan}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]/(4*a^4)$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u]*(n_.))}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)])/u)^n, x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 1804

$\operatorname{Int}[(Pq)*((c_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{Q = \operatorname{PolynomialQuotient}[Pq, a + b*x^2, x], f = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[Pq, a + b*x^2, x], x, 0], g = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[Pq, a + b*x^2, x], x, 1]\}, \operatorname{Simp}[(c*x)^m*(a + b*x^2)^{(p + 1)}*(a*g - b*f*x)/(2*a*b*(p + 1)), x]$

+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 639

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)} x^3 dx &= \int \frac{x^3}{\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax}} dx \\
&= -\frac{4 \operatorname{Subst}\left(\int \frac{(-1+x)^4 x(1+x)^2}{(1+x^2)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^4} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4}{4a^4} + \frac{\operatorname{Subst}\left(\int \frac{8-8x-48x^2+16x^3+16x^4-8x^5}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4}{4a^4} + \frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4} - \frac{\operatorname{Subst}\left(\int \frac{24-144x-96x^2+48x^3}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4}{4a^4} - \frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} + \frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4} + \frac{\operatorname{Subst}\left(\int \frac{24-192x}{(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{48a^4} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4}{4a^4} + \frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} - \frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} + \frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4} + \dots \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4}{4a^4} + \frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} - \frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} + \frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4} + \dots
\end{aligned}$$

Mathematica [C] time = 0.113146, size = 97, normalized size = 0.6

$$\frac{8a^3x^3 + 3a\sqrt{\frac{1-ax}{ax+1}}(-2a^3x^4 - 2a^2x^3 + ax^2 + x) - 3i \log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/E^ArcSech[a*x], x]

[Out] $(8a^3x^3 + 3a\sqrt{(1-ax)/(1+ax)})(x + ax^2 - 2a^2x^3 - 2a^3x^4) - (3I)\operatorname{Log}[(2I)ax + 2\sqrt{(1-ax)/(1+ax)}(1+ax)]/(24a^4)$

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)`

[Out] `int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)`

Fricas [A] time = 2.06981, size = 203, normalized size = 1.25

$$\frac{8a^3x^3 - 3(2a^4x^4 - a^2x^2)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 3\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{24a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")`

[Out] `1/24*(8*a^3*x^3 - 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{x^4}{ax\sqrt{-1 + \frac{1}{ax}}\sqrt{1 + \frac{1}{ax}} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)

[Out] a*Integral(x**4/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")

[Out] integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

3.78 $\int e^{-\operatorname{sech}^{-1}(ax)} x^2 dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^3}{3a^3} + \frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+3\right)(ax+1)^2}{6a^3} - \frac{x}{a^2}$$

[Out] $-(x/a^2) - (\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3)/(3*a^3) + ((1 + a*x)^2*(3 + 4*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(6*a^3)$

Rubi [A] time = 0.513254, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6337, 1804, 1814, 12, 261}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^3}{3a^3} + \frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+3\right)(ax+1)^2}{6a^3} - \frac{x}{a^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/E^{\operatorname{ArcSech}[a*x]}, x]$

[Out] $-(x/a^2) - (\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3)/(3*a^3) + ((1 + a*x)^2*(3 + 4*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(6*a^3)$

Rule 6337

$\operatorname{Int}[E^{\operatorname{ArcSech}[u]}*(n_.)*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /;$ FreeQ[m, x] && Integer Q[n]

Rule 1804

$\operatorname{Int}[(Pq_)*((c_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{Q = \operatorname{PolynomialQuotient}[Pq, a + b*x^2, x], f = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[Pq, a + b*x^2, x], x, 0], g = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[Pq, a + b*x^2, x], x, 1]\}, \operatorname{Simp}[(c*x)^m*(a + b*x^2)^{(p+1)}*(a*g - b*f*x)/(2*a*b*(p+1)), x] + \operatorname{Dist}[c/(2*a*b*(p+1)), \operatorname{Int}[(c*x)^{(m-1)}*(a + b*x^2)^{(p+1)}*\operatorname{ExpandToSum}[2*a*b*(p+1)*x*Q - a*g*m + b*f*(m+2*p+3)*x, x], x]] /;$ FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] / ; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) / ; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] / ; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)} x^2 dx &= \int \frac{x^2}{\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax}} dx \\
&= \frac{4 \operatorname{Subst}\left(\int \frac{(-1+x)^3 x(1+x)}{(1+x^2)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3}{3a^3} - \frac{2 \operatorname{Subst}\left(\int \frac{-4+6x+12x^2-6x^3}{(1+x^2)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} + \frac{\operatorname{Subst}\left(\int \frac{24x}{(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} + \frac{4 \operatorname{Subst}\left(\int \frac{x}{(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3} \\
&= -\frac{x}{a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3}
\end{aligned}$$

Mathematica [A] time = 0.055459, size = 48, normalized size = 0.64

$$\frac{3a^2x^2 - 2(ax - 1)\sqrt{\frac{1-ax}{ax+1}}(ax + 1)^2}{6a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/E^ArcSech[a*x], x]

[Out] (3*a^2*x^2 - 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)

Maple [C] time = 0.496, size = 269, normalized size = 3.6

$$-\frac{ax-1}{6x^5a^8} \left(3a^6x^6 \left(\frac{ax+1}{ax} \right)^{5/2} \left(-\frac{ax-1}{ax} \right)^{3/2} + 3x^4 \ln(a^2x^2) \left(\frac{ax+1}{ax} \right)^{5/2} \left(-\frac{ax-1}{ax} \right)^{3/2} a^4 + 3 \sqrt{-\frac{ax-1}{ax}} \left(\frac{ax+1}{ax} \right)^{5/2} \ln(a^2x^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)), x)

[Out] -1/6*(a*x-1)/x^5*(3*a^6*x^6*((a*x+1)/a/x)^(5/2)*(-(a*x-1)/a/x)^(3/2)+3*x^4*ln(a^2*x^2)*((a*x+1)/a/x)^(5/2)*(-(a*x-1)/a/x)^(3/2)*a^4+3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(5/2)*ln(a^2*x^2)*x^4*a^4-2*a^7*x^7-3*x^3*ln(a^2*x^2)*((a*x+1)/a/x)^(5/2)*(-(a*x-1)/a/x)^(1/2)*a^3-2*x^6*a^6+6*x^5*a^5+6*x^4*a^4-6*x^3*a^3-6*a^2*x^2+2*a*x+2)/a^8/((a*x+1)/a/x)^(5/2)/(-(a*x-1)/a/x)^(5/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)), x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

Fricas [A] time = 1.91037, size = 111, normalized size = 1.48

$$\frac{3ax^2 - 2(a^2x^3 - x)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")

[Out] 1/6*(3*a*x^2 - 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)))/a^2

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{x^3}{ax\sqrt{-1 + \frac{1}{ax}}\sqrt{1 + \frac{1}{ax}} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)

[Out] a*Integral(x**3/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

3.79 $\int e^{-\operatorname{sech}^{-1}(ax)} x dx$

Optimal. Leaf size=94

$$\frac{(ax+1)^2 \left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2}{4a^2} + \frac{(ax+1) \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)}{2a^2} + \frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

[Out] $((1 + a*x)^2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2)/(4*a^2) + ((1 + a*x)*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(2*a^2) + \operatorname{ArcTan}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]/a^2$

Rubi [A] time = 0.308749, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6337, 819, 639, 203}

$$\frac{(ax+1)^2 \left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2}{4a^2} + \frac{(ax+1) \left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)}{2a^2} + \frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x/E^{\operatorname{ArcSech}[a*x]}, x]$

[Out] $((1 + a*x)^2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2)/(4*a^2) + ((1 + a*x)*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))/(2*a^2) + \operatorname{ArcTan}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]/a^2$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*(n_.))}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)])/u)^n, x] /;$ FreeQ[m, x] && Integer Q[n]

Rule 819

$\operatorname{Int}[(d_. + (e_.)*(x_))^{(m_.)}*((f_. + (g_.)*(x_))*((a_. + (c_.)*(x_)^2)^{(p_.)}), x_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{(m - 1)}*(a + c*x^2)^{(p + 1)}*(a*(e*f + d*g) - (c*d*f - a*e*g)*x)]/(2*a*c*(p + 1)), x] - \operatorname{Dist}[1/(2*a*c*(p + 1)), \operatorname{Int}[(d + e*x)^{(m - 2)}*(a + c*x^2)^{(p + 1)}*\operatorname{Simp}[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g])) ||

!ILtQ[m + 2*p + 3, 0])

Rule 639

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int e^{-\operatorname{sech}^{-1}(ax)} x \, dx &= \int \frac{x}{\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax}} \, dx \\ &= -\frac{4 \operatorname{Subst}\left(\int \frac{(-1+x)^2 x}{(1+x^2)^3} \, dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} \\ &= \frac{(1+ax)^2 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2} - \frac{\operatorname{Subst}\left(\int \frac{-2+2x}{(1+x^2)^2} \, dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} \\ &= \frac{(1+ax)^2 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2} + \frac{(1+ax) \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} \\ &= \frac{(1+ax)^2 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2} + \frac{(1+ax) \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2} + \frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} \end{aligned}$$

Mathematica [C] time = 0.0679862, size = 75, normalized size = 0.8

$$\frac{-2ax + ax\sqrt{\frac{1-ax}{ax+1}}(ax+1) + i \log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/E^ArcSech[a*x], x]

[Out] $-\left(-2ax + ax\sqrt{\frac{1-ax}{1+ax}}\right)(1+ax) + I\log\left(\frac{-2Iax + 2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{2a^2}\right)$

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)`

[Out] `int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)`

Fricas [A] time = 2.02363, size = 173, normalized size = 1.84

$$\frac{a^2 x^2 \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 2ax - \arctan\left(\sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")`

[Out] $-\frac{1}{2} \left(a^2 x^2 \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 2ax - \arctan\left(\sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}}\right) \right) / a^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{x^2}{ax \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)

[Out] a*Integral(x**2/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")

[Out] integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

3.80 $\int e^{-\operatorname{sech}^{-1}(ax)} dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{a} + \frac{\log(ax+1)}{a} + \frac{2\log\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{a}$$

[Out] $-\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a}\right) + \frac{\log[1+ax]}{a} + \frac{2\log\left[1+\sqrt{\frac{1-ax}{1+ax}}\right]}{a}$

Rubi [A] time = 0.165642, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6332, 1647, 801, 260}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{a} + \frac{\log(ax+1)}{a} + \frac{2\log\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^{-ArcSech[a*x]}], x]

[Out] $-\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a}\right) + \frac{\log[1+ax]}{a} + \frac{2\log\left[1+\sqrt{\frac{1-ax}{1+ax}}\right]}{a}$

Rule 6332

Int[E^{ArcSech[u]*(n.)}], x_Symbol] := Int[(1/u + Sqrt[(1-u)/(1+u)] + (1*Sqrt[(1-u)/(1+u)]/u)ⁿ], x] /; IntegerQ[n]

Rule 1647

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)²)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x², x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x², x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x², x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x²)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*((a + c*x²)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d² + a*e², 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)} dx &= \int \frac{1}{\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax}} dx \\
&= \frac{4 \operatorname{Subst}\left(\int \frac{(-1+x)x}{(1+x)(1+x^2)^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a} - \frac{2 \operatorname{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a} - \frac{2 \operatorname{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{x}{1+x^2}\right) dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a} + \frac{2 \log\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)}{a} - \frac{2 \operatorname{Subst}\left(\int \frac{x}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a} + \frac{\log(1+ax)}{a} + \frac{2 \log\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0305943, size = 72, normalized size = 1.11

$$\frac{\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) - \sqrt{\frac{1-ax}{ax+1}}(ax+1)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcSech[a*x]), x]
```

```
[Out] (- (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)) + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]
+ a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

