Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/7.4.2-Exponentials-of-inverse-hyperbolic-cotangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [935]. This is test number [199].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (935)	% 0. (0)
Mathematica	% 97.75 (914)	% 2.25 (21)
Maple	% 82.46 (771)	% 17.54 (164)
Maxima	% 48.24 (451)	% 51.76 (484)
Fricas	% 88.66 (829)	% 11.34 (106)
Sympy	% 20.64 (193)	% 79.36 (742)
Giac	% 56.36 (527)	% 43.64 (408)

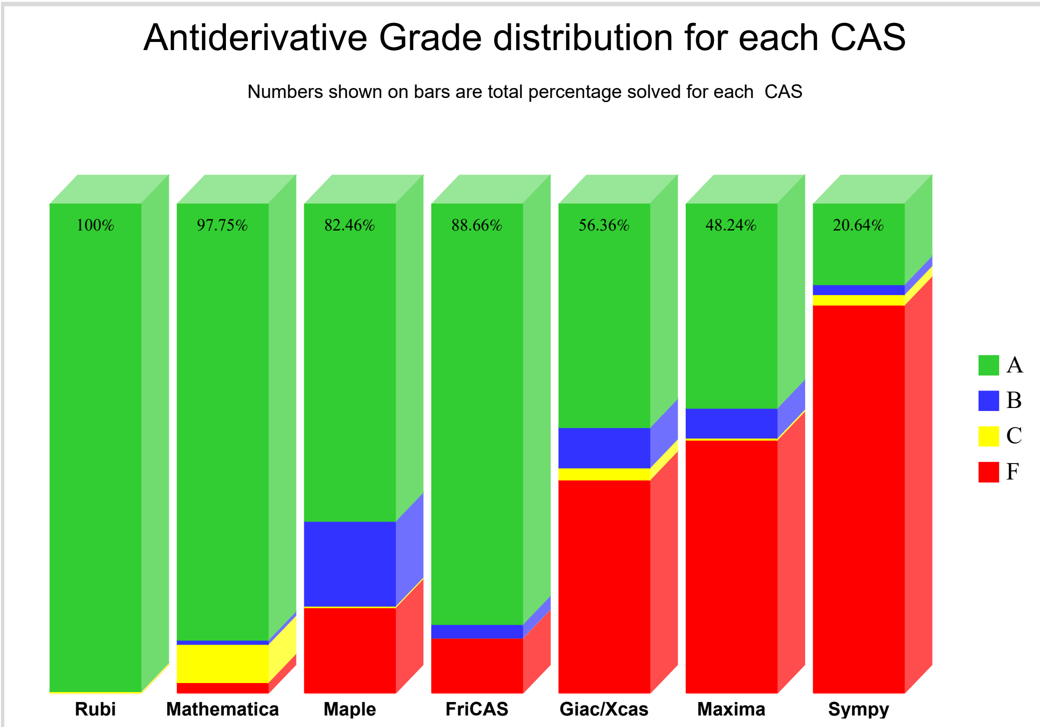
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

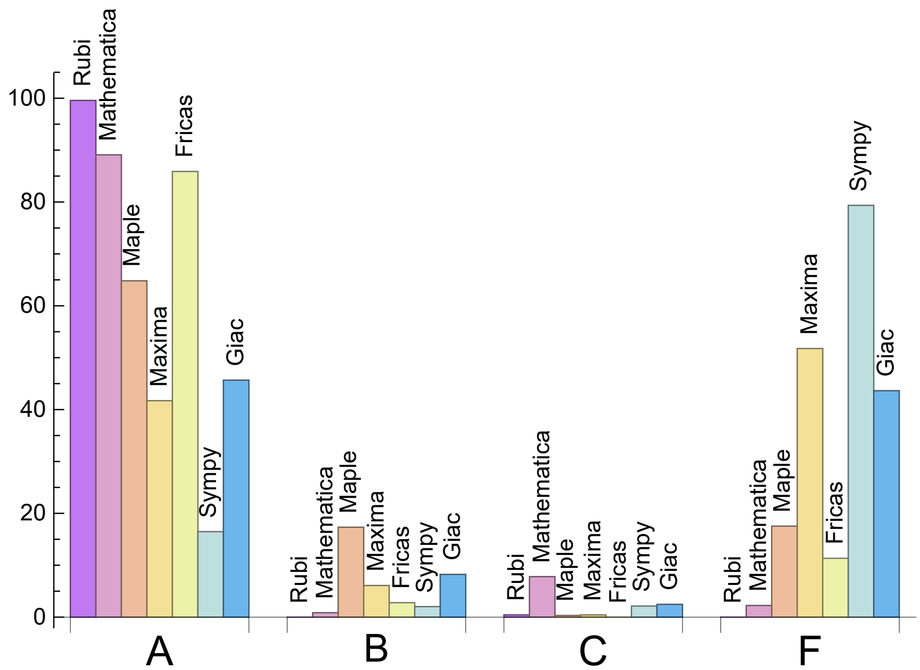
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.57	0.	0.43	0.
Mathematica	89.09	0.86	7.81	2.25
Maple	64.81	17.33	0.32	17.54
Maxima	41.71	6.1	0.43	51.76
Fricas	85.88	2.78	0.	11.34
Sympy	16.47	2.03	2.14	79.36
Giac	45.67	8.24	2.46	43.64

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.2	133.81	1.	110.	1.
Mathematica	0.25	81.17	0.75	70.5	0.67
Maple	0.12	147.23	1.36	92.	0.92
Maxima	1.18	159.29	1.55	131.	1.39
Fricas	1.69	350.99	2.7	231.	2.49
Sympy	8.46	116.52	1.52	54.	0.99
Giac	1.47	171.94	1.6	143.	1.36

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {228, 229, 252, 253, 254, 255, 256, 336, 337, 338, 438, 439, 440, 542, 751, 756, 928, 929, 930, 931}

Mathematica {1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 178, 179, 180, 181, 182, 198, 199, 200, 201, 206, 217, 218, 219, 220, 228, 229, 230, 246, 252, 253, 254, 255, 256, 257, 267, 268, 269, 276, 279, 280, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 298, 326, 331, 332, 334, 336, 337, 338, 355, 356, 357, 358, 361, 365, 366, 367, 368, 379, 380, 381, 382, 383, 387, 388, 389, 396, 397, 398, 399, 400, 404, 406, 407, 413, 414, 415, 416, 421, 422, 423, 426, 429, 430, 431, 432, 433, 438, 439, 440, 441, 442, 451, 452, 453, 454, 457, 458, 466, 467, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 490, 491, 492, 493, 498, 499, 500, 507, 508, 509, 511, 512, 513, 514, 515, 517, 518, 519, 520, 533, 534, 535, 537, 542, 543, 556, 557, 558, 559, 561, 562, 563, 573, 574, 575, 576, 578, 579, 580, 590, 591, 592, 593, 595, 596, 597, 606, 607, 608, 609, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 636, 651, 652, 653, 654, 655, 662, 676, 677, 678, 679, 680, 681, 715, 716, 717, 718, 719, 720, 731, 734, 736, 737, 738, 739, 744, 745, 746, 748, 749, 750, 751, 752, 755, 756, 757, 758, 759, 760, 761, 762, 766, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 796, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 832, 838, 839, 840, 841, 842, 843, 844, 845, 848, 857, 863, 864, 865, 866, 867, 868, 869, 870, 873, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 928, 929, 930, 931, 932}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

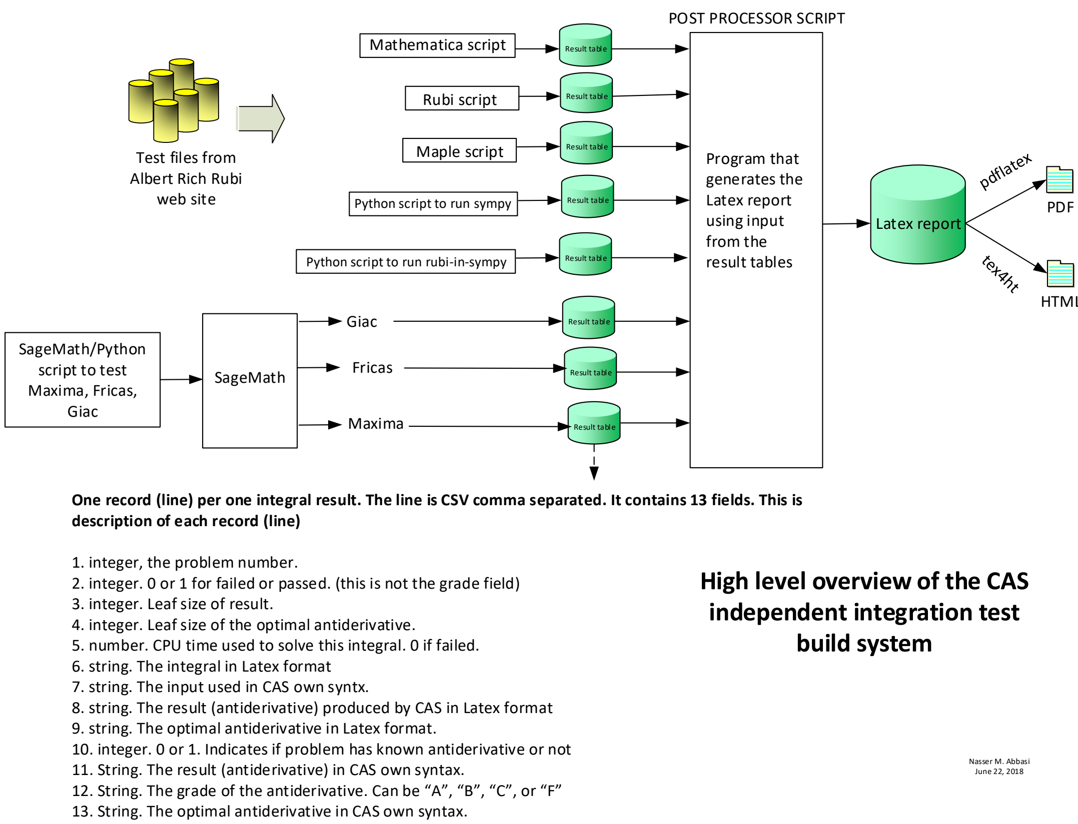
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 932, 933, 934, 935 }

B grade: { }

C grade: { 172, 542, 928, 931 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 65, 66, 68, 69, 70, 71, 77, 78, 80, 81, 83, 84, 86, 87, 89, 95, 96, 98, 99, 101, 102, 104, 105, 107, 113, 114, 121, 122, 124, 127, 132, 134, 137, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 550, 552, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 735, 738, 739, 740, 741, 742, 743, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { 365, 381, 398, 426, 584, 736, 737, 744 }

C grade: { 61, 64, 67, 72, 73, 74, 75, 76, 79, 82, 85, 88, 90, 91, 92, 93, 94, 97, 100, 103, 106, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 123, 125, 126, 128, 129, 130, 131, 133, 135, 136, 138, 172, 267, 268, 269, 429, 430, 431, 432, 451, 452, 453, 454, 458, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 498, 499, 569, 602, 731, 734 }

F grade: { 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 545, 546, 547, 548, 549, 551, 553, 554, 933, 934, 935}

2.1.3 Maple

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 404, 405, 406, 407, 408, 409, 410, 411, 412, 417, 421, 422, 423, 424, 425, 426, 427, 428, 435, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 475, 476, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 676, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 715, 716, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 161, 162, 174, 178, 181, 182, 199, 200, 201, 212, 217, 218, 219, 220, 289, 290, 291, 293, 379, 380, 381, 382, 383, 384, 385, 386, 396, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 418, 419, 420, 429, 430, 431, 432, 433, 434, 436, 437, 450, 451, 452, 453, 454, 471, 472, 473, 474, 477, 478, 479, 500, 501, 502, 526, 527, 528, 529, 530, 531, 532, 586, 623, 624, 625, 626, 651, 674, 675, 677, 678, 679, 680, 681, 714, 717, 718, 719, 720, 776, 777, 778, 779, 792, 793, 794, 795, 796, 810, 811, 812, 813, 825, 826, 827, 828, 829, 838, 839, 840, 843, 844, 845, 863, 864, 865, 868, 869, 870, 890, 891, 892, 893, 894, 914, 915, 916, 917, 918 }

C grade: { 132, 134, 137 }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.4 Maxima

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 270, 271, 272, 273, 274, 275, 276, 279, 280, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 319, 320, 323, 324, 327, 328, 336, 337, 338, 350, 351, 352, 353, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 633, 634, 635, 659, 660, 661, 732, 733, 740, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 880, 904

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 53, 54, 158, 159, 160, 161, 178, 179, 180, 181, 187, 194, 198, 199, 200, 201, 212, 222, 281, 282, 285, 286, 379, 380, 381, 382, 396, 397, 398, 399, 414, 416, 417, 435, 584, 587, 636, 662 }

C grade: { 321, 322, 325, 326 }

F grade: { 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 277, 278, 295, 299, 300, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 369, 370, 371, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 461, 462, 463, 464, 465, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 490, 491, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 512, 513, 514, 515, 517, 518, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade: { 5, 6, 21, 37, 38, 129, 130, 131, 183, 194, 201, 242, 290, 381, 442, 466, 467, 492, 493, 519, 520, 572, 577, 584, 587, 605 }

C grade: { }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 444, 445, 459, 460, 468, 487, 489, 508, 510, 511, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.6 Sympy

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 167, 169, 170, 171, 172, 173, 174, 175, 187, 188, 189, 190, 191, 192, 193, 195, 196, 207, 208, 209, 210, 211, 212, 213, 214, 215, 235, 236, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 266, 267, 268, 269, 288, 292, 301, 302, 303, 304, 305, 324, 327, 328, 341, 342, 343, 344, 345, 387, 388, 389, 390, 391, 392, 393, 394, 395, 404, 405, 406, 407, 408, 409, 410, 411, 412, 421, 422, 423, 424, 425, 427, 428, 502, 528, 564, 565, 566, 567, 568, 569, 570, 571, 572, 585, 588, 589, 598, 599, 600, 601, 602, 603, 604, 605, 780, 781, 782, 783, 784, 785, 786, 787, 788, 797, 798, 799, 800, 801, 802, 803, 804, 805, 814, 815, 816, 817, 818, 819, 820, 821 }

B grade: { 134, 168, 176, 194, 306, 307, 308, 309, 346, 347, 348, 349, 426, 581, 582, 583, 584, 586, 587 }

C grade: { 137, 323, 325, 326, 338, 447, 552, 624, 625, 626, 651, 652, 767, 770, 838, 839, 840, 863, 864, 865 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 396, 397, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 417, 418, 419, 420, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 573, 574, 575, 576, 577, 578, 579, 580, 590, 591, 592, 593, 594, 595, 596, 597, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 796, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.7 Giac

A grade: { 1, 4, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 222, 226, 227, 228, 229, 230, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 287, 288, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 409, 410, 411, 412, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 427, 428, 435, 452, 453, 454, 475, 476, 477, 478, 479, 498, 499, 500, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 587, 588, 589, 590, 591, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 623, 624, 625, 626, 627, 651, 652, 653, 673, 674, 675, 676, 677, 678, 712, 713, 714, 715, 716, 717, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 838, 839, 840, 841, 863, 864, 865, 866, 886, 887, 888, 889, 890, 892, 910, 911, 912, 913, 914, 916 }

B grade: { 2, 3, 5, 6, 7, 8, 9, 20, 21, 29, 37, 38, 39, 40, 41, 158, 159, 160, 161, 174, 178, 179, 180, 181, 191, 194, 212, 235, 236, 281, 282, 285, 286, 289, 379, 380, 381, 382, 383, 396, 397, 398, 399, 404, 405, 406, 407, 408, 413, 414, 426, 450, 451, 501, 530, 531, 532, 583, 584, 586, 610, 629, 655, 679, 680, 681, 718, 719, 720, 799, 800, 891, 893, 894, 915, 917, 918 }

C grade: { 231, 247, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328 }

F grade: { 50, 51, 52, 53, 54, 55, 56, 57, 58, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 177, 187, 197, 202, 206, 216, 217, 218, 219, 220, 221, 223, 224, 225, 295, 329, 330, 331, 332, 335, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 418, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 594, 595, 596, 597, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 628, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 654, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 810, 811, 812, 813, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	68	193	274	227	0	246
normalized size	1	1.	0.6	1.69	2.4	1.99	0.	2.16
time (sec)	N/A	0.123	0.069	0.187	1.03	1.686	0.	1.181

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	60	173	224	204	0	204
normalized size	1	1.	0.67	1.92	2.49	2.27	0.	2.27
time (sec)	N/A	0.093	0.047	0.128	1.088	1.691	0.	1.197

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	49	152	173	180	0	159
normalized size	1	1.	0.78	2.41	2.75	2.86	0.	2.52
time (sec)	N/A	0.064	0.036	0.123	0.982	1.569	0.	1.163

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	36	36	41	97	122	155	0	77
normalized size	1	1.	1.14	2.69	3.39	4.31	0.	2.14
time (sec)	N/A	0.036	0.022	0.12	1.025	1.569	0.	1.157

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	22	22	36	132	93	150	0	95
normalized size	1	1.	1.64	6.	4.23	6.82	0.	4.32
time (sec)	N/A	0.043	0.013	0.128	1.563	1.531	0.	1.174

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	24	24	27	220	72	112	0	72
normalized size	1	1.	1.12	9.17	3.	4.67	0.	3.
time (sec)	N/A	0.025	0.019	0.13	1.543	1.635	0.	1.154

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	42	257	123	144	0	117
normalized size	1	1.	1.11	6.76	3.24	3.79	0.	3.08
time (sec)	N/A	0.031	0.042	0.126	1.525	1.728	0.	1.136

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	51	284	184	161	0	176
normalized size	1	1.	0.68	3.79	2.45	2.15	0.	2.35
time (sec)	N/A	0.062	0.078	0.135	1.552	1.559	0.	1.173

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	59	308	232	185	0	221
normalized size	1	1.	0.67	3.5	2.64	2.1	0.	2.51
time (sec)	N/A	0.083	0.094	0.138	1.561	1.966	0.	1.159

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	58	100	37	63
normalized size	1	1.	1.	0.91	1.35	2.33	0.86	1.47
time (sec)	N/A	0.053	0.019	0.042	1.018	1.872	1.905	1.165

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	46	76	27	51
normalized size	1	1.	1.	0.94	1.39	2.3	0.82	1.55
time (sec)	N/A	0.049	0.014	0.039	1.009	1.802	0.72	1.139

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	35	59	20	41
normalized size	1	1.	1.	0.92	1.35	2.27	0.77	1.58
time (sec)	N/A	0.033	0.012	0.037	1.167	1.825	1.214	1.141

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	18	35	10	19
normalized size	1	1.	1.	1.	1.29	2.5	0.71	1.36
time (sec)	N/A	0.014	0.011	0.042	1.018	1.785	0.098	1.127

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	18	34	10	20
normalized size	1	1.	1.	1.	1.29	2.43	0.71	1.43
time (sec)	N/A	0.04	0.009	0.043	1.007	1.793	0.179	1.164

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	24	58	15	27
normalized size	1	1.	1.	1.	1.26	3.05	0.79	1.42
time (sec)	N/A	0.045	0.011	0.044	1.13	1.942	2.053	1.167

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	41	88	26	43
normalized size	1	1.	1.	0.94	1.24	2.67	0.79	1.3
time (sec)	N/A	0.047	0.014	0.047	0.983	1.837	2.127	1.148

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	38	51	104	34	54
normalized size	1	1.	1.	0.95	1.27	2.6	0.85	1.35
time (sec)	N/A	0.051	0.015	0.044	0.983	1.973	1.614	1.123

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	75	471	246	267	0	231
normalized size	1	1.	0.64	3.99	2.08	2.26	0.	1.96
time (sec)	N/A	1.071	0.08	0.166	1.	1.982	0.	1.177

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	66	421	196	243	0	189
normalized size	1	1.	0.72	4.58	2.13	2.64	0.	2.05
time (sec)	N/A	0.872	0.063	0.165	1.007	1.956	0.	1.189

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	54	247	149	217	0	161
normalized size	1	1.	0.87	3.98	2.4	3.5	0.	2.6
time (sec)	N/A	0.811	0.047	0.136	1.009	1.841	0.	1.167

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	53	363	122	262	0	123
normalized size	1	1.	1.15	7.89	2.65	5.7	0.	2.67
time (sec)	N/A	0.783	0.059	0.173	1.483	1.861	0.	1.207

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	41	593	97	162	0	115
normalized size	1	1.	0.8	11.63	1.9	3.18	0.	2.25
time (sec)	N/A	0.074	0.082	0.169	1.473	1.9	0.	1.186

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	56	642	149	194	0	146
normalized size	1	1.	0.62	7.05	1.64	2.13	0.	1.6
time (sec)	N/A	0.453	0.093	0.171	1.528	1.945	0.	1.189

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	66	666	208	213	0	203
normalized size	1	1.	0.71	7.16	2.24	2.29	0.	2.18
time (sec)	N/A	0.743	0.112	0.171	1.493	1.95	0.	1.214

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	78	155	49	105
normalized size	1	1.	1.	0.91	1.37	2.72	0.86	1.84
time (sec)	N/A	0.066	0.046	0.047	0.985	1.706	0.353	1.21

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	66	130	39	93
normalized size	1	1.	1.	0.94	1.4	2.77	0.83	1.98
time (sec)	N/A	0.06	0.036	0.043	1.072	1.712	0.32	1.118

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	55	109	31	86
normalized size	1	1.	1.	0.92	1.41	2.79	0.79	2.21
time (sec)	N/A	0.041	0.028	0.045	1.082	1.84	0.327	1.129

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	35	81	19	62
normalized size	1	1.	0.96	0.96	1.3	3.	0.7	2.3
time (sec)	N/A	0.019	0.019	0.046	0.998	1.712	0.319	1.124

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	16	46	8	77
normalized size	1	1.	1.	1.	1.23	3.54	0.62	5.92
time (sec)	N/A	0.039	0.01	0.046	1.02	1.787	0.356	1.12

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	46	116	26	54
normalized size	1	1.	1.	0.97	1.44	3.62	0.81	1.69
time (sec)	N/A	0.051	0.024	0.047	0.97	1.786	0.412	1.138

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	65	155	41	84
normalized size	1	1.	1.	0.93	1.41	3.37	0.89	1.83
time (sec)	N/A	0.054	0.03	0.053	1.019	1.813	0.459	1.157

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	76	173	49	100
normalized size	1	1.	1.	0.94	1.41	3.2	0.91	1.85
time (sec)	N/A	0.061	0.042	0.049	1.002	1.893	0.463	1.182

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	68	193	274	220	0	136
normalized size	1	1.	0.6	1.69	2.4	1.93	0.	1.19
time (sec)	N/A	0.127	0.067	0.125	1.052	1.963	0.	1.16

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	60	173	224	198	0	116
normalized size	1	1.	0.67	1.92	2.49	2.2	0.	1.29
time (sec)	N/A	0.1	0.049	0.128	1.015	1.927	0.	1.197

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	49	152	176	177	0	96
normalized size	1	1.	0.77	2.38	2.75	2.77	0.	1.5
time (sec)	N/A	0.068	0.035	0.128	1.002	1.84	0.	1.211

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	42	98	122	155	0	70
normalized size	1	1.	1.14	2.65	3.3	4.19	0.	1.89
time (sec)	N/A	0.039	0.024	0.118	1.048	1.859	0.	1.155

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	20	20	34	130	95	151	0	80
normalized size	1	1.	1.7	6.5	4.75	7.55	0.	4.
time (sec)	N/A	0.048	0.015	0.125	1.599	1.848	0.	1.161

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	25	25	26	220	74	112	0	84
normalized size	1	1.	1.04	8.8	2.96	4.48	0.	3.36
time (sec)	N/A	0.027	0.02	0.128	1.505	1.864	0.	1.152

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	41	260	126	143	0	212
normalized size	1	1.	1.02	6.5	3.15	3.58	0.	5.3
time (sec)	N/A	0.037	0.046	0.138	1.574	1.854	0.	1.194

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	52	284	185	155	0	219
normalized size	1	1.	0.68	3.74	2.43	2.04	0.	2.88
time (sec)	N/A	0.066	0.079	0.13	1.57	1.854	0.	1.169

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	59	308	234	180	0	348
normalized size	1	1.	0.67	3.5	2.66	2.05	0.	3.95
time (sec)	N/A	0.09	0.094	0.138	1.519	1.937	0.	1.188

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	58	100	37	63
normalized size	1	1.	1.	0.93	1.38	2.38	0.88	1.5
time (sec)	N/A	0.057	0.019	0.039	0.986	1.746	0.277	1.137

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	46	76	27	51
normalized size	1	1.	1.	0.97	1.39	2.3	0.82	1.55
time (sec)	N/A	0.051	0.014	0.039	1.02	1.585	0.271	1.147

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	35	59	20	41
normalized size	1	1.	1.	0.96	1.4	2.36	0.8	1.64
time (sec)	N/A	0.032	0.012	0.039	1.052	1.841	0.261	1.118

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	35	10	19
normalized size	1	1.	1.	1.08	1.38	2.69	0.77	1.46
time (sec)	N/A	0.013	0.011	0.039	1.013	1.732	0.094	1.149

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	34	10	20
normalized size	1	1.	1.	1.08	1.38	2.62	0.77	1.54
time (sec)	N/A	0.04	0.007	0.043	1.011	1.754	0.136	1.14

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	59	15	27
normalized size	1	1.	1.	1.06	1.33	3.28	0.83	1.5
time (sec)	N/A	0.043	0.009	0.045	1.001	1.522	0.319	1.145

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	41	88	26	43
normalized size	1	1.	1.	0.97	1.28	2.75	0.81	1.34
time (sec)	N/A	0.046	0.012	0.043	1.021	1.537	0.34	1.144

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	51	105	34	54
normalized size	1	1.	1.	0.98	1.27	2.62	0.85	1.35
time (sec)	N/A	0.05	0.013	0.046	0.985	1.472	0.361	1.151

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	83	539	301	227	0	0
normalized size	1	1.	0.61	3.96	2.21	1.67	0.	0.
time (sec)	N/A	1.019	0.088	0.138	1.043	1.476	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	75	471	251	209	0	0
normalized size	1	1.	0.65	4.06	2.16	1.8	0.	0.
time (sec)	N/A	0.869	0.076	0.132	1.015	1.698	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	66	421	204	186	0	0
normalized size	1	1.	0.73	4.68	2.27	2.07	0.	0.
time (sec)	N/A	0.842	0.062	0.128	0.996	1.729	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	54	248	150	161	0	0
normalized size	1	1.	0.9	4.13	2.5	2.68	0.	0.
time (sec)	N/A	0.779	0.047	0.129	1.032	1.701	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	55	369	120	192	0	0
normalized size	1	1.	1.2	8.02	2.61	4.17	0.	0.
time (sec)	N/A	0.761	0.052	0.132	1.555	1.583	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	41	592	97	116	0	0
normalized size	1	1.	0.77	11.17	1.83	2.19	0.	0.
time (sec)	N/A	0.078	0.067	0.131	1.545	1.511	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	56	642	151	147	0	0
normalized size	1	1.	0.64	7.38	1.74	1.69	0.	0.
time (sec)	N/A	0.445	0.137	0.131	1.499	1.594	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	66	666	212	166	0	0
normalized size	1	1.	0.69	6.94	2.21	1.73	0.	0.
time (sec)	N/A	0.766	0.101	0.147	1.523	1.619	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	75	690	261	184	0	0
normalized size	1	1.	0.56	5.19	1.96	1.38	0.	0.
time (sec)	N/A	0.839	0.05	0.138	1.53	1.515	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	350	331	0	316
normalized size	1	1.	0.68	0.	1.38	1.31	0.	1.25
time (sec)	N/A	0.154	5.216	0.18	1.534	1.63	0.	1.178

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	302	302	0	274
normalized size	1	1.	0.69	0.	1.4	1.4	0.	1.27
time (sec)	N/A	0.119	5.174	0.132	1.523	1.682	0.	1.216

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	399	0	252	275	0	232
normalized size	1	1.	2.23	0.	1.41	1.54	0.	1.3
time (sec)	N/A	0.095	5.133	0.135	1.484	1.762	0.	1.217

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	201	247	0	188
normalized size	1	1.	0.46	0.	1.42	1.74	0.	1.32
time (sec)	N/A	0.06	0.163	0.132	1.485	1.721	0.	1.196

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	51	0	150	225	0	146
normalized size	1	1.	0.53	0.	1.56	2.34	0.	1.52
time (sec)	N/A	0.037	0.088	0.138	1.473	1.609	0.	1.24

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	302	807	0	313
normalized size	1	1.	0.1	0.	1.04	2.77	0.	1.08
time (sec)	N/A	0.246	0.04	0.124	1.568	1.733	0.	1.187

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	148	0	251	1027	0	251
normalized size	1	1.	0.55	0.	0.94	3.85	0.	0.94
time (sec)	N/A	0.225	0.24	0.138	1.56	1.798	0.	1.203

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	173	0	305	1067	0	301
normalized size	1	1.	0.54	0.	0.96	3.34	0.	0.94
time (sec)	N/A	0.249	0.216	0.137	1.486	1.773	0.	1.156

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	365	1160	0	366
normalized size	1	1.	0.26	0.	1.03	3.26	0.	1.03
time (sec)	N/A	0.292	0.106	0.137	1.493	1.67	0.	1.174

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	350	333	0	316
normalized size	1	1.	0.68	0.	1.38	1.32	0.	1.25
time (sec)	N/A	0.14	5.236	0.158	1.471	1.682	0.	1.217

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	302	302	0	274
normalized size	1	1.	0.69	0.	1.4	1.4	0.	1.27
time (sec)	N/A	0.118	5.209	0.148	1.484	1.711	0.	1.221

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	125	0	252	279	0	232
normalized size	1	1.	0.7	0.	1.41	1.56	0.	1.3
time (sec)	N/A	0.094	5.16	0.138	1.503	1.703	0.	1.208

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	205	254	0	190
normalized size	1	1.	0.49	0.	1.44	1.79	0.	1.34
time (sec)	N/A	0.059	0.172	0.13	1.504	1.766	0.	1.214

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	56	0	151	231	0	147
normalized size	1	1.	0.57	0.	1.54	2.36	0.	1.5
time (sec)	N/A	0.037	0.06	0.126	1.518	1.586	0.	1.162

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	302	807	0	313
normalized size	1	1.	0.1	0.	1.04	2.77	0.	1.08
time (sec)	N/A	0.228	0.045	0.122	1.579	1.825	0.	1.157

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	46	0	252	1014	0	252
normalized size	1	1.	0.17	0.	0.94	3.78	0.	0.94
time (sec)	N/A	0.215	0.07	0.136	1.568	1.683	0.	1.137

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	76	0	309	1072	0	304
normalized size	1	1.	0.24	0.	0.97	3.36	0.	0.95
time (sec)	N/A	0.246	0.081	0.132	1.531	1.767	0.	1.161

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	374	1133	0	366
normalized size	1	1.	0.26	0.	1.05	3.18	0.	1.03
time (sec)	N/A	0.282	0.117	0.135	1.496	1.784	0.	1.135

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	371	423	0	343
normalized size	1	1.	0.69	0.	1.29	1.47	0.	1.2
time (sec)	N/A	0.165	5.25	0.319	1.546	1.705	0.	1.204

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	321	389	0	301
normalized size	1	1.	0.64	0.	1.28	1.56	0.	1.2
time (sec)	N/A	0.136	5.239	0.325	1.494	1.704	0.	1.207

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	441	0	274	358	0	259
normalized size	1	1.	2.07	0.	1.29	1.68	0.	1.22
time (sec)	N/A	0.112	9.056	0.322	1.552	1.585	0.	1.182

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	80	0	224	332	0	217
normalized size	1	1.	0.45	0.	1.27	1.89	0.	1.23
time (sec)	N/A	0.068	0.212	0.325	1.487	1.679	0.	1.169

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	67	0	177	304	0	190
normalized size	1	1.	0.52	0.	1.36	2.34	0.	1.46
time (sec)	N/A	0.044	0.133	0.322	1.485	1.601	0.	1.209

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	30	0	329	975	0	340
normalized size	1	1.	0.09	0.	1.03	3.05	0.	1.06
time (sec)	N/A	0.297	0.071	0.135	1.535	1.676	0.	1.194

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	173	0	275	1169	0	293
normalized size	1	1.	0.58	0.	0.92	3.91	0.	0.98
time (sec)	N/A	0.257	0.373	0.355	1.559	1.814	0.	1.209

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	186	0	329	1247	0	328
normalized size	1	1.	0.53	0.	0.94	3.55	0.	0.93
time (sec)	N/A	0.285	0.262	0.336	1.526	1.76	0.	1.189

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	389	1319	0	393
normalized size	1	1.	0.27	0.	1.01	3.43	0.	1.02
time (sec)	N/A	0.313	0.143	0.336	1.678	1.71	0.	1.214

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	350	323	0	316
normalized size	1	1.	0.68	0.	1.38	1.28	0.	1.25
time (sec)	N/A	0.14	5.28	0.137	1.51	1.607	0.	1.231

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	302	296	0	274
normalized size	1	1.	0.69	0.	1.4	1.37	0.	1.27
time (sec)	N/A	0.115	5.229	0.131	1.632	1.657	0.	1.293

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	252	270	0	232
normalized size	1	1.	2.17	0.	1.41	1.51	0.	1.3
time (sec)	N/A	0.092	8.387	0.128	1.538	1.67	0.	1.204

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	204	244	0	189
normalized size	1	1.	0.46	0.	1.44	1.72	0.	1.33
time (sec)	N/A	0.06	0.155	0.128	1.533	1.636	0.	1.192

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	33	0	150	225	0	146
normalized size	1	1.	0.34	0.	1.55	2.32	0.	1.51
time (sec)	N/A	0.036	0.043	0.123	1.484	1.658	0.	1.15

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	302	806	0	313
normalized size	1	1.	0.1	0.	1.04	2.77	0.	1.08
time (sec)	N/A	0.228	0.044	0.124	1.548	1.607	0.	1.174

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	33	0	251	990	0	251
normalized size	1	1.	0.12	0.	0.94	3.69	0.	0.94
time (sec)	N/A	0.218	0.05	0.141	1.504	1.722	0.	1.172

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	56	0	306	1035	0	301
normalized size	1	1.	0.18	0.	0.96	3.24	0.	0.94
time (sec)	N/A	0.25	0.069	0.137	1.541	1.723	0.	1.147

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	374	1106	0	366
normalized size	1	1.	0.26	0.	1.05	3.11	0.	1.03
time (sec)	N/A	0.277	0.127	0.138	1.487	1.668	0.	1.193

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	350	331	0	316
normalized size	1	1.	0.68	0.	1.38	1.31	0.	1.25
time (sec)	N/A	0.14	5.313	0.141	1.503	1.643	0.	1.252

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	302	300	0	274
normalized size	1	1.	0.69	0.	1.4	1.39	0.	1.27
time (sec)	N/A	0.116	5.255	0.135	1.52	1.646	0.	1.23

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	252	277	0	232
normalized size	1	1.	2.17	0.	1.41	1.55	0.	1.3
time (sec)	N/A	0.092	8.141	0.138	1.503	1.617	0.	1.23

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	205	254	0	190
normalized size	1	1.	0.49	0.	1.44	1.79	0.	1.34
time (sec)	N/A	0.059	0.187	0.134	1.503	1.646	0.	1.202

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	55	0	151	231	0	147
normalized size	1	1.	0.56	0.	1.54	2.36	0.	1.5
time (sec)	N/A	0.035	0.11	0.128	1.505	1.669	0.	1.2

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	28	0	302	806	0	313
normalized size	1	1.	0.1	0.	1.04	2.77	0.	1.08
time (sec)	N/A	0.226	0.068	0.128	1.515	1.755	0.	1.238

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	149	0	252	1068	0	252
normalized size	1	1.	0.55	0.	0.94	3.97	0.	0.94
time (sec)	N/A	0.214	0.285	0.134	1.524	1.665	0.	1.214

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	174	0	308	1130	0	304
normalized size	1	1.	0.55	0.	0.97	3.54	0.	0.95
time (sec)	N/A	0.247	0.204	0.134	1.514	1.744	0.	1.213

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	365	1203	0	366
normalized size	1	1.	0.26	0.	1.03	3.38	0.	1.03
time (sec)	N/A	0.281	0.145	0.135	1.57	1.821	0.	1.247

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	377	346	0	343
normalized size	1	1.	0.69	0.	1.31	1.21	0.	1.2
time (sec)	N/A	0.162	5.371	0.33	1.549	1.585	0.	1.215

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	329	313	0	301
normalized size	1	1.	0.64	0.	1.32	1.25	0.	1.2
time (sec)	N/A	0.14	5.306	0.329	1.509	1.704	0.	1.217

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	389	0	279	284	0	259
normalized size	1	1.	1.83	0.	1.31	1.33	0.	1.22
time (sec)	N/A	0.11	8.431	0.331	1.545	1.593	0.	1.189

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	121	0	232	258	0	217
normalized size	1	1.	0.69	0.	1.32	1.47	0.	1.23
time (sec)	N/A	0.071	0.242	0.329	1.603	1.654	0.	1.167

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	31	0	178	232	0	174
normalized size	1	1.	0.24	0.	1.37	1.78	0.	1.34
time (sec)	N/A	0.046	0.066	0.325	1.48	1.791	0.	1.231

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	28	0	329	851	0	340
normalized size	1	1.	0.09	0.	1.03	2.66	0.	1.06
time (sec)	N/A	0.285	0.094	0.13	1.542	1.674	0.	1.218

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	31	0	275	1023	0	275
normalized size	1	1.	0.1	0.	0.92	3.42	0.	0.92
time (sec)	N/A	0.245	0.082	0.353	1.54	1.716	0.	1.197

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	101	0	333	1087	0	328
normalized size	1	1.	0.29	0.	0.95	3.1	0.	0.93
time (sec)	N/A	0.281	0.141	0.332	1.585	1.882	0.	1.162

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	401	1143	0	393
normalized size	1	1.	0.27	0.	1.04	2.97	0.	1.02
time (sec)	N/A	0.309	0.167	0.335	1.529	1.809	0.	1.197

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	285	285	189	0	297	581	0	290
normalized size	1	1.	0.66	0.	1.04	2.04	0.	1.02
time (sec)	N/A	0.249	5.249	0.098	1.537	1.641	0.	1.167

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	167	0	262	547	0	258
normalized size	1	1.	0.65	0.	1.02	2.12	0.	1.
time (sec)	N/A	0.203	0.349	0.069	1.677	1.591	0.	1.167

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	35	0	225	520	0	227
normalized size	1	1.	0.16	0.	1.01	2.33	0.	1.02
time (sec)	N/A	0.18	0.041	0.067	1.532	1.704	0.	1.207

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	26	0	0	1080	0	352
normalized size	1	1.	0.06	0.	0.	2.69	0.	0.88
time (sec)	N/A	0.525	0.032	0.068	0.	1.84	0.	1.184

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	39	0	205	689	0	317
normalized size	1	1.	0.17	0.	0.88	2.96	0.	1.36
time (sec)	N/A	0.367	0.053	0.08	1.584	1.781	0.	1.385

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	124	0	240	720	0	348
normalized size	1	1.	0.48	0.	0.92	2.77	0.	1.34
time (sec)	N/A	0.379	0.701	0.085	1.596	1.76	0.	1.418

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	133	0	277	775	0	381
normalized size	1	1.	0.46	0.	0.97	2.7	0.	1.33
time (sec)	N/A	0.398	0.206	0.083	1.569	1.744	0.	1.372

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	340	0	201	325	0	194
normalized size	1	1.	2.17	0.	1.28	2.07	0.	1.24
time (sec)	N/A	0.07	7.162	0.119	1.54	1.463	0.	1.182

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	165	0	166	301	0	162
normalized size	1	1.	1.27	0.	1.28	2.32	0.	1.25
time (sec)	N/A	0.047	0.429	0.082	1.517	1.56	0.	1.152

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	85	0	130	282	0	131
normalized size	1	1.	0.89	0.	1.35	2.94	0.	1.36
time (sec)	N/A	0.029	0.16	0.069	1.542	1.597	0.	1.186

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	26	0	189	266	0	192
normalized size	1	1.	0.17	0.	1.22	1.72	0.	1.24
time (sec)	N/A	0.051	0.037	0.069	1.516	1.655	0.	1.161

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	87	0	132	293	0	134
normalized size	1	1.	0.88	0.	1.33	2.96	0.	1.35
time (sec)	N/A	0.043	0.144	0.085	1.629	1.641	0.	1.135

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	134	0	167	321	0	165
normalized size	1	1.	1.03	0.	1.28	2.47	0.	1.27
time (sec)	N/A	0.053	0.276	0.082	1.527	1.587	0.	1.155

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	167	0	460	1332	0	450
normalized size	1	1.	0.39	0.	1.07	3.1	0.	1.05
time (sec)	N/A	0.343	5.302	0.168	1.559	1.913	0.	1.225

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	392	392	319	0	410	1269	0	405
normalized size	1	1.	0.81	0.	1.05	3.24	0.	1.03
time (sec)	N/A	0.247	0.724	0.139	1.553	1.734	0.	1.204

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	56	0	358	1224	0	363
normalized size	1	1.	0.16	0.	1.02	3.48	0.	1.03
time (sec)	N/A	0.2	0.047	0.14	1.529	1.813	0.	1.225

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	919	919	30	0	0	6747	0	878
normalized size	1	1.	0.03	0.	0.	7.34	0.	0.96
time (sec)	N/A	0.899	0.039	0.141	0.	2.135	0.	1.253

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	676	676	46	0	0	7880	0	583
normalized size	1	1.	0.07	0.	0.	11.66	0.	0.86
time (sec)	N/A	0.605	0.061	0.148	0.	2.196	0.	1.217

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	731	731	72	0	0	8216	0	639
normalized size	1	1.	0.1	0.	0.	11.24	0.	0.87
time (sec)	N/A	0.659	0.09	0.161	0.	2.275	0.	1.2

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	201	0	0	0	0
normalized size	1	1.	1.04	4.47	0.	0.	0.	0.
time (sec)	N/A	0.057	0.022	0.606	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	228	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	1.231	0.32	0.187	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	106	0	0	100	0
normalized size	1	1.	0.74	3.03	0.	0.	2.86	0.
time (sec)	N/A	0.041	0.009	0.38	0.	0.	2.299	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	128	0	0	0	0	0
normalized size	1	1.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.387	0.184	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	115	0	0	0	0	0
normalized size	1	1.	1.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.242	0.188	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	93	0	0	119	0
normalized size	1	1.	0.75	2.58	0.	0.	3.31	0.
time (sec)	N/A	0.041	0.01	0.412	0.	0.	2.375	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	192	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	1.051	0.241	0.178	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.324	0.132	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.328	0.134	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.263	0.135	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.341	0.139	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.307	0.136	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.347	0.135	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.283	0.093	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.274	0.09	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.276	0.131	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.315	0.091	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	118	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.566	0.067	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	98	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.297	0.066	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.17	0.059	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	142	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.141	0.058	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	44	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.036	0.064	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	107	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.336	0.075	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	132	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.59	0.074	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	148	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.484	0.078	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	131	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.085	0.392	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	80	183	350	296	0	316
normalized size	1	1.	0.61	1.39	2.65	2.24	0.	2.39
time (sec)	N/A	0.303	0.185	0.14	0.999	1.644	0.	1.22

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	73	141	298	263	0	270
normalized size	1	1.	0.7	1.34	2.84	2.5	0.	2.57
time (sec)	N/A	0.226	0.153	0.132	0.992	1.55	0.	1.214

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	64	121	244	231	0	224
normalized size	1	1.	0.82	1.55	3.13	2.96	0.	2.87
time (sec)	N/A	0.136	0.096	0.141	1.117	1.493	0.	1.228

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	51	93	178	180	0	211
normalized size	1	1.	1.09	1.98	3.79	3.83	0.	4.49
time (sec)	N/A	0.071	0.05	0.135	1.045	1.577	0.	1.266

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	60	247	105	205	0	107
normalized size	1	1.	1.18	4.84	2.06	4.02	0.	2.1
time (sec)	N/A	0.206	0.052	0.132	1.032	1.614	0.	1.184

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	34	36	31	122	0	47
normalized size	1	1.	1.03	1.09	0.94	3.7	0.	1.42
time (sec)	N/A	0.099	0.049	0.045	1.013	1.509	0.	1.2

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	42	41	53	161	0	72
normalized size	1	1.	0.63	0.61	0.79	2.4	0.	1.07
time (sec)	N/A	0.125	0.052	0.052	1.096	1.51	0.	1.171

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	51	50	74	205	0	93
normalized size	1	1.	0.51	0.5	0.74	2.05	0.	0.93
time (sec)	N/A	0.23	0.056	0.052	0.996	1.617	0.	1.142

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	59	58	96	247	0	115
normalized size	1	1.	0.44	0.44	0.72	1.86	0.	0.86
time (sec)	N/A	0.342	0.062	0.046	1.025	1.59	0.	1.147

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	29	66	62	124	0
normalized size	1	1.	0.67	0.69	1.57	1.48	2.95	0.
time (sec)	N/A	0.067	0.023	0.043	1.069	1.596	0.835	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	47	81	130	66	81
normalized size	1	1.	0.62	1.27	2.19	3.51	1.78	2.19
time (sec)	N/A	0.06	0.019	0.039	1.041	1.519	0.097	1.143

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	30	50	74	36	50
normalized size	1	1.	0.81	0.81	1.35	2.	0.97	1.35
time (sec)	N/A	0.056	0.015	0.047	1.013	1.508	0.091	1.166

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	31	51	81	37	51
normalized size	1	1.	0.81	0.84	1.38	2.19	1.	1.38
time (sec)	N/A	0.059	0.013	0.046	1.009	1.522	0.084	1.22

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	24	34	15	24
normalized size	1	1.	0.85	0.85	1.2	1.7	0.75	1.2
time (sec)	N/A	0.048	0.008	0.041	1.069	1.414	0.081	1.137

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	26	26	13	16	27	12	16
normalized size	1	1.86	1.86	0.93	1.14	1.93	0.86	1.14
time (sec)	N/A	0.012	0.008	0.039	1.037	1.438	0.072	1.152

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	31	41	63	20	42
normalized size	1	1.	0.94	0.97	1.28	1.97	0.62	1.31
time (sec)	N/A	0.064	0.015	0.056	1.023	1.452	0.352	1.146

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	30	35	49	24	46
normalized size	1	1.	1.79	2.14	2.5	3.5	1.71	3.29
time (sec)	N/A	0.051	0.008	0.047	1.005	1.355	0.371	1.118

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	63	93	46	28
normalized size	1	1.	0.62	0.81	1.7	2.51	1.24	0.76
time (sec)	N/A	0.062	0.014	0.047	0.989	1.456	0.528	1.124

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	77	116	60	28
normalized size	1	1.	0.62	0.81	2.08	3.14	1.62	0.76
time (sec)	N/A	0.061	0.016	0.046	1.024	1.473	0.548	1.164

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	155	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	0.136	0.409	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	80	192	350	285	0	316
normalized size	1	1.	0.76	1.83	3.33	2.71	0.	3.01
time (sec)	N/A	0.166	0.188	0.18	1.107	1.611	0.	1.217

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	64	124	298	246	0	275
normalized size	1	1.	0.82	1.59	3.82	3.15	0.	3.53
time (sec)	N/A	0.141	0.137	0.165	1.066	1.53	0.	1.231

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	64	130	244	232	0	224
normalized size	1	1.	0.82	1.67	3.13	2.97	0.	2.87
time (sec)	N/A	0.17	0.104	0.171	1.015	1.629	0.	1.219

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	53	162	182	197	0	167
normalized size	1	1.	0.82	2.49	2.8	3.03	0.	2.57
time (sec)	N/A	0.185	0.075	0.174	1.02	1.553	0.	1.235

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	63	345	128	284	0	146
normalized size	1	1.	0.79	4.31	1.6	3.55	0.	1.82
time (sec)	N/A	0.284	0.082	0.206	1.028	1.613	0.	1.278

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	36	36	31	159	0	50
normalized size	1	1.	1.09	1.09	0.94	4.82	0.	1.52
time (sec)	N/A	0.105	0.054	0.121	1.012	1.552	0.	1.247

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	41	41	53	201	0	72
normalized size	1	1.	0.61	0.61	0.79	3.	0.	1.07
time (sec)	N/A	0.134	0.061	0.125	1.035	1.636	0.	1.31

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	50	50	74	251	0	93
normalized size	1	1.	0.53	0.53	0.79	2.67	0.	0.99
time (sec)	N/A	0.275	0.062	0.123	1.03	1.575	0.	1.384

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	58	58	96	296	0	115
normalized size	1	1.	0.46	0.46	0.77	2.37	0.	0.92
time (sec)	N/A	0.389	0.068	0.125	1.031	1.549	0.	1.535

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	74	207	161	512	0
normalized size	1	1.	0.76	1.12	3.14	2.44	7.76	0.
time (sec)	N/A	0.081	0.076	0.046	1.077	1.529	1.968	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	45	80	130	63	57
normalized size	1	1.	0.58	0.85	1.51	2.45	1.19	1.08
time (sec)	N/A	0.068	0.021	0.038	0.992	1.478	0.198	1.159

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	38	58	29	57
normalized size	1	1.	0.81	0.72	1.19	1.81	0.91	1.78
time (sec)	N/A	0.055	0.016	0.039	1.038	1.358	0.172	1.168

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	50	81	37	57
normalized size	1	1.	0.86	0.83	1.43	2.31	1.06	1.63
time (sec)	N/A	0.059	0.015	0.04	1.016	1.421	0.177	1.146

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	34	50	24	54
normalized size	1	1.	1.24	0.94	2.	2.94	1.41	3.18
time (sec)	N/A	0.046	0.013	0.039	1.017	1.486	0.156	1.174

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	32	66	26	68
normalized size	1	1.	0.96	0.93	1.19	2.44	0.96	2.52
time (sec)	N/A	0.037	0.012	0.043	1.026	1.508	0.471	1.179

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	46	59	108	36	77
normalized size	1	1.	0.75	0.96	1.23	2.25	0.75	1.6
time (sec)	N/A	0.068	0.021	0.047	1.058	1.462	0.73	1.158

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	42	69	100	51	68
normalized size	1	1.	1.	1.68	2.76	4.	2.04	2.72
time (sec)	N/A	0.05	0.009	0.048	1.021	1.439	1.126	1.153

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	41	88	131	66	57
normalized size	1	1.	0.6	0.79	1.69	2.52	1.27	1.1
time (sec)	N/A	0.068	0.018	0.048	1.024	1.526	0.831	1.153

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	42	104	158	80	57
normalized size	1	1.	0.58	0.79	1.96	2.98	1.51	1.08
time (sec)	N/A	0.067	0.019	0.045	1.023	1.465	1.596	1.154

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	76	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.045	0.356	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	72	196	298	270	0	147
normalized size	1	1.	0.57	1.54	2.35	2.13	0.	1.16
time (sec)	N/A	0.352	0.188	0.132	1.05	1.595	0.	1.15

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	64	176	244	240	0	122
normalized size	1	1.	0.64	1.76	2.44	2.4	0.	1.22
time (sec)	N/A	0.286	0.114	0.132	1.021	1.681	0.	1.164

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	53	153	182	197	0	92
normalized size	1	1.	0.82	2.35	2.8	3.03	0.	1.42
time (sec)	N/A	0.167	0.074	0.124	1.027	1.584	0.	1.17

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	23	23	34	76	74	111	0	45
normalized size	1	1.	1.48	3.3	3.22	4.83	0.	1.96
time (sec)	N/A	0.101	0.038	0.181	1.02	1.548	0.	1.171

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	36	31	78	0	0
normalized size	1	1.	0.96	1.29	1.11	2.79	0.	0.
time (sec)	N/A	0.101	0.051	0.043	1.031	1.582	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	34	41	53	119	0	61
normalized size	1	1.	0.55	0.66	0.85	1.92	0.	0.98
time (sec)	N/A	0.125	0.055	0.044	1.044	1.549	0.	1.197

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	43	50	74	161	0	88
normalized size	1	1.	0.45	0.53	0.78	1.69	0.	0.93
time (sec)	N/A	0.229	0.056	0.05	1.014	1.564	0.	1.192

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	51	58	96	200	0	115
normalized size	1	1.	0.4	0.45	0.75	1.56	0.	0.9
time (sec)	N/A	0.255	0.066	0.049	1.006	1.563	0.	1.222

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.015	0.52	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	64	85	154	68	101
normalized size	1	1.	0.62	0.7	0.93	1.69	0.75	1.11
time (sec)	N/A	0.071	0.021	0.04	1.007	1.52	0.671	1.192

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	53	70	130	56	86
normalized size	1	1.	0.66	0.73	0.96	1.78	0.77	1.18
time (sec)	N/A	0.065	0.018	0.043	1.034	1.456	0.602	1.129

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	39	42	55	97	41	70
normalized size	1	1.	0.71	0.76	1.	1.76	0.75	1.27
time (sec)	N/A	0.058	0.014	0.04	1.016	1.412	0.644	1.136

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	32	66	24	47
normalized size	1	1.	1.	0.96	1.23	2.54	0.92	1.81
time (sec)	N/A	0.035	0.009	0.043	1.024	1.468	1.09	1.123

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	28	12	20
normalized size	1	1.	1.	1.07	1.36	2.	0.86	1.43
time (sec)	N/A	0.055	0.007	0.039	1.017	1.439	0.243	1.138

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	30	39	59	20	34
normalized size	1	1.	1.	2.5	3.25	4.92	1.67	2.83
time (sec)	N/A	0.049	0.009	0.047	1.016	1.581	0.429	1.153

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	45	65	108	39	62
normalized size	1	1.	0.97	1.36	1.97	3.27	1.18	1.88
time (sec)	N/A	0.063	0.02	0.051	1.019	1.529	0.654	1.143

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	85	171	54	69
normalized size	1	1.	0.69	1.18	1.67	3.35	1.06	1.35
time (sec)	N/A	0.068	0.025	0.051	1.006	1.534	0.722	1.143

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	44	75	113	251	76	120
normalized size	1	1.	0.64	1.09	1.64	3.64	1.1	1.74
time (sec)	N/A	0.08	0.031	0.046	1.035	1.554	1.115	1.138

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.061	0.365	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	86	542	329	270	0	0
normalized size	1	1.	0.57	3.57	2.16	1.78	0.	0.
time (sec)	N/A	0.436	0.219	0.138	1.034	1.689	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	78	474	275	246	0	0
normalized size	1	1.	0.6	3.67	2.13	1.91	0.	0.
time (sec)	N/A	0.35	0.151	0.136	1.119	1.623	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	68	422	211	201	0	0
normalized size	1	1.	0.74	4.59	2.29	2.18	0.	0.
time (sec)	N/A	0.244	0.134	0.16	1.063	1.667	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	54	248	105	150	0	0
normalized size	1	1.	1.02	4.68	1.98	2.83	0.	0.
time (sec)	N/A	0.198	0.064	0.141	1.031	1.572	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	30	47	0	0
normalized size	1	1.	0.93	1.25	1.07	1.68	0.	0.
time (sec)	N/A	0.104	0.05	0.066	1.097	1.494	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	33	33	65	61	0	30
normalized size	1	1.	1.57	1.57	3.1	2.9	0.	1.43
time (sec)	N/A	0.101	0.054	0.052	1.081	1.534	0.	1.175

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	50	88	123	0	0
normalized size	1	1.	0.82	0.82	1.44	2.02	0.	0.
time (sec)	N/A	0.137	0.061	0.041	1.016	1.56	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	57	57	111	158	0	0
normalized size	1	1.	0.61	0.61	1.18	1.68	0.	0.
time (sec)	N/A	0.313	0.063	0.05	1.04	1.583	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	66	66	131	204	0	0
normalized size	1	1.	0.53	0.53	1.05	1.63	0.	0.
time (sec)	N/A	0.411	0.071	0.043	1.05	1.581	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	83	72	134	246	0	198
normalized size	1	1.	0.33	0.28	0.53	0.97	0.	0.78
time (sec)	N/A	0.221	0.055	0.049	1.08	1.544	0.	1.277

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	75	64	112	211	0	167
normalized size	1	1.	0.38	0.32	0.57	1.07	0.	0.85
time (sec)	N/A	0.19	0.042	0.048	1.105	1.562	0.	1.257

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	115	137	67	56	90	182	0	134
normalized size	1	1.19	0.58	0.49	0.78	1.58	0.	1.17
time (sec)	N/A	0.171	0.037	0.043	1.103	1.606	0.	1.184

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	77	89	57	48	61	142	0	100
normalized size	1	1.16	0.74	0.62	0.79	1.84	0.	1.3
time (sec)	N/A	0.158	0.03	0.039	1.118	1.645	0.	1.132

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	35	111	0	66
normalized size	1	1.	1.48	1.21	1.21	3.83	0.	2.28
time (sec)	N/A	0.034	0.02	0.04	1.062	1.51	0.	1.166

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	83	0	568	0	123
normalized size	1	1.	0.84	0.7	0.	4.81	0.	1.04
time (sec)	N/A	0.166	0.062	0.154	0.	1.578	0.	1.207

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	116	118	0	664	0	96
normalized size	1	1.	0.91	0.92	0.	5.19	0.	0.75
time (sec)	N/A	0.185	0.108	0.141	0.	1.674	0.	1.167

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	123	165	0	775	0	120
normalized size	1	1.	0.64	0.85	0.	4.02	0.	0.62
time (sec)	N/A	0.202	0.15	0.141	0.	1.691	0.	1.198

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	139	219	0	902	0	157
normalized size	1	1.	0.56	0.88	0.	3.61	0.	0.63
time (sec)	N/A	0.221	0.155	0.141	0.	1.634	0.	1.215

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	43	132	172	234
normalized size	1	1.	0.85	0.52	1.08	3.3	4.3	5.85
time (sec)	N/A	0.088	0.041	0.041	1.01	1.499	26.02	1.155

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	43	103	80	108
normalized size	1	1.	0.85	0.52	1.08	2.58	2.	2.7
time (sec)	N/A	0.088	0.04	0.04	1.003	1.609	12.58	1.122

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	21	43	76	61	76
normalized size	1	1.	0.75	0.52	1.08	1.9	1.52	1.9
time (sec)	N/A	0.089	0.03	0.04	1.02	1.508	10.873	1.103

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	41	46	31	41
normalized size	1	1.	0.61	0.53	1.08	1.21	0.82	1.08
time (sec)	N/A	0.078	0.025	0.046	0.995	1.586	4.981	1.162

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	41	63	51	43
normalized size	1	1.	0.58	0.56	1.14	1.75	1.42	1.19
time (sec)	N/A	0.084	0.025	0.041	1.001	1.598	14.723	1.166

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	21	35	96	29	49
normalized size	1	1.	0.89	0.55	0.92	2.53	0.76	1.29
time (sec)	N/A	0.086	0.045	0.042	1.024	1.562	24.284	1.154

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	117	31	46
normalized size	1	1.	0.85	0.52	0.8	2.92	0.78	1.15
time (sec)	N/A	0.089	0.059	0.045	1.035	1.484	25.011	1.124

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	35	140	31	49
normalized size	1	1.	0.85	0.52	0.88	3.5	0.78	1.22
time (sec)	N/A	0.086	0.057	0.039	1.	1.475	41.25	1.113

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	77	64	143	239	0	317
normalized size	1	1.	0.39	0.32	0.73	1.21	0.	1.61
time (sec)	N/A	0.193	0.046	0.118	1.07	1.609	0.	1.296

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	69	56	122	209	0	251
normalized size	1	1.	0.5	0.41	0.89	1.53	0.	1.83
time (sec)	N/A	0.173	0.039	0.117	1.062	1.555	0.	1.281

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	59	48	100	177	0	188
normalized size	1	1.	0.66	0.54	1.12	1.99	0.	2.11
time (sec)	N/A	0.158	0.039	0.117	1.084	1.508	0.	1.188

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	31	31	43	35	55	136	0	76
normalized size	1	1.	1.39	1.13	1.77	4.39	0.	2.45
time (sec)	N/A	0.037	0.031	0.115	1.069	1.474	0.	1.192

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	598	0	144
normalized size	1	1.	0.64	0.66	0.	3.67	0.	0.88
time (sec)	N/A	0.177	0.069	0.188	0.	1.573	0.	1.17

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	116	135	0	682	0	115
normalized size	1	1.	0.66	0.76	0.	3.85	0.	0.65
time (sec)	N/A	0.18	0.143	0.178	0.	1.631	0.	1.2

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	125	174	0	784	0	120
normalized size	1	1.	0.67	0.93	0.	4.19	0.	0.64
time (sec)	N/A	0.194	0.149	0.213	0.	1.653	0.	1.201

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	142	226	0	900	0	157
normalized size	1	1.	0.57	0.9	0.	3.6	0.	0.63
time (sec)	N/A	0.218	0.138	0.197	0.	1.686	0.	1.205

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	147	278	0	1030	0	189
normalized size	1	1.	0.48	0.91	0.	3.36	0.	0.62
time (sec)	N/A	0.239	0.188	0.25	0.	1.687	0.	1.235

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	194	311	86	80	173	244	0	181
normalized size	1	1.6	0.44	0.41	0.89	1.26	0.	0.93
time (sec)	N/A	0.226	0.051	0.045	1.08	1.875	0.	1.281

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	161	254	78	72	151	216	0	149
normalized size	1	1.58	0.48	0.45	0.94	1.34	0.	0.93
time (sec)	N/A	0.222	0.045	0.044	1.109	1.874	0.	1.231

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	128	197	70	64	130	182	0	116
normalized size	1	1.54	0.55	0.5	1.02	1.42	0.	0.91
time (sec)	N/A	0.203	0.042	0.043	1.118	1.888	0.	1.191

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	95	137	60	56	97	147	0	84
normalized size	1	1.44	0.63	0.59	1.02	1.55	0.	0.88
time (sec)	N/A	0.186	0.034	0.046	1.1	1.89	0.	1.164

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	62	89	50	47	73	111	0	80
normalized size	1	1.44	0.81	0.76	1.18	1.79	0.	1.29
time (sec)	N/A	0.153	0.026	0.046	1.091	1.782	0.	1.142

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	28	35	39	99	0	66
normalized size	1	1.	0.97	1.21	1.34	3.41	0.	2.28
time (sec)	N/A	0.038	0.023	0.041	1.104	1.882	0.	1.138

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	78	0	352	0	86
normalized size	1	1.	1.	1.03	0.	4.63	0.	1.13
time (sec)	N/A	0.18	0.048	0.151	0.	1.928	0.	1.177

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	123	0	664	0	99
normalized size	1	1.	0.85	0.9	0.	4.88	0.	0.73
time (sec)	N/A	0.193	0.114	0.175	0.	1.89	0.	1.214

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	125	172	0	786	0	123
normalized size	1	1.	0.65	0.89	0.	4.07	0.	0.64
time (sec)	N/A	0.203	0.161	0.152	0.	2.011	0.	1.224

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	88	101	0	518	0	217
normalized size	1	1.	0.64	0.74	0.	3.78	0.	1.58
time (sec)	N/A	0.156	0.079	0.046	0.	1.87	0.	1.165

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	87	0	462	110	181
normalized size	1	1.	0.69	0.75	0.	3.98	0.95	1.56
time (sec)	N/A	0.132	0.059	0.048	0.	1.78	84.89	1.137

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	71	73	0	381	92	144
normalized size	1	1.	0.75	0.77	0.	4.01	0.97	1.52
time (sec)	N/A	0.12	0.042	0.048	0.	1.849	43.745	1.148

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	0	313	73	104
normalized size	1	1.	0.8	0.78	0.	4.12	0.96	1.37
time (sec)	N/A	0.102	0.032	0.05	0.	1.849	5.715	1.122

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	45	0	293	60	69
normalized size	1	1.	1.	0.78	0.	5.05	1.03	1.19
time (sec)	N/A	0.099	0.026	0.043	0.	1.955	20.101	1.113

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	0	231	41	49
normalized size	1	1.	1.	0.78	0.	6.24	1.11	1.32
time (sec)	N/A	0.093	0.018	0.052	0.	1.891	19.803	1.158

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	37	50	0	362	61	73
normalized size	1	1.	0.65	0.88	0.	6.35	1.07	1.28
time (sec)	N/A	0.103	0.022	0.049	0.	1.823	15.727	1.164

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	39	64	0	478	82	99
normalized size	1	1.	0.47	0.77	0.	5.76	0.99	1.19
time (sec)	N/A	0.116	0.028	0.053	0.	1.905	48.946	1.124

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	39	78	0	599	100	126
normalized size	1	1.	0.38	0.75	0.	5.76	0.96	1.21
time (sec)	N/A	0.126	0.034	0.051	0.	1.692	28.009	1.144

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	84	88	205	244	0	203
normalized size	1	1.	0.23	0.24	0.56	0.66	0.	0.55
time (sec)	N/A	0.267	0.051	0.051	1.115	1.516	0.	1.391

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	76	80	184	212	0	170
normalized size	1	1.	0.24	0.26	0.59	0.68	0.	0.55
time (sec)	N/A	0.236	0.045	0.042	1.112	1.609	0.	1.309

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	68	72	162	185	0	138
normalized size	1	1.	0.27	0.28	0.64	0.73	0.	0.54
time (sec)	N/A	0.213	0.038	0.052	1.113	1.5	0.	1.294

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	57	63	126	142	0	104
normalized size	1	1.	0.29	0.32	0.65	0.73	0.	0.53
time (sec)	N/A	0.192	0.032	0.043	1.079	1.563	0.	1.203

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	48	55	101	113	0	70
normalized size	1	1.	0.35	0.4	0.74	0.82	0.	0.51
time (sec)	N/A	0.155	0.027	0.049	1.086	1.589	0.	1.19

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	48	47	65	99	0	49
normalized size	1	1.	0.56	0.55	0.76	1.16	0.	0.58
time (sec)	N/A	0.143	0.031	0.041	1.087	1.538	0.	1.18

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	41	35	61	90	0	55
normalized size	1	1.	1.41	1.21	2.1	3.1	0.	1.9
time (sec)	N/A	0.038	0.029	0.046	1.091	1.543	0.	1.159

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	122	85	0	560	0	134
normalized size	1	1.	1.02	0.71	0.	4.67	0.	1.12
time (sec)	N/A	0.18	0.078	0.149	0.	1.662	0.	1.157

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	140	129	0	675	0	122
normalized size	1	1.	0.76	0.7	0.	3.67	0.	0.66
time (sec)	N/A	0.198	0.142	0.167	0.	1.59	0.	1.217

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	41	67	149	158	0	142
normalized size	1	1.	0.41	0.68	1.51	1.6	0.	1.43
time (sec)	N/A	0.07	0.042	0.106	0.996	1.738	0.	1.153

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	40	57	117	158	0	113
normalized size	1	1.	0.51	0.72	1.48	2.	0.	1.43
time (sec)	N/A	0.047	0.024	0.104	1.043	1.913	0.	1.111

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	21	22	68	65	0	39
normalized size	1	1.	1.17	1.22	3.78	3.61	0.	2.17
time (sec)	N/A	0.053	0.02	0.06	1.015	1.838	0.	1.168

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	39	48	112	151	0	149
normalized size	1	1.	1.11	1.37	3.2	4.31	0.	4.26
time (sec)	N/A	0.047	0.02	0.101	1.025	1.824	0.	1.148

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	52	79	186	190	0	176
normalized size	1	1.	0.39	0.59	1.4	1.43	0.	1.32
time (sec)	N/A	0.111	0.044	0.107	1.031	1.884	0.	1.142

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	47	69	151	176	0	144
normalized size	1	1.	0.44	0.65	1.42	1.66	0.	1.36
time (sec)	N/A	0.09	0.032	0.117	1.034	1.832	0.	1.168

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	52	70	186	185	0	176
normalized size	1	1.	0.73	0.99	2.62	2.61	0.	2.48
time (sec)	N/A	0.116	0.039	0.118	1.076	2.006	0.	1.174

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	47	60	151	169	0	144
normalized size	1	1.	0.89	1.13	2.85	3.19	0.	2.72
time (sec)	N/A	0.09	0.032	0.113	1.127	1.576	0.	1.134

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	15	16	35	42	0	39
normalized size	1	1.	0.68	0.73	1.59	1.91	0.	1.77
time (sec)	N/A	0.054	0.023	0.067	1.027	1.535	0.	1.17

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	22	22	18	35	42	88	29	43
normalized size	1	1.	0.82	1.59	1.91	4.	1.32	1.95
time (sec)	N/A	0.066	0.009	0.126	1.004	1.576	20.644	1.15

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	41	106	100	184	0	113
normalized size	1	1.	0.87	2.26	2.13	3.91	0.	2.4
time (sec)	N/A	0.135	0.052	0.118	1.04	1.677	0.	1.153

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	38	106	59	170	0	61
normalized size	1	1.	1.15	3.21	1.79	5.15	0.	1.85
time (sec)	N/A	0.132	0.023	0.12	1.004	1.581	0.	1.165

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	36	110	59	122	0	61
normalized size	1	1.	0.8	2.44	1.31	2.71	0.	1.36
time (sec)	N/A	0.083	0.043	0.123	1.007	1.623	0.	1.133

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	21	21	18	21	15	31	8	15
normalized size	1	1.	0.86	1.	0.71	1.48	0.38	0.71
time (sec)	N/A	0.065	0.011	0.058	1.037	1.594	37.071	1.103

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	43	146	76	223	0	88
normalized size	1	1.	0.78	2.65	1.38	4.05	0.	1.6
time (sec)	N/A	0.155	0.055	0.116	1.009	1.577	0.	1.15

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	18	81	0	28
normalized size	1	1.	1.	0.92	0.75	3.38	0.	1.17
time (sec)	N/A	0.071	0.014	0.061	1.068	1.588	0.	1.141

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.027	0.388	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	64	49	74	151	0	196
normalized size	1	1.	0.46	0.35	0.53	1.08	0.	1.4
time (sec)	N/A	0.215	0.033	0.043	1.109	1.586	0.	1.247

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	56	41	55	131	0	101
normalized size	1	1.	0.61	0.45	0.6	1.42	0.	1.1
time (sec)	N/A	0.173	0.025	0.043	1.059	1.573	0.	1.186

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	35	111	0	66
normalized size	1	1.	1.48	1.21	1.21	3.83	0.	2.28
time (sec)	N/A	0.033	0.018	0.039	1.05	1.605	0.	1.18

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	75	70	0	490	0	119
normalized size	1	1.	0.8	0.74	0.	5.21	0.	1.27
time (sec)	N/A	0.195	0.055	0.135	0.	1.649	0.	1.188

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	76	78	0	514	0	162
normalized size	1	1.	0.78	0.8	0.	5.3	0.	1.67
time (sec)	N/A	0.195	0.039	0.144	0.	1.576	0.	1.2

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	48	45	100	113	83	140
normalized size	1	1.	0.48	0.45	0.99	1.12	0.82	1.39
time (sec)	N/A	0.225	0.067	0.046	0.999	1.596	15.255	1.139

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	40	37	81	93	68	108
normalized size	1	1.	0.5	0.46	1.01	1.16	0.85	1.35
time (sec)	N/A	0.224	0.053	0.044	1.023	1.537	11.462	1.155

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	31	28	59	65	48	74
normalized size	1	1.	0.54	0.49	1.04	1.14	0.84	1.3
time (sec)	N/A	0.152	0.043	0.041	1.013	1.531	6.182	1.173

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	41	46	31	41
normalized size	1	1.	0.61	0.53	1.08	1.21	0.82	1.08
time (sec)	N/A	0.081	0.024	0.049	1.009	1.525	4.194	1.117

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	0	208	39	49
normalized size	1	1.	1.	0.82	0.	5.33	1.	1.26
time (sec)	N/A	0.198	0.03	0.046	0.	1.661	7.972	1.148

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	45	0	235	119	53
normalized size	1	1.	1.	1.07	0.	5.6	2.83	1.26
time (sec)	N/A	0.203	0.031	0.052	0.	1.589	12.198	1.127

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	65	0	289	270	97
normalized size	1	1.	0.81	0.96	0.	4.25	3.97	1.43
time (sec)	N/A	0.21	0.047	0.052	0.	1.595	23.25	1.142

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	80	0	332	439	134
normalized size	1	1.	0.71	0.9	0.	3.73	4.93	1.51
time (sec)	N/A	0.219	0.058	0.059	0.	1.507	18.986	1.125

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	71	93	0	371	639	170
normalized size	1	1.	0.65	0.85	0.	3.37	5.81	1.55
time (sec)	N/A	0.234	0.069	0.051	0.	1.625	27.531	1.18

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	130	161	0	749	0	244
normalized size	1	1.	0.42	0.52	0.	2.42	0.	0.79
time (sec)	N/A	0.324	0.108	0.177	0.	1.7	0.	1.281

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	122	143	0	690	0	182
normalized size	1	1.	0.47	0.55	0.	2.64	0.	0.7
time (sec)	N/A	0.3	0.084	0.191	0.	1.663	0.	1.271

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	114	125	0	655	0	180
normalized size	1	1.	0.54	0.59	0.	3.1	0.	0.85
time (sec)	N/A	0.236	0.074	0.178	0.	1.637	0.	1.202

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	598	0	144
normalized size	1	1.	0.64	0.66	0.	3.67	0.	0.88
time (sec)	N/A	0.187	0.06	0.194	0.	1.591	0.	1.178

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	120	110	0	855	0	189
normalized size	1	1.	0.71	0.65	0.	5.03	0.	1.11
time (sec)	N/A	0.24	0.063	0.182	0.	1.714	0.	1.251

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	120	119	0	900	0	211
normalized size	1	1.	0.7	0.69	0.	5.23	0.	1.23
time (sec)	N/A	0.251	0.077	0.184	0.	1.731	0.	1.29

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	132	144	0	979	0	248
normalized size	1	1.	0.59	0.64	0.	4.37	0.	1.11
time (sec)	N/A	0.265	0.148	0.211	0.	1.691	0.	1.314

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	140	165	0	1022	0	282
normalized size	1	1.	0.51	0.6	0.	3.73	0.	1.03
time (sec)	N/A	0.282	0.203	0.191	0.	1.749	0.	1.325

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	148	186	0	1079	0	315
normalized size	1	1.	0.46	0.58	0.	3.35	0.	0.98
time (sec)	N/A	0.302	0.25	0.196	0.	1.717	0.	1.349

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	46	37	36	93	0	46
normalized size	1	1.	0.32	0.26	0.25	0.65	0.	0.32
time (sec)	N/A	0.128	0.021	0.06	1.076	1.529	0.	1.16

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	41	32	30	81	0	36
normalized size	1	1.	0.38	0.3	0.28	0.76	0.	0.34
time (sec)	N/A	0.107	0.016	0.062	1.023	1.581	0.	1.168

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	46	34	30	120	0	78
normalized size	1	1.	0.44	0.33	0.29	1.15	0.	0.75
time (sec)	N/A	0.126	0.021	0.058	1.066	1.621	0.	1.242

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	29	23	101	0	59
normalized size	1	1.	0.6	0.43	0.34	1.49	0.	0.87
time (sec)	N/A	0.099	0.017	0.058	1.055	1.626	0.	1.171

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	39	30	27	74	133	36
normalized size	1	1.	0.36	0.28	0.25	0.69	1.24	0.34
time (sec)	N/A	0.106	0.015	0.058	1.044	1.555	51.924	1.143

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	34	25	20	63	39	27
normalized size	1	1.	0.49	0.36	0.29	0.9	0.56	0.39
time (sec)	N/A	0.081	0.015	0.058	1.054	1.612	23.293	1.136

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	41	29	23	99	46	58
normalized size	1	1.	0.58	0.41	0.32	1.39	0.65	0.82
time (sec)	N/A	0.103	0.015	0.058	1.052	1.578	130.87	1.164

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	20	20	34	24	16	86	29	36
normalized size	1	1.	1.7	1.2	0.8	4.3	1.45	1.8
time (sec)	N/A	0.02	0.012	0.056	1.097	1.598	48.862	1.14

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	26	25	18	63	48	27
normalized size	1	1.	0.36	0.34	0.25	0.86	0.66	0.37
time (sec)	N/A	0.097	0.015	0.06	1.084	1.555	62.838	1.125

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	21	22	9	50	19	18
normalized size	1	1.	0.64	0.67	0.27	1.52	0.58	0.55
time (sec)	N/A	0.075	0.008	0.058	1.117	1.505	56.893	1.151

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	69	66	0	196	0	0
normalized size	1	1.	0.55	0.52	0.	1.56	0.	0.
time (sec)	N/A	0.115	0.047	0.118	0.	1.575	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	63	55	0	180	0	0
normalized size	1	1.	0.7	0.61	0.	2.	0.	0.
time (sec)	N/A	0.093	0.029	0.121	0.	1.632	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	65	47	0	138	0	0
normalized size	1	1.	0.7	0.51	0.	1.48	0.	0.
time (sec)	N/A	0.117	0.037	0.106	0.	1.49	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	41	37	0	85	0	0
normalized size	1	1.	0.71	0.64	0.	1.47	0.	0.
time (sec)	N/A	0.097	0.019	0.111	0.	1.618	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	75	90	0	221	0	73
normalized size	1	1.	0.58	0.69	0.	1.7	0.	0.56
time (sec)	N/A	0.132	0.075	0.115	0.	1.467	0.	1.146

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	58	79	0	208	0	59
normalized size	1	1.	0.64	0.88	0.	2.31	0.	0.66
time (sec)	N/A	0.11	0.091	0.11	0.	1.576	0.	1.133

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	102	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.071	0.357	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	142	185	67	64	112	159	0	150
normalized size	1	1.3	0.47	0.45	0.79	1.12	0.	1.06
time (sec)	N/A	0.237	0.034	0.043	1.101	1.639	0.	1.174

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	104	137	58	55	93	130	0	120
normalized size	1	1.32	0.56	0.53	0.89	1.25	0.	1.15
time (sec)	N/A	0.194	0.031	0.04	1.126	1.603	0.	1.163

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	62	89	50	47	73	111	66	80
normalized size	1	1.44	0.81	0.76	1.18	1.79	1.06	1.29
time (sec)	N/A	0.153	0.026	0.039	1.102	1.532	164.195	1.176

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	80	0	489	0	103
normalized size	1	1.	0.79	0.85	0.	5.2	0.	1.1
time (sec)	N/A	0.209	0.049	0.155	0.	1.643	0.	1.195

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	78	90	0	520	0	136
normalized size	1	1.	0.81	0.94	0.	5.42	0.	1.42
time (sec)	N/A	0.204	0.039	0.142	0.	1.658	0.	1.162

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	85	101	0	452	126	215
normalized size	1	1.	0.61	0.73	0.	3.25	0.91	1.55
time (sec)	N/A	0.264	0.126	0.046	0.	1.598	8.781	1.145

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	75	0	400	95	142
normalized size	1	1.	0.8	0.77	0.	4.12	0.98	1.46
time (sec)	N/A	0.248	0.09	0.046	0.	1.713	5.769	1.141

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	70	73	0	366	92	142
normalized size	1	1.	0.72	0.75	0.	3.77	0.95	1.46
time (sec)	N/A	0.17	0.082	0.043	0.	1.641	4.815	1.127

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	0	313	73	104
normalized size	1	1.	0.8	0.78	0.	4.12	0.96	1.37
time (sec)	N/A	0.099	0.031	0.043	0.	1.402	3.573	1.184

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	0	416	80	90
normalized size	1	1.	1.	0.78	0.	5.62	1.08	1.22
time (sec)	N/A	0.232	0.033	0.051	0.	1.346	5.387	1.153

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	71	0	455	162	96
normalized size	1	1.	1.	0.91	0.	5.83	2.08	1.23
time (sec)	N/A	0.226	0.041	0.051	0.	1.328	6.876	1.155

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	95	0	524	352	143
normalized size	1	1.	0.88	0.9	0.	4.94	3.32	1.35
time (sec)	N/A	0.26	0.077	0.055	0.	1.251	12.838	1.141

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	110	0	568	614	180
normalized size	1	1.	0.8	0.87	0.	4.47	4.83	1.42
time (sec)	N/A	0.278	0.09	0.056	0.	1.37	16.964	1.14

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	109	123	0	621	991	216
normalized size	1	1.	0.74	0.83	0.	4.2	6.7	1.46
time (sec)	N/A	0.305	0.112	0.055	0.	1.485	44.077	1.171

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	73	80	158	176	0	184
normalized size	1	1.	0.26	0.28	0.56	0.63	0.	0.65
time (sec)	N/A	0.286	0.041	0.051	1.128	1.395	0.	1.309

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	65	72	140	163	0	149
normalized size	1	1.	0.28	0.31	0.61	0.71	0.	0.65
time (sec)	N/A	0.261	0.034	0.042	1.097	1.274	0.	1.256

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	57	64	123	139	0	113
normalized size	1	1.	0.31	0.35	0.68	0.76	0.	0.62
time (sec)	N/A	0.216	0.03	0.043	1.114	1.4	0.	1.193

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	48	55	101	113	0	70
normalized size	1	1.	0.35	0.4	0.74	0.82	0.	0.51
time (sec)	N/A	0.163	0.026	0.048	1.074	1.189	0.	1.173

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	78	80	0	490	0	73
normalized size	1	1.	0.56	0.57	0.	3.5	0.	0.52
time (sec)	N/A	0.221	0.112	0.138	0.	1.382	0.	1.202

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	79	86	0	524	0	85
normalized size	1	1.	0.56	0.61	0.	3.74	0.	0.61
time (sec)	N/A	0.223	0.077	0.144	0.	1.383	0.	1.213

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	90	103	0	587	0	113
normalized size	1	1.	0.47	0.54	0.	3.09	0.	0.59
time (sec)	N/A	0.232	0.083	0.158	0.	1.711	0.	1.233

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	98	114	0	632	0	150
normalized size	1	1.	0.41	0.48	0.	2.66	0.	0.63
time (sec)	N/A	0.246	0.091	0.152	0.	1.638	0.	1.243

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	286	286	106	125	0	687	0	182
normalized size	1	1.	0.37	0.44	0.	2.4	0.	0.64
time (sec)	N/A	0.264	0.092	0.173	0.	1.733	0.	1.297

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	116	104	0	0	0	0
normalized size	1	1.	0.42	0.37	0.	0.	0.	0.
time (sec)	N/A	0.268	0.074	0.054	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	78	61	0	0	0	0
normalized size	1	1.	0.61	0.48	0.	0.	0.	0.
time (sec)	N/A	0.157	0.048	0.04	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	36	36	58	34	0	0	0	0
normalized size	1	1.	1.61	0.94	0.	0.	0.	0.
time (sec)	N/A	0.03	0.017	0.04	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.026	0.353	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.039	0.352	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.048	0.342	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	1.778	0.21	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	0	0	0
normalized size	1	1.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	1.04	0.228	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.494	0.064	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	87	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.182	0.281	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	33	0	122	0	0
normalized size	1	1.	0.69	0.69	0.	2.54	0.	0.
time (sec)	N/A	0.109	0.168	0.041	0.	1.676	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	64	46	0	259	0	0
normalized size	1	1.	0.62	0.44	0.	2.49	0.	0.
time (sec)	N/A	0.149	0.217	0.049	0.	1.63	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	83	68	0	485	0	0
normalized size	1	1.	0.37	0.3	0.	2.17	0.	0.
time (sec)	N/A	0.262	0.275	0.045	0.	1.793	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	103	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.096	0.404	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	101	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.071	0.381	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	0.057	0.367	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	0.053	0.341	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	94	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	0.04	0.367	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	117	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	0.108	0.329	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	138	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.271	0.158	0.339	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	175	224	302	359	0	309
normalized size	1	1.	1.54	1.96	2.65	3.15	0.	2.71
time (sec)	N/A	0.266	0.18	0.131	1.563	1.756	0.	1.208

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	167	200	271	332	0	263
normalized size	1	1.	1.9	2.27	3.08	3.77	0.	2.99
time (sec)	N/A	0.191	0.142	0.131	1.631	1.665	0.	1.213

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	158	166	169	278	0	165
normalized size	1	1.	2.55	2.68	2.73	4.48	0.	2.66
time (sec)	N/A	0.096	0.163	0.126	1.527	1.635	0.	1.208

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	27	27	31	63	89	113	0	117
normalized size	1	1.	1.15	2.33	3.3	4.19	0.	4.33
time (sec)	N/A	0.029	0.037	0.116	1.577	1.589	0.	1.281

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	63	250	157	223	0	173
normalized size	1	1.	0.9	3.57	2.24	3.19	0.	2.47
time (sec)	N/A	0.196	0.09	0.135	1.003	1.539	0.	1.159

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	94	339	185	309	0	200
normalized size	1	1.	0.9	3.23	1.76	2.94	0.	1.9
time (sec)	N/A	0.293	0.063	0.139	1.026	1.606	0.	1.166

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	431	207	390	0	224
normalized size	1	1.	0.75	3.12	1.5	2.83	0.	1.62
time (sec)	N/A	0.393	0.072	0.135	1.029	1.715	0.	1.168

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	112	523	228	475	0	246
normalized size	1	1.	0.65	3.06	1.33	2.78	0.	1.44
time (sec)	N/A	0.501	0.085	0.138	1.02	1.637	0.	1.157