Maple [B] time = 0.475, size = 2632, normalized size = 40.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

```
[Out] -1/2*(a*x-1)/x^3*(x^2*ln(a^2*x^2)*((a*x+1)/a/x)^(3/2)*(-(a*x-1)/a/x)^(1/2)*
(-a^2*x^2+1)^(1/2)*a^2-ln(2*((-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(
1/2)*a-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x)^(1/2)*x^2*(-(a
*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a*x+1)/x^2/(a*x-1
))^(1/2)*x-(-a^2*x^2+1)^(1/2)-1)*a^2/(-a^2*x+(-a^2*((a*x+1)/a/x)^(1/2)*x^2
*(-(a*x-1)/a/x)^(1/2)*a-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x
)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a
*x+1)/x^2/(a*x-1))^(1/2))*x^3*a^3-ln(2*((-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(
a*x-1)/a/x)^(1/2)*a-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x)^(1
/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a*x+1
)/x^2/(a*x-1))^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+(-a^2*((a*x+1)/a/x
)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((
(a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x
)^(1/2))/(a*x+1)/x^2/(a*x-1))^(1/2))*x^3*a^3+2*(-a^2*x^2+1)^(1/2)*x^3*a^3-
ln(2*((-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a-((a*x+1)*x)^(1/
2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a
*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a*x+1)/x^2/(a*x-1))^(1/2)*x-(-a^2*x^2+1
)^(1/2)-1)*a^2/(-a^2*x+(-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a
-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a
/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a*x+1)/x^2/(a*x-1))^(1/2
))*x^2*a^2-ln(2*((-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a-((a
*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(
1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a*x+1)/x^2/(a*x-1))^(1/2)*x+
(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+(-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a
/x)^(1/2)*a-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*((a*x+1)/a/x)^(1/2)*x^2*
(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))/(a*x+1)/x^2/(a
*x-1))^(1/2))*x^2*a^2+2*a^2*x^2*(-a^2*x^2+1)^(1/2)+ln(2*((-a^2*((a*x+1)/a
/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a-((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*
((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)
*x)^(1/2))/(a*x+1)/x^2/(a*x-1))^(1/2)*x-(-a^2*x^2+1)^(1/2)-1)*a^2/(-a^2*x+(
-a^2*((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a-((a*x+1)*x)^(1/2)*(-(a
```

$$\begin{aligned}
 & (x-1)x^{1/2} \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \\
 & \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) x^a + \ln(2 \left(-a^2 \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a - ((ax+1)x)^{1/2} \right. \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) x + (-a^2 x^2 + 1)^{1/2} + 1) a^2 / (a^2 x + (-a^2 \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a - ((ax+1)x)^{1/2} \\
 & \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \right) x^a - 2 \left(-a^2 x^2 + 1 \right)^{1/2} a x + \ln(2 \left(-a^2 \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a - ((ax+1)x)^{1/2} \right. \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) x - (-a^2 x^2 + 1)^{1/2} - 1) a^2 / (-a^2 x + (-a^2 \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a - ((ax+1)x)^{1/2} \\
 & \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) + \ln(2 \left(-a^2 \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a - ((ax+1)x)^{1/2} \right. \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) x + (-a^2 x^2 + 1)^{1/2} + 1) a^2 / (a^2 x + (-a^2 \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a - ((ax+1)x)^{1/2} \\
 & \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \left(\frac{(ax+1)/a}{x} \right)^{1/2} x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) x^2 \left(-\frac{(ax-1)/a}{x} \right)^{1/2} a + ((ax+1)x)^{1/2} \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) \\
 & \left. \left(-\frac{(ax-1)x}{(ax+1)/x^2/(ax-1)} \right)^{1/2} \right) - 2 \left(-a^2 x^2 + 1 \right)^{1/2} \right) / a^4 \left(\frac{(ax+1)/a}{x} \right)^{3/2} / \left(-\frac{(ax-1)/a}{x} \right)^{3/2} / \left(-a^2 x^2 + 1 \right)^{1/2}
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

Fricas [A] time = 2.08143, size = 259, normalized size = 3.98

$$\frac{2ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1\right) + \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 1\right) - 2\log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")

[Out]
$$-1/2*(2*a*x*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} - \log(a*x*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} + 1) + \log(a*x*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} - 1) - 2*\log(x))/a$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{x}{ax\sqrt{-1 + \frac{1}{ax}}\sqrt{1 + \frac{1}{ax}} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)

[Out] a*Integral(x/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)

$$3.81 \quad \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x} dx$$

Optimal. Leaf size=46

$$-\frac{2}{\sqrt{\frac{1-ax}{ax+1}} + 1} - 2 \tan^{-1} \left(\sqrt{\frac{1-ax}{ax+1}} \right)$$

[Out] -2/(1 + Sqrt[(1 - a*x)/(1 + a*x)]) - 2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]

Rubi [A] time = 0.401255, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 801, 203}

$$-\frac{2}{\sqrt{\frac{1-ax}{ax+1}} + 1} - 2 \tan^{-1} \left(\sqrt{\frac{1-ax}{ax+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcSech[a*x]*x), x]

[Out] -2/(1 + Sqrt[(1 - a*x)/(1 + a*x)]) - 2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]

Rule 6337

Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x} dx &= \int \frac{1}{x \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax} \right)} dx \\
&= - \left(4 \operatorname{Subst} \left(\int \frac{x}{(1+x)^2 (1+x^2)} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\
&= - \left(4 \operatorname{Subst} \left(\int \left(-\frac{1}{2(1+x)^2} + \frac{1}{2(1+x^2)} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\
&= -\frac{2}{1 + \sqrt{\frac{1-ax}{1+ax}}} - 2 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\
&= -\frac{2}{1 + \sqrt{\frac{1-ax}{1+ax}}} - 2 \tan^{-1} \left(\sqrt{\frac{1-ax}{1+ax}} \right)
\end{aligned}$$

Mathematica [C] time = 0.0358713, size = 74, normalized size = 1.61

$$\sqrt{\frac{1-ax}{ax+1}} \left(\frac{1}{ax} + 1 \right) - \frac{1}{ax} + i \log \left(2 \sqrt{\frac{1-ax}{ax+1}} (ax+1) - 2iax \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x), x]

[Out] -(1/(a*x)) + (1 + 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] + I*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax}} - 1 \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)

[Out] `int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima")`

[Out] `integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)`

Fricas [A] time = 2.30327, size = 163, normalized size = 3.54

$$\frac{ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - ax \arctan \left(\sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} \right) - 1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas")`

[Out] `(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) - 1)/(a*x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)`

[Out] `a*Integral(1/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")`

[Out] `integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)`

$$3.82 \quad \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2} dx$$

Optimal. Leaf size=72

$$\frac{a}{\sqrt{\frac{1-ax}{ax+1}}+1} - \frac{a}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2} + a \left(-\tanh^{-1} \left(\sqrt{\frac{1-ax}{ax+1}} \right) \right)$$

[Out] $-(a/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^2 + a/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]) - a*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]$

Rubi [A] time = 0.382769, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 77, 207}

$$\frac{a}{\sqrt{\frac{1-ax}{ax+1}}+1} - \frac{a}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2} + a \left(-\tanh^{-1} \left(\sqrt{\frac{1-ax}{ax+1}} \right) \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(E^{\operatorname{ArcSech}[a*x]*x^2}), x]$

[Out] $-(a/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^2 + a/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]) - a*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]]$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u]*(n_.))*(x_)^m_.}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)])/u)^n, x] /;$ FreeQ[m, x] && Integer Q[n]

Rule 77

$\operatorname{Int}(((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^{n_.}*((e_.) + (f_.)*(x_))^{p_.}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2} dx &= \int \frac{1}{x^2 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax} \right)} dx \\
 &= (4a) \operatorname{Subst} \left(\int \frac{x}{(-1+x)(1+x)^3} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\
 &= (4a) \operatorname{Subst} \left(\int \left(\frac{1}{2(1+x)^3} - \frac{1}{4(1+x)^2} + \frac{1}{4(-1+x^2)} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\
 &= -\frac{a}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a}{1 + \sqrt{\frac{1-ax}{1+ax}}} + a \operatorname{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\
 &= -\frac{a}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a}{1 + \sqrt{\frac{1-ax}{1+ax}}} - a \tanh^{-1} \left(\sqrt{\frac{1-ax}{1+ax}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.0573269, size = 92, normalized size = 1.28

$$\frac{1}{2} \left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax^2} - \frac{1}{ax^2} + a \log(x) - a \log \left(ax \sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x^2), x]

[Out] (-(1/(a*x^2)) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) + a*Log[x] - a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)

[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxima")

[Out] integrate(1/(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

Fricas [A] time = 2.02791, size = 279, normalized size = 3.88

$$\frac{a^2 x^2 \log \left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 1 \right) - a^2 x^2 \log \left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 1 \right) - 2 ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 2}{4 ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="fricas")

[Out] -1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2)/(a*x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax^2 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax} + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)

[Out] a*Integral(1/(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac")

[Out] undef

3.83

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3} dx$$

Optimal. Leaf size=116

$$-\frac{a^2}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{a^2}{2\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{a^2}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} - \frac{2a^2}{3\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3}$$

[Out] $-a^2/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (2*a^2)/(3*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^3 + a^2/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]^2 - a^2/(2*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rubi [A] time = 0.423809, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$-\frac{a^2}{2\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{a^2}{2\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{a^2}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} - \frac{2a^2}{3\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3}$$

Antiderivative was successfully verified.

[In] `Int[1/(E^ArcSech[a*x]*x^3), x]`

[Out] $-a^2/(2*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (2*a^2)/(3*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))^3 + a^2/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]^2 - a^2/(2*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rule 6337

`Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] := Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]`

Rule 1612

`Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]`

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3} dx &= \int \frac{1}{x^3 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \sqrt{\frac{1-ax}{1+ax}} \right)} dx \\
&= - \left((4a^2) \operatorname{Subst} \left(\int \frac{x(1+x^2)}{(-1+x)^2(1+x)^4} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\
&= - \left((4a^2) \operatorname{Subst} \left(\int \left(\frac{1}{8(-1+x)^2} - \frac{1}{2(1+x)^4} + \frac{1}{2(1+x)^3} - \frac{1}{8(1+x)^2} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\
&= - \frac{a^2}{2 \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)} - \frac{2a^2}{3 \left(1 + \sqrt{\frac{1-ax}{1+ax}} \right)^3} + \frac{a^2}{\left(1 + \sqrt{\frac{1-ax}{1+ax}} \right)^2} - \frac{a^2}{2 \left(1 + \sqrt{\frac{1-ax}{1+ax}} \right)}
\end{aligned}$$

Mathematica [A] time = 0.0523257, size = 43, normalized size = 0.37

$$-\frac{(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2+1}{3ax^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x^3),x]

[Out] -(1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a*x^3)

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)

[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxima")

[Out] integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

Fricas [A] time = 2.06645, size = 109, normalized size = 0.94

$$\frac{(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="fricas")

[Out] -1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)/(a*x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax^3 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax} + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)

[Out] a*Integral(1/(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

$$3.84 \quad \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4} dx$$

Optimal. Leaf size=200

$$\frac{a^3}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} - \frac{a^3}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} + \frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} - \frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^4} - \frac{1}{4}a^3 \tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

[Out] $-a^3/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) + a^3/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^3/(2*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + a^3/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3 - a^3/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 + a^3/(2*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (a^3*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/4$

Rubi [A] time = 0.500469, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 1612, 207}

$$\frac{a^3}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} - \frac{a^3}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} + \frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} - \frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^4} - \frac{1}{4}a^3 \tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(E^{\operatorname{ArcSech}[a*x]*x^4}), x]$

[Out] $-a^3/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) + a^3/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^3/(2*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + a^3/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3 - a^3/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 + a^3/(2*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (a^3*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/4$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*n_)}*(x_)^{m_}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)])/u)^n, x] \text{ ; FreeQ}[m, x] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 1612

$\operatorname{Int}[(P_x)*((a_.) + (b_.)*(x_))^{m_}*((c_.) + (d_.)*(x_))^{n_}*((e_.) + (f_.)*(x_))^{p_}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[P_x*(a + b*x)^m*(c + d*x)^n, x], x]$

$n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x\} \&\& \text{PolyQ}\{Px, x\} \&\& \text{IntegersQ}\{m, n\}$

Rule 207

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \text{ :> } -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \mid\mid \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4} dx &= \int \frac{1}{x^4 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax} \right)} dx \\ &= (4a) \text{Subst} \left(\int \frac{x(a+ax^2)^2}{(-1+x)^3(1+x)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\ &= (4a) \text{Subst} \left(\int \left(\frac{a^2}{8(-1+x)^3} + \frac{a^2}{16(-1+x)^2} + \frac{a^2}{2(1+x)^5} - \frac{3a^2}{4(1+x)^4} + \frac{a^2}{2(1+x)^3} - \frac{a^2}{8(1+x)^2} + \frac{a^2}{16(-1+x)} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\ &= -\frac{a^3}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^3}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^3}{2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^3}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{a^3}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^3}{2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)} \\ &= -\frac{a^3}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^3}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^3}{2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^3}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{a^3}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^3}{2 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)} \end{aligned}$$

Mathematica [A] time = 0.0996863, size = 110, normalized size = 0.55

$$\frac{\sqrt{\frac{1-ax}{ax+1}} \left(a^3 x^3 + a^2 x^2 - 2ax - 2 \right) - a^4 x^4 \log(x) + a^4 x^4 \log \left(ax \sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right) + 2}{8ax^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x^4), x]

[Out] $-(2 + \text{Sqrt}[(1 - a*x)/(1 + a*x)])*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*x^4*\text{Log}[x] + a^4*x^4*\text{Log}[1 + \text{Sqrt}[(1 - a*x)/(1 + a*x)]] + a*x*\text{Sqrt}[(1 - a*x)/(1 + a*x)]/(8*a*x^4)$

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)

[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxima")

[Out] integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

Fricas [A] time = 2.07651, size = 300, normalized size = 1.5

$$\frac{a^4 x^4 \log \left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 1 \right) - a^4 x^4 \log \left(ax \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} - 1 \right) + 2 \left(a^3 x^3 - 2ax \right) \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 4}{16ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="fricas")

[Out] -1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^3*x^3 - 2*a*x)*sqrt(a*x+1)*sqrt(-a*x-1)/a^2 + 4)

$$x^3 - 2ax \sqrt{(ax + 1)/(ax)} \sqrt{-(ax - 1)/(ax)} + 4)/(ax^4)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax^4 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} + x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)

[Out] a*Integral(1/(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac")

[Out] integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

$$3.85 \quad \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5} dx$$

Optimal. Leaf size=233

$$-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{3a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)} + \frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2} - \frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{4a^4}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3} + \frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}$$

[Out] $-a^4/(6*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^4/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - (3*a^4)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (2*a^4)/(5*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + a^4/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 - (4*a^4)/(3*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^4/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 - (3*a^4)/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rubi [A] time = 0.499871, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{3a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)} + \frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2} - \frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{4a^4}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3} + \frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcSech[a*x]*x^5), x]

[Out] $-a^4/(6*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^4/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - (3*a^4)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (2*a^4)/(5*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + a^4/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4 - (4*a^4)/(3*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^4/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 - (3*a^4)/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rule 6337

Int[E^(ArcSech[u]*(n_.))*(x_)^(m_.), x_Symbol] := Int[x^m*(1/u + Sqrt[(1 - u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && IntegerQ[n]

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p], x]

$n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& \text{PolyQ}[Px, x] \&\& \text{IntegersQ}[m, n]$

Rubi steps

$$\begin{aligned} \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5} dx &= \int \frac{1}{x^5 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax} \right)} dx \\ &= - \left((4a) \operatorname{Subst} \left(\int \frac{x(a+ax^2)^3}{(-1+x)^4(1+x)^6} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\ &= - \left((4a) \operatorname{Subst} \left(\int \left(\frac{a^3}{8(-1+x)^4} + \frac{a^3}{8(-1+x)^3} + \frac{3a^3}{32(-1+x)^2} - \frac{a^3}{2(1+x)^6} + \frac{a^3}{(1+x)^5} - \frac{a^3}{(1+x)^4} + \frac{a^3}{2(1+x)^3} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\ &= - \frac{a^4}{6 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{a^4}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{3a^4}{8 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{2a^4}{5 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^5} + \frac{a^4}{\left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^4} - \frac{4a^4}{3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^3} \end{aligned}$$

Mathematica [A] time = 0.0688688, size = 60, normalized size = 0.26

$$-\frac{\sqrt{\frac{1-ax}{ax+1}} (2a^3x^3 - 2a^2x^2 + 3ax - 3)(ax+1)^2 + 3}{15ax^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x^5), x]

[Out] $-(3 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 + 2*a^3*x^3))/(15*a*x^5)$

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^5} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)`

[Out] `int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxima")`

[Out] `integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)`

Fricas [A] time = 2.03202, size = 130, normalized size = 0.56

$$\frac{(2a^5x^5 + a^3x^3 - 3ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 3}{15ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="fricas")`

[Out] `-1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 3)/(a*x^5)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax^5 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax} + x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

```
[Out] a*Integral(1/(a*x**5*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

$$3.86 \quad \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6} dx$$

Optimal. Leaf size=320

$$\frac{a^5}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{3a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} - \frac{3a^5}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{a^5}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} + \frac{a^5}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} + \frac{19a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} - \frac{a^5}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4}$$

[Out] $-a^5/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + a^5/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) - (3*a^5)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) + a^5/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^5/(3*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6) + a^5/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5 - (13*a^5)/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + (19*a^5)/(12*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) - a^5/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 + (3*a^5)/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (a^5*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/8$

Rubi [A] time = 0.566879, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6337, 1612, 207}

$$\frac{a^5}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} + \frac{3a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} - \frac{3a^5}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} - \frac{a^5}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} + \frac{a^5}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} + \frac{19a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} - \frac{a^5}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(E^{\operatorname{ArcSech}[a*x]}*x^6), x]$

[Out] $-a^5/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + a^5/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) - (3*a^5)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) + a^5/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - a^5/(3*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6) + a^5/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5 - (13*a^5)/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) + (19*a^5)/(12*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) - a^5/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 + (3*a^5)/(8*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (a^5*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]])/8$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*n_)}*(x_)^{m_}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)]) + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{Integer}$

Q[n]

Rule 1612

```
Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6} dx &= \int \frac{1}{x^6 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax} \right)} dx \\ &= (4a) \operatorname{Subst} \left(\int \frac{x (a + ax^2)^4}{(-1+x)^5 (1+x)^7} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \\ &= (4a) \operatorname{Subst} \left(\int \left(\frac{a^4}{8(-1+x)^5} + \frac{3a^4}{16(-1+x)^4} + \frac{3a^4}{16(-1+x)^3} + \frac{a^4}{16(-1+x)^2} + \frac{a^4}{2(1+x)^7} - \frac{5a^4}{4(1+x)^6} + \frac{a^4}{4(1+x)^5} \right) dx, \sqrt{\frac{1-ax}{1+ax}} \right) \\ &= -\frac{a^5}{8 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^5}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{3a^5}{8 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^5}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^5}{3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^6} + \frac{a^5}{4 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^5} \\ &= -\frac{a^5}{8 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^5}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{3a^5}{8 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^5}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^5}{3 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^6} + \frac{a^5}{4 \left(1 + \sqrt{\frac{1-ax}{1+ax}}\right)^5} \end{aligned}$$

Mathematica [A] time = 0.141185, size = 129, normalized size = 0.4

$$\frac{\sqrt{\frac{1-ax}{ax+1}} \left(3a^5x^5 + 3a^4x^4 + 2a^3x^3 + 2a^2x^2 - 8ax - 8 \right) - 3a^6x^6 \log(x) + 3a^6x^6 \log \left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1 \right) + 8}{48ax^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x^6),x]

[Out] $-(8 + \text{Sqrt}[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 + 3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*\text{Log}[x] + 3*a^6*x^6*\text{Log}[1 + \text{Sqrt}[(1 - a*x)/(1 + a*x)]] + a*x*\text{Sqrt}[(1 - a*x)/(1 + a*x)])/(48*a*x^6)$

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)

[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxima")

[Out] integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

Fricas [A] time = 2.09554, size = 325, normalized size = 1.02

$$\frac{3 a^6 x^6 \log \left(a x \sqrt{\frac{a x+1}{a x}} \sqrt{-\frac{a x-1}{a x}} + 1 \right) - 3 a^6 x^6 \log \left(a x \sqrt{\frac{a x+1}{a x}} \sqrt{-\frac{a x-1}{a x}} - 1 \right) + 2 \left(3 a^5 x^5 + 2 a^3 x^3 - 8 a x \right) \sqrt{\frac{a x+1}{a x}} \sqrt{-\frac{a x-1}{a x}} + 16}{96 a x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="fricas")

[Out]
$$-1/96*(3*a^6*x^6*\log(a*x*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} + 1) - 3*a^6*x^6*\log(a*x*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} - 1) + 2*(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*\sqrt{(a*x + 1)/(a*x)}*\sqrt{-(a*x - 1)/(a*x)} + 16)/(a*x^6)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax^6 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax} + x^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)

[Out] a*Integral(1/(a*x**6*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**5), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac")

[Out] integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

$$3.87 \quad \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7} dx$$

Optimal. Leaf size=353

$$-\frac{5a^6}{16\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{5a^6}{16\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{3a^6}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{a^6}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} - \frac{5a^6}{12\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{11a^6}{6\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} + \frac{a^6}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)}$$

[Out] $-a^6/(10*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + a^6/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) - (5*a^6)/(12*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (3*a^6)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - (5*a^6)/(16*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (2*a^6)/(7*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^7) + a^6/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6 - (19*a^6)/(10*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + (9*a^6)/(4*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) - (11*a^6)/(6*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^6/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 - (5*a^6)/(16*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rubi [A] time = 0.604905, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$-\frac{5a^6}{16\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)} - \frac{5a^6}{16\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)} + \frac{3a^6}{8\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^2} + \frac{a^6}{\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^2} - \frac{5a^6}{12\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)^3} - \frac{11a^6}{6\left(\sqrt{\frac{1-ax}{ax+1}} + 1\right)^3} + \frac{a^6}{4\left(1 - \sqrt{\frac{1-ax}{ax+1}}\right)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(E^{\operatorname{ArcSech}[a*x]}*x^7), x]$

[Out] $-a^6/(10*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + a^6/(4*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) - (5*a^6)/(12*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + (3*a^6)/(8*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2) - (5*a^6)/(16*(1 - \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])) - (2*a^6)/(7*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^7) + a^6/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^6 - (19*a^6)/(10*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^5) + (9*a^6)/(4*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^4) - (11*a^6)/(6*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^3) + a^6/(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)])^2 - (5*a^6)/(16*(1 + \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]))$

Rule 6337

$\operatorname{Int}[E^{(\operatorname{ArcSech}[u_]*n_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Int}[x^m*(1/u + \operatorname{Sqrt}[(1 - u)/(1 + u)] + (1*\operatorname{Sqrt}[(1 - u)/(1 + u)]/u)^n, x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{Integer}$

Q[n]

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned}
 \int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7} dx &= \int \frac{1}{x^7 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{1-ax}}{ax} \right)} dx \\
 &= - \left((4a) \operatorname{Subst} \left(\int \frac{x(a+ax^2)^5}{(-1+x)^6(1+x)^8} dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\
 &= - \left((4a) \operatorname{Subst} \left(\int \left(\frac{a^5}{8(-1+x)^6} + \frac{a^5}{4(-1+x)^5} + \frac{5a^5}{16(-1+x)^4} + \frac{3a^5}{16(-1+x)^3} + \frac{5a^5}{64(-1+x)^2} - \frac{a^5}{2(1+x)} \right) dx, x, \sqrt{\frac{1-ax}{1+ax}} \right) \right) \\
 &= - \frac{a^6}{10 \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)^5} + \frac{a^6}{4 \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)^4} - \frac{5a^6}{12 \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)^3} + \frac{3a^6}{8 \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)^2} - \frac{5a^6}{16 \left(1 - \sqrt{\frac{1-ax}{1+ax}} \right)} - \frac{a^6}{7(1+x)}
 \end{aligned}$$

Mathematica [A] time = 0.0969453, size = 76, normalized size = 0.22

$$\frac{\sqrt{\frac{1-ax}{ax+1}} (8a^5x^5 - 8a^4x^4 + 12a^3x^3 - 12a^2x^2 + 15ax - 15)(ax+1)^2 + 15}{105ax^7}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^ArcSech[a*x]*x^7), x]

[Out] -(15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a*x^7)

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^7} \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)

[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxima")

[Out] integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

Fricas [A] time = 2.09432, size = 153, normalized size = 0.43

$$\frac{(8a^7x^7 + 4a^5x^5 + 3a^3x^3 - 15ax) \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}} + 15}{105ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="fricas")

[Out] -1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 15)/(a*x^7)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \int \frac{1}{ax^7 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} + x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)

[Out] a*Integral(1/(a*x**7*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**6), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac")

[Out] integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)

$$3.88 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)} (dx)^m}{1-c^2x^2} dx$$

Optimal. Leaf size=89

$$\frac{\sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (dx)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{m+2}{2}, c^2x^2\right)}{cm} + \frac{(dx)^m \operatorname{Hypergeometric2F1}\left(1, \frac{m}{2}, \frac{m+2}{2}, c^2x^2\right)}{cm}$$

[Out] ((d*x)^m*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, c^2*x^2])/(c*m) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, c^2*x^2])/(c*m)

Rubi [A] time = 0.272119, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6341, 6677, 125, 364}

$$\frac{\sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (dx)^m {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{m+2}{2}; c^2x^2\right)}{cm} + \frac{(dx)^m {}_2F_1\left(1, \frac{m}{2}; \frac{m+2}{2}; c^2x^2\right)}{cm}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2), x]

[Out] ((d*x)^m*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, c^2*x^2])/(c*m) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, c^2*x^2])/(c*m)

Rule 6341

Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x], x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b + a*c^2, 0]