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	63	60	77	142	63	78
normalized size	1	1.	1.03	0.98	1.26	2.33	1.03	1.28
time (sec)	N/A	0.137	0.219	0.046	1.014	1.574	0.463	1.114

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	42	39	50	97	39	51
normalized size	1	1.	1.05	0.98	1.25	2.42	0.98	1.27
time (sec)	N/A	0.13	0.155	0.043	1.039	1.493	0.364	1.155

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	39	39	41	38	50	97	37	51
normalized size	1	1.	1.05	0.97	1.28	2.49	0.95	1.31
time (sec)	N/A	0.128	0.121	0.043	1.042	1.468	0.339	1.122

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	39	15	22
normalized size	1	1.	1.	1.06	1.38	2.44	0.94	1.38
time (sec)	N/A	0.121	0.089	0.043	1.002	1.43	0.26	1.146

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	30	10	16
normalized size	1	1.	1.	1.09	1.36	2.73	0.91	1.45
time (sec)	N/A	0.075	0.036	0.039	1.046	1.402	0.091	1.11

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	36	47	86	26	49
normalized size	1	1.	0.81	0.97	1.27	2.32	0.7	1.32
time (sec)	N/A	0.124	0.028	0.044	1.03	1.546	0.362	1.15

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	74	149	49	57
normalized size	1	1.	0.96	0.96	1.4	2.81	0.92	1.08
time (sec)	N/A	0.155	0.099	0.043	1.049	1.506	0.436	1.142

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	66	101	217	73	68
normalized size	1	1.	0.86	0.9	1.38	2.97	1.	0.93
time (sec)	N/A	0.164	0.12	0.043	1.044	1.505	0.534	1.122

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	81	126	271	94	78
normalized size	1	1.	0.82	0.93	1.45	3.11	1.08	0.9
time (sec)	N/A	0.173	0.149	0.044	1.026	1.553	0.649	1.126

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	175	233	301	358	0	311
normalized size	1	1.	1.7	2.26	2.92	3.48	0.	3.02
time (sec)	N/A	0.133	0.217	0.168	1.591	1.726	0.	1.206

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	51	105	204	192	0	232
normalized size	1	1.	0.84	1.72	3.34	3.15	0.	3.8
time (sec)	N/A	0.056	0.07	0.164	1.525	1.624	0.	1.201

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	154	174	169	262	0	181
normalized size	1	1.	2.44	2.76	2.68	4.16	0.	2.87
time (sec)	N/A	0.131	0.151	0.164	1.643	1.593	0.	1.207

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	73	145	154	223	0	150
normalized size	1	1.	1.49	2.96	3.14	4.55	0.	3.06
time (sec)	N/A	0.174	0.113	0.162	1.524	1.638	0.	1.214

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	70	346	180	304	0	200
normalized size	1	1.	0.67	3.3	1.71	2.9	0.	1.9
time (sec)	N/A	0.302	0.122	0.173	1.053	1.538	0.	1.225

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	438	207	392	0	224
normalized size	1	1.	0.75	3.17	1.5	2.84	0.	1.62
time (sec)	N/A	0.402	0.076	0.172	1.086	1.544	0.	1.237

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	112	530	228	460	0	246
normalized size	1	1.	0.68	3.21	1.38	2.79	0.	1.49
time (sec)	N/A	0.521	0.083	0.174	1.073	1.56	0.	1.334

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	120	622	250	567	0	267
normalized size	1	1.	0.59	3.05	1.23	2.78	0.	1.31
time (sec)	N/A	0.64	0.094	0.18	1.171	1.525	0.	1.418

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	66	61	80	150	63	166
normalized size	1	1.	1.03	0.95	1.25	2.34	0.98	2.59
time (sec)	N/A	0.137	0.278	0.045	1.036	1.456	0.478	1.147

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	42	72	31	80
normalized size	1	1.	1.	0.9	1.4	2.4	1.03	2.67
time (sec)	N/A	0.126	0.181	0.043	1.006	1.542	0.328	1.136

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	40	37	46	97	37	132
normalized size	1	1.	1.05	0.97	1.21	2.55	0.97	3.47
time (sec)	N/A	0.127	0.133	0.043	1.064	1.617	0.349	1.179

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	27	27	29	28	36	65	26	127
normalized size	1	1.	1.07	1.04	1.33	2.41	0.96	4.7
time (sec)	N/A	0.121	0.103	0.045	1.041	1.568	0.293	1.122

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	32	55	17	74
normalized size	1	1.	1.	1.	1.28	2.2	0.68	2.96
time (sec)	N/A	0.082	0.039	0.043	1.122	1.548	0.389	1.156

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	66	140	41	100
normalized size	1	1.	0.96	0.96	1.25	2.64	0.77	1.89
time (sec)	N/A	0.133	0.03	0.045	1.031	1.44	0.43	1.12

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	101	216	73	127
normalized size	1	1.	0.89	0.93	1.42	3.04	1.03	1.79
time (sec)	N/A	0.158	0.106	0.046	1.016	1.719	0.542	1.134

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	81	126	275	94	147
normalized size	1	1.	0.8	0.91	1.42	3.09	1.06	1.65
time (sec)	N/A	0.172	0.139	0.046	1.042	1.841	0.663	1.151

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	79	96	153	347	114	167
normalized size	1	1.	0.75	0.91	1.46	3.3	1.09	1.59
time (sec)	N/A	0.183	0.173	0.047	1.016	1.823	0.797	1.157

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	175	290	301	365	0	358
normalized size	1	1.	1.3	2.15	2.23	2.7	0.	2.65
time (sec)	N/A	0.438	0.145	0.136	1.59	1.826	0.	1.211

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	167	266	271	332	0	313
normalized size	1	1.	1.58	2.51	2.56	3.13	0.	2.95
time (sec)	N/A	0.331	0.122	0.135	1.58	2.025	0.	1.232

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	55	227	170	266	0	176
normalized size	1	1.	0.71	2.95	2.21	3.45	0.	2.29
time (sec)	N/A	0.238	0.198	0.131	1.565	1.911	0.	1.201

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	73	136	154	223	0	115
normalized size	1	1.	1.49	2.78	3.14	4.55	0.	2.35
time (sec)	N/A	0.145	0.102	0.128	1.593	2.003	0.	1.181

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	28	59	58	0	32
normalized size	1	1.	1.	1.47	3.11	3.05	0.	1.68
time (sec)	N/A	0.04	0.067	0.04	1.025	1.837	0.	1.179

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	69	256	162	220	0	0
normalized size	1	1.	0.95	3.51	2.22	3.01	0.	0.
time (sec)	N/A	0.111	0.035	0.137	1.043	1.906	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	94	344	185	309	0	80
normalized size	1	1.	0.9	3.28	1.76	2.94	0.	0.76
time (sec)	N/A	0.293	0.064	0.138	0.994	1.893	0.	1.2

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	436	207	385	0	80
normalized size	1	1.	0.75	3.16	1.5	2.79	0.	0.58
time (sec)	N/A	0.39	0.078	0.148	1.091	1.934	0.	1.198

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	67	64	81	162	56	84
normalized size	1	1.	1.03	0.98	1.25	2.49	0.86	1.29
time (sec)	N/A	0.145	0.179	0.048	1.061	1.854	0.662	1.138

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	56	53	69	140	42	72
normalized size	1	1.	1.04	0.98	1.28	2.59	0.78	1.33
time (sec)	N/A	0.141	0.128	0.046	1.096	1.842	0.568	1.179

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	42	41	54	99	31	57
normalized size	1	1.	1.05	1.02	1.35	2.48	0.78	1.42
time (sec)	N/A	0.135	0.1	0.046	1.043	1.776	0.47	1.193

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	55	17	34
normalized size	1	1.	1.	1.04	1.35	2.39	0.74	1.48
time (sec)	N/A	0.082	0.039	0.045	1.028	1.858	0.381	1.135

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	21	27	38	17	28
normalized size	1	1.	1.1	1.05	1.35	1.9	0.85	1.4
time (sec)	N/A	0.115	0.018	0.037	1.028	1.767	0.269	1.14

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	18	18	39	35	46	69	34	49
normalized size	1	1.	2.17	1.94	2.56	3.83	1.89	2.72
time (sec)	N/A	0.137	0.083	0.046	1.014	1.804	0.329	1.168

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	50	72	136	56	69
normalized size	1	1.	0.98	0.88	1.26	2.39	0.98	1.21
time (sec)	N/A	0.15	0.11	0.044	1.039	1.804	0.506	1.159

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	73	65	93	211	73	77
normalized size	1	1.	0.97	0.87	1.24	2.81	0.97	1.03
time (sec)	N/A	0.162	0.147	0.046	1.036	1.677	0.614	1.15

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	567	672	332	369	0	0
normalized size	1	1.	3.46	4.1	2.02	2.25	0.	0.
time (sec)	N/A	0.552	1.099	0.139	1.566	1.958	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	663	450	304	339	0	0
normalized size	1	1.	4.91	3.33	2.25	2.51	0.	0.
time (sec)	N/A	0.43	0.462	0.134	1.586	1.951	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	424	600	201	286	0	0
normalized size	1	1.	4.04	5.71	1.91	2.72	0.	0.
time (sec)	N/A	0.326	0.389	0.137	1.55	1.87	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	234	376	182	227	0	0
normalized size	1	1.	3.12	5.01	2.43	3.03	0.	0.
time (sec)	N/A	0.224	0.462	0.127	1.553	1.987	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	61	250	162	166	0	0
normalized size	1	1.	0.85	3.47	2.25	2.31	0.	0.
time (sec)	N/A	0.173	0.093	0.132	1.037	1.865	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	69	250	169	163	0	0
normalized size	1	1.	0.93	3.38	2.28	2.2	0.	0.
time (sec)	N/A	0.101	0.037	0.133	1.036	1.931	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	44	124	82	0	51
normalized size	1	1.	0.73	0.98	2.76	1.82	0.	1.13
time (sec)	N/A	0.049	0.022	0.04	1.023	1.778	0.	1.221

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	94	523	216	306	0	0
normalized size	1	1.	0.85	4.71	1.95	2.76	0.	0.
time (sec)	N/A	0.15	0.067	0.141	1.025	1.824	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	615	238	387	0	0
normalized size	1	1.	0.75	4.46	1.72	2.8	0.	0.
time (sec)	N/A	0.39	0.081	0.144	1.058	1.847	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	235	279	109	166	0	926	0	0
normalized size	1	1.19	0.46	0.71	0.	3.94	0.	0.
time (sec)	N/A	0.164	0.115	0.178	0.	2.194	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	196	221	101	149	0	861	0	0
normalized size	1	1.13	0.52	0.76	0.	4.39	0.	0.
time (sec)	N/A	0.148	0.082	0.18	0.	2.221	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	157	189	89	132	0	790	0	0
normalized size	1	1.2	0.57	0.84	0.	5.03	0.	0.
time (sec)	N/A	0.133	0.074	0.18	0.	2.187	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	70	105	0	667	0	0
normalized size	1	1.	0.6	0.9	0.	5.7	0.	0.
time (sec)	N/A	0.223	0.052	0.178	0.	2.218	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	630	0	0
normalized size	1	1.	0.85	1.12	0.	8.08	0.	0.
time (sec)	N/A	0.155	0.044	0.17	0.	2.168	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	93	151	0	1137	0	0
normalized size	1	1.	0.61	0.99	0.	7.48	0.	0.
time (sec)	N/A	0.135	0.065	0.171	0.	2.707	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	122	259	0	0	0	0
normalized size	1	1.	0.57	1.2	0.	0.	0.	0.
time (sec)	N/A	0.155	0.091	0.187	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	135	366	0	0	0	0
normalized size	1	1.	0.49	1.32	0.	0.	0.	0.
time (sec)	N/A	0.171	0.12	0.183	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	91	163	0	516	0	0
normalized size	1	1.	0.64	1.14	0.	3.61	0.	0.
time (sec)	N/A	0.238	0.151	0.164	0.	1.931	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	83	144	0	460	729	0
normalized size	1	1.	0.7	1.22	0.	3.9	6.18	0.
time (sec)	N/A	0.225	0.117	0.164	0.	1.873	19.958	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	108	0	397	0	0
normalized size	1	1.	0.79	1.14	0.	4.18	0.	0.
time (sec)	N/A	0.201	0.065	0.164	0.	1.862	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	55	103	0	298	0	0
normalized size	1	1.	0.79	1.47	0.	4.26	0.	0.
time (sec)	N/A	0.178	0.047	0.163	0.	1.808	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	118	0	273	0	131
normalized size	1	1.	1.	2.36	0.	5.46	0.	2.62
time (sec)	N/A	0.159	0.03	0.166	0.	1.94	0.	1.293

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	43	194	0	377	0	167
normalized size	1	1.	0.61	2.77	0.	5.39	0.	2.39
time (sec)	N/A	0.179	0.03	0.177	0.	1.913	0.	1.31

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	55	260	0	514	0	193
normalized size	1	1.	0.58	2.74	0.	5.41	0.	2.03
time (sec)	N/A	0.2	0.032	0.171	0.	1.78	0.	1.315

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	58	328	0	629	0	220
normalized size	1	1.	0.49	2.78	0.	5.33	0.	1.86
time (sec)	N/A	0.216	0.035	0.173	0.	1.936	0.	1.256

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	46	396	0	774	0	250
normalized size	1	1.	0.32	2.73	0.	5.34	0.	1.72
time (sec)	N/A	0.23	0.041	0.174	0.	1.805	0.	1.319

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	109	178	0	919	0	0
normalized size	1	1.	0.41	0.66	0.	3.43	0.	0.
time (sec)	N/A	0.163	0.128	0.181	0.	2.32	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	101	161	0	859	0	0
normalized size	1	1.	0.43	0.68	0.	3.62	0.	0.
time (sec)	N/A	0.146	0.089	0.181	0.	2.228	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	89	144	0	788	0	0
normalized size	1	1.	0.57	0.92	0.	5.05	0.	0.
time (sec)	N/A	0.268	0.066	0.18	0.	2.138	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	66	118	0	668	0	0
normalized size	1	1.	0.56	1.	0.	5.66	0.	0.
time (sec)	N/A	0.215	0.044	0.181	0.	2.247	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	93	160	0	0	0	0
normalized size	1	1.	0.61	1.05	0.	0.	0.	0.
time (sec)	N/A	0.134	0.073	0.18	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	115	259	0	0	0	0
normalized size	1	1.	0.53	1.2	0.	0.	0.	0.
time (sec)	N/A	0.15	0.1	0.181	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	135	373	0	1455	0	0
normalized size	1	1.	0.49	1.36	0.	5.29	0.	0.
time (sec)	N/A	0.171	0.126	0.186	0.	2.692	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	143	480	0	1625	0	0
normalized size	1	1.	0.43	1.43	0.	4.85	0.	0.
time (sec)	N/A	0.191	0.145	0.191	0.	3.194	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	101	161	0	859	0	0
normalized size	1	1.	0.46	0.73	0.	3.89	0.	0.
time (sec)	N/A	0.144	0.103	0.181	0.	2.632	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	89	144	0	795	0	0
normalized size	1	1.	0.55	0.89	0.	4.94	0.	0.
time (sec)	N/A	0.125	0.064	0.179	0.	2.526	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	70	118	0	667	0	0
normalized size	1	1.	0.5	0.84	0.	4.76	0.	0.
time (sec)	N/A	0.126	0.051	0.184	0.	2.644	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	635	0	0
normalized size	1	1.	0.82	1.28	0.	8.04	0.	0.
time (sec)	N/A	0.156	0.038	0.176	0.	2.569	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	66	104	0	640	0	0
normalized size	1	1.	0.85	1.33	0.	8.21	0.	0.
time (sec)	N/A	0.157	0.046	0.17	0.	2.614	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	91	162	0	0	0	0
normalized size	1	1.	0.6	1.07	0.	0.	0.	0.
time (sec)	N/A	0.139	0.068	0.18	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	123	264	0	1300	0	0
normalized size	1	1.	0.56	1.21	0.	5.94	0.	0.
time (sec)	N/A	0.149	0.099	0.186	0.	3.047	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	135	371	0	1467	0	0
normalized size	1	1.	0.49	1.34	0.	5.3	0.	0.
time (sec)	N/A	0.169	0.156	0.186	0.	3.156	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	281	0	744	0	0
normalized size	1	1.	0.77	1.72	0.	4.56	0.	0.
time (sec)	N/A	0.28	0.26	0.168	0.	2.282	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	116	257	0	657	0	0
normalized size	1	1.	0.84	1.86	0.	4.76	0.	0.
time (sec)	N/A	0.254	0.101	0.164	0.	2.265	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	229	0	541	0	0
normalized size	1	1.	0.84	2.03	0.	4.79	0.	0.
time (sec)	N/A	0.238	0.083	0.163	0.	2.206	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	190	0	508	0	0
normalized size	1	1.	1.	2.07	0.	5.52	0.	0.
time (sec)	N/A	0.209	0.051	0.162	0.	1.977	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	136	0	536	0	177
normalized size	1	1.	1.	1.43	0.	5.64	0.	1.86
time (sec)	N/A	0.206	0.05	0.161	0.	1.952	0.	1.269

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	134	0	531	0	176
normalized size	1	1.	1.	1.43	0.	5.65	0.	1.87
time (sec)	N/A	0.215	0.052	0.17	0.	1.981	0.	1.256

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	70	368	0	652	0	224
normalized size	1	1.	0.6	3.17	0.	5.62	0.	1.93
time (sec)	N/A	0.235	0.055	0.17	0.	2.007	0.	1.26

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	79	497	0	814	0	252
normalized size	1	1.	0.54	3.38	0.	5.54	0.	1.71
time (sec)	N/A	0.264	0.056	0.179	0.	1.994	0.	1.276

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	82	626	0	976	0	279
normalized size	1	1.	0.48	3.64	0.	5.67	0.	1.62
time (sec)	N/A	0.293	0.065	0.174	0.	2.01	0.	1.348

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	140	229	0	918	0	0
normalized size	1	1.	0.42	0.68	0.	2.74	0.	0.
time (sec)	N/A	0.2	0.114	0.184	0.	2.214	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	132	212	0	872	0	0
normalized size	1	1.	0.48	0.77	0.	3.15	0.	0.
time (sec)	N/A	0.171	0.104	0.212	0.	2.303	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	124	195	0	798	0	0
normalized size	1	1.	0.57	0.89	0.	3.64	0.	0.
time (sec)	N/A	0.15	0.094	0.191	0.	2.076	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	71	169	0	675	0	0
normalized size	1	1.	0.45	1.07	0.	4.27	0.	0.
time (sec)	N/A	0.133	0.041	0.189	0.	2.124	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	640	0	0
normalized size	1	1.	0.48	1.04	0.	4.57	0.	0.
time (sec)	N/A	0.118	0.046	0.184	0.	2.104	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	69	149	0	651	0	0
normalized size	1	1.	0.58	1.26	0.	5.52	0.	0.
time (sec)	N/A	0.221	0.04	0.184	0.	2.288	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	64	149	0	662	0	0
normalized size	1	1.	0.55	1.27	0.	5.66	0.	0.
time (sec)	N/A	0.223	0.045	0.182	0.	2.238	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	199	90	262	0	0	0	0
normalized size	1	1.	0.45	1.32	0.	0.	0.	0.
time (sec)	N/A	0.168	0.062	0.187	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	121	290	0	1301	0	0
normalized size	1	1.	0.45	1.09	0.	4.87	0.	0.
time (sec)	N/A	0.172	0.081	0.191	0.	2.698	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.031	0.17	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	147	121	0	717	0	0
normalized size	1	1.	0.9	0.74	0.	4.37	0.	0.
time (sec)	N/A	0.342	0.463	0.175	0.	2.229	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	148	102	0	674	0	0
normalized size	1	1.	1.19	0.82	0.	5.44	0.	0.
time (sec)	N/A	0.24	0.325	0.17	0.	2.205	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	630	0	0
normalized size	1	1.	0.85	1.12	0.	8.08	0.	0.
time (sec)	N/A	0.154	0.047	0.173	0.	2.248	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	132	88	0	603	0	0
normalized size	1	1.	1.74	1.16	0.	7.93	0.	0.
time (sec)	N/A	0.227	0.218	0.178	0.	1.968	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	41	0	122	0	0
normalized size	1	1.	1.22	1.11	0.	3.3	0.	0.
time (sec)	N/A	0.148	0.087	0.117	0.	1.635	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	58	47	0	140	0	0
normalized size	1	1.	0.75	0.61	0.	1.82	0.	0.
time (sec)	N/A	0.189	0.103	0.118	0.	1.628	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	66	55	0	162	0	0
normalized size	1	1.	0.56	0.47	0.	1.38	0.	0.
time (sec)	N/A	0.246	0.109	0.119	0.	1.581	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	74	63	0	178	0	0
normalized size	1	1.	0.47	0.4	0.	1.12	0.	0.
time (sec)	N/A	0.306	0.116	0.117	0.	1.56	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	50	172	0	409	0	193
normalized size	1	1.	0.38	1.32	0.	3.15	0.	1.48
time (sec)	N/A	0.377	0.038	0.166	0.	1.595	0.	1.23

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	50	155	0	370	0	173
normalized size	1	1.	0.48	1.48	0.	3.52	0.	1.65
time (sec)	N/A	0.352	0.036	0.168	0.	1.63	0.	1.259

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	77	139	0	327	0	153
normalized size	1	1.	0.96	1.74	0.	4.09	0.	1.91
time (sec)	N/A	0.241	0.071	0.161	0.	1.545	0.	1.246

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	118	0	273	0	131
normalized size	1	1.	1.	2.36	0.	5.46	0.	2.62
time (sec)	N/A	0.146	0.039	0.165	0.	1.641	0.	1.248

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	99	0	247	39	0
normalized size	1	1.	1.	2.11	0.	5.26	0.83	0.
time (sec)	N/A	0.33	0.033	0.17	0.	1.621	9.748	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	0	58	0	0
normalized size	1	1.	0.67	0.64	0.	1.38	0.	0.
time (sec)	N/A	0.324	0.039	0.122	0.	1.547	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	36	35	0	77	0	0
normalized size	1	1.	0.52	0.51	0.	1.12	0.	0.
time (sec)	N/A	0.333	0.046	0.125	0.	1.575	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	44	43	0	103	0	0
normalized size	1	1.	0.46	0.45	0.	1.07	0.	0.
time (sec)	N/A	0.352	0.051	0.116	0.	1.552	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	52	51	0	123	0	0
normalized size	1	1.	0.43	0.42	0.	1.02	0.	0.
time (sec)	N/A	0.378	0.061	0.119	0.	1.491	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	252	224	0	1285	0	0
normalized size	1	1.	0.81	0.72	0.	4.11	0.	0.
time (sec)	N/A	0.349	0.916	0.179	0.	2.356	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	244	202	0	0	0	0
normalized size	1	1.	0.93	0.77	0.	0.	0.	0.
time (sec)	N/A	0.317	0.719	0.174	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	236	180	0	1187	0	0
normalized size	1	1.	1.13	0.86	0.	5.68	0.	0.
time (sec)	N/A	0.219	0.532	0.176	0.	2.205	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	93	160	0	0	0	0
normalized size	1	1.	0.61	1.05	0.	0.	0.	0.
time (sec)	N/A	0.126	0.079	0.181	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	218	161	0	0	0	0
normalized size	1	1.	1.49	1.1	0.	0.	0.	0.
time (sec)	N/A	0.273	0.369	0.18	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	155	141	0	770	0	0
normalized size	1	1.	1.24	1.13	0.	6.16	0.	0.
time (sec)	N/A	0.239	0.248	0.181	0.	1.951	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	162	165	0	832	0	0
normalized size	1	1.	0.95	0.97	0.	4.89	0.	0.
time (sec)	N/A	0.3	0.27	0.214	0.	1.912	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	170	187	0	867	0	0
normalized size	1	1.	0.81	0.89	0.	4.15	0.	0.
time (sec)	N/A	0.476	0.283	0.187	0.	2.012	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	178	209	0	926	0	0
normalized size	1	1.	0.59	0.69	0.	3.06	0.	0.
time (sec)	N/A	0.32	0.297	0.195	0.	1.942	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	93	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.267	0.066	0.174	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	147	133	0	729	0	0
normalized size	1	1.	0.9	0.81	0.	4.45	0.	0.
time (sec)	N/A	0.36	0.471	0.185	0.	1.869	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	139	116	0	686	0	0
normalized size	1	1.	1.12	0.94	0.	5.53	0.	0.
time (sec)	N/A	0.255	0.494	0.181	0.	1.791	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	635	0	0
normalized size	1	1.	0.82	1.28	0.	8.04	0.	0.
time (sec)	N/A	0.158	0.049	0.188	0.	1.766	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	132	100	0	603	0	0
normalized size	1	1.	1.74	1.32	0.	7.93	0.	0.
time (sec)	N/A	0.244	0.233	0.196	0.	1.737	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	46	54	0	124	0	0
normalized size	1	1.	0.66	0.77	0.	1.77	0.	0.
time (sec)	N/A	0.185	0.082	0.125	0.	1.553	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	58	62	0	142	0	0
normalized size	1	1.	0.51	0.55	0.	1.26	0.	0.
time (sec)	N/A	0.224	0.106	0.118	0.	1.523	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	66	70	0	170	0	0
normalized size	1	1.	0.44	0.47	0.	1.14	0.	0.
time (sec)	N/A	0.303	0.123	0.112	0.	1.555	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	116	259	0	663	0	0
normalized size	1	1.	0.67	1.51	0.	3.85	0.	0.
time (sec)	N/A	0.47	0.155	0.161	0.	1.655	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	108	237	0	612	0	0
normalized size	1	1.	0.73	1.61	0.	4.16	0.	0.
time (sec)	N/A	0.451	0.108	0.158	0.	1.64	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	100	216	0	566	0	0
normalized size	1	1.	0.82	1.77	0.	4.64	0.	0.
time (sec)	N/A	0.324	0.094	0.157	0.	1.723	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	190	0	508	0	0
normalized size	1	1.	1.	2.07	0.	5.52	0.	0.
time (sec)	N/A	0.198	0.048	0.165	0.	1.62	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	228	0	479	80	0
normalized size	1	1.	1.	2.65	0.	5.57	0.93	0.
time (sec)	N/A	0.373	0.042	0.174	0.	1.644	11.3	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	254	0	374	0	0
normalized size	1	1.	0.84	3.1	0.	4.56	0.	0.
time (sec)	N/A	0.376	0.07	0.176	0.	1.598	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	79	278	0	431	0	375
normalized size	1	1.	0.7	2.46	0.	3.81	0.	3.32
time (sec)	N/A	0.394	0.084	0.17	0.	1.616	0.	1.977

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	87	302	0	466	0	481
normalized size	1	1.	0.77	2.67	0.	4.12	0.	4.26
time (sec)	N/A	0.416	0.134	0.168	0.	1.706	0.	2.295

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	95	326	0	517	0	586
normalized size	1	1.	0.58	2.	0.	3.17	0.	3.6
time (sec)	N/A	0.458	0.139	0.186	0.	1.656	0.	2.703

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	306	167	197	0	792	0	0
normalized size	1	1.01	0.55	0.65	0.	2.61	0.	0.
time (sec)	N/A	0.318	0.586	0.188	0.	1.938	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	252	159	180	0	737	0	0
normalized size	1	1.	0.63	0.72	0.	2.94	0.	0.
time (sec)	N/A	0.293	0.491	0.187	0.	1.942	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	199	151	163	0	694	0	0
normalized size	1	1.	0.76	0.82	0.	3.49	0.	0.
time (sec)	N/A	0.2	0.493	0.182	0.	1.914	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	640	0	0
normalized size	1	1.	0.48	1.04	0.	4.57	0.	0.
time (sec)	N/A	0.114	0.048	0.175	0.	1.91	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	131	151	0	609	0	0
normalized size	1	1.	0.98	1.13	0.	4.54	0.	0.
time (sec)	N/A	0.262	0.374	0.182	0.	1.811	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	58	62	0	126	0	0
normalized size	1	1.	0.53	0.57	0.	1.16	0.	0.
time (sec)	N/A	0.216	0.096	0.122	0.	1.532	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	70	70	0	150	0	0
normalized size	1	1.	0.47	0.47	0.	1.	0.	0.
time (sec)	N/A	0.266	0.113	0.128	0.	1.651	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	78	78	0	173	0	0
normalized size	1	1.	0.41	0.41	0.	0.92	0.	0.
time (sec)	N/A	0.414	0.124	0.115	0.	1.706	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	86	86	0	186	0	0
normalized size	1	1.	0.3	0.3	0.	0.64	0.	0.
time (sec)	N/A	0.293	0.167	0.118	0.	1.656	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	185	81	155	0	0	0	0	0
normalized size	1	0.44	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.35	0.083	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.401	0.062	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	113	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.073	0.065	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	180.005	0.175	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	180.008	0.185	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	180.006	0.172	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	180.013	0.184	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.877	0.188	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.028	0.182	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.607	0.191	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	272	0
normalized size	1	1.	0.81	0.	0.	0.	4.77	0.
time (sec)	N/A	0.107	0.024	0.345	0.	0.	7.644	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.817	0.17	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	1.085	0.174	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.051	0.333	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	111	279	560	404	0	500
normalized size	1	1.	0.28	0.71	1.42	1.03	0.	1.27
time (sec)	N/A	0.337	0.179	0.149	1.069	1.397	0.	1.18

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	95	231	455	344	0	408
normalized size	1	1.	0.3	0.74	1.45	1.1	0.	1.3
time (sec)	N/A	0.251	0.188	0.138	1.054	1.32	0.	1.192

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	79	183	350	285	0	316
normalized size	1	1.	0.34	0.79	1.5	1.22	0.	1.36
time (sec)	N/A	0.194	0.097	0.132	1.058	1.322	0.	1.17

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	61	119	231	215	0	211
normalized size	1	1.	0.42	0.82	1.59	1.48	0.	1.46
time (sec)	N/A	0.116	0.091	0.137	1.123	1.378	0.	1.193

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	30	72	0	30
normalized size	1	1.	1.	1.77	2.31	5.54	0.	2.31
time (sec)	N/A	0.029	0.044	0.043	1.085	1.54	0.	1.129

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	50	49	88	123	0	104
normalized size	1	1.	0.98	0.96	1.73	2.41	0.	2.04
time (sec)	N/A	0.06	0.143	0.045	1.061	1.573	0.	1.107

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	134	181	0	186
normalized size	1	1.	0.78	0.76	1.58	2.13	0.	2.19
time (sec)	N/A	0.093	0.167	0.044	0.997	1.637	0.	1.134

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	82	81	178	278	0	259
normalized size	1	1.	0.69	0.68	1.5	2.34	0.	2.18
time (sec)	N/A	0.128	0.247	0.044	1.1	1.557	0.	1.125

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	47	85	153	235	119	153
normalized size	1	1.	0.56	1.01	1.82	2.8	1.42	1.82
time (sec)	N/A	0.099	0.041	0.037	1.125	1.371	0.111	1.15

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	39	63	111	166	87	111
normalized size	1	1.	0.57	0.91	1.61	2.41	1.26	1.61
time (sec)	N/A	0.085	0.032	0.038	1.215	1.552	0.1	1.118

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	54	95	143	70	95
normalized size	1	1.	0.6	1.04	1.83	2.75	1.35	1.83
time (sec)	N/A	0.077	0.023	0.039	1.074	1.532	0.092	1.121

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	31	51	74	36	51
normalized size	1	1.	0.66	0.89	1.46	2.11	1.03	1.46
time (sec)	N/A	0.067	0.016	0.039	1.047	1.448	0.081	1.129

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	14	28	43	20	28
normalized size	1	1.	1.33	0.93	1.87	2.87	1.33	1.87
time (sec)	N/A	0.034	0.012	0.037	1.06	1.474	0.07	1.097

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	18	26	10	19
normalized size	1	1.	1.12	0.94	1.12	1.62	0.62	1.19
time (sec)	N/A	0.065	0.015	0.039	1.022	1.498	0.292	1.122

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	85	171	54	69
normalized size	1	1.	0.69	1.18	1.67	3.35	1.06	1.35
time (sec)	N/A	0.079	0.025	0.048	1.061	1.574	0.465	1.12

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	90	123	266	83	100
normalized size	1	1.	0.73	1.05	1.43	3.09	0.97	1.16
time (sec)	N/A	0.1	0.037	0.05	1.061	1.536	0.676	1.117

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	82	120	189	456	141	123
normalized size	1	1.	0.68	0.99	1.56	3.77	1.17	1.02
time (sec)	N/A	0.123	0.066	0.05	1.031	1.528	0.985	1.114

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	111	288	560	419	0	500
normalized size	1	1.	0.28	0.73	1.42	1.07	0.	1.27
time (sec)	N/A	0.333	0.192	0.207	1.048	1.628	0.	1.209

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	95	240	455	351	0	408
normalized size	1	1.	0.3	0.77	1.45	1.12	0.	1.3
time (sec)	N/A	0.254	0.135	0.19	1.087	1.704	0.	1.205

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	79	192	350	301	0	316
normalized size	1	1.	0.34	0.82	1.5	1.29	0.	1.36
time (sec)	N/A	0.213	0.103	0.184	1.082	1.671	0.	1.187

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	61	183	231	225	0	211
normalized size	1	1.	0.42	1.26	1.59	1.55	0.	1.46
time (sec)	N/A	0.119	0.076	0.175	1.046	1.516	0.	1.188

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	31	112	0	47
normalized size	1	1.	1.	1.33	1.72	6.22	0.	2.61
time (sec)	N/A	0.031	0.046	0.123	1.077	1.583	0.	1.174

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	43	49	74	161	0	93
normalized size	1	1.	0.78	0.89	1.35	2.93	0.	1.69
time (sec)	N/A	0.065	0.149	0.131	1.316	1.497	0.	1.141

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	66	65	131	205	0	150
normalized size	1	1.	0.73	0.71	1.44	2.25	0.	1.65
time (sec)	N/A	0.1	0.205	0.122	1.244	1.609	0.	1.172

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	82	81	177	263	0	230
normalized size	1	1.	0.65	0.64	1.39	2.07	0.	1.81
time (sec)	N/A	0.139	0.307	0.135	1.211	1.537	0.	1.221

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	39	75	136	215	109	138
normalized size	1	1.	0.59	1.14	2.06	3.26	1.65	2.09
time (sec)	N/A	0.085	0.034	0.038	1.072	1.399	0.117	1.148

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	69	124	190	100	122
normalized size	1	1.	0.6	1.33	2.38	3.65	1.92	2.35
time (sec)	N/A	0.079	0.03	0.037	1.113	1.473	0.112	1.17

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	45	80	122	63	105
normalized size	1	1.	0.66	1.29	2.29	3.49	1.8	3.
time (sec)	N/A	0.063	0.021	0.041	0.981	1.515	0.096	1.157

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	63	93	48	86
normalized size	1	1.	2.18	0.94	3.71	5.47	2.82	5.06
time (sec)	N/A	0.057	0.028	0.04	1.081	1.537	0.092	1.116

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	34	45	88	36	81
normalized size	1	1.	0.78	0.74	0.98	1.91	0.78	1.76
time (sec)	N/A	0.048	0.014	0.042	1.097	1.632	0.294	1.126

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	26	39	17	36
normalized size	1	1.	1.92	2.15	2.	3.	1.31	2.77
time (sec)	N/A	0.063	0.01	0.046	1.093	1.484	0.344	1.12

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	16	55	78	42	20
normalized size	1	1.	0.94	0.89	3.06	4.33	2.33	1.11
time (sec)	N/A	0.06	0.018	0.039	1.018	1.395	0.401	1.138

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	90	138	324	99	123
normalized size	1	1.	0.6	1.03	1.59	3.72	1.14	1.41
time (sec)	N/A	0.093	0.038	0.049	1.046	1.581	0.682	1.142

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	120	176	421	129	171
normalized size	1	1.	0.66	0.98	1.44	3.45	1.06	1.4
time (sec)	N/A	0.118	0.061	0.055	1.088	1.487	0.957	1.15

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	111	279	560	401	0	265
normalized size	1	1.	0.28	0.71	1.42	1.02	0.	0.67
time (sec)	N/A	0.322	0.199	0.154	1.127	1.657	0.	1.183

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	95	231	455	343	0	217
normalized size	1	1.	0.3	0.74	1.45	1.1	0.	0.69
time (sec)	N/A	0.255	0.197	0.138	1.075	2.02	0.	1.173

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	79	183	350	285	0	170
normalized size	1	1.	0.34	0.79	1.5	1.22	0.	0.73
time (sec)	N/A	0.199	0.104	0.138	1.14	2.008	0.	1.164

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	61	119	231	216	0	111
normalized size	1	1.	0.42	0.82	1.59	1.49	0.	0.77
time (sec)	N/A	0.115	0.088	0.134	1.072	1.826	0.	1.154

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	31	46	0	0
normalized size	1	1.	1.	1.5	1.94	2.88	0.	0.
time (sec)	N/A	0.03	0.045	0.043	1.044	1.829	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	90	105	0	0
normalized size	1	1.	0.87	0.89	1.64	1.91	0.	0.
time (sec)	N/A	0.062	0.14	0.046	0.995	1.646	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	64	65	138	163	0	0
normalized size	1	1.	0.7	0.71	1.52	1.79	0.	0.
time (sec)	N/A	0.097	0.189	0.046	0.979	1.514	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	80	81	182	223	0	0
normalized size	1	1.	0.63	0.64	1.43	1.76	0.	0.
time (sec)	N/A	0.136	0.255	0.046	1.048	1.598	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	39	61	108	166	87	108
normalized size	1	1.	0.53	0.84	1.48	2.27	1.19	1.48
time (sec)	N/A	0.081	0.028	0.042	1.117	1.528	0.104	1.117

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	31	54	95	143	70	95
normalized size	1	1.	0.56	0.98	1.73	2.6	1.27	1.73
time (sec)	N/A	0.075	0.025	0.042	1.006	1.483	0.095	1.119

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	30	50	74	36	50
normalized size	1	1.	0.81	0.81	1.35	2.	0.97	1.35
time (sec)	N/A	0.063	0.017	0.04	1.13	1.502	0.084	1.134

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	14	27	43	19	27
normalized size	1	1.	1.31	0.88	1.69	2.69	1.19	1.69
time (sec)	N/A	0.034	0.014	0.04	1.02	1.525	0.072	1.147

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	16	26	10	19
normalized size	1	1.	1.29	1.07	1.14	1.86	0.71	1.36
time (sec)	N/A	0.061	0.014	0.039	1.035	1.493	0.292	1.111

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	60	85	171	54	69
normalized size	1	1.	0.67	1.22	1.73	3.49	1.1	1.41
time (sec)	N/A	0.076	0.027	0.05	1.112	1.506	0.464	1.107

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	61	90	123	266	83	100
normalized size	1	1.	0.73	1.07	1.46	3.17	0.99	1.19
time (sec)	N/A	0.1	0.04	0.053	1.094	1.555	0.653	1.113

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	80	120	189	456	141	123
normalized size	1	1.	0.67	1.01	1.59	3.83	1.18	1.03
time (sec)	N/A	0.12	0.062	0.052	1.034	1.645	0.975	1.141

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	111	288	560	416	0	267
normalized size	1	1.	0.28	0.73	1.42	1.06	0.	0.68
time (sec)	N/A	0.337	0.186	0.15	1.068	1.683	0.	1.146

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	95	240	455	347	0	219
normalized size	1	1.	0.3	0.77	1.45	1.11	0.	0.7
time (sec)	N/A	0.264	0.144	0.143	1.092	1.596	0.	1.186

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	79	192	350	296	0	170
normalized size	1	1.	0.34	0.82	1.5	1.27	0.	0.73
time (sec)	N/A	0.204	0.106	0.132	1.022	1.539	0.	1.136

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	61	183	231	223	0	111
normalized size	1	1.	0.42	1.26	1.59	1.54	0.	0.77
time (sec)	N/A	0.12	0.101	0.133	1.117	1.679	0.	1.141

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	31	78	0	66
normalized size	1	1.	1.	1.33	1.72	4.33	0.	3.67
time (sec)	N/A	0.032	0.047	0.043	1.042	1.519	0.	1.207

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	43	49	81	124	0	88
normalized size	1	1.	0.78	0.89	1.47	2.25	0.	1.6
time (sec)	N/A	0.066	0.141	0.043	1.049	1.574	0.	1.178

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	66	65	139	182	0	0
normalized size	1	1.	0.73	0.71	1.53	2.	0.	0.
time (sec)	N/A	0.101	0.185	0.047	1.032	1.537	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	82	81	184	278	0	0
normalized size	1	1.	0.65	0.64	1.45	2.19	0.	0.
time (sec)	N/A	0.138	0.251	0.046	1.057	1.638	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	79	116	0	261	0	0
normalized size	1	1.	0.34	0.51	0.	1.14	0.	0.
time (sec)	N/A	0.2	0.064	0.046	0.	1.659	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	71	100	0	212	0	0
normalized size	1	1.	0.39	0.55	0.	1.16	0.	0.
time (sec)	N/A	0.183	0.052	0.046	0.	1.569	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	63	84	0	153	0	0
normalized size	1	1.	0.46	0.62	0.	1.12	0.	0.
time (sec)	N/A	0.176	0.041	0.043	0.	1.605	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	53	68	0	96	0	0
normalized size	1	1.	0.57	0.73	0.	1.03	0.	0.
time (sec)	N/A	0.166	0.031	0.041	0.	1.564	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	44	0	46	0	0
normalized size	1	1.	0.6	0.65	0.	0.68	0.	0.
time (sec)	N/A	0.109	0.019	0.04	0.	1.555	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	0	49	0	0
normalized size	1	1.	1.	1.34	0.	1.29	0.	0.
time (sec)	N/A	0.146	0.019	0.134	0.	1.462	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	177	0	0
normalized size	1	1.	0.62	0.92	0.	1.95	0.	0.
time (sec)	N/A	0.166	0.037	0.146	0.	1.651	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	278	0	0
normalized size	1	1.	0.45	0.92	0.	1.51	0.	0.
time (sec)	N/A	0.185	0.059	0.147	0.	1.671	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	101	241	0	393	0	0
normalized size	1	1.	0.36	0.87	0.	1.42	0.	0.
time (sec)	N/A	0.21	0.081	0.161	0.	1.58	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	167	350	0	790	0	221
normalized size	1	1.	0.95	1.99	0.	4.49	0.	1.26
time (sec)	N/A	0.165	0.154	0.059	0.	1.918	0.	1.179

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	151	296	0	653	1091	190
normalized size	1	1.	0.99	1.93	0.	4.27	7.13	1.24
time (sec)	N/A	0.153	0.131	0.052	0.	1.681	34.397	1.17

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	135	242	0	544	478	157
normalized size	1	1.	1.04	1.86	0.	4.18	3.68	1.21
time (sec)	N/A	0.138	0.116	0.049	0.	1.715	17.119	1.187

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	117	188	0	412	342	115
normalized size	1	1.	1.09	1.76	0.	3.85	3.2	1.07
time (sec)	N/A	0.127	0.092	0.047	0.	1.716	20.065	1.133

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	76	134	0	306	0	84
normalized size	1	1.	0.88	1.56	0.	3.56	0.	0.98
time (sec)	N/A	0.11	0.052	0.049	0.	1.703	0.	1.152

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	82	79	0	339	0	0
normalized size	1	1.	1.39	1.34	0.	5.75	0.	0.
time (sec)	N/A	0.105	0.037	0.045	0.	1.792	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	64	31	0	97	0	200
normalized size	1	1.	1.25	0.61	0.	1.9	0.	3.92
time (sec)	N/A	0.105	0.035	0.04	0.	1.658	0.	1.202

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	53	47	0	151	0	0
normalized size	1	1.	0.72	0.64	0.	2.04	0.	0.
time (sec)	N/A	0.11	0.038	0.043	0.	2.076	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	64	0	250	0	0
normalized size	1	1.	0.99	0.66	0.	2.58	0.	0.
time (sec)	N/A	0.12	0.054	0.043	0.	3.029	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	112	80	0	317	0	0
normalized size	1	1.	0.93	0.67	0.	2.64	0.	0.
time (sec)	N/A	0.131	0.059	0.057	0.	5.041	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	71	100	275	212	0	0
normalized size	1	1.	0.38	0.54	1.49	1.15	0.	0.
time (sec)	N/A	0.2	0.058	0.128	1.077	1.508	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	63	100	232	209	0	0
normalized size	1	1.	0.45	0.72	1.67	1.5	0.	0.
time (sec)	N/A	0.193	0.049	0.133	1.084	1.515	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	55	84	189	154	0	0
normalized size	1	1.	0.59	0.9	2.03	1.66	0.	0.
time (sec)	N/A	0.185	0.041	0.123	1.068	1.61	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	58	60	131	90	0	0
normalized size	1	1.	1.26	1.3	2.85	1.96	0.	0.
time (sec)	N/A	0.176	0.031	0.116	1.109	1.59	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	57	67	0	77	0	0
normalized size	1	1.	0.5	0.59	0.	0.68	0.	0.
time (sec)	N/A	0.133	0.028	0.166	0.	1.6	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	53	64	0	84	0	0
normalized size	1	1.	0.67	0.81	0.	1.06	0.	0.
time (sec)	N/A	0.167	0.032	0.17	0.	1.898	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	51	39	0	77	0	0
normalized size	1	1.	1.09	0.83	0.	1.64	0.	0.
time (sec)	N/A	0.172	0.045	0.114	0.	1.9	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	71	169	0	290	0	0
normalized size	1	1.	0.38	0.91	0.	1.57	0.	0.
time (sec)	N/A	0.204	0.061	0.175	0.	1.901	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	99	241	0	406	0	0
normalized size	1	1.	0.36	0.87	0.	1.46	0.	0.
time (sec)	N/A	0.225	0.088	0.181	0.	1.97	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	79	116	0	261	0	0
normalized size	1	1.	0.34	0.5	0.	1.12	0.	0.
time (sec)	N/A	0.211	0.061	0.044	0.	1.81	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	71	100	0	212	0	0
normalized size	1	1.	0.38	0.53	0.	1.13	0.	0.
time (sec)	N/A	0.201	0.048	0.043	0.	1.982	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	63	84	0	153	0	0
normalized size	1	1.	0.45	0.6	0.	1.1	0.	0.
time (sec)	N/A	0.193	0.043	0.046	0.	1.856	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	53	68	0	96	0	0
normalized size	1	1.	0.56	0.72	0.	1.01	0.	0.
time (sec)	N/A	0.18	0.032	0.041	0.	1.888	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	46	0	0
normalized size	1	1.	0.59	0.64	0.	0.67	0.	0.
time (sec)	N/A	0.117	0.02	0.041	0.	1.798	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	0	49	0	0
normalized size	1	1.	1.	1.38	0.	1.32	0.	0.
time (sec)	N/A	0.153	0.019	0.134	0.	1.705	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	54	84	0	177	0	0
normalized size	1	1.	0.6	0.93	0.	1.97	0.	0.
time (sec)	N/A	0.184	0.049	0.145	0.	1.843	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	81	169	0	278	0	0
normalized size	1	1.	0.44	0.92	0.	1.52	0.	0.
time (sec)	N/A	0.197	0.066	0.146	0.	1.936	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	99	241	0	393	0	0
normalized size	1	1.	0.36	0.87	0.	1.42	0.	0.
time (sec)	N/A	0.221	0.086	0.151	0.	1.928	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	136	226	0	544	478	158
normalized size	1	1.	1.04	1.73	0.	4.15	3.65	1.21
time (sec)	N/A	0.141	0.136	0.05	0.	1.983	14.121	1.171

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	117	176	0	412	340	115
normalized size	1	1.	1.08	1.63	0.	3.81	3.15	1.06
time (sec)	N/A	0.127	0.089	0.049	0.	1.73	9.26	1.148

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	100	126	0	306	0	84
normalized size	1	1.	1.15	1.45	0.	3.52	0.	0.97
time (sec)	N/A	0.111	0.05	0.05	0.	1.767	0.	1.171

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	100	73	0	339	0	0
normalized size	1	1.	1.67	1.22	0.	5.65	0.	0.
time (sec)	N/A	0.104	0.061	0.05	0.	1.664	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	52	63	31	0	97	0	200
normalized size	1	1.	1.21	0.6	0.	1.87	0.	3.85
time (sec)	N/A	0.104	0.035	0.044	0.	1.705	0.	1.203

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	47	0	151	0	0
normalized size	1	1.	1.05	0.63	0.	2.01	0.	0.
time (sec)	N/A	0.115	0.053	0.043	0.	1.892	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	64	0	250	0	0
normalized size	1	1.	0.98	0.65	0.	2.55	0.	0.
time (sec)	N/A	0.124	0.061	0.043	0.	3.037	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	112	80	0	317	0	0
normalized size	1	1.	0.93	0.66	0.	2.62	0.	0.
time (sec)	N/A	0.141	0.072	0.044	0.	5.072	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	71	100	275	212	0	0
normalized size	1	1.	0.38	0.53	1.46	1.12	0.	0.
time (sec)	N/A	0.202	0.056	0.046	1.095	1.669	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	63	100	232	209	0	0
normalized size	1	1.	0.44	0.7	1.63	1.47	0.	0.
time (sec)	N/A	0.191	0.049	0.046	1.129	1.615	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	84	189	154	0	0
normalized size	1	1.	0.58	0.88	1.99	1.62	0.	0.
time (sec)	N/A	0.179	0.039	0.04	1.102	1.621	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	58	60	131	90	0	0
normalized size	1	1.	1.23	1.28	2.79	1.91	0.	0.
time (sec)	N/A	0.168	0.032	0.042	1.115	1.616	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	56	67	0	77	0	0
normalized size	1	1.	0.5	0.6	0.	0.69	0.	0.
time (sec)	N/A	0.127	0.026	0.131	0.	1.655	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	52	62	0	84	0	0
normalized size	1	1.	0.68	0.81	0.	1.09	0.	0.
time (sec)	N/A	0.163	0.034	0.139	0.	1.596	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	39	0	77	0	0
normalized size	1	1.	1.11	0.85	0.	1.67	0.	0.
time (sec)	N/A	0.169	0.048	0.043	0.	1.538	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	71	169	0	290	0	0
normalized size	1	1.	0.39	0.93	0.	1.59	0.	0.
time (sec)	N/A	0.198	0.063	0.144	0.	1.609	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	99	241	0	406	0	0
normalized size	1	1.	0.36	0.88	0.	1.48	0.	0.
time (sec)	N/A	0.222	0.094	0.151	0.	1.721	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	47	0	53	0	0
normalized size	1	1.	0.59	0.62	0.	0.7	0.	0.
time (sec)	N/A	0.203	0.024	0.04	0.	1.593	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	47	0	51	0	0
normalized size	1	1.	0.58	0.64	0.	0.69	0.	0.
time (sec)	N/A	0.174	0.021	0.042	0.	1.574	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	44	0	46	0	0
normalized size	1	1.	0.6	0.65	0.	0.68	0.	0.
time (sec)	N/A	0.108	0.017	0.04	0.	1.549	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	42	44	0	42	0	0
normalized size	1	1.	0.61	0.64	0.	0.61	0.	0.
time (sec)	N/A	0.139	0.02	0.138	0.	1.723	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	43	48	0	50	0	0
normalized size	1	1.	0.59	0.66	0.	0.68	0.	0.
time (sec)	N/A	0.195	0.024	0.136	0.	1.501	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	96	210	0	431	0	124
normalized size	1	1.	0.7	1.53	0.	3.15	0.	0.91
time (sec)	N/A	0.413	0.121	0.056	0.	1.676	0.	1.177

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	186	0	396	0	113
normalized size	1	1.	0.79	1.66	0.	3.54	0.	1.01
time (sec)	N/A	0.387	0.095	0.052	0.	1.689	0.	1.135

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	79	162	0	344	0	97
normalized size	1	1.	0.93	1.91	0.	4.05	0.	1.14
time (sec)	N/A	0.251	0.079	0.052	0.	1.703	0.	1.146

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	76	134	0	306	0	84
normalized size	1	1.	0.88	1.56	0.	3.56	0.	0.98
time (sec)	N/A	0.114	0.053	0.049	0.	1.647	0.	1.153

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	97	129	0	441	0	128
normalized size	1	1.	1.29	1.72	0.	5.88	0.	1.71
time (sec)	N/A	0.345	0.077	0.055	0.	1.661	0.	1.179

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	104	208	0	477	0	181
normalized size	1	1.	1.27	2.54	0.	5.82	0.	2.21
time (sec)	N/A	0.348	0.091	0.058	0.	1.6	0.	1.191

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	76	239	0	338	0	270
normalized size	1	1.	0.97	3.06	0.	4.33	0.	3.46
time (sec)	N/A	0.339	0.113	0.057	0.	1.585	0.	1.128

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	82	261	0	370	0	338
normalized size	1	1.	0.83	2.64	0.	3.74	0.	3.41
time (sec)	N/A	0.378	0.116	0.065	0.	1.655	0.	1.179

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	95	287	0	416	0	437
normalized size	1	1.	0.73	2.21	0.	3.2	0.	3.36
time (sec)	N/A	0.403	0.137	0.066	0.	1.617	0.	1.183

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	88	92	0	142	0	0
normalized size	1	1.	0.39	0.4	0.	0.62	0.	0.
time (sec)	N/A	0.27	0.059	0.175	0.	1.602	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	74	83	0	112	0	0
normalized size	1	1.	0.4	0.45	0.	0.6	0.	0.
time (sec)	N/A	0.252	0.044	0.181	0.	1.616	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	66	76	0	99	0	0
normalized size	1	1.	0.43	0.5	0.	0.65	0.	0.
time (sec)	N/A	0.224	0.038	0.178	0.	1.635	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	57	67	0	77	0	0
normalized size	1	1.	0.5	0.59	0.	0.68	0.	0.
time (sec)	N/A	0.133	0.024	0.174	0.	1.532	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	53	59	0	65	0	0
normalized size	1	1.	0.46	0.52	0.	0.57	0.	0.
time (sec)	N/A	0.161	0.033	0.187	0.	1.678	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	55	65	0	81	0	0
normalized size	1	1.	0.48	0.57	0.	0.71	0.	0.
time (sec)	N/A	0.238	0.032	0.184	0.	1.629	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	69	77	0	197	0	0
normalized size	1	1.	0.45	0.5	0.	1.29	0.	0.
time (sec)	N/A	0.24	0.038	0.191	0.	1.562	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	79	85	0	216	0	0
normalized size	1	1.	0.41	0.44	0.	1.11	0.	0.
time (sec)	N/A	0.248	0.043	0.195	0.	1.644	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	84	93	0	232	0	0
normalized size	1	1.	0.37	0.41	0.	1.02	0.	0.
time (sec)	N/A	0.253	0.057	0.2	0.	1.624	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	77	106	0	173	0	0
normalized size	1	1.	0.36	0.5	0.	0.82	0.	0.
time (sec)	N/A	0.23	0.082	0.16	0.	1.737	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	71	98	0	157	0	0
normalized size	1	1.	0.41	0.57	0.	0.91	0.	0.
time (sec)	N/A	0.2	0.055	0.152	0.	1.595	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	75	86	0	130	0	0
normalized size	1	1.	0.58	0.66	0.	1.	0.	0.
time (sec)	N/A	0.254	0.044	0.166	0.	1.625	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	59	84	0	177	0	0
normalized size	1	1.	0.68	0.97	0.	2.03	0.	0.
time (sec)	N/A	0.211	0.041	0.153	0.	1.535	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	177	0	0
normalized size	1	1.	0.62	0.92	0.	1.95	0.	0.
time (sec)	N/A	0.171	0.032	0.14	0.	1.628	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	68	92	0	157	0	0
normalized size	1	1.	0.38	0.52	0.	0.89	0.	0.
time (sec)	N/A	0.252	0.059	0.144	0.	1.693	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	79	118	0	197	0	0
normalized size	1	1.	0.37	0.55	0.	0.92	0.	0.
time (sec)	N/A	0.265	0.065	0.136	0.	1.726	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	94	138	0	232	0	0
normalized size	1	1.	0.37	0.55	0.	0.92	0.	0.
time (sec)	N/A	0.272	0.08	0.143	0.	1.724	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	89	185	0	296	0	0
normalized size	1	1.	0.34	0.71	0.	1.13	0.	0.
time (sec)	N/A	0.242	0.109	0.139	0.	1.597	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	84	169	0	257	0	0
normalized size	1	1.	0.39	0.78	0.	1.18	0.	0.
time (sec)	N/A	0.278	0.095	0.143	0.	1.546	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	86	169	0	275	0	0
normalized size	1	1.	0.49	0.96	0.	1.56	0.	0.
time (sec)	N/A	0.268	0.064	0.142	0.	1.613	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	85	164	0	273	0	0
normalized size	1	1.	0.46	0.89	0.	1.48	0.	0.
time (sec)	N/A	0.272	0.06	0.144	0.	1.504	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	60	164	0	270	0	0
normalized size	1	1.	0.44	1.2	0.	1.97	0.	0.
time (sec)	N/A	0.221	0.068	0.15	0.	1.518	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	278	0	0
normalized size	1	1.	0.45	0.92	0.	1.51	0.	0.
time (sec)	N/A	0.197	0.05	0.15	0.	1.571	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	88	196	0	311	0	0
normalized size	1	1.	0.32	0.72	0.	1.15	0.	0.
time (sec)	N/A	0.284	0.096	0.139	0.	1.671	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	99	225	0	360	0	0
normalized size	1	1.	0.32	0.73	0.	1.17	0.	0.
time (sec)	N/A	0.293	0.089	0.143	0.	1.765	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	47	0	53	0	0
normalized size	1	1.	0.59	0.62	0.	0.7	0.	0.
time (sec)	N/A	0.225	0.027	0.04	0.	1.583	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	47	0	51	0	0
normalized size	1	1.	0.58	0.64	0.	0.69	0.	0.
time (sec)	N/A	0.193	0.023	0.038	0.	1.581	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	46	0	0
normalized size	1	1.	0.59	0.64	0.	0.67	0.	0.
time (sec)	N/A	0.12	0.02	0.037	0.	1.594	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	46	0	42	0	0
normalized size	1	1.	0.63	0.66	0.	0.6	0.	0.
time (sec)	N/A	0.146	0.019	0.12	0.	1.637	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	44	48	0	50	0	0
normalized size	1	1.	0.61	0.67	0.	0.69	0.	0.
time (sec)	N/A	0.216	0.022	0.133	0.	1.622	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	96	202	0	431	0	124
normalized size	1	1.	0.7	1.47	0.	3.15	0.	0.91
time (sec)	N/A	0.43	0.127	0.055	0.	1.777	0.	1.168

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	178	0	396	0	113
normalized size	1	1.	0.79	1.59	0.	3.54	0.	1.01
time (sec)	N/A	0.396	0.099	0.049	0.	1.626	0.	1.138

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	156	0	344	0	99
normalized size	1	1.	0.94	1.86	0.	4.1	0.	1.18
time (sec)	N/A	0.254	0.083	0.048	0.	1.737	0.	1.173

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	100	126	0	306	0	84
normalized size	1	1.	1.15	1.45	0.	3.52	0.	0.97
time (sec)	N/A	0.113	0.053	0.048	0.	1.695	0.	1.174

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	97	121	0	443	0	128
normalized size	1	1.	1.29	1.61	0.	5.91	0.	1.71
time (sec)	N/A	0.349	0.082	0.052	0.	1.674	0.	1.137

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	104	200	0	478	0	181
normalized size	1	1.	1.27	2.44	0.	5.83	0.	2.21
time (sec)	N/A	0.347	0.095	0.052	0.	1.681	0.	1.167

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	76	231	0	338	0	270
normalized size	1	1.	0.97	2.96	0.	4.33	0.	3.46
time (sec)	N/A	0.347	0.114	0.056	0.	1.731	0.	1.139

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	82	254	0	371	0	338
normalized size	1	1.	0.81	2.51	0.	3.67	0.	3.35
time (sec)	N/A	0.368	0.114	0.059	0.	1.721	0.	1.141

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	95	279	0	416	0	437
normalized size	1	1.	0.73	2.15	0.	3.2	0.	3.36
time (sec)	N/A	0.402	0.139	0.063	0.	1.668	0.	1.175

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	87	92	0	142	0	0
normalized size	1	1.	0.38	0.41	0.	0.63	0.	0.
time (sec)	N/A	0.261	0.057	0.127	0.	1.644	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	72	83	0	112	0	0
normalized size	1	1.	0.39	0.45	0.	0.6	0.	0.
time (sec)	N/A	0.258	0.051	0.132	0.	1.569	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	65	76	0	99	0	0
normalized size	1	1.	0.43	0.5	0.	0.66	0.	0.
time (sec)	N/A	0.212	0.039	0.131	0.	1.586	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	56	67	0	77	0	0
normalized size	1	1.	0.5	0.6	0.	0.69	0.	0.
time (sec)	N/A	0.129	0.024	0.127	0.	1.638	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	50	57	0	65	0	0
normalized size	1	1.	0.45	0.51	0.	0.58	0.	0.
time (sec)	N/A	0.16	0.026	0.13	0.	1.736	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	56	65	0	81	0	0
normalized size	1	1.	0.49	0.57	0.	0.71	0.	0.
time (sec)	N/A	0.233	0.03	0.132	0.	1.641	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	68	77	0	197	0	0
normalized size	1	1.	0.45	0.51	0.	1.3	0.	0.
time (sec)	N/A	0.239	0.035	0.131	0.	1.639	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	78	85	0	216	0	0
normalized size	1	1.	0.4	0.44	0.	1.12	0.	0.
time (sec)	N/A	0.245	0.042	0.132	0.	1.675	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	83	93	0	232	0	0
normalized size	1	1.	0.37	0.41	0.	1.02	0.	0.
time (sec)	N/A	0.251	0.052	0.133	0.	1.559	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.051	0.357	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	129	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.387	0.209	0.477	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	56	62	73	166	0	0
normalized size	1	1.	0.68	0.76	0.89	2.02	0.	0.
time (sec)	N/A	0.213	0.033	0.039	1.087	1.97	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	64	77	166	0	0
normalized size	1	1.	0.7	0.77	0.93	2.	0.	0.
time (sec)	N/A	0.227	0.034	0.04	1.151	2.109	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	110	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.378	0.149	0.477	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	75	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.051	0.357	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	267	0	0	0	0	0
normalized size	1	1.	3.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	2.281	0.22	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	1.307	0.211	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.704	0.06	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.175	0.037	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	42	54	0	0
normalized size	1	1.	1.	1.	2.33	3.	0.	0.
time (sec)	N/A	0.035	0.049	0.04	1.099	1.671	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	55	0	165	0	0
normalized size	1	1.	0.76	0.76	0.	2.29	0.	0.
time (sec)	N/A	0.078	0.162	0.043	0.	1.738	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	97	101	0	373	0	0
normalized size	1	1.	0.76	0.8	0.	2.94	0.	0.
time (sec)	N/A	0.13	0.216	0.043	0.	1.667	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	152	167	0	683	0	0
normalized size	1	1.	0.77	0.85	0.	3.47	0.	0.
time (sec)	N/A	0.181	0.301	0.046	0.	1.782	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	280	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	2.223	0.306	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	101	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	0.553	0.31	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	81	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.177	0.306	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	153	0	0
normalized size	1	1.	0.93	1.07	0.	3.33	0.	0.
time (sec)	N/A	0.054	0.182	0.041	0.	1.603	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	335	0	0
normalized size	1	1.	1.08	0.82	0.	3.28	0.	0.
time (sec)	N/A	0.112	0.581	0.043	0.	1.569	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	299	140	0	622	0	0
normalized size	1	1.	1.8	0.84	0.	3.75	0.	0.
time (sec)	N/A	0.177	1.461	0.042	0.	1.7	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	260	218	0	1030	0	0
normalized size	1	1.	1.09	0.91	0.	4.31	0.	0.
time (sec)	N/A	0.254	1.515	0.045	0.	1.678	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	359	363	133	0	0	0	0	0
normalized size	1	1.01	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.333	0.553	0.307	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	127	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	0.375	0.309	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	161	0	0
normalized size	1	1.	0.93	1.07	0.	3.5	0.	0.
time (sec)	N/A	0.092	0.17	0.041	0.	1.397	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	153	0	0
normalized size	1	1.	0.93	1.07	0.	3.33	0.	0.
time (sec)	N/A	0.052	0.178	0.043	0.	1.402	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	127	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	0.824	0.309	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	463	467	201	0	0	0	0	0
normalized size	1	1.01	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.535	1.976	0.303	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	110	93	0	354	0	0
normalized size	1	1.	0.33	0.28	0.	1.07	0.	0.
time (sec)	N/A	0.367	0.654	0.043	0.	1.634	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	109	96	0	356	0	0
normalized size	1	1.	1.07	0.94	0.	3.49	0.	0.
time (sec)	N/A	0.189	0.689	0.043	0.	1.328	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	108	86	0	346	0	0
normalized size	1	1.	1.11	0.89	0.	3.57	0.	0.
time (sec)	N/A	0.147	0.586	0.041	0.	1.451	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	335	0	0
normalized size	1	1.	1.08	0.82	0.	3.28	0.	0.
time (sec)	N/A	0.106	0.2	0.041	0.	1.513	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	944	944	220	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.632	1.557	0.342	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	83	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.188	0.378	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	38	49	89	0	0
normalized size	1	1.	0.71	0.75	0.96	1.75	0.	0.
time (sec)	N/A	0.123	0.064	0.039	1.055	1.401	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	40	46	92	0	0
normalized size	1	1.	0.69	0.77	0.88	1.77	0.	0.
time (sec)	N/A	0.122	0.062	0.037	1.073	1.311	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.023	0.677	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.256	0.372	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	67	0	0	0	651	0
normalized size	1	1.	1.24	0.	0.	0.	12.06	0.
time (sec)	N/A	0.088	0.018	0.522	0.	0.	9.65	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.202	0.369	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.22	0.359	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	73	0	0	0	651	0
normalized size	1	1.	1.33	0.	0.	0.	11.84	0.
time (sec)	N/A	0.086	0.024	0.554	0.	0.	8.908	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	119	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.268	0.374	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	120	320	513	489	0	493
normalized size	1	1.	0.35	0.94	1.5	1.43	0.	1.44
time (sec)	N/A	0.246	0.275	0.165	1.603	1.408	0.	1.16

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	104	272	408	423	0	401
normalized size	1	1.	0.39	1.01	1.52	1.58	0.	1.5
time (sec)	N/A	0.188	0.205	0.141	1.494	1.359	0.	1.205

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	94	224	301	358	0	309
normalized size	1	1.	0.48	1.15	1.55	1.85	0.	1.59
time (sec)	N/A	0.134	0.132	0.135	1.53	1.334	0.	1.204

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	53	163	158	248	0	170
normalized size	1	1.	0.5	1.52	1.48	2.32	0.	1.59
time (sec)	N/A	0.071	0.078	0.153	1.519	1.437	0.	1.17

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	56	251	157	215	0	171
normalized size	1	1.	0.54	2.41	1.51	2.07	0.	1.64
time (sec)	N/A	0.075	0.11	0.141	1.019	1.368	0.	1.147

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	83	530	216	306	0	231
normalized size	1	1.	0.46	2.94	1.2	1.7	0.	1.28
time (sec)	N/A	0.116	0.154	0.149	1.007	1.374	0.	1.141

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	99	714	262	405	0	313
normalized size	1	1.	0.39	2.81	1.03	1.59	0.	1.23
time (sec)	N/A	0.166	0.196	0.164	1.031	1.454	0.	1.15

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	115	898	311	602	0	386
normalized size	1	1.	0.35	2.74	0.95	1.84	0.	1.18
time (sec)	N/A	0.228	0.267	0.156	1.012	1.437	0.	1.151

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	116	154	297	124	155
normalized size	1	1.	1.	0.91	1.21	2.34	0.98	1.22
time (sec)	N/A	0.175	0.043	0.055	1.048	1.204	0.901	1.144

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	83	109	207	88	111
normalized size	1	1.	1.	0.92	1.21	2.3	0.98	1.23
time (sec)	N/A	0.16	0.03	0.047	1.027	1.311	0.645	1.102

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	71	95	174	76	96
normalized size	1	1.	1.	0.93	1.25	2.29	1.	1.26
time (sec)	N/A	0.159	0.027	0.047	1.009	1.106	0.51	1.129

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	47	97	39	49
normalized size	1	1.	1.	0.97	1.21	2.49	1.	1.26
time (sec)	N/A	0.147	0.018	0.045	1.094	1.315	0.345	1.1

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	28	57	20	30
normalized size	1	1.	1.	1.05	1.33	2.71	0.95	1.43
time (sec)	N/A	0.085	0.014	0.042	1.044	1.449	0.264	1.136

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	36	47	86	36	49
normalized size	1	1.	0.78	1.	1.31	2.39	1.	1.36
time (sec)	N/A	0.16	0.029	0.044	1.059	1.525	0.316	1.089

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	65	93	211	73	77
normalized size	1	1.	1.	0.87	1.24	2.81	0.97	1.03
time (sec)	N/A	0.176	0.046	0.047	1.052	1.499	0.585	1.131

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	95	131	306	102	108
normalized size	1	1.	0.75	0.86	1.19	2.78	0.93	0.98
time (sec)	N/A	0.199	0.071	0.052	1.06	1.61	0.878	1.135

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	98	125	196	509	156	130
normalized size	1	1.	0.68	0.86	1.35	3.51	1.08	0.9
time (sec)	N/A	0.228	0.103	0.053	1.045	1.591	1.309	1.137

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	126	329	513	485	0	493
normalized size	1	1.	0.37	0.96	1.5	1.41	0.	1.44
time (sec)	N/A	0.257	0.238	0.188	1.616	1.711	0.	1.189

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	110	281	408	419	0	401
normalized size	1	1.	0.41	1.04	1.52	1.56	0.	1.49
time (sec)	N/A	0.192	0.188	0.189	1.532	1.713	0.	1.218

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	94	233	301	362	0	311
normalized size	1	1.	0.48	1.19	1.54	1.86	0.	1.59
time (sec)	N/A	0.135	0.136	0.175	1.568	1.671	0.	1.218

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	57	235	159	266	0	154
normalized size	1	1.	0.75	3.09	2.09	3.5	0.	2.03
time (sec)	N/A	0.058	0.088	0.172	1.557	1.65	0.	1.186

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	69	346	180	301	0	200
normalized size	1	1.	0.48	2.4	1.25	2.09	0.	1.39
time (sec)	N/A	0.101	0.124	0.184	1.068	1.586	0.	1.195

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	78	438	207	385	0	224
normalized size	1	1.	0.43	2.42	1.14	2.13	0.	1.24
time (sec)	N/A	0.128	0.167	0.191	1.12	1.651	0.	1.21

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	101	714	259	468	0	277
normalized size	1	1.	0.4	2.8	1.02	1.84	0.	1.09
time (sec)	N/A	0.172	0.208	0.206	1.027	1.659	0.	1.278

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	117	766	305	571	0	356
normalized size	1	1.	0.36	2.33	0.93	1.74	0.	1.08
time (sec)	N/A	0.236	0.274	0.192	1.107	1.71	0.	1.276

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	116	105	139	269	112	248
normalized size	1	1.	1.	0.91	1.2	2.32	0.97	2.14
time (sec)	N/A	0.171	0.042	0.046	1.004	1.544	0.901	1.127

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	93	124	231	100	216
normalized size	1	1.	1.	0.93	1.24	2.31	1.	2.16
time (sec)	N/A	0.166	0.033	0.053	1.022	1.474	0.722	1.127

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	80	151	65	184
normalized size	1	1.	1.	0.95	1.27	2.4	1.03	2.92
time (sec)	N/A	0.152	0.023	0.046	1.079	1.579	0.498	1.123

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	50	62	122	51	151
normalized size	1	1.	1.	0.98	1.22	2.39	1.	2.96
time (sec)	N/A	0.148	0.021	0.044	1.009	1.495	0.397	1.169

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	33	43	88	26	89
normalized size	1	1.	1.	1.	1.3	2.67	0.79	2.7
time (sec)	N/A	0.093	0.02	0.055	1.027	1.576	0.458	1.113

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	51	66	140	41	100
normalized size	1	1.	1.	0.96	1.25	2.64	0.77	1.89
time (sec)	N/A	0.168	0.034	0.043	1.041	1.582	0.425	1.132

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	101	215	83	126
normalized size	1	1.	0.89	0.93	1.42	3.03	1.17	1.77
time (sec)	N/A	0.172	0.038	0.056	1.036	1.504	0.533	1.122

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	95	144	370	114	176
normalized size	1	1.	0.8	0.86	1.3	3.33	1.03	1.59
time (sec)	N/A	0.196	0.063	0.058	1.088	1.524	0.923	1.124

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	98	125	182	479	144	230
normalized size	1	1.	0.67	0.86	1.25	3.28	0.99	1.58
time (sec)	N/A	0.221	0.096	0.06	1.066	1.281	1.376	1.109

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	120	320	513	490	0	707
normalized size	1	1.	0.35	0.93	1.5	1.43	0.	2.06
time (sec)	N/A	0.257	0.276	0.152	1.629	1.454	0.	1.227

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	110	272	408	424	0	532
normalized size	1	1.	0.41	1.01	1.52	1.58	0.	1.98
time (sec)	N/A	0.186	0.187	0.138	1.621	1.39	0.	1.211

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	94	224	301	358	0	356
normalized size	1	1.	0.48	1.15	1.54	1.84	0.	1.83
time (sec)	N/A	0.132	0.137	0.146	1.535	1.504	0.	1.184

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	55	166	158	262	0	163
normalized size	1	1.	0.51	1.54	1.46	2.43	0.	1.51
time (sec)	N/A	0.076	0.084	0.13	1.514	1.429	0.	1.149

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	57	250	163	161	0	0
normalized size	1	1.	0.54	2.38	1.55	1.53	0.	0.
time (sec)	N/A	0.077	0.114	0.157	1.04	1.463	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	85	530	220	267	0	0
normalized size	1	1.	0.47	2.96	1.23	1.49	0.	0.
time (sec)	N/A	0.118	0.153	0.143	1.023	1.456	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	101	714	266	366	0	0
normalized size	1	1.	0.4	2.8	1.04	1.44	0.	0.
time (sec)	N/A	0.166	0.205	0.154	1.039	1.308	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	117	898	312	471	0	0
normalized size	1	1.	0.36	2.73	0.95	1.43	0.	0.
time (sec)	N/A	0.225	0.255	0.175	1.037	1.345	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	83	109	207	88	111
normalized size	1	1.	1.	0.92	1.21	2.3	0.98	1.23
time (sec)	N/A	0.153	0.033	0.049	1.036	1.344	0.654	1.128

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	71	95	174	76	96
normalized size	1	1.	1.	0.93	1.25	2.29	1.	1.26
time (sec)	N/A	0.156	0.025	0.047	1.042	1.153	0.515	1.11

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	50	97	39	51
normalized size	1	1.	1.	0.98	1.25	2.42	0.98	1.27
time (sec)	N/A	0.141	0.019	0.045	1.08	1.216	0.348	1.133

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	28	57	20	30
normalized size	1	1.	1.	1.05	1.33	2.71	0.95	1.43
time (sec)	N/A	0.085	0.014	0.045	1.	1.32	0.271	1.127

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	46	86	36	49
normalized size	1	1.	0.8	1.03	1.31	2.46	1.03	1.4
time (sec)	N/A	0.155	0.031	0.045	1.026	1.195	0.32	1.156

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	65	93	211	73	77
normalized size	1	1.	0.96	0.89	1.27	2.89	1.	1.05
time (sec)	N/A	0.174	0.047	0.05	1.048	1.383	0.6	1.145

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	104	95	131	306	102	108
normalized size	1	1.	0.96	0.88	1.21	2.83	0.94	1.
time (sec)	N/A	0.202	0.066	0.053	1.027	1.172	0.887	1.116

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	124	125	196	509	156	130
normalized size	1	1.	0.87	0.87	1.37	3.56	1.09	0.91
time (sec)	N/A	0.231	0.095	0.062	1.054	1.336	1.343	1.154

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	126	329	512	485	0	709
normalized size	1	1.	0.37	0.96	1.49	1.41	0.	2.07
time (sec)	N/A	0.25	0.241	0.148	1.573	1.425	0.	1.236

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	110	281	406	416	0	533
normalized size	1	1.	0.41	1.04	1.51	1.55	0.	1.98
time (sec)	N/A	0.184	0.186	0.135	1.557	1.335	0.	1.231

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	94	233	302	359	0	356
normalized size	1	1.	0.48	1.19	1.55	1.84	0.	1.83
time (sec)	N/A	0.131	0.134	0.151	1.548	1.325	0.	1.186

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	57	234	159	252	0	165
normalized size	1	1.	0.75	3.08	2.09	3.32	0.	2.17
time (sec)	N/A	0.057	0.087	0.132	1.553	1.304	0.	1.189

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	69	346	189	235	0	80
normalized size	1	1.	0.48	2.4	1.31	1.63	0.	0.56
time (sec)	N/A	0.098	0.13	0.131	1.074	1.32	0.	1.158

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	78	438	217	315	0	80
normalized size	1	1.	0.43	2.42	1.2	1.74	0.	0.44
time (sec)	N/A	0.124	0.168	0.151	1.033	1.247	0.	1.183

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	101	714	269	416	0	0
normalized size	1	1.	0.4	2.82	1.06	1.64	0.	0.
time (sec)	N/A	0.167	0.208	0.161	1.113	1.366	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	117	766	312	610	0	0
normalized size	1	1.	0.36	2.34	0.95	1.87	0.	0.
time (sec)	N/A	0.222	0.263	0.181	1.072	1.363	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	96	112	0	216	0	0
normalized size	1	1.	0.3	0.35	0.	0.67	0.	0.
time (sec)	N/A	0.15	0.073	0.23	0.	1.275	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	75	96	0	166	0	0
normalized size	1	1.	0.32	0.41	0.	0.7	0.	0.
time (sec)	N/A	0.13	0.057	0.236	0.	1.305	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	64	80	0	103	0	0
normalized size	1	1.	0.44	0.55	0.	0.71	0.	0.
time (sec)	N/A	0.118	0.045	0.237	0.	1.408	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	39	50	0	43	0	0
normalized size	1	1.	0.58	0.75	0.	0.64	0.	0.
time (sec)	N/A	0.1	0.023	0.213	0.	1.391	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	44	57	0	57	0	0
normalized size	1	1.	0.61	0.79	0.	0.79	0.	0.
time (sec)	N/A	0.106	0.03	0.229	0.	1.455	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	68	102	0	155	0	0
normalized size	1	1.	0.39	0.59	0.	0.9	0.	0.
time (sec)	N/A	0.133	0.073	0.228	0.	1.429	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	86	175	0	292	0	0
normalized size	1	1.	0.33	0.67	0.	1.11	0.	0.
time (sec)	N/A	0.159	0.107	0.234	0.	1.468	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	104	247	0	443	0	0
normalized size	1	1.	0.29	0.69	0.	1.23	0.	0.
time (sec)	N/A	0.195	0.148	0.258	0.	1.622	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	372	372	150	795	0	972	1059	757
normalized size	1	1.	0.4	2.14	0.	2.61	2.85	2.03
time (sec)	N/A	0.542	0.159	0.277	0.	1.843	36.142	68.889

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	134	625	0	856	500	562
normalized size	1	1.	0.46	2.13	0.	2.91	1.7	1.91
time (sec)	N/A	0.495	0.132	0.187	0.	1.802	19.401	3.209

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	115	455	0	694	376	359
normalized size	1	1.	0.54	2.14	0.	3.26	1.77	1.69
time (sec)	N/A	0.459	0.106	0.183	0.	1.751	12.21	1.588

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	576	0	207
normalized size	1	1.	0.69	1.7	0.	4.97	0.	1.78
time (sec)	N/A	0.36	0.078	0.191	0.	1.783	0.	1.185

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	68	177	0	448	0	0
normalized size	1	1.	0.61	1.59	0.	4.04	0.	0.
time (sec)	N/A	0.307	0.071	0.184	0.	1.769	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	95	326	0	591	0	0
normalized size	1	1.	0.77	2.65	0.	4.8	0.	0.
time (sec)	N/A	0.436	0.084	0.184	0.	1.799	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	105	462	0	745	0	0
normalized size	1	1.	0.52	2.28	0.	3.67	0.	0.
time (sec)	N/A	0.463	0.098	0.187	0.	2.073	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	133	572	0	1045	0	0
normalized size	1	1.	0.47	2.02	0.	3.69	0.	0.
time (sec)	N/A	0.502	0.121	0.212	0.	2.6	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	228	0	0
normalized size	1	1.	0.3	0.35	0.	0.71	0.	0.
time (sec)	N/A	0.165	0.086	0.231	0.	1.612	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	94	112	0	219	0	0
normalized size	1	1.	0.29	0.35	0.	0.68	0.	0.
time (sec)	N/A	0.159	0.083	0.247	0.	1.654	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	87	96	0	158	0	0
normalized size	1	1.	0.37	0.41	0.	0.68	0.	0.
time (sec)	N/A	0.142	0.053	0.23	0.	1.609	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	59	69	0	103	0	0
normalized size	1	1.	0.4	0.47	0.	0.7	0.	0.
time (sec)	N/A	0.128	0.044	0.244	0.	1.613	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	50	65	0	66	0	0
normalized size	1	1.	0.46	0.6	0.	0.61	0.	0.
time (sec)	N/A	0.12	0.029	0.231	0.	1.679	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	56	85	0	105	0	0
normalized size	1	1.	0.49	0.74	0.	0.91	0.	0.
time (sec)	N/A	0.124	0.048	0.243	0.	1.544	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	64	102	0	176	0	0
normalized size	1	1.	0.37	0.6	0.	1.03	0.	0.
time (sec)	N/A	0.146	0.073	0.25	0.	1.509	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	86	175	0	313	0	0
normalized size	1	1.	0.32	0.66	0.	1.17	0.	0.
time (sec)	N/A	0.177	0.106	0.234	0.	1.605	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	140	247	0	460	0	0
normalized size	1	1.	0.39	0.69	0.	1.28	0.	0.
time (sec)	N/A	0.206	0.126	0.249	0.	1.827	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	216	0	0
normalized size	1	1.	0.3	0.35	0.	0.67	0.	0.
time (sec)	N/A	0.172	0.07	0.236	0.	1.532	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	77	96	0	166	0	0
normalized size	1	1.	0.32	0.4	0.	0.7	0.	0.
time (sec)	N/A	0.146	0.056	0.221	0.	1.838	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	65	80	0	103	0	0
normalized size	1	1.	0.44	0.54	0.	0.7	0.	0.
time (sec)	N/A	0.124	0.044	0.226	0.	1.997	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	52	0	43	0	0
normalized size	1	1.	0.6	0.76	0.	0.63	0.	0.
time (sec)	N/A	0.107	0.025	0.239	0.	1.911	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	59	0	57	0	0
normalized size	1	1.	0.62	0.82	0.	0.79	0.	0.
time (sec)	N/A	0.113	0.03	0.219	0.	1.884	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	65	103	0	155	0	0
normalized size	1	1.	0.38	0.6	0.	0.9	0.	0.
time (sec)	N/A	0.143	0.065	0.281	0.	1.971	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	85	175	0	292	0	0
normalized size	1	1.	0.32	0.67	0.	1.11	0.	0.
time (sec)	N/A	0.169	0.119	0.241	0.	1.817	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	104	247	0	443	0	0
normalized size	1	1.	0.29	0.69	0.	1.24	0.	0.
time (sec)	N/A	0.198	0.15	0.254	0.	1.697	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	375	375	150	795	0	971	1059	757
normalized size	1	1.	0.4	2.12	0.	2.59	2.82	2.02
time (sec)	N/A	0.541	0.171	0.263	0.	1.816	37.551	69.275

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	134	625	0	855	500	562
normalized size	1	1.	0.46	2.13	0.	2.92	1.71	1.92
time (sec)	N/A	0.483	0.136	0.188	0.	1.696	19.781	3.089

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	115	454	0	693	376	359
normalized size	1	1.	0.54	2.13	0.	3.25	1.77	1.69
time (sec)	N/A	0.446	0.109	0.177	0.	1.866	13.243	1.568

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	80	196	0	576	0	205
normalized size	1	1.	0.69	1.69	0.	4.97	0.	1.77
time (sec)	N/A	0.357	0.084	0.181	0.	1.66	0.	1.185