Rule 6677

Int[(u_)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(p_), x_Symbol] :> Dist[(c^IntPart[p]*(c*(a + b*x)^n)^FracPart[p]]/(a + b*x)^(n*FracPart[p]), Int[u*(a + b*x)^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && !IntegerQ[p] && !MatchQ[u, x^(n1_.)*(v_.) /; EqQ[n, n1 + 1]]

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1 - c^2x^2} dx &= \frac{d \int \frac{(dx)^{-1+m} \sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}} dx}{c} + \frac{d \int \frac{(dx)^{-1+m}}{1-c^2x^2} dx}{c} \\ &= \frac{(dx)^m {}_2F_1\left(1, \frac{m}{2}; \frac{2+m}{2}; c^2x^2\right)}{cm} + \frac{\left(d\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(dx)^{-1+m}}{\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\ &= \frac{(dx)^m {}_2F_1\left(1, \frac{m}{2}; \frac{2+m}{2}; c^2x^2\right)}{cm} + \frac{\left(d\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(dx)^{-1+m}}{\sqrt{1-c^2x^2}} dx}{c} \\ &= \frac{(dx)^m \sqrt{\frac{1}{1+cx}}\sqrt{1+cx} {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{2+m}{2}; c^2x^2\right)}{cm} + \frac{(dx)^m {}_2F_1\left(1, \frac{m}{2}; \frac{2+m}{2}; c^2x^2\right)}{cm} \end{aligned}$$

Mathematica [F] time = 0.634565, size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1 - c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2), x]

[Out] Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2), x]

Maple [F] time = 0.434, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{-c^2x^2+1} \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)

[Out] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-d^m \int \frac{\sqrt{cx+1}\sqrt{-cx+1}x^m}{c^3x^3-cx} dx - d^m \int \frac{x^m}{2(cx+1)} dx - d^m \int \frac{x^m}{2(cx-1)} dx + \frac{d^m x^m}{cm}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="maxima")

[Out] -d^m*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(c^3*x^3 - c*x), x) - d^m*integrate(1/2*x^m/(c*x + 1), x) - d^m*integrate(1/2*x^m/(c*x - 1), x) + d^m*x^m/(c*m)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{(dx)^m cx \sqrt{\frac{cx+1}{cx}} \sqrt{-\frac{cx-1}{cx}} + (dx)^m}{c^3x^3 - cx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="fricas")

[Out] integral(-((d*x)^m*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + (d*x)^m)/(c^3*x^3 - c*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(dx)^m}{c^2x^3-x} dx + \int \frac{cx(dx)^m \sqrt{-1+\frac{1}{cx}} \sqrt{1+\frac{1}{cx}}}{c^2x^3-x} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*(d*x)**m/(-c**2*x**2+1),x)

[Out] -(Integral((d*x)**m/(c**2*x**3 - x), x) + Integral(c*x*(d*x)**m*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**3 - x), x))/c

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(dx)^m \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1 + \frac{1}{cx}} \right)}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-(d*x)^m*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x)

$$3.89 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)} x^4}{1-c^2x^2} dx$$

Optimal. Leaf size=88

$$-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{cx+1}}} - \frac{x^2}{2c^3} - \frac{\log(1-c^2x^2)}{2c^5} - \frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{cx+1}}}$$

[Out] $-x^2/(2*c^3) - (2*\text{Sqrt}[1 - c*x])/(3*c^5*\text{Sqrt}[(1 + c*x)^{-1}]) - (x^2*\text{Sqrt}[1 - c*x])/(3*c^3*\text{Sqrt}[(1 + c*x)^{-1}]) - \text{Log}[1 - c^2*x^2]/(2*c^5)$

Rubi [A] time = 0.180897, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6341, 1956, 100, 12, 74, 266, 43}

$$-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{cx+1}}} - \frac{x^2}{2c^3} - \frac{\log(1-c^2x^2)}{2c^5} - \frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(E^{\text{ArcSech}[c*x]}*x^4)/(1 - c^2*x^2), x]$

[Out] $-x^2/(2*c^3) - (2*\text{Sqrt}[1 - c*x])/(3*c^5*\text{Sqrt}[(1 + c*x)^{-1}]) - (x^2*\text{Sqrt}[1 - c*x])/(3*c^3*\text{Sqrt}[(1 + c*x)^{-1}]) - \text{Log}[1 - c^2*x^2]/(2*c^5)$

Rule 6341

$\text{Int}[(E^{\text{ArcSech}[(c_*)*(x_*)]}*((d_*)*(x_*)^{(m_*)})/((a_*) + (b_*)*(x_*)^2), x_Symbol] \rightarrow \text{Dist}[d/(a*c), \text{Int}[(d*x)^{(m-1)}*\text{Sqrt}[1/(1+c*x)]]/\text{Sqrt}[1-c*x], x], x] + \text{Dist}[d/c, \text{Int}[(d*x)^{(m-1)}/(a+b*x^2), x], x] /;$ $\text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{EqQ}[b + a*c^2, 0]$

Rule 1956

$\text{Int}[(x_*)^{(m_*)}*((e_*)*((a_*) + (b_*)*(x_*)^{(n_*)})^{(r_*)})^{(p_*)}*((f_*)*((c_*) + (d_*)*(x_*)^{(n_*)})^{(s_*)})^{(q_*)}, x_Symbol] \rightarrow \text{Dist}[(e*(a + b*x^n)^r)^p*(f*(c + d*x^n)^s)^q/((a + b*x^n)^{(p*r)}*(c + d*x^n)^{(q*s))}, \text{Int}[x^m*(a + b*x^n)^{(p*r)}*(c + d*x^n)^{(q*s)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, m, n, p, q, r, s\}, x]$

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)} x^4}{1-c^2x^2} dx &= \int \frac{x^3 \sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}} dx + \int \frac{x^3}{1-c^2x^2} dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{1-c^2x} dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\
&= -\frac{x^2 \sqrt{1-cx}}{3c^3 \sqrt{\frac{1}{1+cx}}} + \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{c^2} - \frac{1}{c^2(-1+c^2x)}\right) dx, x, x^2\right)}{2c} - \frac{\left(\sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int -\frac{2x}{\sqrt{1-cx}\sqrt{1+cx}} dx}{3c^3} \\
&= -\frac{x^2}{2c^3} - \frac{x^2 \sqrt{1-cx}}{3c^3 \sqrt{\frac{1}{1+cx}}} - \frac{\log(1-c^2x^2)}{2c^5} + \frac{\left(2\sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}} dx}{3c^3} \\
&= -\frac{x^2}{2c^3} - \frac{2\sqrt{1-cx}}{3c^5 \sqrt{\frac{1}{1+cx}}} - \frac{x^2 \sqrt{1-cx}}{3c^3 \sqrt{\frac{1}{1+cx}}} - \frac{\log(1-c^2x^2)}{2c^5}
\end{aligned}$$

Mathematica [A] time = 0.18891, size = 69, normalized size = 0.78

$$\frac{3c^2x^2 + 2\sqrt{\frac{1-cx}{cx+1}}(c^3x^3 + c^2x^2 + 2cx + 2) + 3\log(1-c^2x^2)}{6c^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2), x]

[Out] -(3*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3) + 3*Log[1 - c^2*x^2])/(6*c^5)

Maple [A] time = 0.211, size = 69, normalized size = 0.8

$$-\frac{x(c^2x^2 + 2)}{3c^4} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{x^2}{2c^3} - \frac{\ln(c^2x^2 - 1)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1), x)

[Out] $-1/3*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*(c^2*x^2+2)/c^4-1/2*x^2/c^3-1/2/c^5*\ln(c^2*x^2-1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{1}{2}x^2}{c^3} - \frac{\log(cx+1)}{2c^5} - \frac{\log(cx-1)}{2c^5} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}x^3}{c^3x^2-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algorith="maxima")`

[Out] `-integrate(x, x)/c^3 - 1/2*log(c*x + 1)/c^5 - 1/2*log(c*x - 1)/c^5 - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3/(c^3*x^2 - c), x)`

Fricas [A] time = 2.06668, size = 149, normalized size = 1.69

$$\frac{3c^2x^2 + 2(c^3x^3 + 2cx)\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + 3\log(c^2x^2 - 1)}{6c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algorith="fricas")`

[Out] `-1/6*(3*c^2*x^2 + 2*(c^3*x^3 + 2*c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 3*log(c^2*x^2 - 1))/c^5`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)**(1/2))*(1+1/c/x)**(1/2))*x**4/(-c**2*x**2+1),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-x^4*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x)

$$3.90 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)} x^3}{1-c^2x^2} dx$$

Optimal. Leaf size=75

$$-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{cx+1}}} - \frac{x}{c^3} + \frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{2c^4} + \frac{\tanh^{-1}(cx)}{c^4}$$

[Out] $-(x/c^3) - (x*\text{Sqrt}[1 - c*x])/(2*c^3*\text{Sqrt}[(1 + c*x)^{-1}]) + (\text{Sqrt}[(1 + c*x)^{-1}]*\text{Sqrt}[1 + c*x]*\text{ArcSin}[c*x])/(2*c^4) + \text{ArcTanh}[c*x]/c^4$

Rubi [A] time = 0.165123, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6341, 1956, 90, 41, 216, 321, 206}

$$-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{cx+1}}} - \frac{x}{c^3} + \frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{2c^4} + \frac{\tanh^{-1}(cx)}{c^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(E^{\text{ArcSech}[c*x]}*x^3)/(1 - c^2*x^2), x]$

[Out] $-(x/c^3) - (x*\text{Sqrt}[1 - c*x])/(2*c^3*\text{Sqrt}[(1 + c*x)^{-1}]) + (\text{Sqrt}[(1 + c*x)^{-1}]*\text{Sqrt}[1 + c*x]*\text{ArcSin}[c*x])/(2*c^4) + \text{ArcTanh}[c*x]/c^4$

Rule 6341

$\text{Int}[(E^{\text{ArcSech}[(c_)*(x_)]*((d_)*(x_))^{\text{m}_}})/((a_)+(b_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d/(a*c), \text{Int}[(d*x)^{\text{m}-1}*\text{Sqrt}[1/(1+c*x)]]/\text{Sqrt}[1-c*x], x], x] + \text{Dist}[d/c, \text{Int}[(d*x)^{\text{m}-1}/(a+b*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{EqQ}[b+a*c^2, 0]$

Rule 1956

$\text{Int}[(x_)^{\text{m}_}*((e_)*((a_)+(b_)*(x_)^{\text{n}_}))^{\text{r}_})^{\text{p}_}*((f_)*((c_)+(d_)*(x_)^{\text{n}_}))^{\text{s}_})^{\text{q}_}, x_Symbol] \rightarrow \text{Dist}[(e*(a+b*x^n)^r)^p*(f*(c+d*x^n)^s)^q/((a+b*x^n)^{p*r}*(c+d*x^n)^{q*s}), \text{Int}[x^m*(a+b*x^n)^{p*r}*(c+d*x^n)^{q*s}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p, q, r, s\}, x]$

Rule 90

```
Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)(n + 1)*(e + f*x)(p + 1)]/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)n*(e + f*x)p*Simp[a2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))(m_.)*((c_) + (d_.)*(x_))(m_.), x_Symbol] := Int[(a*c + b*d*x2)m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))(m_)*((a_) + (b_.)*(x_)(n_))(p_), x_Symbol] := Simp[(c(n - 1)*(c*x)(m - n + 1)*(a + b*xn)(p + 1)]/(b*(m + n*p + 1)), x] - Dist[(a*cn*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)(m - n)*(a + b*xn)p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)2)(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)} x^3}{1-c^2 x^2} dx &= \frac{\int \frac{x^2 \sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}} dx}{c} + \frac{\int \frac{x^2}{1-c^2 x^2} dx}{c} \\
&= -\frac{x}{c^3} + \frac{\int \frac{1}{1-c^2 x^2} dx}{c^3} + \frac{\left(\sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{x^2}{\sqrt{1-cx} \sqrt{1+cx}} dx}{c} \\
&= -\frac{x}{c^3} - \frac{x\sqrt{1-cx}}{2c^3 \sqrt{\frac{1}{1+cx}}} + \frac{\tanh^{-1}(cx)}{c^4} + \frac{\left(\sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{1}{\sqrt{1-cx} \sqrt{1+cx}} dx}{2c^3} \\
&= -\frac{x}{c^3} - \frac{x\sqrt{1-cx}}{2c^3 \sqrt{\frac{1}{1+cx}}} + \frac{\tanh^{-1}(cx)}{c^4} + \frac{\left(\sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{1}{\sqrt{1-c^2 x^2}} dx}{2c^3} \\
&= -\frac{x}{c^3} - \frac{x\sqrt{1-cx}}{2c^3 \sqrt{\frac{1}{1+cx}}} + \frac{\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{2c^4} + \frac{\tanh^{-1}(cx)}{c^4}
\end{aligned}$$

Mathematica [C] time = 0.153733, size = 110, normalized size = 1.47

$$\frac{c^2 x^2 \sqrt{\frac{1-cx}{cx+1}} + 2cx + cx \sqrt{\frac{1-cx}{cx+1}} + \log(1-cx) - \log(cx+1) - i \log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2icx\right)}{2c^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2), x]

[Out] $-(2*c*x + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + c^2*x^2*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + \text{Log}[1 - c*x] - \text{Log}[1 + c*x] - I*\text{Log}[(-2*I)*c*x + 2*\text{Sqrt}[(1 - c*x)/(1 + c*x)]]*(1 + c*x))/(2*c^4)$

Maple [C] time = 0.213, size = 117, normalized size = 1.6

$$-\frac{x \operatorname{sgn}(c)}{2c^3} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(x \sqrt{-c^2 x^2 + 1} \operatorname{sgn}(c) c - \arctan\left(x \operatorname{sgn}(c) c \frac{1}{\sqrt{-c^2 x^2 + 1}}\right) \right) \frac{1}{\sqrt{-c^2 x^2 + 1}} - \frac{x}{c^3} - \frac{\ln(cx-1)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1), x)

[Out] $-1/2*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}/c^3*(x*(-c^2*x^2+1)^{(1/2)}*c\text{sgn}(c)*c-\arctan(\text{csgn}(c)*c*x/(-c^2*x^2+1)^{(1/2)}))/(-c^2*x^2+1)^{(1/2)}*c\text{sgn}(c)-x/c^3-1/2/c^4*\ln(c*x-1)+1/2/c^4*\ln(c*x+1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{c^3} + \frac{\log(cx+1)}{2c^4} - \frac{\log(cx-1)}{2c^4} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}x^2}{c^3x^2-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algorithm="maxima")`

[Out] $-x/c^3 + 1/2*\log(c*x + 1)/c^4 - 1/2*\log(c*x - 1)/c^4 - \text{integrate}(\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1)*x^2/(c^3*x^2 - c), x)$

Fricas [B] time = 2.0929, size = 213, normalized size = 2.84

$$\frac{c^2x^2\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + 2cx + \arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right) - \log(cx+1) + \log(cx-1)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algorithm="fricas")`

[Out] $-1/2*(c^2*x^2*\text{sqrt}((c*x + 1)/(c*x))*\text{sqrt}(-(c*x - 1)/(c*x)) + 2*c*x + \arctan(\text{sqrt}((c*x + 1)/(c*x))*\text{sqrt}(-(c*x - 1)/(c*x))) - \log(c*x + 1) + \log(c*x - 1))/c^4$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^3*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

$$3.91 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)} x^2}{1-c^2 x^2} dx$$

Optimal. Leaf size=45

$$-\frac{\log(1-c^2 x^2)}{2c^3} - \frac{\sqrt{1-cx}}{c^3 \sqrt{\frac{1}{cx+1}}}$$

[Out] -(Sqrt[1 - c*x]/(c^3*Sqrt[(1 + c*x)^(-1)])) - Log[1 - c^2*x^2]/(2*c^3)

Rubi [A] time = 0.125038, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6341, 1956, 74, 260}

$$-\frac{\log(1-c^2 x^2)}{2c^3} - \frac{\sqrt{1-cx}}{c^3 \sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]

[Out] -(Sqrt[1 - c*x]/(c^3*Sqrt[(1 + c*x)^(-1)])) - Log[1 - c^2*x^2]/(2*c^3)

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_.)]*((d_.)*(x_.))^(m_.))/((a_.) + (b_.)*(x_.)^2), x_Symbol]
:> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
d, m}, x] && EqQ[b + a*c^2, 0]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(r_.))^(p_.)*((f_.)*((c_.) +
(d_.)*(x_)^(n_.))^(s_.))^(q_.), x_Symbol]
:> Dist[((e*(a + b*x^n)^r)^p*(f*(c + d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 74


```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\operatorname{sech}^{-1}(cx)x^2}}{1 - c^2x^2} dx &= \frac{\int \frac{x\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}} dx}{c} + \frac{\int \frac{x}{1-c^2x^2} dx}{c} \\ &= -\frac{\log(1 - c^2x^2)}{2c^3} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\ &= -\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{1+cx}}} - \frac{\log(1 - c^2x^2)}{2c^3} \end{aligned}$$

Mathematica [A] time = 0.0903014, size = 44, normalized size = 0.98

$$-\frac{\log(1 - c^2x^2) + 2\sqrt{\frac{1-cx}{cx+1}}(cx + 1)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2), x]
```

```
[Out] -(2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + Log[1 - c^2*x^2])/(2*c^3)
```

Maple [A] time = 0.2, size = 52, normalized size = 1.2

$$-\frac{x}{c^2} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{\ln(c^2x^2 - 1)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x)`

[Out] $-\frac{-(c*x-1)/c/x)^{(1/2)*x*((c*x+1)/c/x)^{(1/2)}/c^2-1/2/c^3*\ln(c^2*x^2-1)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(cx+1)}{2c^3} - \frac{\log(cx-1)}{2c^3} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}x}{c^3x^2-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algorithm="maxima")`

[Out] $-1/2*\log(c*x + 1)/c^3 - 1/2*\log(c*x - 1)/c^3 - \text{integrate}(\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1)*x/(c^3*x^2 - c), x)$

Fricas [A] time = 2.11662, size = 111, normalized size = 2.47

$$-\frac{2cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + \log(c^2x^2-1)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algorithm="fricas")`

[Out] $-1/2*(2*c*x*\text{sqrt}((c*x + 1)/(c*x))*\text{sqrt}(-(c*x - 1)/(c*x)) + \log(c^2*x^2 - 1))/c^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{c^2x^2-1} dx + \int \frac{cx^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)**(1/2))*(1+1/c/x)**(1/2))*x**2/(-c**2*x**2+1),x)

[Out] -(Integral(x/(c**2*x**2 - 1), x) + Integral(c*x**2*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-x^2*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x)

$$3.92 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)} x}{1-c^2x^2} dx$$

Optimal. Leaf size=37

$$\frac{\sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sin^{-1}(cx)}{c^2} + \frac{\tanh^{-1}(cx)}{c^2}$$

[Out] (Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c^2 + ArcTanh[c*x]/c^2

Rubi [A] time = 0.113641, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6341, 6677, 41, 216, 206}

$$\frac{\sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sin^{-1}(cx)}{c^2} + \frac{\tanh^{-1}(cx)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]

[Out] (Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c^2 + ArcTanh[c*x]/c^2

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Symbol]
  := Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
  x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
  d, m}, x] && EqQ[b + a*c^2, 0]
```

Rule 6677

```
Int[(u_)*((c_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)), x_Symbol] := Dist[(c^IntPart[p]*
(c*(a + b*x)^n)^FracPart[p])/(a + b*x)^(n*FracPart[p]), Int[u*(a + b*x)^(n*p),
x], x] /; FreeQ[{a, b, c, n, p}, x] && !IntegerQ[p] && !MatchQ[u, x^(n1_.)*(v_.)
]; EqQ[n, n1 + 1]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] := Int[(a*c +
b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{e^{\operatorname{sech}^{-1}(cx)x}}{1-c^2x^2} dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}} dx}{c} + \frac{\int \frac{1}{1-c^2x^2} dx}{c} \\ &= \frac{\tanh^{-1}(cx)}{c^2} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\ &= \frac{\tanh^{-1}(cx)}{c^2} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{c} \\ &= \frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \sin^{-1}(cx)}{c^2} + \frac{\tanh^{-1}(cx)}{c^2} \end{aligned}$$

Mathematica [C] time = 0.044183, size = 68, normalized size = 1.84

$$-\frac{\log(1-cx)}{2c^2} + \frac{\log(cx+1)}{2c^2} + \frac{i \log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2icx\right)}{c^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(E^ArcSech[c*x]*x)/(1 - c^2*x^2), x]

[Out] -Log[1 - c*x]/(2*c^2) + Log[1 + c*x]/(2*c^2) + (I*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^2

Maple [C] time = 0.227, size = 92, normalized size = 2.5

$$\frac{x \operatorname{sgn}(c)}{c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \arctan\left(\operatorname{sgn}(c) cx \frac{1}{\sqrt{-(cx+1)(cx-1)}}\right) \frac{1}{\sqrt{-c^2x^2+1}} - \frac{\ln(cx-1)}{2c^2} + \frac{\ln(cx+1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x)

[Out] $(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*\arctan(\operatorname{sgn}(c)*c*x/(-(c*x+1)*(c*x-1))^{(1/2)})/(-c^2*x^2+1)^{(1/2)}*c\operatorname{sgn}(c)/c-1/2/c^2*\ln(c*x-1)+1/2/c^2*\ln(c*x+1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(cx+1)}{2c^2} - \frac{\log(cx-1)}{2c^2} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^2-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algorithm="maxima")

[Out] $1/2*\log(c*x+1)/c^2 - 1/2*\log(c*x-1)/c^2 - \operatorname{integrate}(\operatorname{sqrt}(c*x+1)*\operatorname{sqrt}(-c*x+1)/(c^3*x^2-c),x)$

Fricas [B] time = 2.1015, size = 131, normalized size = 3.54

$$\frac{2 \arctan\left(\sqrt{\frac{cx+1}{cx}} \sqrt{\frac{-cx-1}{cx}}\right) - \log(cx+1) + \log(cx-1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algorithm="fricas")

[Out] $-1/2*(2*\arctan(\operatorname{sqrt}((c*x+1)/(c*x))*\operatorname{sqrt}(-(c*x-1)/(c*x)))) - \log(c*x+1) + \log(c*x-1))/c^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1} dx + \int \frac{1}{c^2x^2-1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x/(-c**2*x**2+1),x)

[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**2 - 1), x) + Integral(1/(c**2*x**2 - 1), x))/c

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-x*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x)

3.93

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2} dx$$

Optimal. Leaf size=71

$$-\frac{\log(1-c^2x^2)}{2c} + \frac{\log(x)}{c} - \frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1} \tanh^{-1}(\sqrt{1-cx}\sqrt{cx+1})}{c}$$

[Out] -((Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/c) + Log[x]/c - Log[1 - c^2*x^2]/(2*c)

Rubi [A] time = 0.119409, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6339, 1956, 92, 208, 266, 36, 29, 31}

$$-\frac{\log(1-c^2x^2)}{2c} + \frac{\log(x)}{c} - \frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1} \tanh^{-1}(\sqrt{1-cx}\sqrt{cx+1})}{c}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[c*x]/(1 - c^2*x^2), x]

[Out] -((Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/c) + Log[x]/c - Log[1 - c^2*x^2]/(2*c)

Rule 6339