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	68	176	0	447	0	0
normalized size	1	1.	0.61	1.57	0.	3.99	0.	0.
time (sec)	N/A	0.291	0.073	0.178	0.	1.706	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	95	326	0	590	0	0
normalized size	1	1.	0.77	2.63	0.	4.76	0.	0.
time (sec)	N/A	0.422	0.086	0.198	0.	1.778	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	105	462	0	744	0	0
normalized size	1	1.	0.54	2.37	0.	3.82	0.	0.
time (sec)	N/A	0.451	0.098	0.184	0.	1.885	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	131	572	0	1044	0	0
normalized size	1	1.	0.49	2.12	0.	3.87	0.	0.
time (sec)	N/A	0.489	0.117	0.227	0.	2.655	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	228	0	0
normalized size	1	1.	0.3	0.35	0.	0.71	0.	0.
time (sec)	N/A	0.156	0.081	0.243	0.	1.565	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	94	112	0	219	0	0
normalized size	1	1.	0.29	0.35	0.	0.68	0.	0.
time (sec)	N/A	0.156	0.08	0.237	0.	1.623	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	81	96	0	158	0	0
normalized size	1	1.	0.34	0.41	0.	0.67	0.	0.
time (sec)	N/A	0.139	0.063	0.227	0.	1.696	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	59	69	0	103	0	0
normalized size	1	1.	0.4	0.47	0.	0.7	0.	0.
time (sec)	N/A	0.123	0.04	0.233	0.	1.575	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	47	63	0	66	0	0
normalized size	1	1.	0.44	0.59	0.	0.62	0.	0.
time (sec)	N/A	0.112	0.03	0.239	0.	1.711	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	54	87	0	105	0	0
normalized size	1	1.	0.48	0.77	0.	0.93	0.	0.
time (sec)	N/A	0.125	0.048	0.256	0.	1.635	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	63	102	0	176	0	0
normalized size	1	1.	0.38	0.61	0.	1.05	0.	0.
time (sec)	N/A	0.141	0.073	0.232	0.	1.756	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	85	175	0	313	0	0
normalized size	1	1.	0.32	0.66	0.	1.19	0.	0.
time (sec)	N/A	0.166	0.102	0.24	0.	1.833	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	105	247	0	460	0	0
normalized size	1	1.	0.29	0.69	0.	1.29	0.	0.
time (sec)	N/A	0.199	0.179	0.263	0.	1.655	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	63	59	154	0	0
normalized size	1	1.	0.66	0.79	0.74	1.92	0.	0.
time (sec)	N/A	0.245	0.035	0.13	1.24	1.756	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	53	0	0
normalized size	1	1.	0.59	0.7	0.	0.7	0.	0.
time (sec)	N/A	0.261	0.025	0.125	0.	1.573	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	43	52	0	47	0	0
normalized size	1	1.	0.61	0.73	0.	0.66	0.	0.
time (sec)	N/A	0.172	0.023	0.121	0.	1.563	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	39	50	0	43	0	0
normalized size	1	1.	0.58	0.75	0.	0.64	0.	0.
time (sec)	N/A	0.098	0.021	0.217	0.	1.677	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	43	50	0	51	0	0
normalized size	1	1.	0.61	0.71	0.	0.73	0.	0.
time (sec)	N/A	0.25	0.024	0.225	0.	1.585	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	47	53	0	54	0	0
normalized size	1	1.	1.02	1.15	0.	1.17	0.	0.
time (sec)	N/A	0.244	0.023	0.135	0.	1.58	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	93	196	0	487	0	173
normalized size	1	1.	0.58	1.22	0.	3.04	0.	1.08
time (sec)	N/A	0.558	0.091	0.191	0.	1.697	0.	1.159

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	84	173	0	436	0	157
normalized size	1	1.	0.68	1.41	0.	3.54	0.	1.28
time (sec)	N/A	0.472	0.072	0.195	0.	1.714	0.	1.203

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	77	147	0	404	0	143
normalized size	1	1.	0.79	1.5	0.	4.12	0.	1.46
time (sec)	N/A	0.31	0.064	0.206	0.	1.759	0.	1.143

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	80	196	0	576	0	207
normalized size	1	1.	0.69	1.69	0.	4.97	0.	1.78
time (sec)	N/A	0.341	0.071	0.195	0.	1.604	0.	1.152

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	82	306	0	555	0	171
normalized size	1	1.	0.7	2.62	0.	4.74	0.	1.46
time (sec)	N/A	0.551	0.082	0.179	0.	1.66	0.	1.314

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	78	347	0	393	0	262
normalized size	1	1.	0.7	3.13	0.	3.54	0.	2.36
time (sec)	N/A	0.515	0.074	0.192	0.	1.683	0.	1.577

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	86	378	0	441	0	312
normalized size	1	1.	0.63	2.76	0.	3.22	0.	2.28
time (sec)	N/A	0.528	0.083	0.186	0.	1.646	0.	1.814

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	487	0	427
normalized size	1	1.	0.6	2.63	0.	3.12	0.	2.74
time (sec)	N/A	0.556	0.093	0.198	0.	1.712	0.	2.978

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	522	0	489
normalized size	1	1.	0.56	2.47	0.	2.88	0.	2.7
time (sec)	N/A	0.575	0.1	0.178	0.	1.638	0.	3.287

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	71	89	0	111	0	0
normalized size	1	1.	0.38	0.48	0.	0.6	0.	0.
time (sec)	N/A	0.293	0.051	0.231	0.	1.662	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	63	82	0	97	0	0
normalized size	1	1.	0.41	0.54	0.	0.64	0.	0.
time (sec)	N/A	0.286	0.043	0.21	0.	1.619	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	54	73	0	76	0	0
normalized size	1	1.	0.48	0.65	0.	0.67	0.	0.
time (sec)	N/A	0.187	0.028	0.249	0.	1.525	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	50	65	0	66	0	0
normalized size	1	1.	0.46	0.6	0.	0.61	0.	0.
time (sec)	N/A	0.111	0.029	0.227	0.	1.709	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	52	67	0	82	0	0
normalized size	1	1.	0.48	0.62	0.	0.76	0.	0.
time (sec)	N/A	0.272	0.039	0.216	0.	1.645	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	66	82	0	194	0	0
normalized size	1	1.	0.45	0.56	0.	1.32	0.	0.
time (sec)	N/A	0.277	0.046	0.23	0.	1.573	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	76	90	0	213	0	0
normalized size	1	1.	0.4	0.48	0.	1.13	0.	0.
time (sec)	N/A	0.288	0.049	0.275	0.	1.666	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	81	98	0	230	0	0
normalized size	1	1.	0.36	0.44	0.	1.04	0.	0.
time (sec)	N/A	0.295	0.061	0.224	0.	1.683	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	90	106	0	257	0	0
normalized size	1	1.	0.34	0.4	0.	0.97	0.	0.
time (sec)	N/A	0.304	0.072	0.237	0.	1.626	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	50	65	62	154	0	0
normalized size	1	1.	0.62	0.8	0.77	1.9	0.	0.
time (sec)	N/A	0.249	0.041	0.114	1.212	1.693	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	53	0	0
normalized size	1	1.	0.59	0.7	0.	0.7	0.	0.
time (sec)	N/A	0.263	0.027	0.115	0.	1.633	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	42	52	0	47	0	0
normalized size	1	1.	0.58	0.72	0.	0.65	0.	0.
time (sec)	N/A	0.176	0.023	0.116	0.	1.519	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	41	52	0	43	0	0
normalized size	1	1.	0.6	0.76	0.	0.63	0.	0.
time (sec)	N/A	0.099	0.02	0.203	0.	1.477	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	50	0	51	0	0
normalized size	1	1.	0.59	0.72	0.	0.74	0.	0.
time (sec)	N/A	0.256	0.023	0.212	0.	1.684	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	47	53	0	54	0	0
normalized size	1	1.	1.	1.13	0.	1.15	0.	0.
time (sec)	N/A	0.246	0.024	0.121	0.	1.55	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	93	196	0	487	0	173
normalized size	1	1.	0.57	1.2	0.	2.99	0.	1.06
time (sec)	N/A	0.547	0.09	0.19	0.	1.726	0.	1.14

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	84	173	0	436	0	158
normalized size	1	1.	0.68	1.4	0.	3.52	0.	1.27
time (sec)	N/A	0.47	0.069	0.174	0.	1.667	0.	1.18

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	100	147	0	404	0	143
normalized size	1	1.	1.01	1.48	0.	4.08	0.	1.44
time (sec)	N/A	0.305	0.063	0.172	0.	1.725	0.	1.174

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	576	0	205
normalized size	1	1.	0.69	1.7	0.	4.97	0.	1.77
time (sec)	N/A	0.34	0.076	0.197	0.	1.687	0.	1.174

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	82	306	0	554	0	171
normalized size	1	1.	0.7	2.62	0.	4.74	0.	1.46
time (sec)	N/A	0.525	0.077	0.186	0.	1.742	0.	1.32

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	78	348	0	393	0	262
normalized size	1	1.	0.7	3.11	0.	3.51	0.	2.34
time (sec)	N/A	0.523	0.064	0.182	0.	1.712	0.	1.514

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	86	378	0	440	0	312
normalized size	1	1.	0.61	2.7	0.	3.14	0.	2.23
time (sec)	N/A	0.534	0.075	0.18	0.	1.694	0.	1.897

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	487	0	427
normalized size	1	1.	0.6	2.63	0.	3.12	0.	2.74
time (sec)	N/A	0.554	0.084	0.207	0.	1.64	0.	2.981

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	521	0	489
normalized size	1	1.	0.56	2.47	0.	2.88	0.	2.7
time (sec)	N/A	0.585	0.089	0.198	0.	1.651	0.	3.146

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	69	89	0	111	0	0
normalized size	1	1.	0.37	0.48	0.	0.6	0.	0.
time (sec)	N/A	0.29	0.051	0.219	0.	1.627	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	62	82	0	97	0	0
normalized size	1	1.	0.41	0.54	0.	0.64	0.	0.
time (sec)	N/A	0.287	0.043	0.237	0.	1.672	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	53	73	0	76	0	0
normalized size	1	1.	0.47	0.65	0.	0.68	0.	0.
time (sec)	N/A	0.189	0.03	0.224	0.	1.699	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	47	63	0	66	0	0
normalized size	1	1.	0.44	0.59	0.	0.62	0.	0.
time (sec)	N/A	0.111	0.025	0.223	0.	1.748	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	53	67	0	82	0	0
normalized size	1	1.	0.49	0.62	0.	0.76	0.	0.
time (sec)	N/A	0.272	0.036	0.23	0.	1.666	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	65	82	0	194	0	0
normalized size	1	1.	0.45	0.56	0.	1.33	0.	0.
time (sec)	N/A	0.278	0.046	0.229	0.	1.694	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	75	90	0	213	0	0
normalized size	1	1.	0.4	0.48	0.	1.14	0.	0.
time (sec)	N/A	0.28	0.047	0.23	0.	1.569	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	80	98	0	230	0	0
normalized size	1	1.	0.36	0.44	0.	1.04	0.	0.
time (sec)	N/A	0.287	0.052	0.246	0.	1.588	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	89	106	0	257	0	0
normalized size	1	1.	0.34	0.4	0.	0.98	0.	0.
time (sec)	N/A	0.295	0.071	0.237	0.	1.71	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	154	81	123	0	0	0	0	0
normalized size	1	0.53	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.24	0.066	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	150	164	94	0	0	0	0	0
normalized size	1	1.09	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.252	0.068	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	289	303	142	0	0	0	0	0
normalized size	1	1.05	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.431	0.081	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	295	111	146	0	0	0	0	0
normalized size	1	0.38	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	0.463	0.181	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	112	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	0.341	0.175	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.473	0.202	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.455	0.189	0.	0.	0.	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.463	0.182	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [129] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.	10	0.6
2	A	7	6	1.	10	0.6
3	A	6	6	1.	8	0.75
4	A	5	5	1.	6	0.833
5	A	6	6	1.	10	0.6
6	A	3	3	1.	10	0.3
7	A	3	3	1.	10	0.3
8	A	7	5	1.	10	0.5
9	A	5	4	1.	10	0.4
10	A	4	3	1.	12	0.25
11	A	4	3	1.	12	0.25
12	A	4	3	1.	10	0.3
13	A	4	3	1.	8	0.375
14	A	4	3	1.	12	0.25
15	A	4	3	1.	12	0.25
16	A	4	3	1.	12	0.25
17	A	4	3	1.	12	0.25
18	A	14	9	1.	12	0.75
19	A	12	8	1.	10	0.8
20	A	8	7	1.	8	0.875
21	A	8	7	1.	12	0.583
22	A	5	5	1.	12	0.417
23	A	9	7	1.	12	0.583

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	10	9	1.	12	0.75
25	A	4	3	1.	12	0.25
26	A	4	3	1.	12	0.25
27	A	4	3	1.	10	0.3
28	A	4	3	1.	8	0.375
29	A	4	3	1.	12	0.25
30	A	4	3	1.	12	0.25
31	A	4	3	1.	12	0.25
32	A	4	3	1.	12	0.25
33	A	8	6	1.	12	0.5
34	A	7	6	1.	12	0.5
35	A	6	6	1.	10	0.6
36	A	5	5	1.	8	0.625
37	A	6	6	1.	12	0.5
38	A	3	3	1.	12	0.25
39	A	3	3	1.	12	0.25
40	A	7	5	1.	12	0.417
41	A	5	4	1.	12	0.333
42	A	4	3	1.	12	0.25
43	A	4	3	1.	12	0.25
44	A	4	3	1.	10	0.3
45	A	4	3	1.	8	0.375
46	A	4	3	1.	12	0.25
47	A	4	3	1.	12	0.25
48	A	4	3	1.	12	0.25
49	A	4	3	1.	12	0.25
50	A	19	9	1.	12	0.75
51	A	14	9	1.	12	0.75
52	A	12	8	1.	10	0.8
53	A	8	7	1.	8	0.875
54	A	8	7	1.	12	0.583
55	A	5	5	1.	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	9	7	1.	12	0.583
57	A	10	9	1.	12	0.75
58	A	14	11	1.	12	0.917
59	A	11	8	1.	14	0.571
60	A	10	8	1.	14	0.571
61	A	9	8	1.	14	0.571
62	A	7	7	1.	12	0.583
63	A	6	6	1.	10	0.6
64	A	17	14	1.	14	1.
65	A	13	10	1.	14	0.714
66	A	14	11	1.	14	0.786
67	A	15	12	1.	14	0.857
68	A	11	8	1.	14	0.571
69	A	10	8	1.	14	0.571
70	A	9	8	1.	14	0.571
71	A	7	7	1.	12	0.583
72	A	6	6	1.	10	0.6
73	A	17	14	1.	14	1.
74	A	13	10	1.	14	0.714
75	A	14	11	1.	14	0.786
76	A	15	12	1.	14	0.857
77	A	12	9	1.	14	0.643
78	A	11	9	1.	14	0.643
79	A	10	9	1.	14	0.643
80	A	8	7	1.	12	0.583
81	A	7	6	1.	10	0.6
82	A	19	16	1.	14	1.143
83	A	14	11	1.	14	0.786
84	A	15	11	1.	14	0.786
85	A	16	12	1.	14	0.857
86	A	11	8	1.	14	0.571
87	A	10	8	1.	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	9	8	1.	14	0.571
89	A	7	7	1.	12	0.583
90	A	6	6	1.	10	0.6
91	A	17	14	1.	14	1.
92	A	13	10	1.	14	0.714
93	A	14	11	1.	14	0.786
94	A	15	12	1.	14	0.857
95	A	11	8	1.	14	0.571
96	A	10	8	1.	14	0.571
97	A	9	8	1.	14	0.571
98	A	7	7	1.	12	0.583
99	A	6	6	1.	10	0.6
100	A	17	14	1.	14	1.
101	A	13	10	1.	14	0.714
102	A	14	11	1.	14	0.786
103	A	15	12	1.	14	0.857
104	A	12	9	1.	14	0.643
105	A	11	9	1.	14	0.643
106	A	10	9	1.	14	0.643
107	A	8	7	1.	12	0.583
108	A	7	6	1.	10	0.6
109	A	19	16	1.	14	1.143
110	A	14	11	1.	14	0.786
111	A	15	11	1.	14	0.786
112	A	16	12	1.	14	0.857
113	A	16	11	1.	12	0.917
114	A	14	10	1.	10	1.
115	A	13	9	1.	8	1.125
116	A	25	13	1.	12	1.083
117	A	14	10	1.	12	0.833
118	A	15	11	1.	12	0.917
119	A	16	12	1.	12	1.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	6	5	1.	12	0.417
121	A	4	4	1.	10	0.4
122	A	3	3	1.	8	0.375
123	A	4	4	1.	12	0.333
124	A	3	3	1.	12	0.25
125	A	4	4	1.	12	0.333
126	A	19	15	1.	14	1.071
127	A	17	14	1.	12	1.167
128	A	16	13	1.	10	1.3
129	A	39	20	1.	14	1.429
130	A	25	11	1.	14	0.786
131	A	26	12	1.	14	0.857
132	A	5	5	1.	12	0.417
133	A	9	5	1.	12	0.417
134	A	4	4	1.	12	0.333
135	A	4	3	1.	10	0.3
136	A	4	3	1.	12	0.25
137	A	4	4	1.	12	0.333
138	A	9	5	1.	12	0.417
139	A	2	2	1.	14	0.143
140	A	2	2	1.	14	0.143
141	A	2	2	1.	14	0.143
142	A	2	2	1.	14	0.143
143	A	2	2	1.	14	0.143
144	A	2	2	1.	14	0.143
145	A	2	2	1.	12	0.167
146	A	2	2	1.	12	0.167
147	A	2	2	1.	14	0.143
148	A	2	2	1.	12	0.167
149	A	5	5	1.	12	0.417
150	A	3	3	1.	10	0.3
151	A	2	2	1.	8	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	4	4	1.	12	0.333
153	A	2	2	1.	12	0.167
154	A	3	3	1.	12	0.25
155	A	4	4	1.	12	0.333
156	A	4	4	1.	12	0.333
157	A	4	4	1.	16	0.25
158	A	9	8	1.	16	0.5
159	A	8	8	1.	16	0.5
160	A	7	7	1.	16	0.438
161	A	6	6	1.	14	0.429
162	A	7	7	1.	16	0.438
163	A	3	3	1.	16	0.188
164	A	4	4	1.	16	0.25
165	A	6	6	1.	16	0.375
166	A	8	6	1.	16	0.375
167	A	5	4	1.	18	0.222
168	A	4	3	1.	18	0.167
169	A	4	3	1.	18	0.167
170	A	4	3	1.	18	0.167
171	A	4	3	1.	18	0.167
172	C	1	1	1.86	16	0.062
173	A	4	3	1.	18	0.167
174	A	3	3	1.	18	0.167
175	A	4	3	1.	18	0.167
176	A	4	3	1.	18	0.167
177	A	5	4	1.	18	0.222
178	A	8	7	1.	18	0.389
179	A	7	6	1.	18	0.333
180	A	8	8	1.	18	0.444
181	A	8	8	1.	16	0.5
182	A	9	8	1.	18	0.444
183	A	3	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	4	4	1.	18	0.222
185	A	6	6	1.	18	0.333
186	A	7	6	1.	18	0.333
187	A	5	4	1.	18	0.222
188	A	4	3	1.	18	0.167
189	A	5	4	1.	18	0.222
190	A	4	3	1.	18	0.167
191	A	3	3	1.	18	0.167
192	A	4	3	1.	16	0.188
193	A	4	3	1.	18	0.167
194	A	3	3	1.	18	0.167
195	A	4	3	1.	18	0.167
196	A	4	3	1.	18	0.167
197	A	3	3	1.	18	0.167
198	A	9	7	1.	18	0.389
199	A	8	7	1.	18	0.389
200	A	7	7	1.	16	0.438
201	A	5	5	1.	18	0.278
202	A	3	3	1.	18	0.167
203	A	4	4	1.	18	0.222
204	A	6	6	1.	18	0.333
205	A	7	6	1.	18	0.333
206	A	4	4	1.	18	0.222
207	A	4	3	1.	18	0.167
208	A	4	3	1.	18	0.167
209	A	4	3	1.	18	0.167
210	A	4	3	1.	16	0.188
211	A	3	3	1.	18	0.167
212	A	4	4	1.	18	0.222
213	A	5	4	1.	18	0.222
214	A	5	4	1.	18	0.222
215	A	5	4	1.	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	3	3	1.	18	0.167
217	A	10	8	1.	18	0.444
218	A	9	8	1.	18	0.444
219	A	8	8	1.	16	0.5
220	A	6	6	1.	18	0.333
221	A	3	3	1.	18	0.167
222	A	3	3	1.	18	0.167
223	A	5	5	1.	18	0.278
224	A	6	5	1.	18	0.278
225	A	7	5	1.	18	0.278
226	A	7	6	1.	18	0.333
227	A	6	6	1.	18	0.333
228	A	5	5	1.19	18	0.278
229	A	4	4	1.16	18	0.222
230	A	1	1	1.	18	0.056
231	A	5	5	1.	18	0.278
232	A	5	5	1.	18	0.278
233	A	6	5	1.	18	0.278
234	A	7	5	1.	18	0.278
235	A	5	4	1.	20	0.2
236	A	5	4	1.	20	0.2
237	A	5	4	1.	20	0.2
238	A	5	4	1.	20	0.2
239	A	5	4	1.	20	0.2
240	A	5	4	1.	20	0.2
241	A	5	4	1.	20	0.2
242	A	5	4	1.	20	0.2
243	A	6	6	1.	20	0.3
244	A	5	5	1.	20	0.25
245	A	4	4	1.	20	0.2
246	A	1	1	1.	20	0.05
247	A	6	5	1.	20	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
248	A	6	5	1.	20	0.25
249	A	6	5	1.	20	0.25
250	A	7	5	1.	20	0.25
251	A	8	5	1.	20	0.25
252	A	8	6	1.6	20	0.3
253	A	7	6	1.58	20	0.3
254	A	6	6	1.54	20	0.3
255	A	5	5	1.44	20	0.25
256	A	4	4	1.44	20	0.2
257	A	1	1	1.	20	0.05
258	A	4	4	1.	20	0.2
259	A	5	5	1.	20	0.25
260	A	6	5	1.	20	0.25
261	A	10	6	1.	20	0.3
262	A	9	6	1.	20	0.3
263	A	8	6	1.	20	0.3
264	A	7	6	1.	20	0.3
265	A	6	6	1.	20	0.3
266	A	5	5	1.	20	0.25
267	A	6	6	1.	20	0.3
268	A	7	6	1.	20	0.3
269	A	8	6	1.	20	0.3
270	A	9	6	1.	20	0.3
271	A	8	6	1.	20	0.3
272	A	7	6	1.	20	0.3
273	A	6	6	1.	20	0.3
274	A	5	5	1.	20	0.25
275	A	4	4	1.	20	0.2
276	A	1	1	1.	20	0.05
277	A	5	5	1.	20	0.25
278	A	6	5	1.	20	0.25
279	A	7	6	1.	9	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	6	5	1.	8	0.625
281	A	3	3	1.	11	0.273
282	A	6	6	1.	10	0.6
283	A	8	6	1.	11	0.546
284	A	7	5	1.	10	0.5
285	A	8	8	1.	13	0.615
286	A	7	7	1.	12	0.583
287	A	3	3	1.	11	0.273
288	A	4	4	1.	10	0.4
289	A	8	8	1.	13	0.615
290	A	7	7	1.	12	0.583
291	A	5	5	1.	11	0.454
292	A	3	3	1.	10	0.3
293	A	9	9	1.	13	0.692
294	A	3	3	1.	12	0.25
295	A	3	3	1.	21	0.143
296	A	5	4	1.	21	0.19
297	A	4	4	1.	19	0.21
298	A	1	1	1.	18	0.056
299	A	5	5	1.	21	0.238
300	A	5	5	1.	21	0.238
301	A	5	4	1.	23	0.174
302	A	5	4	1.	23	0.174
303	A	5	4	1.	21	0.19
304	A	5	4	1.	20	0.2
305	A	6	6	1.	23	0.261
306	A	6	6	1.	23	0.261
307	A	7	7	1.	23	0.304
308	A	8	7	1.	23	0.304
309	A	9	7	1.	23	0.304
310	A	10	7	1.	23	0.304
311	A	9	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	7	6	1.	21	0.286
313	A	6	5	1.	20	0.25
314	A	8	8	1.	23	0.348
315	A	8	8	1.	23	0.348
316	A	9	9	1.	23	0.391
317	A	10	9	1.	23	0.391
318	A	11	9	1.	23	0.391
319	A	6	6	1.	13	0.462
320	A	5	5	1.	12	0.417
321	A	5	5	1.	15	0.333
322	A	4	4	1.	14	0.286
323	A	5	5	1.	13	0.385
324	A	4	4	1.	12	0.333
325	A	4	4	1.	15	0.267
326	A	1	1	1.	14	0.071
327	A	4	4	1.	13	0.308
328	A	3	3	1.	12	0.25
329	A	6	6	1.	15	0.4
330	A	5	5	1.	14	0.357
331	A	5	5	1.	13	0.385
332	A	4	4	1.	12	0.333
333	A	6	6	1.	15	0.4
334	A	5	5	1.	14	0.357
335	A	4	4	1.	23	0.174
336	A	6	5	1.3	23	0.217
337	A	5	5	1.32	21	0.238
338	A	4	4	1.44	20	0.2
339	A	5	5	1.	23	0.217
340	A	5	5	1.	23	0.217
341	A	9	7	1.	23	0.304
342	A	9	7	1.	23	0.304
343	A	8	7	1.	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
344	A	7	6	1.	20	0.3
345	A	9	8	1.	23	0.348
346	A	9	8	1.	23	0.348
347	A	10	9	1.	23	0.391
348	A	11	9	1.	23	0.391
349	A	12	9	1.	23	0.391
350	A	8	6	1.	23	0.261
351	A	7	6	1.	23	0.261
352	A	6	6	1.	21	0.286
353	A	5	5	1.	20	0.25
354	A	6	6	1.	23	0.261
355	A	6	6	1.	23	0.261
356	A	7	7	1.	23	0.304
357	A	8	7	1.	23	0.304
358	A	9	7	1.	23	0.304
359	A	6	6	1.	24	0.25
360	A	4	4	1.	24	0.167
361	A	1	1	1.	22	0.045
362	A	3	3	1.	24	0.125
363	A	3	3	1.	24	0.125
364	A	3	3	1.	18	0.167
365	A	3	3	1.	18	0.167
366	A	3	3	1.	18	0.167
367	A	3	3	1.	16	0.188
368	A	3	3	1.	18	0.167
369	A	3	3	1.	18	0.167
370	A	4	4	1.	18	0.222
371	A	6	6	1.	18	0.333
372	A	3	3	1.	20	0.15
373	A	3	3	1.	20	0.15
374	A	3	3	1.	20	0.15
375	A	3	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	3	3	1.	20	0.15
377	A	4	4	1.	20	0.2
378	A	5	4	1.	20	0.2
379	A	9	9	1.	20	0.45
380	A	8	8	1.	20	0.4
381	A	7	7	1.	20	0.35
382	A	3	3	1.	18	0.167
383	A	7	7	1.	20	0.35
384	A	8	7	1.	20	0.35
385	A	9	7	1.	20	0.35
386	A	10	7	1.	20	0.35
387	A	5	4	1.	22	0.182
388	A	5	4	1.	22	0.182
389	A	5	4	1.	22	0.182
390	A	6	5	1.	22	0.227
391	A	5	4	1.	20	0.2
392	A	5	4	1.	22	0.182
393	A	5	4	1.	22	0.182
394	A	5	4	1.	22	0.182
395	A	5	4	1.	22	0.182
396	A	8	8	1.	22	0.364
397	A	4	4	1.	22	0.182
398	A	8	8	1.	22	0.364
399	A	8	8	1.	20	0.4
400	A	8	7	1.	22	0.318
401	A	9	7	1.	22	0.318
402	A	10	7	1.	22	0.318
403	A	11	7	1.	22	0.318
404	A	5	4	1.	22	0.182
405	A	6	5	1.	22	0.227
406	A	5	4	1.	22	0.182
407	A	5	4	1.	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
408	A	5	4	1.	20	0.2
409	A	5	4	1.	22	0.182
410	A	5	4	1.	22	0.182
411	A	5	4	1.	22	0.182
412	A	5	4	1.	22	0.182
413	A	10	8	1.	22	0.364
414	A	9	8	1.	22	0.364
415	A	8	8	1.	22	0.364
416	A	7	7	1.	20	0.35
417	A	2	2	1.	22	0.091
418	A	6	6	1.	22	0.273
419	A	8	7	1.	22	0.318
420	A	9	7	1.	22	0.318
421	A	5	4	1.	22	0.182
422	A	5	4	1.	22	0.182
423	A	5	4	1.	22	0.182
424	A	5	4	1.	20	0.2
425	A	5	4	1.	22	0.182
426	A	6	5	1.	22	0.227
427	A	5	4	1.	22	0.182
428	A	5	4	1.	22	0.182
429	A	11	9	1.	22	0.409
430	A	10	9	1.	22	0.409
431	A	9	9	1.	22	0.409
432	A	8	8	1.	20	0.4
433	A	6	6	1.	22	0.273
434	A	6	6	1.	22	0.273
435	A	3	3	1.	22	0.136
436	A	7	7	1.	22	0.318
437	A	9	7	1.	22	0.318
438	A	8	7	1.19	22	0.318
439	A	7	7	1.13	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
440	A	7	7	1.2	22	0.318
441	A	5	5	1.	22	0.227
442	A	4	4	1.	22	0.182
443	A	8	7	1.	22	0.318
444	A	9	8	1.	22	0.364
445	A	10	8	1.	22	0.364
446	A	12	9	1.	24	0.375
447	A	11	9	1.	24	0.375
448	A	10	9	1.	24	0.375
449	A	9	9	1.	24	0.375
450	A	8	8	1.	24	0.333
451	A	9	9	1.	24	0.375
452	A	10	9	1.	24	0.375
453	A	11	9	1.	24	0.375
454	A	12	9	1.	24	0.375
455	A	8	8	1.	24	0.333
456	A	8	7	1.	24	0.292
457	A	6	5	1.	24	0.208
458	A	5	5	1.	24	0.208
459	A	8	7	1.	24	0.292
460	A	9	8	1.	24	0.333
461	A	10	8	1.	24	0.333
462	A	11	8	1.	24	0.333
463	A	7	7	1.	24	0.292
464	A	6	6	1.	24	0.25
465	A	6	6	1.	24	0.25
466	A	4	4	1.	24	0.167
467	A	4	4	1.	24	0.167
468	A	9	8	1.	24	0.333
469	A	10	9	1.	24	0.375
470	A	11	10	1.	24	0.417
471	A	14	10	1.	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	13	10	1.	24	0.417
473	A	12	10	1.	24	0.417
474	A	11	9	1.	24	0.375
475	A	11	9	1.	24	0.375
476	A	12	10	1.	24	0.417
477	A	12	10	1.	24	0.417
478	A	13	10	1.	24	0.417
479	A	14	10	1.	24	0.417
480	A	9	8	1.	24	0.333
481	A	8	8	1.	24	0.333
482	A	7	7	1.	24	0.292
483	A	6	6	1.	24	0.25
484	A	6	6	1.	24	0.25
485	A	5	5	1.	24	0.208
486	A	5	5	1.	24	0.208
487	A	9	8	1.	24	0.333
488	A	10	9	1.	24	0.375
489	A	3	3	1.	25	0.12
490	A	6	5	1.	25	0.2
491	A	5	5	1.	23	0.217
492	A	4	4	1.	22	0.182
493	A	4	4	1.	25	0.16
494	A	2	2	1.	25	0.08
495	A	3	3	1.	25	0.12
496	A	4	4	1.	25	0.16
497	A	5	4	1.	25	0.16
498	A	11	9	1.	27	0.333
499	A	10	9	1.	27	0.333
500	A	9	9	1.	25	0.36
501	A	8	8	1.	24	0.333
502	A	8	8	1.	27	0.296
503	A	7	6	1.	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	7	6	1.	27	0.222
505	A	7	6	1.	27	0.222
506	A	7	6	1.	27	0.222
507	A	11	8	1.	27	0.296
508	A	10	8	1.	27	0.296
509	A	9	8	1.	25	0.32
510	A	8	7	1.	24	0.292
511	A	8	7	1.	27	0.259
512	A	5	4	1.	27	0.148
513	A	6	5	1.	27	0.185
514	A	7	6	1.	27	0.222
515	A	8	6	1.	27	0.222
516	A	4	4	1.	27	0.148
517	A	6	5	1.	27	0.185
518	A	5	5	1.	25	0.2
519	A	4	4	1.	24	0.167
520	A	4	4	1.	27	0.148
521	A	3	3	1.	27	0.111
522	A	4	4	1.	27	0.148
523	A	5	5	1.	27	0.185
524	A	14	10	1.	27	0.37
525	A	13	10	1.	27	0.37
526	A	12	10	1.	25	0.4
527	A	11	9	1.	24	0.375
528	A	11	9	1.	27	0.333
529	A	9	8	1.	27	0.296
530	A	10	9	1.	27	0.333
531	A	11	9	1.	27	0.333
532	A	11	9	1.	27	0.333
533	A	9	7	1.01	27	0.259
534	A	8	7	1.	27	0.259
535	A	7	7	1.	25	0.28

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	6	6	1.	24	0.25
537	A	6	5	1.	27	0.185
538	A	4	3	1.	27	0.111
539	A	5	4	1.	27	0.148
540	A	6	5	1.	27	0.185
541	A	4	3	1.	27	0.111
542	C	2	2	0.44	20	0.1
543	A	3	3	1.	22	0.136
544	A	5	5	1.	22	0.227
545	A	3	3	1.	24	0.125
546	A	3	3	1.	24	0.125
547	A	3	3	1.	24	0.125
548	A	3	3	1.	24	0.125
549	A	3	3	1.	22	0.136
550	A	3	3	1.	23	0.13
551	A	3	3	1.	23	0.13
552	A	7	7	1.	22	0.318
553	A	3	3	1.	20	0.15
554	A	3	3	1.	22	0.136
555	A	9	9	1.	22	0.409
556	A	13	5	1.	20	0.25
557	A	11	5	1.	20	0.25
558	A	9	5	1.	20	0.25
559	A	7	5	1.	18	0.278
560	A	1	1	1.	20	0.05
561	A	2	2	1.	20	0.1
562	A	3	2	1.	20	0.1
563	A	4	2	1.	20	0.1
564	A	4	3	1.	22	0.136
565	A	4	3	1.	22	0.136
566	A	4	3	1.	22	0.136
567	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	3	3	1.	20	0.15
569	A	3	3	1.	22	0.136
570	A	5	4	1.	22	0.182
571	A	5	4	1.	22	0.182
572	A	5	4	1.	22	0.182
573	A	13	5	1.	22	0.227
574	A	11	5	1.	22	0.227
575	A	9	5	1.	22	0.227
576	A	7	5	1.	20	0.25
577	A	1	1	1.	22	0.045
578	A	2	2	1.	22	0.091
579	A	3	2	1.	22	0.091
580	A	4	2	1.	22	0.091
581	A	4	3	1.	22	0.136
582	A	4	3	1.	22	0.136
583	A	4	3	1.	22	0.136
584	A	3	3	1.	22	0.136
585	A	4	3	1.	20	0.15
586	A	3	3	1.	22	0.136
587	A	3	3	1.	22	0.136
588	A	5	4	1.	22	0.182
589	A	5	4	1.	22	0.182
590	A	13	5	1.	22	0.227
591	A	11	5	1.	22	0.227
592	A	9	5	1.	22	0.227
593	A	7	5	1.	20	0.25
594	A	1	1	1.	22	0.045
595	A	2	2	1.	22	0.091
596	A	3	2	1.	22	0.091
597	A	4	2	1.	22	0.091
598	A	4	3	1.	22	0.136
599	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
600	A	4	3	1.	22	0.136
601	A	3	3	1.	20	0.15
602	A	3	3	1.	22	0.136
603	A	5	4	1.	22	0.182
604	A	5	4	1.	22	0.182
605	A	5	4	1.	22	0.182
606	A	13	5	1.	22	0.227
607	A	11	5	1.	22	0.227
608	A	9	5	1.	22	0.227
609	A	7	5	1.	20	0.25
610	A	1	1	1.	22	0.045
611	A	2	2	1.	22	0.091
612	A	3	2	1.	22	0.091
613	A	4	2	1.	22	0.091
614	A	4	3	1.	22	0.136
615	A	4	3	1.	22	0.136
616	A	4	3	1.	22	0.136
617	A	4	3	1.	22	0.136
618	A	3	2	1.	22	0.091
619	A	3	3	1.	22	0.136
620	A	5	4	1.	22	0.182
621	A	5	4	1.	22	0.182
622	A	5	4	1.	22	0.182
623	A	10	7	1.	24	0.292
624	A	9	7	1.	24	0.292
625	A	8	7	1.	24	0.292
626	A	7	7	1.	24	0.292
627	A	6	6	1.	24	0.25
628	A	5	5	1.	24	0.208
629	A	4	4	1.	24	0.167
630	A	5	5	1.	24	0.208
631	A	6	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
632	A	7	5	1.	24	0.208
633	A	4	3	1.	24	0.125
634	A	4	3	1.	24	0.125
635	A	4	3	1.	24	0.125
636	A	3	3	1.	24	0.125
637	A	4	3	1.	24	0.125
638	A	4	3	1.	24	0.125
639	A	3	3	1.	24	0.125
640	A	5	4	1.	24	0.167
641	A	5	4	1.	24	0.167
642	A	4	3	1.	24	0.125
643	A	4	3	1.	24	0.125
644	A	4	3	1.	24	0.125
645	A	4	3	1.	24	0.125
646	A	3	2	1.	24	0.083
647	A	3	3	1.	24	0.125
648	A	5	4	1.	24	0.167
649	A	5	4	1.	24	0.167
650	A	5	4	1.	24	0.167
651	A	8	7	1.	24	0.292
652	A	7	7	1.	24	0.292
653	A	6	6	1.	24	0.25
654	A	5	5	1.	24	0.208
655	A	4	4	1.	24	0.167
656	A	5	5	1.	24	0.208
657	A	6	5	1.	24	0.208
658	A	7	5	1.	24	0.208
659	A	4	3	1.	24	0.125
660	A	4	3	1.	24	0.125
661	A	4	3	1.	24	0.125
662	A	3	3	1.	24	0.125
663	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
664	A	4	3	1.	24	0.125
665	A	3	3	1.	24	0.125
666	A	5	4	1.	24	0.167
667	A	5	4	1.	24	0.167
668	A	4	3	1.	25	0.12
669	A	4	3	1.	23	0.13
670	A	3	2	1.	22	0.091
671	A	4	3	1.	25	0.12
672	A	4	3	1.	25	0.12
673	A	8	7	1.	27	0.259
674	A	7	7	1.	27	0.259
675	A	6	6	1.	25	0.24
676	A	6	6	1.	24	0.25
677	A	9	9	1.	27	0.333
678	A	9	9	1.	27	0.333
679	A	7	7	1.	27	0.259
680	A	8	8	1.	27	0.296
681	A	9	8	1.	27	0.296
682	A	4	3	1.	27	0.111
683	A	4	3	1.	27	0.111
684	A	4	3	1.	25	0.12
685	A	4	3	1.	24	0.125
686	A	4	3	1.	27	0.111
687	A	4	3	1.	27	0.111
688	A	4	3	1.	27	0.111
689	A	4	3	1.	27	0.111
690	A	4	3	1.	27	0.111
691	A	4	3	1.	25	0.12
692	A	4	3	1.	25	0.12
693	A	4	3	1.	25	0.12
694	A	5	4	1.	23	0.174
695	A	5	4	1.	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
696	A	4	3	1.	25	0.12
697	A	4	3	1.	25	0.12
698	A	4	3	1.	25	0.12
699	A	4	3	1.	25	0.12
700	A	4	3	1.	25	0.12
701	A	5	4	1.	25	0.16
702	A	5	4	1.	25	0.16
703	A	5	4	1.	23	0.174
704	A	5	4	1.	22	0.182
705	A	4	3	1.	25	0.12
706	A	4	3	1.	25	0.12
707	A	4	3	1.	27	0.111
708	A	4	3	1.	25	0.12
709	A	3	2	1.	24	0.083
710	A	4	3	1.	27	0.111
711	A	4	3	1.	27	0.111
712	A	8	7	1.	27	0.259
713	A	7	7	1.	27	0.259
714	A	6	6	1.	25	0.24
715	A	6	6	1.	24	0.25
716	A	9	9	1.	27	0.333
717	A	9	9	1.	27	0.333
718	A	7	7	1.	27	0.259
719	A	8	8	1.	27	0.296
720	A	9	8	1.	27	0.296
721	A	4	3	1.	27	0.111
722	A	4	3	1.	27	0.111
723	A	4	3	1.	25	0.12
724	A	4	3	1.	24	0.125
725	A	4	3	1.	27	0.111
726	A	4	3	1.	27	0.111
727	A	4	3	1.	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
728	A	4	3	1.	27	0.111
729	A	4	3	1.	27	0.111
730	A	5	4	1.	27	0.148
731	A	8	6	1.	27	0.222
732	A	4	3	1.	25	0.12
733	A	4	3	1.	27	0.111
734	A	8	6	1.	27	0.222
735	A	5	4	1.	27	0.148
736	A	3	3	1.	22	0.136
737	A	3	3	1.	22	0.136
738	A	3	3	1.	20	0.15
739	A	2	2	1.	8	0.25
740	A	1	1	1.	22	0.045
741	A	2	2	1.	22	0.091
742	A	3	2	1.	22	0.091
743	A	4	2	1.	22	0.091
744	A	3	3	1.	24	0.125
745	A	3	3	1.	24	0.125
746	A	3	3	1.	24	0.125
747	A	1	1	1.	24	0.042
748	A	2	2	1.	24	0.083
749	A	3	2	1.	24	0.083
750	A	4	2	1.	24	0.083
751	A	7	6	1.01	27	0.222
752	A	4	4	1.	27	0.148
753	A	1	1	1.	25	0.04
754	A	1	1	1.	24	0.042
755	A	5	5	1.	27	0.185
756	A	8	6	1.01	27	0.222
757	A	6	4	1.	27	0.148
758	A	2	2	1.	27	0.074
759	A	2	2	1.	25	0.08

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	2	2	1.	24	0.083
761	A	15	6	1.	27	0.222
762	A	3	3	1.	22	0.136
763	A	3	3	1.	23	0.13
764	A	3	3	1.	23	0.13
765	A	4	4	1.	22	0.182
766	A	3	3	1.	22	0.136
767	A	4	4	1.	22	0.182
768	A	3	3	1.	20	0.15
769	A	3	3	1.	22	0.136
770	A	4	4	1.	22	0.182
771	A	3	3	1.	22	0.136
772	A	14	8	1.	20	0.4
773	A	12	8	1.	20	0.4
774	A	10	8	1.	20	0.4
775	A	9	9	1.	18	0.5
776	A	6	6	1.	20	0.3
777	A	8	6	1.	20	0.3
778	A	10	6	1.	20	0.3
779	A	12	6	1.	20	0.3
780	A	5	4	1.	22	0.182
781	A	5	4	1.	22	0.182
782	A	5	4	1.	22	0.182
783	A	5	4	1.	22	0.182
784	A	5	4	1.	20	0.2
785	A	5	4	1.	22	0.182
786	A	5	4	1.	22	0.182
787	A	5	4	1.	22	0.182
788	A	5	4	1.	22	0.182
789	A	14	8	1.	22	0.364
790	A	12	8	1.	22	0.364
791	A	10	8	1.	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
792	A	8	8	1.	20	0.4
793	A	7	6	1.	22	0.273
794	A	9	8	1.	22	0.364
795	A	10	6	1.	22	0.273
796	A	12	6	1.	22	0.273
797	A	5	4	1.	22	0.182
798	A	5	4	1.	22	0.182
799	A	5	4	1.	22	0.182
800	A	5	4	1.	22	0.182
801	A	5	4	1.	20	0.2
802	A	5	4	1.	22	0.182
803	A	5	4	1.	22	0.182
804	A	5	4	1.	22	0.182
805	A	5	4	1.	22	0.182
806	A	14	8	1.	22	0.364
807	A	12	8	1.	22	0.364
808	A	10	8	1.	22	0.364
809	A	9	9	1.	20	0.45
810	A	6	6	1.	22	0.273
811	A	8	6	1.	22	0.273
812	A	10	6	1.	22	0.273
813	A	12	6	1.	22	0.273
814	A	5	4	1.	22	0.182
815	A	5	4	1.	22	0.182
816	A	5	4	1.	22	0.182
817	A	5	4	1.	20	0.2
818	A	5	4	1.	22	0.182
819	A	5	4	1.	22	0.182
820	A	5	4	1.	22	0.182
821	A	5	4	1.	22	0.182
822	A	14	8	1.	22	0.364
823	A	12	8	1.	22	0.364

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
824	A	10	8	1.	22	0.364
825	A	8	8	1.	20	0.4
826	A	7	6	1.	22	0.273
827	A	9	8	1.	22	0.364
828	A	10	6	1.	22	0.273
829	A	12	6	1.	22	0.273
830	A	4	3	1.	22	0.136
831	A	4	3	1.	22	0.136
832	A	4	3	1.	22	0.136
833	A	4	3	1.	22	0.136
834	A	4	3	1.	22	0.136
835	A	4	3	1.	22	0.136
836	A	4	3	1.	22	0.136
837	A	4	3	1.	22	0.136
838	A	15	11	1.	24	0.458
839	A	13	11	1.	24	0.458
840	A	11	11	1.	24	0.458
841	A	9	9	1.	24	0.375
842	A	7	7	1.	24	0.292
843	A	7	7	1.	24	0.292
844	A	9	8	1.	24	0.333
845	A	11	8	1.	24	0.333
846	A	4	3	1.	24	0.125
847	A	4	3	1.	24	0.125
848	A	4	3	1.	24	0.125
849	A	4	3	1.	24	0.125
850	A	4	3	1.	24	0.125
851	A	4	3	1.	24	0.125
852	A	4	3	1.	24	0.125
853	A	4	3	1.	24	0.125
854	A	4	3	1.	24	0.125
855	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
856	A	4	3	1.	24	0.125
857	A	4	3	1.	24	0.125
858	A	4	3	1.	24	0.125
859	A	4	3	1.	24	0.125
860	A	4	3	1.	24	0.125
861	A	4	3	1.	24	0.125
862	A	4	3	1.	24	0.125
863	A	15	11	1.	24	0.458
864	A	13	11	1.	24	0.458
865	A	11	11	1.	24	0.458
866	A	9	9	1.	24	0.375
867	A	7	7	1.	24	0.292
868	A	7	7	1.	24	0.292
869	A	9	8	1.	24	0.333
870	A	11	8	1.	24	0.333
871	A	4	3	1.	24	0.125
872	A	4	3	1.	24	0.125
873	A	4	3	1.	24	0.125
874	A	4	3	1.	24	0.125
875	A	4	3	1.	24	0.125
876	A	4	3	1.	24	0.125
877	A	4	3	1.	24	0.125
878	A	4	3	1.	24	0.125
879	A	4	3	1.	24	0.125
880	A	4	3	1.	25	0.12
881	A	4	3	1.	25	0.12
882	A	3	2	1.	23	0.087
883	A	4	3	1.	22	0.136
884	A	4	3	1.	25	0.12
885	A	3	3	1.	25	0.12
886	A	9	8	1.	27	0.296
887	A	8	7	1.	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
888	A	7	6	1.	25	0.24
889	A	9	9	1.	24	0.375
890	A	9	9	1.	27	0.333
891	A	7	6	1.	27	0.222
892	A	8	7	1.	27	0.259
893	A	10	8	1.	27	0.296
894	A	11	8	1.	27	0.296
895	A	4	3	1.	27	0.111
896	A	4	3	1.	27	0.111
897	A	4	3	1.	25	0.12
898	A	4	3	1.	24	0.125
899	A	4	3	1.	27	0.111
900	A	4	3	1.	27	0.111
901	A	4	3	1.	27	0.111
902	A	4	3	1.	27	0.111
903	A	4	3	1.	27	0.111
904	A	4	3	1.	27	0.111
905	A	4	3	1.	27	0.111
906	A	3	2	1.	25	0.08
907	A	4	3	1.	24	0.125
908	A	4	3	1.	27	0.111
909	A	3	3	1.	27	0.111
910	A	9	8	1.	27	0.296
911	A	8	7	1.	27	0.259
912	A	7	6	1.	25	0.24
913	A	9	9	1.	24	0.375
914	A	9	9	1.	27	0.333
915	A	7	6	1.	27	0.222
916	A	8	7	1.	27	0.259
917	A	10	8	1.	27	0.296
918	A	11	8	1.	27	0.296
919	A	4	3	1.	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
920	A	4	3	1.	27	0.111
921	A	4	3	1.	25	0.12
922	A	4	3	1.	24	0.125
923	A	4	3	1.	27	0.111
924	A	4	3	1.	27	0.111
925	A	4	3	1.	27	0.111
926	A	4	3	1.	27	0.111
927	A	4	3	1.	27	0.111
928	C	2	2	0.53	20	0.1
929	A	5	5	1.09	22	0.227
930	A	7	5	1.05	22	0.227
931	C	3	3	0.38	24	0.125
932	A	4	4	1.	24	0.167
933	A	3	3	1.	22	0.136
934	A	3	3	1.	23	0.13
935	A	3	3	1.	23	0.13

Chapter 3

Listing of integrals

3.1 $\int e^{\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=114

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2)
 + (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rubi [A] time = 0.123382, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2)
 + (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{4}\,\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{12}\,\text{Subst}\left(\int \frac{\frac{9}{a^2}+\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{24}\,\text{Subst}\left(\int \frac{-\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^4}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a^4}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2}}\right.}{8a^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Mathematica [A] time = 0.0688987, size = 68, normalized size = 0.6

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3+8a^2x^2+9ax+16\right)+9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Maple [B] time = 0.187, size = 193, normalized size = 1.7

$$\frac{ax-1}{24\,a^4}\left(6\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+15\sqrt{a^2}\sqrt{a^2x^2-1}xa+8\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}+24\sqrt{a^2}\sqrt{(ax-1)(ax+1)}-15\ln\left(\frac{a^2 \cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x)
```

```
[Out] 1/24*(a*x-1)*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+15*(a^2)^(1/2)*(a^2*x^2-1
)^(1/2)*x*a+8*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+24*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2)-15*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+24
*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a
*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^4/(a^2)^(1/2)
```

Maxima [B] time = 1.02973, size = 274, normalized size = 2.4

$$\frac{1}{24}a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-49\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-39\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(2*(9*((a*x - 1)/(a*x + 1))^(7/2) - 49*((a*x - 1)/(a*x + 1))^(5/2) +
 31*((a*x - 1)/(a*x + 1))^(3/2) - 39*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1
)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 9*log(sqrt((a*x - 1)/(a*x + 1))
 + 1)/a^5 - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Fricas [A] time = 1.68643, size = 227, normalized size = 1.99

$$\frac{\left(6\,a^4x^4+14\,a^3x^3+17\,a^2x^2+25\,ax+16\right)\sqrt{\frac{ax-1}{ax+1}}+9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 + 14*a^3*x^3 + 17*a^2*x^2 + 25*a*x + 16)*sqrt((a*x - 1)/(a
*x + 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3,x)
```

```
[Out] Integral(x**3/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.18111, size = 246, normalized size = 2.16

$$\frac{1}{24}\,a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^5}-\frac{2\left(\frac{31\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{49\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{9\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}-39\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^5 - 9*log(abs(sqrt((a*x - 1)
/(a*x + 1)) - 1))/a^5 - 2*(31*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)
 - 49*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 9*(a*x - 1)^3*sqr
t((a*x - 1)/(a*x + 1))/(a*x + 1)^3 - 39*sqrt((a*x - 1)/(a*x + 1)))/(a^5*((a
*x - 1)/(a*x + 1) - 1)^4))
```

3.2 $\int e^{\coth^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=90

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rubi [A] time = 0.093424, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{4}{a^2}+\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] time = 0.0470575, size = 60, normalized size = 0.67

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+3ax+4\right)+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + 3*a*x + 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]])/(6*a^3)
```

Maple [B] time = 0.128, size = 173, normalized size = 1.9

$$\frac{ax-1}{6a^3}\left(3\sqrt{a^2}\sqrt{a^2x^2-1}xa+2\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a+6\sqrt{a^2}\sqrt{(ax-1)(ax+1)}+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x)
```

```
[Out] 1/6*(a*x-1)*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+6*(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+6*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^3/
(a^2)^(1/2)
```

Maxima [B] time = 1.08841, size = 224, normalized size = 2.49

$$-\frac{1}{6}a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(3*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
9*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4
/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^4 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

Fricas [A] time = 1.69133, size = 204, normalized size = 2.27

$$\frac{\left(2a^3x^3+5a^2x^2+7ax+4\right)\sqrt{\frac{ax-1}{ax+1}}+3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 + 5*a^2*x^2 + 7*a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) + 3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2,x)
```

```
[Out] Integral(x**2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [B] time = 1.19661, size = 204, normalized size = 2.27

$$\frac{1}{6}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^4}+\frac{2\left(\frac{4\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{3\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-9\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 - 3*log(abs(sqrt((a*x - 1)/
(a*x + 1)) - 1))/a^4 + 2*(4*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) -
 3*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 9*sqrt((a*x - 1)/(a*
x + 1)))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.3 $\int e^{\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=63

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rubi [A] time = 0.0644179, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x\,dx = -\,\mathrm{Subst}\left(\int \frac{1+\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{-\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] time = 0.0360467, size = 49, normalized size = 0.78

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(ax+2) + \log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/
(2*a^2)
```

Maple [B] time = 0.123, size = 152, normalized size = 2.4

$$\frac{ax-1}{2\,a^2}\left(\sqrt{a^2}\sqrt{a^2x^2-1}xa - \ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)a + 2\sqrt{a^2}\sqrt{(ax-1)(ax+1)} + 2\,a\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x,x)
```

```
[Out] 1/2*(a*x-1)*((a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-ln((a^2*x+(a^2*x^2-1)^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*a+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+2*a*ln((a
^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(
1/2)/((a*x-1)*(a*x+1))^(1/2)/a^2/(a^2)^(1/2)
```

Maxima [B] time = 0.982333, size = 173, normalized size = 2.75

$$\frac{1}{2}\,a\left(\frac{2\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(((a*x - 1)/(a*x + 1))^(3/2) - 3*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^3 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Fricas [A] time = 1.5686, size = 180, normalized size = 2.86

$$\frac{\left(a^2x^2+3\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 3*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(
a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x,x)
```

```
[Out] Integral(x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [B] time = 1.16286, size = 159, normalized size = 2.52

$$\frac{1}{2}\,a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^3}-\frac{2\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - log(abs(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^3 - 2*((a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 3*sqr
t((a*x - 1)/(a*x + 1)))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.4 $\int e^{\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=36

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rubi [A] time = 0.0358052, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6168, 807, 266, 63, 208}

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x],x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\,dx &= -\operatorname{Subst}\left(\int \frac{1+\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x + a\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0218327, size = 41, normalized size = 1.14

$$
x\sqrt{1-\frac{1}{a^2x^2}}+\frac{\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x],x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Maple [B] time = 0.12, size = 97, normalized size = 2.7

$$\frac{ax-1}{a}\left(\sqrt{a^2}\sqrt{(ax-1)(ax+1)}+a\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax-1)(ax+1)}}\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] (a*x-1)*((a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^
(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.02547, size = 122, normalized size = 3.39

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.5687, size = 155, normalized size = 4.31

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.15749, size = 77, normalized size = 2.14

$$-\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2 - 1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2 - 1}}{a\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x
^2 - 1)/(a*sgn(a*x + 1))
```

3.5 $\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=22

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

```
[Out] -ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rubi [A] time = 0.0434277, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6169, 844, 216, 266, 63, 208}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x,x]
```

```
[Out] -ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{1+\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\csc^{-1}(ax) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\csc^{-1}(ax) + a^2\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0131496, size = 36, normalized size = 1.64

$$\log\left(x\left(\sqrt{\frac{a^2x^2-1}{a^2x^2}}+1\right)\right) - \sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x,x]
```

```
[Out] -ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Maple [B] time = 0.128, size = 132, normalized size = 6.

$$-(ax-1)\left(\sqrt{a^2x^2-1}\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-a\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)-\sqrt{a^2}\sqrt{(ax-1)(ax+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] -(a*x-1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(
1/2)-a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2
)/(a^2)^(1/2)
```

Maxima [B] time = 1.56288, size = 93, normalized size = 4.23

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Fricas [B] time = 1.53101, size = 150, normalized size = 6.82

$$2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(1/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] time = 1.17373, size = 95, normalized size = 4.32

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a)
```

3.6 $\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=24

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

```
[Out] a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcCsc[a*x]
```

Rubi [A] time = 0.0247176, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6169, 641, 216}

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^2,x]
```

```
[Out] a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcCsc[a*x]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)$$
$$= a\sqrt{1-\frac{1}{a^2x^2}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)$$
$$= a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

Mathematica [A] time = 0.0192168, size = 27, normalized size = 1.12

$$a\left(\sqrt{1-\frac{1}{a^2x^2}} - \sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^2,x]
```

```
[Out] a*(Sqrt[1 - 1/(a^2*x^2)] - ArcSin[1/(a*x)])
```

Maple [B] time = 0.13, size = 220, normalized size = 9.2

$$-\frac{ax-1}{x}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 + \left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2} + \sqrt{a^2}\sqrt{a^2x^2-1}xa + \ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)xa^2 + ax\sqrt{a^2}\,\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] -(a*x-1)*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^2)^(1
/2)+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/
2))/(a^2)^(1/2))*x*a^2+a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2))/(a^2)^(1/2))*x*a^2)/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/x/(
a^2)^(1/2)
```

Maxima [B] time = 1.54295, size = 72, normalized size = 3.

$$2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}+\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*x
 - 1)/(a*x + 1))))
```

Fricas [B] time = 1.63493, size = 112, normalized size = 4.67

$$\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] time = 1.15409, size = 72, normalized size = 3.

$$2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}+\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*x
 - 1)/(a*x + 1))))
```

3.7 $\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=38

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a + x^(-1)))/2 - (a^2*ArcCsc[a*x])/2
```

Rubi [A] time = 0.0308692, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6169, 780, 216}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^3,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a + x^(-1)))/2 - (a^2*ArcCsc[a*x])/2
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = -\operatorname{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right) - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right) - \frac{1}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] time = 0.0419218, size = 42, normalized size = 1.11

$$\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(2ax+1) - ax\sin^{-1}\left(\frac{1}{ax}\right)\right)}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^3,x]
```

```
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a*x) - a*x*ArcSin[1/(a*x)]))/(2*x)
```

Maple [B] time = 0.126, size = 257, normalized size = 6.8

$$-\frac{ax-1}{2x^2}\left(-2\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3 + 2\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa + \sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 + 2\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3 + a^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

```
[Out] -1/2*(a*x-1)*(-2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+2*(a^2)^(1/2)*(a^2*x
^2-1)^(3/2)*x*a+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+2*ln((a^2*x+(a^2*x^2-
1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^
2*x^2-1)^(1/2))-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-2*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+(a^2*x^2-1)^(3/2)*
(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/x^2/(a^2)^(1/2
```

)

Maxima [B] time = 1.52503, size = 123, normalized size = 3.24

$$\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] (a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*((a*x - 1)/(a*x + 1))^(3/2) + 3*a
*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^
2 + 1))*a
```

Fricas [A] time = 1.72788, size = 144, normalized size = 3.79

$$\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(2\,a^2x^2+3\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (2*a^2*x^2 + 3*a*x + 1)*
sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] time = 1.13562, size = 117, normalized size = 3.08

$$\left(a \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{\frac{(ax-1)a\sqrt{\frac{ax-1}{ax+1}}}{ax+1} + 3\,a\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt((a*x - 1)/(a*x + 1))) + ((a*x - 1)*a*sqrt((a*x - 1)/(a*x + 1
))/(a*x + 1) + 3*a*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)/(a*x + 1) + 1)^2)*
a
```

3.8 $\int \frac{e^{\coth^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=75

$$-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

```
[Out] a^3*Sqrt[1 - 1/(a^2*x^2)] - (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[1 -
 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rubi [A] time = 0.0618059, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6169, 797, 641, 195, 216}

$$-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] a^3*Sqrt[1 - 1/(a^2*x^2)] - (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[1 -
 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx &= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(a^2\,\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + a^2\,\text{Subst}\left(\int \left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} - a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + a^2\,\text{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - a^3\csc^{-1}(ax) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0783049, size = 51, normalized size = 0.68

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a^2x^2+3ax+2\right)}{x^2} - 3a^2\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x + 4*a^2*x^2))/x^2 - 3*a^2*ArcSin[1/(a
*x)]))/6
```

Maple [B] time = 0.135, size = 284, normalized size = 3.8

$$-\frac{ax-1}{6\,x^{3}}\left(-6\,\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}x^{4}a^{4}+6\,\left(a^{2}x^{2}-1\right)^{3/2}\sqrt{a^{2}}x^{2}a^{2}+3\,\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}x^{3}a^{3}+6\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{3}a^{4}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

```
[Out] -1/6*(a*x-1)*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*(a^2*x^2-1)^(3/2)*
(a^2)^(1/2)*x^2*a^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+6*ln((a^2*x+(a^
2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+3*a^3*x^3*(a^2)^(1/2)*arct
an(1/(a^2*x^2-1)^(1/2))-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-6*ln(
(a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+3*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1)
)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/x^3/(a^2)^(1/2)
```

Maxima [B] time = 1.55237, size = 184, normalized size = 2.45

$$\frac{1}{3}\left(3\,a^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{3\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{(ax-1)^{3}}{(ax+1)^{3}}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (3*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 9*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

Fricas [A] time = 1.55917, size = 161, normalized size = 2.15

$$\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(4\,a^3x^3+7\,a^2x^2+5\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*a^3*x^3 + 7*a^2*x^2 +
 5*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] time = 1.17312, size = 176, normalized size = 2.35

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{4\,(ax-1)a^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{3\,(ax-1)^2a^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*(a*x - 1)*a^2*sqrt((a*x -
 1)/(a*x + 1))/(a*x + 1) + 3*(a*x - 1)^2*a^2*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1)^2 + 9*a^2*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

3.9 $\int \frac{e^{\coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=88

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{3}{8}a^4\csc^{-1}(ax)$$

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a + 9/x))/24 + (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - (3*a^4*ArcCsc[a*x])/8
```

Rubi [A] time = 0.0828003, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6169, 833, 780, 216}

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{3}{8}a^4\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a + 9/x))/24 + (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - (3*a^4*ArcCsc[a*x])/8
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx &= -\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{12}a^4\,\text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}+\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{3}{8}a^4\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0942163, size = 59, normalized size = 0.67

$$\frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(16a^3x^3+9a^2x^2+8ax+6\right)}{x^3} - 9a^3\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^3 - 9*a^
3*ArcSin[1/(a*x)]))/24
```

Maple [B] time = 0.138, size = 308, normalized size = 3.5

$$-\frac{ax-1}{24\,x^4}\left(-24\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5+24\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3+9\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+9\,a^4x^4\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x)
```

```
[Out] -1/24*(a*x-1)*(-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+24*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^3*a^3+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+9*a^4*x^4*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^
(1/2))/(a^2)^(1/2))*x^4*a^5-24*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-
24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+15*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+8*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+6*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1
/2)/x^4/(a^2)^(1/2)
```

Maxima [B] time = 1.56124, size = 232, normalized size = 2.64

$$\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{9\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (9*a^3*((a*x - 1)/(a*x + 1)
)^(7/2) + 49*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 31*a^3*((a*x - 1)/(a*x + 1))
^(3/2) + 39*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
- 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
1))*a
```

Fricas [A] time = 1.96609, size = 185, normalized size = 2.1

$$\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(16\,a^4x^4+25\,a^3x^3+17\,a^2x^2+14\,ax+6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) + (16*a^4*x^4 + 25*a^3*x
^3 + 17*a^2*x^2 + 14*a*x + 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15896, size = 221, normalized size = 2.51

$$\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{31\,(ax-1)a^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{49\,(ax-1)^2a^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{9\,(ax-1)^3a^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (31*(a*x - 1)*a^3*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1) + 49*(a*x - 1)^2*a^3*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^2 + 9*(a*x - 1)^3*a^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 39*a
^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)/(a*x + 1) + 1)^4)*a
```

3.10 $\int e^{2 \coth^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=43

$$\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\log(1-ax)}{a^4}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Rubi [A] time = 0.0526951, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\log(1-ax)}{a^4}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^3\,dx &= -\int e^{2\tanh^{-1}(ax)}x^3\,dx\\ &= -\int \frac{x^3(1+ax)}{1-ax}\,dx\\ &= -\int \left(-\frac{2}{a^3}-\frac{2x}{a^2}-\frac{2x^2}{a}-x^3-\frac{2}{a^3(-1+ax)}\right)dx\\ &= \frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0189926, size = 43, normalized size = 1.

$$\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\log(1-ax)}{a^4}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Maple [A] time = 0.042, size = 39, normalized size = 0.9

$$\frac{x^4}{4}+\frac{2\,x^3}{3\,a}+\frac{x^2}{a^2}+2\,\frac{x}{a^3}+2\,\frac{\ln(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3,x)
```

```
[Out] 1/4*x^4+2/3*x^3/a+x^2/a^2+2*x/a^3+2/a^4*ln(a*x-1)
```

Maxima [A] time = 1.01774, size = 58, normalized size = 1.35

$$\frac{3\,a^3x^4+8\,a^2x^3+12\,ax^2+24\,x}{12\,a^3}+\frac{2\,\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 + 2*log(a*x - 1)/a^4
```

Fricas [A] time = 1.8717, size = 100, normalized size = 2.33

$$\frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax + 24\,\log\left(ax - 1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

Sympy [A] time = 1.90545, size = 37, normalized size = 0.86

$$\frac{x^4}{4} + \frac{2x^3}{3a} + \frac{x^2}{a^2} + \frac{2x}{a^3} + \frac{2\log\left(ax - 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3,x)
```

```
[Out] x**4/4 + 2*x**3/(3*a) + x**2/a**2 + 2*x/a**3 + 2*log(a*x - 1)/a**4
```

Giac [A] time = 1.16499, size = 63, normalized size = 1.47

$$\frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax - 1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 + 2*log(abs(a*x - 1)
)/a^4
```

3.11 $\int e^{2\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=33

$$\frac{2x}{a^2}+\frac{2\log(1-ax)}{a^3}+\frac{x^2}{a}+\frac{x^3}{3}$$

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Rubi [A] time = 0.0493616, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$\frac{2x}{a^2}+\frac{2\log(1-ax)}{a^3}+\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^2\,dx &= -\int e^{2\tanh^{-1}(ax)}x^2\,dx\\ &= -\int \frac{x^2(1+ax)}{1-ax}\,dx\\ &= -\int \left(-\frac{2}{a^2}-\frac{2x}{a}-x^2-\frac{2}{a^2(-1+ax)}\right)dx\\ &= \frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0140906, size = 33, normalized size = 1.

$$\frac{2x}{a^2}+\frac{2\log(1-ax)}{a^3}+\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Maple [A] time = 0.039, size = 31, normalized size = 0.9

$$\frac{x^3}{3}+\frac{x^2}{a}+2\,\frac{x}{a^2}+2\,\frac{\ln(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2,x)
```

```
[Out] 1/3*x^3+x^2/a+2*x/a^2+2/a^3*ln(a*x-1)
```

Maxima [A] time = 1.00945, size = 46, normalized size = 1.39

$$\frac{a^2x^3+3\,ax^2+6\,x}{3\,a^2}+\frac{2\,\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 + 2*log(a*x - 1)/a^3
```

Fricas [A] time = 1.80224, size = 76, normalized size = 2.3

$$\frac{a^3x^3 + 3\,a^2x^2 + 6\,ax + 6\,\log\left(ax - 1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

Sympy [A] time = 0.720187, size = 27, normalized size = 0.82

$$\frac{x^3}{3} + \frac{x^2}{a} + \frac{2x}{a^2} + \frac{2\log\left(ax - 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2,x)
```

```
[Out] x**3/3 + x**2/a + 2*x/a**2 + 2*log(a*x - 1)/a**3
```

Giac [A] time = 1.13857, size = 51, normalized size = 1.55

$$\frac{a^3x^3 + 3\,a^2x^2 + 6\,ax}{3\,a^3} + \frac{2\,\log\left(|ax - 1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 + 2*log(abs(a*x - 1))/a^3
```

3.12 $\int e^{2\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=26

$$\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

```
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Rubi [A] time = 0.0327405, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6126, 77}

$$\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x,x]
```

```
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x\,dx &= -\int e^{2\tanh^{-1}(ax)}x\,dx\\ &= -\int \frac{x(1+ax)}{1-ax}\,dx\\ &= -\int \left(-\frac{2}{a}-x-\frac{2}{a(-1+ax)}\right)dx\\ &= \frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0122553, size = 26, normalized size = 1.

$$\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x,x]
```

```
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Maple [A] time = 0.037, size = 24, normalized size = 0.9

$$\frac{x^2}{2}+2\frac{x}{a}+2\frac{\ln(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x,x)
```

```
[Out] 1/2*x^2+2*x/a+2/a^2*ln(a*x-1)
```

Maxima [A] time = 1.16666, size = 35, normalized size = 1.35

$$\frac{ax^2+4x}{2a}+\frac{2\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 4*x)/a + 2*log(a*x - 1)/a^2
```

Fricas [A] time = 1.82504, size = 59, normalized size = 2.27

$$\frac{a^2x^2 + 4\,ax + 4\,\log\left(ax - 1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

Sympy [A] time = 1.21425, size = 20, normalized size = 0.77

$$\frac{x^2}{2} + \frac{2x}{a} + \frac{2\log\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x)
```

```
[Out] x**2/2 + 2*x/a + 2*log(a*x - 1)/a**2
```

Giac [A] time = 1.14079, size = 41, normalized size = 1.58

$$\frac{a^2x^2 + 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax - 1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*x^2 + 4*a*x)/a^2 + 2*log(abs(a*x - 1))/a^2
```

3.13 $\int e^{2\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$\frac{2\log(1-ax)}{a}+x$$

```
[Out] x + (2*Log[1 - a*x])/a
```

Rubi [A] time = 0.0141243, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6125, 43}

$$\frac{2\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x]),x]
```

```
[Out] x + (2*Log[1 - a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\,dx &= -\int e^{2\tanh^{-1}(ax)}\,dx \\
&= -\int \frac{1+ax}{1-ax}\,dx \\
&= -\int \left(-1-\frac{2}{-1+ax}\right)dx \\
&= x+\frac{2\log(1-ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0111536, size = 14, normalized size = 1.

$$\frac{2\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x]),x]
```

```
[Out] x + (2*Log[1 - a*x])/a
```

Maple [A] time = 0.042, size = 14, normalized size = 1.

$$x+2\,\frac{\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1),x)
```

```
[Out] x+2/a*ln(a*x-1)
```

Maxima [A] time = 1.0179, size = 18, normalized size = 1.29

$$x+\frac{2\,\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="maxima")
```

```
[Out] x + 2*log(a*x - 1)/a
```

Fricas [A] time = 1.78469, size = 35, normalized size = 2.5

$$\frac{ax + 2 \log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="fricas")
```

```
[Out] (a*x + 2*log(a*x - 1))/a
```

Sympy [A] time = 0.09782, size = 10, normalized size = 0.71

$$x + \frac{2 \log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x)
```

```
[Out] x + 2*log(a*x - 1)/a
```

Giac [A] time = 1.12736, size = 19, normalized size = 1.36

$$x + \frac{2 \log(|ax - 1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="giac")
```

```
[Out] x + 2*log(abs(a*x - 1))/a
```

3.14 $\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=14

$$2\log(1-ax)-\log(x)$$

```
[Out] -Log[x] + 2*Log[1 - a*x]
```

Rubi [A] time = 0.0396086, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 72}

$$2\log(1-ax)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x,x]
```

```
[Out] -Log[x] + 2*Log[1 - a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x}\,dx\\ &= -\int \frac{1+ax}{x(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x}-\frac{2a}{-1+ax}\right)dx\\ &= -\log(x)+2\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.008856, size = 14, normalized size = 1.

$$2\log(1-ax)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x,x]
```

```
[Out] -Log[x] + 2*Log[1 - a*x]
```

Maple [A] time = 0.043, size = 14, normalized size = 1.

$$-\ln(x)+2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x,x)
```

```
[Out] -ln(x)+2*ln(a*x-1)
```

Maxima [A] time = 1.00709, size = 18, normalized size = 1.29

$$2\log(ax-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] 2*log(a*x - 1) - log(x)
```

Fricas [A] time = 1.79277, size = 34, normalized size = 2.43

$$2 \log(ax-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] 2*log(a*x - 1) - log(x)
```

Sympy [A] time = 0.179207, size = 10, normalized size = 0.71

$$-\log(x) + 2\log\left(x - \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x)
```

```
[Out] -log(x) + 2*log(x - 1/a)
```

Giac [A] time = 1.16421, size = 20, normalized size = 1.43

$$2 \log(|ax-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="giac")
```

```
[Out] 2*log(abs(a*x - 1)) - log(abs(x))
```

3.15 $\int \frac{e^{2\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=19

$$-2a\log(x) + 2a\log(1-ax) + \frac{1}{x}$$

```
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Rubi [A] time = 0.0445641, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$-2a\log(x) + 2a\log(1-ax) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x^2,x]
```

```
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^2}\,dx\\ &= -\int \frac{1+ax}{x^2(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x^2}+\frac{2a}{x}-\frac{2a^2}{-1+ax}\right)dx\\ &= \frac{1}{x}-2a\log(x)+2a\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0110726, size = 19, normalized size = 1.

$$-2a\log(x)+2a\log(1-ax)+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^2,x]
```

```
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Maple [A] time = 0.044, size = 19, normalized size = 1.

$$x^{-1}-2\,a\ln(x)+2\,a\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x^2,x)
```

```
[Out] 1/x-2*a*ln(x)+2*a*ln(a*x-1)
```

Maxima [A] time = 1.13019, size = 24, normalized size = 1.26

$$2\,a\log(ax-1)-2\,a\log(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*log(a*x - 1) - 2*a*log(x) + 1/x
```

Fricas [A] time = 1.9424, size = 58, normalized size = 3.05

$$\frac{2\,ax\log\left(ax-1\right)-2\,ax\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

Sympy [A] time = 2.05294, size = 15, normalized size = 0.79

$$2a\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**2,x)
```

```
[Out] 2*a*(-log(x) + log(x - 1/a)) + 1/x
```

Giac [A] time = 1.16738, size = 27, normalized size = 1.42

$$2\,a\log\left(|ax-1|\right)-2\,a\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(a*x - 1)) - 2*a*log(abs(x)) + 1/x
```

3.16 $\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=33

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

```
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Rubi [A] time = 0.0474221, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x^3,x]
```

```
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx\\ &= -\int \frac{1+ax}{x^3(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{-1+ax}\right)dx\\ &= \frac{1}{2x^2}+\frac{2a}{x}-2a^2\log(x)+2a^2\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0136995, size = 33, normalized size = 1.

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^3,x]
```

```
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Maple [A] time = 0.047, size = 31, normalized size = 0.9

$$\frac{1}{2\,x^2}+2\,\frac{a}{x}-2\,a^2\ln(x)+2\,a^2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x^3,x)
```

```
[Out] 1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x-1)
```

Maximal [A] time = 0.982735, size = 41, normalized size = 1.24

$$2\,a^2\log(ax-1)-2\,a^2\log(x)+\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 2*a^2*log(a*x - 1) - 2*a^2*log(x) + 1/2*(4*a*x + 1)/x^2
```

Fricas [A] time = 1.83743, size = 88, normalized size = 2.67

$$\frac{4\,a^2x^2\log\left(ax-1\right)-4\,a^2x^2\log\left(x\right)+4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

Sympy [A] time = 2.12677, size = 26, normalized size = 0.79

$$2a^2\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**3,x)
```

```
[Out] 2*a**2*(-log(x) + log(x - 1/a)) + (4*a*x + 1)/(2*x**2)
```

Giac [A] time = 1.1481, size = 43, normalized size = 1.3

$$2\,a^2\log\left(|ax-1|\right)-2\,a^2\log\left(|x|\right)+\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(abs(a*x - 1)) - 2*a^2*log(abs(x)) + 1/2*(4*a*x + 1)/x^2
```

3.17 $\int \frac{e^{2\coth^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=40

$$\frac{2a^2}{x} - 2a^3\log(x) + 2a^3\log(1-ax) + \frac{a}{x^2} + \frac{1}{3x^3}$$

```
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Rubi [A] time = 0.0514506, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$\frac{2a^2}{x} - 2a^3\log(x) + 2a^3\log(1-ax) + \frac{a}{x^2} + \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x^4,x]
```

```
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx\\ &= -\int \frac{1+ax}{x^4(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}-\frac{2a^4}{-1+ax}\right)dx\\ &= \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0146343, size = 40, normalized size = 1.

$$\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)+\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^4,x]
```

```
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Maple [A] time = 0.044, size = 38, normalized size = 1.

$$\frac{1}{3\,x^3}+\frac{a}{x^2}+2\,\frac{a^2}{x}-2\,a^3\ln(x)+2\,a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x^4,x)
```

```
[Out] 1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(a*x-1)
```

Maxima [A] time = 0.983053, size = 51, normalized size = 1.27

$$2\,a^3\log(ax-1)-2\,a^3\log(x)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*log(a*x - 1) - 2*a^3*log(x) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

Fricas [A] time = 1.97263, size = 104, normalized size = 2.6

$$\frac{6\,a^3x^3\log\left(ax-1\right)-6\,a^3x^3\log\left(x\right)+6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^3
```

Sympy [A] time = 1.61375, size = 34, normalized size = 0.85

$$2a^3\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**4,x)
```

```
[Out] 2*a**3*(-log(x) + log(x - 1/a)) + (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

Giac [A] time = 1.12271, size = 54, normalized size = 1.35

$$2\,a^3\log\left(|ax-1|\right)-2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*log(abs(a*x - 1)) - 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x
^3
```

3.18 $\int e^{3\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=118

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rubi [A] time = 1.07141, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6169, 6742, 651, 271, 264, 266, 51, 63, 208}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2,x]
```


```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^4\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a^3(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}} + \frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}}\right) dx, x,$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} -$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^3} - \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-}}\right.}{3a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3} +$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3} + \frac{3\,\text{Subs}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] time = 0.0800755, size = 75, normalized size = 0.64

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3+7a^2x^2+19ax-52\right)}{ax-1} + 33\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 + a
*x) + 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Maple [B] time = 0.166, size = 471, normalized size = 4.

$$\frac{1}{6\,a^3\,(ax+1)}\left(9\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+2\,\sqrt{a^2}\,((ax-1)\,(ax+1))^{3/2}\,x^2a^2-18\,\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2-9\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x)
```

```
[Out] 1/6/a^3*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+2*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^2*a^2-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-9*ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-4*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(3/2)*x*a+42*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+42*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+9*(a^2)^(1/2)*(
a^2*x^2-1)^(1/2)*x*a+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2
))*x*a^2-10*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-84*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2)*x*a-84*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/
2))*x*a^2-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+42*(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+42*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2)))/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)
/(a*x+1))^(3/2)
```

Maxima [A] time = 1.00047, size = 246, normalized size = 2.08

$$-\frac{1}{6}\,a\left(\frac{2\left(\frac{75\,(ax-1)}{ax+1}-\frac{88\,(ax-1)^2}{(ax+1)^2}+\frac{33\,(ax-1)^3}{(ax+1)^3}-12\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(75*(a*x - 1)/(a*x + 1) - 88*(a*x - 1)^2/(a*x + 1)^2 + 33*(a*x -
1)^3/(a*x + 1)^3 - 12)/(a^4*((a*x - 1)/(a*x + 1))^(7/2) - 3*a^4*((a*x - 1)/
(a*x + 1))^(5/2) + 3*a^4*((a*x - 1)/(a*x + 1))^(3/2) - a^4*sqrt((a*x - 1)/(
```

```
a*x + 1))) - 33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 33*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^4)
```

Fricas [A] time = 1.98187, size = 267, normalized size = 2.26

$$\frac{33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4x^4+9\,a^3x^3+26\,a^2x^2-33\,ax-52\right)\sqrt{\frac{ax-1}{ax+1}}}{6\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(33*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 33*(a*x - 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (2*a^4*x^4 + 9*a^3*x^3 + 26*a^2*x^2 - 33*a*x
- 52)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.17711, size = 231, normalized size = 1.96

$$\frac{1}{6}\,a\left(\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^4}-\frac{24}{a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{2\left(\frac{52\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{21\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-39\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*a*(33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 - 33*log(abs(sqrt((a*x - 1
)/(a*x + 1)) - 1))/a^4 - 24/(a^4*sqrt((a*x - 1)/(a*x + 1))) + 2*(52*(a*x -
1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 21*(a*x - 1)^2*sqrt((a*x - 1)/(a*x
 + 1))/(a*x + 1)^2 - 39*sqrt((a*x - 1)/(a*x + 1)))/(a^4*((a*x - 1)/(a*x + 1
) - 1)^3))
```

3.19 $\int e^{3\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rubi [A] time = 0.871573, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {6169, 6742, 651, 266, 51, 63, 208, 264}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a^2(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{S}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 4\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) - \frac{\text{Subst}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] time = 0.0632278, size = 66, normalized size = 0.72

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(a^2x^2+5ax-14\right)}{ax-1} + 9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 + 5*a*x + a^2*x^2))/(-1 + a*x) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Maple [B] time = 0.165, size = 421, normalized size = 4.6

$$\frac{1}{2\,a^{2}\,(ax+1)}\left(\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}x^{3}a^{3}-2\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}x^{2}a^{2}-\ln\left(\left(a^{2}x+\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}\right)\frac{1}{\sqrt{a^{2}}}\right)x^{2}a^{3}+10\sqrt{a^{2}}\sqrt{(ax-1)\,(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x,x)
```

```
[Out] 1/2/a^2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-2*(a^2*x^2-1)^(1/2)*(a^2)^(1
/2)*x^2*a^2-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+1
0*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+10*ln((a^2*x+(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a
+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-4*((a*x-1)*(
a*x+1))^(3/2)*(a^2)^(1/2)-20*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-20*ln(
(a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+10*(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)+10*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/(
a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.00739, size = 196, normalized size = 2.13

$$\frac{1}{2}\,a\left(\frac{2\left(\frac{15\,(ax-1)}{ax+1}-\frac{9\,(ax-1)^{2}}{(ax+1)^{2}}-4\right)}{a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-2\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{3}}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(15*(a*x - 1)/(a*x + 1) - 9*(a*x - 1)^2/(a*x + 1)^2 - 4)/(a^3*((a*
x - 1)/(a*x + 1))^(5/2) - 2*a^3*((a*x - 1)/(a*x + 1))^(3/2) + a^3*sqrt((a*x
 - 1)/(a*x + 1))) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Fricas [A] time = 1.95602, size = 243, normalized size = 2.64

$$\frac{9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^3x^3+6\,a^2x^2-9\,ax-14\right)\sqrt{\frac{ax-1}{ax+1}}}{2\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(9*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x - 1)*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1) + (a^3*x^3 + 6*a^2*x^2 - 9*a*x - 14)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.18944, size = 189, normalized size = 2.05

$$\frac{1}{2}a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^3}-\frac{8}{a^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{2\left(\frac{5\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-7\sqrt{\frac{ax-1}{ax+1}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(abs(sqrt((a*x - 1)/
(a*x + 1)) - 1))/a^3 - 8/(a^3*sqrt((a*x - 1)/(a*x + 1))) - 2*(5*(a*x - 1)*s
```

```
qrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 7*sqrt((a*x - 1)/(a*x + 1)))/(a^3*((a*
x - 1)/(a*x + 1) - 1)^2))
```

3.20 $\int e^{3\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=62

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.810646, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6168, 6742, 651, 264, 266, 63, 208}

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^2\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \frac{4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + (3a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.0471783, size = 54, normalized size = 0.87

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax-5)}{ax-1} + \frac{3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + a*x))/(-1 + a*x) + (3*Log[a*(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/a
```

Maple [B] time = 0.136, size = 247, normalized size = 4.

$$\frac{1}{a(ax+1)}\left(3\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^2a^3 + 3\sqrt{a^2}\sqrt{(ax-1)(ax+1)}x^2a^2 - 6\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/a*(3*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+
3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-6*ln((a^2*x+(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2
)-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+3*a*ln((a^2*x+(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^
2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.00855, size = 149, normalized size = 2.4

$$-a\left(\frac{2\left(\frac{3(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*sq
rt((a*x - 1)/(a*x + 1))) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*log
(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.84083, size = 217, normalized size = 3.5

$$\frac{3(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-4ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 4*a*x - 5)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/2), x)
```

Giac [B] time = 1.16671, size = 161, normalized size = 2.6

$$a\left(\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{3(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\sqrt{\frac{ax-1}{ax+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^2 - 2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)*sqrt((a*x
- 1)/(a*x + 1))/(a*x + 1) - sqrt((a*x - 1)/(a*x + 1)))))
```

3.21 $\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcCsc[a*x] + ArcTanh[Sqrt[1 -
1/(a^2*x^2)]]
```

Rubi [A] time = 0.782756, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 6742, 216, 651, 266, 63, 208}

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x,x]
```


```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcCsc[a*x] + ArcTanh[Sqrt[1 -
1/(a^2*x^2)]]
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x,\right.$$
$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + a^2\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [A] time = 0.058624, size = 53, normalized size = 1.15

$$-\frac{4ax\sqrt{1-\frac{1}{a^2x^2}}}{ax-1} + \log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) + \sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x,x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]
```

Maple [B] time = 0.173, size = 363, normalized size = 7.9

$$\frac{1}{ax+1}\left(\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)x^2a^3 + \sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 + a^2x^2\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) + \sqrt{a^2}\sqrt{(ax\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] (ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+(a^2*x
^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1
/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-2*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x
*a-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+a*ln((a^2*x+(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arct
an(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a
^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [B] time = 1.4833, size = 122, normalized size = 2.65

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}+\frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a + 4/(a*sqrt((a*x - 1)/(a*x + 1
))))
```

Fricas [B] time = 1.86066, size = 262, normalized size = 5.7

$$-\frac{2\,(ax-1)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(2*(a*x - 1)*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x - 1)*log(sqrt((a*x -
 1)/(a*x + 1)) + 1) + (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + 4*(a*x
```

```
+ 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.20681, size = 123, normalized size = 2.67

$$-a\left(\frac{2\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a}+\frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a + 4/(a*sqrt((a*x - 1)/(a*
x + 1))))
```

3.22 $\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=51

$$-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}-3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

```
[Out] -3*a*Sqrt[1 - 1/(a^2*x^2)] - (2*(a + x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
 3*a*ArcCsc[a*x]
```

Rubi [A] time = 0.0740333, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6169, 853, 669, 641, 216}

$$-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}-3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^2,x]
```



```
[Out] -3*a*Sqrt[1 - 1/(a^2*x^2)] - (2*(a + x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
 3*a*ArcCsc[a*x]
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3\,\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -3a\sqrt{1-\frac{1}{a^2x^2}}-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -3a\sqrt{1-\frac{1}{a^2x^2}}-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.0817021, size = 41, normalized size = 0.8

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}(1-5ax)}{ax-1}+3a\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^2,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x))/(-1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Maple [B] time = 0.169, size = 593, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x
+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-5*(a
^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2
-1)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+7*
(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+6*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x
^2-1)^(1/2))+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-2*(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2)*x^2*a^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*x^2*a^3+ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*
x*a^2-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-3*a
*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x*a-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/x/(a^2
)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.47312, size = 97, normalized size = 1.9

$$-2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*((3*(a*x - 1)/(a*x + 1) + 2)/(((a*x - 1)/(a*x + 1))^(3/2) + sqrt((a*x
- 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A] time = 1.89969, size = 162, normalized size = 3.18

$$-\frac{6\left(a^{2}x^{2}-ax\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(5\,a^{2}x^{2}+4\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{ax^{2}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*(a^2*x^2 - a*x)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (5*a^2*x^2 + 4*a*x
- 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [A] time = 1.18608, size = 115, normalized size = 2.25

$$-2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*((3*(a*x - 1)/(a*x + 1) + 2)/((a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1) + sqrt((a*x - 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

3.23 $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=91

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{9}{2}a^2\csc^{-1}(ax)$$

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a - x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a - x^(-1))) + (9*a^2*ArcCsc[a*x
])/2
```

Rubi [A] time = 0.453295, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 1633, 1593, 12, 793, 665, 216}

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{9}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^3,x]
```


```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a - x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a - x^(-1))) + (9*a^2*ArcCsc[a*x
])/2
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(-ax-x^2)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} + (3a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] time = 0.0930147, size = 56, normalized size = 0.62

$$\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-14a^2x^2+5ax+1\right)}{x(ax-1)} + 9a\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^3,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x - 14*a^2*x^2))/(x*(-1 + a*x)) + 9*a*A
rcSin[1/(a*x)]))/2
```

Maple [B] time = 0.171, size = 642, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] -1/2*(6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+6*ln((a^2*x+(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^
3*a^3-21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-9*a^4*x^4*(a^2)^(1/2)*arctan
(1/(a^2*x^2-1)^(1/2))+6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-6*ln((a
^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-12*ln((a^2*x+(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+11*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*x^2*a^2+24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+18*a^3*x^3*(a^2
)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x
^2*a^2-12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+12*ln((a^2*x+(a^2*x^2
-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+6*ln((a^2*x+(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-4*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-
9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-9*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2
*x^2-1)^(1/2))+6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-6*ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(
1/2))/x^2/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/
2)
```

Maxima [A] time = 1.52817, size = 149, normalized size = 1.64

$$-\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{15\,(ax-1)a}{ax+1}+\frac{9\,(ax-1)^2a}{(ax+1)^2}+4\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (15*(a*x - 1)*a/(a*x + 1) + 9*(a*
x - 1)^2*a/(a*x + 1)^2 + 4*a)/(((a*x - 1)/(a*x + 1))^(5/2) + 2*((a*x - 1)/(
a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Fricas [A] time = 1.94519, size = 194, normalized size = 2.13

$$-\frac{18\left(a^{3}x^{3}-a^{2}x^{2}\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(14\,a^{3}x^{3}+9\,a^{2}x^{2}-6\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*(a^3*x^3 - a^2*x^2)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (14*a^3*x^
3 + 9*a^2*x^2 - 6*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [A] time = 1.18949, size = 146, normalized size = 1.6

$$-\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{4\,a}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\frac{5\,(ax-1)a\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+7\,a\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a/sqrt((a*x - 1)/(a*x + 1)) + (
5*(a*x - 1)*a*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 7*a*sqrt((a*x - 1)/(a*x
 + 1)))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.24 $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right)-\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2-\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{11}{2}a^3\csc^{-1}(ax)$$

```
[Out] -((a + x^(-1))^3/Sqrt[1 - 1/(a^2*x^2)]) - (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a + x
^(-1))^2)/3 - (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a + 3/x))/6 + (11*a^3*ArcCsc[a
*x])/2
```

Rubi [A] time = 0.743133, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6169, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$-\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right)-\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2-\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{11}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] -((a + x^(-1))^3/Sqrt[1 - 1/(a^2*x^2)]) - (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a + x
^(-1))^2)/3 - (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a + 3/x))/6 + (11*a^3*ArcCsc[a
*x])/2
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(-ax^2-x^3\right)}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2\left(3a^2+ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5-\frac{3x}{a}\right)\left(3a^2+ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,d\right.$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{11}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] time = 0.111733, size = 66, normalized size = 0.71

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-52a^3x^3+19a^2x^2+7ax+2\right)}{x^2(ax-1)}+33a^2\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^2*(-1
+ a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Maple [B] time = 0.171, size = 666, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] -1/6*(30*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x^6*a^6+30*ln((a^2*x+(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6-30*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)
*x^4*a^4-93*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5-33*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2))*x^5*a^5+30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-30
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-60*ln((a^2*x
+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+51*(a^2*x^2-1)^(
3/2)*(a^2)^(1/2)*x^3*a^3+96*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+66*a^4*x^
4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(3/2)*x^3*a^3-60*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+60*ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+30*ln((a^2*x+(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-14*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^2*a^2-33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-33*a^3*x^3*(a^2)^(1/2
)*arctan(1/(a^2*x^2-1)^(1/2))+30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^
3-30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-5*(a^2)^
(1/2)*(a^2*x^2-1)^(3/2)*x*a-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/x^3/(a^2)^(1/2
)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.49323, size = 208, normalized size = 2.24

$$-\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{75\,(ax-1)a^2}{ax+1}+\frac{88\,(ax-1)^2a^2}{(ax+1)^2}+\frac{33\,(ax-1)^3a^2}{(ax+1)^3}+12\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (75*(a*x - 1)*a^2/(a*x + 1
) + 88*(a*x - 1)^2*a^2/(a*x + 1)^2 + 33*(a*x - 1)^3*a^2/(a*x + 1)^3 + 12*a^
2)/(((a*x - 1)/(a*x + 1))^(7/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + 3*((a*x -
 1)/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Fricas [A] time = 1.94999, size = 213, normalized size = 2.29

$$-\frac{66\left(a^4x^4-a^3x^3\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(52\,a^4x^4+33\,a^3x^3-26\,a^2x^2-9\,ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*(a^4*x^4 - a^3*x^3)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (52*a^4*x^
4 + 33*a^3*x^3 - 26*a^2*x^2 - 9*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4
- x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21361, size = 203, normalized size = 2.18

$$-\frac{1}{3}\left(33\,a^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{12\,a^{2}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\frac{52\,(ax-1)a^{2}\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{21\,(ax-1)^{2}a^{2}\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^{2}}+39\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^{3}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2/sqrt((a*x - 1)/(a*x
 + 1)) + (52*(a*x - 1)*a^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 21*(a*x -
1)^2*a^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 39*a^2*sqrt((a*x - 1)/(a*x
 + 1)))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

3.25 $\int e^{4\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=57

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rubi [A] time = 0.0658147, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 88}

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^3,x]
```



```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}x^3\,dx &= \int e^{4\tanh^{-1}(ax)}x^3\,dx\\ &= \int \frac{x^3(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3+\frac{4}{a^3(-1+ax)^2}+\frac{16}{a^3(-1+ax)}\right)dx\\ &= \frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.045886, size = 57, normalized size = 1.