```
Int[E^ArcSech[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] := Dist[1/(a*c),
  Int[Sqrt[1/(1 + c*x)]/(x*Sqrt[1 - c*x]), x], x] + Dist[1/c, Int[1/(x*(a +
b*x^2)), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + a*c^2, 0]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_.)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_.))^(q_.), x_Symbol] := Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q]/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 92


```
Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2} dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x\sqrt{1-cx}} dx}{c} + \frac{\int \frac{1}{x(1-c^2x^2)} dx}{c} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(1-c^2x)} dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^2\right)}{2c} + \frac{1}{2}c \operatorname{Subst}\left(\int \frac{1}{1-c^2x} dx, x, x^2\right) - \left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{c-cx^2} dx, x, x^2\right) \\
&= -\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right)}{c} + \frac{\log(x)}{c} - \frac{\log(1-c^2x^2)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.0398584, size = 73, normalized size = 1.03

$$-\frac{\log(1-c^2x^2)}{2c} + \frac{2\log(x)}{c} - \frac{\log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[c*x]/(1 - c^2*x^2), x]

[Out] (2*Log[x])/c - Log[1 - c^2*x^2]/(2*c) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/c

Maple [A] time = 0.204, size = 87, normalized size = 1.2

$$-x\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{Arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\frac{1}{\sqrt{-c^2x^2+1}} - \frac{\ln(cx-1)}{2c} + \frac{\ln(x)}{c} - \frac{\ln(cx+1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1), x)

[Out] -((-c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))-1/2/c*ln(c*x-1)+ln(x)/c-1/2/c*ln(c*x+1)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(x)}{c} - \frac{\log(cx+1)}{2c} - \frac{\log(cx-1)}{2c} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^3 - cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorithm="maxima")

[Out] integrate(1/x, x)/c - 1/2*log(c*x + 1)/c - 1/2*log(c*x - 1)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^3 - c*x), x)

Fricas [B] time = 2.07632, size = 213, normalized size = 3.

$$\frac{\log(c^2x^2 - 1) + \log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + 1\right) - \log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} - 1\right) - 2\log(x)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorithm="fricas")

[Out] -1/2*(log(c^2*x^2 - 1) + log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 1) - log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 1) - 2*log(x))/c

Sympy [A] time = 15.4875, size = 48, normalized size = 0.68

$$\frac{\log\left(-1 + \frac{1}{cx}\right)}{2c} - \frac{\log\left(\sqrt{1 + \frac{1}{cx}}\right)}{c} - \frac{2\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{1 + \frac{1}{cx}}}{2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/(-c**2*x**2+1),x)

[Out] $-\log(-1 + 1/(c*x))/(2*c) - \log(\sqrt{1 + 1/(c*x)})/c - 2*\operatorname{acosh}(\sqrt{2}*\sqrt{1 + 1/(c*x)})/2/c$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorithm="giac")`

[Out] `integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x)`

$$3.94 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)} dx$$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{cx+1}}} - \frac{1}{cx} + \tanh^{-1}(cx)$$

[Out] $-(1/(c*x)) - \operatorname{Sqrt}[1 - c*x]/(c*x*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + \operatorname{ArcTanh}[c*x]$

Rubi [A] time = 0.142613, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6341, 1956, 95, 325, 206}

$$-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{cx+1}}} - \frac{1}{cx} + \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[c*x]}/(x*(1 - c^2*x^2)), x]$

[Out] $-(1/(c*x)) - \operatorname{Sqrt}[1 - c*x]/(c*x*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + \operatorname{ArcTanh}[c*x]$

Rule 6341

$\operatorname{Int}[(E^{\operatorname{ArcSech}[(c_.)*(x_)]})*((d_.)*(x_))^{(m_.)}]/((a_.) + (b_.)*(x_)^2), x_Symbol] \rightarrow \operatorname{Dist}[d/(a*c), \operatorname{Int}[(d*x)^{(m-1)}*\operatorname{Sqrt}[1/(1+c*x)]]/\operatorname{Sqrt}[1-c*x], x], x] + \operatorname{Dist}[d/c, \operatorname{Int}[(d*x)^{(m-1)}/(a+b*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{EqQ}[b + a*c^2, 0]$

Rule 1956

$\operatorname{Int}[(x_)^{(m_.)}*((e_.)*((a_.) + (b_.)*(x_)^{(n_.)})^{(r_.)})^{(p_.)}*((f_.)*((c_.) + (d_.)*(x_)^{(n_.)})^{(s_.)})^{(q_.)}], x_Symbol] \rightarrow \operatorname{Dist}[(e*(a + b*x^n)^r)^p*(f*(c + d*x^n)^s)^q/((a + b*x^n)^{(p*r})*(c + d*x^n)^{(q*s))}, \operatorname{Int}[x^m*(a + b*x^n)^{(p*r})*(c + d*x^n)^{(q*s)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, n, p, q, r, s\}, x]$

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)} dx &= \int \frac{\sqrt{\frac{1}{1+cx}}}{x^2\sqrt{1-cx}} dx + \int \frac{1}{x^2(1-c^2x^2)} dx \\ &= -\frac{1}{cx} + c \int \frac{1}{1-c^2x^2} dx + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\ &= -\frac{1}{cx} - \frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{1+cx}}} + \tanh^{-1}(cx) \end{aligned}$$

Mathematica [A] time = 0.163771, size = 59, normalized size = 1.4

$$-\sqrt{\frac{1-cx}{cx+1}} \left(\frac{1}{cx} + 1\right) - \frac{1}{cx} - \frac{1}{2} \log(1-cx) + \frac{1}{2} \log(cx+1)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x*(1 - c^2*x^2)), x]
```

[Out] $-(1/(c*x)) - (1 + 1/(c*x))*\text{Sqrt}[(1 - c*x)/(1 + c*x)] - \text{Log}[1 - c*x]/2 + \text{Log}[1 + c*x]/2$

Maple [C] time = 0.21, size = 61, normalized size = 1.5

$$-\sqrt{\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}(\text{csgn}(c))^2 - \frac{\ln(cx-1)}{2} - \frac{1}{cx} + \frac{\ln(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x)`

[Out] $-(-(c*x-1)/c/x)^{1/2} * ((c*x+1)/c/x)^{1/2} * \text{csgn}(c)^{2-1/2} * \ln(c*x-1) - 1/c/x + 1/2 * \ln(c*x+1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{c} \int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^4 - cx^2} dx + \frac{1}{2} \log(cx+1) - \frac{1}{2} \log(cx-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algorithm="maxima")`

[Out] `integrate(x^(-2), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^4 - c*x^2), x) + 1/2*log(c*x + 1) - 1/2*log(c*x - 1)`

Fricas [A] time = 2.16174, size = 144, normalized size = 3.43

$$\frac{2cx\sqrt{\frac{cx+1}{cx}}\sqrt{\frac{-cx-1}{cx}} - cx\log(cx+1) + cx\log(cx-1) + 2}{2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algorithm="fricas")`

[Out] $-1/2*(2*c*x*\sqrt{(c*x + 1)/(c*x)}*\sqrt{-(c*x - 1)/(c*x)} - c*x*\log(c*x + 1) + c*x*\log(c*x - 1) + 2)/(c*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^4-x^2} dx + \int \frac{1}{c^2x^4-x^2} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x/(-c**2*x**2+1),x)`

[Out] $-(\text{Integral}(c*x*\sqrt{-1 + 1/(c*x)}*\sqrt{1 + 1/(c*x)})/(c**2*x**4 - x**2), x) + \text{Integral}(1/(c**2*x**4 - x**2), x))/c$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1 + \frac{1}{cx}}}{(c^2x^2 - 1)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algorithm="giac")`

[Out] `integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x), x)`

$$3.95 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)} dx$$

Optimal. Leaf size=108

$$-\frac{1}{2}c \log(1-c^2x^2) - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{cx+1}}} - \frac{1}{2cx^2} + c \log(x) - \frac{1}{2}c\sqrt{\frac{1}{cx+1}}\sqrt{cx+1} \tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)$$

[Out] $-1/(2*c*x^2) - \operatorname{Sqrt}[1 - c*x]/(2*c*x^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (c*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]])/2 + c*\operatorname{Log}[x] - (c*\operatorname{Log}[1 - c^2*x^2])/2$

Rubi [A] time = 0.176461, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6341, 1956, 103, 12, 92, 208, 266, 44}

$$-\frac{1}{2}c \log(1-c^2x^2) - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{cx+1}}} - \frac{1}{2cx^2} + c \log(x) - \frac{1}{2}c\sqrt{\frac{1}{cx+1}}\sqrt{cx+1} \tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[c*x]}/(x^2*(1 - c^2*x^2)), x]$

[Out] $-1/(2*c*x^2) - \operatorname{Sqrt}[1 - c*x]/(2*c*x^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (c*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]])/2 + c*\operatorname{Log}[x] - (c*\operatorname{Log}[1 - c^2*x^2])/2$

Rule 6341

$\operatorname{Int}[(E^{\operatorname{ArcSech}[(c_*)*(x_*)]}*((d_*)*(x_*)^{(m_*)})/((a_*) + (b_*)*(x_*)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[d/(a*c), \operatorname{Int}[(d*x)^{(m-1)}*\operatorname{Sqrt}[1/(1+c*x)]]/\operatorname{Sqrt}[1-c*x], x], x] + \operatorname{Dist}[d/c, \operatorname{Int}[(d*x)^{(m-1)}/(a+b*x^2), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \operatorname{EqQ}[b + a*c^2, 0]$

Rule 1956

$\operatorname{Int}[(x_*)^{(m_*)}*((e_*)*((a_*) + (b_*)*(x_*)^{(n_*)})^{(r_*)})^{(p_*)}*((f_*)*((c_*) + (d_*)*(x_*)^{(n_*)})^{(s_*)})^{(q_*)}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(e*(a + b*x^n)^r)^p*(f*(c + d*x^n)^s)^q/((a + b*x^n)^{(p*r})*(c + d*x^n)^{(q*s))}, \operatorname{Int}[x^m*(a + b*x^n)^{(p*r})*(c + d*x^n)^{(q*s)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, m, n, p, q, r, s$

}, x]

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)} dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^3\sqrt{1-cx}} dx}{c} + \frac{\int \frac{1}{x^3(1-c^2x^2)} dx}{c} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-c^2x)} dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\
&= -\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^2} + \frac{c^2}{x} - \frac{c^4}{-1+c^2x}\right) dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{c^2}{x\sqrt{1-cx}\sqrt{1+cx}} dx}{2c} \\
&= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + c \log(x) - \frac{1}{2}c \log(1-c^2x^2) + \frac{1}{2} \left(c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}} dx \\
&= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + c \log(x) - \frac{1}{2}c \log(1-c^2x^2) - \frac{1}{2} \left(c^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \operatorname{Subst}\left(\int \frac{1}{c-cx^2} dx, x, x^2\right) \\
&= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} - \frac{1}{2}c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right) + c \log(x) - \frac{1}{2}c \log(1-c^2x^2)
\end{aligned}$$

Mathematica [A] time = 0.151469, size = 108, normalized size = 1.

$$\frac{1}{2} \left(-c \log(1-c^2x^2) - \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx^2} - \frac{1}{cx^2} + 3c \log(x) - c \log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)), x]

[Out] $\left(-\frac{1}{c*x^2}\right) - \left(\frac{\sqrt{\frac{1-cx}{1+cx}}*(1+cx)}{c*x^2} + 3*c*\log[x] - c*\log[1 - c^2*x^2] - c*\log[1 + \sqrt{\frac{1-cx}{1+cx}}] + c*x*\sqrt{\frac{1-cx}{1+cx}}\right)/2$

Maple [A] time = 0.227, size = 111, normalized size = 1.

$$-\frac{1}{2x}\sqrt{\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(c^2x^2\operatorname{Artanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) + \sqrt{-c^2x^2+1}\right)\frac{1}{\sqrt{-c^2x^2+1}} - \frac{c \ln(cx-1)}{2} - \frac{1}{2cx^2} + c \ln(x) - \frac{c \ln(1-c^2x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x)`

[Out] $-1/2*(-(c*x-1)/c/x)^{(1/2)}/x*((c*x+1)/c/x)^{(1/2)}*(c^2*x^2*\operatorname{arctanh}(1/(-c^2*x^2+1)^{(1/2)}))+(-c^2*x^2+1)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}-1/2*c*\ln(c*x-1)-1/2/c/x^2+c*\ln(x)-1/2*c*\ln(c*x+1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c \int \frac{1}{x} dx - \frac{1}{2} c \log(cx + 1) - \frac{1}{2} c \log(cx - 1) + \frac{-\frac{1}{2x^2}}{c} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^5 - cx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algorithm="maxima")`

[Out] `c*integrate(1/x, x) - 1/2*c*log(c*x + 1) - 1/2*c*log(c*x - 1) + integrate(x^(-3), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^5 - c*x^3), x)`

Fricas [B] time = 2.12026, size = 344, normalized size = 3.19

$$\frac{2c^2x^2 \log(c^2x^2 - 1) + c^2x^2 \log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + 1\right) - c^2x^2 \log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} - 1\right) - 4c^2x^2 \log(x) + 2cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}}{4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algorithm="fricas")`

[Out] $-1/4*(2*c^2*x^2*\log(c^2*x^2 - 1) + c^2*x^2*\log(c*x*\sqrt{(c*x + 1)/(c*x)}*\sqrt{-(c*x - 1)/(c*x)} + 1) - c^2*x^2*\log(c*x*\sqrt{(c*x + 1)/(c*x)}*\sqrt{-(c*x - 1)/(c*x)} - 1) - 4*c^2*x^2*\log(x) + 2*c*x*\sqrt{(c*x + 1)/(c*x)}*\sqrt{-(c*x - 1)/(c*x)} + 2)/(c*x^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^5-x^3} dx + \int \frac{1}{c^2x^5-x^3} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**2/(-c**2*x**2+1),x)

[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**5 - x**3), x) + Integral(1/(c**2*x**5 - x**3), x))/c

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1+\frac{1}{cx}}}{(c^2x^2-1)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x^2), x)

$$3.96 \quad \int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3(1-c^2x^2)} dx$$

Optimal. Leaf size=85

$$c^2 \tanh^{-1}(cx) - \frac{\sqrt{1-cx}}{3cx^3 \sqrt{\frac{1}{cx+1}}} - \frac{1}{3cx^3} - \frac{2c\sqrt{1-cx}}{3x \sqrt{\frac{1}{cx+1}}} - \frac{c}{x}$$

[Out] $-1/(3*c*x^3) - c/x - \operatorname{Sqrt}[1 - c*x]/(3*c*x^3*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (2*c*\operatorname{Sqrt}[1 - c*x])/(3*x*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + c^2*\operatorname{ArcTanh}[c*x]$