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Maple [A] time = 0.047, size = 52, normalized size = 0.9

$$\frac{x^4}{4}+\frac{4\,x^3}{3\,a}+4\,\frac{x^2}{a^2}+12\,\frac{x}{a^3}-4\,\frac{1}{a^4\,(ax-1)}+16\,\frac{\ln(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^3,x)
```

```
[Out] 1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3-4/a^4/(a*x-1)+16/a^4*ln(a*x-1)
```

Maxima [A] time = 0.984836, size = 78, normalized size = 1.37

$$-\frac{4}{a^5x-a^4}+\frac{3\,a^3x^4+16\,a^2x^3+48\,ax^2+144\,x}{12\,a^3}+\frac{16\,\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 + 1
6*log(a*x - 1)/a^4
```

Fricas [A] time = 1.70644, size = 155, normalized size = 2.72

$$\frac{3\,a^5x^5 + 13\,a^4x^4 + 32\,a^3x^3 + 96\,a^2x^2 - 144\,ax + 192\,(ax-1)\log\left(ax-1\right) - 48}{12\left(a^5x - a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*x
 - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

Sympy [A] time = 0.353346, size = 49, normalized size = 0.86

$$\frac{x^4}{4} - \frac{4}{a^5x - a^4} + \frac{4x^3}{3a} + \frac{4x^2}{a^2} + \frac{12x}{a^3} + \frac{16\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**3,x)
```

```
[Out] x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*lo
g(a*x - 1)/a**4
```

Giac [A] time = 1.21016, size = 105, normalized size = 1.84

$$\frac{(ax-1)^4\left(\frac{28}{ax-1} + \frac{114}{(ax-1)^2} + \frac{300}{(ax-1)^3} + 3\right)}{12\,a^4} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^4} - \frac{4}{(ax-1)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(a*x - 1)^4*(28/(a*x - 1) + 114/(a*x - 1)^2 + 300/(a*x - 1)^3 + 3)/a^4
 - 16*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a^4 - 4/((a*x - 1)*a^4)
```

3.26 $\int e^{4\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=47

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rubi [A] time = 0.0595973, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 88}

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \int e^{4\tanh^{-1}(ax)}x^2\,dx$$
$$= \int \frac{x^2(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \int \left(\frac{8}{a^2} + \frac{4x}{a} + x^2 + \frac{4}{a^2(-1+ax)^2} + \frac{12}{a^2(-1+ax)}\right)dx$$
$$= \frac{8x}{a^2} + \frac{2x^2}{a} + \frac{x^3}{3} + \frac{4}{a^3(1-ax)} + \frac{12\log(1-ax)}{a^3}$$

Mathematica [A] time = 0.0361972, size = 47, normalized size = 1.

$$\frac{8x}{a^2} + \frac{4}{a^3(1-ax)} + \frac{12\log(1-ax)}{a^3} + \frac{2x^2}{a} + \frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Maple [A] time = 0.043, size = 44, normalized size = 0.9

$$\frac{x^3}{3} + 2\,\frac{x^2}{a} + 8\,\frac{x}{a^2} - 4\,\frac{1}{a^3\,(ax-1)} + 12\,\frac{\ln(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^2,x)
```

```
[Out] 1/3*x^3+2*x^2/a+8*x/a^2-4/a^3/(a*x-1)+12/a^3*ln(a*x-1)
```

Maxima [A] time = 1.07234, size = 66, normalized size = 1.4

$$-\frac{4}{a^4x - a^3} + \frac{a^2x^3 + 6\,ax^2 + 24\,x}{3\,a^2} + \frac{12\,\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^3
```

Fricas [A] time = 1.71211, size = 130, normalized size = 2.77

$$\frac{a^4x^4 + 5\,a^3x^3 + 18\,a^2x^2 - 24\,ax + 36\,(ax - 1)\log{(ax - 1)} - 12}{3\left(a^4x - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
- 12)/(a^4*x - a^3)
```

Sympy [A] time = 0.320149, size = 39, normalized size = 0.83

$$\frac{x^3}{3} - \frac{4}{a^4x - a^3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\log{(ax - 1)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**2,x)
```

```
[Out] x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

Giac [A] time = 1.1178, size = 93, normalized size = 1.98

$$\frac{(ax - 1)^3\left(\frac{9}{ax-1} + \frac{39}{(ax-1)^2} + 1\right)}{3\,a^3} - \frac{12\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^3} - \frac{4}{(ax - 1)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(a*x - 1)^3*(9/(a*x - 1) + 39/(a*x - 1)^2 + 1)/a^3 - 12*log(abs(a*x - 1
)/((a*x - 1)^2*abs(a)))/a^3 - 4/((a*x - 1)*a^3)
```

3.27 $\int e^{4\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=39

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rubi [A] time = 0.0413822, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6126, 77}

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}x\,dx &= \int e^{4\tanh^{-1}(ax)}x\,dx\\ &= \int \frac{x(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{4}{a}+x+\frac{4}{a(-1+ax)^2}+\frac{8}{a(-1+ax)}\right)dx\\ &= \frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0283569, size = 39, normalized size = 1.

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Maple [A] time = 0.045, size = 36, normalized size = 0.9

$$\frac{x^2}{2}+4\,\frac{x}{a}-4\,\frac{1}{a^2\,(ax-1)}+8\,\frac{\ln(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x,x)
```

```
[Out] 1/2*x^2+4*x/a-4/a^2/(a*x-1)+8/a^2*ln(a*x-1)
```

Maxima [A] time = 1.08211, size = 55, normalized size = 1.41

$$\frac{ax^2+8\,x}{2\,a}-\frac{4}{a^3x-a^2}+\frac{8\,\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

Fricas [A] time = 1.83994, size = 109, normalized size = 2.79

$$\frac{a^3x^3 + 7\,a^2x^2 - 8\,ax + 16\,(ax-1)\log{(ax-1)} - 8}{2\left(a^3x - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
a^2)
```

Sympy [A] time = 0.327189, size = 31, normalized size = 0.79

$$\frac{x^2}{2} - \frac{4}{a^3x - a^2} + \frac{4x}{a} + \frac{8\log{(ax-1)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x,x)
```

```
[Out] x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

Giac [A] time = 1.12868, size = 86, normalized size = 2.21

$$\frac{\frac{(ax-1)^2\left(\frac{10}{ax-1}+1\right)}{a} - \frac{16\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{8}{(ax-1)a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="giac")
```

```
[Out] 1/2*((a*x - 1)^2*(10/(a*x - 1) + 1)/a - 16*log(abs(a*x - 1)/((a*x - 1)^2*ab
s(a)))/a - 8/((a*x - 1)*a))/a
```

3.28 $\int e^{4\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rubi [A] time = 0.0191366, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6125, 43}

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\,dx = \int e^{4\tanh^{-1}(ax)}\,dx$$
$$= \int \frac{(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \int \left(1 + \frac{4}{(-1+ax)^2} + \frac{4}{-1+ax}\right)dx$$
$$= x + \frac{4}{a(1-ax)} + \frac{4\log(1-ax)}{a}$$

Mathematica [A] time = 0.0185358, size = 26, normalized size = 0.96

$$-\frac{4}{a(ax-1)} + \frac{4\log(1-ax)}{a} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

Maple [A] time = 0.046, size = 26, normalized size = 1.

$$x - 4\,\frac{1}{a\,(ax-1)} + 4\,\frac{\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2,x)
```

```
[Out] x-4/a/(a*x-1)+4/a*ln(a*x-1)
```

Maxima [A] time = 0.998486, size = 35, normalized size = 1.3

$$x + \frac{4\,\log(ax-1)}{a} - \frac{4}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="maxima")
```

```
[Out] x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

Fricas [A] time = 1.71151, size = 81, normalized size = 3.

$$\frac{a^2x^2 - ax + 4\,(ax - 1)\log\left(ax - 1\right) - 4}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

Sympy [A] time = 0.319388, size = 19, normalized size = 0.7

$$x - \frac{4}{a^2x - a} + \frac{4\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2,x)
```

```
[Out] x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

Giac [A] time = 1.12444, size = 62, normalized size = 2.3

$$\frac{ax - 1}{a} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4}{(ax - 1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/a - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4/((a*x - 1)*a)
```

3.29 $\int \frac{e^{4 \coth^{-1}(ax)}}{x} dx$

Optimal. Leaf size=13

$$\frac{4}{1-ax}+\log(x)$$

```
[Out] 4/(1 - a*x) + Log[x]
```

Rubi [A] time = 0.0392385, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 88}

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x}\,dx \\
&= \int \frac{(1+ax)^2}{x(1-ax)^2}\,dx \\
&= \int \left(\frac{1}{x} + \frac{4a}{(-1+ax)^2}\right)dx \\
&= \frac{4}{1-ax} + \log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0097751, size = 13, normalized size = 1.

$$\frac{4}{1-ax} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Maple [A] time = 0.046, size = 13, normalized size = 1.

$$-4\,(ax-1)^{-1} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x,x)
```

```
[Out] -4/(a*x-1)+ln(x)
```

Maxima [A] time = 1.01998, size = 16, normalized size = 1.23

$$-\frac{4}{ax-1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="maxima")
```

```
[Out] -4/(a*x - 1) + log(x)
```

Fricas [A] time = 1.78683, size = 46, normalized size = 3.54

$$\frac{(ax-1)\log(x)-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(x) - 4)/(a*x - 1)
```

Sympy [A] time = 0.355664, size = 8, normalized size = 0.62

$$\log(x)-\frac{4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x,x)
```

```
[Out] log(x) - 4/(a*x - 1)
```

Giac [B] time = 1.12011, size = 77, normalized size = 5.92

$$-a\left(\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{4}{(ax-1)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="giac")
```

```
[Out] -a*(log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - log(abs(-1/(a*x - 1) - 1))/a
 + 4/((a*x - 1)*a))
```

3.30 $\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=32

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rubi [A] time = 0.0505939, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 88}

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x^2}\,dx\\
&= \int \frac{(1+ax)^2}{x^2(1-ax)^2}\,dx\\
&= \int \left(\frac{1}{x^2}+\frac{4a}{x}+\frac{4a^2}{(-1+ax)^2}-\frac{4a^2}{-1+ax}\right)dx\\
&= -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)
\end{aligned}
$$

Mathematica [A] time = 0.0235628, size = 32, normalized size = 1.

$$
\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Maple [A] time = 0.047, size = 31, normalized size = 1.

$$
-x^{-1}+4\,a\ln(x)-4\,\frac{a}{ax-1}-4\,a\ln(ax-1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^2,x)
```

```
[Out] -1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

Maxima [A] time = 0.969968, size = 46, normalized size = 1.44

$$
-4\,a\log(ax-1)+4\,a\log(x)-\frac{5\,ax-1}{ax^2-x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

Fricas [A] time = 1.78587, size = 116, normalized size = 3.62

$$-\frac{5\,ax+4\left(a^2x^2-ax\right)\log\left(ax-1\right)-4\left(a^2x^2-ax\right)\log\left(x\right)-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(a
*x^2 - x)
```

Sympy [A] time = 0.412016, size = 26, normalized size = 0.81

$$4a\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)-\frac{5ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**2,x)
```

```
[Out] 4*a*(log(x) - log(x - 1/a)) - (5*a*x - 1)/(a*x**2 - x)
```

Giac [A] time = 1.13818, size = 54, normalized size = 1.69

$$4\,a\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a}{ax-1}+\frac{a}{\frac{1}{ax-1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="giac")
```

```
[Out] 4*a*log(abs(-1/(a*x - 1) - 1)) - 4*a/(a*x - 1) + a/(1/(a*x - 1) + 1)
```

3.31 $\int \frac{e^{4\coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=46

$$\frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax) - \frac{4a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rubi [A] time = 0.054295, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 88}

$$\frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax) - \frac{4a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx\\
&= \int \frac{(1+ax)^2}{x^3(1-ax)^2}\,dx\\
&= \int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{(-1+ax)^2}-\frac{8a^3}{-1+ax}\right)dx\\
&= -\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)
\end{aligned}
$$

Mathematica [A] time = 0.0300503, size = 46, normalized size = 1.

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Maple [A] time = 0.053, size = 43, normalized size = 0.9

$$-\frac{1}{2\,x^2}-4\,\frac{a}{x}+8\,a^2\ln(x)-4\,\frac{a^2}{ax-1}-8\,a^2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^3,x)
```

```
[Out] -1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

Maxima [A] time = 1.01859, size = 65, normalized size = 1.41

$$-8\,a^2\log(ax-1)+8\,a^2\log(x)-\frac{16\,a^2x^2-7\,ax-1}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
x^2)
```

Fricas [A] time = 1.81316, size = 155, normalized size = 3.37

$$-\frac{16\,a^{2}x^{2}-7\,ax+16\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(ax-1\right)-16\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(x\right)-1}{2\left(ax^{3}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x^
3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

Sympy [A] time = 0.459218, size = 41, normalized size = 0.89

$$8a^{2}\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)-\frac{16a^{2}x^{2}-7ax-1}{2ax^{3}-2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**3,x)
```

```
[Out] 8*a**2*(log(x) - log(x - 1/a)) - (16*a**2*x**2 - 7*a*x - 1)/(2*a*x**3 - 2*x
**2)
```

Giac [A] time = 1.15712, size = 84, normalized size = 1.83

$$8\,a^{2}\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^{2}}{ax-1}+\frac{9\,a^{2}+\frac{10\,a^{2}}{ax-1}}{2\left(\frac{1}{ax-1}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="giac")
```

```
[Out] 8*a^2*log(abs(-1/(a*x - 1) - 1)) - 4*a^2/(a*x - 1) + 1/2*(9*a^2 + 10*a^2/(a
*x - 1))/(1/(a*x - 1) + 1)^2
```

3.32 $\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=54

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rubi [A] time = 0.0612882, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 88}

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx\\ &= \int \frac{(1+ax)^2}{x^4(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{4a}{x^3}+\frac{8a^2}{x^2}+\frac{12a^3}{x}+\frac{4a^4}{(-1+ax)^2}-\frac{12a^4}{-1+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.0424887, size = 54, normalized size = 1.

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Maple [A] time = 0.049, size = 51, normalized size = 0.9

$$-\frac{1}{3\,x^3}-2\,\frac{a}{x^2}-8\,\frac{a^2}{x}+12\,a^3\ln(x)-4\,\frac{a^3}{ax-1}-12\,a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

Maxima [A] time = 1.0022, size = 76, normalized size = 1.41

$$-12\,a^3\log(ax-1)+12\,a^3\log(x)-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax-1}{3\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] -12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x
 - 1)/(a*x^4 - x^3)
```

Fricas [A] time = 1.89311, size = 173, normalized size = 3.2

$$-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax+36\left(a^4x^4-a^3x^3\right)\log\left(ax-1\right)-36\left(a^4x^4-a^3x^3\right)\log\left(x\right)-1}{3\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
 - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

Sympy [A] time = 0.462664, size = 49, normalized size = 0.91

$$12a^3\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)-\frac{36a^3x^3-18a^2x^2-5ax-1}{3ax^4-3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**4,x)
```

```
[Out] 12*a**3*(log(x) - log(x - 1/a)) - (36*a**3*x**3 - 18*a**2*x**2 - 5*a*x - 1)
/(3*a*x**4 - 3*x**3)
```

Giac [A] time = 1.1818, size = 100, normalized size = 1.85

$$12\,a^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^3}{ax-1}+\frac{31\,a^3+\frac{69\,a^3}{ax-1}+\frac{39\,a^3}{(ax-1)^2}}{3\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*a^3*log(abs(-1/(a*x - 1) - 1)) - 4*a^3/(a*x - 1) + 1/3*(31*a^3 + 69*a^3/
(a*x - 1) + 39*a^3/(a*x - 1)^2)/(1/(a*x - 1) + 1)^3
```

3.33 $\int e^{-\coth^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=114

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] (-2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2
) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rubi [A] time = 0.127282, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2
) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}-\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^4}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{16a^4}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\right.}{8a^2}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Mathematica [A] time = 0.0665498, size = 68, normalized size = 0.6

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-8a^2x^2+9ax-16\right)+9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Maple [B] time = 0.125, size = 193, normalized size = 1.7

$$\frac{ax+1}{24\,a^{4}}\sqrt{\frac{ax-1}{ax+1}}\left(6\,\sqrt{a^{2}}\left(a^{2}x^{2}-1\right)^{3/2}xa+15\,\sqrt{a^{2}}\sqrt{a^{2}x^{2}-1}xa-8\,\left(\left(ax-1\right)\left(ax+1\right)\right)^{3/2}\sqrt{a^{2}}-15\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}}{\sqrt{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+1
5*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-8*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-1
5*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+24*a*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-24*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^4/(a^2)^(1/2)
```

Maxima [B] time = 1.05245, size = 274, normalized size = 2.4

$$-\frac{1}{24}\,a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-31\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^{5}}{ax+1}-\frac{6(ax-1)^{2}a^{5}}{(ax+1)^{2}}+\frac{4(ax-1)^{3}a^{5}}{(ax+1)^{3}}-\frac{(ax-1)^{4}a^{5}}{(ax+1)^{4}}-a^{5}}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{5}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*a*(2*(39*((a*x - 1)/(a*x + 1))^(7/2) - 31*((a*x - 1)/(a*x + 1))^(5/2)
 + 49*((a*x - 1)/(a*x + 1))^(3/2) - 9*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x -
1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^5 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Fricas [A] time = 1.96291, size = 220, normalized size = 1.93

$$\frac{\left(6\,a^{4}x^{4}-2\,a^{3}x^{3}+a^{2}x^{2}-7\,ax-16\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 7*a*x - 16)*sqrt((a*x - 1)/(a*x +
1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.16011, size = 136, normalized size = 1.19

$$\frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\,x\left(\frac{3\,x\mathrm{sgn}\,(ax+1)}{a}-\frac{4\,\mathrm{sgn}\,(ax+1)}{a^2}\right)+\frac{9\,\mathrm{sgn}\,(ax+1)}{a^3}\right)x-\frac{16\,\mathrm{sgn}\,(ax+1)}{a^4}\right)-\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-\right.\right.}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*x^2 - 1)*((2*x*(3*x*sgn(a*x + 1)/a - 4*sgn(a*x + 1)/a^2) + 9*
sgn(a*x + 1)/a^3)*x - 16*sgn(a*x + 1)/a^4) - 3/8*log(abs(-x*abs(a) + sqrt(a
^2*x^2 - 1)))*sgn(a*x + 1)/(a^3*abs(a))
```

3.34 $\int e^{-\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=90

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rubi [A] time = 0.100033, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcCoth[a*x],x]
```


```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = -\mathrm{Subst}\left(\int \frac{1-\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{3}\mathrm{Subst}\left(\int \frac{\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{6}\mathrm{Subst}\left(\int \frac{\frac{4}{a^2}-\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\mathrm{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] time = 0.0486425, size = 60, normalized size = 0.67

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-3ax+4\right)-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 - 3*a*x + 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(6*a^3)
```

Maple [B] time = 0.128, size = 173, normalized size = 1.9

$$-\frac{ax+1}{6\,a^{3}}\sqrt{\frac{ax-1}{ax+1}}\left(3\,\sqrt{a^{2}}\sqrt{a^{2}x^{2}-1}xa-2\,\left(\left(ax-1\right)\left(ax+1\right)\right)^{3/2}\sqrt{a^{2}}-3\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a-6\,\sqrt{a^{2}}\sqrt{(ax-1)\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-2
*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))*a-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+6*a*ln((a^2*x+(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2)/a^3
/(a^2)^(1/2)
```

Maxima [B] time = 1.01539, size = 224, normalized size = 2.49

$$-\frac{1}{6}\,a\left(\frac{2\left(9\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^{4}}{ax+1}-\frac{3\,(ax-1)^{2}a^{4}}{(ax+1)^{2}}+\frac{(ax-1)^{3}a^{4}}{(ax+1)^{3}}-a^{4}}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{4}}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(9*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
3*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4
/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^4 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

Fricas [A] time = 1.92685, size = 198, normalized size = 2.2

$$\frac{\left(2\,a^{3}x^{3}-a^{2}x^{2}+ax+4\right)\sqrt{\frac{ax-1}{ax+1}}-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - a^2*x^2 + a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt
((a*x - 1)/(a*x + 1)) + 1) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.19654, size = 116, normalized size = 1.29

$$\frac{1}{6}\sqrt{a^2x^2-1}\left(x\left(\frac{2\,x\mathrm{sgn}\,(ax+1)}{a}-\frac{3\,\mathrm{sgn}\,(ax+1)}{a^2}\right)+\frac{4\,\mathrm{sgn}\,(ax+1)}{a^3}\right)+\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{2\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*(x*(2*x*sgn(a*x + 1)/a - 3*sgn(a*x + 1)/a^2) + 4*sgn(
a*x + 1)/a^3) + 1/2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a
^2*abs(a))
```

3.35 $\int e^{-\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=64

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/a) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rubi [A] time = 0.0677775, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcCoth[a*x],x]
```



```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/a) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]]/(2*a^2)
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}x\,dx &= -\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{2}\operatorname{Subst}\left(\int \frac{\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0354753, size = 49, normalized size = 0.77

$$
\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(ax-2)+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])
/(2*a^2)
```

Maple [B] time = 0.128, size = 152, normalized size = 2.4

$$
\frac{ax+1}{2\,a^2}\sqrt{\frac{ax-1}{ax+1}}\left(\sqrt{a^2}\sqrt{a^2x^2-1}xa-\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)a-2\sqrt{a^2}\sqrt{(ax-1)(ax+1)}+2\,a\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{}}{}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-ln((
a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-2*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/
2)))/((a*x-1)*(a*x+1))^(1/2)/a^2/(a^2)^(1/2)
```

Maxima [B] time = 1.00163, size = 176, normalized size = 2.75

$$-\frac{1}{2}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*((a*x - 1)/(a*x + 1))^(3/2) - sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^3 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Fricas [A] time = 1.83951, size = 177, normalized size = 2.77

$$\frac{\left(a^2x^2-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.21142, size = 96, normalized size = 1.5

$$\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x\mathrm{sgn}\,(ax+1)}{a}-\frac{2\,\mathrm{sgn}\,(ax+1)}{a^2}\right)-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*(x*sgn(a*x + 1)/a - 2*sgn(a*x + 1)/a^2) - 1/2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a*abs(a))
```

3.36 $\int e^{-\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=37

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rubi [A] time = 0.0385505, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6168, 807, 266, 63, 208}

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcCoth[a*x]),x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\,dx &= -\,\mathrm{Subst}\left(\int \frac{1-\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x - a\,\mathrm{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0243888, size = 42, normalized size = 1.14

$$
x\sqrt{1-\frac{1}{a^2x^2}} - \frac{\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcCoth[a*x]),x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Maple [B] time = 0.118, size = 98, normalized size = 2.7

$$\frac{ax+1}{a}\sqrt{\frac{ax-1}{ax+1}}\left(\sqrt{a^2}\sqrt{(ax-1)(ax+1)}-a\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)\right)\frac{1}{\sqrt{a^2}}\frac{1}{\sqrt{(ax-1)(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)-a*ln((
a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^
(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.04831, size = 122, normalized size = 3.3

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.85872, size = 155, normalized size = 4.19

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.15539, size = 70, normalized size = 1.89

$$\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2
- 1)*sgn(a*x + 1)/a
```

3.37 $\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

```
[Out] ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rubi [A] time = 0.0475338, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6169, 844, 216, 266, 63, 208}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x),x]
```

```
[Out] ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}-\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \csc^{-1}(ax)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)\\
&= \csc^{-1}(ax)+a^2\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \csc^{-1}(ax)+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0145184, size = 34, normalized size = 1.7

$$\log\left(x\left(\sqrt{\frac{a^2x^2-1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x),x]
```

```
[Out] ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Maple [B] time = 0.125, size = 130, normalized size = 6.5

$$(ax+1)\sqrt{\frac{ax-1}{ax+1}}\left(\sqrt{a^2x^2-1}\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}+a\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)-\sqrt{a^2}\sqrt{(ax-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^
2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)
)/(a^2)^(1/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1))^(1/2)
/(a^2)^(1/2)
```

Maxima [B] time = 1.59906, size = 95, normalized size = 4.75

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Fricas [B] time = 1.84833, size = 151, normalized size = 7.55

$$-2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [B] time = 1.16105, size = 80, normalized size = 4.

$$-2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)-\frac{a\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - a*log(abs(-x*abs(a)
 + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a)
```

3.38 $\int \frac{e^{-\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=25

$$a\left(-\csc^{-1}(ax)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcCsc[a*x]
```

Rubi [A] time = 0.0267834, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6169, 641, 216}

$$a\left(-\csc^{-1}(ax)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcCsc[a*x]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -a\sqrt{1-\frac{1}{a^2x^2}} - \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

Mathematica [A] time = 0.0201107, size = 26, normalized size = 1.04

$$-a\left(\sqrt{1-\frac{1}{a^2x^2}} + \sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -(a*(Sqrt[1 - 1/(a^2*x^2)] + ArcSin[1/(a*x)]))
```

Maple [B] time = 0.128, size = 220, normalized size = 8.8

$$\frac{ax+1}{x}\sqrt{\frac{ax-1}{ax+1}}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 + \left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2} - \sqrt{a^2}\sqrt{a^2x^2-1}xa + \ln\left(\left(a^2x + \sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)xa^2 - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^
2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-a*x*(a^2)^(1/2)*arctan(1/(a^2
*x^2-1)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-ln((a^2*x+(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-1)*(a*x+1))^(1/2)/x/(a
^2)^(1/2)
```

Maxima [B] time = 1.5047, size = 74, normalized size = 2.96

$$-2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}-\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) - arctan(sqrt((a*
x - 1)/(a*x + 1))))
```

Fricas [B] time = 1.86385, size = 112, normalized size = 4.48

$$\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**2, x)
```

Giac [B] time = 1.1525, size = 84, normalized size = 3.36

$$2\,a\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) - \frac{2\,|a|\operatorname{sgn}(ax+1)}{\left(x|a| - \sqrt{a^2x^2-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - 2*abs(a)*sgn(a*x +
 1)/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)
```

3.39 $\int \frac{e^{-\coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=40

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a - x^(-1)))/2 + (a^2*ArcCsc[a*x])/2
```

Rubi [A] time = 0.036613, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6169, 780, 216}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^3),x]
```



```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a - x^(-1)))/2 + (a^2*ArcCsc[a*x])/2
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] time = 0.0464496, size = 41, normalized size = 1.02

$$\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(2ax-1)+ax\sin^{-1}\left(\frac{1}{ax}\right)\right)}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + 2*a*x) + a*x*ArcSin[1/(a*x)]))/(2*x)
```

Maple [B] time = 0.138, size = 260, normalized size = 6.5

$$-\frac{ax+1}{2x^2}\sqrt{\frac{ax-1}{ax+1}}\left(-2\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+2\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+2\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*
a^3+2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a
^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-a^2*x^2*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2)*x^2*a^2-2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^
2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/x^2/(a^2)^(1/2
```

)

Maxima [B] time = 1.57412, size = 126, normalized size = 3.15

$$-\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(a*arctan(sqrt((a*x - 1)/(a*x + 1))) - (3*a*((a*x - 1)/(a*x + 1))^(3/2) +
a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)
^2 + 1))*a
```

Fricas [A] time = 1.85432, size = 143, normalized size = 3.58

$$-\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(2\,a^2x^2+ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^2*x^2 + a*x - 1)*s
qrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**3, x)
```

<u>Giac</u> [B] time = 1.1941, size = 212, normalized size = 5.3

$$-a^2 \arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) + \frac{\left(x|a| - \sqrt{a^2x^2-1}\right)^3 a^2 \operatorname{sgn}(ax+1) + 2\left(x|a| - \sqrt{a^2x^2-1}\right)^2 a|a|\operatorname{sgn}(ax+1}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + ((x*abs(a) - sqrt
(a^2*x^2 - 1))^3*a^2*sgn(a*x + 1) + 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*ab
s(a)*sgn(a*x + 1) - (x*abs(a) - sqrt(a^2*x^2 - 1))*a^2*sgn(a*x + 1) + 2*a*a
bs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2
```

3.40 $\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=76

$$\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

```
[Out] -(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[
1 - 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rubi [A] time = 0.0656624, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6169, 797, 641, 195, 216}

$$\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[
1 - 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx &= -\mathrm{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\ &= -\left(a^2\,\mathrm{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)+a^2\,\mathrm{Subst}\left(\int \left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\ &= -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)+a^2\,\mathrm{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,dx, x\right.\\ &= -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-a^3\csc^{-1}(ax)+\frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x,\right.\\ &= -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.0786569, size = 52, normalized size = 0.68

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(4a^2x^2-3ax+2\right)}{6x^2}-\frac{1}{2}a^3\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]*(2 - 3*a*x + 4*a^2*x^2))/(6*x^2) - (a^3*ArcSin[1/
(a*x)])/2
```

Maple [B] time = 0.13, size = 284, normalized size = 3.7

$$\frac{ax+1}{6\,x^3}\sqrt{\frac{ax-1}{ax+1}}\left(-6\,\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+6\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-3\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+6\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a
^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x^3*a^3+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-3*a
^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2)*x^3*a^3-6*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1
/2))*x^3*a^4-3*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2))/((a*x-1)*(a*x+1))^(1/2)/x^3/(a^2)^(1/2)
```

Maxima [B] time = 1.57039, size = 185, normalized size = 2.43

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (9*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

Fricas [A] time = 1.85405, size = 155, normalized size = 2.04

$$\frac{6\,a^{3}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(4\,a^{3}x^{3}+a^{2}x^{2}-ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (4*a^3*x^3 + a^2*x^2 - a
*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**4, x)
```

Giac [B] time = 1.16876, size = 219, normalized size = 2.88

$$a^{3}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\operatorname{sgn}\left(ax+1\right)-\frac{3\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{5}a^{3}\operatorname{sgn}\left(ax+1\right)+12\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}a^{2}|a|\operatorname{sgn}\left(ax\right.}{3\left(\left(x|a|-\sqrt{a^{2}x^{2}-1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] a^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - 1/3*(3*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^5*a^3*sgn(a*x + 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^
2*a^2*abs(a)*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*a^3*sgn(a*x +
1) + 4*a^2*abs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3
```

3.41 $\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=88

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{3}{8}a^4\csc^{-1}(ax)$$

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a - 9/x))/24 - (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + (3*a^4*ArcCsc[a*x])/8
```

Rubi [A] time = 0.0895651, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6169, 833, 780, 216}

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{3}{8}a^4\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a - 9/x))/24 - (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + (3*a^4*ArcCsc[a*x])/8
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx &= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{4}a^2\,\text{Subst}\left(\int \frac{x^2\left(\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{12}a^4\,\text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}-\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3}{8}a^4\csc^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.0939583, size = 59, normalized size = 0.67

$$\frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(16a^3x^3-9a^2x^2+8ax-6\right)}{x^3}+9a^3\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^3 + 9*a
^3*ArcSin[1/(a*x)]))/24
```

Maple [B] time = 0.138, size = 308, normalized size = 3.5

$$-\frac{ax+1}{24\,x^4}\sqrt{\frac{ax-1}{ax+1}}\left(-24\,\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5+24\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3-9\,\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4-9\,a^4x^4\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^5,x)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^
5*a^5+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-9*(a^2*x^2-1)^(1/2)*(a^2)^(1
/2)*x^4*a^4-9*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+24*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x^4*a^4-24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(
a^2)^(1/2))*x^4*a^5-15*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+8*(a^2)^(1/2)*
(a^2*x^2-1)^(3/2)*x*a-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1
/2)/x^4/(a^2)^(1/2)
```

Maxima [B] time = 1.51904, size = 234, normalized size = 2.66

$$-\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{39\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (39*a^3*((a*x - 1)/(a*x +
1))^(7/2) + 31*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 49*a^3*((a*x - 1)/(a*x + 1
))^(3/2) + 9*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
 - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
 1))*a
```

Fricas [A] time = 1.93733, size = 180, normalized size = 2.05

$$-\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(16\,a^4x^4+7\,a^3x^3-a^2x^2+2\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (16*a^4*x^4 + 7*a^3*x
^3 - a^2*x^2 + 2*a*x - 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**5, x)
```

Giac [B] time = 1.18818, size = 348, normalized size = 3.95

$$-\frac{3}{4}\,a^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}\left(ax+1\right)+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^7a^4\operatorname{sgn}\left(ax+1\right)+33\left(x|a|-\sqrt{a^2x^2-1}\right)^5a^4\operatorname{sgn}\left(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + 1/12*(9*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^7*a^4*sgn(a*x + 1) + 33*(x*abs(a) - sqrt(a^2*x^2
- 1))^5*a^4*sgn(a*x + 1) + 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a)*s
gn(a*x + 1) - 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*a^4*sgn(a*x + 1) + 64*(x*
```

```
abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a)*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(
a^2*x^2 - 1))*a^4*sgn(a*x + 1) + 16*a^3*abs(a)*sgn(a*x + 1))/((x*abs(a) - s
qrt(a^2*x^2 - 1))^2 + 1)^4
```

3.42 $\int e^{-2\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=42

$$\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\log(ax+1)}{a^4}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

```
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Rubi [A] time = 0.0570863, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\log(ax+1)}{a^4}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^3\,dx\\ &= -\int \frac{x^3(1-ax)}{1+ax}\,dx\\ &= -\int \left(\frac{2}{a^3}-\frac{2x}{a^2}+\frac{2x^2}{a}-x^3-\frac{2}{a^3(1+ax)}\right)dx\\ &= -\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0193023, size = 42, normalized size = 1.

$$\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\log(ax+1)}{a^4}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Maple [A] time = 0.039, size = 39, normalized size = 0.9

$$-2\,\frac{x}{a^3}+\frac{x^2}{a^2}-\frac{2\,x^3}{3\,a}+\frac{x^4}{4}+2\,\frac{\ln(ax+1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

Maxima [A] time = 0.985613, size = 58, normalized size = 1.38

$$\frac{3\,a^3x^4-8\,a^2x^3+12\,ax^2-24\,x}{12\,a^3}+\frac{2\,\log(ax+1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 + 2*log(a*x + 1)/a^4
```

Fricas [A] time = 1.7463, size = 100, normalized size = 2.38

$$\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax + 24\,\log\left(ax + 1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

Sympy [A] time = 0.276682, size = 37, normalized size = 0.88

$$\frac{x^4}{4} - \frac{2x^3}{3a} + \frac{x^2}{a^2} - \frac{2x}{a^3} + \frac{2\log\left(ax + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x-1)/(a*x+1),x)
```

```
[Out] x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 + 2*log(a*x + 1)/a**4
```

Giac [A] time = 1.13709, size = 63, normalized size = 1.5

$$\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax + 1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x)/a^4 + 2*log(abs(a*x + 1)
)/a^4
```

3.43 $\int e^{-2\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=33

$$\frac{2x}{a^2}-\frac{2\log(ax+1)}{a^3}-\frac{x^2}{a}+\frac{x^3}{3}$$

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Rubi [A] time = 0.050608, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$\frac{2x}{a^2}-\frac{2\log(ax+1)}{a^3}-\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^2\,dx\\
&= -\int \frac{x^2(1-ax)}{1+ax}\,dx\\
&= -\int \left(-\frac{2}{a^2}+\frac{2x}{a}-x^2+\frac{2}{a^2(1+ax)}\right)dx\\
&= \frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0137995, size = 33, normalized size = 1.

$$\frac{2x}{a^2}-\frac{2\log(ax+1)}{a^3}-\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Maple [A] time = 0.039, size = 32, normalized size = 1.

$$2\frac{x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-2\frac{\ln(ax+1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)*(a*x-1),x)
```

```
[Out] 2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Maxima [A] time = 1.02015, size = 46, normalized size = 1.39

$$\frac{a^2x^3-3\,ax^2+6\,x}{3\,a^2}-\frac{2\,\log(ax+1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 - 2*log(a*x + 1)/a^3
```

Fricas [A] time = 1.58506, size = 76, normalized size = 2.3

$$\frac{a^3x^3 - 3\,a^2x^2 + 6\,ax - 6\,\log\left(ax + 1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

Sympy [A] time = 0.270656, size = 27, normalized size = 0.82

$$\frac{x^3}{3} - \frac{x^2}{a} + \frac{2x}{a^2} - \frac{2\log\left(ax + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x-1)/(a*x+1),x)
```

```
[Out] x**3/3 - x**2/a + 2*x/a**2 - 2*log(a*x + 1)/a**3
```

Giac [A] time = 1.14682, size = 51, normalized size = 1.55

$$\frac{a^3x^3 - 3\,a^2x^2 + 6\,ax}{3\,a^3} - \frac{2\,\log\left(|ax + 1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x + 1))/a^3
```

3.44 $\int e^{-2\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=25

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Rubi [A] time = 0.0323883, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6126, 77}

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x\,dx &= -\int e^{-2\tanh^{-1}(ax)}x\,dx\\ &= -\int \frac{x(1-ax)}{1+ax}\,dx\\ &= -\int \left(\frac{2}{a}-x-\frac{2}{a(1+ax)}\right)dx\\ &= -\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.012387, size = 25, normalized size = 1.

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Maple [A] time = 0.039, size = 24, normalized size = 1.

$$-2\,\frac{x}{a}+\frac{x^2}{2}+2\,\frac{\ln(ax+1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)*(a*x-1),x)
```

```
[Out] -2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Maxima [A] time = 1.05236, size = 35, normalized size = 1.4

$$\frac{ax^2-4\,x}{2\,a}+\frac{2\,\log(ax+1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 - 4*x)/a + 2*log(a*x + 1)/a^2
```

Fricas [A] time = 1.84129, size = 59, normalized size = 2.36

$$\frac{a^2x^2 - 4\,ax + 4\,\log\left(ax + 1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

Sympy [A] time = 0.260727, size = 20, normalized size = 0.8

$$\frac{x^2}{2} - \frac{2x}{a} + \frac{2\log\left(ax + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x)
```

```
[Out] x**2/2 - 2*x/a + 2*log(a*x + 1)/a**2
```

Giac [A] time = 1.11769, size = 41, normalized size = 1.64

$$\frac{a^2x^2 - 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax + 1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*x^2 - 4*a*x)/a^2 + 2*log(abs(a*x + 1))/a^2
```

3.45 $\int e^{-2\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$x-\frac{2\log(ax+1)}{a}$$

```
[Out] x - (2*Log[1 + a*x])/a
```

Rubi [A] time = 0.013253, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6125, 43}

$$x-\frac{2\log(ax+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-2*ArcCoth[a*x]),x]
```

```
[Out] x - (2*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\,dx\\ &= -\int \frac{1-ax}{1+ax}\,dx\\ &= -\int \left(-1+\frac{2}{1+ax}\right)dx\\ &= x-\frac{2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.010677, size = 13, normalized size = 1.

$$x-\frac{2\log(ax+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-2*ArcCoth[a*x]),x]
```

```
[Out] x - (2*Log[1 + a*x])/a
```

Maple [A] time = 0.039, size = 14, normalized size = 1.1

$$x-2\,{\frac {\ln \left(ax+1 \right) }{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1),x)
```

```
[Out] x-2*ln(a*x+1)/a
```

Maxima [A] time = 1.01314, size = 18, normalized size = 1.38

$$x-\frac{2\,\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] x - 2*log(a*x + 1)/a
```

Fricas [A] time = 1.73219, size = 35, normalized size = 2.69

$$\frac{ax - 2\log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] (a*x - 2*log(a*x + 1))/a
```

Sympy [A] time = 0.09364, size = 10, normalized size = 0.77

$$x - \frac{2\log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x)
```

```
[Out] x - 2*log(a*x + 1)/a
```

Giac [A] time = 1.1491, size = 19, normalized size = 1.46

$$x - \frac{2\log(|ax + 1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] x - 2*log(abs(a*x + 1))/a
```

3.46 $\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$2\log(ax+1)-\log(x)$$

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

Rubi [A] time = 0.0401803, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 72}

$$2\log(ax+1)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx\\ &= -\int \frac{1-ax}{x(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x}-\frac{2a}{1+ax}\right)dx\\ &= -\log(x)+2\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0071152, size = 13, normalized size = 1.

$$2\log(ax+1)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

Maple [A] time = 0.043, size = 14, normalized size = 1.1

$$-\ln(x)+2\,\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -ln(x)+2*ln(a*x+1)
```

Maxima [A] time = 1.01063, size = 18, normalized size = 1.38

$$2\,\log(ax+1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] 2*log(a*x + 1) - log(x)
```

Fricas [A] time = 1.75448, size = 34, normalized size = 2.62

$$2 \log(ax+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] 2*log(a*x + 1) - log(x)
```

Sympy [A] time = 0.135858, size = 10, normalized size = 0.77

$$-\log(x) + 2\log\left(x + \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x)
```

```
[Out] -log(x) + 2*log(x + 1/a)
```

Giac [A] time = 1.1399, size = 20, normalized size = 1.54

$$2 \log(|ax+1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] 2*log(abs(a*x + 1)) - log(abs(x))
```

3.47 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=18

$$2a\log(x)-2a\log(ax+1)+\frac{1}{x}$$

```
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Rubi [A] time = 0.0432672, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$2a\log(x)-2a\log(ax+1)+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2}\,dx\\ &= -\int \frac{1-ax}{x^2(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x^2}-\frac{2a}{x}+\frac{2a^2}{1+ax}\right)dx\\ &= \frac{1}{x}+2a\log(x)-2a\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0092316, size = 18, normalized size = 1.

$$2a\log(x)-2a\log(ax+1)+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Maple [A] time = 0.045, size = 19, normalized size = 1.1

$$x^{-1}+2\,a\ln(x)-2\,a\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

Maxima [A] time = 1.00079, size = 24, normalized size = 1.33

$$-2\,a\log(ax+1)+2\,a\log(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*log(a*x + 1) + 2*a*log(x) + 1/x
```

Fricas [A] time = 1.52237, size = 59, normalized size = 3.28

$$-\frac{2\,ax\log\left(ax+1\right)-2\,ax\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

Sympy [A] time = 0.318615, size = 15, normalized size = 0.83

$$2a\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] 2*a*(log(x) - log(x + 1/a)) + 1/x
```

Giac [A] time = 1.14546, size = 27, normalized size = 1.5

$$-2\,a\log\left(|ax+1|\right)+2\,a\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*log(abs(a*x + 1)) + 2*a*log(abs(x)) + 1/x
```

3.48 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=32

$$-2a^2\log(x) + 2a^2\log(ax+1) - \frac{2a}{x} + \frac{1}{2x^2}$$

```
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Rubi [A] time = 0.0457361, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$-2a^2\log(x) + 2a^2\log(ax+1) - \frac{2a}{x} + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx\\ &= -\int \frac{1-ax}{x^3(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x^3}-\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{1+ax}\right)dx\\ &= \frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0117426, size = 32, normalized size = 1.

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Maple [A] time = 0.043, size = 31, normalized size = 1.

$$\frac{1}{2\,x^2}-2\,\frac{a}{x}-2\,a^2\ln(x)+2\,a^2\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] 1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Maxima [A] time = 1.02089, size = 41, normalized size = 1.28

$$2\,a^2\log(ax+1)-2\,a^2\log(x)-\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 2*a^2*log(a*x + 1) - 2*a^2*log(x) - 1/2*(4*a*x - 1)/x^2
```

Fricas [A] time = 1.53662, size = 88, normalized size = 2.75

$$\frac{4\,a^2x^2\log\left(ax+1\right)-4\,a^2x^2\log\left(x\right)-4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

Sympy [A] time = 0.33974, size = 26, normalized size = 0.81

$$2a^2\left(-\log\left(x\right)+\log\left(x+\frac{1}{a}\right)\right)-\frac{4ax-1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] 2*a**2*(-log(x) + log(x + 1/a)) - (4*a*x - 1)/(2*x**2)
```

Giac [A] time = 1.14377, size = 43, normalized size = 1.34

$$2\,a^2\log\left(|ax+1|\right)-2\,a^2\log\left(|x|\right)-\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(abs(a*x + 1)) - 2*a^2*log(abs(x)) - 1/2*(4*a*x - 1)/x^2
```

3.49 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=40

$$\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(ax+1)-\frac{a}{x^2}+\frac{1}{3x^3}$$

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Rubi [A] time = 0.0496812, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6126, 77}

$$\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(ax+1)-\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx\\ &= -\int \frac{1-ax}{x^4(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^2}{x^2}-\frac{2a^3}{x}+\frac{2a^4}{1+ax}\right)dx\\ &= \frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)\end{aligned}$$

Mathematica [A] time = 0.0132844, size = 40, normalized size = 1.

$$\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(ax+1)-\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Maple [A] time = 0.046, size = 39, normalized size = 1.

$$\frac{1}{3\,x^3}-\frac{a}{x^2}+2\,\frac{a^2}{x}+2\,a^3\ln(x)-2\,a^3\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Maxima [A] time = 0.985085, size = 51, normalized size = 1.27

$$-2\,a^3\log(ax+1)+2\,a^3\log(x)+\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*a^3*log(a*x + 1) + 2*a^3*log(x) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

Fricas [A] time = 1.47225, size = 105, normalized size = 2.62

$$-\frac{6\,a^3x^3\log\left(ax+1\right)-6\,a^3x^3\log\left(x\right)-6\,a^2x^2+3\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x^
3
```

Sympy [A] time = 0.360706, size = 34, normalized size = 0.85

$$2a^3\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{6a^2x^2-3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] 2*a**3*(log(x) - log(x + 1/a)) + (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

Giac [A] time = 1.15059, size = 54, normalized size = 1.35

$$-2\,a^3\log\left(|ax+1|\right)+2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(a*x + 1)) + 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/
x^3
```

3.50 $\int e^{-3\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=136

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}+\frac{51\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x)
/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)
/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8
*a^4)
```

Rubi [A] time = 1.01928, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6169, 6742, 266, 51, 63, 208, 271, 264, 651}

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}+\frac{51\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x)
/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)
/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8
*a^4)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^5\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^4\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^4x\sqrt{1-\frac{x^2}{a^2}}} - \frac{}{a^4(a+}\right.\right.$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\,\text{Subst}\left(\int \frac{}{x}\right.}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-}}\right.}{a^4}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{4\tanh^{-1}\left(\sqrt{1\right.}{a^4}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{6\tanh^{-1}\left(\sqrt{1\right.}{a^4}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{51\tanh^{-1}\left(\sqrt{\right.}{8a^4}$$

Mathematica [A] time = 0.0882918, size = 83, normalized size = 0.61

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^4x^4-6a^3x^3+11a^2x^2-29ax-80\right)}{ax+1}+51\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{8a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-80 - 29*a*x + 11*a^2*x^2 - 6*a^3*x^3 + 2*a^4*
x^4))/(1 + a*x) + 51*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(8*a^4)
```

Maple [B] time = 0.138, size = 539, normalized size = 4.

$$\frac{1}{8\,a^4\,(ax-1)}\left(2\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3+4\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+21\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3-8\,\sqrt{a^2}\,((ax-1)\,(ax+1))^{3/2}\,x^2a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8*(2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)*x^2*a^2+21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-8*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(3/2)*x^2*a^2+2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+42*(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)*x^2*a^2-21*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x^2*a^3-16*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-72*(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(1/2)*x^2*a^2+72*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2))/(a^2)^(1/2))*x^2*a^3+21*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-42*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+8*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)-144*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+144*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-21*ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-72*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+
72*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a^4*((a*x
-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.04308, size = 301, normalized size = 2.21

$$-\frac{1}{8}a\left(\frac{2\left(77\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-149\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-35\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{51\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{51\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}+\frac{32\sqrt{\frac{ax}{ax}}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*(77*((a*x - 1)/(a*x + 1))^(7/2) - 149*((a*x - 1)/(a*x + 1))^(5/2)
 + 123*((a*x - 1)/(a*x + 1))^(3/2) - 35*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x
- 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x
 + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 51*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^5 + 51*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5 + 32*sqrt((a*x -
1)/(a*x + 1))/a^5)
```

Fricas [A] time = 1.47619, size = 227, normalized size = 1.67

$$\frac{\left(2a^4x^4-6a^3x^3+11a^2x^2-29ax-80\right)\sqrt{\frac{ax-1}{ax+1}}+51\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-51\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^4*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt((a*x - 1)/(a*x
 + 1)) + 51*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.51 $\int e^{-3\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=116

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}-\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x)
/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^3
)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rubi [A] time = 0.869178, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6169, 6742, 271, 264, 266, 51, 63, 208, 651}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}-\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x)
/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^3
)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

0]

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^3(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx,\right.$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} +$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^3} - \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1}}\right.}{3a^2}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3} -$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3} - \frac{3\,\text{Sub}}{}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] time = 0.0761989, size = 75, normalized size = 0.65

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-7a^2x^2+19ax+52\right)}{ax+1} - 33\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*x
) - 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Maple [B] time = 0.132, size = 471, normalized size = 4.1

$$-\frac{1}{6\,a^3\,(ax-1)}\left(9\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3-2\,\sqrt{a^2}\,((ax-1)\,(ax+1))^{3/2}\,x^2a^2+18\,\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2-9\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-2*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(3/2)*x^2*a^2+18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-9*ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-4*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x*a-42*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+42*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+9*(a^2)^(1/2)*(a^2
*x^2-1)^(1/2)*x*a-18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*
x*a^2+10*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-84*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(1/2)*x*a+84*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))
*x*a^2-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-42*(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2)+42*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/(a^2)^(1/2)))/a^3*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1
))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.01547, size = 251, normalized size = 2.16

$$-\frac{1}{6}\,a\left(\frac{2\left(39\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-52\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\,\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}-\frac{24\,\sqrt{\frac{ax-1}{ax+1}}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(39*((a*x - 1)/(a*x + 1))^(5/2) - 52*((a*x - 1)/(a*x + 1))^(3/2)
+ 21*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*
a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^4 - 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4 - 24*sqrt
((a*x - 1)/(a*x + 1))/a^4)
```

Fricas [A] time = 1.69842, size = 209, normalized size = 1.8

$$\frac{\left(2\,a^{3}x^{3}-7\,a^{2}x^{2}+19\,ax+52\right)\sqrt{\frac{ax-1}{ax+1}}-33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt((a*x - 1)/(a*x + 1)) - 33*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.52 $\int e^{-3\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=90

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rubi [A] time = 0.842227, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {6169, 6742, 266, 51, 63, 208, 264, 651}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^2(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} -$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 4\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) - \text{Subst}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,d\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] time = 0.0616541, size = 66, normalized size = 0.73

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(a^2x^2-5ax-14\right)}{ax+1} + 9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 - 5*a*x + a^2*x^2))/(1 + a*x) + 9*Log[(1 +
 Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Maple [B] time = 0.128, size = 421, normalized size = 4.7

$$\frac{1}{2\,a^{2}\left(ax-1\right)}\left(\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}x^{3}a^{3}-10\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}x^{2}a^{2}+2\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}x^{2}a^{2}-\ln\left(\left(a^{2}x+\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}\right)\frac{1}{\sqrt{\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-10*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+10*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+4*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-20*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x
*a-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+20*ln((a^2
*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-10*(a^2)^(1/2)*(
(a*x-1)*(a*x+1))^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
)*a+10*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a^2*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 0.995983, size = 204, normalized size = 2.27

$$-\frac{1}{2}\,a\left(\frac{2\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^{3}}{ax+1}-\frac{(ax-1)^{2}a^{3}}{(ax+1)^{2}}-a^{3}}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{3}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{3}}+\frac{8\sqrt{\frac{ax-1}{ax+1}}}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(7*((a*x - 1)/(a*x + 1))^(3/2) - 5*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 9*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^3 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3 + 8
*sqrt((a*x - 1)/(a*x + 1))/a^3)
```

Fricas [A] time = 1.72874, size = 186, normalized size = 2.07

$$\frac{\left(a^2x^2-5\,ax-14\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - 5*a*x - 14)*sqrt((a*x - 1)/(a*x + 1)) + 9*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.53 $\int e^{-3\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.778659, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6168, 6742, 264, 266, 63, 208, 651}

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcCoth[a*x]),x]
```


```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \frac{4\operatorname{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$
$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - (3a)\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.0466342, size = 54, normalized size = 0.9

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+5)}{ax+1} - \frac{3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(5 + a*x))/(1 + a*x) - (3*Log[a*(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/a
```

Maple [B] time = 0.129, size = 248, normalized size = 4.1

$$-\frac{1}{(ax-1)\,a}\left(3\ \ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^2a^3 - 3\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^2a^2 + 6\ \ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -(3*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-3*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+6*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-6
*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+3*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*
x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [B] time = 1.03164, size = 150, normalized size = 2.5

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + 3*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- 4*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Fricas [A] time = 1.70101, size = 161, normalized size = 2.68

$$\frac{(ax+5)\sqrt{\frac{ax-1}{ax+1}}-3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 5)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.54 $\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcCsc[a*x] + ArcTanh[Sqrt[1 -
1/(a^2*x^2)]]
```

Rubi [A] time = 0.760731, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 6742, 216, 266, 63, 208, 651}

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x),x]
```


```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcCsc[a*x] + ArcTanh[Sqrt[1 -
1/(a^2*x^2)]]
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int \left(\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}-\frac{4}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= 4\operatorname{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + a^2\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [A] time = 0.0523513, size = 55, normalized size = 1.2

$$-\frac{4ax\sqrt{1-\frac{1}{a^2x^2}}}{ax+1} + \log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) - \sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - ArcSin[1/(a*x)] + Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]
```

Maple [B] time = 0.132, size = 369, normalized size = 8.

$$\frac{1}{ax-1}\left(\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)x^2a^3 - \sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 - a^2x^2\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) - \sqrt{a^2}\sqrt{(ax-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] (ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^2*x
^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1
/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+2*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x
*a-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+2*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+a*ln((a^2*x+(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arct
an(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))*((
a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [B] time = 1.55525, size = 120, normalized size = 2.61

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a - 4*sqrt((a*x - 1)/(a*x + 1))/a
)
```

Fricas [A] time = 1.58301, size = 192, normalized size = 4.17

$$-4\sqrt{\frac{ax-1}{ax+1}}+2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -4*sqrt((a*x - 1)/(a*x + 1)) + 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

3.55 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=53

$$\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

```
[Out] 3*a*Sqrt[1 - 1/(a^2*x^2)] + (2*(a - x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
3*a*ArcCsc[a*x]
```

Rubi [A] time = 0.0775462, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6169, 853, 669, 641, 216}

$$\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] 3*a*Sqrt[1 - 1/(a^2*x^2)] + (2*(a - x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
3*a*ArcCsc[a*x]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx &= -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.0673987, size = 41, normalized size = 0.77

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}(5ax+1)}{ax+1}+3a\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x))/(1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Maple [B] time = 0.131, size = 592, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-5*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x^3*a^3+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-
3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-ln((a^2*x+(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(3/2)*x*a+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+2*(a^2)^(1/2)*(a^2*
x^2-1)^(3/2)*x*a-7*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+2*ln((a^2*x+(a^2*x
^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-6*a^2*x^2*(a^2)^(1/2)*arctan(
1/(a^2*x^2-1)^(1/2))-2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)
^(1/2))*x^2*a^3+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(a^2*x^2-1)^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-3*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2
))-ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2)*((a*x
-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.5446, size = 97, normalized size = 1.83

$$2a\left(2\sqrt{\frac{ax-1}{ax+1}}+\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}-3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(2*sqrt((a*x - 1)/(a*x + 1)) + sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*
x + 1) + 1) - 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A] time = 1.51105, size = 116, normalized size = 2.19

$$-\frac{6\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (5*a*x + 1)*sqrt((a*x - 1)/(a*x
 + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

3.56 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{2}a^2\csc^{-1}(ax)$$

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a + x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a + x^(-1))) - (9*a^2*ArcCsc[a*x
])/2
```

Rubi [A] time = 0.444977, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 1633, 1593, 12, 793, 665, 216}

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a + x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a + x^(-1))) - (9*a^2*ArcCsc[a*x
])/2
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(ax-x^2\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - (3a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] time = 0.136651, size = 56, normalized size = 0.64

$$\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-14a^2x^2-5ax+1\right)}{x(ax+1)} - 9a\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x - 14*a^2*x^2))/(x*(1 + a*x)) - 9*a*Ar
cSin[1/(a*x)]))/2
```

Maple [B] time = 0.131, size = 642, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x^3*a^3-21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*ln((a^2*x+(a^2*x^2-1)
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-9*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^
2*x^2-1)^(1/2))+6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-6*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+11*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^2*a^2-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+12*ln((a^2*
x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-18*a^3*x^3*(a^2)^(1/2
)*arctan(1/(a^2*x^2-1)^(1/2))+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2
+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-12*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+4*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)
*x*a-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-9*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1
)^(1/2))+6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-6*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(
1/2))*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x^2/(a*x-1)/((a*x-1)*(a*x+1))^(1/
2)
```

Maxima [A] time = 1.49877, size = 151, normalized size = 1.74

$$\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-4\,a\sqrt{\frac{ax-1}{ax+1}}-\frac{7\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+5\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] (9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) - 4*a*sqrt((a*x - 1)/(a*x + 1)) - (7
*a*((a*x - 1)/(a*x + 1))^(3/2) + 5*a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1
)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Fricas [A] time = 1.59415, size = 147, normalized size = 1.69

$$\frac{18\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(14\,a^2x^2+5\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(18*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (14*a^2*x^2 + 5*a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

3.57 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{11}{2}a^3\csc^{-1}(ax)$$

```
[Out] (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a - 3/x))/6 + (a - x^(-1))^3/Sqrt[1 - 1/(a^2
*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a - x^(-1))^2)/3 + (11*a^3*ArcCsc[a*x]
)/2
```

Rubi [A] time = 0.766351, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6169, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{11}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^4),x]
```


```
[Out] (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a - 3/x))/6 + (a - x^(-1))^3/Sqrt[1 - 1/(a^2
*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a - x^(-1))^2)/3 + (11*a^3*ArcCsc[a*x]
)/2
```


Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(ax^2-x^3\right)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\left(3a^2-ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5+\frac{3x}{a}\right)\left(3a^2-ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{11}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] time = 0.101494, size = 66, normalized size = 0.69

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(52a^3x^3+19a^2x^2-7ax+2\right)}{x^2(ax+1)}+33a^2\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^2*(1 +
 a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Maple [B] time = 0.147, size = 666, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] -1/6*(-30*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x^6*a^6+30*(a^2)^(1/2)*(a^2*x^2-1)^
(3/2)*x^4*a^4-93*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+30*ln((a^2*x+(a^2*x^
2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-33*(a^2)^(1/2)*arctan(1/(a^2*x
^2-1)^(1/2))*x^5*a^5+30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-30*ln((
a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+51*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+60*l
n((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-66*a^4*x^4*(a^
2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)
*x^3*a^3+60*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-60*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+14*(a^2*x^2-1)^(3/2)*(a
^2)^(1/2)*x^2*a^2-33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+30*ln((a^2*x+(a^
2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-33*a^3*x^3*(a^2)^(1/2)*arc
tan(1/(a^2*x^2-1)^(1/2))+30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-30*
ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-5*(a^2)
^(1/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))*((a*x-1)/(a*x
+1))^(3/2)/(a^2)^(1/2)/x^3/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.52274, size = 212, normalized size = 2.21

$$-\frac{1}{3}\left(33\,a^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-12\,a^{2}\sqrt{\frac{ax-1}{ax+1}}-\frac{39\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+52\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{(ax-1)^{3}}{(ax+1)^{3}}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 12*a^2*sqrt((a*x - 1)/(a*x
 + 1)) - (39*a^2*((a*x - 1)/(a*x + 1))^(5/2) + 52*a^2*((a*x - 1)/(a*x + 1))
^(3/2) + 21*a^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x
- 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Fricas [A] time = 1.61868, size = 166, normalized size = 1.73

$$-\frac{66\,a^{3}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(52\,a^{3}x^{3}+19\,a^{2}x^{2}-7\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (52*a^3*x^3 + 19*a^2*x
^2 - 7*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

3.58 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=133

$$-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{51}{8}a^4\csc^{-1}(ax)$$

```
[Out] (-27*a^4*Sqrt[1 - 1/(a^2*x^2)])/4 - (9*a^3*Sqrt[1 - 1/(a^2*x^2)]*(2*a - 3/x
))/8 - (a*(a - x^(-1))^3)/Sqrt[1 - 1/(a^2*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)])
/(4*x^3) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/x^2 - (51*a^4*ArcCsc[a*x])/8
```

Rubi [A] time = 0.8388, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6169, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 216}

$$-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{51}{8}a^4\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-27*a^4*Sqrt[1 - 1/(a^2*x^2)])/4 - (9*a^3*Sqrt[1 - 1/(a^2*x^2)]*(2*a - 3/x
))/8 - (a*(a - x^(-1))^3)/Sqrt[1 - 1/(a^2*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)])
/(4*x^3) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/x^2 - (51*a^4*ArcCsc[a*x])/8
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(ax^3-x^4\right)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\left(-3a^3+a^2x-ax^2\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{12a-28x+\frac{27x^2}{a}-\frac{12x^3}{a^2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\,\text{Subst}\left(\int \frac{-\frac{36}{a}+\frac{108x}{a^2}-\frac{81x^2}{a^3}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\,\text{Subst}\left(\int -\frac{9(2a-3x)^2}{a^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{(2a-3x)^2}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{-17+\frac{18\ldots}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\ldots\right.$$

Mathematica [A] time = 0.0503231, size = 75, normalized size = 0.56

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(80a^4x^4+29a^3x^3-11a^2x^2+6ax-2\right)}{8x^3(ax+1)}-\frac{51}{8}a^4\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a^4*x^
4))/(8*x^3*(1 + a*x)) - (51*a^4*ArcSin[1/(a*x)])/8
```

Maple [B] time = 0.138, size = 690, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] 1/8*(-56*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^7*a^7+56*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^5*a^5-163*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x^6*a^6-51*arctan(1/(a^2*x^2
-1)^(1/2))*(a^2)^(1/2)*x^6*a^6+56*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*x^6*a^7+56*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a^6-56*ln((
a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7+91*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-158*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5-102
*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*x^5*a^5+112*ln((a^2*x+(a^2*x^2-1)^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+16*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3
/2)*x^4*a^4+112*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-112*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+22*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^3*a^3-51*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-51*a^4*x^4*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+56*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*x^4*a^5+56*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^
4-56*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-7*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+4*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-2
*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x^4/(a*
x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.53044, size = 261, normalized size = 1.96

$$\frac{1}{4}\left(51\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-16\,a^3\sqrt{\frac{ax-1}{ax+1}}-\frac{77\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+149\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+35\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(51*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 16*a^3*sqrt((a*x - 1)/(a*x
+ 1)) - (77*a^3*((a*x - 1)/(a*x + 1))^(7/2) + 149*a^3*((a*x - 1)/(a*x + 1))
^(5/2) + 123*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 35*a^3*sqrt((a*x - 1)/(a*x +
 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a
*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + 1))*a
```

Fricas [A] time = 1.51496, size = 184, normalized size = 1.38

$$\frac{102\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(80\,a^4x^4+29\,a^3x^3-11\,a^2x^2+6\,ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(102*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (80*a^4*x^4 + 29*a^3*x
^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

3.59 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{11x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{48a^2}+\frac{269x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{960a^3}+\frac{611x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{1920a^4}+\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] (611*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(1920*a^4) + (269*(1 - 1/(a
*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(3/4)*(1
+ 1/(a*x))^(1/4)*x^3)/(48*a^2) + (9*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rubi [A] time = 0.153866, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{11x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{48a^2}+\frac{269x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{960a^3}+\frac{611x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{1920a^4}+\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^4,x]
```


```
[Out] (611*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(1920*a^4) + (269*(1 - 1/(a
*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(3/4)*(1
+ 1/(a*x))^(1/4)*x^3)/(48*a^2) + (9*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^6\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{5}\operatorname{Subst}\left(\int \frac{\frac{9}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{1}{20}\operatorname{Subst}\left(\int \frac{-\frac{55}{4a^2}-\frac{27x}{2a^3}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{60}\operatorname{Subst}\left(\int\right.$$

$$= \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}}{40}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}}{40}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}}{40}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}}{40}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}}{40}$$

Mathematica [A] time = 5.21619, size = 173, normalized size = 0.68

$$\frac{\frac{9620e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{34000e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{64640e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{62976e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}+\frac{24576e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5}-465\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+465\,l}{3840a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^4,x]
```

```
[Out] ((24576*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (62976*E^(ArcCoth
[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^(ArcCoth[a*x]/2))/(-1 + E^
(2*ArcCoth[a*x]))^3 + (34000*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^
2 + (9620*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 930*ArcTan[E^(Arc
Coth[a*x]/2)] - 465*Log[1 - E^(ArcCoth[a*x]/2)] + 465*Log[1 + E^(ArcCoth[a*
x]/2)])/(3840*a^5)
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

Maxima [A] time = 1.53406, size = 350, normalized size = 1.38

$$-\frac{1}{3840}a\left(\frac{4\left(465\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(465*((a*x - 1)/(a*x + 1))^(19/4) - 696*((a*x - 1)/(a*x + 1))^
(15/4) + 5090*((a*x - 1)/(a*x + 1))^(11/4) - 1120*((a*x - 1)/(a*x + 1))^(7/
4) + 2405*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/
(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 465*
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A] time = 1.62991, size = 331, normalized size = 1.31

$$\frac{2 \left(384 \, a^{5} x^{5} + 816 \, a^{4} x^{4} + 872 \, a^{3} x^{3} + 978 \, a^{2} x^{2} + 1149 \, a x + 611\right) \left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{4}} - 930 \, \arctan\left(\left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{4}}\right) + 465 \, \log\left(\left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{4}} + \cdots}{3840 \, a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 + 816*a^4*x^4 + 872*a^3*x^3 + 978*a^2*x^2 + 1149*a*x
 + 611)*((a*x - 1)/(a*x + 1))^(3/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 465*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt[4]{\frac{a x - 1}{a x + 1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.17775, size = 316, normalized size = 1.25

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{1120\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{5090\,(ax-1)^2\left(\frac{a}{a}\right.}{(ax+1)^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 + 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(1120*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5090*(a*x -
 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 696*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(3/4)/(a*x + 1)^3 - 465*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/
(a*x + 1)^4 - 2405*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

3.60 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{29x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{96a^2}+\frac{83x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{192a^3}+\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}$$

```
[Out] (83*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(192*a^3) + (29*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(96*a^2) + (7*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
 (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rubi [A] time = 0.119058, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{29x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{96a^2}+\frac{83x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{192a^3}+\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] (83*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(192*a^3) + (29*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(96*a^2) + (7*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
 (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^5\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{29}{4a^2}-\frac{7x}{a^3}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{1}{24}\text{Subst}\left(\int\right.$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

Mathematica [A] time = 5.17397, size = 149, normalized size = 0.69

$$\frac{\frac{980e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{2512e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{3200e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}-33\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+33\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}+1\right)+ \cdots}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] ((1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (3200*E^(ArcCoth[a
*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^(ArcCoth[a*x]/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (980*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 66*
ArcTan[E^(ArcCoth[a*x]/2)] - 33*Log[1 - E^(ArcCoth[a*x]/2)] + 33*Log[1 + E^
(ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

Maxima [A] time = 1.52343, size = 302, normalized size = 1.4

$$\frac{1}{384}a\left(\frac{4\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-279\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+107\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}- \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/384*a*(4*(33*((a*x - 1)/(a*x + 1))^(15/4) - 279*((a*x - 1)/(a*x + 1))^(11
/4) + 107*((a*x - 1)/(a*x + 1))^(7/4) - 245*((a*x - 1)/(a*x + 1))^(3/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A] time = 1.68203, size = 302, normalized size = 1.4

$$\frac{2\left(48\,a^4x^4+104\,a^3x^3+114\,a^2x^2+141\,ax+83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-33\,\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 + 104*a^3*x^3 + 114*a^2*x^2 + 141*a*x + 83)*((a*x - 1)
/(a*x + 1))^(3/4) - 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x -
 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.21551, size = 274, normalized size = 1.27

$$-\frac{1}{384}\,a\left(\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{33\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{107\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{279\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}-\right.}{a^5\left(\frac{ax-1}{ax+1}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^5 + 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5
+ 4*(107*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 279*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 33*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(3/4)/(a*x + 1)^3 - 245*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a
*x + 1) - 1)^4))
```

3.61 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{11x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{5x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

```
[Out] (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) + (5*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
+ (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rubi [A] time = 0.0946851, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{11x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{5x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) + (5*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
+ (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^4\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}-\frac{5x}{2a^3}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{}{8a}\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{3\,\text{Subst}\left(\int \frac{}{x\sqrt[4]{1}}\right.}{1}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+}\right.}{4}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x}\right.}{8a}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right.}{8a^3}$$

Mathematica [C] time = 5.13347, size = 399, normalized size = 2.23

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(1280e^{8\coth^{-1}(ax)}\left(1346e^{2\coth^{-1}(ax)}+557e^{4\coth^{-1}(ax)}+821\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{9}{4}\right\},\left\{1,1,\frac{21}{4}\right\}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] -(-1070609085 - 946471617*E^(2*ArcCoth[a*x]) + 369641285*E^(4*ArcCoth[a*x])
 + 351173641*E^(6*ArcCoth[a*x]) - 23818496*E^(8*ArcCoth[a*x]) + 1070609085*
Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 732349800*E^(2*ArcCoth
[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 635067810*E^(4*
ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 38483172
0*E^(6*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 6
0913125*E^(8*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x]
)] + 1280*E^(8*ArcCoth[a*x])*(821 + 1346*E^(2*ArcCoth[a*x]) + 557*E^(4*ArcC
oth[a*x]))*HypergeometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(2*ArcCoth[a*x
])] + 10240*E^(8*ArcCoth[a*x])*(23 + 42*E^(2*ArcCoth[a*x]) + 19*E^(4*ArcCot
h[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^(2*ArcCoth
[a*x])] + 20480*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4},
{1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])] + 40960*E^(10*ArcCoth[a*x])*Hyperge
ometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])] +
20480*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1,
 1, 21/4}, E^(2*ArcCoth[a*x])])/(1909440*a^3*E^((7*ArcCoth[a*x])/2))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int x^{2} \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

Maxima [A] time = 1.48375, size = 252, normalized size = 1.41

$$-\frac{1}{48}\, a \left(\frac{4 \left(9 \left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}} - 6 \left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}} + 29 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} \right)}{\frac{3\,(ax-1)a^4}{ax+1} - \frac{3\,(ax-1)^2 a^4}{(ax+1)^2} + \frac{(ax-1)^3 a^4}{(ax+1)^3} - a^4} + \frac{18 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} \right)}{a^4} - \frac{9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1 \right)}{a^4} + \frac{9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1 \right)}{a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(9*((a*x - 1)/(a*x + 1))^(11/4) - 6*((a*x - 1)/(a*x + 1))^(7/4)
+ 29*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^
2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 9*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A] time = 1.76249, size = 275, normalized size = 1.54

$$\frac{2\left(8\,a^{3}x^{3}+18\,a^{2}x^{2}+21\,ax+11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 18*a^2*x^2 + 21*a*x + 11)*((a*x - 1)/(a*x + 1))^(3/4)
- 18*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.21693, size = 232, normalized size = 1.3

$$-\frac{1}{48}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{9\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}-29\,\cdots\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 9*(a*x - 1)^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 29*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

3.62 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

```
[Out] ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(3/4)*(1
 + 1/(a*x))^(5/4)*x^2)/2 + ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/
(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rubi [A] time = 0.0604824, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(3/4)*(1
 + 1/(a*x))^(5/4)*x^2)/2 + ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/
(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
```

```
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^3\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{1}{1\cdots}\right.}{\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.163456, size = 66, normalized size = 0.46

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(5e^{2\coth^{-1}(ax)}-1\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2)*(-1 + 5*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x]
))^2 + ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

Maxima [A] time = 1.48495, size = 201, normalized size = 1.42

$$\frac{1}{8}\,a\left(\frac{4\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(((a*x - 1)/(a*x + 1))^(7/4) - 5*((a*x - 1)/(a*x + 1))^(3/4))/(2*(
a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A] time = 1.72144, size = 247, normalized size = 1.74

$$\frac{2\left(2\,a^2x^2+5\,ax+3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 + 5*a*x + 3)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x
 - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x
- 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.19638, size = 188, normalized size = 1.32

$$-\frac{1}{8}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^3 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*((a*x
 - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5*((a*x - 1)/(a*x + 1))^(3/4)
)/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.63 $\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=96

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rubi [A] time = 0.0368659, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6170, 94, 93, 212, 206, 203}

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.0882639, size = 51, normalized size = 0.53

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + ArcTan[E^(ArcCoth[a*x]/
2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A] time = 1.47326, size = 150, normalized size = 1.56

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A] time = 1.60895, size = 225, normalized size = 2.34

$$\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/4), x)
```

Giac [A] time = 1.24035, size = 146, normalized size = 1.52

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.64 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)$$

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rubi [A] time = 0.24617, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6171, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x,x]
```


```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1+x}{1+x}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+\frac{1}{a}}}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.0395665, size = 30, normalized size = 0.1

$$\frac{8}{5}e^{\frac{5}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x,x]
```

```
[Out] (8*E^((5*ArcCoth[a*x])/2)*Hypergeometric2F1[5/8, 1, 13/8, E^(4*ArcCoth[a*x]
)])/5
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Maxima [A] time = 1.56849, size = 302, normalized size = 1.04

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
```

```
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

Fricas [A] time = 1.73327, size = 807, normalized size = 2.77

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A] time = 1.18663, size = 313, normalized size = 1.08

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

3.65 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=267

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\tan}{}$$

```
[Out] a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (a*ArcTan[1 - (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 - 1/
(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*Log[1 + Sqrt[1 - 1/(a*x)]/
Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*
Sqrt[2]) - (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rubi [A] time = 0.225057, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^2,x]
```


```
[Out] a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (a*ArcTan[1 - (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 - 1/
(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*Log[1 + Sqrt[1 - 1/(a*x)]/
Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*
Sqrt[2]) - (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\mathrm{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - a\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\,\mathrm{Subst}}{}$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.239578, size = 148, normalized size = 0.55

$$a\left(\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} - \frac{\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1-\sqrt{2}e^{\frac{1}{2}\mathrm{c}}\right.}{\sqrt{2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*((2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + ArcTan[1 - Sqrt[2]*E^(
ArcCoth[a*x]/2)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]/Sqrt[2] +
```

```
Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]) - Log[1 +
Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Maxima [A] time = 1.56037, size = 251, normalized size = 0.94

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Fricas [A] time = 1.79753, size = 1027, normalized size = 3.85

$$-\frac{4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^4\right)^{\frac{1}{4}}}{a^4}\right)+4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-}{ax+}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*a^3*((a*x
- 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a
^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4))
/a^4) + 4*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(1/4)*a^3*((a*x
 - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(
a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4)
)/a^4) + sqrt(2)*(a^4)^(1/4)*x*log(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^4
)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - sqrt(2)*(a^4
)^(1/4)*x*log(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^4)*a^4 - sqrt(2)*(a^4)
^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))
^(3/4))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A] time = 1.20252, size = 251, normalized size = 0.94

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

3.66 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.}{8\sqrt{2}}$$

```
[Out] (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(3/4)*
(1 + 1/(a*x))^(5/4))/2 - (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rubi [A] time = 0.249052, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(3/4)*
(1 + 1/(a*x))^(5/4))/2 - (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\,\text{Subst}\left(\int \frac{x\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{4}$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log\left(1\right.}{}$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\tan^{-1}\left(1+\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.216427, size = 173, normalized size = 0.54

$$\frac{1}{16}a^2\left(\frac{40e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} - \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}+1\right)^2} + \sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right) - \sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*((-32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (40*E^(ArcCoth[
a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[
1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] - Sqrt[2]*Log[1 + Sqrt[2]*
E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Maxima [A] time = 1.48622, size = 305, normalized size = 0.96

$$\frac{1}{16}\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1))*a + 8*(a*((a*x - 1)/(a*x + 1))^(7/4) + 5*a*((a*x - 1)
/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Fricas [A] time = 1.77314, size = 1067, normalized size = 3.34

$$-\frac{4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(1/4)*a^6*((a
*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1)) + sq
rt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8)^(1
/4))/a^8) + 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(1/4)*a^6
*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8
)^(1/4))/a^8) + sqrt(2)*(a^8)^(1/4)*x^2*log(a^12*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - sq
rt(2)*(a^8)^(1/4)*x^2*log(a^12*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^8)*a^8 -
sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(3*a^2*x^2 + 5*a*x
 + 2)*((a*x - 1)/(a*x + 1))^(3/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A] time = 1.15616, size = 301, normalized size = 0.94

$$\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a
*x + 1) + 5*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.67 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-$$

```
[Out] (3*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 + (a^3*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(5/4))/12 + (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(
3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x
))^(1/4)])/(8*Sqrt[2]) + (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (3*a^
3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rubi [A] time = 0.291752, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^4,x]
```


```
[Out] (3*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 + (a^3*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(5/4))/12 + (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(
3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x
))^(1/4)])/(8*Sqrt[2]) + (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (3*a^
3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\left(-1-\frac{x}{2a}\right)\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{16}\left(3a^2\right)\text{Su}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{4}\left(3a^3\right)\text{Su}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{4}\left(3a^3\right)\text{Su}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a^3\right)\text{Su}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{16}\left(3a^3\right)\text{Su}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{3a^3\log\left(1 -\right.}{}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{3a^3\tan^{-1}\left(\right.}{8}$$

Mathematica [C] time = 0.106194, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(9\text{RootSum}\left[\#1^4+1\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]+\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(6e^{2\coth^{-1}(ax)}+29e^{4\coth^{-1}(ax)}+9\right)}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^4,x]
```

```
[Out] (a^3*((8*E^(ArcCoth[a*x]/2)*(9 + 6*E^(2*ArcCoth[a*x]) + 29*E^(4*ArcCoth[a*x
])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x] - 2*
Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Maxima [A] time = 1.49274, size = 365, normalized size = 1.03

$$\frac{1}{96}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
```

```
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))*a^2 + 8*(9*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 6*a^2
*((a*x - 1)/(a*x + 1))^(7/4) + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x
- 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*
a
```

Fricas [A] time = 1.66981, size = 1160, normalized size = 3.26

$$-\frac{36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(1/4)*a^9
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*(
a^12)^(1/4))/a^12) + 36*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^
12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^18*sqrt((a*x - 1
)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x +
 1))^(1/4))*(a^12)^(1/4))/a^12) + 9*sqrt(2)*(a^12)^(1/4)*x^3*log(729*a^18*s
qrt((a*x - 1)/(a*x + 1)) + 729*sqrt(a^12)*a^12 + 729*sqrt(2)*(a^12)^(3/4)*a
^9*((a*x - 1)/(a*x + 1))^(1/4)) - 9*sqrt(2)*(a^12)^(1/4)*x^3*log(729*a^18*s
qrt((a*x - 1)/(a*x + 1)) + 729*sqrt(a^12)*a^12 - 729*sqrt(2)*(a^12)^(3/4)*a
^9*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(11*a^3*x^3 + 21*a^2*x^2 + 18*a*x + 8)*
((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A] time = 1.17358, size = 366, normalized size = 1.03

$$\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-9\sqrt{2}a^2\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) - 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(3/4)/(a*x + 1) + 9*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(
a*x + 1)^2 + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

3.68 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{5x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{16a^2}+\frac{157x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{320a^3}+\frac{557x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{640a^4}-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\tanh^{-1}\left(\cdots\right.}{128a^5}$$

```
[Out] (557*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(640*a^4) + (157*(1 - 1/(a*
x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(1/4)*(1 +
1/(a*x))^(3/4)*x^3)/(16*a^2) + (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rubi [A] time = 0.139706, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{5x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{16a^2}+\frac{157x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{320a^3}+\frac{557x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{640a^4}-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\tanh^{-1}\left(\cdots\right.}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^4,x]
```


```
[Out] (557*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(640*a^4) + (157*(1 - 1/(a*
x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(1/4)*(1 +
1/(a*x))^(3/4)*x^3)/(16*a^2) + (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^6\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{5}\operatorname{Subst}\left(\int \frac{\frac{11}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{1}{20}\operatorname{Subst}\left(\int \frac{-\frac{75}{4a^2}-\frac{33x}{2a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{60}\operatorname{Subst}\left(\int \right.$$

$$= \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\right.$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\right.}{40a}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\right.}{40a}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\right.}{40a}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\right.}{40a}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\right.}{40a}$$

Mathematica [A] time = 5.23589, size = 173, normalized size = 0.68

$$\frac{\frac{5500e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{14032e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{23936e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{22016e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}+\frac{8192e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5}-1185\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+118}{1280a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] ((8192*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (22016*E^((3*A
rcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((3*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^3 + (14032*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (5500*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))
- 2370*ArcTan[E^(ArcCoth[a*x]/2)] - 1185*Log[1 - E^(ArcCoth[a*x]/2)] + 1185
*Log[1 + E^(ArcCoth[a*x]/2)])/(1280*a^5)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

Maxima [A] time = 1.47139, size = 350, normalized size = 1.38

$$-\frac{1}{1280}a\left(\frac{4\left(395\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(395*((a*x - 1)/(a*x + 1))^(17/4) - 1440*((a*x - 1)/(a*x + 1))
^(13/4) + 3710*((a*x - 1)/(a*x + 1))^(9/4) - 1992*((a*x - 1)/(a*x + 1))^(5/
4) + 1375*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 11
85*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A] time = 1.68208, size = 333, normalized size = 1.32

$$\frac{2\left(128\,a^5x^5 + 304\,a^4x^4 + 376\,a^3x^3 + 514\,a^2x^2 + 871\,ax + 557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + \cdots\right.}{1280\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 + 304*a^4*x^4 + 376*a^3*x^3 + 514*a^2*x^2 + 871*a*x
+ 557)*((a*x - 1)/(a*x + 1))^(1/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1185*log(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A] time = 1.21739, size = 316, normalized size = 1.25

$$\frac{1}{1280}\,a\left(\frac{2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{1185\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1992\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{3710\,(ax-1}{(ax\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 - 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1992*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1440*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 395*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/
4)/(a*x + 1)^4 - 1375*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

3.69 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{15x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{32a^2}+\frac{63x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{64a^3}-\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right.$$

```
[Out] (63*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(64*a^3) + (15*(1 - 1/(a*x))
^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(32*a^2) + (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a
*x))^(3/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (
123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rubi [A] time = 0.117535, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{15x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{32a^2}+\frac{63x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{64a^3}-\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^3,x]
```


```
[Out] (63*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(64*a^3) + (15*(1 - 1/(a*x))
^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(32*a^2) + (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a
*x))^(3/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (
123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^5\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{9}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}-\frac{9x}{a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{24}\text{Subst}\left(\int \ldots\right.$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\ldots\right.$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\ldots\right.$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\ldots\right.$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\ldots\right.$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\ldots\right.$$

Mathematica [A] time = 5.20868, size = 149, normalized size = 0.69

$$\frac{\frac{532e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{1008e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{1152e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{512e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}-123\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+123\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}+1\right.}{128a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] ((512*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (1152*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((3*ArcCoth[a*x])/2))/
(-1 + E^(2*ArcCoth[a*x]))^2 + (532*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x])) - 246*ArcTan[E^(ArcCoth[a*x]/2)] - 123*Log[1 - E^(ArcCoth[a*x]/2)
] + 123*Log[1 + E^(ArcCoth[a*x]/2)])/(128*a^4)
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

Maxima [A] time = 1.48425, size = 302, normalized size = 1.4

$$\frac{1}{128}a\left(\frac{4\left(41\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-183\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+147\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/128*a*(4*(41*((a*x - 1)/(a*x + 1))^(13/4) - 183*((a*x - 1)/(a*x + 1))^(9/
4) + 147*((a*x - 1)/(a*x + 1))^(5/4) - 133*((a*x - 1)/(a*x + 1))^(1/4))/(4*
(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5
/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 123*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A] time = 1.71072, size = 302, normalized size = 1.4

$$\frac{2\left(16\,a^4x^4+40\,a^3x^3+54\,a^2x^2+93\,ax+63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-123\,\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 54*a^2*x^2 + 93*a*x + 63)*((a*x - 1)/(a
*x + 1))^(1/4) + 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x -
1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A] time = 1.22118, size = 274, normalized size = 1.27

$$\frac{1}{128}a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{147(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{183(ax-1)^2\left(\frac{ax-1}{ax+1}\right.}{(ax+1)^2}\right.}{a^5\left(\frac{ax-}{ax+}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="giac")
```

```
[Out] 1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^5 - 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
5 - 4*(147*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 183*(a*x - 1)^
2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 41*(a*x - 1)^3*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1)^3 - 133*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/
(a*x + 1) - 1)^4))
```

3.70 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{23x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{7x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

```
[Out] (23*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) + (7*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 + (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rubi [A] time = 0.0936718, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{23x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{7x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (23*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) + (7*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 + (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^4\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{23}{4a^2}-\frac{7x}{2a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{}{8a^3}\right.$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\,\text{Subst}\left(\int \frac{}{x(1-}\right.}{1}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\,\text{Subst}\left(\int \frac{x^2}{-1+}\right.}{4a}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{17\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{8a}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right.}{8a^3}$$

Mathematica [A] time = 5.15991, size = 125, normalized size = 0.7

$$\frac{\frac{180e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{240e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{128e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} - 51\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 51\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}+1\right) - 102\tan^{-1}\left(e^{\frac{1}{2}\coth\right.}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] ((128*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (240*E^((3*ArcC
oth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (180*E^((3*ArcCoth[a*x])/2))/(-
1 + E^(2*ArcCoth[a*x])) - 102*ArcTan[E^(ArcCoth[a*x]/2)] - 51*Log[1 - E^(Ar
cCoth[a*x]/2)] + 51*Log[1 + E^(ArcCoth[a*x]/2)])/(48*a^3)
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

Maxima [A] time = 1.50305, size = 252, normalized size = 1.41

$$-\frac{1}{48}a\left(\frac{4\left(17\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(17*((a*x - 1)/(a*x + 1))^(9/4) - 30*((a*x - 1)/(a*x + 1))^(5/4)
 + 45*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)
^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 +
51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A] time = 1.70283, size = 279, normalized size = 1.56

$$\frac{2\left(8a^3x^3+22a^2x^2+37ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 37*a*x + 23)*((a*x - 1)/(a*x + 1))^(1/4)
+ 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 51*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A] time = 1.20814, size = 232, normalized size = 1.3

$$\frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{17(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}-45\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 - 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 17*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 45*((a*x - 1)/(a*x + 1))^(1/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.71 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$-\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

```
[Out] (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(1/4)*
(1 + 1/(a*x))^(7/4)*x^2)/2 - (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2
)
```

Rubi [A] time = 0.0590618, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 298, 203, 206}

$$-\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(1/4)*
(1 + 1/(a*x))^(7/4)*x^2)/2 - (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2
)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)} x\,dx = -\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} x^2 - \frac{3\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} x^2 - \frac{9\operatorname{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} x^2 - \frac{9\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} x^2 + \frac{9\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - \frac{9\operatorname{Subst}\left(\int \frac{1}{\cdots}\right.}{}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} x^2 - \frac{9\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.171909, size = 70, normalized size = 0.49

$$\frac{\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(7e^{2\coth^{-1}(ax)}-3\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - 9\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] ((2*E^((3*ArcCoth[a*x])/2)*(-3 + 7*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[
a*x]))^2 - 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4
*a^2)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int x\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

Maxima [A] time = 1.50407, size = 205, normalized size = 1.44

$$\frac{1}{8}\,a\left(\frac{4\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(3*((a*x - 1)/(a*x + 1))^(5/4) - 7*((a*x - 1)/(a*x + 1))^(1/4))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A] time = 1.76629, size = 254, normalized size = 1.79

$$\frac{2\left(2\,a^2x^2+7\,ax+5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 + 7*a*x + 5)*((a*x - 1)/(a*x + 1))^(1/4) + 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A] time = 1.21384, size = 190, normalized size = 1.34

$$\frac{1}{8}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{3\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="giac")
```

```
[Out] 1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^3 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*(
3*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 7*((a*x - 1)/(a*x + 1))
^(1/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.72 $\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a + (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rubi [A] time = 0.0371912, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6170, 94, 93, 298, 203, 206}

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a + (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = -\mathrm{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{6\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.0596493, size = 56, normalized size = 0.57

$$\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(\left(e^{2\coth^{-1}(ax)}-1\right)\mathrm{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)+1\right)}{a\left(e^{2\coth^{-1}(ax)}-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (8*E^((3*ArcCoth[a*x])/2)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[
3/4, 2, 7/4, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A] time = 1.51758, size = 151, normalized size = 1.54

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A] time = 1.58572, size = 231, normalized size = 2.36

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/4), x)
```

Giac [A] time = 1.16182, size = 147, normalized size = 1.5

$$\frac{1}{2}a\left(\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 - 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.73 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)$$

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rubi [A] time = 0.228126, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6171, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-}{-1+\sqrt{2}}\right.}{}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

Mathematica [C] time = 0.044619, size = 30, normalized size = 0.1

$$\frac{8}{7}e^{\frac{7}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (8*E^((7*ArcCoth[a*x])/2)*Hypergeometric2F1[7/8, 1, 15/8, E^(4*ArcCoth[a*x]
)])/7
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Maxima [A] time = 1.57938, size = 302, normalized size = 1.04

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
```

```
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

Fricas [A] time = 1.82518, size = 807, normalized size = 2.77

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [A] time = 1.15654, size = 313, normalized size = 1.08

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

3.74 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=268

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+$$

```
[Out] a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - (3*a*ArcTan[1 - (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (3*a*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(2*Sqrt[2]) + (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rubi [A] time = 0.215381, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^2,x]
```


```
[Out] a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - (3*a*ArcTan[1 - (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (3*a*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(2*Sqrt[2]) + (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (3a)\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x}\right.$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{(3a)\operatorname{Sub}}{}$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.0703909, size = 46, normalized size = 0.17

$$-8ae^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)-\frac{1}{e^{2\coth^{-1}(ax)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -8*a*E^((3*ArcCoth[a*x])/2)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometri
c2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])])
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Maxima [A] time = 1.5684, size = 252, normalized size = 0.94

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Fricas [A] time = 1.68286, size = 1014, normalized size = 3.78

$$-\frac{12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)+12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-\cdots}{ax+\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1
)) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
 12*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a
*x + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - s
qrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - 3*sqr
t(2)*(a^4)^(1/4)*x*log(9*a^2*sqrt((a*x - 1)/(a*x + 1)) + 9*sqrt(2)*(a^4)^(1
/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt(a^4)) + 3*sqrt(2)*(a^4)^(1/4)*x*
log(9*a^2*sqrt((a*x - 1)/(a*x + 1)) - 9*sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a
*x + 1))^(1/4) + 9*sqrt(a^4)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [A] time = 1.13653, size = 252, normalized size = 0.94

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
```

```
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

3.75 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[Out] (3*a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(7/4))/2 - (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rubi [A] time = 0.246147, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^3,x]
```


```
[Out] (3*a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(7/4))/2 - (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{1}{8}(9a)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) -$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{8}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{9a^2\log\left(\right.}{}$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{9a^2\tan^{-1}\left(1+\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.0812046, size = 76, normalized size = 0.24

$$-\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) - 3\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + 2\text{Hy}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] - 3*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])] + 2*Hyperg
eometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Maxima [A] time = 1.53114, size = 309, normalized size = 0.97

$$\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*a*((a*x - 1)/(a*x + 1))^(5/4) + 7*
a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x +
1)^2 + 1))*a
```

Fricas [A] time = 1.76729, size = 1072, normalized size = 3.36

$$-\frac{36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(36*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*((
a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*
x + 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))
/a^8) + 36*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*((
a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*
x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))
/a^8) - 9*sqrt(2)*(a^8)^(1/4)*x^2*log(81*a^4*sqrt((a*x - 1)/(a*x + 1)) + 81
*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 81*sqrt(a^8)) + 9*sq
rt(2)*(a^8)^(1/4)*x^2*log(81*a^4*sqrt((a*x - 1)/(a*x + 1)) - 81*sqrt(2)*(a^
8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 81*sqrt(a^8)) - 4*(5*a^2*x^2 + 7
*a*x + 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [A] time = 1.16103, size = 304, normalized size = 0.95

$$\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1) + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1
)^2)*a
```

3.76 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+ 17\cdots$$

```
[Out] (17*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/24 + (a^3*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(7/4))/4 + (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/
(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(
a*x))^(1/4)])/(8*Sqrt[2]) - (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a
*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (
17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rubi [A] time = 0.281635, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+ 17\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (17*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/24 + (a^3*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(7/4))/4 + (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/
(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(
a*x))^(1/4)])/(8*Sqrt[2]) - (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a
*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (
17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\left(-1-\frac{3x}{2a}\right)\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{1}{16}\left(17a^2\right)\text{Subst}\Bigg($$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{4}\left(17a^3\right)\text{Subst}\Bigg(\int$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{4}\left(17a^3\right)\text{Subst}\Bigg(\int$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{8}\left(17a^3\right)\text{Subst}\Bigg(\int$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{16}\left(17a^3\right)\text{Subst}\Bigg($$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{17a^3\log\Bigg(1+\frac{\sqrt{1-}}{\sqrt{1+}}}{16\sqrt{2}}$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{17a^3\tan^{-1}\Bigg(1-\frac{\sqrt{2}}{}}{8\sqrt{2}}$$

Mathematica [C] time = 0.117398, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(51\text{RootSum}\left[\#1^4+1\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]+\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(30e^{2\coth^{-1}(ax)}+45e^{4\coth^{-1}(ax)}\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (a^3*((8*E^((3*ArcCoth[a*x])/2)*(17 + 30*E^(2*ArcCoth[a*x]) + 45*E^(4*ArcCo
th[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*
x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int\frac{1}{x^4}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Maxima [A] time = 1.49628, size = 374, normalized size = 1.05

$$\frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+51\sqrt{2}a^2\log\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
```

```
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(17*a^2*((a*x - 1)/(a*x +
 1))^(9/4) + 30*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 45*a^2*((a*x - 1)/(a*x +
1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3
/(a*x + 1)^3 + 1))*a
```

Fricas [A] time = 1.78408, size = 1133, normalized size = 3.18

$$-\frac{204\,\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^{3}\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+204\,\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^{3}\arctan\left(\frac{a^{12}-\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^
3*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1
)/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
a^12)))/a^12) + 204*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^
(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt(
(a*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt(a^12)))/a^12) - 51*sqrt(2)*(a^12)^(1/4)*x^3*log(289*a^6*sqrt((a*x -
 1)/(a*x + 1)) + 289*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 289*sqrt(a^12)) + 51*sqrt(2)*(a^12)^(1/4)*x^3*log(289*a^6*sqrt((a*x - 1)/(
a*x + 1)) - 289*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 289*
sqrt(a^12)) - 4*(23*a^3*x^3 + 37*a^2*x^2 + 22*a*x + 8)*((a*x - 1)/(a*x + 1)
)^(1/4))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

[Out] Timed out

Giac [A] time = 1.13451, size = 366, normalized size = 1.03

$$\frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+51\sqrt{2}a^2\log\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1) + 17*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1)^2 + 45*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

3.77 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=287

$$\frac{181x^3\sqrt[4]{\frac{1}{ax}+1}}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189x^2\sqrt[4]{\frac{1}{ax}+1}}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533x\sqrt[4]{\frac{1}{ax}+1}}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}-\frac{26111\sqrt[4]{\frac{1}{ax}+1}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+$$

```
[Out] (-26111*(1 + 1/(a*x))^(1/4))/(1920*a^5*(1 - 1/(a*x))^(1/4)) + (5533*(1 + 1/
(a*x))^(1/4)*x)/(1920*a^4*(1 - 1/(a*x))^(1/4)) + (1189*(1 + 1/(a*x))^(1/4)*
x^2)/(960*a^3*(1 - 1/(a*x))^(1/4)) + (181*(1 + 1/(a*x))^(1/4)*x^3)/(240*a^2
*(1 - 1/(a*x))^(1/4)) + (21*(1 + 1/(a*x))^(1/4)*x^4)/(40*a*(1 - 1/(a*x))^(1
/4)) + ((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) + (1003*ArcTan[(1
+ 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (1003*ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rubi [A] time = 0.16532, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{181x^3\sqrt[4]{\frac{1}{ax}+1}}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189x^2\sqrt[4]{\frac{1}{ax}+1}}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533x\sqrt[4]{\frac{1}{ax}+1}}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}-\frac{26111\sqrt[4]{\frac{1}{ax}+1}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^4,x]
```


```
[Out] (-26111*(1 + 1/(a*x))^(1/4))/(1920*a^5*(1 - 1/(a*x))^(1/4)) + (5533*(1 + 1/
(a*x))^(1/4)*x)/(1920*a^4*(1 - 1/(a*x))^(1/4)) + (1189*(1 + 1/(a*x))^(1/4)*
x^2)/(960*a^3*(1 - 1/(a*x))^(1/4)) + (181*(1 + 1/(a*x))^(1/4)*x^3)/(240*a^2
*(1 - 1/(a*x))^(1/4)) + (21*(1 + 1/(a*x))^(1/4)*x^4)/(40*a*(1 - 1/(a*x))^(1
/4)) + ((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) + (1003*ArcTan[(1
+ 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (1003*ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^6\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}+\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{-\frac{1189}{8a^3}-\frac{543x}{4a^4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{120}\text{Subst}\left(\int \frac{\frac{5533}{16a^4}+\frac{118}{4a}}{x^2\left(1-\frac{x}{a}\right)^{5/4}(1}\right.$$

$$= \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{120}\text{Subst}\left(\right.$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+}}{5\sqrt[4]{1}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+}}{5\sqrt[4]{1}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+}}{5\sqrt[4]{1}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+}}{5\sqrt[4]{1}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+}}{5\sqrt[4]{1}}$$

Mathematica [A] time = 5.2503, size = 198, normalized size = 0.69

$$\frac{-8e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{4117e^{\frac{1}{2}\coth^{-1}(ax)}}{192\left(e^{2\coth^{-1}(ax)}-1\right)}+\frac{1661e^{\frac{1}{2}\coth^{-1}(ax)}}{48\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{233e^{\frac{1}{2}\coth^{-1}(ax)}}{6\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{122e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{2\coth^{-1}(ax)}-1\right)^4}+\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{2\coth^{-1}(ax)}-1\right)^5}-\frac{1003}{256}\log\left(1-\right.}{a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-8*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]
))^5) + (122*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]))^4) + (233*E^(
ArcCoth[a*x]/2))/(6*(-1 + E^(2*ArcCoth[a*x]))^3) + (1661*E^(ArcCoth[a*x]/2)
)/(48*(-1 + E^(2*ArcCoth[a*x]))^2) + (4117*E^(ArcCoth[a*x]/2))/(192*(-1 + E
^(2*ArcCoth[a*x]))) + (1003*ArcTan[E^(ArcCoth[a*x]/2)])/128 - (1003*Log[1 -
 E^(ArcCoth[a*x]/2)])/256 + (1003*Log[1 + E^(ArcCoth[a*x]/2)])/256)/a^5
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

Maxima [A] time = 1.54584, size = 371, normalized size = 1.29

$$-\frac{1}{3840}a\left(\frac{4\left(\frac{58985(ax-1)}{ax+1}-\frac{125920(ax-1)^2}{(ax+1)^2}+\frac{137930(ax-1)^3}{(ax+1)^3}-\frac{72216(ax-1)^4}{(ax+1)^4}+\frac{15045(ax-1)^5}{(ax+1)^5}-7680\right)}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{21}{4}}-5a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}+10a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-10a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+5a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{30090\arctan\left(\left(\frac{ax-}{ax+}\right.\right.}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(58985*(a*x - 1)/(a*x + 1) - 125920*(a*x - 1)^2/(a*x + 1)^2 +
137930*(a*x - 1)^3/(a*x + 1)^3 - 72216*(a*x - 1)^4/(a*x + 1)^4 + 15045*(a*x
 - 1)^5/(a*x + 1)^5 - 7680)/(a^6*((a*x - 1)/(a*x + 1))^(21/4) - 5*a^6*((a*x
 - 1)/(a*x + 1))^(17/4) + 10*a^6*((a*x - 1)/(a*x + 1))^(13/4) - 10*a^6*((a*
x - 1)/(a*x + 1))^(9/4) + 5*a^6*((a*x - 1)/(a*x + 1))^(5/4) - a^6*((a*x - 1
)/(a*x + 1))^(1/4)) + 30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^6)
```

Fricas [A] time = 1.70466, size = 423, normalized size = 1.47

$$-\frac{30090\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(384\,a^6x^6\right.}{3840\left(a^6x-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(30090*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 15045*(a*x -
 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*(a*x - 1)*log(((a*x - 1)/(
a*x + 1))^(1/4) - 1) - 2*(384*a^6*x^6 + 1392*a^5*x^5 + 2456*a^4*x^4 + 3826*
a^3*x^3 + 7911*a^2*x^2 - 20578*a*x - 26111)*((a*x - 1)/(a*x + 1))^(3/4))/(a
^6*x - a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A] time = 1.20408, size = 343, normalized size = 1.2

$$-\frac{1}{3840}a\left(\frac{30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{15045\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{30720}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4\left(\frac{49120(ax-1}{ax+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 + 30720/(a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(49120*(a*x - 1)*((
a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1
))^(3/4)/(a*x + 1)^2 + 33816*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^3 - 7365*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^4 - 20585*((
a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

3.78 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=250

$$\frac{113x^2\sqrt[4]{\frac{1}{ax}+1}}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{521x\sqrt[4]{\frac{1}{ax}+1}}{192a^3\sqrt[4]{1-\frac{1}{ax}}}-\frac{2467\sqrt[4]{\frac{1}{ax}+1}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{17x^3\sqrt[4]{\frac{1}{ax}+1}}{24a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] (-2467*(1 + 1/(a*x))^(1/4))/(192*a^4*(1 - 1/(a*x))^(1/4)) + (521*(1 + 1/(a*
x))^(1/4)*x)/(192*a^3*(1 - 1/(a*x))^(1/4)) + (113*(1 + 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 - 1/(a*x))^(1/4)) + (17*(1 + 1/(a*x))^(1/4)*x^3)/(24*a*(1 - 1/(a
*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rubi [A] time = 0.135991, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{113x^2\sqrt[4]{\frac{1}{ax}+1}}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{521x\sqrt[4]{\frac{1}{ax}+1}}{192a^3\sqrt[4]{1-\frac{1}{ax}}}-\frac{2467\sqrt[4]{\frac{1}{ax}+1}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{17x^3\sqrt[4]{\frac{1}{ax}+1}}{24a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-2467*(1 + 1/(a*x))^(1/4))/(192*a^4*(1 - 1/(a*x))^(1/4)) + (521*(1 + 1/(a*
x))^(1/4)*x)/(192*a^3*(1 - 1/(a*x))^(1/4)) + (113*(1 + 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 - 1/(a*x))^(1/4)) + (17*(1 + 1/(a*x))^(1/4)*x^3)/(24*a*(1 - 1/(a
*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^5\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}+\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{24}\text{Subst}\left(\int \frac{\frac{1425}{16a^4}+\frac{521x}{8a^5}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)}\right.$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{12}a\,\text{Subst}\left(\int\right.$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{475\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{475\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{475\,\text{Subst}\left(\int\right.}{6}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{475\tan^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{64a^4}$$

Mathematica [A] time = 5.23914, size = 161, normalized size = 0.64

$$\frac{-3072e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{6292e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{7376e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{5248e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}-1425\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+14}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-3072*E^(ArcCoth[a*x]/2) + (1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*
x]))^4 + (5248*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (7376*E^(A
rcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (6292*E^(ArcCoth[a*x]/2))/(-1
 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(ArcCoth[a*x]/2)] - 1425*Log[1 - E^(
ArcCoth[a*x]/2)] + 1425*Log[1 + E^(ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F] time = 0.325, size = 0, normalized size = 0.

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

Maxima [A] time = 1.4941, size = 321, normalized size = 1.28

$$\frac{1}{384}a\left(\frac{4\left(\frac{4645(ax-1)}{ax+1}-\frac{7483(ax-1)^2}{(ax+1)^2}+\frac{5415(ax-1)^3}{(ax+1)^3}-\frac{1425(ax-1)^4}{(ax+1)^4}-768\right)}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-4a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+6a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-4a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\log\left(\left(\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/384*a*(4*(4645*(a*x - 1)/(a*x + 1) - 7483*(a*x - 1)^2/(a*x + 1)^2 + 5415*
(a*x - 1)^3/(a*x + 1)^3 - 1425*(a*x - 1)^4/(a*x + 1)^4 - 768)/(a^5*((a*x -
1)/(a*x + 1))^(17/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 6*a^5*((a*x - 1
)/(a*x + 1))^(9/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(5/4) + a^5*((a*x - 1)/(a*
x + 1))^(1/4)) - 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^5)
```

Fricas [A] time = 1.70379, size = 389, normalized size = 1.56

$$-\frac{2850\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(48\,a^5x^5+184\,a\right.}{384\left(a^5x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="fricas")
```

```
[Out] -1/384*(2850*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 1425*(a*x - 1)
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1425*(a*x - 1)*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1) - 2*(48*a^5*x^5 + 184*a^4*x^4 + 362*a^3*x^3 + 747*a^2*x^2
- 1946*a*x - 2467)*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*x - a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A] time = 1.20687, size = 301, normalized size = 1.2

$$-\frac{1}{384}\,a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{3072}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{2875\,(ax-1)\left(\frac{ax-}{ax+}\right.}{ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^5 + 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^5 + 3072/(a^5*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(2875*(a*x - 1)*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1) - 2343*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4
)/(a*x + 1)^2 + 657*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1
573*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

3.79 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=213

$$\frac{61x\sqrt[4]{\frac{1}{ax}+1}}{24a^2\sqrt[4]{1-\frac{1}{ax}}}-\frac{287\sqrt[4]{\frac{1}{ax}+1}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{13x^2\sqrt[4]{\frac{1}{ax}+1}}{12a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] (-287*(1 + 1/(a*x))^(1/4))/(24*a^3*(1 - 1/(a*x))^(1/4)) + (61*(1 + 1/(a*x))
^(1/4)*x)/(24*a^2*(1 - 1/(a*x))^(1/4)) + (13*(1 + 1/(a*x))^(1/4)*x^2)/(12*a
*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) +
 (55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) + (55*ArcTanh
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rubi [A] time = 0.112202, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{61x\sqrt[4]{\frac{1}{ax}+1}}{24a^2\sqrt[4]{1-\frac{1}{ax}}}-\frac{287\sqrt[4]{\frac{1}{ax}+1}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{13x^2\sqrt[4]{\frac{1}{ax}+1}}{12a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^2,x]
```


```
[Out] (-287*(1 + 1/(a*x))^(1/4))/(24*a^3*(1 - 1/(a*x))^(1/4)) + (61*(1 + 1/(a*x))
^(1/4)*x)/(24*a^2*(1 - 1/(a*x))^(1/4)) + (13*(1 + 1/(a*x))^(1/4)*x^2)/(12*a
*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) +
 (55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) + (55*ArcTanh
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^4\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}+\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a\,\text{Subst}\left(\int \frac{165}{16a^4x\sqrt[4]{1-\frac{x}{a}}\left(1+\right.}\right.$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x,\right.}{16a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{55\tanh^{-1}\left(\right.}{8a^3}$$

Mathematica [C] time = 9.0558, size = 441, normalized size = 2.07

$$-\frac{8e^{\frac{9}{2}\coth^{-1}(ax)}\left(\frac{e^{2\coth^{-1}(ax)}\left(1906e^{2\coth^{-1}(ax)}+821e^{4\coth^{-1}(ax)}+1117\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{13}{4}\right\},\left\{1,1,\frac{25}{4}\right\},e^{2\coth^{-1}(ax)}\right)}{3094} + \frac{4e^{2\coth^{-1}(ax)}\left(50e^{2}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (-8*E^((9*ArcCoth[a*x])/2)*(-27653/195 - 899079/(512*E^(8*ArcCoth[a*x])) -
3309759/(2560*E^(6*ArcCoth[a*x])) + 8521937/(7680*E^(4*ArcCoth[a*x])) + 695
71361/(99840*E^(2*ArcCoth[a*x])) - (653*E^(2*ArcCoth[a*x]))/390 + (133407*H
ypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/512 + (899079*Hypergeome
tric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(512*E^(8*ArcCoth[a*x])) + (60267
*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(64*E^(6*ArcCoth[a*x])
) - (382227*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(256*E^(4*A
rcCoth[a*x])) - (40827*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/
(64*E^(2*ArcCoth[a*x])) + (E^(2*ArcCoth[a*x])*(1117 + 1906*E^(2*ArcCoth[a*x
]) + 821*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 25/4
}, E^(2*ArcCoth[a*x])])/3094 + (4*E^(2*ArcCoth[a*x])*(27 + 50*E^(2*ArcCoth[
a*x]) + 23*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 13/4}, {1, 1,
 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(2*ArcCoth[a*x])*Hypergeometric
PFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 +
(16*E^(4*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1
, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(6*ArcCoth[a*x])*HypergeometricPF
Q[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547))/(9
*a^3)
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

Maxima [A] time = 1.55226, size = 274, normalized size = 1.29

$$-\frac{1}{48}\,a\left(\frac{4\left(\frac{425\,(ax-1)}{ax+1}-\frac{462\,(ax-1)^2}{(ax+1)^2}+\frac{165\,(ax-1)^3}{(ax+1)^3}-96\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(425*(a*x - 1)/(a*x + 1) - 462*(a*x - 1)^2/(a*x + 1)^2 + 165*(a*
x - 1)^3/(a*x + 1)^3 - 96)/(a^4*((a*x - 1)/(a*x + 1))^(13/4) - 3*a^4*((a*x
- 1)/(a*x + 1))^(9/4) + 3*a^4*((a*x - 1)/(a*x + 1))^(5/4) - a^4*((a*x - 1)/
(a*x + 1))^(1/4)) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^4)
```

Fricas [A] time = 1.58515, size = 358, normalized size = 1.68

$$-\frac{330\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(8\,a^4x^4+34\,a^3x^3\cdots\right.}{48\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="fricas")
```

```
[Out] -1/48*(330*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*(a*x - 1)*lo
g(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*(a*x - 1)*log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1) - 2*(8*a^4*x^4 + 34*a^3*x^3 + 87*a^2*x^2 - 226*a*x - 287)*((a*
x - 1)/(a*x + 1))^(3/4))/(a^4*x - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A] time = 1.18243, size = 259, normalized size = 1.22

$$-\frac{1}{48}a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{384}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4\left(\frac{174(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\cdots\right.}{a^4(\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 + 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 + 384/(a^4*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(174*(a*x - 1)*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1) - 69*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^2 - 137*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.80 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=176

$$-\frac{25\sqrt[4]{\frac{1}{ax}+1}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5x\left(\frac{1}{ax}+1\right)^{5/4}}{4a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] (-25*(1 + 1/(a*x))^(1/4))/(2*a^2*(1 - 1/(a*x))^(1/4)) + (5*(1 + 1/(a*x))^(5
/4)*x)/(4*a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x
))^(1/4)) + (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rubi [A] time = 0.0676022, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 212, 206, 203}

$$-\frac{25\sqrt[4]{\frac{1}{ax}+1}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5x\left(\frac{1}{ax}+1\right)^{5/4}}{4a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x,x]
```


```
[Out] (-25*(1 + 1/(a*x))^(1/4))/(2*a^2*(1 - 1/(a*x))^(1/4)) + (5*(1 + 1/(a*x))^(5
/4)*x)/(4*a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x
))^(1/4)) + (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{25\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} + \frac{25\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{25\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.212247, size = 80, normalized size = 0.45

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-45e^{2\coth^{-1}(ax)}+16e^{4\coth^{-1}(ax)}+25\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + 25\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 25\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x,x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(25 - 45*E^(2*ArcCoth[a*x]) + 16*E^(4*ArcCoth[a*x])
))/(-1 + E^(2*ArcCoth[a*x]))^2 + 25*ArcTan[E^(ArcCoth[a*x]/2)] + 25*ArcTanh
[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F] time = 0.325, size = 0, normalized size = 0.

$$\int x\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

Maxima [A] time = 1.48703, size = 224, normalized size = 1.27

$$\frac{1}{8}a\left(\frac{4\left(\frac{45(ax-1)}{ax+1}-\frac{25(ax-1)^2}{(ax+1)^2}-16\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-2a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(45*(a*x - 1)/(a*x + 1) - 25*(a*x - 1)^2/(a*x + 1)^2 - 16)/(a^3*((
a*x - 1)/(a*x + 1))^(9/4) - 2*a^3*((a*x - 1)/(a*x + 1))^(5/4) + a^3*((a*x -
 1)/(a*x + 1))^(1/4)) - 50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4)
- 1)/a^3)
```

Fricas [A] time = 1.67888, size = 332, normalized size = 1.89

$$-\frac{50(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-25(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+25(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(2a^3x^3+11a^2x^2-34a\right.}{8\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(50*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 25*(a*x - 1)*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) + 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1) - 2*(2*a^3*x^3 + 11*a^2*x^2 - 34*a*x - 43)*((a*x - 1)/(a*x + 1))^(
3/4))/(a^3*x - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A] time = 1.16932, size = 217, normalized size = 1.23

$$-\frac{1}{8}\,a\left(\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{64}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{9\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-13\left(\frac{ax}{ax}\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 + 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 +
64/(a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(9*(a*x - 1)*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) - 13*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*((a*x - 1)/(a*x + 1
) - 1)^2))
```

3.81 $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=130

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{10\sqrt[4]{\frac{1}{ax}+1}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (-10*(1 + 1/(a*x))^(1/4))/(a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(5/4)*x)
/(1 - 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/
a + (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rubi [A] time = 0.0440145, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6170, 94, 93, 212, 206, 203}

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{10\sqrt[4]{\frac{1}{ax}+1}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2),x]
```


```
[Out] (-10*(1 + 1/(a*x))^(1/4))/(a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(5/4)*x)
/(1 - 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/
a + (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```


Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{10\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.133485, size = 67, normalized size = 0.52

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(4e^{2\coth^{-1}(ax)}-5\right)}{e^{2\coth^{-1}(ax)}-1} + 5\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 5\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-5 + 4*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
])) + 5*ArcTan[E^(ArcCoth[a*x]/2)] + 5*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A] time = 1.48489, size = 177, normalized size = 1.36

$$-\frac{1}{2}a\left(\frac{4\left(\frac{5(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)/(a*x + 1))^(5/4) - a^
2*((a*x - 1)/(a*x + 1))^(1/4)) + 10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2
 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 5*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^2)
```

Fricas [A] time = 1.60109, size = 304, normalized size = 2.34

$$-\frac{10(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(a^2x^2-8ax-9\right)\left(\frac{ax-1}{ax+1}\right)}{2\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(10*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 5*(a*x - 1)*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1) - 2*(a^2*x^2 - 8*a*x - 9)*((a*x - 1)/(a*x + 1))^(3/4))/(a^2*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-5/4), x)
```

Giac [A] time = 1.20936, size = 190, normalized size = 1.46

$$-\frac{1}{2}a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{5(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 + 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*
(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))))
```

3.82 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=320

$$-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\right.$$

```
[Out] (-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.297283, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6171, 98, 21, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + (4a)\,\text{Subst}\left(\int \frac{-\frac{1}{4a}+\frac{x}{4a^2}}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\,\text{Subst}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Su}}{}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\right.}{}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2$$

Mathematica [C] time = 0.0710473, size = 30, normalized size = 0.09

$$8e^{\frac{1}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)-1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] 8*E^(ArcCoth[a*x]/2)*(-1 + Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x]
)])
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Maxima [A] time = 1.53527, size = 329, normalized size = 1.03

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
```

```
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Fricas [A] time = 1.67634, size = 975, normalized size = 3.05

$$\frac{4\sqrt{2}(ax-1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}(ax-1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*(a*x - 1)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) - 1) + 4*sqrt(2)*(a*x - 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a
*x + 1))^(1/4) + 1) + sqrt(2)*(a*x - 1)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*(a*x - 1)*log(-4*sqrt(2
)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - 4*(a*x -
 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 2*(a*x - 1)*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1) - 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) - 16*(a*
x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Giac [A] time = 1.1944, size = 340, normalized size = 1.06

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

3.83 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=299

$$-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\tan}{}$$

```
[Out] -5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (4*a*(1 + 1/(a*x))^(5/4))/(1
 - 1/(a*x))^(1/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a
*x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1
/(a*x))^(1/4)])/Sqrt[2] - (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
- (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) + (5*a*Lo
g[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rubi [A] time = 0.256712, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (4*a*(1 + 1/(a*x))^(5/4))/(1
 - 1/(a*x))^(1/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a
*x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1
/(a*x))^(1/4)])/Sqrt[2] - (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
- (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) + (5*a*Lo
g[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\,\mathrm{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + 5\,\mathrm{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - (10a)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - (10a)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - (5a)\,\mathrm{Subst}\left(\int\right.$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}(5a)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{2}(5a)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \right.}{2\sqrt{2}}$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - 5$$

Mathematica [A] time = 0.372751, size = 173, normalized size = 0.58

$$a\left(-\frac{10e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} - \frac{8e^{\frac{5}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} - \frac{5\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} + \frac{5\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] a*((-10*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - (8*E^((5*ArcCoth[a*x
])/2))/(1 + E^(2*ArcCoth[a*x])) - (5*ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)]
)/Sqrt[2] + (5*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (5*Log[1 -
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]) + (5*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Maxima [A] time = 1.5586, size = 275, normalized size = 0.92

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/(((a*x - 1)/(a*x
+ 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Fricas [A] time = 1.8141, size = 1169, normalized size = 3.91

$$\frac{20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\left(ax^2-x\right)\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^4\right)^{\frac{1}{4}}}{a^4}\right)+20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\left(ax^2-x\right)\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*
a^3*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(a^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a
^4)^(1/4))/a^4) + 20*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*arctan((a^4 - sqrt(2)*
(a^4)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x -
1)/(a*x + 1)) + sqrt(a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1
))^(1/4))*(a^4)^(1/4))/a^4) + 5*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*log(15625*a
^6*sqrt((a*x - 1)/(a*x + 1)) + 15625*sqrt(a^4)*a^4 + 15625*sqrt(2)*(a^4)^(3
/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 5*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*lo
g(15625*a^6*sqrt((a*x - 1)/(a*x + 1)) + 15625*sqrt(a^4)*a^4 - 15625*sqrt(2)
*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(9*a^2*x^2 + 8*a*x - 1)*(
(a*x - 1)/(a*x + 1))^(3/4))/(a*x^2 - x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20863, size = 293, normalized size = 0.98

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((a*x - 1)*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

3.84 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=351

$$-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{25a^2\,\mathrm{l}\cdots}{\cdots}$$

```
[Out] (-25*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 - (5*a^2*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(5/4))/2 - (2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1
/4) + (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) - (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rubi [A] time = 0.284915, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 78, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{25a^2\,\mathrm{l}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-25*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 - (5*a^2*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(5/4))/2 - (2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1
/4) + (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) - (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\mathrm{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\,\mathrm{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}(25a)\,\mathrm{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{8}(25a)\,\mathrm{Subst}\Bigg(\int$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}\left(25a^2\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}\left(25a^2\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\left(25a^2\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{8}\left(25a^2\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{25a^2\log\Bigg(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}}{8\sqrt{2}}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{25a^2\tan^{-1}\Bigg(1-\frac{\sqrt{2}}{\sqrt[4]{}}}{4\sqrt{2}}$$

Mathematica [A] time = 0.2621, size = 186, normalized size = 0.53

$$\frac{1}{16}a^2\left(-128e^{\frac{1}{2}\coth^{-1}(ax)}-\frac{104e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1}+\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}+1\right)^2}-25\sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)+25\sqrt{2}\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (a^2*(-128*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a
*x]))^2 - (104*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 50*Sqrt[2]*Ar
cTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)] + 50*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(Arc
Coth[a*x]/2)] - 25*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a
*x]] + 25*Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Maxima [A] time = 1.52576, size = 329, normalized size = 0.94

$$-\frac{1}{16}\left(25\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(25*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1))*a + 8*(45*(a*x - 1)*a/(a*x + 1) + 25*(a*x - 1)^2*
a/(a*x + 1)^2 + 16*a)/(((a*x - 1)/(a*x + 1))^(9/4) + 2*((a*x - 1)/(a*x + 1)
)^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Fricas [A] time = 1.7602, size = 1247, normalized size = 3.55

$$100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}\left(ax^3-x^2\right)\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}\left(ax^3-x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(100*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)*arctan(-(a^8 + sqrt(2)*(a^8)^(1
/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x
 + 1)) + sqrt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4
))*(a^8)^(1/4))/a^8) + 100*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)*arctan((a^8 -
sqrt(2)*(a^8)^(1/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqr
t((a*x - 1)/(a*x + 1)) + sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)
/(a*x + 1))^(1/4))*(a^8)^(1/4))/a^8) + 25*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)
*log(244140625*a^12*sqrt((a*x - 1)/(a*x + 1)) + 244140625*sqrt(a^8)*a^8 + 2
44140625*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 25*sqrt(2)*
(a^8)^(1/4)*(a*x^3 - x^2)*log(244140625*a^12*sqrt((a*x - 1)/(a*x + 1)) + 24
4140625*sqrt(a^8)*a^8 - 244140625*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x +
 1))^(1/4)) - 4*(43*a^3*x^3 + 34*a^2*x^2 - 11*a*x - 2)*((a*x - 1)/(a*x + 1)
)^(3/4))/(a*x^3 - x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18918, size = 328, normalized size = 0.93

$$-\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-25\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 128*a/((a*x - 1)/(a*x + 1))^(1/4) +
8*(9*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 13*a*((a*x - 1)/(a
*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.85 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=385

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{55a^3\log}{}$$

```
[Out] (-55*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 - (11*a^3*(1 - 1/(a*x))
^(3/4)*(1 + 1/(a*x))^(5/4))/4 - (2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(
1/4) - (a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(9/4))/3 + (55*a^3*ArcTan[1 -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
- (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*
x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (55*a^3*Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(16*Sqrt[2])
```

Rubi [A] time = 0.31253, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 89, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{55a^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^4,x]
```


```
[Out] (-55*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 - (11*a^3*(1 - 1/(a*x))
^(3/4)*(1 + 1/(a*x))^(5/4))/4 - (2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(
1/4) - (a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(9/4))/3 + (55*a^3*ArcTan[1 -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
- (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*
x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (55*a^3*Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
```

```
)])/(16*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \left(2a^3\right)\text{Subst}\left(\int \frac{\left(\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{8}\left(55a^2\right)\text{Subst}\Bigg(\int$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\Big(1$$

Mathematica [C] time = 0.143025, size = 104, normalized size = 0.27

$$a^3\left(-\frac{55}{32}\text{RootSum}\left[\#1^4+1\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]-\frac{e^{\frac{1}{2}\coth^{-1}(ax)}\left(462e^{2\coth^{-1}(ax)}+425e^{4\coth^{-1}(ax)}+\right.}{12\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] a^3*(-(E^(ArcCoth[a*x]/2)*(165 + 462*E^(2*ArcCoth[a*x]) + 425*E^(4*ArcCoth[
a*x]) + 96*E^(6*ArcCoth[a*x])))/(12*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootS
um[1 + #1^4 & , (ArcCoth[a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ])/3
2)
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Maxima [A] time = 1.67789, size = 389, normalized size = 1.01

$$-\frac{1}{96}\left(165\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(165*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(425*(a*x - 1)*a^2/(a*x + 1) + 462*(a*x
- 1)^2*a^2/(a*x + 1)^2 + 165*(a*x - 1)^3*a^2/(a*x + 1)^3 + 96*a^2)/(((a*x -
 1)/(a*x + 1))^(13/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + 3*((a*x - 1)/(a*x +
 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Fricas [A] time = 1.71029, size = 1319, normalized size = 3.43

$$660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}\left(ax^4-x^3\right)\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}\left(ax^{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*arctan(-(a^12 + sqrt(2)*(a^12)
^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(
a*x + 1)) + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1)
)^(1/4))*(a^12)^(1/4))/a^12) + 660*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*arcta
n((a^12 - sqrt(2)*(a^12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sq
rt(a^18*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*
a^9*((a*x - 1)/(a*x + 1))^(1/4))*(a^12)^(1/4))/a^12) + 165*sqrt(2)*(a^12)^(
1/4)*(a*x^4 - x^3)*log(27680640625*a^18*sqrt((a*x - 1)/(a*x + 1)) + 2768064
0625*sqrt(a^12)*a^12 + 27680640625*sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x
 + 1))^(1/4)) - 165*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*log(27680640625*a^18
*sqrt((a*x - 1)/(a*x + 1)) + 27680640625*sqrt(a^12)*a^12 - 27680640625*sqrt
(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(287*a^4*x^4 + 226*a^
3*x^3 - 87*a^2*x^2 - 34*a*x - 8)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^4 - x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21429, size = 393, normalized size = 1.02

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-165\sqrt{2}a^2\log\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) - 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 768*a^2/((a*x - 1)/(a*x
+ 1))^(1/4) + 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) +
69*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 137*a^2*((a*x
- 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

3.86 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{11x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{48a^2}-\frac{269x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{960a^3}+\frac{611x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{1920a^4}+\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] (611*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(1920*a^4) - (269*(1 - 1/(a
*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4)*x^3)/(48*a^2) - (9*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rubi [A] time = 0.139715, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{11x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{48a^2}-\frac{269x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{960a^3}+\frac{611x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{1920a^4}+\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(ArcCoth[a*x]/2),x]
```


```
[Out] (611*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(1920*a^4) - (269*(1 - 1/(a
*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4)*x^3)/(48*a^2) - (9*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^6\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{55}{4a^2}+\frac{27x}{2a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x,$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{60}\text{Subst}\left(\int\right.$$

$$= -\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1\right.}{40a}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1\right.}{40a}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1\right.}{40a}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1\right.}{40a}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1\right.}{40a}$$

Mathematica [A] time = 5.28007, size = 173, normalized size = 0.68

$$\frac{\frac{9620e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}-\frac{34000e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^{2}}+\frac{64640e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^{3}}-\frac{62976e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^{4}}+\frac{24576e^{\frac{19}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^{5}}+465\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-}{3840a^{5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((24576*E^((19*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (62976*E^((1
5*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^((11*ArcCoth[a*x
])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (34000*E^((7*ArcCoth[a*x])/2))/(-1 + E
^(2*ArcCoth[a*x]))^2 + (9620*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x
])) - 930*ArcTan[E^(-ArcCoth[a*x]/2)] + 465*Log[1 - E^(-ArcCoth[a*x]/2)] -
465*Log[1 + E^(-ArcCoth[a*x]/2)])/(3840*a^5)
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int x^{4}\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A] time = 1.50989, size = 350, normalized size = 1.38

$$-\frac{1}{3840}a\left(\frac{4\left(2405\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^{6}}{ax+1}-\frac{10(ax-1)^{2}a^{6}}{(ax+1)^{2}}+\frac{10(ax-1)^{3}a^{6}}{(ax+1)^{3}}-\frac{5(ax-1)^{4}a^{6}}{(ax+1)^{4}}+\frac{(ax-1)^{5}a^{6}}{(ax+1)^{5}}-a^{6}}+\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{6}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(2405*((a*x - 1)/(a*x + 1))^(17/4) - 1120*((a*x - 1)/(a*x + 1)
)^(13/4) + 5090*((a*x - 1)/(a*x + 1))^(9/4) - 696*((a*x - 1)/(a*x + 1))^(5/
4) + 465*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
- 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^6
/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 465*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A] time = 1.60675, size = 323, normalized size = 1.28

$$\frac{2\left(384\,a^5x^5 - 48\,a^4x^4 + 8\,a^3x^3 - 98\,a^2x^2 + 73\,ax + 611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 465}{3840\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 - 48*a^4*x^4 + 8*a^3*x^3 - 98*a^2*x^2 + 73*a*x + 611
)*((a*x - 1)/(a*x + 1))^(1/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 4
65*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 465*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1))/a^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**4*((a*x - 1)/(a*x + 1))**(1/4), x)
```

<u>Giac</u> [A] time = 1.23068, size = 316, normalized size = 1.25

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{696\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{5090\,(ax-1)^2\left(\right.}{(ax+1)^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 - 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(696*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 5090*(a*x -
1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1120*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1)^3 - 2405*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1)^4 - 465*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

3.87 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{29x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{96a^2}-\frac{83x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{192a^3}-\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/}$$

```
[Out] (-83*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(192*a^3) + (29*(1 - 1/(a*x
))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(96*a^2) - (7*(1 - 1/(a*x))^(1/4)*(1 + 1/
(a*x))^(3/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4
- (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rubi [A] time = 0.114805, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{29x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{96a^2}-\frac{83x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{192a^3}-\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (-83*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(192*a^3) + (29*(1 - 1/(a*x
))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(96*a^2) - (7*(1 - 1/(a*x))^(1/4)*(1 + 1/
(a*x))^(3/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4
- (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^5\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{29}{4a^2}+\frac{7x}{a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x,\right.$$

$$= \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{24}\text{Subst}\left(\int\right.$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1\right.$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1\right.$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1\right.$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1\right.$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1\right.$$

Mathematica [A] time = 5.22897, size = 149, normalized size = 0.69

$$\frac{-\frac{980e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{2512e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}-\frac{3200e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}-33\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+33\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}+\cdots\right.}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (3200*E^((11*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^((7*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^2 - (980*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*Ar
cCoth[a*x])) + 66*ArcTan[E^(-ArcCoth[a*x]/2)] - 33*Log[1 - E^(-ArcCoth[a*x]
/2)] + 33*Log[1 + E^(-ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int x^3\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A] time = 1.63207, size = 302, normalized size = 1.4

$$-\frac{1}{384}a\left(\frac{4\left(245\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-107\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+279\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(245*((a*x - 1)/(a*x + 1))^(13/4) - 107*((a*x - 1)/(a*x + 1))^(
9/4) + 279*((a*x - 1)/(a*x + 1))^(5/4) - 33*((a*x - 1)/(a*x + 1))^(1/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A] time = 1.65704, size = 296, normalized size = 1.37

$$\frac{2\left(48\,a^4x^4 - 8\,a^3x^3 + 2\,a^2x^2 - 25\,ax - 83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 - 8*a^3*x^3 + 2*a^2*x^2 - 25*a*x - 83)*((a*x - 1)/(a*x
 + 1))^(1/4) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.29344, size = 274, normalized size = 1.27

$$\frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{279(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{107(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}-\cdots\right.}{a^5\left(\frac{ax-1}{ax+1}-\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^5 - 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5 +
 4*(279*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 107*(a*x - 1)^2*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 245*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(1/4)/(a*x + 1)^3 - 33*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*
x + 1) - 1)^4))
```

3.88 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{11x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{5x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

```
[Out] (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) - (5*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
- (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rubi [A] time = 0.0915285, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{11x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{5x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) - (5*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
- (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^4\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{5}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}+\frac{5x}{2a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{6}\text{Subst}\left(\int -\ldots\right.$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\,\text{Subst}\left(\int \frac{\ldots}{x(1-\ldots}\right.}{1\ldots}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+\ldots}\right.}{4a\ldots}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\ldots\right.}{8a\ldots}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\ldots\right.}{8a^3}$$

Mathematica [C] time = 8.38715, size = 389, normalized size = 2.17

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(256e^{6\coth^{-1}(ax)}\left(1090e^{2\coth^{-1}(ax)}+437e^{4\coth^{-1}(ax)}+685\right)\text{HypergeometricPFQ}\left(\left\{\frac{7}{4},2,2,2\right\},\left\{1,1,\frac{19}{4}\right\},e\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -(-22034705 - 26688365*E^(2*ArcCoth[a*x]) - 3731255*E^(4*ArcCoth[a*x]) + 31
22405*E^(6*ArcCoth[a*x]) + 22034705*Hypergeometric2F1[3/4, 1, 7/4, E^(2*Arc
Coth[a*x])] + 17244920*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^
(2*ArcCoth[a*x])] - 9077530*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/
4, E^(2*ArcCoth[a*x])] - 7043960*E^(6*ArcCoth[a*x])*Hypergeometric2F1[3/4,
1, 7/4, E^(2*ArcCoth[a*x])] + 446985*E^(8*ArcCoth[a*x])*Hypergeometric2F1[3
/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(685 + 1090*E^(2*A
rcCoth[a*x]) + 437*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2, 2, 2}, {1
, 1, 19/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(21 + 38*E^(2*Arc
Coth[a*x]) + 17*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2, 2, 2, 2}, {1
, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*Hypergeometric
PFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 8192*E^
(8*ArcCoth[a*x])*HypergeometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}
, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ[{7/4, 2,
 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])])/(221760*a^3*E^((5*Ar
cCoth[a*x])/2))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int x^2 \sqrt[4]{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A] time = 1.53793, size = 252, normalized size = 1.41

$$-\frac{1}{48}\,a\left(\frac{4\left(29\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(29*((a*x - 1)/(a*x + 1))^(9/4) - 6*((a*x - 1)/(a*x + 1))^(5/4)
+ 9*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2
*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 - 9*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A] time = 1.66984, size = 270, normalized size = 1.51

$$\frac{2\left(8\,a^3x^3-2\,a^2x^2+ax+11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 2*a^2*x^2 + a*x + 11)*((a*x - 1)/(a*x + 1))^(1/4) - 18
*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) +
1) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.20354, size = 232, normalized size = 1.3

$$-\frac{1}{48}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{29\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}-9\,\left(\right.\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 29*(a*x - 1)^2*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

3.89 $\int e^{-\frac{1}{2}\coth^{-1}(ax)} x\,dx$

Optimal. Leaf size=142

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

```
[Out] -((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(5/4)*(
1 + 1/(a*x))^(3/4)*x^2)/2 - ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rubi [A] time = 0.0602726, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(ArcCoth[a*x]/2),x]
```


```
[Out] -((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(5/4)*(
1 + 1/(a*x))^(3/4)*x^2)/2 - ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
```

```
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^3\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.154832, size = 66, normalized size = 0.46

$$\frac{-\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}-5\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - \tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((-2*E^((3*ArcCoth[a*x])/2)*(-5 + E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a
*x]))^2 - ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A] time = 1.53275, size = 204, normalized size = 1.44

$$-\frac{1}{8}a\left(\frac{4\left(5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(5*((a*x - 1)/(a*x + 1))^(5/4) - ((a*x - 1)/(a*x + 1))^(1/4))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 +
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A] time = 1.63593, size = 244, normalized size = 1.72

$$\frac{2\left(2a^2x^2-ax-3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - a*x - 3)*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x -
 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.19221, size = 189, normalized size = 1.33

$$\frac{1}{8}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{5\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1)/a^3 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*(5*(a*
x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))
/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.90 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=97

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rubi [A] time = 0.0355157, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6170, 94, 93, 298, 203, 206}

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcCoth[a*x]/2),x]
```


```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```


Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$
$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.0430654, size = 33, normalized size = 0.34

$$-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4}, 2, \frac{7}{4}, e^{2\coth^{-1}(ax)}\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcCoth[a*x]/2),x]
```

```
[Out] (-8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, E^(2*ArcCoth[a*x]
)])/(3*a)
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int \sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A] time = 1.48446, size = 150, normalized size = 1.55

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A] time = 1.65827, size = 225, normalized size = 2.32

$$\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A] time = 1.14995, size = 146, normalized size = 1.51

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.91 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)-$$

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rubi [A] time = 0.228066, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6171, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)-$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\right.}{\sqrt{2}}$$

$$= \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.0437114, size = 30, normalized size = 0.1

$$\frac{8}{3}e^{\frac{3}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{8},1,\frac{11}{8},e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x),x]
```

```
[Out] (8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/8, 1, 11/8, E^(4*ArcCoth[a*x]
)])/3
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Maxima [A] time = 1.54838, size = 302, normalized size = 1.04

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
```

```
x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

Fricas [A] time = 1.60693, size = 806, normalized size = 2.77

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 2*sqrt
(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt
((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/2
*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
+ 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x, x)
```

Giac [A] time = 1.17378, size = 313, normalized size = 1.08

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

3.92 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=268

$$-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\,\text{t}}{}$$

```
[Out] -(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) - (a*ArcTan[1 - (Sqrt[2]*(1 -
1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (a*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(2*Sqrt[2]) + (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rubi [A] time = 0.218081, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\,\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```


```
[Out] -(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) - (a*ArcTan[1 - (Sqrt[2]*(1 -
1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (a*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(2*Sqrt[2]) + (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + a\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+\right.$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\operatorname{Subst}}{}$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.050332, size = 33, normalized size = 0.12

$$-\frac{8}{3}ae^{\frac{3}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

```
[Out] (-8*a*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a
*x])])/3
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Maxima [A] time = 1.50447, size = 251, normalized size = 0.94

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Fricas [A] time = 1.7223, size = 990, normalized size = 3.69

$$-\frac{4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)+4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
4*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - sqr
t(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - sqrt(2)
*(a^4)^(1/4)*x*log(a^2*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a^4)^(1/4)*a*((
a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)) + sqrt(2)*(a^4)^(1/4)*x*log(a^2*sqrt
((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt(a^4)) + 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**2, x)
```

Giac [A] time = 1.17205, size = 251, normalized size = 0.94

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
```

```
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

3.93 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[Out] (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(5/4)*
(1 + 1/(a*x))^(3/4))/2 + (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rubi [A] time = 0.250436, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(5/4)*
(1 + 1/(a*x))^(3/4))/2 + (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\operatorname{Subst}\left(\int \frac{x\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a\operatorname{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}a\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x,$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}a^2\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) -$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}a^2\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2\log}{}$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\tan^{-1}\left(1\right.}{4\sqrt{}}$$

Mathematica [C] time = 0.0693259, size = 56, normalized size = 0.18

$$-\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)-2\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCot
h[a*x])] - 2*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Maxima [A] time = 1.54085, size = 306, normalized size = 0.96

$$-\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*a*((a*x - 1)/(a*x + 1))^(5/4) + a*((a*x
 - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

Fricas [A] time = 1.72257, size = 1035, normalized size = 3.24

$$4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax}{ax}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*((a*
x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x
+ 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a
^8) + 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*((a*x
 - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x +
 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a^
8) - sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a
^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)) + sqrt(2)*(a^8)^(1/4
)*x^2*log(a^4*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1
)/(a*x + 1))^(1/4) + sqrt(a^8)) + 4*(3*a^2*x^2 + a*x - 2)*((a*x - 1)/(a*x +
 1))^(1/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**3, x)
```

Giac [A] time = 1.1465, size = 301, normalized size = 0.94

$$-\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1) + a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.94 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}$$

```
[Out] (-3*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/8 - (a^3*(1 - 1/(a*x))^(5/
4)*(1 + 1/(a*x))^(3/4))/12 + (a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/
(3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(8*Sqrt[2]) - (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)
] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (3*a
^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rubi [A] time = 0.27745, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```


```
[Out] (-3*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/8 - (a^3*(1 - 1/(a*x))^(5/
4)*(1 + 1/(a*x))^(3/4))/12 + (a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/
(3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(8*Sqrt[2]) - (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)
] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (3*a
^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}\left(-1+\frac{x}{2a}\right)}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{1}{16}\left(3a^2\right)\text{S}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{4}\left(3a^3\right)\text{Su}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{4}\left(3a^3\right)\text{Su}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{8}\left(3a^3\right)\text{Su}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{16}\left(3a^3\right)\text{S}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{3a^3\log\left(1\right.}{}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{3a^3\tan^{-1}\left(\right.}{8}$$

Mathematica [C] time = 0.126709, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(9\text{RootSum}\left[\#1^4+1\&,\frac{2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)+\coth^{-1}(ax)}{\#1^3}\&\right]-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(6e^{2\coth^{-1}(ax)}+9e^{4\coth^{-1}(ax)}+\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^((3*ArcCoth[a*x])/2)*(29 + 6*E^(2*ArcCoth[a*x]) + 9*E^(4*ArcCot
h[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x]
 + 2*Log[E^(-ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int\frac{1}{x^4}\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Maxima [A] time = 1.48684, size = 374, normalized size = 1.05

$$\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}a^2\log\left(\sqrt{2}\left(\frac{ax}{ax}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
```

```
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(29*a^2*((a*x - 1)/(a*x + 1))
^(9/4) + 6*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 9*a^2*((a*x - 1)/(a*x + 1))^(1
/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
+ 1)^3 + 1))*a
```

Fricas [A] time = 1.66775, size = 1106, normalized size = 3.11

$$-\frac{36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1)
/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a
^12)))/a^12) + 36*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^(3
/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a
*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt(a^12)))/a^12) - 9*sqrt(2)*(a^12)^(1/4)*x^3*log(9*a^6*sqrt((a*x - 1)/(
a*x + 1)) + 9*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt
(a^12)) + 9*sqrt(2)*(a^12)^(1/4)*x^3*log(9*a^6*sqrt((a*x - 1)/(a*x + 1)) -
9*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt(a^12)) + 4*
(11*a^3*x^3 + a^2*x^2 - 2*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**4, x)
```

Giac [A] time = 1.19298, size = 366, normalized size = 1.03

$$\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}a^2\log\left(\sqrt{2}\left(\frac{ax}{ax}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(1/4)/(a*x + 1) + 29*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/
(a*x + 1)^2 + 9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

3.95 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{5x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{16a^2}-\frac{157x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{320a^3}+\frac{557x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{640a^4}-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\tanh^{-1}\left(\cdots\right)}{128a^5}$$

```
[Out] (557*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(640*a^4) - (157*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(3/4)*(1 +
1/(a*x))^(1/4)*x^3)/(16*a^2) - (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rubi [A] time = 0.14026, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{5x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{16a^2}-\frac{157x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{320a^3}+\frac{557x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{640a^4}-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\tanh^{-1}\left(\cdots\right)}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((3*ArcCoth[a*x])/2),x]
```



```
[Out] (557*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(640*a^4) - (157*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(3/4)*(1 +
1/(a*x))^(1/4)*x^3)/(16*a^2) - (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^6\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{11}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{75}{4a^2}+\frac{33x}{2a^3}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,\right.$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5 - \frac{1}{60}\text{Subst}\left(\int\right.$$

$$= -\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/}}{40}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/}}{40}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/}}{40}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/}}{40}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/}}{40}$$

Mathematica [A] time = 5.31345, size = 173, normalized size = 0.68

$$\frac{\frac{5500e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}-\frac{14032e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+\frac{23936e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}-\frac{22016e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}+\frac{8192e^{\frac{17}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5}+1185\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-118}{1280a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((8192*E^((17*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (22016*E^((13
*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((9*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (14032*E^((5*ArcCoth[a*x])/2))/(-1 + E^(
2*ArcCoth[a*x]))^2 + (5500*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) +
2370*ArcTan[E^(-ArcCoth[a*x]/2)] + 1185*Log[1 - E^(-ArcCoth[a*x]/2)] - 1185
*Log[1 + E^(-ArcCoth[a*x]/2)])/(1280*a^5)
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A] time = 1.50333, size = 350, normalized size = 1.38

$$-\frac{1}{1280}a\left(\frac{4\left(1375\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+395\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(1375*((a*x - 1)/(a*x + 1))^(19/4) - 1992*((a*x - 1)/(a*x + 1)
)^(15/4) + 3710*((a*x - 1)/(a*x + 1))^(11/4) - 1440*((a*x - 1)/(a*x + 1))^(
7/4) + 395*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*
x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a
^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 1
185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A] time = 1.64321, size = 331, normalized size = 1.31

$$\frac{2\left(128\,a^5x^5-48\,a^4x^4+24\,a^3x^3-114\,a^2x^2+243\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right.}{1280\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 - 48*a^4*x^4 + 24*a^3*x^3 - 114*a^2*x^2 + 243*a*x +
557)*((a*x - 1)/(a*x + 1))^(3/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))
 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1185*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2517, size = 316, normalized size = 1.25

$$\frac{1}{1280}a\left(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1440\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{3710\,(ax-1)^2}{(ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 + 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1440*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 1992*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1375*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3
/4)/(a*x + 1)^4 - 395*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

3.96 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{15x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{32a^2}-\frac{63x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{64a^3}+\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{}$$

```
[Out] (-63*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(64*a^3) + (15*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(32*a^2) - (3*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
(123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rubi [A] time = 0.116205, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{15x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{32a^2}-\frac{63x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{64a^3}+\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*ArcCoth[a*x])/2),x]
```


```
[Out] (-63*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(64*a^3) + (15*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(32*a^2) - (3*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
(123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^5\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4-\frac{1}{4}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4+\frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}+\frac{9x}{a^3}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x$$

$$= \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4-\frac{1}{24}\text{Subst}\left(\int\right.$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}$$

Mathematica [A] time = 5.25472, size = 149, normalized size = 0.69

$$\frac{-\frac{532e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{1008e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}-\frac{1152e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{512e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}-123\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+123\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}+\right.}{128a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((512*E^((13*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (1152*E^((9*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((5*ArcCoth[a*x])/2))
/(-1 + E^(2*ArcCoth[a*x]))^2 - (532*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[
a*x])) - 246*ArcTan[E^(-ArcCoth[a*x]/2)] - 123*Log[1 - E^(-ArcCoth[a*x]/2)]
 + 123*Log[1 + E^(-ArcCoth[a*x]/2)])/(128*a^4)
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A] time = 1.51981, size = 302, normalized size = 1.4

$$-\frac{1}{128}a\left(\frac{4\left(133\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-147\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+183\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-41\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/128*a*(4*(133*((a*x - 1)/(a*x + 1))^(15/4) - 147*((a*x - 1)/(a*x + 1))^(
11/4) + 183*((a*x - 1)/(a*x + 1))^(7/4) - 41*((a*x - 1)/(a*x + 1))^(3/4))/(
4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a
^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 123
*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A] time = 1.64639, size = 300, normalized size = 1.39

$$\frac{2\left(16\,a^4x^4-8\,a^3x^3+6\,a^2x^2-33\,ax-63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(2*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 33*a*x - 63)*((a*x - 1)/(a*x
 + 1))^(3/4) - 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22975, size = 274, normalized size = 1.27

$$-\frac{1}{128}a\left(\frac{246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{123\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{183\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{147\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)}{(ax+1)^2}\right.}{a^5\left(\frac{ax-}{ax+}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^5 + 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a
^5 - 4*(183*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 147*(a*x - 1)
^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 133*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1)^3 - 41*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)
/(a*x + 1) - 1)^4))
```

3.97 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{23x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{7x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}}}{12a}$$

```
[Out] (23*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) - (7*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 - (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rubi [A] time = 0.0919374, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{23x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{7x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (23*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) - (7*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 - (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^4\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{23}{4a^2}+\frac{7x}{2a^3}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x,$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{1}{6}\text{Subst}\left(\int -\right.$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{17\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 + \frac{17\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{17\,\text{Subst}\left(\int \right.}{8}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3 - \frac{17\tan^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.}{8a^3}$$

Mathematica [C] time = 8.14073, size = 389, normalized size = 2.17

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(256e^{6\coth^{-1}(ax)}\left(850e^{2\coth^{-1}(ax)}+325e^{4\coth^{-1}(ax)}+557\right)\text{HypergeometricPFQ}\left(\left\{\frac{5}{4},2,2,2\right\},\left\{1,1,\frac{17}{4}\right\},\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] -(-15779205 - 17312841*E^(2*ArcCoth[a*x]) - 1213875*E^(4*ArcCoth[a*x]) + 21
99249*E^(6*ArcCoth[a*x]) + 15779205*Hypergeometric2F1[1/4, 1, 5/4, E^(2*Arc
Coth[a*x])] + 14157000*E^(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^
(2*ArcCoth[a*x])] - 2472210*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/
4, E^(2*ArcCoth[a*x])] - 3598920*E^(6*ArcCoth[a*x])*Hypergeometric2F1[1/4,
1, 5/4, E^(2*ArcCoth[a*x])] + 21645*E^(8*ArcCoth[a*x])*Hypergeometric2F1[1/
4, 1, 5/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(557 + 850*E^(2*Arc
Coth[a*x]) + 325*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{5/4, 2, 2, 2}, {1,
1, 17/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(19 + 34*E^(2*ArcCo
th[a*x]) + 15*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{5/4, 2, 2, 2, 2}, {1,
1, 1, 17/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*HypergeometricPF
Q[{5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])] + 8192*E^(8
*ArcCoth[a*x])*HypergeometricPFQ[{5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 17/4},
E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ[{5/4, 2, 2
, 2, 2, 2}, {1, 1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])])/(112320*a^3*E^((7*ArcC
oth[a*x])/2))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A] time = 1.50274, size = 252, normalized size = 1.41

$$-\frac{1}{48}\,a\left(\frac{4\left(45\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3\,(ax-1)a^{4}}{ax+1}-\frac{3\,(ax-1)^{2}a^{4}}{(ax+1)^{2}}+\frac{(ax-1)^{3}a^{4}}{(ax+1)^{3}}-a^{4}}-\frac{102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}}+\frac{51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{4}}-\frac{51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(45*((a*x - 1)/(a*x + 1))^(11/4) - 30*((a*x - 1)/(a*x + 1))^(7/4
) + 17*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1
)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 -
 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A] time = 1.61666, size = 277, normalized size = 1.55

$$\frac{2\left(8\,a^3x^3 - 6\,a^2x^2 + 9\,ax + 23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 6*a^2*x^2 + 9*a*x + 23)*((a*x - 1)/(a*x + 1))^(3/4) +
102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 51*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23022, size = 232, normalized size = 1.3

$$\frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{45(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}-17\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 + 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 45*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 17*((a*x - 1)/(a*x + 1))^(3/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.98 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

```
[Out] (-3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(7/4)
*(1 + 1/(a*x))^(1/4)*x^2)/2 + (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^
2)
```

Rubi [A] time = 0.0586925, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 212, 206, 203}

$$\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(7/4)
*(1 + 1/(a*x))^(1/4)*x^2)/2 + (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^
2)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^3\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 + \frac{3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 - \frac{9\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 - \frac{9\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 + \frac{9\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\,\text{Subst}}{}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}x^2 + \frac{9\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.186756, size = 70, normalized size = 0.49

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(3e^{2\coth^{-1}(ax)}-7\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + 9\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-7 + 3*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
]))^2 + 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^
2)
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A] time = 1.50299, size = 205, normalized size = 1.44

$$-\frac{1}{8}a\left(\frac{4\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(7*((a*x - 1)/(a*x + 1))^(7/4) - 3*((a*x - 1)/(a*x + 1))^(3/4))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan(
((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A] time = 1.64627, size = 254, normalized size = 1.79

$$\frac{2\left(2a^2x^2-3ax-5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 3*a*x - 5)*((a*x - 1)/(a*x + 1))^(3/4) - 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20189, size = 190, normalized size = 1.34

$$-\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{7(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*
(7*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3*((a*x - 1)/(a*x + 1)
)^(3/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.99 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a - (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rubi [A] time = 0.0349385, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6170, 94, 93, 212, 206, 203}

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a - (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{6\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.110326, size = 55, normalized size = 0.56

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} - 3\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 3\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) - 3*ArcTan[E^(ArcCoth[a*x
]/2)] - 3*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A] time = 1.50452, size = 151, normalized size = 1.54

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A] time = 1.66902, size = 231, normalized size = 2.36

$$\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A] time = 1.19967, size = 147, normalized size = 1.5

$$\frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 + 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.100 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right.$$

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rubi [A] time = 0.225859, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6171, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\operatorname{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, \ldots\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\operatorname{Subst}\left(\int \frac{1+\ldots}{1+\ldots}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}-2x\ldots}{-1+\sqrt{2}x-\ldots}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{1+\ldots}}\right.}{\sqrt{2}}$$

$$= \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\ldots}}{\sqrt[4]{1-\ldots}}\right.$$

Mathematica [C] time = 0.0679206, size = 28, normalized size = 0.1

$$8e^{\frac{1}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

```
[Out] 8*E^(ArcCoth[a*x]/2)*Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x])]
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Maxima [A] time = 1.51463, size = 302, normalized size = 1.04

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
```

```
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

Fricas [A] time = 1.75478, size = 806, normalized size = 2.77

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{a}{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 2*sqrt
(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt
((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/2
*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
+ 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x, x)
```

Giac [A] time = 1.23801, size = 313, normalized size = 1.08

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

3.101 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=269

$$-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+$$

```
[Out] -(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - (3*a*ArcTan[1 - (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*Log[1 + Sqrt[1 -
1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1
/4)])/(2*Sqrt[2]) - (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rubi [A] time = 0.214073, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```


```
[Out] -(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - (3*a*ArcTan[1 - (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*Log[1 + Sqrt[1 -
1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1
/4)])/(2*Sqrt[2]) - (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\operatorname{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - (3a)\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \right.$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{}{1+}\right.$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{(3a)}{}$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.28488, size = 149, normalized size = 0.55

$$a\left(-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{3\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} - \frac{3\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} + \frac{3\tan^{-1}\left(1-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] a*((-2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + (3*ArcTan[1 - Sqrt[2]
*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/
```

```
Sqrt[2] + (3*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[
2]) - (3*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Maxima [A] time = 1.5244, size = 252, normalized size = 0.94

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Fricas [A] time = 1.66496, size = 1068, normalized size = 3.97

$$-\frac{12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^4\right)^{\frac{1}{4}}}{a^4}\right)+12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*a^3*((a*x
 - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(
a^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4)
)/a^4) + 12*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(1/4)*a^3*((a
*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqr
t(a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/
4))/a^4) + 3*sqrt(2)*(a^4)^(1/4)*x*log(729*a^6*sqrt((a*x - 1)/(a*x + 1)) +
729*sqrt(a^4)*a^4 + 729*sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
) - 3*sqrt(2)*(a^4)^(1/4)*x*log(729*a^6*sqrt((a*x - 1)/(a*x + 1)) + 729*sqr
t(a^4)*a^4 - 729*sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(
a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21375, size = 252, normalized size = 0.94

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
```

```
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

3.102 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.}{8\sqrt{2}}$$

```
[Out] (3*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4
)*(1 + 1/(a*x))^(1/4))/2 + (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rubi [A] time = 0.246536, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```


```
[Out] (3*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4
)*(1 + 1/(a*x))^(1/4))/2 + (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{8}(9a)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{8}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{9a^2\log\left(\right.}{}$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\tan^{-1}\left(1+\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.203784, size = 174, normalized size = 0.55

$$\frac{1}{16}a^2\left(\frac{24e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}+1\right)^2} - 9\sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right) + 9\sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\mathrm{co}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (a^2*((32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (24*E^(ArcCoth[a
*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 18*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] + 18*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] - 9*Sqrt[2]*L
og[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] + 9*Sqrt[2]*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Maxima [A] time = 1.51377, size = 308, normalized size = 0.97

$$-\frac{1}{16}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a - 8*(7*a*((a*x - 1)/(a*x + 1))^(7/4) + 3*a*((a*x
 - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

Fricas [A] time = 1.74379, size = 1130, normalized size = 3.54

$$36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(36*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(1/4)*a^6*((a
*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1)) + sq
rt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8)^(1
/4))/a^8) + 36*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(1/4)*a^
6*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1))
 + sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^
8)^(1/4))/a^8) + 9*sqrt(2)*(a^8)^(1/4)*x^2*log(531441*a^12*sqrt((a*x - 1)/(
a*x + 1)) + 531441*sqrt(a^8)*a^8 + 531441*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1
)/(a*x + 1))^(1/4)) - 9*sqrt(2)*(a^8)^(1/4)*x^2*log(531441*a^12*sqrt((a*x -
 1)/(a*x + 1)) + 531441*sqrt(a^8)*a^8 - 531441*sqrt(2)*(a^8)^(3/4)*a^6*((a*
x - 1)/(a*x + 1))^(1/4)) + 4*(5*a^2*x^2 + 3*a*x - 2)*((a*x - 1)/(a*x + 1))^
(3/4))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21282, size = 304, normalized size = 0.95

$$-\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-9\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(7*(a*x - 1)*a*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) + 3*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) +
1)^2)*a
```

3.103 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\cdots$$

```
[Out] (-17*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/24 - (a^3*(1 - 1/(a*x))^(
7/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))
/(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/
4)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(
a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) -
(17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rubi [A] time = 0.28086, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (-17*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/24 - (a^3*(1 - 1/(a*x))^(
7/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))
/(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/
4)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(
a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) -
(17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\operatorname{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{3}a^2\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}\left(-1+\frac{3x}{2a}\right)}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{24}\left(17a^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{16}\left(17a^2\right)\mathrm{Su}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{4}\left(17a^3\right)\mathrm{Sub}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{4}\left(17a^3\right)\mathrm{Sub}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{8}\left(17a^3\right)\mathrm{Sub}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{16}\left(17a^3\right)\mathrm{Su}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{17a^3\log\left(1+\right.}{}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{17a^3\tan^{-1}\left(1\right.}{8\sqrt{}}$$

Mathematica [C] time = 0.144604, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(51\text{RootSum}\left[\#1^4+1\&,\frac{2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)+\coth^{-1}(ax)}{\#1}\&\right]-\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(30e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\cdot\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^(ArcCoth[a*x]/2)*(45 + 30*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[
a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*x]
+ 2*Log[E^(-ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Maxima [A] time = 1.56982, size = 365, normalized size = 1.03

$$\frac{1}{96}\left(51\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(51*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
```

```
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a^2 - 8*(45*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 30*
a^2*((a*x - 1)/(a*x + 1))^(7/4) + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a
*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1
))*a
```

Fricas [A] time = 1.82051, size = 1203, normalized size = 3.38

$$-\frac{204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(1/4)*a^
9*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1))
 + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*
(a^12)^(1/4))/a^12) + 204*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(
a^12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^18*sqrt((a*x -
 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x
 + 1))^(1/4))*(a^12)^(1/4))/a^12) + 51*sqrt(2)*(a^12)^(1/4)*x^3*log(2413756
9*a^18*sqrt((a*x - 1)/(a*x + 1)) + 24137569*sqrt(a^12)*a^12 + 24137569*sqrt
(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) - 51*sqrt(2)*(a^12)^(1/4)
*x^3*log(24137569*a^18*sqrt((a*x - 1)/(a*x + 1)) + 24137569*sqrt(a^12)*a^12
 - 24137569*sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(23*a
^3*x^3 + 9*a^2*x^2 - 6*a*x + 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24661, size = 366, normalized size = 1.03

$$\frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-51\sqrt{2}a^2\log\left(\sqrt{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) - 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1) + 45*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^2 + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

3.104 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=287

$$\frac{181x^3\sqrt[4]{1-\frac{1}{ax}}}{240a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{1189x^2\sqrt[4]{1-\frac{1}{ax}}}{960a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{5533x\sqrt[4]{1-\frac{1}{ax}}}{1920a^4\sqrt[4]{\frac{1}{ax}+1}}+\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{\frac{1}{ax}+1}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

```
[Out] (26111*(1 - 1/(a*x))^(1/4))/(1920*a^5*(1 + 1/(a*x))^(1/4)) + (5533*(1 - 1/(
a*x))^(1/4)*x)/(1920*a^4*(1 + 1/(a*x))^(1/4)) - (1189*(1 - 1/(a*x))^(1/4)*x
^2)/(960*a^3*(1 + 1/(a*x))^(1/4)) + (181*(1 - 1/(a*x))^(1/4)*x^3)/(240*a^2*
(1 + 1/(a*x))^(1/4)) - (21*(1 - 1/(a*x))^(1/4)*x^4)/(40*a*(1 + 1/(a*x))^(1/
4)) + ((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + (1003*ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (1003*ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rubi [A] time = 0.162028, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{181x^3\sqrt[4]{1-\frac{1}{ax}}}{240a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{1189x^2\sqrt[4]{1-\frac{1}{ax}}}{960a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{5533x\sqrt[4]{1-\frac{1}{ax}}}{1920a^4\sqrt[4]{\frac{1}{ax}+1}}+\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{\frac{1}{ax}+1}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((5*ArcCoth[a*x])/2),x]
```


```
[Out] (26111*(1 - 1/(a*x))^(1/4))/(1920*a^5*(1 + 1/(a*x))^(1/4)) + (5533*(1 - 1/(
a*x))^(1/4)*x)/(1920*a^4*(1 + 1/(a*x))^(1/4)) - (1189*(1 - 1/(a*x))^(1/4)*x
^2)/(960*a^3*(1 + 1/(a*x))^(1/4)) + (181*(1 - 1/(a*x))^(1/4)*x^3)/(240*a^2*
(1 + 1/(a*x))^(1/4)) - (21*(1 - 1/(a*x))^(1/4)*x^4)/(40*a*(1 + 1/(a*x))^(1/
4)) + ((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + (1003*ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (1003*ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^6\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{5}\mathrm{Subst}\left(\int \frac{\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{20}\mathrm{Subst}\left(\int \frac{\frac{181}{4a^2}-\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{60}\mathrm{Subst}\left(\int \frac{\frac{1189}{8a^3}-\frac{543x}{4a^4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{120}\mathrm{Subst}\left(\int \frac{\frac{5533}{16a^4}-\frac{118}{4a}}{x^2\left(1-\frac{x}{a}\right)^{3/4}(1}\right.$$

$$= \frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{120}\mathrm{Subst}\Big($$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{a}}}{5\sqrt[4]{1+}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{a}}}{5\sqrt[4]{1+}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{a}}}{5\sqrt[4]{1+}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{a}}}{5\sqrt[4]{1+}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{a}}}{5\sqrt[4]{1+}}$$