Rubi [A] time = 0.165316, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6341, 1956, 103, 12, 95, 325, 206}

$$c^2 \tanh^{-1}(cx) - \frac{\sqrt{1-cx}}{3cx^3 \sqrt{\frac{1}{cx+1}}} - \frac{1}{3cx^3} - \frac{2c\sqrt{1-cx}}{3x \sqrt{\frac{1}{cx+1}}} - \frac{c}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[E^{\operatorname{ArcSech}[c*x]}/(x^3*(1 - c^2*x^2)), x]$

[Out] $-1/(3*c*x^3) - c/x - \operatorname{Sqrt}[1 - c*x]/(3*c*x^3*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (2*c*\operatorname{Sqrt}[1 - c*x])/(3*x*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + c^2*\operatorname{ArcTanh}[c*x]$

Rule 6341

$\operatorname{Int}[(E^{\operatorname{ArcSech}[(c_*)*(x_*)]}*((d_*)*(x_*)^{(m_*)})/((a_*) + (b_*)*(x_*)^2), x_Symbol] \rightarrow \operatorname{Dist}[d/(a*c), \operatorname{Int}[(d*x)^{(m-1)}*\operatorname{Sqrt}[1/(1 + c*x)]]/\operatorname{Sqrt}[1 - c*x], x], x] + \operatorname{Dist}[d/c, \operatorname{Int}[(d*x)^{(m-1)}/(a + b*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \operatorname{EqQ}[b + a*c^2, 0]$

Rule 1956

$\operatorname{Int}[(x_*)^{(m_*)}*((e_*)*((a_*) + (b_*)*(x_*)^{(n_*)})^{(r_*)})^{(p_*)}*((f_*)*((c_*) + (d_*)*(x_*)^{(n_*)})^{(s_*)})^{(q_*)}, x_Symbol] \rightarrow \operatorname{Dist}[(e*(a + b*x^n)^r)^p*(f*(c + d*x^n)^s)^q/((a + b*x^n)^{(p*r)}*(c + d*x^n)^{(q*s))}, \operatorname{Int}[x^m*(a + b*x^n)^{(p*r)}*(c + d*x^n)^{(q*s)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, n, p, q, r, s\}, x]$

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3(1-c^2x^2)} dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^4\sqrt{1-cx}} dx}{c} + \frac{\int \frac{1}{x^4(1-c^2x^2)} dx}{c} \\
&= -\frac{1}{3cx^3} + c \int \frac{1}{x^2(1-c^2x^2)} dx + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}} dx}{c} \\
&= -\frac{1}{3cx^3} - \frac{c}{x} - \frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}} + c^3 \int \frac{1}{1-c^2x^2} dx - \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int -\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{1+cx}} dx}{3c} \\
&= -\frac{1}{3cx^3} - \frac{c}{x} - \frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}} + c^2 \tanh^{-1}(cx) + \frac{1}{3} \left(2c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}} dx \\
&= -\frac{1}{3cx^3} - \frac{c}{x} - \frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}} - \frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{1+cx}}} + c^2 \tanh^{-1}(cx)
\end{aligned}$$

Mathematica [A] time = 0.241577, size = 90, normalized size = 1.06

$$\frac{6c^2x^2 + 2\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) + 3c^3x^3 \log(1-cx) - 3c^3x^3 \log(cx+1) + 2}{6cx^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)), x]

[Out] -(2 + 6*c^2*x^2 + 2*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3) + 3*c^3*x^3*Log[1 - c*x] - 3*c^3*x^3*Log[1 + c*x])/(6*c*x^3)

Maple [C] time = 0.22, size = 86, normalized size = 1.

$$-\frac{(\operatorname{csgn}(c))^2(2c^2x^2 + 1)}{3x^2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{c^2 \ln(cx-1)}{2} - \frac{1}{3cx^3} - \frac{c}{x} + \frac{c^2 \ln(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1), x)

[Out] $-1/3*(-(c*x-1)/c/x)^{(1/2)}/x^2*((c*x+1)/c/x)^{(1/2)}*c\text{sgn}(c)^2*(2*c^2*x^2+1)-1/2*c^2*\ln(c*x-1)-1/3/c/x^3-c/x+1/2*c^2*\ln(c*x+1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}c^2 \log(cx+1) - \frac{1}{2}c^2 \log(cx-1) + c \int \frac{1}{x^2} dx + \frac{-\frac{1}{3x^3}}{c} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^6 - cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algorithm="maxima")`

[Out] $1/2*c^2*\log(c*x + 1) - 1/2*c^2*\log(c*x - 1) + c*\text{integrate}(x^{(-2)}, x) + \text{integrate}(x^{(-4)}, x)/c - \text{integrate}(\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1)/(c^3*x^6 - c*x^4), x)$

Fricas [A] time = 2.20589, size = 197, normalized size = 2.32

$$\frac{3c^3x^3 \log(cx+1) - 3c^3x^3 \log(cx-1) - 6c^2x^2 - 2(2c^3x^3 + cx)\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} - 2}{6cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algorithm="fricas")`

[Out] $1/6*(3*c^3*x^3*\log(c*x + 1) - 3*c^3*x^3*\log(c*x - 1) - 6*c^2*x^2 - 2*(2*c^3*x^3 + c*x)*\text{sqrt}((c*x + 1)/(c*x))*\text{sqrt}(-(c*x - 1)/(c*x)) - 2)/(c*x^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^6-x^4} dx + \int \frac{1}{c^2x^6-x^4} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**3/(-c**2*x**2+1),x)

[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**6 - x**4), x)
+ Integral(1/(c**2*x**6 - x**4), x))/c

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1+\frac{1}{cx}}}{(c^2x^2-1)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="giac")

[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x
^3), x)

$$3.97 \quad \int \frac{x \left(-1 + a e^{\operatorname{sech}^{-1}(ax)x} \right)}{1 - a^2 x^2} dx$$

Optimal. Leaf size=12

$$-\frac{x e^{\operatorname{sech}^{-1}(ax)}}{a}$$

[Out] $-(E^{\operatorname{ArcSech}[a*x]*x})/a$

Rubi [B] time = 1.05158, antiderivative size = 26, normalized size of antiderivative = 2.17, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {6725, 260, 6341, 1956, 74}

$$-\frac{\sqrt{1-ax}}{a^2 \sqrt{\frac{1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

[In] $\operatorname{Int}[(x*(-1 + a*E^{\operatorname{ArcSech}[a*x]*x)})/(1 - a^2*x^2), x]$

[Out] $-(\operatorname{Sqrt}[1 - a*x]/(a^2*\operatorname{Sqrt}[(1 + a*x)^{-1}])))$

Rule 6725

$\operatorname{Int}[(u_)/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{RationalFunctionE} \operatorname{x} \operatorname{p} \operatorname{a} \operatorname{n} \operatorname{d}[\operatorname{u}/(a + b*x^n), x]\}, \operatorname{Int}[v, x] /; \operatorname{SumQ}[v]] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{IGtQ}[n, 0]$

Rule 260

$\operatorname{Int}[(x_)^{(m_)} / ((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \operatorname{FreeQ}\{a, b, m, n, x\} \ \&\& \ \operatorname{EqQ}[m, n - 1]$

Rule 6341

$\operatorname{Int}[(E^{\operatorname{ArcSech}[c_*](x_)} * ((d_)*(x_)^{(m_)})) / ((a_) + (b_)*(x_)^2), x_Symbol] \rightarrow \operatorname{Dist}[d/(a*c), \operatorname{Int}[(d*x)^{(m-1)} * \operatorname{Sqrt}[1/(1 + c*x)] / \operatorname{Sqrt}[1 - c*x], x, x] + \operatorname{Dist}[d/c, \operatorname{Int}[(d*x)^{(m-1)} / (a + b*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \operatorname{EqQ}[b + a*c^2, 0]$

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] := Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q]/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(-1 + ae^{\operatorname{sech}^{-1}(ax)x})}{1 - a^2x^2} dx &= \int \left(\frac{x}{-1 + a^2x^2} - \frac{ae^{\operatorname{sech}^{-1}(ax)x^2}}{-1 + a^2x^2} \right) dx \\
&= - \left(a \int \frac{e^{\operatorname{sech}^{-1}(ax)x^2}}{-1 + a^2x^2} dx \right) + \int \frac{x}{-1 + a^2x^2} dx \\
&= \frac{\log(1 - a^2x^2)}{2a^2} + \int \frac{x\sqrt{\frac{1}{1+ax}}}{\sqrt{1-ax}} dx - \int \frac{x}{-1 + a^2x^2} dx \\
&= \left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax} \right) \int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}} dx \\
&= -\frac{\sqrt{1-ax}}{a^2\sqrt{\frac{1}{1+ax}}}
\end{aligned}$$

Mathematica [B] time = 0.243834, size = 28, normalized size = 2.33

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2), x]
```

[Out] $-\left(\sqrt{\frac{1 - ax}{1 + ax}}\right) \cdot (1 + ax) / a^2$

Maple [A] time = 0.15, size = 36, normalized size = 3.

$$-\frac{x}{a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x`

[Out] $-1/a*x*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(ax\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)-1\right)x}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x, algorithm="maxima")`

[Out] `-integrate((a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^2*x^2 - 1), x)`

Fricas [A] time = 2.04182, size = 69, normalized size = 5.75

$$-\frac{x \sqrt{\frac{ax+1}{ax}} \sqrt{\frac{ax-1}{ax}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x, algorithm="fricas")`

[Out] $-x\sqrt{(ax + 1)/(ax)}\sqrt{-(ax - 1)/(ax)}/a$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a \int \frac{x^2 \sqrt{-1 + \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}}{a^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-1+a*(1/a/x+(1/a/x-1)**(1/2))*(1+1/a/x)**(1/2))*x)/(-a**2*x**2+1),x)`

[Out] `-a*Integral(x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/(a**2*x**2 - 1), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(ax\left(\sqrt{\frac{1}{ax} + 1}\sqrt{\frac{1}{ax} - 1 + \frac{1}{ax}}\right) - 1\right)x}{a^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x, algorithm="giac")`

[Out] `integrate(-(a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^2*x^2 - 1), x)`

$$3.98 \quad \int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$$

Optimal. Leaf size=61

$$-\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d} + \frac{\operatorname{sech}^{-1}(a+bx)^2}{2d} - \frac{\operatorname{sech}^{-1}(a+bx) \log\left(e^{2\operatorname{sech}^{-1}(a+bx)} + 1\right)}{d}$$

[Out] ArcSech[a + b*x]^2/(2*d) - (ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])])/d - PolyLog[2, -E^(2*ArcSech[a + b*x])]/(2*d)

Rubi [A] time = 0.0977936, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6319, 12, 6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d} + \frac{\operatorname{sech}^{-1}(a+bx)^2}{2d} - \frac{\operatorname{sech}^{-1}(a+bx) \log\left(e^{2\operatorname{sech}^{-1}(a+bx)} + 1\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]/((a*d)/b + d*x), x]

[Out] ArcSech[a + b*x]^2/(2*d) - (ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])])/d - PolyLog[2, -E^(2*ArcSech[a + b*x])]/(2*d)

Rule 6319

Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] & IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b} + dx} dx &= \frac{\operatorname{Subst}\left(\int \frac{b \operatorname{sech}^{-1}(x)}{dx} dx, x, a+bx\right)}{b} \\
&= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}^{-1}(x)}{x} dx, x, a+bx\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}(x)}{x} dx, x, \frac{1}{a+bx}\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int x \tanh(x) dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d} \\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{2 \operatorname{Subst}\left(\int \frac{e^{2x}}{1+e^{2x}} dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d} \\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\operatorname{Subst}\left(\int \log(1+e^{2x}) dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d} \\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2 \cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d} \\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right) \log\left(1 + e^{2 \cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} - \frac{\operatorname{Li}_2\left(-e^{2 \cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0552964, size = 52, normalized size = 0.85

$$\frac{\operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{sech}^{-1}(a+bx) \left(\operatorname{sech}^{-1}(a+bx) + 2 \log\left(e^{-2 \operatorname{sech}^{-1}(a+bx)} + 1\right)\right)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a + b*x]/((a*d)/b + d*x), x]

[Out] (-(ArcSech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x]))]) + PolyLog[2, -E^(-2*ArcSech[a + b*x])])/(2*d)

Maple [A] time = 0.283, size = 104, normalized size = 1.7

$$\frac{(\operatorname{arcsech}(bx+a))^2}{2d} - \frac{\operatorname{arcsech}(bx+a)}{d} \ln \left(1 + \left((bx+a)^{-1} + \sqrt{(bx+a)^{-1} - 1} \sqrt{(bx+a)^{-1} + 1} \right)^2 \right) - \frac{1}{2d} \operatorname{polylog} \left(2, - \left((bx+a)^{-1} + \sqrt{(bx+a)^{-1} - 1} \sqrt{(bx+a)^{-1} + 1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)/(a*d/b+d*x),x)

[Out] 1/2*arcsech(b*x+a)^2/d-arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2))*(1/(b*x+a)+1)^(1/2))^2)/d-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2))*(1/(b*x+a)+1)^(1/2))^2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \log(\sqrt{bx+a+1}\sqrt{-bx-a+1}bx + \sqrt{bx+a+1}\sqrt{-bx-a+1}a + bx + a) \log(bx+a) - 3 \log(bx+a)^2}{2d} - \frac{\log(bx+a + \dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")

[Out] 1/2*(2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)*log(b*x + a) - 3*log(b*x + a)^2)/d - 1/2*(log(b*x + a + 1)*log(b*x + a) + dilog(-b*x - a))/d - 1/2*(log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))/d + integrate((b^2*x + a*b)*log(b*x + a)/(b^2*d*x^2 + 2*a*b*d*x + a^2*d + (b^2*d*x^2 + 2*a*b*d*x + a^2*d - d)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1) - d), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{b \operatorname{arcsech}(bx+a)}{bdx+ad}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")

[Out] `integral(b*arcsech(b*x + a)/(b*d*x + a*d), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{\operatorname{asech}(a+bx)}{a+bx} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asech(b*x+a)/(a*d/b+d*x), x)`

[Out] `b*Integral(asech(a + b*x)/(a + b*x), x)/d`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(bx + a)}{dx + \frac{ad}{b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsech(b*x+a)/(a*d/b+d*x), x, algorithm="giac")`

[Out] `integrate(arcsech(b*x + a)/(d*x + a*d/b), x)`

3.99 $\int x^3 \operatorname{sech}^{-1}(a + bx^4) dx$

Optimal. Leaf size=57

$$\frac{(a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b} - \frac{\tan^{-1}\left(\sqrt{\frac{-a-bx^4+1}{a+bx^4+1}}\right)}{2b}$$

[Out] ((a + b*x^4)*ArcSech[a + b*x^4])/(4*b) - ArcTan[Sqrt[(1 - a - b*x^4)/(1 + a + b*x^4)]]/(2*b)

Rubi [A] time = 0.115012, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6715, 6313, 1961, 12, 203}

$$\frac{(a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b} - \frac{\tan^{-1}\left(\sqrt{\frac{-a-bx^4+1}{a+bx^4+1}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcSech[a + b*x^4],x]

[Out] ((a + b*x^4)*ArcSech[a + b*x^4])/(4*b) - ArcTan[Sqrt[(1 - a - b*x^4)/(1 + a + b*x^4)]]/(2*b)

Rule 6715

Int[(u_)*(x_)^(m_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rule 6313

Int[ArcSech[(c_) + (d_)*(x_)], x_Symbol] := Simp[((c + d*x)*ArcSech[c + d*x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; FreeQ[{c, d}, x]

Rule 1961

Int[(u_)^(r_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,