Mathematica [A] time = 5.37064, size = 198, normalized size = 0.69

$$\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)} - \frac{4117e^{-\frac{1}{2}\coth^{-1}(ax)}}{192\left(e^{-2\coth^{-1}(ax)}-1\right)} - \frac{1661e^{-\frac{1}{2}\coth^{-1}(ax)}}{48\left(e^{-2\coth^{-1}(ax)}-1\right)^2} - \frac{233e^{-\frac{1}{2}\coth^{-1}(ax)}}{6\left(e^{-2\coth^{-1}(ax)}-1\right)^3} - \frac{122e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{-2\coth^{-1}(ax)}-1\right)^4} - \frac{32e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{-2\coth^{-1}(ax)}-1\right)^5} + \frac{1003}{256}\log}{a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (8/E^(ArcCoth[a*x]/2) - 32/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))
^5) - 122/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^4) - 233/(6*E^(A
rcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^3) - 1661/(48*E^(ArcCoth[a*x]/2)*
(-1 + E^(-2*ArcCoth[a*x]))^2) - 4117/(192*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*Ar
cCoth[a*x]))) - (1003*ArcTan[E^(-ArcCoth[a*x]/2)])/128 + (1003*Log[1 - E^(-
ArcCoth[a*x]/2)])/256 - (1003*Log[1 + E^(-ArcCoth[a*x]/2)])/256)/a^5
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A] time = 1.5489, size = 377, normalized size = 1.31

$$-\frac{1}{3840}a\left(\frac{4\left(20585\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}} - 49120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}} + 61130\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}} - 33816\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} + 7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1} - \frac{10(ax-1)^2a^6}{(ax+1)^2} + \frac{10(ax-1)^3a^6}{(ax+1)^3} - \frac{5(ax-1)^4a^6}{(ax+1)^4} + \frac{(ax-1)^5a^6}{(ax+1)^5} - a^6} + \frac{30090\arctan\left(\left(\right.\right.}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(20585*((a*x - 1)/(a*x + 1))^(17/4) - 49120*((a*x - 1)/(a*x +
1))^(13/4) + 61130*((a*x - 1)/(a*x + 1))^(9/4) - 33816*((a*x - 1)/(a*x + 1)
)^(5/4) + 7365*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10
*(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)
^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 30090*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^6 - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6 - 30720*((a*x - 1)/(a*x
 + 1))^(1/4)/a^6)
```

Fricas [A] time = 1.58489, size = 346, normalized size = 1.21

$$\frac{2\left(384\,a^5x^5-1008\,a^4x^4+1448\,a^3x^3-2378\,a^2x^2+5533\,ax+26111\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-15045\,\log\left(\left(\frac{a}{a}\right.\right.}{3840\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 - 1008*a^4*x^4 + 1448*a^3*x^3 - 2378*a^2*x^2 + 5533*
a*x + 26111)*((a*x - 1)/(a*x + 1))^(1/4) - 30090*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21479, size = 343, normalized size = 1.2

$$-\frac{1}{3840}\,a\left(\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{30720\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^6}-\frac{4\left(\frac{338\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 - 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 - 30720*((a*x - 1)/(a*x + 1))^(1/4)/a^6 - 4*(33816*(a*x - 1)*((a*
x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1))
^(1/4)/(a*x + 1)^2 + 49120*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1
)^3 - 20585*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^4 - 7365*((a*
x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

3.105 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=250

$$\frac{113x^2\sqrt[4]{1-\frac{1}{ax}}}{96a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{521x\sqrt[4]{1-\frac{1}{ax}}}{192a^3\sqrt[4]{\frac{1}{ax}+1}}-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}-\frac{17x^3\sqrt[4]{1-\frac{1}{ax}}}{24a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[Out] (-2467*(1 - 1/(a*x))^(1/4))/(192*a^4*(1 + 1/(a*x))^(1/4)) - (521*(1 - 1/(a*
x))^(1/4)*x)/(192*a^3*(1 + 1/(a*x))^(1/4)) + (113*(1 - 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 + 1/(a*x))^(1/4)) - (17*(1 - 1/(a*x))^(1/4)*x^3)/(24*a*(1 + 1/(a
*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rubi [A] time = 0.139989, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{113x^2\sqrt[4]{1-\frac{1}{ax}}}{96a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{521x\sqrt[4]{1-\frac{1}{ax}}}{192a^3\sqrt[4]{\frac{1}{ax}+1}}-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}-\frac{17x^3\sqrt[4]{1-\frac{1}{ax}}}{24a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-2467*(1 - 1/(a*x))^(1/4))/(192*a^4*(1 + 1/(a*x))^(1/4)) - (521*(1 - 1/(a*
x))^(1/4)*x)/(192*a^3*(1 + 1/(a*x))^(1/4)) + (113*(1 - 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 + 1/(a*x))^(1/4)) - (17*(1 - 1/(a*x))^(1/4)*x^3)/(24*a*(1 + 1/(a
*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^5\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}-\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{24}\text{Subst}\left(\int \frac{\frac{1425}{16a^4}-\frac{521x}{8a^5}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\right.}\right.$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}a\,\text{Subst}\left(\right.$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{475\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475\tan^{-1}\left(\right.}{64a^4}$$

Mathematica [A] time = 5.30586, size = 161, normalized size = 0.64

$$\frac{-3072e^{-\frac{1}{2}\coth^{-1}(ax)}-\frac{6292e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}+\frac{7376e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}-\frac{5248e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4}-1425\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+\cdots}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-3072/E^(ArcCoth[a*x]/2) + (1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x]))^4 - (5248*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 +
(7376*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (6292*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(-ArcCoth[a*x]/2)]
 - 1425*Log[1 - E^(-ArcCoth[a*x]/2)] + 1425*Log[1 + E^(-ArcCoth[a*x]/2)])/(
384*a^4)
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A] time = 1.50864, size = 329, normalized size = 1.32

$$-\frac{1}{384}a\left(\frac{4\left(1573\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-2875\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2343\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\cdots\right.\right.}{a^5}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(1573*((a*x - 1)/(a*x + 1))^(13/4) - 2875*((a*x - 1)/(a*x + 1))
^(9/4) + 2343*((a*x - 1)/(a*x + 1))^(5/4) - 657*((a*x - 1)/(a*x + 1))^(1/4)
)/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^
3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 2850*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5
+ 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5 + 3072*((a*x - 1)/(a*x + 1)
)^(1/4)/a^5)
```

Fricas [A] time = 1.7037, size = 313, normalized size = 1.25

$$\frac{2\left(48\,a^4x^4 - 136\,a^3x^3 + 226\,a^2x^2 - 521\,ax - 2467\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 1425}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*((a*x -
1)/(a*x + 1))^(1/4) + 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 1425*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21674, size = 301, normalized size = 1.2

$$\frac{1}{384}a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}+\frac{4\left(\frac{2343\,(ax-1)\left(\frac{ax-}{ax+}\right.}{ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))
/a^5 - 3072*((a*x - 1)/(a*x + 1))^(1/4)/a^5 + 4*(2343*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1) - 2875*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(
a*x + 1)^2 + 1573*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 657
*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

3.106 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=213

$$\frac{61x\sqrt[4]{1-\frac{1}{ax}}}{24a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}-\frac{13x^2\sqrt[4]{1-\frac{1}{ax}}}{12a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[Out] (287*(1 - 1/(a*x))^(1/4))/(24*a^3*(1 + 1/(a*x))^(1/4)) + (61*(1 - 1/(a*x))^
(1/4)*x)/(24*a^2*(1 + 1/(a*x))^(1/4)) - (13*(1 - 1/(a*x))^(1/4)*x^2)/(12*a*
(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) +
(55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) - (55*ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rubi [A] time = 0.110327, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{61x\sqrt[4]{1-\frac{1}{ax}}}{24a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}-\frac{13x^2\sqrt[4]{1-\frac{1}{ax}}}{12a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (287*(1 - 1/(a*x))^(1/4))/(24*a^3*(1 + 1/(a*x))^(1/4)) + (61*(1 - 1/(a*x))^
(1/4)*x)/(24*a^2*(1 + 1/(a*x))^(1/4)) - (13*(1 - 1/(a*x))^(1/4)*x^2)/(12*a*
(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) +
(55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) - (55*ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^4\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{\frac{61}{4a^2}-\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a\operatorname{Subst}\left(\int \frac{165}{16a^4x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\right.}}\right.$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\operatorname{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{55\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \cdots$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - \frac{55\tanh^{-1}\left(\cdots\right.}{8a^3}$$

Mathematica [C] time = 8.43072, size = 389, normalized size = 1.83

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(256e^{4\coth^{-1}(ax)}\left(626e^{2\coth^{-1}(ax)}+221e^{4\coth^{-1}(ax)}+437\right)\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},2,2,2\right\},\left\{1,1,\frac{15}{4}\right\},e^{2}\cdots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] -(-818741 - 1530529*E^(2*ArcCoth[a*x]) - 266035*E^(4*ArcCoth[a*x]) + 7161*E
^(6*ArcCoth[a*x]) + 818741*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x]
)] + 824824*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[
a*x])] + 248094*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcC
oth[a*x])] - 85624*E^(6*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*A
rcCoth[a*x])] - 2387*E^(8*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2
*ArcCoth[a*x])] + 256*E^(4*ArcCoth[a*x])*(437 + 626*E^(2*ArcCoth[a*x]) + 22
1*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4}, E^(2*
ArcCoth[a*x])] + 2048*E^(4*ArcCoth[a*x])*(17 + 30*E^(2*ArcCoth[a*x]) + 13*E
^(4*ArcCoth[a*x]))*HypergeometricPFQ[{3/4, 2, 2, 2, 2}, {1, 1, 1, 15/4}, E^
(2*ArcCoth[a*x])] + 4096*E^(4*ArcCoth[a*x])*HypergeometricPFQ[{3/4, 2, 2, 2
, 2, 2}, {1, 1, 1, 1, 15/4}, E^(2*ArcCoth[a*x])] + 8192*E^(6*ArcCoth[a*x])*
HypergeometricPFQ[{3/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 15/4}, E^(2*ArcCoth[a*
x])] + 4096*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{3/4, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 15/4}, E^(2*ArcCoth[a*x])])/(44352*a^3*E^((5*ArcCoth[a*x])/2))
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int x^{2} \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A] time = 1.54513, size = 279, normalized size = 1.31

$$-\frac{1}{48}\,a\left(\frac{4\left(137\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-174\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{165\log\left(\left(\frac{ax}{ax}\right.\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(137*((a*x - 1)/(a*x + 1))^(9/4) - 174*((a*x - 1)/(a*x + 1))^(5/
4) + 69*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 330*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4
 - 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4 - 384*((a*x - 1)/(a*x + 1))
^(1/4)/a^4)
```

Fricas [A] time = 1.59335, size = 284, normalized size = 1.33

$$\frac{2\left(8\,a^3x^3-26\,a^2x^2+61\,ax+287\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*((a*x - 1)/(a*x + 1))^(1/4)
 - 330*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*log(((a*x - 1)/(a*x + 1))^
(1/4) + 1) + 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18864, size = 259, normalized size = 1.22

$$-\frac{1}{48}a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{384\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^4}-\frac{4\left(\frac{174(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 - 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4 - 4*(174*(a*x - 1)*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1) - 137*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x +
1)^2 - 69*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.107 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=176

$$-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}-\frac{5x\left(1-\frac{1}{ax}\right)^{5/4}}{4a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[Out] (-25*(1 - 1/(a*x))^(1/4))/(2*a^2*(1 + 1/(a*x))^(1/4)) - (5*(1 - 1/(a*x))^(5
/4)*x)/(4*a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x
))^(1/4)) - (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rubi [A] time = 0.0714863, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 298, 203, 206}

$$-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}-\frac{5x\left(1-\frac{1}{ax}\right)^{5/4}}{4a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-25*(1 - 1/(a*x))^(1/4))/(2*a^2*(1 + 1/(a*x))^(1/4)) - (5*(1 - 1/(a*x))^(5
/4)*x)/(4*a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x
))^(1/4)) - (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^3\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - \frac{25\,\text{Subst}\left(\int \frac{1}{\ldots}\right.}{\ldots}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{25\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.241956, size = 121, normalized size = 0.69

$$-\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(-90e^{2\coth^{-1}(ax)} + 50e^{4\coth^{-1}(ax)} + 25e^{\frac{1}{2}\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}-1\right)^2\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 25e^{\frac{1}{2}\coth^{-1}(ax)}\left(e^{2}\ldots\right.\right.}{4a^2\left(e^{2\coth^{-1}(ax)}-1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] -(32 - 90*E^(2*ArcCoth[a*x]) + 50*E^(4*ArcCoth[a*x]) + 25*E^(ArcCoth[a*x]/2
)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTan[E^(ArcCoth[a*x]/2)] - 25*E^(ArcCoth[a*
x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2*E^(Ar
cCoth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2)
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A] time = 1.60295, size = 232, normalized size = 1.32

$$-\frac{1}{8}\,a\left(\frac{4\left(13\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}+\frac{64\left(\frac{ax-1}{ax+1}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(13*((a*x - 1)/(a*x + 1))^(5/4) - 9*((a*x - 1)/(a*x + 1))^(1/4))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 50*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^3 + 25*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3 + 64*((a*x - 1)/(a*x + 1
))^(1/4)/a^3)
```

Fricas [A] time = 1.65404, size = 258, normalized size = 1.47

$$\frac{2\left(2\,a^2x^2-9\,ax-43\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 9*a*x - 43)*((a*x - 1)/(a*x + 1))^(1/4) + 50*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 25*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16659, size = 217, normalized size = 1.23

$$\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}+\frac{4\left(\frac{13(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-9\left(\frac{ax-1}{ax+1}\ldots\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 - 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 6
4*((a*x - 1)/(a*x + 1))^(1/4)/a^3 + 4*(13*(a*x - 1)*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^3*((a*x - 1)/(a*x + 1) -
 1)^2))
```

3.108 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=130

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{\frac{1}{ax}+1}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (10*(1 - 1/(a*x))^(1/4))/(a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(5/4)*x)/
(1 + 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
 - (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rubi [A] time = 0.0464704, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6170, 94, 93, 298, 203, 206}

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{\frac{1}{ax}+1}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-5*ArcCoth[a*x])/2),x]
```


```
[Out] (10*(1 - 1/(a*x))^(1/4))/(a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(5/4)*x)/
(1 + 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
 - (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```


Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\operatorname{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{10\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.0659613, size = 31, normalized size = 0.24

$$\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{1}{4}, 2, \frac{3}{4}, e^{2\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-5*ArcCoth[a*x])/2),x]
```

```
[Out] (8*Hypergeometric2F1[-1/4, 2, 3/4, E^(2*ArcCoth[a*x])])/(a*E^(ArcCoth[a*x]/
2))
```

Maple [F] time = 0.325, size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A] time = 1.47956, size = 178, normalized size = 1.37

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 10*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x + 1))^(1/4)
 + 1)/a^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2 - 16*((a*x - 1)/(a*x
 + 1))^(1/4)/a^2)
```

Fricas [A] time = 1.79123, size = 232, normalized size = 1.78

$$\frac{2\,(ax+9)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 9)*((a*x - 1)/(a*x + 1))^(1/4) - 10*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) - 1))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23107, size = 174, normalized size = 1.34

$$-\frac{1}{2}a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 - 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 16
*((a*x - 1)/(a*x + 1))^(1/4)/a^2 + 4*((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x
 - 1)/(a*x + 1) - 1)))
```

3.109 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=320

$$-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.$$

```
[Out] (-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.284588, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6171, 98, 21, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```


```
[Out] (-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - (4a)\,\text{Subst}\left(\int \frac{\frac{1}{4a}+\frac{x}{4a^2}}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\Bigg(\cdots$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\cdots$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \cdots$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\cdots\right.}{\cdots}$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{t}\cdots$$

Mathematica [C] time = 0.0944097, size = 28, normalized size = 0.09

$$-8e^{-\frac{1}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{1}{8},1,\frac{7}{8},e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

```
[Out] (-8*Hypergeometric2F1[-1/8, 1, 7/8, E^(4*ArcCoth[a*x])])/E^(ArcCoth[a*x]/2)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Maxima [A] time = 1.54197, size = 329, normalized size = 1.03

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
```

```
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Fricas [A] time = 1.67414, size = 851, normalized size = 2.66

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) - 8*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*
x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21825, size = 340, normalized size = 1.06

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

3.110 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=299

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+5a\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\tan^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] (4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*a*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/Sqrt[2] + (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) - (5*a*Log
[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rubi [A] time = 0.244575, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+5a\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\tan^{-1}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*a*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/Sqrt[2] + (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) - (5*a*Log
[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5}{2}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (5a)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - (5a)\,\text{Subst}\Bigg(\int$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}(5a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \frac{1}{2}(5a$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\right.}{2\sqrt{2}}$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + $$

Mathematica [C] time = 0.0820784, size = 31, normalized size = 0.1

$$8ae^{-\frac{1}{2}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{1}{4}, 2, \frac{3}{4}, -e^{2\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (8*a*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcCoth[a*x])])/E^(ArcCoth[a*x]/
2)
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Maxima [A] time = 1.5397, size = 275, normalized size = 0.92

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Fricas [A] time = 1.71596, size = 1023, normalized size = 3.42

$$20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)+20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
20*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a*
x + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - sq
rt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - 5*sqrt
(2)*(a^4)^(1/4)*x*log(25*a^2*sqrt((a*x - 1)/(a*x + 1)) + 25*sqrt(2)*(a^4)^(
1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + 25*sqrt(a^4)) + 5*sqrt(2)*(a^4)^(1/4)*
x*log(25*a^2*sqrt((a*x - 1)/(a*x + 1)) - 25*sqrt(2)*(a^4)^(1/4)*a*((a*x - 1
)/(a*x + 1))^(1/4) + 25*sqrt(a^4)) + 4*(9*a*x + 1)*((a*x - 1)/(a*x + 1))^(1
/4))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19687, size = 275, normalized size = 0.92

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

3.111 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=351

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{5}{2}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}-\frac{25}{4}a^2\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{25a^2}{}$$

```
[Out] (-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - (25*a^2*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4))/4 - (5*a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/2 - (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) + (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rubi [A] time = 0.280799, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 78, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{5}{2}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}-\frac{25}{4}a^2\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{25a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - (25*a^2*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4))/4 - (5*a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/2 - (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) + (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\operatorname{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - (5a)\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}(25a)\operatorname{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}(25a)\operatorname{Subst}\left(\int \frac{}{(1}\right.$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(25a^2\right)\operatorname{Subst}\left(\int\right.$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(25a^2\right)\operatorname{Subst}\left(\int\right.$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}\left(25a^2\right)\operatorname{Subst}\left(\int\right.$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}\left(25a^2\right)\operatorname{Subst}\left(\int\right.$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right.}{8\sqrt{2}}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.141258, size = 101, normalized size = 0.29

$$-\frac{8}{3}a^2e^{-\frac{1}{2}\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (-8*a^2*(3 + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[
a*x])] + 2*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[
a*x])]))/(3*E^(ArcCoth[a*x]/2))
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Maxima [A] time = 1.58494, size = 333, normalized size = 0.95

$$\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+25\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
```

```
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*a*((a*x - 1)/(a*x + 1))^(5/4) + 9*a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a
*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Fricas [A] time = 1.88164, size = 1087, normalized size = 3.1

$$-\frac{100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(100*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*(
(a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a
*x + 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8))
)/a^8) + 100*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*
((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(
a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)
))/a^8) - 25*sqrt(2)*(a^8)^(1/4)*x^2*log(625*a^4*sqrt((a*x - 1)/(a*x + 1))
+ 625*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 625*sqrt(a^8))
+ 25*sqrt(2)*(a^8)^(1/4)*x^2*log(625*a^4*sqrt((a*x - 1)/(a*x + 1)) - 625*sq
rt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 625*sqrt(a^8)) + 4*(43*
a^2*x^2 + 9*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16193, size = 328, normalized size = 0.93

$$\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+25\sqrt{2}a\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 9*a*((a*x - 1)/(a*
x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.112 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=385

$$\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{55a^3\log\left(\sqrt{\right.}\right.}{}$$

```
[Out] (2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + (55*a^3*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4))/8 + (11*a^3*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/4 + (a^3*(1 - 1/(a*x))^(9/4)*(1 + 1/(a*x))^(3/4))/3 + (55*a^3*ArcTan[1 -
(Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*A
rcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) +
 (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (55*a^3*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(16*Sqrt[2])
```

Rubi [A] time = 0.309019, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 89, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{55a^3\log\left(\sqrt{\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + (55*a^3*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4))/8 + (11*a^3*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/4 + (a^3*(1 - 1/(a*x))^(9/4)*(1 + 1/(a*x))^(3/4))/3 + (55*a^3*ArcTan[1 -
(Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*A
rcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) +
 (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (55*a^3*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
```

```
])/(16*Sqrt[2])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\operatorname{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \left(2a^3\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(11a^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}\left(55a^2\right)\operatorname{Subst}\left(\int\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\right.$$

Mathematica [C] time = 0.166763, size = 104, normalized size = 0.27

$$a^3\left(\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(425e^{2\coth^{-1}(ax)}+462e^{4\coth^{-1}(ax)}+165e^{6\coth^{-1}(ax)}+96\right)}{12\left(e^{2\coth^{-1}(ax)}+1\right)^3}-\frac{55}{32}\text{RootSum}\left[\#1^4+1\&,\frac{2\log\left(e^{-\frac{1}{2}\coth^{-1}(a\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] a^3*((96 + 425*E^(2*ArcCoth[a*x]) + 462*E^(4*ArcCoth[a*x]) + 165*E^(6*ArcCo
th[a*x]))/(12*E^(ArcCoth[a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootSum[
1 + #1^4 & , (ArcCoth[a*x] + 2*Log[E^(-ArcCoth[a*x]/2) - #1])/#1^3 & ])/32)
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Maxima [A] time = 1.52914, size = 401, normalized size = 1.04

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+165\sqrt{2}a^2\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
```

```
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(137*a^2*((a*x - 1)/(a*x + 1))^(9/4) + 174*a^2*((a*x - 1)/(
a*x + 1))^(5/4) + 69*a^2*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)/(a*x + 1
) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Fricas [A] time = 1.80943, size = 1143, normalized size = 2.97

$$660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{\ }}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1)
/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a
^12)))/a^12) + 660*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^(
3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((
a*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt(a^12)))/a^12) - 165*sqrt(2)*(a^12)^(1/4)*x^3*log(3025*a^6*sqrt((a*x
- 1)/(a*x + 1)) + 3025*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
 + 3025*sqrt(a^12)) + 165*sqrt(2)*(a^12)^(1/4)*x^3*log(3025*a^6*sqrt((a*x -
 1)/(a*x + 1)) - 3025*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ 3025*sqrt(a^12)) + 4*(287*a^3*x^3 + 61*a^2*x^2 - 26*a*x + 8)*((a*x - 1)/(
a*x + 1))^(1/4))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19702, size = 393, normalized size = 1.02

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+165\sqrt{2}a^2\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) +
137*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 69*a^2*((a*x
- 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

3.113 $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

Optimal. Leaf size=285

$$\frac{1}{3}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^3+\frac{7}{18}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{11}{27}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{19}{324}\text{l}$$

```
[Out] (11*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/27 + (7*(1 + x^(-1))^(1/6)*((-
1 + x)/x)^(5/6)*x^2)/18 + ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x^3)/3 - (
19*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])/(54*Sqr
t[3]) + (19*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]]
)/(54*Sqrt[3]) + (19*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/81 - (
19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/324 + (19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (
1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/324
```

Rubi [A] time = 0.249146, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6171, 99, 151, 12, 93, 210, 634, 618, 204, 628, 206}

$$\frac{1}{3}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^3+\frac{7}{18}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{11}{27}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{19}{324}\text{l}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] (11*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/27 + (7*(1 + x^(-1))^(1/6)*((-
1 + x)/x)^(5/6)*x^2)/18 + ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x^3)/3 - (
19*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])/(54*Sqr
t[3]) + (19*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]]
```

```
)/(54*Sqrt[3]) + (19*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/81 - (
19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/324 + (19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (
1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/324
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\mathrm{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{3}\mathrm{Subst}\left(\int \frac{\frac{7}{3}+2x}{\sqrt[6]{1-x}x^3(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{1}{6}\mathrm{Subst}\left(\int \frac{-\frac{22}{9}-\frac{7x}{3}}{\sqrt[6]{1-x}x^2(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{6}\mathrm{Subst}\Bigg(\int$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19}{162}\mathrm{Subst}\Bigg(\int$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19}{27}\mathrm{Subst}\Bigg(\int$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{19}{81}\mathrm{Subst}\Bigg(\int$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{19}{81}\tanh^{-1}\Bigg($$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{19}{81}\tanh^{-1}\Bigg($$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19\tan^{-1}\Bigg(1-\frac{2}{6}\cdots}{54\sqrt{3}}$$

Mathematica [A] time = 5.249, size = 189, normalized size = 0.66

$$\frac{1}{324}\left(\frac{732e^{\frac{1}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1} + \frac{1368e^{\frac{1}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^2} + \frac{864e^{\frac{1}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^3} - 38\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) + 38\log\left(e^{\frac{1}{3}\coth^{-1}(x)}+1\right) - 19\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] ((864*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^3 + (1368*E^(ArcCoth[x]/3))
/(-1 + E^(2*ArcCoth[x]))^2 + (732*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))
 + 38*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] + 38*Sqrt[3]*ArcTan
[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 38*Log[1 - E^(ArcCoth[x]/3)] + 38*Log[
1 + E^(ArcCoth[x]/3)] - 19*Log[1 - E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)]
 + 19*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/324
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x^2,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)*x^2,x)
```

Maxima [A] time = 1.53666, size = 297, normalized size = 1.04

$$-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}-8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+61\left(\frac{x-}{x+}\right.}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="maxima")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/27*(19*((
x - 1)/(x + 1))^(17/6) - 8*((x - 1)/(x + 1))^(11/6) + 61*((x - 1)/(x + 1))^
(5/6))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1
) + 19/324*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/
324*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log
```

```
(((x - 1)/(x + 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Fricas [A] time = 1.64059, size = 581, normalized size = 2.04

$$\frac{1}{54}\left(18\,x^{3}+39\,x^{2}+43\,x+22\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right)-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="fricas")
```

```
[Out] 1/54*(18*x^3 + 39*x^2 + 43*x + 22)*((x - 1)/(x + 1))^(5/6) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 19/324*log(((x
 - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x - 1)/(
x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x + 1))
^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/6), x)
```

Giac [A] time = 1.16715, size = 290, normalized size = 1.02

$$-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)+\frac{\frac{8\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-\frac{19\,(x-1)^{2}\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^{2}}-\cdots}{27\left(\frac{x-1}{x+1}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="giac")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/27*(8*(x
- 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) - 19*(x - 1)^2*((x - 1)/(x + 1))^(5/6)
/(x + 1)^2 - 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) - 1)^3 + 19/324*l
og(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x
- 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x
 + 1))^(1/6) + 1) - 19/162*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

3.114 $\int e^{\frac{1}{3}\coth^{-1}(x)} x\,dx$

Optimal. Leaf size=258

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{7/6}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{1}{6}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right.$$

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/6 + ((1 + x^(-1))^(7/6)*((-1 + x)
/x)^(5/6)*x^2)/2 - ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/S
qrt[3]]/(6*Sqrt[3]) + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6)
)/Sqrt[3]]/(6*Sqrt[3]) + ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/9 -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/36 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-
1))^(1/6)/((-1 + x)/x)^(1/6)]/36
```

Rubi [A] time = 0.203263, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6171, 96, 94, 93, 210, 634, 618, 204, 628, 206}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{7/6}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{1}{6}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/6 + ((1 + x^(-1))^(7/6)*((-1 + x)
/x)^(5/6)*x^2)/2 - ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/S
qrt[3]]/(6*Sqrt[3]) + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6)
)/Sqrt[3]]/(6*Sqrt[3]) + ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/9 -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
```

```
x)/x)^(1/6)]/36 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-
1))^(1/6)/((-1 + x)/x)^(1/6)]/36
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{9}\text{Sub}$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\text{Subst}\left(\int \frac{-1+}{1-x+}\right.$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}\right.$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\text{t}$$

Mathematica [A] time = 0.348568, size = 167, normalized size = 0.65

$$\frac{1}{36}\left(\frac{84e^{\frac{1}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1} + \frac{72e^{\frac{1}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^2} - 2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) + 2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}+1\right) - \log\left(-e^{\frac{1}{3}\coth^{-1}(x)} + e^{\frac{2}{3}\coth^{-1}(x)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((72*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^2 + (84*E^(ArcCoth[x]/3))/(-
1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]]
 + 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E^(ArcCot
h[x]/3)] + 2*Log[1 + E^(ArcCoth[x]/3)] - Log[1 - E^(ArcCoth[x]/3) + E^((2*A
rcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/36
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)*x,x)
```

Maxima [A] time = 1.67654, size = 262, normalized size = 1.02

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{36}\log\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/3*(((x - 1)/(
x + 1))^(11/6) - 7*((x - 1)/(x + 1))^(5/6))/(2*(x - 1)/(x + 1) - (x - 1)^2/
(x + 1)^2 - 1) + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6)
 + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1
/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(((x - 1)/(x + 1))^(1/6) - 1
)
```

Fricas [A] time = 1.59146, size = 547, normalized size = 2.12

$$\frac{1}{6}\left(3x^2+7x+4\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{1}{18}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{18}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{36}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 7*x + 4)*((x - 1)/(x + 1))^(5/6) - 1/18*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/18*sqrt(3)*arctan(2/3*sqrt(
3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/36*log(((x - 1)/(x + 1))^(1/3
) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x -
 1)/(x + 1))^(1/6) + 1) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(
((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/6), x)
```

Giac [A] time = 1.167, size = 258, normalized size = 1.

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}-1\right)^{2}}+\frac{1}{36}\log\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/3*((x - 1)*((
x - 1)/(x + 1))^(5/6)/(x + 1) - 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1)
 - 1)^2 + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) -
 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1/18*log
(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

3.115 $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

Optimal. Leaf size=223

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right)}{\sqrt{3}}\cdots$$

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x - ArcTan[(1 - (2*(1 + x^(-1))^(1/6)
)/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))
/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + (2*ArcTanh[(1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/3 - Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^
(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)
^(1/3) + (1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6
```

Rubi [A] time = 0.179596, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6170, 94, 93, 210, 634, 618, 204, 628, 206}

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right)}{\sqrt{3}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3),x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x - ArcTan[(1 - (2*(1 + x^(-1))^(1/6)
)/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))
/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + (2*ArcTanh[(1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/3 - Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^
(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)
^(1/3) + (1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x - 2\operatorname{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x + \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{2}{3}\operatorname{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right.$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{6}\operatorname{Subst}\Bigg(\int$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}\right.$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x + \frac{\tan^{-1}\left(\frac{-1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\log\Bigg(1+$$

Mathematica [C] time = 0.0413237, size = 35, normalized size = 0.16

$$2e^{\frac{1}{3}\coth^{-1}(x)}\left(\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{2\coth^{-1}(x)}\right)+\frac{1}{e^{2\coth^{-1}(x)}-1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3),x]
```

```
[Out] 2*E^(ArcCoth[x]/3)*((-1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6, 1
, 7/6, E^(2*ArcCoth[x])])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6),x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6),x)
```

Maxima [A] time = 1.53153, size = 225, normalized size = 1.01

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1
))^(1/6) - 1)
```

Fricas [A] time = 1.70431, size = 520, normalized size = 2.33

$$(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="fricas")
```

```
[Out] (x + 1)*((x - 1)/(x + 1))^(5/6) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x +
 1))^(1/6) - 1/3*sqrt(3)) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x +
 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6)
 + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1))^(1
/6) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/6), x)
```

Giac [A] time = 1.20744, size = 227, normalized size = 1.02

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{2\,\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-}{x+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(abs(((x - 1)/(x
 + 1))^(1/6) - 1))
```

3.116 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=402

$$-\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{\frac{1}{x}}}\right.$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])
+ Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]] -
 ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + ArcTan[Sqrt[
3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + 2*ArcTan[((-1 + x)/x)^(1/
6)/(1 + x^(-1))^(1/6)] + 2*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)] -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/2 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-1
))^(1/6)/((-1 + x)/x)^(1/6)]/2 + (Sqrt[3]*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1/
6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - (Sqrt[
3]*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(
1/3)/(1 + x^(-1))^(1/3)])/2
```

Rubi [A] time = 0.525113, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6171, 105, 63, 331, 295, 634, 618, 204, 628, 203, 93, 210, 206}

$$-\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{\frac{1}{x}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])
+ Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]] -
 ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + ArcTan[Sqrt[
3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + 2*ArcTan[((-1 + x)/x)^(1/
```

```
6)/(1 + x^(-1))^(1/6)] + 2*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)] -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/2 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-1
))^(1/6)/((-1 + x)/x)^(1/6)]/2 + (Sqrt[3]*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1/
6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - (Sqrt[
3]*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(
1/3)/(1 + x^(-1))^(1/3)])/2
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
```

```
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}x}\,dx,x,\frac{1}{x}\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx,x,\frac{1}{x}\right)-\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}x(1+x)^{5/6}}\,dx,x,\frac{1}{x}\right)$$
$$= 6\operatorname{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx,x,\sqrt[6]{\frac{-1+x}{x}}\right)-6\operatorname{Subst}\left(\int \frac{1}{-1+x^6}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$
$$= 2\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+2\operatorname{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+2\operatorname{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\right.$$
$$= 2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right.$$
$$= 2\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)-\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)+\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}}+\right.$$
$$= -\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right)-\frac{1}{2}\log$$
$$= -\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)-\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right.$$

Mathematica [C] time = 0.0317729, size = 26, normalized size = 0.06

$$\frac{12}{7}e^{\frac{7}{3}\coth^{-1}(x)}\text{Hypergeometric2F1}\left(\frac{7}{12},1,\frac{19}{12},e^{4\coth^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] (12*E^((7*ArcCoth[x])/3)*Hypergeometric2F1[7/12, 1, 19/12, E^(4*ArcCoth[x])
])/7
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((x - 1)/(x + 1))^(1/6)), x)
```

Fricas [A] time = 1.83976, size = 1080, normalized size = 2.69

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - sqrt(3
)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) - 1/2*sqrt(3)*l
og(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) +
1/2*sqrt(3)*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^
```

```
(1/3) + 16) - 2*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3)*((x - 1)/(x + 1))^(1/
6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/(x + 1))^(1/6)) - 2*arct
an(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) +
 1/2*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log((
(x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1
))^(1/6) + 1) - log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A] time = 1.18374, size = 352, normalized size = 0.88

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/2*sqrt(3)*log(sqrt(3)*(
(x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*sqrt(3)*log(-sq
rt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + arctan(sqrt(
3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(1/3) +
 ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/
(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(abs(((x - 1)/(
x + 1))^(1/6) - 1))
```

3.117 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=233

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{3}\tan^{-1}\Bigl(\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}$$

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6) - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6)]/3 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x
^(-1))^(1/6)]/3 + (2*ArcTan[((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/3 + Log
[1 - (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(
1 + x^(-1))^(1/3)]/(2*Sqrt[3]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 +
x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(2*Sqrt[3])
```

Rubi [A] time = 0.367476, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6171, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{3}\tan^{-1}\Bigl(\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6) - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6)]/3 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x
^(-1))^(1/6)]/3 + (2*ArcTan[((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/3 + Log
[1 - (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(
1 + x^(-1))^(1/3)]/(2*Sqrt[3]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 +
x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(2*Sqrt[3])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\operatorname{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\operatorname{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\operatorname{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right.$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{6}\operatorname{Subst}\left(\int\right.$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}} - \frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right.}{2\sqrt{3}}$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{3}\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) +$$

Mathematica [C] time = 0.0525046, size = 39, normalized size = 0.17

$$-2e^{\frac{1}{3}\coth^{-1}(x)}\left(\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{2\coth^{-1}(x)}\right)-\frac{1}{e^{2\coth^{-1}(x)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] -2*E^(ArcCoth[x]/3)*(-(1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
1, 7/6, -E^(2*ArcCoth[x])])
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^2,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)/x^2,x)
```

Maxima [A] time = 1.58391, size = 205, normalized size = 0.88

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Fricas [A] time = 1.78059, size = 689, normalized size = 2.96

$$-\frac{\sqrt{3}x\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-\sqrt{3}x\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+4x\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x*log(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1
))^(1/3) + 16) - sqrt(3)*x*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x
 - 1)/(x + 1))^(1/3) + 16) + 4*x*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3)*((x
- 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/(x + 1)
)^(1/6)) + 4*x*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^(1/6) + (
(x - 1)/(x + 1))^(1/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) - 4*x*arctan(((x -
 1)/(x + 1))^(1/6)) - 6*(x + 1)*((x - 1)/(x + 1))^(5/6))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**2,x)
```

```
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A] time = 1.385, size = 317, normalized size = 1.36

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1} - \frac{2\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \cdots\right.}{3\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)\left(\left(\frac{x-}{x+}\right.\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - sqrt(3)*a
rctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1)/(x + 1))^(5
/6)/((x - 1)/(x + 1) + 1) - 2/3*(2*((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x +
1))^(1/3) - 1)/((((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1)*(((
x - 1)/(x + 1))^(1/3) - 1)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6)) + 5/18*lo
g(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 1/2*log(((x - 1)
/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) + 4/9*log(abs(((x - 1)/(x +
1))^(1/3) - 1))
```

3.118 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=260

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{1}{6}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\tan^{-1}($$

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/6 + ((1 + x^(-1))^(7/6)*((-1 + x)/x
)^(5/6))/2 - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18
 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18 + ArcTan[
((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)]/9 + Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3
]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)
^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3])
```

Rubi [A] time = 0.379102, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6171, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{1}{6}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/6 + ((1 + x^(-1))^(7/6)*((-1 + x)/x
)^(5/6))/2 - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18
 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18 + ArcTan[
((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)]/9 + Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3
]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)
^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
```

```
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{9}\text{Subst}\Bigg(\int$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{36}\text{Subst}\Bigg(\int \frac{1}{1-\sqrt{3}x+x^2}$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{12\sqrt{3}}$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{18}\tan^{-1}\Bigg(\sqrt{3}+\frac{2\sqrt[6]{}}{\sqrt[6]{}}$$

Mathematica [C] time = 0.7009, size = 124, normalized size = 0.48

$$\frac{1}{54}\Bigg(\text{RootSum}\Bigg[\#1^4-\#1^2+1\&, \frac{\#1^2\left(-\coth^{-1}(x)\right)+3\#1^2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+2\coth^{-1}(x)}{2\#1^3-\#1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(1 + 7*E^(2*ArcCoth[x])))/(1 + E^(2*ArcCoth[x]))^2 -
6*ArcTan[E^(ArcCoth[x]/3)] + RootSum[1 - #1^2 + #1^4 & , (2*ArcCoth[x] - 6*
Log[E^(ArcCoth[x]/3) - #1] - ArcCoth[x]*#1^2 + 3*Log[E^(ArcCoth[x]/3) - #1]
*#1^2)/(-#1 + 2*#1^3) & ])/54
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^3,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)/x^3,x)
```

Maxima [A] time = 1.59628, size = 240, normalized size = 0.92

$$-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}} + 7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1} + \frac{(x-1)^2}{(x+1)^2} + 1\right)} + \frac{1}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="maxima")
```

```
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*(((x - 1)/(x + 1))^(11/6) + 7*((x - 1)/(x + 1))^(5/6))/
(2*(x - 1)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) + 1/18*arctan(sqrt(3) + 2*((x
 - 1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 1/9*arctan(((x - 1)/(x + 1))^(1/6))
```

Fricas [A] time = 1.76004, size = 720, normalized size = 2.77

$$-\frac{\sqrt{3}x^2\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-\sqrt{3}x^2\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+4x^2\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-1\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(sqrt(3)*x^2*log(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x
+ 1))^(1/3) + 16) - sqrt(3)*x^2*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1
6*((x - 1)/(x + 1))^(1/3) + 16) + 4*x^2*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(
3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/
(x + 1))^(1/6)) + 4*x^2*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^
(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) - 4*x^2*a
rctan(((x - 1)/(x + 1))^(1/6)) - 6*(4*x^2 + 7*x + 3)*((x - 1)/(x + 1))^(5/6
))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A] time = 1.41753, size = 348, normalized size = 1.34

$$\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1}{9\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 1/6*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) - 1/9*(2*((x - 1)/(
x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) - 1)/((((x - 1)/(x + 1))^(2/3) + ((
x - 1)/(x + 1))^(1/3) + 1)*(((x - 1)/(x + 1))^(1/3) - 1)) + 1/3*((x - 1)*((
x - 1)/(x + 1))^(5/6)/(x + 1) + 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1)
 + 1)^2 + 1/9*arctan(((x - 1)/(x + 1))^(1/6)) + 5/108*log(((x - 1)/(x + 1))
^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 1/12*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) + 2/27*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.119 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

Optimal. Leaf size=287

$$\frac{1}{18}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}+\frac{19}{54}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}-\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\right.}{108\sqrt{}}$$

```
[Out] (19*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/54 + ((1 + x^(-1))^(7/6)*((-1 +
x)/x)^(5/6))/18 + ((1 + x^(-1))^(7/6)*((-1 + x)/x)^(5/6))/(3*x) - (19*ArcTa
n[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[Sq
rt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[((-1 +
 x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/81 + (19*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt
[3]) - (19*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 +
 x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt[3])
```

Rubi [A] time = 0.398351, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6171, 90, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{18}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}+\frac{19}{54}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}-\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\right.}{108\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^4,x]
```



```
[Out] (19*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/54 + ((1 + x^(-1))^(7/6)*((-1 +
x)/x)^(5/6))/18 + ((1 + x^(-1))^(7/6)*((-1 + x)/x)^(5/6))/(3*x) - (19*ArcTa
n[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[Sq
rt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[((-1 +
 x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/81 + (19*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt
[3]) - (19*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 +
 x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt[3])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-1-\frac{x}{3}\right)\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{54}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}\text{Subst}\left(\int \frac{}{\sqrt[6]{1-x}}\right.$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)}\right.$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{Subst}\left(\int \frac{x^4}{1+x^6}\right.$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\text{Subst}\left(\int \frac{1}{1+x^2}\right.$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) +$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) +$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{-}}{\sqrt[6]{1}}\right.$$

Mathematica [C] time = 0.206159, size = 133, normalized size = 0.46

$$\frac{1}{486}\left(-19\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{\#1^2\coth^{-1}(x)-3\#1^2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-2\coth^{-1}(x}{2\#1^3-\#1}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(19 + 8*E^(2*ArcCoth[x]) + 61*E^(4*ArcCoth[x])))/(1 +
 E^(2*ArcCoth[x]))^3 - 114*ArcTan[E^(ArcCoth[x]/3)] - 19*RootSum[1 - #1^2 +
 #1^4 & , (-2*ArcCoth[x] + 6*Log[E^(ArcCoth[x]/3) - #1] + ArcCoth[x]*#1^2 -
 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-#1 + 2*#1^3) & ])/486
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^4,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)/x^4,x)
```

Maxima [A] time = 1.56902, size = 277, normalized size = 0.97

$$-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}+8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+}{27\left(\frac{3(x-1)}{x+1}+\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-}{(x-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="maxima")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
```

```
 + 1))^(1/3) + 1) + 1/27*(19*((x - 1)/(x + 1))^(17/6) + 8*((x - 1)/(x + 1))
^(11/6) + 61*((x - 1)/(x + 1))^(5/6))/(3*(x - 1)/(x + 1) + 3*(x - 1)^2/(x +
 1)^2 + (x - 1)^3/(x + 1)^3 + 1) + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x +
1))^(1/6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

Fricas [A] time = 1.74442, size = 775, normalized size = 2.7

$$-\frac{19\,\sqrt{3}x^3\log\left(5776\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+5776\right)-19\,\sqrt{3}x^3\log\left(-5776\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+5776\right)+76}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="fricas")
```

```
[Out] -1/324*(19*sqrt(3)*x^3*log(5776*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 5776*((x
- 1)/(x + 1))^(1/3) + 5776) - 19*sqrt(3)*x^3*log(-5776*sqrt(3)*((x - 1)/(x
+ 1))^(1/6) + 5776*((x - 1)/(x + 1))^(1/3) + 5776) + 76*x^3*arctan(sqrt(3)
+ 1/38*sqrt(-5776*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 5776*((x - 1)/(x + 1))^
(1/3) + 5776) - 2*((x - 1)/(x + 1))^(1/6)) + 76*x^3*arctan(-sqrt(3) + 2*sqr
t(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) - 2*((x -
1)/(x + 1))^(1/6)) - 76*x^3*arctan(((x - 1)/(x + 1))^(1/6)) - 6*(22*x^3 + 4
3*x^2 + 39*x + 18)*((x - 1)/(x + 1))^(5/6))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**4,x)
```

```
[Out] Integral(1/(x**4*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A] time = 1.37161, size = 381, normalized size = 1.33

$$\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{19}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)-\frac{19\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)}{81\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="giac")
```

```
[Out] 19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 19/54*
sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) - 19/81*(2*((x
- 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) - 1)/((((x - 1)/(x + 1))^(2/3
) + ((x - 1)/(x + 1))^(1/3) + 1)*(((x - 1)/(x + 1))^(1/3) - 1)) + 1/27*(8*(
x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 19*(x - 1)^2*((x - 1)/(x + 1))^(5/
6)/(x + 1)^2 + 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) + 1)^3 + 19/81*
arctan(((x - 1)/(x + 1))^(1/6)) + 95/972*log(((x - 1)/(x + 1))^(2/3) + ((x
- 1)/(x + 1))^(1/3) + 1) - 19/108*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x
 + 1))^(1/3) + 1) + 38/243*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.120 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx$

Optimal. Leaf size=157

$$\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^3+\frac{4}{9}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{14}{27}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{11}{27}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{11\log(x)}{81}$$

```
[Out] (14*(1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/27 + (4*(1 + x^(-1))^(1/3)*((-
1 + x)/x)^(2/3)*x^2)/9 + ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x^3)/3 - (2
2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/
(27*Sqrt[3]) - (11*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/27 - (11*L
og[x])/81
```

Rubi [A] time = 0.0701047, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6171, 99, 151, 12, 91}

$$\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^3+\frac{4}{9}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{14}{27}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{11}{27}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{11\log(x)}{81}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x^2,x]
```


```
[Out] (14*(1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/27 + (4*(1 + x^(-1))^(1/3)*((-
1 + x)/x)^(2/3)*x^2)/9 + ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x^3)/3 - (2
2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/
(27*Sqrt[3]) - (11*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/27 - (11*L
og[x])/81
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{8}{3}+2x}{\sqrt[3]{1-x}x^3(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{28}{9}-\frac{8x}{3}}{\sqrt[3]{1-x}x^2(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{6}\text{Subst}\Big(\int \frac{}{27}$$
$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{22}{81}\text{Subst}\Big(\int$$
$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{22\tan^{-1}\Big(\frac{1}{\sqrt{3}}+}{27\sqrt{3}}$$

Mathematica [C] time = 7.16166, size = 340, normalized size = 2.17

$$-\frac{e^{-\frac{10}{3}\coth^{-1}(x)}\left(54e^{8\coth^{-1}(x)}\left(782e^{2\coth^{-1}(x)}+325e^{4\coth^{-1}(x)}+475\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{16}{3}\right\},e^{2\coth}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] -(-22750000 - 20915440*E^(2*ArcCoth[x]) + 7026175*E^(4*ArcCoth[x]) + 739414
0*E^(6*ArcCoth[x]) - 433485*E^(8*ArcCoth[x]) + 22750000*Hypergeometric2F1[1
/3, 1, 4/3, E^(2*ArcCoth[x])] + 15227940*E^(2*ArcCoth[x])*Hypergeometric2F1
[1/3, 1, 4/3, E^(2*ArcCoth[x])] - 14083160*E^(4*ArcCoth[x])*Hypergeometric2
F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] - 8250060*E^(6*ArcCoth[x])*Hypergeometric
2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 1456000*E^(8*ArcCoth[x])*Hypergeometri
c2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 54*E^(8*ArcCoth[x])*(475 + 782*E^(2*A
rcCoth[x]) + 325*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1,
 16/3}, E^(2*ArcCoth[x])] + 162*E^(8*ArcCoth[x])*(35 + 64*E^(2*ArcCoth[x])
+ 29*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 16/3}
, E^(2*ArcCoth[x])] + 486*E^(8*ArcCoth[x])*HypergeometricPFQ[{2, 2, 2, 2, 2
, 7/3}, {1, 1, 1, 1, 16/3}, E^(2*ArcCoth[x])] + 972*E^(10*ArcCoth[x])*Hyper
geometricPFQ[{2, 2, 2, 2, 2, 7/3}, {1, 1, 1, 1, 16/3}, E^(2*ArcCoth[x])] +
```

```
486*E^(12*ArcCoth[x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/3}, {1, 1, 1, 1,
16/3}, E^(2*ArcCoth[x])])/(49140*E^((10*ArcCoth[x])/3))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int x^{2} \frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x^2,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3)*x^2,x)
```

Maxima [A] time = 1.54007, size = 201, normalized size = 1.28

$$-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(11\left(\frac{x-1}{x+1}\right)^{\frac{8}{3}}-10\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^{2}}{(x+1)^{2}}+\frac{(x-1)^{3}}{(x+1)^{3}}-1\right)}+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/27*(
11*((x - 1)/(x + 1))^(8/3) - 10*((x - 1)/(x + 1))^(5/3) + 35*((x - 1)/(x +
1))^(2/3))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3
 - 1) + 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) -
22/81*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Fricas [A] time = 1.46335, size = 325, normalized size = 2.07

$$\frac{1}{27}\left(9x^{3}+21x^{2}+26x+14\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{22}{81}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^3 + 21*x^2 + 26*x + 14)*((x - 1)/(x + 1))^(2/3) - 22/81*sqrt(3)*a
rctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 11/81*log(((x -
1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(((x - 1)/(x +
1))^(1/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/3), x)
```

Giac [A] time = 1.18165, size = 194, normalized size = 1.24

$$-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\frac{10\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-\frac{11\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{(x+1)^2}-35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{x-1}{x+1}-1\right)^3}+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/27*(
10*(x - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 11*(x - 1)^2*((x - 1)/(x + 1))
^(2/3)/(x + 1)^2 - 35*((x - 1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)^3 + 11
/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(
abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.121 $\int e^{\frac{2}{3}\coth^{-1}(x)} x\,dx$

Optimal. Leaf size=130

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{4/3}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{1}{3}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/3 + ((1 + x^(-1))^(4/3)*((-1 + x)
/x)^(2/3)*x^2)/2 - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1
 + x^(-1))^(1/3))])/(3*Sqrt[3]) - Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/
3)]/3 - Log[x]/9
```

Rubi [A] time = 0.0466163, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6171, 96, 94, 91}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{4/3}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{1}{3}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/3 + ((1 + x^(-1))^(4/3)*((-1 + x)
/x)^(2/3)*x^2)/2 - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1
 + x^(-1))^(1/3))])/(3*Sqrt[3]) - Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/
3)]/3 - Log[x]/9
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^3}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2-\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2-\frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(\sqrt[3]{1+\frac{1}{x}}\right.\end{aligned}$$

Mathematica [A] time = 0.429379, size = 165, normalized size = 1.27

$$\frac{1}{9}\left(\frac{24e^{\frac{2}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1}+\frac{18e^{\frac{2}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^2}-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}+1\right)+\log\left(-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((18*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (24*E^((2*ArcCoth[x]
)/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/
Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E
^(ArcCoth[x]/3)] - 2*Log[1 + E^(ArcCoth[x]/3)] + Log[1 - E^(ArcCoth[x]/3) +
 E^((2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/
9
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3)*x,x)
```

Maxima [A] time = 1.51746, size = 166, normalized size = 1.28

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/3*(((x
 - 1)/(x + 1))^(5/3) - 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^
(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Fricas [A] time = 1.56023, size = 301, normalized size = 2.32

$$\frac{1}{6}\left(3\,x^2+8\,x+5\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)+\frac{1}{9}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\,\log\left(\left(\frac{x-}{x+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 8*x + 5)*((x - 1)/(x + 1))^(2/3) - 2/9*sqrt(3)*arctan(2/3*sqrt
(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/9*log(((x - 1)/(x + 1))^(2/3
) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/3), x)
```

Giac [A] time = 1.1516, size = 162, normalized size = 1.25

$$-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-4\,\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^2}+\frac{1}{9}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\,\log\left(\left|\left(\frac{x}{x}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/3*((x
- 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/
(x + 1) - 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3)
+ 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.122 $\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx$

Optimal. Leaf size=96

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{3}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x
)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[(1 + x^(-1))^(1/3)
 - ((-1 + x)/x)^(1/3)] - Log[x]/3
```

Rubi [A] time = 0.0286798, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6170, 94, 91}

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{3}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3),x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x
)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[(1 + x^(-1))^(1/3)
 - ((-1 + x)/x)^(1/3)] - Log[x]/3
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx &= -\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{-\frac{1-x}{x}}\right)-\frac{\log(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.160126, size = 85, normalized size = 0.89

$$\frac{1}{3}\left(\frac{6e^{\frac{2}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1}-2\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{2e^{\frac{2}{3}\coth^{-1}(x)}+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3),x]
```

```
[Out] ((6*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(1 + 2
*E^((2*ArcCoth[x])/3))/Sqrt[3]] - 2*Log[1 - E^((2*ArcCoth[x])/3)] + Log[1 +
 E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)])/3
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3),x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3),x)
```

Maxima [A] time = 1.54199, size = 130, normalized size = 1.35

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Fricas [A] time = 1.5966, size = 282, normalized size = 2.94

$$(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="fricas")
```

```
[Out] (x + 1)*((x - 1)/(x + 1))^(2/3) - 2/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/3) + 1/3*sqrt(3)) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(
x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/3), x)
```

Giac [A] time = 1.1857, size = 131, normalized size = 1.36

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.123 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=155

$$-\frac{3}{2}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)-\frac{\log(x)}{2}-\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}\tan$$

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(
1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x
^(-1))^(1/3))] - (3*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/2 - (3*Lo
g[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 - Log[x
]/2
```

Rubi [A] time = 0.0512448, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 105, 60, 91}

$$-\frac{3}{2}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)-\frac{\log(x)}{2}-\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(
1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x
^(-1))^(1/3))] - (3*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/2 - (3*Lo
g[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 - Log[x
]/2
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}x}\,dx,x,\frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx,x,\frac{1}{x}\right)-\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)^{2/3}}\,dx,x,\frac{1}{x}\right)\\ &= -\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)-\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{3}{2}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{3}{2}\text{lo}\end{aligned}$$

Mathematica [C] time = 0.0370847, size = 26, normalized size = 0.17

$$\frac{3}{2}e^{\frac{8}{3}\coth^{-1}(x)}\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},e^{4\coth^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x,x]
```

```
[Out] (3*E^((8*ArcCoth[x])/3)*Hypergeometric2F1[2/3, 1, 5/3, E^(4*ArcCoth[x])])/2
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x} \frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3)/x,x)
```

Maxima [A] time = 1.51635, size = 189, normalized size = 1.22

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1))
^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3) - (
(x - 1)/(x + 1))^(1/3) + 1) - log(((x - 1)/(x + 1))^(1/3) + 1) - log(((x -
1)/(x + 1))^(1/3) - 1)
```

Fricas [A] time = 1.6545, size = 266, normalized size = 1.72

$$\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right)+\frac{1}{2}\log\left(\frac{(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+x+1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(2/3) + 1/3*sqrt(3)) - log(((x
 - 1)/(x + 1))^(2/3) - 1) + 1/2*log(((x + 1)*((x - 1)/(x + 1))^(2/3) + (x -
 1)*((x - 1)/(x + 1))^(1/3) + x + 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/3)), x)
```

Giac [A] time = 1.16116, size = 192, normalized size = 1.24

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1))
^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3) - (
(x - 1)/(x + 1))^(1/3) + 1) - log(abs(((x - 1)/(x + 1))^(1/3) + 1)) - log(a
bs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.124 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=99

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}-\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{\sqrt{3}}$$

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3) - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/
x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[1 + ((-1 + x)/x)^(1/
3)/(1 + x^(-1))^(1/3)] - Log[1 + x^(-1)]/3
```

Rubi [A] time = 0.0429347, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6171, 50, 60}

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}-\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x^2,x]
```


```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3) - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/
x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[1 + ((-1 + x)/x)^(1/
3)/(1 + x^(-1))^(1/3)] - Log[1 + x^(-1)]/3
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx,x,\frac{1}{x}\right)\\ &= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx,x,\frac{1}{x}\right)\\ &= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(1+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{1}{3}\log\left(1+\frac{1}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.144131, size = 87, normalized size = 0.88

$$\frac{2e^{\frac{2}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}+1}-\frac{2}{3}\log\left(e^{\frac{2}{3}\coth^{-1}(x)}+1\right)+\frac{1}{3}\log\left(-e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}+1\right)-\frac{2\tan^{-1}\left(\frac{2e^{\frac{2}{3}\coth^{-1}(x)}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (2*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x])) - (2*ArcTan[(-1 + 2*E^((2*A
rcCoth[x])/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 + E^((2*ArcCoth[x])/3)])/3 + Lo
g[1 - E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)]/3
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}} \frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x^2,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3)/x^2,x)
```

Maxima [A] time = 1.62854, size = 132, normalized size = 1.33

$$\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\,\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Fricas [A] time = 1.64112, size = 293, normalized size = 2.96

$$\frac{2\,\sqrt{3}x\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-2\,x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3))
+ x*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 2*x*log(((
```

```
x - 1)/(x + 1))^(1/3) + 1) + 3*(x + 1)*((x - 1)/(x + 1))^(2/3))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**2,x)
```

```
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/3)), x)
```

Giac [A] time = 1.13534, size = 134, normalized size = 1.35

$$\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\,\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\,\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

3.125 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=130

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}+\frac{1}{3}\left(\frac{x-1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{1}{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{9}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{3\sqrt{3}}$$

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3))/3 + ((1 + x^(-1))^(4/3)*((-1 + x)/x
)^(2/3))/2 - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(
-1))^(1/3))])/(3*Sqrt[3]) - Log[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/
3 - Log[1 + x^(-1)]/9
```

Rubi [A] time = 0.0533082, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 80, 50, 60}

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}+\frac{1}{3}\left(\frac{x-1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{1}{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{9}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3))/3 + ((1 + x^(-1))^(4/3)*((-1 + x)/x
)^(2/3))/2 - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(
-1))^(1/3))])/(3*Sqrt[3]) - Log[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/
3 - Log[1 + x^(-1)]/9
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx &= -\operatorname{Subst}\left(\int \frac{x\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) -
\end{aligned}
$$

Mathematica [C] time = 0.275655, size = 134, normalized size = 1.03

$$-\frac{2}{27}\left(-\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{\#1^2\coth^{-1}(x)-3\#1^2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+\coth^{-1}(x)}{\#1^2-2}\&\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] (-2*((27*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x]))^2 - (36*E^((2*ArcCoth
[x])/3))/(1 + E^(2*ArcCoth[x])) - 2*ArcCoth[x] + 3*Log[1 + E^((2*ArcCoth[x]
)/3)] - RootSum[1 - #1^2 + #1^4 & , (ArcCoth[x] - 3*Log[E^(ArcCoth[x]/3) -
#1] + ArcCoth[x]*#1^2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-2 + #1^2) & ])
)/27
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x^3,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3)/x^3,x)
```

Maxima [A] time = 1.52712, size = 167, normalized size = 1.28

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*(((x
- 1)/(x + 1))^(5/3) + 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) + (x -
1)^2/(x + 1)^2 + 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(
1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Fricas [A] time = 1.58706, size = 321, normalized size = 2.47

$$\frac{4\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-4x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\left(5x^2+8x+3\right)\left(\frac{x-1}{x+1}\right.}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3
)) + 2*x^2*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 4*x
^2*log(((x - 1)/(x + 1))^(1/3) + 1) + 3*(5*x^2 + 8*x + 3)*((x - 1)/(x + 1))
^(2/3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/3)), x)
```

Giac [A] time = 1.15536, size = 165, normalized size = 1.27

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}+1\right)^2}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*((x -
 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) + 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/(
x + 1) + 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) +
 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

3.126 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=429

$$\frac{37x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{96a^2}-\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}+\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}-\frac{11\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+$$

```
[Out] (37*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(96*a^2) + (3*(1 - 1/(a*x))^
(7/8)*(1 + 1/(a*x))^(1/8)*x^2)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x^3)/3 - (11*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1
/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 -
1/(a*x))^(1/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(
a*x))^(1/8)])/(64*a^3) + (11*ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8
)])/(64*a^3) - (11*Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8
) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3) + (11*Log[1
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3)
```

Rubi [A] time = 0.343278, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6171, 99, 151, 12, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{37x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{96a^2}-\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}+\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}-\frac{11\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x^2,x]
```


```
[Out] (37*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(96*a^2) + (3*(1 - 1/(a*x))^
(7/8)*(1 + 1/(a*x))^(1/8)*x^2)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x^3)/3 - (11*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1
/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 -
1/(a*x))^(1/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(
a*x))^(1/8)])/(64*a^3) + (11*ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8
)])/(64*a^3) - (11*Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8
) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3) + (11*Log[1
```

```
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^4\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{9}{4a}+\frac{2x}{a^2}}{x^3\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{37}{16a^2}-\frac{9x}{4a^3}}{x^2\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\cdots}{64a\cdots}\cdots\right.$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\,\text{Subst}\left(\int \frac{\cdots}{x\sqrt[8]{1\cdots}}\cdots\right.}{12\cdots}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\,\text{Subst}\left(\int \frac{1}{-1+\cdots}\cdots\right.}{16a\cdots}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\,\text{Subst}\left(\int \frac{1}{1-x^4\cdots}\cdots\right.}{32a\cdots}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\,\text{Subst}\left(\int \frac{1}{1-x^2\cdots}\cdots\right.}{64a\cdots}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\cdots\right.}{64a^3}\cdots$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\cdots\right.}{64a^3}\cdots$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[8]{\cdots}}\cdots\right.}{64\sqrt{2}a^3}\cdots$$

Mathematica [C] time = 5.30153, size = 167, normalized size = 0.39

$$\frac{-33\text{RootSum}\left[\#1^4+1\&,\frac{\coth^{-1}(ax)-4\log\left(e^{\frac{1}{4}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]-4\left(-\frac{840e^{\frac{1}{4}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}-\frac{1600e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}-\frac{1024e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3}+33\,\text{lo}\right.}{1536a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] (-4*((-1024*E^(ArcCoth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^3 - (1600*E^(ArcC
oth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^2 - (840*E^(ArcCoth[a*x]/4))/(-1 + E
^(2*ArcCoth[a*x])) - 66*ArcTan[E^(ArcCoth[a*x]/4)] + 33*Log[1 - E^(ArcCoth[
a*x]/4)] - 33*Log[1 + E^(ArcCoth[a*x]/4)]) - 33*RootSum[1 + #1^4 & , (ArcCo
th[a*x] - 4*Log[E^(ArcCoth[a*x]/4) - #1])/#1^3 & ])/(1536*a^3)
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int x^2\frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

Maxima [A] time = 1.55936, size = 460, normalized size = 1.07

$$-\frac{1}{768}a\left(\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="maxima")
```

```
[Out] -1/768*a*(16*(33*((a*x - 1)/(a*x + 1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(1
5/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x
 - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*l
og(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1))/a^4 + 132*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x
 - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a
^4)
```

Fricas [A] time = 1.91345, size = 1332, normalized size = 3.1

$$132\,\sqrt{2}a^3\frac{1}{a^{12}}^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^{12}}^{\frac{3}{4}}+a^6\sqrt{\frac{1}{a^{12}}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}a^3\frac{1}{a^{12}}^{\frac{1}{4}}-\sqrt{2}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^{12}}^{\frac{1}{4}}-1\right)+132\,\sqrt{2}a^3\frac{1}{a^{12}}^{\frac{1}{4}}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="fricas")
```

```
[Out] 1/768*(132*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^9*((a*
x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-12)) + ((a*x - 1)/(a
*x + 1))^(1/4))*a^3*(a^(-12))^(1/4) - sqrt(2)*a^3*((a*x - 1)/(a*x + 1))^(1/
8)*(a^(-12))^(1/4) - 1) + 132*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*sq
rt(-sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-
12)) + ((a*x - 1)/(a*x + 1))^(1/4))*a^3*(a^(-12))^(1/4) - sqrt(2)*a^3*((a*x
 - 1)/(a*x + 1))^(1/8)*(a^(-12))^(1/4) + 1) + 33*sqrt(2)*a^3*(a^(-12))^(1/4
)*log(sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^
(-12)) + ((a*x - 1)/(a*x + 1))^(1/4)) - 33*sqrt(2)*a^3*(a^(-12))^(1/4)*log(
-sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-12)
) + ((a*x - 1)/(a*x + 1))^(1/4)) + 8*(32*a^3*x^3 + 68*a^2*x^2 + 73*a*x + 37
)*((a*x - 1)/(a*x + 1))^(7/8) - 132*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 6
6*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 66*log(((a*x - 1)/(a*x + 1))^(1/8)
 - 1))/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22453, size = 450, normalized size = 1.05

$$-\frac{1}{768}a\left(\frac{66\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}+\frac{66\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}-\frac{33\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="giac")
```

```
[Out] -1/768*a*(66*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/8)))/a^4 + 66*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/8)))/a^4 - 33*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 33*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 132*arctan(((a*x - 1)
/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*l
og(abs(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^4 - 16*(10*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(7/8)/(a*x + 1) - 33*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(7/8)/(a*
x + 1)^2 - 105*((a*x - 1)/(a*x + 1))^(7/8))/(a^4*((a*x - 1)/(a*x + 1) - 1)^
3))
```

3.127 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=392

$$-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{16\sqrt{2}a^2}+\frac{\tan^{-1}\left(\ldots\right)}{16a\ldots}$$

```
[Out] ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1
 + 1/(a*x))^(9/8)*x^2)/2 - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/
(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(
1 - 1/(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a
*x))^(1/8)]/(16*a^2) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/(16
*a^2) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(32*Sqrt[2]*a^2) + Log[1 + (Sqrt[2]*(1 +
1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/(32*Sqrt[2]*a^2)
```

Rubi [A] time = 0.247184, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6171, 96, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{16\sqrt{2}a^2}+\frac{\tan^{-1}\left(\ldots\right)}{16a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1
 + 1/(a*x))^(9/8)*x^2)/2 - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/
(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(
1 - 1/(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a
*x))^(1/8)]/(16*a^2) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/(16
*a^2) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(32*Sqrt[2]*a^2) + Log[1 + (Sqrt[2]*(1 +
1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/(32*Sqrt[2]*a^2)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

[a/b, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^3\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{32a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{8a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+x\cdots}\right.}{8\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+x\cdots}\right.}{1\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} - \frac{\log\left(1\cdots\right.}{\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{16\sqrt{2}a^2}$$

Mathematica [A] time = 0.724106, size = 319, normalized size = 0.81

$$\frac{6}{e^{\frac{1}{4}\coth^{-1}(ax)}-1} + \frac{6}{e^{\frac{1}{4}\coth^{-1}(ax)}+1} - \frac{12e^{\frac{1}{4}\coth^{-1}(ax)}}{e^{\frac{1}{2}\coth^{-1}(ax)}+1} - \frac{40e^{\frac{1}{4}\coth^{-1}(ax)}}{e^{\coth^{-1}(ax)}+1} + \frac{2}{\left(e^{\frac{1}{4}\coth^{-1}(ax)}-1\right)^2} - \frac{2}{\left(e^{\frac{1}{4}\coth^{-1}(ax)}+1\right)^2} + \frac{8e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(e^{\frac{1}{2}\coth^{-1}(ax)}+1\right)^2} + \frac{32e^{\frac{1}{4}\cdots}}{\left(e^{\coth^{-1}\cdots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] (2/(-1 + E^(ArcCoth[a*x]/4))^2 + 6/(-1 + E^(ArcCoth[a*x]/4)) - 2/(1 + E^(Ar
cCoth[a*x]/4))^2 + 6/(1 + E^(ArcCoth[a*x]/4)) + (8*E^(ArcCoth[a*x]/4))/(1 +
 E^(ArcCoth[a*x]/2))^2 - (12*E^(ArcCoth[a*x]/4))/(1 + E^(ArcCoth[a*x]/2)) +
 (32*E^(ArcCoth[a*x]/4))/(1 + E^ArcCoth[a*x])^2 - (40*E^(ArcCoth[a*x]/4))/(
1 + E^ArcCoth[a*x]) + 4*ArcTan[E^(ArcCoth[a*x]/4)] - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^(ArcCoth[a*x]/4)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/4
)] + 4*ArcTanh[E^(ArcCoth[a*x]/4)] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x
]/4) + E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/4) + E
^(ArcCoth[a*x]/2)])/(64*a^2)
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

Maxima [A] time = 1.55323, size = 410, normalized size = 1.05

$$\frac{1}{64}\,a\left(\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)-\sqrt{}}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="maxima")
```

```
[Out] 1/64*a*(16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x - 1)/(a*x + 1))^(7/8))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - (2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*lo
```

```
g(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1))/a^3 - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^3 + 2*log(((a*x - 1
)/(a*x + 1))^(1/8) + 1)/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a^3)
```

Fricas [A] time = 1.73388, size = 1269, normalized size = 3.24

$$4\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^8}^{\frac{3}{4}}+a^4\sqrt{\frac{1}{a^8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}a^2\frac{1}{a^8}^{\frac{1}{4}}-\sqrt{2}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^8}^{\frac{1}{4}}-1\right)+4\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="fricas")
```

```
[Out] 1/64*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^6*((a*x -
1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(a*x + 1
))^(1/4))*a^2*(a^(-8))^(1/4) - sqrt(2)*a^2*((a*x - 1)/(a*x + 1))^(1/8)*(a^(
-8))^(1/4) - 1) + 4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*sqrt(-sqrt(2)
*a^6*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x
- 1)/(a*x + 1))^(1/4))*a^2*(a^(-8))^(1/4) - sqrt(2)*a^2*((a*x - 1)/(a*x + 1
))^(1/8)*(a^(-8))^(1/4) + 1) + sqrt(2)*a^2*(a^(-8))^(1/4)*log(sqrt(2)*a^6*(
(a*x - 1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(
a*x + 1))^(1/4)) - sqrt(2)*a^2*(a^(-8))^(1/4)*log(-sqrt(2)*a^6*((a*x - 1)/(
a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(a*x + 1))^(
1/4)) + 8*(4*a^2*x^2 + 9*a*x + 5)*((a*x - 1)/(a*x + 1))^(7/8) - 4*arctan(((
a*x - 1)/(a*x + 1))^(1/8)) + 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log
(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/8), x)
```

Giac [A] time = 1.20437, size = 405, normalized size = 1.03

$$-\frac{1}{64}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right.\right.}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="giac")
```

```
[Out] -1/64*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/8)))/a^3 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8)))/a^3 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^3 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 + 4*arctan(((a*x - 1)/(a*x + 1))
^(1/8))/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 2*log(abs(((a*x
- 1)/(a*x + 1))^(1/8) - 1))/a^3 + 16*((a*x - 1)*((a*x - 1)/(a*x + 1))^(7/8)
/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(7/8))/(a^3*((a*x - 1)/(a*x + 1) - 1)^
2))
```

3.128 $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=352

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}a}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(\sqrt{\ldots}\right.}{2\ldots}$$

```
[Out] (1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x
))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*(1 + 1/(
a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)]/(2*a) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/(2*a) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1
 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*Sqrt[2]*a) + Log[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/(4*Sqrt[2]*a)
```

Rubi [A] time = 0.200465, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.3, Rules used = {6170, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}a}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(\sqrt{\ldots}\right.}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x
))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*(1 + 1/(
a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)]/(2*a) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/(2*a) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1
 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*Sqrt[2]*a) + Log[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/(4*Sqrt[2]*a)
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x - \frac{2\operatorname{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{1}{1\cdots}\right.}{}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4a} + \cdots$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a} + \frac{\log\left(1\cdots\right.}{}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a} + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\cdots}{}$$

Mathematica [C] time = 0.0473046, size = 56, normalized size = 0.16

$$\frac{2e^{\frac{1}{4}\coth^{-1}(ax)}\left(\left(e^{2\coth^{-1}(ax)}-1\right)\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{2\coth^{-1}(ax)}\right)+1\right)}{a\left(e^{2\coth^{-1}(ax)}-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (2*E^(ArcCoth[a*x]/4)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[1/8,
 1, 9/8, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

Maxima [A] time = 1.52903, size = 358, normalized size = 1.02

$$-\frac{1}{8}\,a\left(\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+\cdots}\right.\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="maxima")
```

```
[Out] -1/8*a*(16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)*a^2/(a*x + 1) - a^2) + (2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sq
rt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
 + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
+ 1))^(1/4) + 1))/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^2 - 2*log((
(a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1
)/a^2)
```

Fricas [A] time = 1.81266, size = 1224, normalized size = 3.48

$$4\sqrt{2}a\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^4}^{\frac{3}{4}}+a^2\sqrt{\frac{1}{a^4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}a\frac{1}{a^4}^{\frac{1}{4}}-\sqrt{2}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^4}^{\frac{1}{4}}-1\right)+4\sqrt{2}a\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^3*((a*x - 1)/
(a*x + 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^
(1/4))*a*(a^(-4))^(1/4) - sqrt(2)*a*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-4))^(1
/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^3*((a*
x - 1)/(a*x + 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x
 + 1))^(1/4))*a*(a^(-4))^(1/4) - sqrt(2)*a*((a*x - 1)/(a*x + 1))^(1/8)*(a^(
-4))^(1/4) + 1) + sqrt(2)*a*(a^(-4))^(1/4)*log(sqrt(2)*a^3*((a*x - 1)/(a*x
+ 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^(1/4)
) - sqrt(2)*a*(a^(-4))^(1/4)*log(-sqrt(2)*a^3*((a*x - 1)/(a*x + 1))^(1/8)*(
a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^(1/4)) + 8*(a*x +
1)*((a*x - 1)/(a*x + 1))^(7/8) - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 2*
log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log(((a*x - 1)/(a*x + 1))^(1/8) -
1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/8), x)
```

Giac [A] time = 1.22475, size = 363, normalized size = 1.03

$$-\frac{1}{8}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^2}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^2}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")
```

```
[Out] -1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
8)))/a^2 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/8)))/a^2 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^2 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^
(1/8))/a^2 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(abs(((a*x -
 1)/(a*x + 1))^(1/8) - 1))/a^2 + 16*((a*x - 1)/(a*x + 1))^(7/8)/(a^2*((a*x
- 1)/(a*x + 1) - 1)))
```

3.129 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=919

result too large to display

```
[Out] -(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]]) - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 +
Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]]
+ Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + S
qrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]] -
 Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + Sq
rt[2]*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + 2*Arc
Tan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] + 2*ArcTanh[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)] + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1
+ 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1
/8)])/2 - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4
) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 + (Sqrt
[2 + Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + S
qrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8
))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2] +
 Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.899297, antiderivative size = 919, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6171, 105, 63, 331, 299, 1122, 1169, 634, 618, 204, 628, 93, 214, 212, 206, 203, 211, 1165, 1162, 617}

$$-\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] -(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]]) - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 +
Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]]
+ Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + S
qrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]] -
 Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + Sq
rt[2]*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + 2*Arc
Tan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] + 2*ArcTanh[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)] + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1
+ 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1
/8)])/2 - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4
) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 + (Sqrt
[2 + Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + S
qrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8
))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2] +
 Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= 8\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right) - 8\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 8\,\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x\right.$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x\cdots}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\cdots}}{\sqrt[4]{1-\cdots}}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\cdots}}{\sqrt[8]{1-\cdots}}\right.$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\cdots}}{\sqrt[8]{1-\cdots}}\right.$$

$$= -\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2+\sqrt{2}}\tan^{-1}\left(\cdots\right.$$

Mathematica [C] time = 0.0390659, size = 30, normalized size = 0.03

$$\frac{16}{9}e^{\frac{9}{4}\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{9}{16},1,\frac{25}{16},e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] (16*E^((9*ArcCoth[a*x])/4)*Hypergeometric2F1[9/16, 1, 25/16, E^(4*ArcCoth[a
*x])])/9
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Fricas [B] time = 2.13455, size = 6747, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-((sqr
t(2) + 2)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(2*(sqrt(2
)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2
) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)
/(a*x + 1))^(1/4) + 4) + 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 3*sqrt(sqr
t(2) + 2))/((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt
(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) +
2))*arctan(((sqrt(2) + 2)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + sqrt(2
)*sqrt(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*
sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/
8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt
(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)
*sqrt(-sqrt(2) + 2))*arctan(((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt
(2) + 2) - sqrt(2)*sqrt(2*(sqrt(2)*(sqrt(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2)
+ 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1
)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) + 2*sqrt(2)*((a*x -
 1)/(a*x + 1))^(1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2)^(3/2) - (sqrt(2)
 + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2
) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-((sqrt(2) + 2)^(3/2) + (sqrt(2
) + 1)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) +
(sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(
2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) -
 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2
)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/8*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*(sqrt
(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*s
qrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x +
 1))^(1/4) + 4) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) +
2))*log(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2) + 2) - sqrt(2))
*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1
/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) -
 sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*
(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*
((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 1/8*(sqr
t(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sqrt(2)*(sqrt(
2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sq
rt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x +
1))^(1/4) + 4) + 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^
```

```
(1/8) - 1) + 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x +
 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - sqrt(2)*((a*x - 1)/(a*x +
 1))^(1/8) + 1) - sqrt(-sqrt(2) + 2)*arctan(-((sqrt(2) + 1)*sqrt(-sqrt(2) +
 2) - 2*sqrt((sqrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) +
 ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 2*((a*x - 1)/(a*x + 1))^(1/8))/((sqrt(2
) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))) - sqrt(-sqrt(2) + 2)*arctan(((sqrt(2)
+ 1)*sqrt(-sqrt(2) + 2) + 2*sqrt(-(sqrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 2*((a*x - 1)/(a*x
+ 1))^(1/8))/((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))) - sqrt(sqrt(2) +
2)*arctan(-((sqrt(2) + 2)^(3/2) - 2*sqrt(((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt
(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) -
3*sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/((sqrt(2) + 1)*sqrt(-s
qrt(2) + 2))) - sqrt(sqrt(2) + 2)*arctan(((sqrt(2) + 2)^(3/2) + 2*sqrt(-((s
qrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) - 3*sqrt(sqrt(2) + 2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8))/((sqrt(2) + 1)*sqrt(-sqrt(2) + 2))) - 1/4*sqrt(sqrt(2) + 2)*log((s
qrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*
x + 1))^(1/4) + 1) + 1/4*sqrt(sqrt(2) + 2)*log(-(sqrt(2) + 1)*sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/4*
sqrt(-sqrt(2) + 2)*log(((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/4*sqrt(-sqrt(2)
+ 2)*log(-((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))
^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/2*sqrt(2)*log(4*sqrt(2)*((a*x
 - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 1/2*sqrt(2)*l
og(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) +
 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + log(((a*x - 1)/(a*x + 1))^(1/
8) + 1) - log(((a*x - 1)/(a*x + 1))^(1/8) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x,x)

[Out] Timed out
```

Giac [A] time = 1.25315, size = 878, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
8)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/8)))/a - 2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a
*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/a - 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(
sqrt(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/a - 2*sqr
t(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/
sqrt(sqrt(2) + 2))/a - 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*
((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2))/a - sqrt(2)*log(sqrt(2)*((a
*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + sqrt(2)*log
(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
+ sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/a - sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*
((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + sqrt(-sq
rt(2) + 2)*log(sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a - sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x
- 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 4*arctan(((a*x
 - 1)/(a*x + 1))^(1/8))/a - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a + 2*lo
g(abs(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a)
```

3.130 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=676

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{\sqrt{2-\sqrt{2}}}{\sqrt[8]{\frac{1}{ax}}}\right.$$

```
[Out] a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) - (Sqrt[2 + Sqrt[2]]*a*ArcTan[(Sq
rt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/4 - (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x
))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqrt[2]]*a
*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/S
qrt[2 + Sqrt[2]]])/4 + (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*
(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 -
 Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 - Sqrt[2]]*a*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 + (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1
/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))
/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)
/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x)
)^(1/8)])/8
```

Rubi [A] time = 0.605244, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{\sqrt{2-\sqrt{2}}}{\sqrt[8]{\frac{1}{ax}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) - (Sqrt[2 + Sqrt[2]]*a*ArcTan[(Sq
rt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/4 - (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x
))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqrt[2]]*a
*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/S
qrt[2 + Sqrt[2]]])/4 + (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*
(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 -
 Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 - Sqrt[2]]*a*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 + (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1
/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))
/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)
/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x)
)^(1/8)])/8
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\operatorname{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\operatorname{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\operatorname{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\operatorname{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{a\operatorname{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\operatorname{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, \ldots\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a\right)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) + \frac{1}{4}\left(\ldots\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1 \ldots\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\sqrt{2+\sqrt{2}}a\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2-\sqrt{2}}a\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}} \ldots}{\sqrt{2-\ldots}}\right.$$

Mathematica [C] time = 0.0608243, size = 46, normalized size = 0.07

$$-2ae^{\frac{1}{4}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)-\frac{1}{e^{2\coth^{-1}(ax)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] -2*a*E^(ArcCoth[a*x]/4)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometric2F1
[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])])
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Fricas [B] time = 2.19607, size = 7880, normalized size = 11.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^8)^(1/8)*a^7*((a*x
- 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - 2*sq
rt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 + (a^8)^(7/8)*(a^7*(s
qrt(2) + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1
/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2))) + 8*(a^8)^(1/8)*x
*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8)
- (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - 2*sqrt(a^14*((a*x - 1)/(a*
x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqr
t(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2
)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*arc
tan(-(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*(a^8)^(1/8)*a^7
*((a*x - 1)/(a*x + 1))^(1/8) - 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a
^8)^(3/4)*a^8 + (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(
7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/((a^8*(sqrt(2) + 2) - a^8)*s
qrt(-sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*arctan((a^8*(sqrt(2
) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x
+ 1))^(1/8) + 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (
a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(
a*x + 1))^(1/8))*(a^8)^(1/8))/((a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2)
)) + 2*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4)
 + (a^8)^(3/4)*a^8 + (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqrt(-sqrt(2) +
2)*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*log(a^1
4*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^8)^(7/8)*(a^7*(sqrt(2)
 + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) + 2*(a^8)^(1/8
)*x*sqrt(-sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a
^8 + (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x
- 1)/(a*x + 1))^(1/8)) - 2*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*log(a^14*((a*x
- 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*
sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) + 4*(a^8)^(1/8)
*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(a^8*
(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^8)^(1/8)*a^7*(
(a*x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) -
 sqrt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(s
qrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)
*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1
)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt
(2) + 2) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(
sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((a^8*(sq
rt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*
x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) + sq
```

```
rt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt
(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^
7*(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(
a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2)
 + 2) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqr
t(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((a^8*(sqrt(
2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x -
 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - sqrt(
2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)
*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(
sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x
 + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) +
2) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqrt(2
)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(a^8*(sqrt(2)
 + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x - 1
)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) + sqrt(2)
*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2)*a
^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sq
rt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x +
 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2)
 - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + (a^8)^(1/8)*(sqrt(2)*x*
sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4*a^14*((a*x - 1)/(a*
x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*
sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*s
qrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) - (a^8)^(1/8)*(
sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4*a^14*((a*
x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2)*a^7*(sqrt(2) + 2)^
(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt
(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) - (a^
8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4
*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqr
t(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^7*(sqrt(2) +
 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1
/8)) + (a^8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) +
 2))*log(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2
)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^7*
(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*
x + 1))^(1/8)) - 32*(a*x + 1)*((a*x - 1)/(a*x + 1))^(7/8))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2168, size = 583, normalized size = 0.86

$$\frac{1}{8}\left(2\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+2}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1
))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2)
+ 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(sqrt(2)
+ 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(
2) + 2)) + 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a*x - 1)/(
a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) +
2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(sq
rt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1) - sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*((a*x - 1
)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(-sqrt(2) + 2)*
log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))
^(1/4) + 1) + 16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)/(a*x + 1) + 1))*a
```

3.131 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=731

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{64}\surd$$

```
[Out] (a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8))/8 + (a^2*(1 - 1/(a*x))^(7/8)*
(1 + 1/(a*x))^(9/8))/2 - (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sqrt[2 - Sqrt[2]] -
 (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]])/32 - (Sqr
t[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 +
 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sq
rt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/32 + (Sqrt[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(
a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 - Sqrt[2
]]*a^2*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]
*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 - Sqrt[2]]*a^2*Log
[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a
*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 + (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 -
1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8)
)/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 - 1/(a*x))^(
1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(
a*x))^(1/8)])/64
```

Rubi [A] time = 0.658648, antiderivative size = 731, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{64}\surd$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8))/8 + (a^2*(1 - 1/(a*x))^(7/8)*
(1 + 1/(a*x))^(9/8))/2 - (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sqrt[2 - Sqrt[2]] -
 (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]])/32 - (Sqr
t[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 +
 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sq
rt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/32 + (Sqrt[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(
a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 - Sqrt[2
]]*a^2*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]
*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 - Sqrt[2]]*a^2*Log
[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a
*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 + (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 -
1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8)
)/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 - 1/(a*x))^(
1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(
a*x))^(1/8)])/64
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\operatorname{Subst}\left(\int \frac{x\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{8}a\operatorname{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}a\operatorname{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\operatorname{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\operatorname{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\operatorname{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{a^2}{}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{a^2\operatorname{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{a^2}{}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\operatorname{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{32}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a^2\right)\operatorname{Subst}\left(\int \frac{}{1-\sqrt{2}}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{64}\sqrt{2-\sqrt{2}}a^2\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\sqrt{2-}}{\sqrt[8]{}}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}\sqrt{2+\sqrt{2}}a^2\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right.$$

Mathematica [C] time = 0.0900633, size = 72, normalized size = 0.1

$$-\frac{a^2 e^{\frac{1}{4}\coth^{-1}(ax)}\left(\left(e^{2\coth^{-1}(ax)}+1\right)^2\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)-9e^{2\coth^{-1}(ax)}-1\right)}{4\left(e^{2\coth^{-1}(ax)}+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] -(a^2*E^(ArcCoth[a*x]/4)*(-1 - 9*E^(2*ArcCoth[a*x]) + (1 + E^(2*ArcCoth[a*x
]))^2*Hypergeometric2F1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])]))/(4*(1 + E^(2*Ar
cCoth[a*x]))^2)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Fricas [B] time = 2.27531, size = 8216, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="fricas")
```

```
[Out] -1/256*(8*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^16)^(1/8)*a^14*
((a*x - 1)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2
) - 2*sqrt(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 + (a^16)^(7
/8)*(a^14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1
/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2))) +
 8*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^16)^(1/8)*a^14*((a*x -
 1)/(a*x + 1))^(1/8) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) - 2*s
qrt(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^
14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a
^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2))) + 8*(a^1
6)^(1/8)*x^2*sqrt(sqrt(2) + 2)*arctan(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*s
qrt(sqrt(2) + 2) + 2*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) - 2*sqrt
(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 + (a^16)^(7/8)*(a^14*
(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8)
)*(a^16)^(1/8))/((a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 8*(a^16
)^(1/8)*x^2*sqrt(sqrt(2) + 2)*arctan((a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqr
t(sqrt(2) + 2) - 2*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) + 2*sqrt(a
^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14*(s
qrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*
(a^16)^(1/8))/((a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 2*(a^16)^
(1/8)*x^2*sqrt(sqrt(2) + 2)*log(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(
3/4)*a^16 + (a^16)^(7/8)*(a^14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a
*x - 1)/(a*x + 1))^(1/8)) - 2*(a^16)^(1/8)*x^2*sqrt(sqrt(2) + 2)*log(a^28*(
(a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14*(sqrt(2
) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) + 2*(a^16)^(
1/8)*x^2*sqrt(-sqrt(2) + 2)*log(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(
3/4)*a^16 + (a^16)^(7/8)*(a^14*(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) +
2))*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*lo
g(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14
*(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8
)) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + sqrt(2)*x^2*sqrt(-sqrt
(2) + 2))*arctan(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + 2*
sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (a^16*(sqrt(2) + 2)
 - a^16)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/
4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2)
```

```
 - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)
*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16
*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + (a^16*(sqrt(2) + 2) - a^1
6)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + s
qrt(2)*x^2*sqrt(-sqrt(2) + 2))*arctan((a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sq
rt(sqrt(2) + 2) - 2*sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) -
 (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^28*((a*x
 - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14
*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sq
rt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))
*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + (a^16
*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*s
qrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*arctan((a^16*(sqrt(2) +
2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1
)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) - sqrt(
2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)
^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2)
 + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x -
1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(
sqrt(2) + 2) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^
(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*arct
an(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^16)
^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(
-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^
(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a
^14*sqrt(sqrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-s
qrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)
^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(
2) + 2))) + (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + sqrt(2)*x^2*sqrt(
-sqrt(2) + 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16
 + 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(s
qrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2
))*((a*x - 1)/(a*x + 1))^(1/8)) - (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) +
2) + sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4)
 + 4*(a^16)^(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) -
 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a
^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8)) - (a^16)^(1/8)*(sqrt(
2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*log(4*a^28*((a*x
 - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)^(7/8)*(sqrt(2)*a^14
*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sq
rt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))
 + (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2)
+ 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 - 2*(a^1
6)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) +
2) - (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x
```

```
- 1)/(a*x + 1))^(1/8)) - 32*(5*a^2*x^2 + 9*a*x + 4)*((a*x - 1)/(a*x + 1))^(
7/8))/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19996, size = 639, normalized size = 0.87

$$\frac{1}{64}\left(2\,a\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\sqrt{\sqrt{2}+2}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(2*a*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x
+ 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqr
t(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(s
qrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqr
t(sqrt(2) + 2)) + 2*a*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a
*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - a*sqrt(sqrt(2) + 2)*log(sqrt
(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1
) + a*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)
+ ((a*x - 1)/(a*x + 1))^(1/4) + 1) - a*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + a*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) + 16*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(7/8)/
(a*x + 1) + 9*a*((a*x - 1)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.132 $\int e^{4\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rubi [A] time = 0.0570997, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6126, 89, 80, 64}

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}x^m\,dx &= \int e^{4\tanh^{-1}(ax)}x^m\,dx\\
&= \int \frac{x^m(1+ax)^2}{(1-ax)^2}\,dx\\
&= \frac{4x^{1+m}}{1-ax}-\frac{\int \frac{x^m\left(a^2(3+4m)+a^3x\right)}{1-ax}\,dx}{a^2}\\
&= \frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-(4(1+m))\int \frac{x^m}{1-ax}\,dx\\
&= \frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-4x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)
\end{aligned}
$$

Mathematica [A] time = 0.0224629, size = 47, normalized size = 1.04

$$\frac{x^{m+1}(-4(m+1)(ax-1)\text{Hypergeometric2F1}(1,m+1,m+2,ax)+ax-4m-5)}{(m+1)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 +
m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

Maple [C] time = 0.606, size = 201, normalized size = 4.5

$$-\frac{(-a)^{-m}}{a}\left(\frac{x^m(-a)^m\left(a^2mx^2+amx+2\,ax-m^2-3\,m-2\right)}{m\,(1+m)\,(-ax+1)}+x^m\,(-a)^m\,(2+m)\,\text{LerchPhi}\,(ax,1,m)\right)+2\,\frac{(-a)^{-m}}{a}\left(-\frac{x^m\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^m,x)
```

```
[Out] -(-a)^(-m)/a*(x^m*(-a)^m*(a^2*m*x^2+a*m*x+2*a*x-m^2-3*m-2)/(1+m)/m/(-a*x+1)
+x^m*(-a)^m*(2+m)*LerchPhi(a*x,1,m))+2*(-a)^(-m)/a*(-x^m*(-a)^m*(a*x-m-1)/m
/(-a*x+1)-x^m*(-a)^m*(1+m)*LerchPhi(a*x,1,m))-(-a)^(-m)/a*(1/(1+m)*x^m*(-a)
^m*(-1-m)/(-a*x+1)+x^m*(-a)^m*m*LerchPhi(a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 x^m}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

3.133 $\int e^{3\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=151

$$-\frac{x^m\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}+\frac{4x^m\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}-\frac{3x^{m+1}\text{Hypergeom}}{}$$

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m) + (
4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1
+ m) + (4*x^m*Hypergeometric2F1[3/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rubi [A] time = 1.23086, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6172, 6742, 364, 850, 808}

$$-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{4x^m\,{}_2F_1\left(\frac{3}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-\right.}{m\cdot}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m) + (
4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1
+ m) + (4*x^m*Hypergeometric2F1[3/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}} - \frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}} + \frac{4x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\right)$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + \cdots$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1-m); \frac{1-m}{2}; \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1-\frac{m}{2}; \frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \cdots\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1-m); \frac{1-m}{2}; \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1-\frac{m}{2}; \frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \cdots\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1-m); \frac{1-m}{2}; \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1-\frac{m}{2}; \frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2}, \frac{1}{2}(-1-m); \cdots\right.}{1+m}$$

<u>Mathematica</u> [C] time = 0.319526, size = 228, normalized size = 1.51

$$\frac{x^{m+1}\left(m\sqrt{ax-1}\sqrt{ax+1}\sqrt{x^2-\frac{1}{a^2}}\text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{m}{2}-\frac{1}{2}, \frac{1}{2}-\frac{m}{2}, \frac{1}{a^2x^2}\right) + 3(m+1)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{ax+1}{a^2}}\cdots\right.}{m(m+1)\sqrt{ax-1}\sqrt{ax+1}\sqrt{x^2\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^
2]*AppellF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - 2*(1 + m)*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, -(a*x)
, a*x] + m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]*Hypergeometric2
F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]))/(m*(1 + m)*Sqrt[-1 + a*x]*Sq
```

```
rt[1 + a*x]*Sqrt[-a^(-2) + x^2])
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2 + 2\,ax + 1\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2 - 2\,ax + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.134 $\int e^{2\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=35

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(
1 + m)
```

Rubi [A] time = 0.0412795, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6126, 80, 64}

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(
1 + m)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^m\,dx &= -\int e^{2\tanh^{-1}(ax)}x^m\,dx\\ &= -\int \frac{x^m(1+ax)}{1-ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-2\int \frac{x^m}{1-ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0089821, size = 26, normalized size = 0.74

$$\frac{x^{m+1}(1-2\text{Hypergeometric2F1}(1,m+1,m+2,ax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

Maple [C] time = 0.38, size = 106, normalized size = 3.

$$\frac{(-a)^{-m}}{a}\left(-\frac{x^m(-a)^m(-1-m)}{(1+m)m}-x^m(-a)^m\,\mathrm{LerchPhi}(ax,1,m)\right)-\frac{(-a)^{-m}}{a}\left(-\frac{x^m(-a)^m(amx+m+1)}{(1+m)m}+x^m(-a)^m\,\mathrm{Lerch}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^m,x)
```

```
[Out] (-a)^(-m)/a*(-1/(1+m)*x^m*(-a)^m*(-1-m)/m-x^m*(-a)^m*LerchPhi(a*x,1,m))-(-a
)^(-m)/a*(-x^m*(-a)^m*(a*m*x+m+1)/(1+m)/m+x^m*(-a)^m*LerchPhi(a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(ax+1)x^m}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [B] time = 2.29895, size = 100, normalized size = 2.86

$$-\frac{amx^2x^m\Phi(ax,1,m+2)\Gamma(m+2)}{\Gamma(m+3)}-\frac{2ax^2x^m\Phi(ax,1,m+2)\Gamma(m+2)}{\Gamma(m+3)}-\frac{mxx^m\Phi(ax,1,m+1)\Gamma(m+1)}{\Gamma(m+2)}-\frac{xx^m\Phi(ax,1,}{\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m,x)
```

```
[Out] -a*m*x**2*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - 2*a*x**2
*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - m*x*x**m*lerchphi
(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - x*x**m*lerchphi(a*x, 1, m + 1)*
```

```
gamma(m + 1)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)x^m}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

3.135 $\int e^{\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=74

$$\frac{x^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}+\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) + (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rubi [A] time = 0.0615149, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6172, 808, 364}

$$\frac{x^m \, _2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{x^{m+1} \, _2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m,x]
```



```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) + (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```


Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) - \frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Mathematica [C] time = 0.38729, size = 128, normalized size = 1.73

$$x^{m+1}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{m}{2}-\frac{1}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m+1} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{x^2-\frac{1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};m+1;-ax,ax\right)}{m\sqrt{ax-1}\sqrt{\frac{ax+1}{a^2}}\sqrt{1-a^2x^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^m,x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[-a^(-2) + x^2]*AppellF1[m, -1/2, 1
/2, 1 + m, -(a*x), a*x])/(m*Sqrt[-1 + a*x]*Sqrt[(1 + a*x)/a^2]*Sqrt[1 - a^2
*x^2])) + Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 +
m))
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax+1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m,x)
```

```
[Out] Integral(x**m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.136 $\int e^{-\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=75

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) - (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rubi [A] time = 0.0601647, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6172, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^ArcCoth[a*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) - (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + \frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1-m); \frac{1-m}{2}; \frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1-\frac{m}{2}; \frac{1}{a^2x^2}\right)}{am}$$

Mathematica [C] time = 0.242454, size = 115, normalized size = 1.53

$$x^{m+1}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{m}{2}-\frac{1}{2}, \frac{1}{2}-\frac{m}{2}, \frac{1}{a^2x^2}\right)}{m+1} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{ax-1}{a^2}}F_1\left(m; -\frac{1}{2}, \frac{1}{2}; m+1; ax, -ax\right)}{m\sqrt{1-ax}\sqrt{x^2-\frac{1}{a^2}}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^ArcCoth[a*x],x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[m, -1/2,
1/2, 1 + m, a*x, -(a*x)])/(m*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])) + Hypergeo
metric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 + m))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \sqrt{\frac{ax-1}{ax+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.137 $\int e^{-2\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{m+1}$$

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]
)/(1 + m)
```

Rubi [A] time = 0.0412358, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6126, 80, 64}

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(2*ArcCoth[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]
)/(1 + m)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^m\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^m\,dx\\ &= -\int \frac{x^m(1-ax)}{1+ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-2\int \frac{x^m}{1+ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0103053, size = 27, normalized size = 0.75

$$\frac{x^{m+1}(1-2\text{Hypergeometric2F1}(1,m+1,m+2,-ax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

Maple [C] time = 0.412, size = 93, normalized size = 2.6

$$a^{-1-m}\left(\frac{x^m a^m\,(amx-m-1)}{(1+m)\,m}+x^m a^m \text{LerchPhi}\,(-ax,1,m)\right)-a^{-1-m}\left(\frac{x^m a^m}{m}+\frac{x^m a^m\,(-1-m)\,\text{LerchPhi}\,(-ax,1,m)}{1+m}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)*(a*x-1),x)
```

```
[Out] a^(-1-m)*(x^m*a^m*(a*m*x-m-1)/(1+m)/m+x^m*a^m*LerchPhi(-a*x,1,m))-a^(-1-m)*
(x^m*a^m/m+1/(1+m)*x^m*a^m*(-1-m)*LerchPhi(-a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax-1)x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [C] time = 2.37547, size = 119, normalized size = 3.31

$$\frac{amx^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}+\frac{2ax^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}-\frac{mxx^m\Phi\left(axe^{i\pi},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}-\frac{xx^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*x-1)/(a*x+1),x)
```

```
[Out] a*m*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(m
+ 3) + 2*a*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/g
amma(m + 3) - m*x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1)
```

```
/gamma(m + 2) - x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1)
/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

3.138 $\int e^{-3\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=150

$$\frac{x^m\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}-\frac{4x^m\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}-\frac{3x^{m+1}\text{Hypergeomet}}{}$$

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m) + (
4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1
+ m) - (4*x^m*Hypergeometric2F1[3/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rubi [A] time = 1.05072, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6172, 6742, 364, 850, 808}

$$\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{4x^m\,{}_2F_1\left(\frac{3}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-m\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m) + (
4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1
+ m) - (4*x^m*Hypergeometric2F1[3/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}+\frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\right)$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - -$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2}}{(1}\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x}{(1-}\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1-m);\frac{1-}{}\right.}{1+m}$$

Mathematica [C] time = 0.240829, size = 192, normalized size = 1.28

$$\frac{x^{m+1}\left(m\sqrt{1-ax}\sqrt{x^2-\frac{1}{a^2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{m}{2}-\frac{1}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)-3(m+1)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{ax-1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};m+1\right.\right.}{m(m+1)\sqrt{1-ax}\sqrt{x^2-\frac{1}{a^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*(-3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[
m, -1/2, 1/2, 1 + m, a*x, -(a*x)] + 2*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-
1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, a*x, -(a*x)] + m*Sqrt[1 - a*x]*
Sqrt[-a^(-2) + x^2]*Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x
^2)]))/(m*(1 + m)*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax-1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.139 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;\frac{5}{4},-\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0379282, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{5}{4},-\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m;\frac{5}{4},-\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.324445, size = 0, normalized size = 0.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((5*ArcCoth[a*x])/2)*x^m, x]
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}x^{2}+2\,ax+1\right)x^{m}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^{2}x^{2}-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a^2*x^2 - 2
*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.140 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0381142, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m; \frac{3}{4}, -\frac{3}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.328393, size = 0, normalized size = 0.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((3*ArcCoth[a*x])/2)*x^m, x]
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax+1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.141 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{4},-\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0374394, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{4},-\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=-\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$
$$=\frac{x^{1+m}F_1\left(-1-m;\frac{1}{4},-\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.263129, size = 0, normalized size = 0.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[a*x]/2)*x^m, x]
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int x^m\frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax+1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.036163, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(ArcCoth[a*x]/2),x]

[Out] (x^(1 + m)*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\mathrm{Subst}\left(\int \frac{x^{-2-m}\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m; -\frac{1}{4}, \frac{1}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.340857, size = 0, normalized size = 0.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(ArcCoth[a*x]/2),x]
```

```
[Out] Integrate[x^m/E^(ArcCoth[a*x]/2), x]
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int x^m\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.143 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0372282, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\mathrm{Subst}\left(\int\frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.306669, size = 0, normalized size = 0.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] Integrate[x^m/E^((3*ArcCoth[a*x])/2), x]
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")

[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")

[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/4),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")

[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)

3.144 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -5/4, 5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0378658, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -5/4, 5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.346765, size = 0, normalized size = 0.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] Integrate[x^m/E^((5*ArcCoth[a*x])/2), x]
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax-1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.145 $\int e^{\frac{2}{3}\coth^{-1}(x)} x^m \, dx$

Optimal. Leaf size=34

$$\frac{x^{m+1} F_1\left(-m-1; \frac{1}{3}, -\frac{1}{3}; -m; \frac{1}{x}, -\frac{1}{x}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rubi [A] time = 0.0329036, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6173, 133}

$$\frac{x^{m+1} F_1\left(-m-1; \frac{1}{3}, -\frac{1}{3}; -m; \frac{1}{x}, -\frac{1}{x}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m; \frac{1}{3}, -\frac{1}{3}; -m; \frac{1}{x}, -\frac{1}{x}\right)}{1+m}$$

Mathematica [F] time = 0.283343, size = 0, normalized size = 0.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^m,x]
```

```
[Out] Integrate[E^((2*ArcCoth[x])/3)*x^m, x]
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt[3]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x^m,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/3)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x+1)x^m\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="fricas")
```

```
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(2/3)/(x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**m,x)
```

```
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

3.146 $\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$

Optimal. Leaf size=34

$$\frac{x^{m+1} F_1\left(-m-1; \frac{1}{6}, -\frac{1}{6}; -m; \frac{1}{x}, -\frac{1}{x}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rubi [A] time = 0.0332118, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6173, 133}

$$\frac{x^{m+1} F_1\left(-m-1; \frac{1}{6}, -\frac{1}{6}; -m; \frac{1}{x}, -\frac{1}{x}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m; \frac{1}{6}, -\frac{1}{6}; -m; \frac{1}{x}, -\frac{1}{x}\right)}{1+m}$$

Mathematica [F] time = 0.274042, size = 0, normalized size = 0.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcCoth[x]/3)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[x]/3)*x^m, x]
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt[6]{\frac{-1+x}{1+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x^m,x)
```

```
[Out] int(1/((-1+x)/(1+x))^(1/6)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x+1)x^m\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="fricas")
```

```
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(5/6)/(x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

3.147 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0352318, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.275806, size = 0, normalized size = 0.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[a*x]/4)*x^m, x]
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax+1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(7/8)/(a*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

3.148 $\int e^{n\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$\frac{x^{m+1}F_1\left(-m-1;\frac{n}{2},-\frac{n}{2};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] (x^(1 + m)*AppellF1[-1 - m, n/2, -n/2, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rubi [A] time = 0.0390925, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{n}{2},-\frac{n}{2};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, n/2, -n/2, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{x^{1+m}F_1\left(-1-m; \frac{n}{2}, -\frac{n}{2}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

Mathematica [F] time = 0.314741, size = 0, normalized size = 0.

$$\int e^{n\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] Integrate[E^(n*ArcCoth[a*x])*x^m, x]
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^m,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m e^{n \operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**m,x)
```

```
[Out] Integral(x**m*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.149 $\int e^{n\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=174

$$\frac{2\left(n^2+2\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{nx^2\left(\right.}{}$$

```
[Out] (n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/(6*a) + ((1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 + (2*(2 + n^2)*(1 - 1/(a*x
))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (a - x^(-1))/(a + x^(-1))])/(3*a^3*(2 - n))
```

Rubi [A] time = 0.0923241, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6171, 129, 151, 12, 131}

$$\frac{2\left(n^2+2\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{nx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right.}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/(6*a) + ((1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 + (2*(2 + n^2)*(1 - 1/(a*x
))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (a - x^(-1))/(a + x^(-1))])/(3*a^3*(2 - n))
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\left(2+n^2\right)\left(1-\frac{x}{a}\right)^{-n/2}}{a^2x^2}\right.$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 - \frac{\left(2+n^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/}}{x^2}\right.}{6a^2}$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{2\left(2+n^2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(}}{3a^3(2}$$

Mathematica [A] time = 0.565801, size = 118, normalized size = 0.68

$$\frac{e^{n\coth^{-1}(ax)}\left((n+2)\left(\left(n^2+2\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\coth^{-1}(ax)}\right)+n\left(a^2x^2-1\right)+2a^3x^3+an^2x\right)+n\left(n^2+\right.\right.}{6a^3(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F1[1, 1
+ n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^2*x + 2*a^3*x^3 + n*(-1
+ a^2*x^2) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)])))/(6*a^3*(2 + n))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{n \operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.150 $\int e^{n\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=122

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[Out] ((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a*
x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

Rubi [A] time = 0.0565528, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6171, 96, 131}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] ((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a*
x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 - \frac{n\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}$$

Mathematica [A] time = 0.297065, size = 98, normalized size = 0.8

$$\frac{e^{n\coth^{-1}(ax)}\left((n+2)\left(n\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\coth^{-1}(ax)}\right)+a^2x^2+anx-1\right)+n^2e^{2\coth^{-1}(ax)}\text{Hypergeomet}}{2a^2(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + a^2*x^2 + n*Hypergeomet
ric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a^2*(2 + n))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n \operatorname{arccoth}(ax)} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x e^{n \operatorname{acoth}{\left(ax \right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x,x)
```

```
[Out] Integral(x*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.151 $\int e^{n\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=78

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rubi [A] time = 0.0230922, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6170, 131}

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2, 1-\frac{n}{2}; 2-\frac{n}{2}; \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Mathematica [A] time = 0.170018, size = 82, normalized size = 1.05

$$\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1, \frac{n}{2}+1, \frac{n}{2}+2, e^{2\coth^{-1}(ax)}\right)+(n+2)\left(\text{Hypergeometric2F1}\left(1, \frac{n}{2}, \frac{n}{2}+\right.\right.\right.}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int e^{n\,\text{arccoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x)),x)
```

```
[Out] int(exp(n*arccoth(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n \operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x)),x)
```

```
[Out] Integral(exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.152 $\int \frac{e^{n \coth^{-1}(ax)}}{x} dx$

Optimal. Leaf size=127

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2} \text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1\right.}{n}$$

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
 + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-n/2,
 -n/2, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.0614067, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 105, 69, 131}

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2} {}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
 + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-n/2,
 -n/2, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{n}$$

Mathematica [A] time = 0.14105, size = 142, normalized size = 1.12

$$\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\coth^{-1}(ax)}\right)+ne^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, -E^(2*ArcCoth[a*x])] + E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] - (2 + n)*(Hypergeometric2F1[1, n/2, 1 +
n/2, -E^(2*ArcCoth[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth
[a*x])])))/(n*(2 + n))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x, x)
```

3.153 $\int \frac{e^{n \coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=70

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[Out] (2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 -
 n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

Rubi [A] time = 0.0336342, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6171, 69}

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 -
 n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$
$$= \frac{2^{1+\frac{n}{2}}a\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Mathematica [A] time = 0.0357531, size = 44, normalized size = 0.63

$$-\frac{4ae^{(n+2)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(2,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\coth^{-1}(ax)}\right)}{n+2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (-4*a*E^((2 + n)*ArcCoth[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2
*ArcCoth[a*x])])/(2 + n)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{e^{n\operatorname{arccoth}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**2,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="giac")

[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^2, x)
```

3.154 $\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=114

$$\frac{a^2 2^{n/2} n \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[Out] (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n*
(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^(-
1))/(2*a)])/(2 - n)
```

Rubi [A] time = 0.0560491, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6171, 80, 69}

$$\frac{a^2 2^{n/2} n \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^3,x]
```


```
[Out] (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n*
(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^(-
1))/(2*a)])/(2 - n)
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx &= -\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} - \frac{1}{2}(an)\,\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.33554, size = 107, normalized size = 0.94

$$-\frac{a^2e^{n\coth^{-1}(ax)}\left((n+2)\left(n\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,-e^{2\coth^{-1}(ax)}\right)+\frac{1}{a^2x^2}+\frac{n}{ax}-1\right)-n^2e^{2\coth^{-1}(ax)}\text{Hypergeon}\right.}{2(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^3,x]
```

```
[Out] -(a^2*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 +
 n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-1 + 1/(a^2*x^2) + n/(a*x)
+ n*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2*(2 + n))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**3,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^3, x)
```

3.155 $\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=167

$$\frac{a^3 2^{n/2}\left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{6}a^3 n\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2\left(\frac{1}{ax}+1\right.}{}$$

```
[Out] (a^3*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/6 + (a^2*(1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) + (2^(n/2)*a^3*(2 + n^2)*(
1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^(-1
))/(2*a)])/(3*(2 - n))
```

Rubi [A] time = 0.110067, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 90, 80, 69}

$$\frac{a^3 2^{n/2}\left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{6}a^3 n\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^4,x]
```


```
[Out] (a^3*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/6 + (a^2*(1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) + (2^(n/2)*a^3*(2 + n^2)*(
1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^(-1
))/(2*a)])/(3*(2 - n))
```


Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = -\operatorname{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} + \frac{1}{3}a^2\operatorname{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{nx}{a}\right)dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} - \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\operatorname{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n}\right.$$

$$= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} + \frac{2^{n/2}a^3\left(2+n^2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\right.}{3(2-n)}$$

Mathematica [A] time = 0.589756, size = 132, normalized size = 0.79

$$-\frac{a^3e^{n\coth^{-1}(ax)}\left((n+2)\left(\left(n^2+2\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,-e^{2\coth^{-1}(ax)}\right)-\left(1-\frac{1}{a^2x^2}\right)\left(\frac{2}{ax}+n\right)+\frac{n^2+2}{ax}\right)-n\left(n^2\right.}{6(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] -(a^3*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F
1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-((1 - 1/(a^2*x^2))
*(n + 2/(a*x))) + (2 + n^2)/(a*x) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 +
 n/2, -E^(2*ArcCoth[a*x])])))/(6*(2 + n))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^4,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**4,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^4, x)
```

3.156 $\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=183

$$\frac{a^4 2^{\frac{n}{2}-2} n \left(n^2+8\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)} + \frac{1}{24}a^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a\left(n^2+6\right)+\frac{2n}{x}\right)\left(1-\right.$$

```
[Out] (a^3*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*n)
/x))/24 + (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) +
 (2^(-2 + n/2)*a^4*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1
- n/2, -n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))
```

Rubi [A] time = 0.123382, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 100, 147, 69}

$$\frac{a^4 2^{\frac{n}{2}-2} n \left(n^2+8\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)} + \frac{1}{24}a^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a\left(n^2+6\right)+\frac{2n}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} + \frac{a^2\left(\frac{1}{ax}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] (a^3*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*n)
/x))/24 + (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) +
 (2^(-2 + n/2)*a^4*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1
- n/2, -n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = -\operatorname{Subst}\left(\int x^3\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{1}{4}a^2\operatorname{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-2-\frac{nx}{a}\right)dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right) + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} - \frac{1}{24}\left(a^3n\left(8+n^2\right)\right)$$

$$= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right) + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{2^{-2+\frac{n}{2}}a^4n\left(8+n^2\right.}{}$$

Mathematica [A] time = 0.484103, size = 148, normalized size = 0.81

$$-\frac{1}{24}a^4e^{n\coth^{-1}(ax)}\left(-\frac{\left(n^2+8\right)n^2e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\coth^{-1}(ax)}\right)}{n+2}+\left(n^2+8\right)n\text{Hypergeon}\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(n*ArcCoth[a*x])/x^5,x]

[Out] -(a^4*E^(n*ArcCoth[a*x])*(-6 - n^2 + 6/(a^4*x^4) + (2*n)/(a^3*x^3) + n^2/(a^2*x^2) + (6*n)/(a*x) + n^3/(a*x) - (E^(2*ArcCoth[a*x])*n^2*(8 + n^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])])/(2 + n) + n*(8 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])]))/24

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*arccoth(a*x))/x^5,x)

[Out] int(exp(n*arccoth(a*x))/x^5,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="maxima")

[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**5,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^5, x)
```

3.157 $\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}(c-acx)^p\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}(c-acx)^p}{p+1}$$

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^p)/(1 + p) + (((a - x^(-
1))/(a + x^(-1)))^(1/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric2
F1[1/2 - p, -p, 1 - p, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.157184, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 132}

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}(c-acx)^p}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```


```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^p)/(1 + p) + (((a - x^(-
1))/(a + x^(-1)))^(1/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric2
F1[1/2 - p, -p, 1 - p, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$=-\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p}{1+p}-\frac{\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-1-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a(1+p)}$$

$$=\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p}{1+p}+\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0849435, size = 131, normalized size = 0.92

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{ax-1}{ax+1}\right)^{-p}(c-acx)^p\left(\sqrt{\frac{ax-1}{ax+1}}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{ax+1}\right)+p(ax-1)\left(\frac{ax-1}{ax+1}\right)^p\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*(p*(-1 + a*x)*((-1 + a*x)/(1 + a*x))^p + S
qrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p, 2/(1 + a*x)
]))/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^p)
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int (-acx+c)^p \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax+1)(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c\,(ax-1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.158 $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=132

$$\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{17}{15}a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{7}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] (-7*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (17*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2
)*x^3)/15 - (3*a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (a^4*c^4*(1 - 1/(a^
2*x^2))^(3/2)*x^5)/5 + (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rubi [A] time = 0.30285, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6175, 6178, 1807, 807, 266, 47, 63, 208}

$$\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{17}{15}a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{7}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (-7*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (17*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2
)*x^3)/15 - (3*a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (a^4*c^4*(1 - 1/(a^
2*x^2))^(3/2)*x^5)/5 + (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \left(a^4c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left(\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{5}\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(\frac{15}{a}-\frac{17x}{a^2}+\frac{5x^2}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5-\frac{1}{20}\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(\frac{68}{a^2}-\frac{35x}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\right.$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{4}\left(7ac^4\right)\text{Subst}$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{8}\left(7ac^4\right)\text{Subst}$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/}$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/}$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/}$$

Mathematica [A] time = 0.184801, size = 80, normalized size = 0.61

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+105\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A] time = 0.14, size = 183, normalized size = 1.4

$$\frac{(ax-1)\,c^4}{120\,a}\left(24\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-90\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+16\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}-105\,\sqrt{a^2}\sqrt{a^2x^2-1}xa+120\,((a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x)
```

```
[Out] 1/120*(a*x-1)*c^4/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-90*(a^2)^(1/2
)*(a^2*x^2-1)^(3/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-105*(a^2)^(1/2)*(a
^2*x^2-1)^(1/2)*x*a+120*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+105*ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x
-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] time = 0.998874, size = 350, normalized size = 2.65

$$\frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+490\,c^4\left(\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^4*((a*x - 1)/(a*x + 1))^(9/2) + 790*
c^4*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 490
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a
*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2
/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
```

```
- a^2))*a
```

Fricas [A] time = 1.64434, size = 296, normalized size = 2.24

$$\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^4x^5-66\,a^4c^4x^4+22\,a^3c^4x^3+97\,a^2c^4x^2-151\,ac^4x-136\,c^4\right)\sqrt{}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^4*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^4*x^5 - 66*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 9
7*a^2*c^4*x^2 - 151*a*c^4*x - 136*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\int -\frac{4ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{6a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int -\frac{4a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a*
*2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4/sqrt(a*x/(a*x + 1) -
1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [B] time = 1.22029, size = 316, normalized size = 2.39

$$\frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{490\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{896\,(ax-1)^2c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{790\,(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+\frac{105}{}\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(abs(sqr
t((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(490*(a*x - 1)*c^4*sqrt((a*x - 1)/(a*x
 + 1))/(a*x + 1) - 896*(a*x - 1)^2*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^
2 + 790*(a*x - 1)^3*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 105*(a*x -
1)^4*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 105*c^4*sqrt((a*x - 1)/(a*
x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^5))*a
```

3.159 $\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=105

$$-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{2}{3}a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{5}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] (-5*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (2*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)
*x^3)/3 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (5*c^3*ArcTanh[Sqrt[1 -
 1/(a^2*x^2)]])/(8*a)
```

Rubi [A] time = 0.225517, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6175, 6178, 1807, 807, 266, 47, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{2}{3}a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{5}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (-5*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (2*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)
*x^3)/3 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (5*c^3*ArcTanh[Sqrt[1 -
 1/(a^2*x^2)]])/(8*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(\frac{8}{a}-\frac{5x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{4}\left(5ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}\left(5ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{\left(5c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1}} \cdots\right.}{16a}$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}\left(5ac^3\right)\text{Subst}\left(\int \cdots\right.$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{5c^3\tanh^{-1}\left(\sqrt{1-\cdots}\right.}{8a}$$

Mathematica [A] time = 0.153045, size = 73, normalized size = 0.7

$$\frac{c^3\left(15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-16a^2x^2+9ax+16\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3)) +
15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Maple [A] time = 0.132, size = 141, normalized size = 1.3

$$-\frac{(ax-1)c^3}{24a}\left(6\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+15\sqrt{a^2}\sqrt{a^2x^2-1}xa-16\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x)
```

```
[Out] -1/24*(a*x-1)*c^3/a*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+15*(a^2)^(1/2)*(a^
2*x^2-1)^(1/2)*x*a-16*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-15*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)
*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] time = 0.991732, size = 298, normalized size = 2.84

$$\frac{1}{24}\left(\frac{15c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(15c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+73c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^2}{ax+1}-\frac{6(ax-1)^2a^2}{(ax+1)^2}+\frac{4(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 + 2*(15*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 73*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - 55*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^3*sq
rt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*
x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^
2))*a
```

Fricas [A] time = 1.5496, size = 263, normalized size = 2.5

$$\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^3x^4-10\,a^3c^3x^3-7\,a^2c^3x^2+25\,ac^3x+16\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (6*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 7*a^2*c^3*x^2 + 25*a*c
^3*x + 16*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int\frac{3ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3/sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(a*x +
1)), x))
```

Giac [B] time = 1.21364, size = 270, normalized size = 2.57

$$\frac{1}{24}\left(\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{55\,(ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{73\,(ax-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{15\,(ax-1)^3c^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}-15\,c^3\sqrt{\frac{ax-1}{ax+1}}\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/a^2 + 2*(55*(a*x - 1)*c^3*sqrt((a*x - 1)/(a*x + 1
))/(a*x + 1) - 73*(a*x - 1)^2*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 1
5*(a*x - 1)^3*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 - 15*c^3*sqrt((a*x
- 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^4))*a
```

3.160 $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=78

$$\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -(a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3
)/3 + (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rubi [A] time = 0.136212, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6175, 6178, 807, 266, 47, 63, 208}

$$\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -(a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3
)/3 + (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$

$$= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] time = 0.0961057, size = 64, normalized size = 0.82

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-3ax-2\right)+3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - 3*a*x + 2*a^2*x^2) + 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] time = 0.141, size = 121, normalized size = 1.6

$$-\frac{(ax-1)\,c^2}{6\,a}\left(3\,\sqrt{a^2}\sqrt{a^2x^2-1}xa-2\,\left((ax-1)\,(ax+1)\right)^{3/2}\sqrt{a^2}-3\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax-1)\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/6*(a*x-1)*c^2*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-2*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)
/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.117, size = 244, normalized size = 3.13

$$\frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 8*c^2*((a*
x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2
/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
a^2))
```

Fricas [A] time = 1.49306, size = 231, normalized size = 2.96

$$\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2a^3c^2x^3-a^2c^2x^2-5ac^2x-2c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (2*a^3*c^2*x^3 - a^2*c^2*x^2 - 5*a*c^2*x - 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x))
```

Giac [B] time = 1.22836, size = 224, normalized size = 2.87

$$\frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{8(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{3(ax-1)^2c^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(abs(sqrt((a
*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(8*(a*x - 1)*c^2*sqrt((a*x - 1)/(a*x + 1))
/(a*x + 1) + 3*(a*x - 1)^2*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 3*c^
2*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.161 $\int e^{\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=47

$$\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -(a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a)
```

Rubi [A] time = 0.0706478, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6175, 6178, 266, 47, 63, 208}

$$\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] -(a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\ &= (ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}(ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\ &= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}\\ &= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{2}(ac)\,\mathrm{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\ &= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0502585, size = 51, normalized size = 1.09

$$\frac{c\left(\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] (c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]
))/(2*a)
```

Maple [B] time = 0.135, size = 93, normalized size = 2.

$$-\frac{c\,(ax-1)}{2}\left(x\sqrt{a^2x^2-1}\sqrt{a^2}-\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax-1)\,(ax+1)}}\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x)
```

```
[Out] -1/2*(a*x-1)*c*(x*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)
*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)
/(a^2)^(1/2)
```

Maxima [B] time = 1.0445, size = 178, normalized size = 3.79

$$\frac{1}{2}\,a\left(\frac{2\left(c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(c*((a*x - 1)/(a*x + 1))^(3/2) + c*sqrt((a*x - 1)/(a*x + 1)))/(2*(
a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + c*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.57671, size = 180, normalized size = 3.83

$$\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2+acx\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - c*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (a^2*c*x^2 + a*c*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int -\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a
*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [B] time = 1.26605, size = 211, normalized size = 4.49

$$\frac{1}{4}a\left(\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}+2\right)}{a^2}-\frac{c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}-2\right|\right)}{a^2}-\frac{4c\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{\left(\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2-4\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/4*a*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)) + 2)/a
^2 - c*log(abs(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)) - 2)
)/a^2 - 4*c*(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)))/(((sq
rt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)))^2 - 4)*a^2))
```

3.162 $\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=51

$$\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] (2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rubi [A] time = 0.206206, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6175, 6178, 852, 1805, 266, 63, 208}

$$\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$
$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$
$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.0521113, size = 60, normalized size = 1.18

$$\frac{2ax\sqrt{1-\frac{1}{a^2x^2}} + (1-ax)\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac(ax-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*x + (1 - a*x)*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x])/(a*c*(-1 + a*x))
```

Maple [B] time = 0.132, size = 247, normalized size = 4.8

$$-\frac{1}{ac\,(ax-1)}\left(\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)x^2a^3+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^2a^2-2\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(a}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x)
```

```
[Out] -1/a*(ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-2*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)/
(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.03232, size = 105, normalized size = 2.06

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A] time = 1.61373, size = 205, normalized size = 4.02

$$-\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - 2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))), x)/c
```

Giac [A] time = 1.18387, size = 107, normalized size = 2.1

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(abs(sqrt((a*x - 1)/(a*
x + 1)) - 1))/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

3.163 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2))/(3*c^2*(a - x^(-1))^3)
```

Rubi [A] time = 0.0990949, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6175, 6178, 651}

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2))/(3*c^2*(a - x^(-1))^3)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2c^2}\\ &= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.0488138, size = 34, normalized size = 1.03

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+1)}{3c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x))/(3*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.045, size = 36, normalized size = 1.1

$$-\frac{ax+1}{(3\,ax-3)\,c^2a}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/3*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

Maxima [A] time = 1.0126, size = 31, normalized size = 0.94

$$-\frac{1}{3\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

Fricas [A] time = 1.50868, size = 122, normalized size = 3.7

$$-\frac{\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*c
^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Giac [A] time = 1.19968, size = 47, normalized size = 1.42

$$-\frac{ax+1}{3\,(ax-1)ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(a*x + 1)/((a*x - 1)*a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

3.164 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=67

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2))/(5*c^3*(a - x^(-1))^4) - (4*a^2*(1 - 1/(a^2*x
^2))^(3/2))/(15*c^3*(a - x^(-1))^3)
```

Rubi [A] time = 0.124528, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6175, 6178, 793, 651}

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2))/(5*c^3*(a - x^(-1))^4) - (4*a^2*(1 - 1/(a^2*x
^2))^(3/2))/(15*c^3*(a - x^(-1))^3)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
+ 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4\operatorname{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{5a^2c^3}\\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}
\end{aligned}
$$

Mathematica [A] time = 0.052161, size = 42, normalized size = 0.63

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a^2x^2-3ax-4\right)}{15c^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-4 - 3*a*x + a^2*x^2))/(15*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.052, size = 41, normalized size = 0.6

$$-\frac{(ax-4)(ax+1)}{15c^3(ax-1)^2a}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(1/2)/a
```

Maxima [A] time = 1.0957, size = 53, normalized size = 0.79

$$-\frac{\frac{5(ax-1)}{ax+1}-3}{30ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(a*x - 1)/(a*x + 1) - 3)/(a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

Fricas [A] time = 1.50954, size = 161, normalized size = 2.4

$$-\frac{\left(a^{3}x^{3}-2\,a^{2}x^{2}-7\,ax-4\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(a^3*x^3 - 2*a^2*x^2 - 7*a*x - 4)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3*
x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Giac [A] time = 1.17073, size = 72, normalized size = 1.07

$$-\frac{(ax+1)^{2}\left(\frac{5\,(ax-1)}{ax+1}-3\right)}{30\,(ax-1)^{2}ac^{3}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/30*(a*x + 1)^2*(5*(a*x - 1)/(a*x + 1) - 3)/((a*x - 1)^2*a*c^3*sqrt((a*x
 - 1)/(a*x + 1)))
```

3.165 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=100

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(3/2))/(7*c^4*(a - x^(-1))^5) + (12*a^3*(1 - 1/(a^2
*x^2))^(3/2))/(35*c^4*(a - x^(-1))^4) - (23*a^2*(1 - 1/(a^2*x^2))^(3/2))/(1
05*c^4*(a - x^(-1))^3)
```

Rubi [A] time = 0.229582, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(3/2))/(7*c^4*(a - x^(-1))^5) + (12*a^3*(1 - 1/(a^2
*x^2))^(3/2))/(35*c^4*(a - x^(-1))^4) - (23*a^2*(1 - 1/(a^2*x^2))^(3/2))/(1
05*c^4*(a - x^(-1))^3)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
```

```
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{3x}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{7a^2c^4}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{35a^2c^4}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] time = 0.0562056, size = 51, normalized size = 0.51

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-8a^2x^2+13ax+23\right)}{105c^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(23 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(105*c^4*(
-1 + a*x)^4)
```

Maple [A] time = 0.052, size = 50, normalized size = 0.5

$$-\frac{\left(2 a^{2} x^{2}-10 a x+23\right) \left(a x+1\right)}{105 c^{4} \left(a x-1\right)^{3} a}\frac{1}{\sqrt{\frac{a x-1}{a x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/
a
```

Maxima [A] time = 0.995723, size = 74, normalized size = 0.74

$$\frac{\frac{42 \left(a x-1\right)}{a x+1}-\frac{35 \left(a x-1\right)^{2}}{\left(a x+1\right)^{2}}-15}{420 a c^{4} \left(\frac{a x-1}{a x+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/420*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15)/(a*c^4*((a
*x - 1)/(a*x + 1))^(7/2))
```

Fricas [A] time = 1.61714, size = 205, normalized size = 2.05

$$-\frac{\left(2 a^{4} x^{4}-6 a^{3} x^{3}+5 a^{2} x^{2}+36 a x+23\right) \sqrt{\frac{a x-1}{a x+1}}}{105 \left(a^{5} c^{4} x^{4}-4 a^{4} c^{4} x^{3}+6 a^{3} c^{4} x^{2}-4 a^{2} c^{4} x+a c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + 36*a*x + 23)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))), x)/c**4
```

Giac [A] time = 1.14222, size = 93, normalized size = 0.93

$$\frac{(ax+1)^3\left(\frac{42\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}-15\right)}{420\,(ax-1)^3ac^4\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/420*(a*x + 1)^3*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15
)/((a*x - 1)^3*a*c^4*sqrt((a*x - 1)/(a*x + 1)))
```

3.166 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=133

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(3/2))/(9*c^5*(a - x^(-1))^6) - (8*a^4*(1 - 1/(a^2*x
^2))^(3/2))/(21*c^5*(a - x^(-1))^5) + (47*a^3*(1 - 1/(a^2*x^2))^(3/2))/(105
*c^5*(a - x^(-1))^4) - (58*a^2*(1 - 1/(a^2*x^2))^(3/2))/(315*c^5*(a - x^(-1
))^3)
```

Rubi [A] time = 0.34239, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```


```
[Out] (a^5*(1 - 1/(a^2*x^2))^(3/2))/(9*c^5*(a - x^(-1))^6) - (8*a^4*(1 - 1/(a^2*x
^2))^(3/2))/(21*c^5*(a - x^(-1))^5) + (47*a^3*(1 - 1/(a^2*x^2))^(3/2))/(105
*c^5*(a - x^(-1))^4) - (58*a^2*(1 - 1/(a^2*x^2))^(3/2))/(315*c^5*(a - x^(-1
))^3)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{7x}{a^3}+\frac{2x^2}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{a^4\,\text{Subst}\left(\int \frac{\left(\frac{18}{a^6}-\frac{20x}{a^7}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{29\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{3a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{21a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{105a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] time = 0.0616556, size = 59, normalized size = 0.44

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^4x^4-10a^3x^3+21a^2x^2-25ax-58\right)}{315c^5(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-58 - 25*a*x + 21*a^2*x^2 - 10*a^3*x^3 + 2*a^4*x
^4))/(315*c^5*(-1 + a*x)^5)
```

Maple [A] time = 0.046, size = 58, normalized size = 0.4

$$-\frac{\left(2\,x^3a^3-12\,a^2x^2+33\,ax-58\right)(ax+1)}{315\,c^5\,(ax-1)^4\,a}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a*x
+1))^(1/2)/a
```

Maxima [A] time = 1.02514, size = 96, normalized size = 0.72

$$-\frac{\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{105\,(ax-1)^3}{(ax+1)^3}-35}{2520\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/2520*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 105*(a*x -
 1)^3/(a*x + 1)^3 - 35)/(a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

Fricas [A] time = 1.59009, size = 247, normalized size = 1.86

$$-\frac{\left(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3-4\,a^2x^2-83\,ax-58\right)\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^6c^5x^5-5\,a^5c^5x^4+10\,a^4c^5x^3-10\,a^3c^5x^2+5\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 - 4*a^2*x^2 - 83*a*x - 58)*sqrt(
(a*x - 1)/(a*x + 1))/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a^3
*c^5*x^2 + 5*a^2*c^5*x - a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14683, size = 115, normalized size = 0.86

$$-\frac{(ax+1)^4\left(\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{105\,(ax-1)^3}{(ax+1)^3}-35\right)}{2520\,(ax-1)^4ac^5\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/2520*(a*x + 1)^4*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2
+ 105*(a*x - 1)^3/(a*x + 1)^3 - 35)/((a*x - 1)^4*a*c^5*sqrt((a*x - 1)/(a*x
+ 1)))
```

3.167 $\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=42

$$\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[Out] (2*(c - a*c*x)^p)/(a*p) - (c - a*c*x)^(1 + p)/(a*c*(1 + p))
```

Rubi [A] time = 0.0666535, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6130, 21, 43}

$$\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (2*(c - a*c*x)^p)/(a*p) - (c - a*c*x)^(1 + p)/(a*c*(1 + p))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx \\
&= -\int \frac{(1+ax)(c-acx)^p}{1-ax}\,dx \\
&= -\left(c\int (1+ax)(c-acx)^{-1+p}\,dx\right) \\
&= -\left(c\int \left(2(c-acx)^{-1+p}-\frac{(c-acx)^p}{c}\right)dx\right) \\
&= \frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}
$$

Mathematica [A] time = 0.022598, size = 28, normalized size = 0.67

$$\frac{(apx+p+2)(c-acx)^p}{ap(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p))
```

Maple [A] time = 0.043, size = 29, normalized size = 0.7

$$\frac{\left(apx+p+2\right)(-acx+c)^p}{ap\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^p,x)
```

[Out] (a*p*x+p+2)*(-a*c*x+c)^p/a/p/(1+p)

Maxima [A] time = 1.06866, size = 66, normalized size = 1.57

$$\frac{\left(ac^{p}px+c^{p}\right)(-ax+1)^{p}}{\left(p^{2}+p\right)a}+\frac{(-ax+1)^{p}c^{p}}{ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="maxima")

[Out] (a*c^p*p*x + c^p)*(-a*x + 1)^p/((p^2 + p)*a) + (-a*x + 1)^p*c^p/(a*p)

Fricas [A] time = 1.59576, size = 62, normalized size = 1.48

$$\frac{\left(apx+p+2\right)(-acx+c)^{p}}{ap^{2}+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="fricas")

[Out] (a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)

Sympy [A] time = 0.83473, size = 124, normalized size = 2.95

$$\begin{cases} -c^{p}x & \text{for}\ a=0 \\ -\frac{ax\log\left(x-\frac{1}{a}\right)}{a^{2}cx-ac}+\frac{\log\left(x-\frac{1}{a}\right)}{a^{2}cx-ac}+\frac{2}{a^{2}cx-ac} & \text{for}\ p=-1 \\ x+\frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for}\ p=0 \\ \frac{apx(-acx+c)^{p}}{ap^{2}+ap}+\frac{p(-acx+c)^{p}}{ap^{2}+ap}+\frac{2(-acx+c)^{p}}{ap^{2}+ap} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**p,x)

```
[Out] Piecewise((-c**p*x, Eq(a, 0)), (-a*x*log(x - 1/a)/(a**2*c*x - a*c) + log(x
- 1/a)/(a**2*c*x - a*c) + 2/(a**2*c*x - a*c), Eq(p, -1)), (x + 2*log(x - 1/
a)/a, Eq(p, 0)), (a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) + p*(-a*c*x + c)**p/
(a*p**2 + a*p) + 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)(-acx+c)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

3.168 $\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=37

$$\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[Out] (2*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a)
```

Rubi [A] time = 0.0597232, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (2*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx\\ &= -\left(c^5\int (1-ax)^4(1+ax)\,dx\right)\\ &= -\left(c^5\int \left(2(1-ax)^4-(1-ax)^5\right)\,dx\right)\\ &= \frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] time = 0.0191041, size = 23, normalized size = 0.62

$$-\frac{c^5(ax-1)^5(5ax+7)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -(c^5*(-1 + a*x)^5*(7 + 5*a*x))/(30*a)
```

Maple [A] time = 0.039, size = 47, normalized size = 1.3

$$c^5\left(-\frac{x^6a^5}{6}+\frac{3\,x^5a^4}{5}-\frac{x^4a^3}{2}-\frac{2\,x^3a^2}{3}+\frac{3\,ax^2}{2}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(-1/6*x^6*a^5+3/5*x^5*a^4-1/2*x^4*a^3-2/3*x^3*a^2+3/2*a*x^2-x)
```

Maxima [A] time = 1.0412, size = 81, normalized size = 2.19

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Fricas [A] time = 1.51856, size = 130, normalized size = 3.51

$$-\frac{1}{6}\,a^{5}c^{5}x^{6}+\frac{3}{5}\,a^{4}c^{5}x^{5}-\frac{1}{2}\,a^{3}c^{5}x^{4}-\frac{2}{3}\,a^{2}c^{5}x^{3}+\frac{3}{2}\,ac^{5}x^{2}-c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Sympy [B] time = 0.096712, size = 66, normalized size = 1.78

$$-\frac{a^{5}c^{5}x^{6}}{6}+\frac{3a^{4}c^{5}x^{5}}{5}-\frac{a^{3}c^{5}x^{4}}{2}-\frac{2a^{2}c^{5}x^{3}}{3}+\frac{3ac^{5}x^{2}}{2}-c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + 3*a**4*c**5*x**5/5 - a**3*c**5*x**4/2 - 2*a**2*c**5*x**
3/3 + 3*a*c**5*x**2/2 - c**5*x
```

Giac [A] time = 1.14329, size = 81, normalized size = 2.19

$$-\frac{1}{6}\,a^{5}c^{5}x^{6}+\frac{3}{5}\,a^{4}c^{5}x^{5}-\frac{1}{2}\,a^{3}c^{5}x^{4}-\frac{2}{3}\,a^{2}c^{5}x^{3}+\frac{3}{2}\,ac^{5}x^{2}-c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

3.169 $\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=37

$$\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

```
[Out] (c^4*(1 - a*x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a)
```

Rubi [A] time = 0.0555531, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(1 - a*x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx\\ &= -\left(c^4\int (1-ax)^3(1+ax)\,dx\right)\\ &= -\left(c^4\int \left(2(1-ax)^3-(1-ax)^4\right)\,dx\right)\\ &= \frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.0150451, size = 30, normalized size = 0.81

$$\frac{1}{10}c^4x\left(2a^4x^4-5a^3x^3+10ax-10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Maple [A] time = 0.047, size = 30, normalized size = 0.8

$$c^4\left(\frac{x^5a^4}{5}-\frac{x^4a^3}{2}+ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^4,x)
```

```
[Out] c^4*(1/5*x^5*a^4-1/2*x^4*a^3+a*x^2-x)
```

Maxima [A] time = 1.01339, size = 50, normalized size = 1.35

$$\frac{1}{5}\,a^4c^4x^5-\frac{1}{2}\,a^3c^4x^4+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Fricas [A] time = 1.50833, size = 74, normalized size = 2.

$$\frac{1}{5}a^4c^4x^5 - \frac{1}{2}a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Sympy [A] time = 0.091233, size = 36, normalized size = 0.97

$$\frac{a^4c^4x^5}{5} - \frac{a^3c^4x^4}{2} + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**4,x)
```

```
[Out] a**4*c**4*x**5/5 - a**3*c**4*x**4/2 + a*c**4*x**2 - c**4*x
```

Giac [A] time = 1.16564, size = 50, normalized size = 1.35

$$\frac{1}{5}a^4c^4x^5 - \frac{1}{2}a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

3.170 $\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=37

$$\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

```
[Out] (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x)^4)/(4*a)
```

Rubi [A] time = 0.0591795, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x)^4)/(4*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx \\
&= -\left(c^3\int (1-ax)^2(1+ax)\,dx\right) \\
&= -\left(c^3\int \left(2(1-ax)^2-(1-ax)^3\right)\,dx\right) \\
&= \frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.0133888, size = 30, normalized size = 0.81

$$-\frac{1}{12}c^3x\left(3a^3x^3-4a^2x^2-6ax+12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))/12
```

Maple [A] time = 0.046, size = 31, normalized size = 0.8

$$c^3\left(-\frac{x^4a^3}{4}+\frac{x^3a^2}{3}+\frac{ax^2}{2}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^3,x)
```

```
[Out] c^3*(-1/4*x^4*a^3+1/3*x^3*a^2+1/2*a*x^2-x)
```

Maxima [A] time = 1.00857, size = 51, normalized size = 1.38

$$-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Fricas [A] time = 1.52194, size = 81, normalized size = 2.19

$$-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Sympy [A] time = 0.084199, size = 37, normalized size = 1.

$$-\frac{a^3c^3x^4}{4}+\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**3,x)
```

```
[Out] -a**3*c**3*x**4/4 + a**2*c**3*x**3/3 + a*c**3*x**2/2 - c**3*x
```

Giac [A] time = 1.22017, size = 51, normalized size = 1.38

$$-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

3.171 $\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}a^2c^2x^3-c^2x$$

```
[Out] -(c^2*x) + (a^2*c^2*x^3)/3
```

Rubi [A] time = 0.0482971, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 41}

$$\frac{1}{3}a^2c^2x^3-c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x) + (a^2*c^2*x^3)/3
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx\\ &= -\left(c^2\int(1-ax)(1+ax)\,dx\right)\\ &= -\left(c^2\int\left(1-a^2x^2\right)\,dx\right)\\ &= -c^2x+\frac{1}{3}a^2c^2x^3\end{aligned}$$

Mathematica [A] time = 0.0077956, size = 17, normalized size = 0.85

$$-c^2\left(x-\frac{a^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(x - (a^2*x^3)/3))
```

Maple [A] time = 0.041, size = 17, normalized size = 0.9

$$c^2\left(\frac{x^3a^2}{3}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^2,x)
```

```
[Out] c^2*(1/3*x^3*a^2-x)
```

Maxima [A] time = 1.06875, size = 24, normalized size = 1.2

$$\frac{1}{3}\,a^2c^2x^3-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Fricas [A] time = 1.41356, size = 34, normalized size = 1.7

$$\frac{1}{3}a^2c^2x^3 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Sympy [A] time = 0.080905, size = 15, normalized size = 0.75

$$\frac{a^2c^2x^3}{3} - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 - c**2*x
```

Giac [A] time = 1.13679, size = 24, normalized size = 1.2

$$\frac{1}{3}a^2c^2x^3 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

3.172 $\int e^{2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}acx^2-cx$$

```
[Out] -(c*x) - (a*c*x^2)/2
```

Rubi [C] time = 0.0120579, antiderivative size = 26, normalized size of antiderivative = 1.86, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$\frac{c\left(1-a^2x^2\right)e^{2\coth^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\coth^{-1}(ax)}\left(1-a^2x^2\right)}{2a}$$

Mathematica [C] time = 0.0080872, size = 26, normalized size = 1.86

$$\frac{c\left(1-a^2x^2\right)e^{2\coth^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

Maple [A] time = 0.039, size = 13, normalized size = 0.9

$$c\left(-\frac{ax^2}{2}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c),x)
```

```
[Out] c*(-1/2*a*x^2-x)
```

Maxima [A] time = 1.03664, size = 16, normalized size = 1.14

$$-\frac{1}{2}\,acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

Fricas [A] time = 1.43824, size = 27, normalized size = 1.93

$$-\frac{1}{2}\,acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

Sympy [A] time = 0.071802, size = 12, normalized size = 0.86

$$-\frac{acx^2}{2} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - c*x
```

Giac [A] time = 1.15182, size = 16, normalized size = 1.14

$$-\frac{1}{2}\,acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

3.173 $\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=32

$$-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[Out] -2/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rubi [A] time = 0.0637809, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -2/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx\\ &= -\frac{\int \frac{1+ax}{(1-ax)^2}\,dx}{c}\\ &= -\frac{\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{c}\\ &= -\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0149316, size = 30, normalized size = 0.94

$$-\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

Maple [A] time = 0.056, size = 31, normalized size = 1.

$$2\,\frac{1}{ac\,(ax-1)}-\frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c),x)
```

```
[Out] 2/c/a/(a*x-1)-1/c/a*ln(a*x-1)
```

Maxima [A] time = 1.02256, size = 41, normalized size = 1.28

$$\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 2/(a^2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.45197, size = 63, normalized size = 1.97

$$-\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] time = 0.352277, size = 20, normalized size = 0.62

$$\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
```

```
[Out] 2/(a**2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Giac [A] time = 1.14623, size = 42, normalized size = 1.31

$$-\frac{\log(|ax-1|)}{ac}+\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

3.174 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{x}{c^2(1-ax)^2}$$

```
[Out] -(x/(c^2*(1 - a*x)^2))
```

Rubi [A] time = 0.0510269, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 34}

$$-\frac{x}{c^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(x/(c^2*(1 - a*x)^2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$$
$$= -\frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c^2}$$
$$= -\frac{x}{c^2(1-ax)^2}$$

Mathematica [A] time = 0.0081405, size = 25, normalized size = 1.79

$$-\frac{(ax+1)^2}{4ac^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(1 + a*x)^2/(4*a*c^2*(1 - a*x)^2)
```

Maple [B] time = 0.047, size = 30, normalized size = 2.1

$$\frac{1}{c^2}\left(-\frac{1}{a(ax-1)^2} - \frac{1}{a(ax-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-1/a/(a*x-1)^2-1/a/(a*x-1))
```

Maxima [A] time = 1.00542, size = 35, normalized size = 2.5

$$-\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Fricas [A] time = 1.35504, size = 49, normalized size = 3.5

$$-\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Sympy [A] time = 0.370919, size = 24, normalized size = 1.71

$$-\frac{x}{a^2c^2x^2 - 2ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**2,x)
```

```
[Out] -x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

Giac [B] time = 1.11755, size = 46, normalized size = 3.29

$$-\frac{1}{(acx - c)^2a} - \frac{1}{(acx - c)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c)
```

3.175 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}$$

```
[Out] -2/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rubi [A] time = 0.0623262, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -2/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx \\
&= -\frac{\int \frac{1+ax}{(1-ax)^4}\,dx}{c^3} \\
&= -\frac{\int \left(\frac{2}{(-1+ax)^4}+\frac{1}{(-1+ax)^3}\right)dx}{c^3} \\
&= -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}
$$

Mathematica [A] time = 0.013904, size = 23, normalized size = 0.62

$$
\frac{3ax+1}{6ac^3(ax-1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.047, size = 30, normalized size = 0.8

$$
\frac{1}{c^3}\left(\frac{2}{3\,a\,(ax-1)^3}+\frac{1}{2\,a\,(ax-1)^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(2/3/a/(a*x-1)^3+1/2/a/(a*x-1)^2)
```

Maxima [A] time = 0.988694, size = 63, normalized size = 1.7

$$
\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Fricas [A] time = 1.45622, size = 93, normalized size = 2.51

$$\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.52817, size = 46, normalized size = 1.24

$$\frac{3ax+1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**3,x)
```

```
[Out] (3*a*x + 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a*c*
*3)
```

Giac [A] time = 1.12439, size = 28, normalized size = 0.76

$$\frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

3.176 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}$$

```
[Out] -1/(2*a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.0607729, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/(2*a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx\\
&= -\frac{\int \frac{1+ax}{(1-ax)^5}\,dx}{c^4}\\
&= -\frac{\int \left(-\frac{2}{(-1+ax)^5}-\frac{1}{(-1+ax)^4}\right)dx}{c^4}\\
&= -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}
\end{aligned}
$$

Mathematica [A] time = 0.0162178, size = 23, normalized size = 0.62

$$-\frac{2ax+1}{6ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -(1 + 2*a*x)/(6*a*c^4*(-1 + a*x)^4)
```

Maple [A] time = 0.046, size = 30, normalized size = 0.8

$$\frac{1}{c^4}\left(-\frac{1}{3\,a\,(ax-1)^3}-\frac{1}{2\,a\,(ax-1)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/3/a/(a*x-1)^3-1/2/a/(a*x-1)^4)
```

Maxima [A] time = 1.02396, size = 77, normalized size = 2.08

$$-\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Fricas [A] time = 1.47311, size = 116, normalized size = 3.14

$$-\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Sympy [B] time = 0.548321, size = 60, normalized size = 1.62

$$-\frac{2ax+1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] -(2*a*x + 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 24
*a**2*c**4*x + 6*a*c**4)
```

Giac [A] time = 1.16371, size = 28, normalized size = 0.76

$$-\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

3.177 $\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=202

$$-\frac{3\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}(c-acx)^p\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{a^2p\left(1-p^2\right)x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^p}{(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{3\sqrt{\frac{1}{ax}+}}{ap(p+}$$

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p)/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]) + ((1 +
 1/(a*x))^(3/2)*x*(c - a*c*x)^p)/((1 + p)*Sqrt[1 - 1/(a*x)]) - (3*((a - x^(
-1))/(a + x^(-1)))^(3/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric
2F1[1 - p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a^2*p*(1 - p^2)*(1 - 1/(a*
x))^(3/2)*x)
```

Rubi [A] time = 0.206049, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6176, 6181, 94, 132}

$$-\frac{3\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}(c-acx)^p\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^p}{(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{3\sqrt{\frac{1}{ax}+1}(c-acx)^p}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```


```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p)/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]) + ((1 +
 1/(a*x))^(3/2)*x*(c - a*c*x)^p)/((1 + p)*Sqrt[1 - 1/(a*x)]) - (3*((a - x^(
-1))/(a + x^(-1)))^(3/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric
2F1[1 - p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a^2*p*(1 - p^2)*(1 - 1/(a*
x))^(3/2)*x)
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{3/2}dx,x,\frac{1}{x}\right)\right)$$
$$= \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-1-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\sqrt{1+\right.}{a(1+p)}$$
$$= \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int \frac{x^{-}}{-}\right.}{a^2p(1+p)}$$
$$= \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,{}_2F_1\left(1-p,\frac{3}{2}\right.}{a^2p\left(1-p^2\right)\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.136312, size = 155, normalized size = 0.77

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{ax-1}{ax+1}\right)^{-p}(c-acx)^p\left(3\sqrt{\frac{ax-1}{ax+1}}\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{ax+1}\right)+(p-1)(ax+1)(apx+p+3)\left(\frac{ax}{ax}\right.\right.}{a(p-1)p(p+1)\sqrt{1-\frac{1}{ax}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*((-1 + p)*((-1 + a*x)/(1 + a*x))^p*(1 + a*
x)*(3 + p + a*p*x) + 3*Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1 - p,
3/2 - p, 2 - p, 2/(1 + a*x)]))/(a*(-1 + p)*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1
 + a*x)/(1 + a*x))^p*(1 + a*x))
```

Maple [F] time = 0.409, size = 0, normalized size = 0.

$$\int (-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x^2 - 2*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.178 $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=105

$$\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}-\frac{1}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] (3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 + (a^4*c^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/5 - (3*c^4*ArcTanh[Sqrt[1 - 1/
(a^2*x^2)]])/(8*a)
```

Rubi [A] time = 0.165661, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6175, 6178, 807, 266, 47, 63, 208}

$$\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}-\frac{1}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```


```
[Out] (3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 + (a^4*c^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/5 - (3*c^4*ArcTanh[Sqrt[1 - 1/
(a^2*x^2)]])/(8*a)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = \left(a^4c^4\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left(\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5+\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5+\frac{1}{2}\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)$$

$$= -\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{1}{8}\left(3ac^4\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5+\frac{\left(3c^4\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1\cdots}}\right.}{16a}$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{1}{8}\left(3ac^4\right)\text{Subst}\left(\int\right.$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\cdots}\right.}{8a}$$

Mathematica [A] time = 0.18773, size = 80, normalized size = 0.76

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [B] time = 0.18, size = 192, normalized size = 1.8

$$\frac{(ax-1)^2 c^4}{120\,a\,(ax+1)}\left(24\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-30\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+16\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}+45\sqrt{a^2}\sqrt{a^2x^2-1}xa-40\,((\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x)
```

```
[Out] 1/120*(a*x-1)^2*c^4/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-30*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+45*(a^2)^(1/2)*(
a^2*x^2-1)^(1/2)*x*a-40*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1
)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] time = 1.10712, size = 350, normalized size = 3.33

$$-\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\,c^4\left(\frac{ax-1}{ax+1}\right)\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5}{(ax+1)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^4*
((a*x - 1)/(a*x + 1))^(7/2) - 128*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^4*
((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x - 1
)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
+ 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2)
)*a
```

Fricas [A] time = 1.61108, size = 285, normalized size = 2.71

$$-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(8\,a^5c^4x^5-2\,a^4c^4x^4-26\,a^3c^4x^3+9\,a^2c^4x^2+33\,ac^4x+8\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^4*x^5 - 2*a^4*c^4*x^4 - 26*a^3*c^4*x^3 + 9*a^2
*c^4*x^2 + 33*a*c^4*x + 8*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^4\left(\int -\frac{4ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{6a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{4a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{}{\frac{ax\sqrt{\frac{ax}{ax+}}}{ax}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2*x**2/(a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x) + Integral(-4*a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Inte
gral(a**4*x**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x))
```

Giac [B] time = 1.21687, size = 316, normalized size = 3.01

$$-\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{70\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{128\,(ax-1)^2c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{70\,(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+\frac{15\,(ax-1)}{(ax}\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(abs(sqrt(
(a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(70*(a*x - 1)*c^4*sqrt((a*x - 1)/(a*x +
1))/(a*x + 1) - 128*(a*x - 1)^2*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 -
 70*(a*x - 1)^3*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 15*(a*x - 1)^4*
c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 15*c^4*sqrt((a*x - 1)/(a*x + 1)
))/(a^2*((a*x - 1)/(a*x + 1) - 1)^5))*a
```

3.179 $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=78

$$-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] (3*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 - (3*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rubi [A] time = 0.141355, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 266, 47, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (3*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 - (3*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3x^3\,dx\right)$$
$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx,x,\frac{1}{x}\right)$$
$$= \frac{1}{2}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)$$
$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)$$
$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{16a}$$
$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A] time = 0.136916, size = 64, normalized size = 0.82

$$\frac{c^3\left(a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(5-2a^2x^2\right)-3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{8a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(5 - 2*a^2*x^2) - 3*Log[a*(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(8*a)
```

Maple [A] time = 0.165, size = 124, normalized size = 1.6

$$-\frac{(ax-1)^2c^3}{8ax+8}\left(2x\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}-3x\sqrt{a^2x^2-1}\sqrt{a^2}+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)\right)\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\frac{1}{\sqrt{(ax-1)(ax+1)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x)
```

```
[Out] -1/8*(a*x-1)^2*c^3*(2*x*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*x*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)+3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x
-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] time = 1.06614, size = 298, normalized size = 3.82

$$-\frac{1}{8}\left(\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1)/a^2 + 2*(3*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 11*c^3*((a*
x - 1)/(a*x + 1))^(5/2) - 11*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((
a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x +
1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*
a
```

Fricas [A] time = 1.52967, size = 246, normalized size = 3.15

$$-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4+2\,a^3c^3x^3-5\,a^2c^3x^2-5\,ac^3x\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) + (2*a^4*c^3*x^4 + 2*a^3*c^3*x^3 - 5*a^2*c^3*x^2 - 5*a*c^3*x
```

```
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int \frac{3ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{}{\frac{ax\sqrt{\frac{}{a}}}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2*x**2/(a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x) + Integral(a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [B] time = 1.23131, size = 275, normalized size = 3.53

$$-\frac{1}{16}\left(\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}+2\right)}{a^2}-\frac{3\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}-2\right|\right)}{a^2}-\frac{4\left(3\,c^3\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^3-20\,c^3\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right.\right.}{\left(\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2-4\right)^2a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)) +
2)/a^2 - 3*c^3*log(abs(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x +
1)) - 2))/a^2 - 4*(3*c^3*(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x
 + 1)))^3 - 20*c^3*(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1))
))/(((sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)))^2 - 4)^2*a^2
))*a
```

3.180 $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=78

$$\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] (a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)
/3 - (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rubi [A] time = 0.170004, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 850, 807, 266, 47, 63, 208}

$$\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```


```
[Out] (a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)
/3 - (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\left(a^2c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2x^2\,dx$$
$$=-\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)\right)$$
$$=-\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx,x,\frac{1}{x}\right)\right)$$
$$=\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\left(ac^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)$$
$$=\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a}$$
$$=\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2\ldots}}\right.$$
$$=\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] time = 0.104261, size = 64, normalized size = 0.82

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+3ax-2\right)-3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*a*x + 2*a^2*x^2) - 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] time = 0.171, size = 130, normalized size = 1.7

$$\frac{c^2 (ax-1)^2}{(6ax+6)a}\left(3\sqrt{a^2}\sqrt{a^2x^2-1}xa+2\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\frac{1}{\sqrt{(ax-1)(ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x)
```

```
[Out] 1/6*(a*x-1)^2*c^2*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a
)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.01472, size = 244, normalized size = 3.13

$$-\frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 8*c^2*((a
*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^
2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
 a^2))
```

Fricas [A] time = 1.62937, size = 232, normalized size = 2.97

$$-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2a^3c^2x^3+5a^2c^2x^2+ac^2x-2c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) - (2*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + a*c^2*x - 2*c^2)*sqrt((a*
x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int -\frac{2ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{a^{2}x^{2}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2*x**2/(a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [B] time = 1.21913, size = 224, normalized size = 2.87

$$-\frac{1}{6}\,a\left(\frac{3\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}}-\frac{3\,c^{2}\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}}+\frac{2\left(\frac{8\,(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{3\,(ax-1)^{2}c^{2}\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^{2}}+3\,c^{2}\sqrt{\frac{ax-1}{ax+1}}\right)}{a^{2}\left(\frac{ax-1}{ax+1}-1\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/a^2 + 2*(8*(a*x - 1)*c^2*sqrt((a*x - 1)/(a*x + 1)
)/(a*x + 1) - 3*(a*x - 1)^2*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 3*c
^2*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.181 $\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}-2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rubi [A] time = 0.18547, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6175, 6178, 852, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}-2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```



```
[Out] -2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$
$$= (ac)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(3ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] time = 0.0746661, size = 53, normalized size = 0.82

$$-\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}(ax+4)+3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2
)])*x]))/(2*a)
```

Maple [B] time = 0.174, size = 162, normalized size = 2.5

$$-\frac{(ax-1)^2 c}{(2ax+2)a}\left(\sqrt{a^2}\sqrt{a^2x^2-1}\,xa+4\sqrt{a^2}\sqrt{(ax-1)(ax+1)}-\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)a+4a\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(a}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x)
```

```
[Out] -1/2*(a*x-1)^2*c*((a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+4*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+4*a*
ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+
1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.02008, size = 182, normalized size = 2.8

$$-\frac{1}{2}a\left(\frac{2\left(3c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(3/2) - 5*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 3*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2)
```

Fricas [A] time = 1.55294, size = 197, normalized size = 3.03

$$-\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+5acx+4c\right)\sqrt{\frac{ax-1}{ax+1}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 + 5*a*c*x + 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x))
```

Giac [B] time = 1.23544, size = 167, normalized size = 2.57

$$-\frac{1}{2}a\left(\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{3\,(ax-1)c\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-5\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(abs(sqrt((a*x
- 1)/(a*x + 1)) - 1))/a^2 - 2*(3*(a*x - 1)*c*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1) - 5*c*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.182 $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=80

$$\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2cx\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] (8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) + 4/(3*a^2*c*Sqrt[1 - 1/
(a^2*x^2)]*x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c)
```

Rubi [A] time = 0.284056, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 852, 1805, 12, 266, 63, 208}

$$\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2cx\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```


```
[Out] (8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) + 4/(3*a^2*c*Sqrt[1 - 1/
(a^2*x^2)]*x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
+ 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^4}{x\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3-\frac{4x}{a}+\frac{3x^2}{a^2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{3}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.0819563, size = 63, normalized size = 0.79

$$\frac{\frac{4x\sqrt{1-\frac{1}{a^2x^2}}(2ax-1)}{(ax-1)^2}-\frac{3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] ((4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x))/(-1 + a*x)^2 - (3*Log[a*(1 + Sqrt
[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [B] time = 0.206, size = 345, normalized size = 4.3

$$-\frac{1}{3\,ac\,(ax-1)\,(ax+1)}\left(3\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^3a^4+3\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^3a^3-9\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x)
```

```
[Out] -1/3/a*(3*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a
^4+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-9*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(3/2)*x*a-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+9*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(
a^2)^(1/2)+9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-3*a*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1
))^(3/2)
```

Maxima [A] time = 1.0276, size = 128, normalized size = 1.6

$$-\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1)/(a^2*c) - 2*(3*(a*x - 1)/(a*x + 1) + 1)/(a^2*c*((a*x - 1)/
(a*x + 1))^(3/2)))
```

Fricas [A] time = 1.6133, size = 284, normalized size = 3.55

$$-\frac{3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-4\left(2\,a^{2}x^{2}+ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - 4*(2*a^2*x^2 + a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{\frac{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x)/c
```

Giac [A] time = 1.27822, size = 146, normalized size = 1.82

$$-\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}c}-\frac{2\,(ax+1)\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^{2}c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(abs(sqrt((a*x
- 1)/(a*x + 1)) - 1))/(a^2*c) - 2*(a*x + 1)*(3*(a*x - 1)/(a*x + 1) + 1)/((a
*x - 1)*a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

3.183 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2))/(5*c^2*(a - x^(-1))^5)
```

Rubi [A] time = 0.10519, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 651}

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2))/(5*c^2*(a - x^(-1))^5)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

Mathematica [A] time = 0.0542461, size = 36, normalized size = 1.09

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+1)^2}{5c^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2)/(5*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.121, size = 36, normalized size = 1.1

$$-\frac{ax+1}{(5\,ax-5)\,c^2a}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/5*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

Maxima [A] time = 1.01188, size = 31, normalized size = 0.94

$$-\frac{1}{5\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Fricas [B] time = 1.55231, size = 159, normalized size = 4.82

$$-\frac{\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*x
^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x
)/c**2
```

Giac [A] time = 1.24705, size = 50, normalized size = 1.52

$$-\frac{(ax+1)^2}{5\,(ax-1)^2ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/5*(a*x + 1)^2/((a*x - 1)^2*a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

3.184 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=67

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2))/(7*c^3*(a - x^(-1))^6) - (6*a^4*(1 - 1/(a^2*x
^2))^(5/2))/(35*c^3*(a - x^(-1))^5)
```

Rubi [A] time = 0.134002, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6175, 6178, 793, 651}

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2))/(7*c^3*(a - x^(-1))^6) - (6*a^4*(1 - 1/(a^2*x
^2))^(5/2))/(35*c^3*(a - x^(-1))^5)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{7a^2c^3}\\
&= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}
\end{aligned}
$$

Mathematica [A] time = 0.0612204, size = 41, normalized size = 0.61

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax-6)(ax+1)^2}{35c^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-6 + a*x)*(1 + a*x)^2)/(35*c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.125, size = 41, normalized size = 0.6

$$-\frac{(ax-6)(ax+1)}{35c^3(ax-1)^2a}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/35*(a*x-6)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(3/2)/a
```

Maxima [A] time = 1.03509, size = 53, normalized size = 0.79

$$-\frac{\frac{7(ax-1)}{ax+1}-5}{70\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/70*(7*(a*x - 1)/(a*x + 1) - 5)/(a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

Fricas [A] time = 1.63554, size = 201, normalized size = 3.

$$-\frac{\left(a^4x^4-3\,a^3x^3-15\,a^2x^2-17\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(a^4*x^4 - 3*a^3*x^3 - 15*a^2*x^2 - 17*a*x - 6)*sqrt((a*x - 1)/(a*x +
 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30981, size = 72, normalized size = 1.07

$$-\frac{(ax+1)^3\left(\frac{7\,(ax-1)}{ax+1}-5\right)}{70\,(ax-1)^3ac^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/70*(a*x + 1)^3*(7*(a*x - 1)/(a*x + 1) - 5)/((a*x - 1)^3*a*c^3*sqrt((a*x
- 1)/(a*x + 1)))
```

3.185 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=94

$$-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] (-47*(a + x^(-1))^5)/(315*a^6*c^4*(1 - 1/(a^2*x^2))^(5/2)) + (16*(a + x^(-1
))^6)/(63*a^7*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (a + x^(-1))^7/(9*a^8*c^4*(1 -
 1/(a^2*x^2))^(9/2))
```

Rubi [A] time = 0.275223, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 852, 1635, 789, 651}

$$-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```


```
[Out] (-47*(a + x^(-1))^5)/(315*a^6*c^4*(1 - 1/(a^2*x^2))^(5/2)) + (16*(a + x^(-1
))^6)/(63*a^7*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (a + x^(-1))^7/(9*a^8*c^4*(1 -
 1/(a^2*x^2))^(9/2))
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
```

```
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^7}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^7}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6(7a^2+9ax)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9a^4c^4}$$

$$= \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{47\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63a^2c^4}$$

$$= -\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Mathematica [A] time = 0.0615454, size = 50, normalized size = 0.53

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+1)^2\left(2a^2x^2-14ax+47\right)}{315c^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(47 - 14*a*x + 2*a^2*x^2))/(315*c^4*(
-1 + a*x)^5)
```

Maple [A] time = 0.123, size = 50, normalized size = 0.5

$$-\frac{\left(2 a^{2} x^{2}-14 a x+47\right) \left(a x+1\right)}{315 c^{4} \left(a x-1\right)^{3} a} \left(\frac{a x-1}{a x+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/
a
```

Maxima [A] time = 1.02957, size = 74, normalized size = 0.79

$$\frac{\frac{90 (ax-1)}{ax+1} - \frac{63 (ax-1)^2}{(ax+1)^2} - 35}{1260\, ac^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 35)/(a*c^4*((
a*x - 1)/(a*x + 1))^(9/2))
```

Fricas [A] time = 1.57475, size = 251, normalized size = 2.67

$$-\frac{\left(2 a^{5} x^{5}-8 a^{4} x^{4}+11 a^{3} x^{3}+101 a^{2} x^{2}+127 a x+47\right) \sqrt{\frac{ax-1}{ax+1}}}{315 \left(a^{6} c^{4} x^{5}-5 a^{5} c^{4} x^{4}+10 a^{4} c^{4} x^{3}-10 a^{3} c^{4} x^{2}+5 a^{2} c^{4} x-a c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 + 101*a^2*x^2 + 127*a*x + 47)*sq
rt((a*x - 1)/(a*x + 1))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*
```

```
a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38352, size = 93, normalized size = 0.99

$$\frac{(ax+1)^4\left(\frac{90\,(ax-1)}{ax+1}-\frac{63\,(ax-1)^2}{(ax+1)^2}-35\right)}{1260\,(ax-1)^4ac^4\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/1260*(a*x + 1)^4*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 3
5)/((a*x - 1)^4*a*c^4*sqrt((a*x - 1)/(a*x + 1)))
```

3.186 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=125

$$\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] (-152*(a + x^(-1))^5)/(1155*a^6*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (79*(a + x^(
-1))^6)/(231*a^7*c^5*(1 - 1/(a^2*x^2))^(7/2)) - (10*(a + x^(-1))^7)/(33*a^8
*c^5*(1 - 1/(a^2*x^2))^(9/2)) + (a + x^(-1))^8/(11*a^9*c^5*(1 - 1/(a^2*x^2)
)^(11/2))
```

Rubi [A] time = 0.389062, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 852, 1635, 789, 651}

$$\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```



```
[Out] (-152*(a + x^(-1))^5)/(1155*a^6*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (79*(a + x^(
-1))^6)/(231*a^7*c^5*(1 - 1/(a^2*x^2))^(7/2)) - (10*(a + x^(-1))^7)/(33*a^8
*c^5*(1 - 1/(a^2*x^2))^(9/2)) + (a + x^(-1))^8/(11*a^9*c^5*(1 - 1/(a^2*x^2)
)^(11/2))
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^8}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^8}{\left(1-\frac{x^2}{a^2}\right)^{13/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^7\left(8a^3+11a^2x+11ax^2\right)}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{11a^5c^5}$$

$$= -\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6\left(138a^3+99a^2x\right)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{99a^5c^5}$$

$$= \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{152\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{231a^2c^5}$$

$$= -\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}$$

Mathematica [A] time = 0.0680123, size = 58, normalized size = 0.46

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+1)^2\left(2a^3x^3-16a^2x^2+61ax-152\right)}{1155c^5(ax-1)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(-152 + 61*a*x - 16*a^2*x^2 + 2*a^3*x
^3))/(1155*c^5*(-1 + a*x)^6)
```

Maple [A] time = 0.125, size = 58, normalized size = 0.5

$$-\frac{\left(2\,x^{3}a^{3}-16\,a^{2}x^{2}+61\,ax-152\right)(ax+1)}{1155\,c^{5}\,(ax-1)^{4}\,a}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a
*x+1))^(3/2)/a
```

Maxima [A] time = 1.03058, size = 96, normalized size = 0.77

$$-\frac{\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{231\,(ax-1)^{3}}{(ax+1)^{3}}-105}{9240\,ac^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/9240*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2 + 231*(a*x -
 1)^3/(a*x + 1)^3 - 105)/(a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

Fricas [A] time = 1.54921, size = 296, normalized size = 2.37

$$-\frac{\left(2\,a^{6}x^{6}-10\,a^{5}x^{5}+19\,a^{4}x^{4}-15\,a^{3}x^{3}-289\,a^{2}x^{2}-395\,ax-152\right)\sqrt{\frac{ax-1}{ax+1}}}{1155\left(a^{7}c^{5}x^{6}-6\,a^{6}c^{5}x^{5}+15\,a^{5}c^{5}x^{4}-20\,a^{4}c^{5}x^{3}+15\,a^{3}c^{5}x^{2}-6\,a^{2}c^{5}x+ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/1155*(2*a^6*x^6 - 10*a^5*x^5 + 19*a^4*x^4 - 15*a^3*x^3 - 289*a^2*x^2 - 3
95*a*x - 152)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15*a
^5*c^5*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53506, size = 115, normalized size = 0.92

$$-\frac{(ax+1)^5\left(\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^2}{(ax+1)^2}+\frac{231\,(ax-1)^3}{(ax+1)^3}-105\right)}{9240\,(ax-1)^5ac^5\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/9240*(a*x + 1)^5*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2
+ 231*(a*x - 1)^3/(a*x + 1)^3 - 105)/((a*x - 1)^5*a*c^5*sqrt((a*x - 1)/(a*x
 + 1)))
```

3.187 $\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=66

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rubi [A] time = 0.0810893, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6130, 21, 43}

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx\\ &= \int \frac{(1+ax)^2(c-acx)^p}{(1-ax)^2}\,dx\\ &= c^2\int (1+ax)^2(c-acx)^{-2+p}\,dx\\ &= c^2\int \left(4(c-acx)^{-2+p}-\frac{4(c-acx)^{-1+p}}{c}+\frac{(c-acx)^p}{c^2}\right)dx\\ &= \frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0763295, size = 50, normalized size = 0.76

$$\frac{\left(\frac{ax}{p+1}+\frac{4}{(p-1)(ax-1)}+\frac{3p+4}{p(p+1)}\right)(c-acx)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 + a
*x))))/a
```