```
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
+ 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int x^3 \operatorname{sech}^{-1}(a + bx^4) dx &= \frac{1}{4} \operatorname{Subst} \left(\int \operatorname{sech}^{-1}(a + bx) dx, x, x^4 \right) \\
 &= \frac{(a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b} + \frac{1}{4} \operatorname{Subst} \left(\int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx} dx, x, x^4 \right) \\
 &= \frac{(a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b} - b \operatorname{Subst} \left(\int \frac{1}{2b^2(1+x^2)} dx, x, \sqrt{\frac{1-a-bx^4}{1+a+bx^4}} \right) \\
 &= \frac{(a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b} - \frac{\operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-a-bx^4}{1+a+bx^4}} \right)}{2b} \\
 &= \frac{(a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b} - \frac{\tan^{-1} \left(\sqrt{\frac{1-a-bx^4}{1+a+bx^4}} \right)}{2b}
 \end{aligned}$$

Mathematica [A] time = 0.202167, size = 84, normalized size = 1.47

$$\frac{\sqrt{1-(a+bx^4)^2} \sin^{-1}(a+bx^4)}{\sqrt{-\frac{a+bx^4-1}{a+bx^4+1}}(a+bx^4+1)} + (a + bx^4) \operatorname{sech}^{-1}(a + bx^4)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcSech[a + b*x^4],x]

[Out] ((a + b*x^4)*ArcSech[a + b*x^4] + (Sqrt[1 - (a + b*x^4)^2]*ArcSin[a + b*x^4])/Sqrt[-((-1 + a + b*x^4)/(1 + a + b*x^4))]*(1 + a + b*x^4))/(4*b)

Maple [A] time = 0.289, size = 62, normalized size = 1.1

$$\frac{\operatorname{arcsech}(bx^4 + a)x^4}{4} + \frac{\operatorname{arcsech}(bx^4 + a)a}{4b} - \frac{1}{4b} \arctan\left(\sqrt{(bx^4 + a)^{-1} - 1}\sqrt{(bx^4 + a)^{-1} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arcsech(b*x^4+a),x)

[Out] 1/4*arcsech(b*x^4+a)*x^4+1/4/b*arcsech(b*x^4+a)*a-1/4/b*arctan((1/(b*x^4+a)-1)^(1/2)*(1/(b*x^4+a)+1)^(1/2))

Maxima [A] time = 0.993601, size = 51, normalized size = 0.89

$$\frac{(bx^4 + a) \operatorname{arsech}(bx^4 + a) - \arctan\left(\sqrt{\frac{1}{(bx^4 + a)^2} - 1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="maxima")

[Out] 1/4*((b*x^4 + a)*arcsech(b*x^4 + a) - arctan(sqrt(1/(b*x^4 + a)^2 - 1)))/b

Fricas [B] time = 2.47328, size = 613, normalized size = 10.75

$$\frac{2bx^4 \log\left(\frac{(bx^4+a)\sqrt{\frac{b^2x^8+2abx^4+a^2-1}{b^2x^8+2abx^4+a^2}+1}}{bx^4+a}\right) + a \log\left(\frac{(bx^4+a)\sqrt{\frac{b^2x^8+2abx^4+a^2-1}{b^2x^8+2abx^4+a^2}+1}}{x^4}\right) - a \log\left(\frac{(bx^4+a)\sqrt{\frac{b^2x^8+2abx^4+a^2-1}{b^2x^8+2abx^4+a^2}-1}}{x^4}\right) - 2 \arctan\left(\frac{(bx^4+a)\sqrt{\frac{b^2x^8+2abx^4+a^2-1}{b^2x^8+2abx^4+a^2}-1}}{x^4}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x^4*log(((b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 1)/(b*x^4 + a)) + a*log(((b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 1)/x^4) - a*log(((b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 1)/x^4) - 2*arctan((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsech}(bx^4 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(b*x^4 + a), x)
```

3.100 $\int x^{-1+n} \operatorname{sech}^{-1}(a + bx^n) dx$

Optimal. Leaf size=58

$$\frac{(a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn} - \frac{2 \tan^{-1}\left(\sqrt{\frac{-a-bx^n+1}{a+bx^n+1}}\right)}{bn}$$

[Out] ((a + b*x^n)*ArcSech[a + b*x^n])/(b*n) - (2*ArcTan[Sqrt[(1 - a - b*x^n)/(1 + a + b*x^n)]])/(b*n)

Rubi [A] time = 0.113096, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6715, 6313, 1961, 12, 203}

$$\frac{(a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn} - \frac{2 \tan^{-1}\left(\sqrt{\frac{-a-bx^n+1}{a+bx^n+1}}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*ArcSech[a + b*x^n], x]

[Out] ((a + b*x^n)*ArcSech[a + b*x^n])/(b*n) - (2*ArcTan[Sqrt[(1 - a - b*x^n)/(1 + a + b*x^n)]])/(b*n)

Rule 6715

Int[(u_)*(x_)^(m_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rule 6313

Int[ArcSech[(c_) + (d_)*(x_)], x_Symbol] := Simp[((c + d*x)*ArcSech[c + d*x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; FreeQ[{c, d}, x]

Rule 1961

Int[(u_)^(r_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)*

u /. x -> (- (a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n)^r/(b*e - d*x^q)^(1/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegerQ[r]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x^{-1+n} \operatorname{sech}^{-1}(a + bx^n) dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{-1}(a + bx) dx, x, x^n\right)}{n} \\
 &= \frac{(a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx} dx, x, x^n\right)}{n} \\
 &= \frac{(a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn} - \frac{(4b) \operatorname{Subst}\left(\int \frac{1}{2b^2(1+x^2)} dx, x, \sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{n} \\
 &= \frac{(a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn} \\
 &= \frac{(a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn} - \frac{2 \tan^{-1}\left(\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}
 \end{aligned}$$

Mathematica [A] time = 0.247847, size = 84, normalized size = 1.45

$$\frac{\frac{\sqrt{1-(a+bx^n)^2} \sin^{-1}(a+bx^n)}{\sqrt{\frac{a+bx^n-1}{a+bx^n+1}}(a+bx^n+1)} + (a + bx^n) \operatorname{sech}^{-1}(a + bx^n)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)*ArcSech[a + b*x^n], x]

[Out] ((a + b*x^n)*ArcSech[a + b*x^n] + (Sqrt[1 - (a + b*x^n)^2]*ArcSin[a + b*x^n])/((Sqrt[-((-1 + a + b*x^n)/(1 + a + b*x^n))]*(1 + a + b*x^n)))/(b*n)

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int x^{n-1} \operatorname{arcsech}(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)*arcsech(a+b*x^n), x)

[Out] int(x^(n-1)*arcsech(a+b*x^n), x)

Maxima [A] time = 0.99902, size = 54, normalized size = 0.93

$$\frac{(bx^n + a) \operatorname{arsech}(bx^n + a) - \arctan\left(\sqrt{\frac{1}{(bx^n + a)^2} - 1}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*arcsech(a+b*x^n), x, algorithm="maxima")

[Out] ((b*x^n + a)*arcsech(b*x^n + a) - arctan(sqrt(1/(b*x^n + a)^2 - 1)))/(b*n)

Fricas [B] time = 2.60826, size = 1138, normalized size = 19.62

$$2(b \cosh(n \log(x)) + b \sinh(n \log(x))) \log\left(\frac{\sqrt{\frac{2ab+(a^2+b^2-1) \cosh(n \log(x))-(a^2-b^2-1) \sinh(n \log(x))}{\cosh(n \log(x))-\sinh(n \log(x))}+1}}{b \cosh(n \log(x))+b \sinh(n \log(x))+a}}\right) + a \log\left(\frac{\sqrt{\frac{2ab+(a^2+b^2-1) \cosh(n \log(x))}{\cosh(n \log(x))}}}{\cosh(n \log(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="fricas")

[Out] $\frac{1}{2} * (2 * (b * \cosh(n * \log(x)) + b * \sinh(n * \log(x))) * \log\left(\frac{\sqrt{-(2 * a * b + (a^2 + b^2 - 1) * \cosh(n * \log(x)) - (a^2 - b^2 - 1) * \sinh(n * \log(x)))}}{\cosh(n * \log(x)) - \sinh(n * \log(x))} + 1\right) + 1) / (b * \cosh(n * \log(x)) + b * \sinh(n * \log(x)) + a) + a * \log\left(\frac{\sqrt{-(2 * a * b + (a^2 + b^2 - 1) * \cosh(n * \log(x)) - (a^2 - b^2 - 1) * \sinh(n * \log(x)))}}{\cosh(n * \log(x)) - \sinh(n * \log(x))} + 1\right) / (\cosh(n * \log(x)) + \sinh(n * \log(x))) - a * \log\left(\frac{\sqrt{-(2 * a * b + (a^2 + b^2 - 1) * \cosh(n * \log(x)) - (a^2 - b^2 - 1) * \sinh(n * \log(x)))}}{\cosh(n * \log(x)) - \sinh(n * \log(x))} - 1\right) / (\cosh(n * \log(x)) + \sinh(n * \log(x))) - 2 * \arctan\left(\frac{b * \cosh(n * \log(x)) + b * \sinh(n * \log(x)) + a * \sqrt{-(2 * a * b + (a^2 + b^2 - 1) * \cosh(n * \log(x)) - (a^2 - b^2 - 1) * \sinh(n * \log(x)))}}{\cosh(n * \log(x)) - \sinh(n * \log(x))}\right) / (b^2 * \cosh(n * \log(x))^2 + b^2 * \sinh(n * \log(x))^2 + 2 * a * b * \cosh(n * \log(x)) + a^2 + 2 * (b^2 * \cosh(n * \log(x)) + a * b) * \sinh(n * \log(x)) - 1)) / (b * n)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)*asech(a+b*x**n),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{n-1} \operatorname{ar} \operatorname{sech}(bx^n + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="giac")

[Out] integrate(x^(n - 1)*arcsech(b*x^n + a), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```



```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```



```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```