Maple [A] time = 0.046, size = 74, normalized size = 1.1

$$\frac{(-acx+c)^p\left(a^2p^2x^2-a^2x^2p+2\,ap^2x+2\,apx-4\,ax+p^2+3\,p+4\right)}{\left(p^2-1\right)ap\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x)
```

```
[Out] (-a*c*x+c)^p*(a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)/(p^2
-1)/a/p/(a*x-1)
```

Maxima [B] time = 1.07685, size = 207, normalized size = 3.14

$$\frac{\left(\left(p^2-p\right)a^2c^px^2+2\,ac^p\left(p-1\right)x+2\,c^p\right)(-ax+1)^pa^2}{\left(p^3-p\right)a^4x-\left(p^3-p\right)a^3}+\frac{2\left(ac^p\left(p-1\right)x+c^p\right)(-ax+1)^pa}{\left(p^2-p\right)a^3x-\left(p^2-p\right)a^2}+\frac{(-ax+1)^pc^p}{a^2\left(p-1\right)x-a\left(p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 - p)*a^2*c^p*x^2 + 2*a*c^p*(p - 1)*x + 2*c^p)*(-a*x + 1)^p*a^2/((p^3
- p)*a^4*x - (p^3 - p)*a^3) + 2*(a*c^p*(p - 1)*x + c^p)*(-a*x + 1)^p*a/((p^
2 - p)*a^3*x - (p^2 - p)*a^2) + (-a*x + 1)^p*c^p/(a^2*(p - 1)*x - a*(p - 1)
)
```

Fricas [A] time = 1.52886, size = 161, normalized size = 2.44

$$-\frac{\left(\left(a^2p^2-a^2p\right)x^2+p^2+2\left(ap^2+ap-2\,a\right)x+3\,p+4\right)(-acx+c)^p}{ap^3-ap-\left(a^2p^3-a^2p\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] -((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
+ c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

Sympy [A] time = 1.96831, size = 512, normalized size = 7.76

$$\begin{cases} c^p x \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} + \frac{2a^2x^2}{a^3cx^2-2a^2cx+ac} + \frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} - \frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a} + \frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a} - \frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a} - \frac{5}{a^2x-a} \\ -\frac{acx^2}{2} - 3cx - \frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} - \frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} - \frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{p^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \ldots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a**
2*c*x + a*c) + 2*a**2*x**2/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 2*a*x*log(x -
 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2
*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**2*x - a) + 4*a*x*log(x - 1/a)/(a**2
*x - a) - 4*log(x - 1/a)/(a**2*x - a) - 5/(a**2*x - a), Eq(p, 0)), (-a*c*x*
*2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(p, 1)), (a**2*p**2*x**2*(-a*c*x + c)*
*p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - a**2*p*x**2*(-a*c*x + c)**p/(a
**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p**2*x*(-a*c*x + c)**p/(a**2*p*
*3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p*x*(-a*c*x + c)**p/(a**2*p**3*x - a*
*2*p*x - a*p**3 + a*p) - 4*a*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*
p**3 + a*p) + p**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p)
+ 3*p*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 4*(-a*c*x +
 c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2(-acx+c)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*(-a*c*x + c)^p/(a*x - 1)^2, x)
```

3.188 $\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=53

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rubi [A] time = 0.0680539, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx \\
&= c^5\int (1-ax)^3(1+ax)^2\,dx \\
&= c^5\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)dx \\
&= -\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}
\end{aligned}
$$

Mathematica [A] time = 0.0208371, size = 31, normalized size = 0.58

$$
-\frac{c^5(ax-1)^4\left(5a^2x^2+14ax+11\right)}{30a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/(30*a)
```

Maple [A] time = 0.038, size = 45, normalized size = 0.9

$$
c^5\left(-\frac{x^6a^5}{6}+\frac{x^5a^4}{5}+\frac{x^4a^3}{2}-\frac{2\,x^3a^2}{3}-\frac{ax^2}{2}+x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(-1/6*x^6*a^5+1/5*x^5*a^4+1/2*x^4*a^3-2/3*x^3*a^2-1/2*a*x^2+x)
```

Maxima [A] time = 0.992122, size = 80, normalized size = 1.51

$$
-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Fricas [A] time = 1.47756, size = 130, normalized size = 2.45

$$-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Sympy [A] time = 0.19778, size = 63, normalized size = 1.19

$$-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3/
3 - a*c**5*x**2/2 + c**5*x
```

Giac [A] time = 1.15922, size = 57, normalized size = 1.08

$$-\frac{\left(5\,c^5+\frac{24\,c^5}{ax-1}+\frac{30\,c^5}{(ax-1)^2}\right)(ax-1)^6}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/30*(5*c^5 + 24*c^5/(a*x - 1) + 30*c^5/(a*x - 1)^2)*(a*x - 1)^6/a
```

3.189 $\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=32

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rubi [A] time = 0.0550847, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 41, 194}

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx\\ &= c^4\int (1-ax)^2(1+ax)^2\,dx\\ &= c^4\int \left(1-a^2x^2\right)^2\,dx\\ &= c^4\int \left(1-2a^2x^2+a^4x^4\right)\,dx\\ &= c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5\end{aligned}$$

Mathematica [A] time = 0.0161992, size = 26, normalized size = 0.81

$$c^4\left(\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

Maple [A] time = 0.039, size = 23, normalized size = 0.7

$$c^4\left(\frac{x^5a^4}{5}-\frac{2\,x^3a^2}{3}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x)
```

```
[Out] c^4*(1/5*x^5*a^4-2/3*x^3*a^2+x)
```

Maxima [A] time = 1.03821, size = 38, normalized size = 1.19

$$\frac{1}{5}a^4c^4x^5 - \frac{2}{3}a^2c^4x^3 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Fricas [A] time = 1.35828, size = 58, normalized size = 1.81

$$\frac{1}{5}a^4c^4x^5 - \frac{2}{3}a^2c^4x^3 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Sympy [A] time = 0.172421, size = 29, normalized size = 0.91

$$\frac{a^4c^4x^5}{5} - \frac{2a^2c^4x^3}{3} + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**4,x)
```

```
[Out] a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

Giac [A] time = 1.16814, size = 57, normalized size = 1.78

$$\frac{\left(3\,c^4 + \frac{15\,c^4}{ax-1} + \frac{20\,c^4}{(ax-1)^2}\right)(ax-1)^5}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^4 + 15*c^4/(a*x - 1) + 20*c^4/(a*x - 1)^2)*(a*x - 1)^5/a
```

3.190 $\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=35

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rubi [A] time = 0.0585238, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = \int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx$$
$$= c^3\int (1-ax)(1+ax)^2\,dx$$
$$= c^3\int \left(2(1+ax)^2-(1+ax)^3\right)dx$$
$$= \frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}$$

Mathematica [A] time = 0.0147218, size = 30, normalized size = 0.86

$$-\frac{1}{12}c^3x\left(3a^3x^3+4a^2x^2-6ax-12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))/12
```

Maple [A] time = 0.04, size = 29, normalized size = 0.8

$$c^3\left(-\frac{x^4a^3}{4}-\frac{x^3a^2}{3}+\frac{ax^2}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x)
```

```
[Out] c^3*(-1/4*x^4*a^3-1/3*x^3*a^2+1/2*a*x^2+x)
```

Maxima [A] time = 1.01588, size = 50, normalized size = 1.43

$$-\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Fricas [A] time = 1.42124, size = 81, normalized size = 2.31

$$-\frac{1}{4}\,a^3c^3x^4 - \frac{1}{3}\,a^2c^3x^3 + \frac{1}{2}\,ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Sympy [A] time = 0.177027, size = 37, normalized size = 1.06

$$-\frac{a^3c^3x^4}{4} - \frac{a^2c^3x^3}{3} + \frac{ac^3x^2}{2} + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**3,x)
```

```
[Out] -a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

Giac [A] time = 1.14638, size = 57, normalized size = 1.63

$$-\frac{\left(3\,c^3 + \frac{16\,c^3}{ax-1} + \frac{24\,c^3}{(ax-1)^2}\right)(ax-1)^4}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*c^3 + 16*c^3/(a*x - 1) + 24*c^3/(a*x - 1)^2)*(a*x - 1)^4/a
```

3.191 $\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^3}{3a}$$

```
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rubi [A] time = 0.0464065, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 32}

$$\frac{c^2(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx \\
&= c^2\int (1+ax)^2\,dx \\
&= \frac{c^2(1+ax)^3}{3a}
\end{aligned}
$$

Mathematica [A] time = 0.0125278, size = 21, normalized size = 1.24

$$
c^2\left(\frac{a^2x^3}{3}+ax^2+x\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] c^2*(x + a*x^2 + (a^2*x^3)/3)
```

Maple [A] time = 0.039, size = 16, normalized size = 0.9

$$
\frac{c^2\,(ax+1)^3}{3\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x)
```

```
[Out] 1/3*c^2*(a*x+1)^3/a
```

Maxima [A] time = 1.0172, size = 34, normalized size = 2.

$$
\frac{1}{3}\,a^2c^2x^3+ac^2x^2+c^2x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Fricas [A] time = 1.48649, size = 50, normalized size = 2.94

$$\frac{1}{3}\,a^{2}c^{2}x^{3}+ac^{2}x^{2}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Sympy [A] time = 0.156255, size = 24, normalized size = 1.41

$$\frac{a^{2}c^{2}x^{3}}{3}+ac^{2}x^{2}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

Giac [B] time = 1.17359, size = 54, normalized size = 3.18

$$\frac{\left(c^{2}+\frac{6\,c^{2}}{ax-1}+\frac{12\,c^{2}}{(ax-1)^{2}}\right)(ax-1)^{3}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(c^2 + 6*c^2/(a*x - 1) + 12*c^2/(a*x - 1)^2)*(a*x - 1)^3/a
```

3.192 $\int e^{4\coth^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}acx^2 - \frac{4c\log(1-ax)}{a} - 3cx$$

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0366372, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6167, 6129, 43}

$$-\frac{1}{2}acx^2 - \frac{4c\log(1-ax)}{a} - 3cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}(c-acx)\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)\,dx\\ &= c\int \frac{(1+ax)^2}{1-ax}\,dx\\ &= c\int \left(-3-ax+\frac{4}{1-ax}\right)dx\\ &= -3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0122167, size = 26, normalized size = 0.96

$$c\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

Maple [A] time = 0.043, size = 25, normalized size = 0.9

$$-\frac{acx^2}{2}-3\,cx-4\,\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x)
```

```
[Out] -1/2*a*c*x^2-3*c*x-4*c/a*ln(a*x-1)
```

Maxima [A] time = 1.02561, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,acx^2-3\,cx-\frac{4\,c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Fricas [A] time = 1.50781, size = 66, normalized size = 2.44

$$-\frac{a^2cx^2 + 6\,acx + 8\,c\log\left(ax - 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

Sympy [A] time = 0.470965, size = 26, normalized size = 0.96

$$-\frac{acx^2}{2} - 3cx - \frac{4c\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Giac [A] time = 1.17871, size = 68, normalized size = 2.52

$$-\frac{(ax-1)^2\left(c + \frac{8\,c}{ax-1}\right)}{2\,a} + \frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*x - 1)^2*(c + 8*c/(a*x - 1))/a + 4*c*log(abs(a*x - 1)/((a*x - 1)^2*
abs(a)))/a
```

3.193 $\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=48

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rubi [A] time = 0.0677062, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{c-acx}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^3}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{1-ax} - \frac{4}{(-1+ax)^3} - \frac{4}{(-1+ax)^2}\right)dx}{c}$$
$$= \frac{2}{ac(1-ax)^2} - \frac{4}{ac(1-ax)} - \frac{\log(1-ax)}{ac}$$

Mathematica [A] time = 0.0206116, size = 36, normalized size = 0.75

$$\frac{4ax + (ax-1)^2(-\log(1-ax)) - 2}{ac(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

Maple [A] time = 0.047, size = 46, normalized size = 1.

$$2\,\frac{1}{ac\,(ax-1)^2} + 4\,\frac{1}{ac\,(ax-1)} - \frac{\ln(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x)
```

```
[Out] 2/c/a/(a*x-1)^2+4/c/a/(a*x-1)-1/c/a*ln(a*x-1)
```

Maxima [A] time = 1.05803, size = 59, normalized size = 1.23

$$\frac{2\,(2\,ax-1)}{a^3cx^2 - 2\,a^2cx + ac} - \frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.46157, size = 108, normalized size = 2.25

$$\frac{4\,ax - \left(a^2x^2 - 2\,ax + 1\right)\log\left(ax - 1\right) - 2}{a^3cx^2 - 2\,a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x + a
*c)
```

Sympy [A] time = 0.729809, size = 36, normalized size = 0.75

$$\frac{4ax - 2}{a^3cx^2 - 2a^2cx + ac} - \frac{\log\left(ax - 1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c),x)
```

```
[Out] (4*a*x - 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Giac [A] time = 1.15814, size = 77, normalized size = 1.6

$$\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac} + \frac{2\left(\frac{2\,ac}{ax-1} + \frac{ac}{(ax-1)^2}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) + 2*(2*a*c/(a*x - 1) + a*c/(a*
x - 1)^2)/(a^2*c^2)
```

3.194 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rubi [A] time = 0.0504695, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 37}

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

Mathematica [A] time = 0.0089624, size = 25, normalized size = 1.

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Maple [A] time = 0.048, size = 42, normalized size = 1.7

$$\frac{1}{c^2}\left(-\frac{4}{3\,a\,(ax-1)^3}-2\,\frac{1}{a\,(ax-1)^2}-\frac{1}{a\,(ax-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-4/3/a/(a*x-1)^3-2/a/(a*x-1)^2-1/a/(a*x-1))
```

Maxima [B] time = 1.02094, size = 69, normalized size = 2.76

$$-\frac{3\,a^2x^2+1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Fricas [B] time = 1.43907, size = 100, normalized size = 4.

$$-\frac{3\,a^2x^2+1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] time = 1.1261, size = 51, normalized size = 2.04

$$-\frac{3a^2x^2+1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**2,x)
```

```
[Out] -(3*a**2*x**2 + 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3
*a*c**2)
```

Giac [B] time = 1.15261, size = 68, normalized size = 2.72

$$-\frac{2}{(acx-c)^2a}-\frac{1}{(acx-c)ac}-\frac{4\,c}{3\,(acx-c)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

3.195 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=52

$$\frac{1}{2ac^3(1-ax)^2} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{ac^3(1-ax)^4}$$

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rubi [A] time = 0.0680686, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{2ac^3(1-ax)^2} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3}\,dx\\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^5}\,dx}{c^3}\\
&= \frac{\int \left(-\frac{4}{(-1+ax)^5}-\frac{4}{(-1+ax)^4}-\frac{1}{(-1+ax)^3}\right)dx}{c^3}\\
&= \frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}
$$

Mathematica [A] time = 0.0183849, size = 31, normalized size = 0.6

$$
\frac{3a^2x^2+2ax+1}{6ac^3(ax-1)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.048, size = 41, normalized size = 0.8

$$
\frac{1}{c^3}\left(\frac{4}{3\,a\,(ax-1)^3}+\frac{1}{a\,(ax-1)^4}+\frac{1}{2\,a\,(ax-1)^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(4/3/a/(a*x-1)^3+1/a/(a*x-1)^4+1/2/a/(a*x-1)^2)
```

Maxima [A] time = 1.02431, size = 88, normalized size = 1.69

$$
\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Fricas [A] time = 1.52557, size = 131, normalized size = 2.52

$$\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Sympy [A] time = 0.830525, size = 66, normalized size = 1.27

$$\frac{3a^2x^2+2ax+1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**3,x)
```

```
[Out] (3*a**2*x**2 + 2*a*x + 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3*c
**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

Giac [A] time = 1.15269, size = 57, normalized size = 1.1

$$\frac{\frac{3}{(ax-1)^2a}+\frac{8}{(ax-1)^3a}+\frac{6}{(ax-1)^4a}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="giac")

[Out] 1/6*(3/((a*x - 1)^2*a) + 8/((a*x - 1)^3*a) + 6/((a*x - 1)^4*a))/c^3
```

3.196 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rubi [A] time = 0.0669647, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx\\ &= \frac{\int \frac{(1+ax)^2}{(1-ax)^6}\,dx}{c^4}\\ &= \frac{\int \left(\frac{4}{(-1+ax)^6}+\frac{4}{(-1+ax)^5}+\frac{1}{(-1+ax)^4}\right)dx}{c^4}\\ &= \frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0187029, size = 31, normalized size = 0.58

$$-\frac{5a^2x^2+5ax+2}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -(2 + 5*a*x + 5*a^2*x^2)/(15*a*c^4*(-1 + a*x)^5)
```

Maple [A] time = 0.045, size = 42, normalized size = 0.8

$$\frac{1}{c^4}\left(-\frac{1}{3\,a\,(ax-1)^3}-\frac{1}{a\,(ax-1)^4}-\frac{4}{5\,a\,(ax-1)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/3/a/(a*x-1)^3-1/a/(a*x-1)^4-4/5/a/(a*x-1)^5)
```

Maxima [A] time = 1.02298, size = 104, normalized size = 1.96

$$-\frac{5\,a^2x^2+5\,ax+2}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Fricas [A] time = 1.46517, size = 158, normalized size = 2.98

$$-\frac{5\,a^{2}x^{2}+5\,ax+2}{15\left(a^{6}c^{4}x^{5}-5\,a^{5}c^{4}x^{4}+10\,a^{4}c^{4}x^{3}-10\,a^{3}c^{4}x^{2}+5\,a^{2}c^{4}x-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] time = 1.59646, size = 80, normalized size = 1.51

$$-\frac{5a^{2}x^{2}+5ax+2}{15a^{6}c^{4}x^{5}-75a^{5}c^{4}x^{4}+150a^{4}c^{4}x^{3}-150a^{3}c^{4}x^{2}+75a^{2}c^{4}x-15ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**4,x)
```

```
[Out] -(5*a**2*x**2 + 5*a*x + 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**
4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

Giac [A] time = 1.15388, size = 57, normalized size = 1.08

$$-\frac{\frac{5}{(ax-1)^{3}a}+\frac{15}{(ax-1)^{4}a}+\frac{12}{(ax-1)^{5}a}}{15\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/15*(5/((a*x - 1)^3*a) + 15/((a*x - 1)^4*a) + 12/((a*x - 1)^5*a))/c^4
```

3.197 $\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=94

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\text{Hypergeometric2F1}\left(-p-1,-p-\frac{1}{2},-p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*x)
])/(1 + p)
```

Rubi [A] time = 0.121731, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 132}

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,{}_2F_1\left(-p-1,-p-\frac{1}{2};-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*x)
])/(1 + p)
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx\\
&= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\operatorname{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p\,{}_2F_1\left(-1-p,-\frac{1}{2}-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}
\end{aligned}$$

Mathematica [A] time = 0.0446656, size = 76, normalized size = 0.81

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{-p-\frac{1}{2}}(c-acx)^p\text{Hypergeometric2F1}\left(-p-1,-p-\frac{1}{2},-p,\frac{2}{ax+1}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(c - a*c*x)^p*Hy
pergeometric2F1[-1 - p, -1/2 - p, -p, 2/(1 + a*x)])/(1 + p)
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int (-acx + c)^p \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^p \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((-acx + c)^p \sqrt{\frac{ax - 1}{ax + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^p \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.198 $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=127

$$-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{4}{3}a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{27}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{20}{3}c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{35c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] (20*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (27*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8
 + (4*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]
*x^4)/4 - (35*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rubi [A] time = 0.351761, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6175, 6178, 1807, 807, 266, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{4}{3}a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{27}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{20}{3}c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{35c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```


```
[Out] (20*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (27*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8
 + (4*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]
*x^4)/4 - (35*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$
$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{16}{a}-\frac{27x}{a^2}+\frac{16x^2}{a^3}-\frac{4x^3}{a^4}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{81}{a^2}-\frac{80x}{a^3}+\frac{12x^2}{a^4}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{24}\left(a^3c^3\right)\text{Subst}\left(\int$$
$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{(3}{}$$
$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{(3}{}$$
$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{8}($$
$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{35}{}$$

Mathematica [A] time = 0.188479, size = 72, normalized size = 0.57

$$\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-6a^3x^3+32a^2x^2-81ax+160\right)-105\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(160 - 81*a*x + 32*a^2*x^2 - 6*a^3*x^3) - 1
05*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Maple [A] time = 0.132, size = 196, normalized size = 1.5

$$-\frac{(ax+1)\,c^3}{24\,a}\sqrt{\frac{ax-1}{ax+1}}\left(6\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+87\sqrt{a^2}\sqrt{a^2x^2-1}xa-32\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-87\ln\left(\frac{a^2x+\sqrt{a^2x^2}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3/a*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2
)*x*a+87*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-32*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)-87*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-192*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2)+192*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] time = 1.05021, size = 298, normalized size = 2.35

$$-\frac{1}{24}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(279\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-511\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+385\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^3\sqrt{\cdots}\right.}{\frac{4(ax-1)a^2}{ax+1}-\frac{6(ax-1)^2a^2}{(ax+1)^2}+\frac{4(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(279*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 511*
c^3*((a*x - 1)/(a*x + 1))^(5/2) + 385*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105
*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*
a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)
^4 - a^2))*a
```

Fricas [A] time = 1.59482, size = 270, normalized size = 2.13

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^3x^4-26\,a^3c^3x^3+49\,a^2c^3x^2-79\,ac^3x-160\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (6*a^4*c^3*x^4 - 26*a^3*c^3*x^3 + 49*a^2*c^3*x^2 - 79
*a*c^3*x - 160*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int 3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -\sqrt{\frac{ax}{ax+1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
), x))
```

Giac [A] time = 1.14968, size = 147, normalized size = 1.16

$$\frac{35\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}+\frac{1}{24}\sqrt{a^2x^2-1}\left(\frac{160\,c^3\operatorname{sgn}(ax+1)}{a}-\left(81\,c^3\operatorname{sgn}(ax+1)+2\left(3\,a^2c^3x\operatorname{sgn}(a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 35/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/24
*sqrt(a^2*x^2 - 1)*(160*c^3*sgn(a*x + 1)/a - (81*c^3*sgn(a*x + 1) + 2*(3*a^
2*c^3*x*sgn(a*x + 1) - 16*a*c^3*sgn(a*x + 1))*x)*x)
```

3.199 $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=100

$$\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{3}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{11}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] (11*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (3*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2
+ (a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (5*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(2*a)
```

Rubi [A] time = 0.286164, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6175, 6178, 1807, 807, 266, 63, 208}

$$\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{3}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{11}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (11*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (3*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2
+ (a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (5*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(2*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{1}{3}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\frac{9}{a}-\frac{11x}{a^2}+\frac{3x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\frac{22}{a^2}-\frac{15x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\left(5c^2\right)\text{Subst}\Bigg(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x}{2a}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 + \frac{\left(5c^2\right)\text{Subst}\Bigg(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x}{4a}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{2}\left(5ac^2\right)\text{Subst}\Bigg(\int \frac{1}{a^2-a^2x}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] time = 0.113812, size = 64, normalized size = 0.64

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-9ax+22\right)-15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 - 9*a*x + 2*a^2*x^2) - 15*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [B] time = 0.132, size = 176, normalized size = 1.8

$$\frac{(ax+1)\,c^{2}}{6\,a}\sqrt{\frac{ax-1}{ax+1}}\left(2\,\left((ax-1)\,(ax+1)\right)^{3/2}\sqrt{a^{2}}-9\sqrt{a^{2}}\sqrt{a^{2}x^{2}-1}xa+24\sqrt{a^{2}}\sqrt{(ax-1)\,(ax+1)}+9\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}}}{\sqrt{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1
/2)-9*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+24*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)+9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-24*a*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2
)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.02074, size = 244, normalized size = 2.44

$$-\frac{1}{6}\,a\left(\frac{15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}}-\frac{15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}}+\frac{2\left(33\,c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^{2}\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^{2}}{ax+1}-\frac{3\,(ax-1)^{2}a^{2}}{(ax+1)^{2}}+\frac{(ax-1)^{3}a^{2}}{(ax+1)^{3}}-a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 + 2*(33*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 40*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x -
1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1
)^3 - a^2))
```

Fricas [A] time = 1.6813, size = 240, normalized size = 2.4

$$-\frac{15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^{3}c^{2}x^{3}-7\,a^{2}c^{2}x^{2}+13\,ac^{2}x+22\,c^{2}\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 7*a^2*c^2*x^2 + 13*a*c^2*x + 22*c^2)*sq
rt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int \sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) -
 1/(a*x + 1)), x))
```

Giac [A] time = 1.16438, size = 122, normalized size = 1.22

$$\frac{5\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}+\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,ac^2x\operatorname{sgn}(ax+1)-9\,c^2\operatorname{sgn}(ax+1)\right)x+\frac{22\,c^2\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/6*s
qrt(a^2*x^2 - 1)*((2*a*c^2*x*sgn(a*x + 1) - 9*c^2*sgn(a*x + 1))*x + 22*c^2*
sgn(a*x + 1)/a)
```

3.200 $\int e^{-\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] 2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rubi [A] time = 0.167051, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6175, 6178, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] 2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$
$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{1}{2}(3ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= 2c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2 - \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] time = 0.0737628, size = 53, normalized size = 0.82

$$-\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}(ax-4)+3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] -(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^2*x^
2)])*x]))/(2*a)
```

Maple [B] time = 0.124, size = 153, normalized size = 2.4

$$-\frac{c\,(ax+1)}{2\,a}\sqrt{\frac{ax-1}{ax+1}}\left(\sqrt{a^2}\sqrt{a^2x^2-1}xa-\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)a-4\sqrt{a^2}\sqrt{(ax-1)(ax+1)}+4\,a\ln\left(\frac{a^2x+\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*((a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-l
n((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-4*(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2)+4*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^
(1/2)))/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.02672, size = 182, normalized size = 2.8

$$\frac{1}{2}\,a\left(\frac{2\left(5\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(5*c*((a*x - 1)/(a*x + 1))^(3/2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 3*c*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2)
```

Fricas [A] time = 1.58394, size = 197, normalized size = 3.03

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-3\,acx-4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 - 3*a*c*x - 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A] time = 1.17012, size = 92, normalized size = 1.42

$$\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}-\frac{1}{2}\,\sqrt{a^2x^2-1}\left(cx\operatorname{sgn}(ax+1)-\frac{4\,c\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/2*sqr
t(a^2*x^2 - 1)*(c*x*sgn(a*x + 1) - 4*c*sgn(a*x + 1)/a)
```

3.201 $\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))
```

Rubi [A] time = 0.100635, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx &= -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.0383503, size = 34, normalized size = 1.48

$$
-\frac{\log\left(ax\left(\sqrt{\frac{a^2x^2-1}{a^2x^2}}+1\right)\right)}{ac}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

```
[Out] -(Log[a*x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]/(a*c))
```

Maple [B] time = 0.181, size = 76, normalized size = 3.3

$$-\frac{ax+1}{c}\sqrt{\frac{ax-1}{ax+1}}\ln\left(\left(a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\right)\frac{1}{\sqrt{a^2}}\right)\frac{1}{\sqrt{a^2}}\frac{1}{\sqrt{(ax-1)(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2))/(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c/(a^2)^(1/2)
```

Maxima [B] time = 1.01962, size = 74, normalized size = 3.22

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c))
```

Fricas [B] time = 1.54787, size = 111, normalized size = 4.83

$$-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -(log(sqrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Giac [A] time = 1.17058, size = 45, normalized size = 1.96

$$\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a))
```

3.202 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

Rubi [A] time = 0.101127, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 651}

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

Mathematica [A] time = 0.050955, size = 27, normalized size = 0.96

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(-1 + a*x)))
```

Maple [A] time = 0.043, size = 36, normalized size = 1.3

$$-\frac{ax+1}{(ax-1)\,ac^2}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/(a*x-1)/a/c^2
```

Maxima [A] time = 1.03071, size = 31, normalized size = 1.11

$$-\frac{1}{ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/(a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

Fricas [A] time = 1.58156, size = 78, normalized size = 2.79

$$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.203 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=62

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)])/(3*c^3*(a - x^(-1))^2) - (2*Sqrt[1 - 1/(a^2*x^2)]
)/(3*c^3*(a - x^(-1)))
```

Rubi [A] time = 0.125238, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6175, 6178, 793, 651}

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```


```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)])/(3*c^3*(a - x^(-1))^2) - (2*Sqrt[1 - 1/(a^2*x^2)]
)/(3*c^3*(a - x^(-1)))
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3a^2c^3}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}
\end{aligned}$$

Mathematica [A] time = 0.0549729, size = 34, normalized size = 0.55

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax-2)}{3c^3(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(3*c^3*(-1 + a*x)^2)
```

Maple [A] time = 0.044, size = 41, normalized size = 0.7

$$-\frac{(ax-2)(ax+1)}{3(ax-1)^2c^3a}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(a*x-2)*(a*x+1)/(a*x-1)^2/c^3/a
```

Maxima [A] time = 1.04444, size = 53, normalized size = 0.85

$$-\frac{\frac{3(ax-1)}{ax+1}-1}{6ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a*x - 1)/(a*x + 1) - 1)/(a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

Fricas [A] time = 1.54921, size = 119, normalized size = 1.92

$$-\frac{\left(a^{2} x^{2}-a x-2\right) \sqrt{\frac{a x-1}{a x+1}}}{3\left(a^{3} c^{3} x^{2}-2 a^{2} c^{3} x+a c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^3*x^2 - 2*a^2*c^3
*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{\frac{a x}{a x+1}-\frac{1}{a x+1}}}{a^{3} x^{3}-3 a^{2} x^{2}+3 a x-1}\, d x}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x**2 + 3*a*
x - 1), x)/c**3
```

Giac [A] time = 1.19694, size = 61, normalized size = 0.98

$$\frac{2\left(3\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right) x-1\right)}{3\left(\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right) x-1\right)^{3} a c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^3*a*
c^3)
```

3.204 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=95

$$-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

```
[Out] -(a^2*Sqrt[1 - 1/(a^2*x^2)])/(5*c^4*(a - x^(-1))^3) + (8*a*Sqrt[1 - 1/(a^2*
x^2)])/(15*c^4*(a - x^(-1))^2) - (7*Sqrt[1 - 1/(a^2*x^2)])/(15*c^4*(a - x^(
-1)))
```

Rubi [A] time = 0.229206, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

```
[Out] -(a^2*Sqrt[1 - 1/(a^2*x^2)])/(5*c^4*(a - x^(-1))^3) + (8*a*Sqrt[1 - 1/(a^2*
x^2)])/(15*c^4*(a - x^(-1))^2) - (7*Sqrt[1 - 1/(a^2*x^2)])/(15*c^4*(a - x^(
-1)))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
```

```
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$
$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{x}{a^3}}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^4}$$
$$= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5a^2c^4}$$
$$= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15a^2c^4}$$
$$= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

Mathematica [A] time = 0.0562102, size = 43, normalized size = 0.45

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-6ax+7\right)}{15c^4(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]

[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(15*c^4*(-1 + a*x)^3)
```

Maple [A] time = 0.05, size = 50, normalized size = 0.5

$$-\frac{\left(2 a^{2} x^{2}-6 a x+7\right) \left(a x+1\right)}{15 \left(a x-1\right)^{3} c^{4} a}\sqrt{\frac{a x-1}{a x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2-6*a*x+7)*(a*x+1)/(a*x-1)^3/c^4/a
```

Maxima [A] time = 1.01394, size = 74, normalized size = 0.78

$$\frac{\frac{10 \left(a x-1\right)}{a x+1}-\frac{15 \left(a x-1\right)^{2}}{\left(a x+1\right)^{2}}-3}{60 \, a c^{4} \left(\frac{a x-1}{a x+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^4*((a*x
 - 1)/(a*x + 1))^(5/2))
```

Fricas [A] time = 1.56422, size = 161, normalized size = 1.69

$$-\frac{\left(2 a^{3} x^{3}-4 a^{2} x^{2}+a x+7\right) \sqrt{\frac{a x-1}{a x+1}}}{15 \left(a^{4} c^{4} x^{3}-3 a^{3} c^{4} x^{2}+3 a^{2} c^{4} x-a c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^4*
x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x**3 + 6*a**
2*x**2 - 4*a*x + 1), x)/c**4
```

Giac [A] time = 1.19153, size = 88, normalized size = 0.93

$$-\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^4)
```

3.205 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=128

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)])/(7*c^5*(a - x^(-1))^4) - (18*a^2*Sqrt[1 - 1/(a^
2*x^2)])/(35*c^5*(a - x^(-1))^3) + (23*a*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a
- x^(-1))^2) - (12*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a - x^(-1)))
```

Rubi [A] time = 0.254514, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)])/(7*c^5*(a - x^(-1))^4) - (18*a^2*Sqrt[1 - 1/(a^
2*x^2)])/(35*c^5*(a - x^(-1))^3) + (23*a*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a
- x^(-1))^2) - (12*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a - x^(-1)))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
```

```
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{3x}{a^3}}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{18\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{7a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{36\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

Mathematica [A] time = 0.0655887, size = 51, normalized size = 0.4

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-8a^2x^2+13ax-12\right)}{35c^5(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(35*c^5*(
-1 + a*x)^4)
```

Maple [A] time = 0.049, size = 58, normalized size = 0.5

$$-\frac{\left(2\,x^3a^3-8\,a^2x^2+13\,ax-12\right)(ax+1)}{35\,(ax-1)^4\,c^5a}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)*(a*x+1)/(a*x-
1)^4/c^5/a
```

Maxima [A] time = 1.00579, size = 96, normalized size = 0.75

$$-\frac{\frac{21\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}+\frac{35\,(ax-1)^3}{(ax+1)^3}-5}{280\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/280*(21*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 + 35*(a*x - 1)^
3/(a*x + 1)^3 - 5)/(a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

Fricas [A] time = 1.56308, size = 200, normalized size = 1.56

$$-\frac{\left(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+ax-12\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/35*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + a*x - 12)*sqrt((a*x - 1)/(a*x +
1))/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22216, size = 115, normalized size = 0.9

$$\frac{4\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 4/35*(35*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 7*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^5)
```

3.206 $\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=44

$$\frac{(c-acx)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

```
[Out] ((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a*
c^2*(2 + p))
```

Rubi [A] time = 0.0617144, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6130, 21, 68}

$$\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a*
c^2*(2 + p))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx \\
&= -\int \frac{(1-ax)(c-acx)^p}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{1+p}}{1+ax}\,dx}{c} \\
&= \frac{(c-acx)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}
\end{aligned}
$$

Mathematica [A] time = 0.0152882, size = 44, normalized size = 1.

$$
-\frac{(ax-1)(c-acx)^p\left(\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{1}{2}(1-ax)\right)-1\right)}{a(p+1)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 - a
*x)/2]))/(a*(1 + p)))
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$
\int \frac{(-acx+c)^p\,(ax-1)}{ax+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)*(a*x-1),x)
```

```
[Out] int((-a*c*x+c)^p/(a*x+1)*(a*x-1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax-1)(-acx+c)^p}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c\,(ax-1))^p\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(a*x - 1))**p*(a*x - 1)/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

3.207 $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=91

$$-\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}-\frac{4c^4(1-ax)^3}{3a}-\frac{4c^4(1-ax)^2}{a}-\frac{32c^4\log(ax+1)}{a}+16c^4x$$

```
[Out] 16*c^4*x - (4*c^4*(1 - a*x)^2)/a - (4*c^4*(1 - a*x)^3)/(3*a) - (c^4*(1 - a*
x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a) - (32*c^4*Log[1 + a*x])/a
```

Rubi [A] time = 0.0706506, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}-\frac{4c^4(1-ax)^3}{3a}-\frac{4c^4(1-ax)^2}{a}-\frac{32c^4\log(ax+1)}{a}+16c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 16*c^4*x - (4*c^4*(1 - a*x)^2)/a - (4*c^4*(1 - a*x)^3)/(3*a) - (c^4*(1 - a*
x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a) - (32*c^4*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx\\ &= -\left(c^4\int \frac{(1-ax)^5}{1+ax}\,dx\right)\\ &= -\left(c^4\int \left(-16-8(1-ax)-4(1-ax)^2-2(1-ax)^3-(1-ax)^4+\frac{32}{1+ax}\right)dx\right)\\ &= 16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0205326, size = 56, normalized size = 0.62

$$\frac{c^4\left(6a^5x^5-45a^4x^4+160a^3x^3-390a^2x^2+930ax-960\log(ax+1)-181\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5 -
 960*Log[1 + a*x]))/(30*a)
```

Maple [A] time = 0.04, size = 64, normalized size = 0.7

$$\frac{a^4c^4x^5}{5}-\frac{3\,c^4x^4a^3}{2}+\frac{16\,a^2c^4x^3}{3}-13\,c^4x^2a+31\,c^4x-32\,\frac{c^4\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/5*a^4*c^4*x^5-3/2*c^4*x^4*a^3+16/3*a^2*c^4*x^3-13*c^4*x^2*a+31*c^4*x-32*c
^4*ln(a*x+1)/a
```

Maxima [A] time = 1.00712, size = 85, normalized size = 0.93

$$\frac{1}{5}\,a^4c^4x^5 - \frac{3}{2}\,a^3c^4x^4 + \frac{16}{3}\,a^2c^4x^3 - 13\,ac^4x^2 + 31\,c^4x - \frac{32\,c^4\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 3/2*a^3*c^4*x^4 + 16/3*a^2*c^4*x^3 - 13*a*c^4*x^2 + 31*c^
4*x - 32*c^4*log(a*x + 1)/a
```

Fricas [A] time = 1.51959, size = 154, normalized size = 1.69

$$\frac{6\,a^5c^4x^5 - 45\,a^4c^4x^4 + 160\,a^3c^4x^3 - 390\,a^2c^4x^2 + 930\,ac^4x - 960\,c^4\log\left(ax+1\right)}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

Sympy [A] time = 0.670782, size = 68, normalized size = 0.75

$$\frac{a^4c^4x^5}{5} - \frac{3a^3c^4x^4}{2} + \frac{16a^2c^4x^3}{3} - 13ac^4x^2 + 31c^4x - \frac{32c^4\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**4*x**5/5 - 3*a**3*c**4*x**4/2 + 16*a**2*c**4*x**3/3 - 13*a*c**4*x**
2 + 31*c**4*x - 32*c**4*log(a*x + 1)/a
```

Giac [A] time = 1.19164, size = 101, normalized size = 1.11

$$-\frac{32\,c^4\log\left(|ax+1|\right)}{a}+\frac{6\,a^9c^4x^5-45\,a^8c^4x^4+160\,a^7c^4x^3-390\,a^6c^4x^2+930\,a^5c^4x}{30\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*c^4*log(abs(a*x + 1))/a + 1/30*(6*a^9*c^4*x^5 - 45*a^8*c^4*x^4 + 160*a^
7*c^4*x^3 - 390*a^6*c^4*x^2 + 930*a^5*c^4*x)/a^5
```

3.208 $\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=73

$$-\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}-\frac{2c^3(1-ax)^2}{a}-\frac{16c^3\log(ax+1)}{a}+8c^3x$$

```
[Out] 8*c^3*x - (2*c^3*(1 - a*x)^2)/a - (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x
)^4)/(4*a) - (16*c^3*Log[1 + a*x])/a
```

Rubi [A] time = 0.0645011, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}-\frac{2c^3(1-ax)^2}{a}-\frac{16c^3\log(ax+1)}{a}+8c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 8*c^3*x - (2*c^3*(1 - a*x)^2)/a - (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x
)^4)/(4*a) - (16*c^3*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx \\
&= -\left(c^3\int \frac{(1-ax)^4}{1+ax}\,dx\right) \\
&= -\left(c^3\int \left(-8-4(1-ax)-2(1-ax)^2-(1-ax)^3+\frac{16}{1+ax}\right)dx\right) \\
&= 8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0176466, size = 48, normalized size = 0.66

$$-\frac{c^3\left(3a^4x^4-20a^3x^3+66a^2x^2-180ax+192\log(ax+1)+35\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1 + a*x
]))/(12*a)
```

Maple [A] time = 0.043, size = 53, normalized size = 0.7

$$-\frac{c^3x^4a^3}{4}+\frac{5\,c^3x^3a^2}{3}-\frac{11\,c^3x^2a}{2}+15\,c^3x-16\,\frac{c^3\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/4*c^3*x^4*a^3+5/3*c^3*x^3*a^2-11/2*c^3*x^2*a+15*c^3*x-16*c^3*ln(a*x+1)/a
```

Maxima [A] time = 1.03387, size = 70, normalized size = 0.96

$$-\frac{1}{4}\,a^{3}c^{3}x^{4}+\frac{5}{3}\,a^{2}c^{3}x^{3}-\frac{11}{2}\,ac^{3}x^{2}+15\,c^{3}x-\frac{16\,c^{3}\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 + 5/3*a^2*c^3*x^3 - 11/2*a*c^3*x^2 + 15*c^3*x - 16*c^3*log
(a*x + 1)/a
```

Fricas [A] time = 1.45557, size = 130, normalized size = 1.78

$$-\frac{3\,a^{4}c^{3}x^{4}-20\,a^{3}c^{3}x^{3}+66\,a^{2}c^{3}x^{2}-180\,ac^{3}x+192\,c^{3}\log\left(ax+1\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192*
c^3*log(a*x + 1))/a
```

Sympy [A] time = 0.601988, size = 56, normalized size = 0.77

$$-\frac{a^{3}c^{3}x^{4}}{4}+\frac{5a^{2}c^{3}x^{3}}{3}-\frac{11ac^{3}x^{2}}{2}+15c^{3}x-\frac{16c^{3}\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**3*c**3*x**4/4 + 5*a**2*c**3*x**3/3 - 11*a*c**3*x**2/2 + 15*c**3*x - 16*
c**3*log(a*x + 1)/a
```

Giac [A] time = 1.12859, size = 86, normalized size = 1.18

$$-\frac{16\,c^3\log\left(|ax+1|\right)}{a}-\frac{3\,a^7c^3x^4-20\,a^6c^3x^3+66\,a^5c^3x^2-180\,a^4c^3x}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*c^3*log(abs(a*x + 1))/a - 1/12*(3*a^7*c^3*x^4 - 20*a^6*c^3*x^3 + 66*a^5
*c^3*x^2 - 180*a^4*c^3*x)/a^4
```

3.209 $\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=55

$$-\frac{c^2(1-ax)^3}{3a}-\frac{c^2(1-ax)^2}{a}-\frac{8c^2\log(ax+1)}{a}+4c^2x$$

```
[Out] 4*c^2*x - (c^2*(1 - a*x)^2)/a - (c^2*(1 - a*x)^3)/(3*a) - (8*c^2*Log[1 + a*
x])/a
```

Rubi [A] time = 0.0582331, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^2(1-ax)^3}{3a}-\frac{c^2(1-ax)^2}{a}-\frac{8c^2\log(ax+1)}{a}+4c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 4*c^2*x - (c^2*(1 - a*x)^2)/a - (c^2*(1 - a*x)^3)/(3*a) - (8*c^2*Log[1 + a*
x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx\\ &= -\left(c^2\int\frac{(1-ax)^3}{1+ax}\,dx\right)\\ &= -\left(c^2\int\left(-4-2(1-ax)-(1-ax)^2+\frac{8}{1+ax}\right)dx\right)\\ &= 4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.014293, size = 39, normalized size = 0.71

$$\frac{c^2\left(a^3x^3-6a^2x^2+21ax-24\log(ax+1)-4\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

Maple [A] time = 0.04, size = 42, normalized size = 0.8

$$\frac{a^2c^2x^3}{3}-2\,c^2x^2a+7\,xc^2-8\,\frac{c^2\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*a^2*c^2*x^3-2*c^2*x^2*a+7*x*c^2-8*c^2*ln(a*x+1)/a
```

Maxima [A] time = 1.016, size = 55, normalized size = 1.

$$\frac{1}{3}a^2c^2x^3 - 2\,ac^2x^2 + 7\,c^2x - \frac{8\,c^2\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 - 2*a*c^2*x^2 + 7*c^2*x - 8*c^2*log(a*x + 1)/a
```

Fricas [A] time = 1.41193, size = 97, normalized size = 1.76

$$\frac{a^3c^2x^3 - 6\,a^2c^2x^2 + 21\,ac^2x - 24\,c^2\log(ax+1)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

Sympy [A] time = 0.644037, size = 41, normalized size = 0.75

$$\frac{a^2c^2x^3}{3} - 2ac^2x^2 + 7c^2x - \frac{8c^2\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*(a*x-1)/(a*x+1),x)
```

```
[Out] a**2*c**2*x**3/3 - 2*a*c**2*x**2 + 7*c**2*x - 8*c**2*log(a*x + 1)/a
```

Giac [A] time = 1.13615, size = 70, normalized size = 1.27

$$-\frac{8\,c^2\log(|ax+1|)}{a} + \frac{a^5c^2x^3 - 6\,a^4c^2x^2 + 21\,a^3c^2x}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -8*c^2*log(abs(a*x + 1))/a + 1/3*(a^5*c^2*x^3 - 6*a^4*c^2*x^2 + 21*a^3*c^2*
x)/a^3
```

3.210 $\int e^{-2\coth^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}acx^2 - \frac{4c\log(ax+1)}{a} + 3cx$$

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Rubi [A] time = 0.0350002, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6167, 6129, 43}

$$-\frac{1}{2}acx^2 - \frac{4c\log(ax+1)}{a} + 3cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx\\ &= -\left(c\int\frac{(1-ax)^2}{1+ax}\,dx\right)\\ &= -\left(c\int\left(-3+ax+\frac{4}{1+ax}\right)dx\right)\\ &= 3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0092851, size = 26, normalized size = 1.

$$-\frac{1}{2}acx^2-\frac{4c\log(ax+1)}{a}+3cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Maple [A] time = 0.043, size = 25, normalized size = 1.

$$3\,cx-\frac{acx^2}{2}-4\,\frac{c\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)*(a*x-1),x)
```

```
[Out] 3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a
```

Maxima [A] time = 1.02372, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,acx^2+3\,cx-\frac{4\,c\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Fricas [A] time = 1.46784, size = 66, normalized size = 2.54

$$-\frac{a^2cx^2 - 6\,acx + 8\,c\log\left(ax + 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

Sympy [A] time = 1.09001, size = 24, normalized size = 0.92

$$-\frac{acx^2}{2} + 3cx - \frac{4c\log\left(ax + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a*c*x**2/2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Giac [A] time = 1.12306, size = 47, normalized size = 1.81

$$-\frac{4\,c\log\left(|ax + 1|\right)}{a} - \frac{a^3cx^2 - 6\,a^2cx}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*c*log(abs(a*x + 1))/a - 1/2*(a^3*c*x^2 - 6*a^2*c*x)/a^2
```

3.211 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=14

$$-\frac{\log(ax+1)}{ac}$$

```
[Out] -(Log[1 + a*x]/(a*c))
```

Rubi [A] time = 0.0546414, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 31}

$$-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] -(Log[1 + a*x]/(a*c))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx$$
$$= -\frac{\int \frac{1}{1+ax}\,dx}{c}$$
$$= -\frac{\log(1+ax)}{ac}$$

Mathematica [A] time = 0.0074188, size = 14, normalized size = 1.

$$-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] -(Log[1 + a*x]/(a*c))
```

Maple [A] time = 0.039, size = 15, normalized size = 1.1

$$-\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c),x)
```

```
[Out] -ln(a*x+1)/a/c
```

Maxima [A] time = 1.01747, size = 19, normalized size = 1.36

$$-\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -log(a*x + 1)/(a*c)
```

Fricas [A] time = 1.43935, size = 28, normalized size = 2.

$$-\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -log(a*x + 1)/(a*c)
```

Sympy [A] time = 0.242826, size = 12, normalized size = 0.86

$$-\frac{\log(acx+c)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x)
```

```
[Out] -log(a*c*x + c)/(a*c)
```

Giac [A] time = 1.1383, size = 20, normalized size = 1.43

$$-\frac{\log(|ax+1|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -log(abs(a*x + 1))/(a*c)
```

3.212 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] -(ArcTanh[a*x]/(a*c^2))
```

Rubi [A] time = 0.0487918, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 35, 206}

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]

[Out] -(ArcTanh[a*x]/(a*c^2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx\\ &= -\frac{\int \frac{1}{(1-ax)(1+ax)}\,dx}{c^2}\\ &= -\frac{\int \frac{1}{1-a^2x^2}\,dx}{c^2}\\ &= -\frac{\tanh^{-1}(ax)}{ac^2}\end{aligned}$$

Mathematica [A] time = 0.0085515, size = 12, normalized size = 1.

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] -(ArcTanh[a*x]/(a*c^2))
```

Maple [B] time = 0.047, size = 30, normalized size = 2.5

$$-\frac{\ln(ax+1)}{2\,ac^2}+\frac{\ln(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^2,x)
```

```
[Out] -1/2*ln(a*x+1)/a/c^2+1/2/c^2/a*ln(a*x-1)
```

Maxima [B] time = 1.016, size = 39, normalized size = 3.25

$$-\frac{\log\left(ax+1\right)}{2\,ac^2}+\frac{\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.581, size = 59, normalized size = 4.92

$$-\frac{\log\left(ax+1\right)-\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

Sympy [A] time = 0.428533, size = 20, normalized size = 1.67

$$\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**2,x)
```

```
[Out] (log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

Giac [B] time = 1.15308, size = 34, normalized size = 2.83

$$-\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

3.213 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}$$

```
[Out] -1/(2*a*c^3*(1 - a*x)) - ArcTanh[a*x]/(2*a*c^3)
```

Rubi [A] time = 0.0626256, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 44, 207}

$$-\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] -1/(2*a*c^3*(1 - a*x)) - ArcTanh[a*x]/(2*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c^3} \\
&= -\frac{\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c^3} \\
&= -\frac{1}{2ac^3(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{2c^3} \\
&= -\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}
\end{aligned}$$

Mathematica [A] time = 0.0199579, size = 32, normalized size = 0.97

$$-\frac{\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] -((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

Maple [A] time = 0.051, size = 45, normalized size = 1.4

$$-\frac{\ln(ax+1)}{4\,ac^3}+\frac{1}{2\,ac^3\,(ax-1)}+\frac{\ln(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^3,x)
```

```
[Out] -1/4*ln(a*x+1)/a/c^3+1/2/c^3/a/(a*x-1)+1/4/c^3/a*ln(a*x-1)
```

Maxima [A] time = 1.01879, size = 65, normalized size = 1.97

$$\frac{1}{2\left(a^2c^3x-ac^3\right)}-\frac{\log\left(ax+1\right)}{4\,ac^3}+\frac{\log\left(ax-1\right)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(a^2*c^3*x - a*c^3) - 1/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c^
3)
```

Fricas [A] time = 1.52915, size = 108, normalized size = 3.27

$$-\frac{(ax-1)\log\left(ax+1\right)-(ax-1)\log\left(ax-1\right)-2}{4\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*c
^3)
```

Sympy [A] time = 0.653654, size = 39, normalized size = 1.18

$$\frac{1}{2a^2c^3x-2ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**3,x)
```

```
[Out] 1/(2*a**2*c**3*x - 2*a*c**3) - (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

Giac [A] time = 1.1426, size = 62, normalized size = 1.88

$$-\frac{\log(|ax+1|)}{4\,ac^3} + \frac{\log(|ax-1|)}{4\,ac^3} + \frac{1}{2\,(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) + 1/2/((a*x
- 1)*a*c^3)
```

3.214 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

```
[Out] -1/(4*a*c^4*(1 - a*x)^2) - 1/(4*a*c^4*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^4)
```

Rubi [A] time = 0.0675194, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 44, 207}

$$-\frac{1}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] -1/(4*a*c^4*(1 - a*x)^2) - 1/(4*a*c^4*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^4}\\
&= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^4}\\
&= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^4}\\
&= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^4}
\end{aligned}
$$

Mathematica [A] time = 0.0245119, size = 35, normalized size = 0.69

$$\frac{ax+(ax-1)^2\left(-\tanh^{-1}(ax)\right)-2}{4ac^4(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

Maple [A] time = 0.051, size = 60, normalized size = 1.2

$$-\frac{\ln(ax+1)}{8\,ac^4}-\frac{1}{4\,ac^4\,(ax-1)^2}+\frac{1}{4\,ac^4\,(ax-1)}+\frac{\ln(ax-1)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^4,x)
```

```
[Out] -1/8*ln(a*x+1)/a/c^4-1/4/c^4/a/(a*x-1)^2+1/4/c^4/a/(a*x-1)+1/8/c^4/a*ln(a*x
-1)
```

Maxima [A] time = 1.00564, size = 85, normalized size = 1.67

$$\frac{ax-2}{4\left(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4}\right)}-\frac{\log\left(ax+1\right)}{8\,ac^{4}}+\frac{\log\left(ax-1\right)}{8\,ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - 1/8*log(a*x + 1)/(a*c^4
) + 1/8*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.53384, size = 171, normalized size = 3.35

$$\frac{2\,ax-\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax+1\right)+\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.72217, size = 54, normalized size = 1.06

$$\frac{ax-2}{4a^{3}c^{4}x^{2}-8a^{2}c^{4}x+4ac^{4}}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**4)
```

Giac [A] time = 1.14275, size = 69, normalized size = 1.35

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^4}+\frac{\log\left(|ax-1|\right)}{8\,ac^4}+\frac{ax-2}{4\,(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^4) + 1/8*log(abs(a*x - 1))/(a*c^4) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^4)
```

3.215 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=69

$$-\frac{1}{8ac^5(1-ax)}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{6ac^5(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8ac^5}$$

```
[Out] -1/(6*a*c^5*(1 - a*x)^3) - 1/(8*a*c^5*(1 - a*x)^2) - 1/(8*a*c^5*(1 - a*x))
- ArcTanh[a*x]/(8*a*c^5)
```

Rubi [A] time = 0.0796972, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 44, 207}

$$-\frac{1}{8ac^5(1-ax)}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{6ac^5(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] -1/(6*a*c^5*(1 - a*x)^3) - 1/(8*a*c^5*(1 - a*x)^2) - 1/(8*a*c^5*(1 - a*x))
- ArcTanh[a*x]/(8*a*c^5)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^5}\\
&= -\frac{\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^5}\\
&= -\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8c^5}\\
&= -\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}-\frac{\tanh^{-1}(ax)}{8ac^5}
\end{aligned}$$

Mathematica [A] time = 0.0312029, size = 44, normalized size = 0.64

$$\frac{3a^2x^2-9ax-3(ax-1)^3\tanh^{-1}(ax)+10}{24ac^5(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (10 - 9*a*x + 3*a^2*x^2 - 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x)
^3)
```

Maple [A] time = 0.046, size = 75, normalized size = 1.1

$$-\frac{\ln(ax+1)}{16\,c^5a}+\frac{1}{6\,c^5a\,(ax-1)^3}-\frac{1}{8\,c^5a\,(ax-1)^2}+\frac{1}{8\,c^5a\,(ax-1)}+\frac{\ln(ax-1)}{16\,c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^5,x)
```

```
[Out] -1/16/c^5/a*ln(a*x+1)+1/6/c^5/a/(a*x-1)^3-1/8/c^5/a/(a*x-1)^2+1/8/c^5/a/(a*
x-1)+1/16/c^5/a*ln(a*x-1)
```

Maxima [A] time = 1.03516, size = 113, normalized size = 1.64

$$\frac{3\,a^2x^2 - 9\,ax + 10}{24\left(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5\right)} - \frac{\log\left(ax+1\right)}{16\,ac^5} + \frac{\log\left(ax-1\right)}{16\,ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
a*c^5) - 1/16*log(a*x + 1)/(a*c^5) + 1/16*log(a*x - 1)/(a*c^5)
```

Fricas [A] time = 1.55361, size = 251, normalized size = 3.64

$$\frac{6\,a^2x^2 - 18\,ax - 3\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax+1\right) + 3\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax-1\right) + 20}{48\left(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1)
 + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 - 3*
a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [A] time = 1.11456, size = 76, normalized size = 1.1

$$\frac{3a^2x^2 - 9ax + 10}{24a^4c^5x^3 - 72a^3c^5x^2 + 72a^2c^5x - 24ac^5} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16} + \frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**5,x)
```

```
[Out] (3*a**2*x**2 - 9*a*x + 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a**2
*c**5*x - 24*a*c**5) - (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

Giac [A] time = 1.13845, size = 120, normalized size = 1.74

$$-\frac{\log\left(\left|-\frac{2c}{acx-c}-1\right|\right)}{16\,ac^5}+\frac{\frac{3\,a^2c^2}{acx-c}-\frac{3\,a^2c^3}{(acx-c)^2}+\frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) + 1/24*(3*a^2*c^2/(a*c*x - c)
- 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)
```

3.216 $\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=94

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\text{Hypergeometric2F1}\left(-p-\frac{3}{2},-p-1,-p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^p
*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sqrt
[1 + 1/(a*x)])
```

Rubi [A] time = 0.125535, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 132}

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,{}_2F_1\left(-p-\frac{3}{2},-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^p
*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sqrt
[1 + 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx\\
&= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,{}_2F_1\left(-\frac{3}{2}-p,-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A] time = 0.0613283, size = 96, normalized size = 1.02

$$\frac{\sqrt{1-\frac{1}{ax}}(ax+1)\left(\frac{ax-1}{ax+1}\right)^{-p-\frac{1}{2}}(c-acx)^p\text{Hypergeometric2F1}\left(-p-\frac{3}{2},-p-1,-p,\frac{2}{ax+1}\right)}{a(p+1)\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(1 + a*x)*(c - a*c*x)^
p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*Sqrt[1 +
 1/(a*x)])
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax-1)(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^p \left(\frac{ax - 1}{ax + 1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.217 $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=152

$$-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{67}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{315c^3\tanh^{-1}\left(\sqrt{1\right.}}{8a}$$

```
[Out] (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 30*c^3*Sqrt[1 - 1/(a^2*
x^2)]*x - (67*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + 2*a^2*c^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - (315*c^3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/(8*a)
```

Rubi [A] time = 0.43646, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 1805, 1807, 807, 266, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{67}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{315c^3\tanh^{-1}\left(\sqrt{1\right.}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 30*c^3*Sqrt[1 - 1/(a^2*
x^2)]*x - (67*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + 2*a^2*c^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - (315*c^3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/(8*a)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
```

```
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^6}{x^5\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \left(a^3c^3\right)\text{Subst}\left(\int \frac{-1+\frac{6x}{a}-\frac{16x^2}{a^2}+\frac{26x^3}{a^3}-\frac{31x^4}{a^4}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{24}{a}+\frac{67x}{a^2}-\frac{104x^2}{a^3}+\frac{124x^3}{a^4}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \cdots\right.$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 - \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{201}{a^2}+\cdots}{x^3\sqrt{\cdots}}\right.$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{1}{24}\left(a^3c\cdots\right.$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1\cdots}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1\cdots}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1\cdots}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3 - \frac{1}{4}a^3c^3\sqrt{1\cdots}$$

Mathematica [A] time = 0.218798, size = 86, normalized size = 0.57

$$\frac{1}{8}c^3\left(\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^4x^4+14a^3x^3-51a^2x^2+173ax+496\right)}{ax+1}-\frac{315\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*x*(496 + 173*a*x - 51*a^2*x^2 + 14*a^3*x^3 - 2
*a^4*x^4))/(1 + a*x) - (315*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/8
```

Maple [B] time = 0.138, size = 542, normalized size = 3.6

$$-\frac{c^3}{8\,(ax-1)\,a}\left(2\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3+4\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+69\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3-16\,\sqrt{a^2}\,((ax-1)\,(ax+1))^{3/2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/8*(2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+4*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x^2*a^2+69*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-16*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(3/2)*x^2*a^2+2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+138*(a^2*x^2-1)
^(1/2)*(a^2)^(1/2)*x^2*a^2-69*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2
)^(1/2))*x^2*a^3-32*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-384*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+384*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2))*x^2*a^3+69*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-138*ln((
a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+112*((a*x-1)*(a*x+1
))^(3/2)*(a^2)^(1/2)-768*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+768*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-69*ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-384*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(1/2)+384*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))
/a*c^3*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.03389, size = 329, normalized size = 2.16

$$-\frac{1}{8}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{256\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}-\frac{2\left(325\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-765\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+643\,c^3\left(\frac{ax-1}{ax+1}\right)\cdots\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 256*c^3*sqrt((a*x - 1)/(a*x + 1))/a^2 - 2*(325
*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 765*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 64
3*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 187*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(
a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/
(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*a
```

Fricas [A] time = 1.68883, size = 270, normalized size = 1.78

$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4-14\,a^3c^3x^3+51\,a^2c^3x^2-173\,ac^3x-496\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 173
*a*c^3*x - 496*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.218 $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=129

$$\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{5}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{35}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{35c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (35*c^2*Sqrt[1 - 1/(a^2
*x^2)]*x)/3 - (5*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*Sqrt[1 - 1/(
a^2*x^2)]*x^3)/3 - (35*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rubi [A] time = 0.34967, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 1805, 1807, 807, 266, 63, 208}

$$\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{5}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{35}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{35c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (35*c^2*Sqrt[1 - 1/(a^2
*x^2)]*x)/3 - (5*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*Sqrt[1 - 1/(
a^2*x^2)]*x^3)/3 - (35*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\left(a^2c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2x^2\,dx$$

$$=-\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^4\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\left(a^2c^2\right)\text{Subst}\left(\int\frac{-1+\frac{5x}{a}-\frac{11x^2}{a^2}+\frac{15x^3}{a^3}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{3}\left(a^2c^2\right)\text{Subst}\left(\int\frac{-\frac{15}{a}+\frac{35x}{a^2}-\frac{45x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3+\frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int\frac{-\frac{70}{a^2}+\frac{105x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3+\frac{\left(35c^2\right)\text{Su}}{}$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3+\frac{\left(35c^2\right)\text{Su}}{}$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{2}\left(35ac^2\right)$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{35c^2\tanh}{}$$

Mathematica [A] time = 0.150984, size = 78, normalized size = 0.6

$$\frac{1}{6}c^2\left(\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-13a^2x^2+55ax+166\right)}{ax+1}-\frac{105\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*((Sqrt[1 - 1/(a^2*x^2)]*x*(166 + 55*a*x - 13*a^2*x^2 + 2*a^3*x^3))/(1
+ a*x) - (105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/6
```

Maple [B] time = 0.136, size = 474, normalized size = 3.7

$$-\frac{c^2}{6\,(ax-1)\,a}\left(15\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3-2\,\sqrt{a^2}\,((ax-1)\,(ax+1))^{3/2}\,x^2a^2+30\,\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2-15\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-2*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^2*a^2+30*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-15*ln((a^2*x+(a^
2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-4*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x*a-120*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+120*ln((a^2
*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+15*(a^2)^(1/2)
*(a^2*x^2-1)^(1/2)*x*a-30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*x*a^2+46*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-240*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2)*x*a+240*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)
^(1/2))*x*a^2-15*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-12
0*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+120*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2))/(a^2)^(1/2)))/a*c^2*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a
*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.11875, size = 275, normalized size = 2.13

$$-\frac{1}{6}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{96\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{2\left(87\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-136\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+57\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 96*c^2*sqrt((a*x - 1)/(a*x + 1))/a^2 + 2*(87
*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 136*c^2*((a*x - 1)/(a*x + 1))^(3/2) + 57
*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*
a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2))
```

Fricas [A] time = 1.62261, size = 246, normalized size = 1.91

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.219 $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+4cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 4*c*Sqrt[1 - 1/(a^2*x^2)]*
x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (15*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(2*a)
```

Rubi [A] time = 0.243798, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6175, 6178, 1805, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+4cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 4*c*Sqrt[1 - 1/(a^2*x^2)]*
x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (15*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(2*a)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\
&= (ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-(ac)\,\text{Subst}\left(\int \frac{-1+\frac{4x}{a}-\frac{7x^2}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{-\frac{8}{a}+\frac{15x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{(15c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{(15c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a}\\
&= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{2}(15ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{}\right.\\
&= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.133689, size = 68, normalized size = 0.74

$$
\frac{1}{2}c\left(\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-a^2x^2+7ax+24\right)}{ax+1}-\frac{15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*((Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 7*a*x - a^2*x^2))/(1 + a*x) - (15*Log[a*
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/2
```

Maple [B] time = 0.16, size = 422, normalized size = 4.6

$$-\frac{c}{2\,(ax-1)\,a}\left(\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+2\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2-\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)x^2a^3-16\sqrt{a^2}\sqrt{(ax-1)\,(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)
*x^2*a^2-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-16*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+16*ln((a^2*x+(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-2*
ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+8*((a*x-1)*(a*x
+1))^(3/2)*(a^2)^(1/2)-32*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+32*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-16*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2)+16*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a*c*
((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maple [A] time = 1.06275, size = 211, normalized size = 2.29

$$\frac{1}{2}\,a\left(\frac{2\left(9\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-7\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{16\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(9*c*((a*x - 1)/(a*x + 1))^(3/2) - 7*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 15*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/a^2 + 16*c*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Fricas [A] time = 1.66708, size = 201, normalized size = 2.18

$$-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-7\,acx-24\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (a^2*c*x^2 - 7*a*c*x - 24*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.220 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=53

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rubi [A] time = 0.198296, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 1805, 266, 63, 208}

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.0643677, size = 54, normalized size = 1.02

$$\frac{\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{ax+1} - \frac{\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] ((2*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x]/a)/c
```

Maple [B] time = 0.141, size = 248, normalized size = 4.7

$$-\frac{1}{ac\left(ax-1\right)}\left(\ln\left(\left(a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}\right)\frac{1}{\sqrt{a^{2}}}\right)x^{2}a^{3}-\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}x^{2}a^{2}+2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(a\right.}}{\sqrt{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x)
```

```
[Out] -(ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-2*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+
1))^(3/2)/(a^2)^(1/2)/c/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.03066, size = 105, normalized size = 1.98

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c}-\frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2*sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Fricas [A] time = 1.57184, size = 150, normalized size = 2.83

$$\frac{2\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sqr
t((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Integra
l(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] undef
```

3.221 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=28

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.103764, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 637}

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] (a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= \frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0502169, size = 26, normalized size = 0.93

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(1 + a*x))
```

Maple [A] time = 0.066, size = 35, normalized size = 1.3

$$\frac{ax+1}{(ax-1)\,ac^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)/a/c^2
```

Maxima [A] time = 1.09743, size = 30, normalized size = 1.07

$$\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

Fricas [A] time = 1.49414, size = 47, normalized size = 1.68

$$\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x +
 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*
```

```
x**2 - a*x + 1), x))/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.222 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=21

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.100688, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 261}

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] 1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$
$$= \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0541357, size = 33, normalized size = 1.57

$$\frac{ax^2\sqrt{1-\frac{1}{a^2x^2}}}{c^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(c^3*(-1 + a^2*x^2))
```

Maple [A] time = 0.052, size = 33, normalized size = 1.6

$$\frac{(ax+1)\,x}{(ax-1)^2\,c^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*x/(a*x-1)^2/c^3
```

Maxima [B] time = 1.08119, size = 65, normalized size = 3.1

$$\frac{1}{2}\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*(sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + 1/(a^2*c^3*sqrt((a*x - 1)/(a*x
 + 1))))
```

Fricas [A] time = 1.5342, size = 61, normalized size = 2.9

$$\frac{x\sqrt{\frac{ax-1}{ax+1}}}{ac^3x-c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] x*sqrt((a*x - 1)/(a*x + 1))/(a*c^3*x - c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*x**3 + 2*
a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 -
2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Giac [A] time = 1.17534, size = 30, normalized size = 1.43

$$\frac{x\text{sgn}\left(ax+1\right)}{\sqrt{a^2x^2-1}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] x*sgn(a*x + 1)/(sqrt(a^2*x^2 - 1)*c^3)
```

3.223 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=61

$$\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}$$

```
[Out] 2/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) - 1/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a -
 x^(-1))*x^2)
```

Rubi [A] time = 0.136697, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 855, 12, 261}

$$\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] 2/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) - 1/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a -
 x^(-1))*x^2)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2} + \frac{\text{Subst}\left(\int \frac{2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}$$

$$= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2} + \frac{2\,\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}$$

$$= \frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}$$

Mathematica [A] time = 0.0609999, size = 50, normalized size = 0.82

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-2ax-1\right)}{3c^4(ax-1)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^4*(-1 + a*x)^2*(1 +
 a*x))
```

Maple [A] time = 0.041, size = 50, normalized size = 0.8

$$\frac{\left(2\,a^2x^2-2\,ax-1\right)(ax+1)}{3\,(ax-1)^3\,c^4a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] 1/3*((a*x-1)/(a*x+1))^(3/2)*(2*a^2*x^2-2*a*x-1)*(a*x+1)/(a*x-1)^3/c^4/a
```

Maxima [A] time = 1.01646, size = 88, normalized size = 1.44

$$\frac{1}{12}\,a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A] time = 1.55957, size = 123, normalized size = 2.02

$$\frac{\left(2\,a^2x^2-2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^4*x^2 - 2*a^2*
c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^4, x)
```

3.224 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=94

$$\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) + (a + x^(-1))^2/(5*a
^3*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (5*a + 2/x)/(5*a^2*c^5*Sqrt[1 - 1/(a^2*x^
2)])
```

Rubi [A] time = 0.312766, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 852, 1635, 637}

$$\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) + (a + x^(-1))^2/(5*a
^3*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (5*a + 2/x)/(5*a^2*c^5*Sqrt[1 - 1/(a^2*x^
2)])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
```

```
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\left(2a^3+5a^2x+5ax^2\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5a^5c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{6a^3+15a^2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15a^5c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0632543, size = 57, normalized size = 0.61

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-4a^2x^2+ax+2\right)}{5c^5(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3))/(5*c^5*(-1 + a*
x)^3*(1 + a*x))
```

Maple [A] time = 0.05, size = 57, normalized size = 0.6

$$\frac{(2x^3a^3 - 4a^2x^2 + ax + 2)(ax + 1)}{5(ax - 1)^4 c^5 a}\left(\frac{ax - 1}{ax + 1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] 1/5*((a*x-1)/(a*x+1))^(3/2)*(2*a^3*x^3-4*a^2*x^2+a*x+2)*(a*x+1)/(a*x-1)^4/c
^5/a
```

Maxima [A] time = 1.03963, size = 111, normalized size = 1.18

$$\frac{1}{40}a\left(\frac{5\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5} - \frac{\frac{5(ax-1)}{ax+1} - \frac{15(ax-1)^2}{(ax+1)^2} - 1}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/40*a*(5*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^5) - (5*(a*x - 1)/(a*x + 1) - 15
*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(5/2)))
```

Fricas [A] time = 1.58291, size = 158, normalized size = 1.68

$$\frac{(2a^3x^3 - 4a^2x^2 + ax + 2)\sqrt{\frac{ax-1}{ax+1}}}{5(a^4c^5x^3 - 3a^3c^5x^2 + 3a^2c^5x - ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^5*x^
3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^5, x)
```

3.225 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal. Leaf size=125

$$-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-46*(a + x^(-1)))/(35*a^2*c^6*(1 - 1/(a^2*x^2))^(3/2)) + (24*(a + x^(-1))^
2)/(35*a^3*c^6*(1 - 1/(a^2*x^2))^(5/2)) - (a + x^(-1))^3/(7*a^4*c^6*(1 - 1/
(a^2*x^2))^(7/2)) + (35*a + 13/x)/(35*a^2*c^6*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.410555, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 852, 1635, 637}

$$-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```


```
[Out] (-46*(a + x^(-1)))/(35*a^2*c^6*(1 - 1/(a^2*x^2))^(3/2)) + (24*(a + x^(-1))^
2)/(35*a^3*c^6*(1 - 1/(a^2*x^2))^(5/2)) - (a + x^(-1))^3/(7*a^4*c^6*(1 - 1/
(a^2*x^2))^(7/2)) + (35*a + 13/x)/(35*a^2*c^6*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
```

```
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^6 x^6}\,dx}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-\frac{x}{a}\right)^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2\left(3a^4+7a^3x+7a^2x^2+7ax^3\right)}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7a^6c^6}$$

$$= \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\left(33a^4+70a^3x+35a^2x^2\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{39a^4+105a^3x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{105a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0705729, size = 66, normalized size = 0.53

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13\right)}{35c^6(ax-1)^4(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^6*(-1 + a*x)^4*(1 + a*x))
```

Maple [A] time = 0.043, size = 66, normalized size = 0.5

$$\frac{\left(8\,x^4a^4-24\,x^3a^3+20\,a^2x^2+4\,ax-13\right)(ax+1)}{35\,(ax-1)^5\,c^6a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)*(a*
x+1)/(a*x-1)^5/c^6/a
```

Maxima [A] time = 1.05046, size = 131, normalized size = 1.05

$$\frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^6}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^6) + (28*(a*x - 1)/(a*x + 1) -
 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^6*((a
*x - 1)/(a*x + 1))^(7/2)))
```

Fricas [A] time = 1.5806, size = 204, normalized size = 1.63

$$\frac{\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^6x^4-4\,a^4c^6x^3+6\,a^3c^6x^2-4\,a^2c^6x+ac^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^6*x^4 - 4*a^4*c^6*x^3 + 6*a^3*c^6*x^2 - 4*a^2*c^6*x + a*c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^6, x)
```

3.226 $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=254

$$-\frac{768\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{385a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{3465a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{32\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^4\left(\frac{1}{ax}+1\right)^{3}}{11a^4\left(1-\frac{1}{ax}\right.}$$

```
[Out] (-32*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1/(
a*x))^(9/2)) + (9088*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(3465*a^4*(1 -
1/(a*x))^(9/2)*x^3) - (768*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(385*a^3*
(1 - 1/(a*x))^(9/2)*x^2) + (128*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(231
*a^2*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2)*x*(c -
a*c*x)^(9/2))/(11*a^4*(1 - 1/(a*x))^(9/2))
```

Rubi [A] time = 0.220808, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{768\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{385a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{3465a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{32\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^4\left(\frac{1}{ax}+1\right)^{3}}{11a^4\left(1-\frac{1}{ax}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (-32*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1/(
a*x))^(9/2)) + (9088*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(3465*a^4*(1 -
1/(a*x))^(9/2)*x^3) - (768*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(385*a^3*
(1 - 1/(a*x))^(9/2)*x^2) + (128*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(231
*a^2*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2)*x*(c -
a*c*x)^(9/2))/(11*a^4*(1 - 1/(a*x))^(9/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^{13/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(16\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(64\left(\frac{1}{x}\right)^{9/2}(c-\right.}{}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x}{11a^4\left(1-\frac{1}{ax}\right.}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-ac}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3} - \frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-a}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] time = 0.0551238, size = 83, normalized size = 0.33

$$\frac{2c^4\sqrt{\frac{1}{ax}+1}(ax+1)\left(315a^4x^4-1820a^3x^3+4530a^2x^2-6396ax+5419\right)\sqrt{c-acx}}{3465a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(5419 - 6396*a*x + 4530*
a^2*x^2 - 1820*a^3*x^3 + 315*a^4*x^4))/(3465*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.049, size = 72, normalized size = 0.3

$$\frac{(2\,ax+2)\left(315\,x^4a^4-1820\,x^3a^3+4530\,a^2x^2-6396\,ax+5419\right)}{3465\,\left(ax-1\right)^4a}(-acx+c)^{\frac{9}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/3465*(a*x+1)*(315*a^4*x^4-1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)*(-a*c*
x+c)^(9/2)/a/(a*x-1)^4/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.07954, size = 134, normalized size = 0.53

$$\frac{2\left(315\,a^5\sqrt{-c}c^4x^5-1505\,a^4\sqrt{-c}c^4x^4+2710\,a^3\sqrt{-c}c^4x^3-1866\,a^2\sqrt{-c}c^4x^2-977\,a\sqrt{-c}c^4x+5419\,\sqrt{-c}c^4\right)\sqrt{ax+1}}{3465\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*a^5*sqrt(-c)*c^4*x^5 - 1505*a^4*sqrt(-c)*c^4*x^4 + 2710*a^3*sqr
t(-c)*c^4*x^3 - 1866*a^2*sqrt(-c)*c^4*x^2 - 977*a*sqrt(-c)*c^4*x + 5419*sqr
t(-c)*c^4)*sqrt(a*x + 1)/a
```

Fricas [A] time = 1.54384, size = 246, normalized size = 0.97

$$\frac{2\left(315\,a^6c^4x^6-1190\,a^5c^4x^5+1205\,a^4c^4x^4+844\,a^3c^4x^3-2843\,a^2c^4x^2+4442\,ac^4x+5419\,c^4\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3465\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*a^6*c^4*x^6 - 1190*a^5*c^4*x^5 + 1205*a^4*c^4*x^4 + 844*a^3*c^4
*x^3 - 2843*a^2*c^4*x^2 + 4442*a*c^4*x + 5419*c^4)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27681, size = 198, normalized size = 0.78

$$\frac{2\left(\frac{4096\,\sqrt{2}\sqrt{-c}c^4}{\mathrm{sgn}(c)}+\frac{315\,(acx+c)^5\sqrt{-acx-c}-3080\,(acx+c)^4\sqrt{-acx-c}c+11880\,(acx+c)^3\sqrt{-acx-c}c^2-22176\,(acx+c)^2\sqrt{-acx-c}c^3-18480\,(-acx-c)^{\frac{3}{2}}c^4}{c\,\mathrm{sgn}(-acx-c)}\right)}{3465\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3465*(4096*sqrt(2)*sqrt(-c)*c^4/sgn(c) + (315*(a*c*x + c)^5*sqrt(-a*c*x -
 c) - 3080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*c
*x - c)*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c)
^(3/2)*c^4)/(c*sgn(-a*c*x - c)))/a
```

3.227 $\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=197

$$-\frac{568\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{315a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] (-8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(21*a*(1 - 1/(a*x))^(7/2)) - (56
8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(315*a^3*(1 - 1/(a*x))^(7/2)*x^2)
+ (48*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(35*a^2*(1 - 1/(a*x))^(7/2)*x)
 + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(7/2))/(9*a^3*(1 - 1
/(a*x))^(7/2))
```

Rubi [A] time = 0.189735, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{568\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{315a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```


```
[Out] (-8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(21*a*(1 - 1/(a*x))^(7/2)) - (56
8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(315*a^3*(1 - 1/(a*x))^(7/2)*x^2)
+ (48*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(35*a^2*(1 - 1/(a*x))^(7/2)*x)
 + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(7/2))/(9*a^3*(1 - 1
/(a*x))^(7/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3}\sqrt{1+\frac{x}{a}}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^{3}\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2}\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{3a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^{3}\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(8\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{S}\cdots}{21a\cdots}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^{2}\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)\cdots}{9a^{3}\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^{3}\left(1-\frac{1}{ax}\right)^{7/2}x^{2}} + \frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^{2}\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\cdots}{\cdots}$$

Mathematica [A] time = 0.0420794, size = 75, normalized size = 0.38

$$-\frac{2c^{3}\sqrt{\frac{1}{ax}+1}(ax+1)\left(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319\right)\sqrt{c-acx}}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(-319 + 321*a*x - 165*a
^2*x^2 + 35*a^3*x^3))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.048, size = 64, normalized size = 0.3

$$\frac{(2\,ax+2)\left(35\,x^3a^3-165\,a^2x^2+321\,ax-319\right)}{315\,\left(ax-1\right)^3a}\left(-acx+c\right)^{\frac{7}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)*(35*a^3*x^3-165*a^2*x^2+321*a*x-319)*(-a*c*x+c)^(7/2)/a/(a*x-
1)^3/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.10513, size = 112, normalized size = 0.57

$$-\frac{2\left(35\,a^4\sqrt{-c}c^3x^4-130\,a^3\sqrt{-c}c^3x^3+156\,a^2\sqrt{-c}c^3x^2+2\,a\sqrt{-c}c^3x-319\,\sqrt{-c}c^3\right)\sqrt{ax+1}}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 130*a^3*sqrt(-c)*c^3*x^3 + 156*a^2*sqrt(-
c)*c^3*x^2 + 2*a*sqrt(-c)*c^3*x - 319*sqrt(-c)*c^3)*sqrt(a*x + 1)/a
```

Fricas [A] time = 1.56228, size = 211, normalized size = 1.07

$$-\frac{2\left(35\,a^5c^3x^5-95\,a^4c^3x^4+26\,a^3c^3x^3+158\,a^2c^3x^2-317\,ac^3x-319\,c^3\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 95*a^4*c^3*x^4 + 26*a^3*c^3*x^3 + 158*a^2*c^3*x^2
- 317*a*c^3*x - 319*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
```

```
- a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25713, size = 167, normalized size = 0.85

$$\frac{2\left(\frac{256\sqrt{2}\sqrt{-c}c^3}{\text{sgn}(c)}-\frac{35\,(acx+c)^4\sqrt{-acx-c}-270\,(acx+c)^3\sqrt{-acx-c}c+756\,(acx+c)^2\sqrt{-acx-c}c^2+840\,(-acx-c)^{\frac{3}{2}}c^3}{c\text{sgn}(-acx-c)}\right)}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/315*(256*sqrt(2)*sqrt(-c)*c^3/sgn(c) - (35*(a*c*x + c)^4*sqrt(-a*c*x - c)
 - 270*(a*c*x + c)^3*sqrt(-a*c*x - c)*c + 756*(a*c*x + c)^2*sqrt(-a*c*x - c
)*c^2 + 840*(-a*c*x - c)^(3/2)*c^3)/(c*sgn(-a*c*x - c)))/a
```

3.228 $\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=115

$$\frac{64a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105(c-acx)^{3/2}}+\frac{16a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-acx}}+\frac{2}{7}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-acx}$$

```
[Out] (64*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(105*(c - a*c*x)^(3/2)) + (16*a^2*
c^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(35*Sqrt[c - a*c*x]) + (2*a^2*c^2*(1 - 1/(
a^2*x^2))^(3/2)*x^3*Sqrt[c - a*c*x])/7
```

Rubi [A] time = 0.171192, antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{142\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{105a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{36\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (-36*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (1
42*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(105*a^2*(1 - 1/(a*x))^(5/2)*x) +
 (2*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) && !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a}+\frac{7x}{2a^2}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(71\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int \cdots\right.}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{142\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{105a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] time = 0.0369356, size = 67, normalized size = 0.58

$$\frac{2c^2\sqrt{\frac{1}{ax}+1}(ax+1)\left(15a^2x^2-54ax+71\right)\sqrt{c-acx}}{105a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(71 - 54*a*x + 15*a^2*x^
2))/(105*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.043, size = 56, normalized size = 0.5

$$\frac{(2ax+2)\left(15a^2x^2-54ax+71\right)}{105\,(ax-1)^2\,a}(-acx+c)^{\frac{5}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^2*x^2-54*a*x+71)*(-a*c*x+c)^(5/2)/a/(a*x-1)^2/((a*x-1)/
(a*x+1))^(1/2)
```

Maxima [A] time = 1.10341, size = 90, normalized size = 0.78

$$\frac{2\left(15\,a^3\sqrt{-c}c^2x^3-39\,a^2\sqrt{-c}c^2x^2+17\,a\sqrt{-c}c^2x+71\,\sqrt{-c}c^2\right)\sqrt{ax+1}}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*c^2*x^3 - 39*a^2*sqrt(-c)*c^2*x^2 + 17*a*sqrt(-c)*c^
2*x + 71*sqrt(-c)*c^2)*sqrt(a*x + 1)/a
```

Fricas [A] time = 1.60649, size = 182, normalized size = 1.58

$$\frac{2\left(15\,a^4c^2x^4-24\,a^3c^2x^3-22\,a^2c^2x^2+88\,ac^2x+71\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^4*c^2*x^4 - 24*a^3*c^2*x^3 - 22*a^2*c^2*x^2 + 88*a*c^2*x + 71*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18411, size = 134, normalized size = 1.17

$$\frac{2\left(\frac{64\sqrt{2}\sqrt{-c}c^2}{\operatorname{sgn}(c)}+\frac{15(acx+c)^3\sqrt{-acx-c}-84(acx+c)^2\sqrt{-acx-c}c-140(-acx-c)^{\frac{3}{2}}c^2}{c\operatorname{sgn}(-acx-c)}\right)}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(64*sqrt(2)*sqrt(-c)*c^2/sgn(c) + (15*(a*c*x + c)^3*sqrt(-a*c*x - c)
- 84*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)/(c*sgn(
-a*c*x - c)))/a
```

3.229 $\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=77

$$\frac{2a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-acx}}+\frac{8a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15(c-acx)^{3/2}}$$

```
[Out] (8*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(15*(c - a*c*x)^(3/2)) + (2*a^2*c^2
*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(5*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.15801, antiderivative size = 89, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-14*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (2
*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= \frac{(c-acx)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\left(7\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}\\
&= -\frac{14\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0297813, size = 57, normalized size = 0.74

$$
-\frac{2c\sqrt{\frac{1}{ax}+1}(ax+1)(3ax-7)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt[
1 - 1/(a*x)])
```

Maple [A] time = 0.039, size = 48, normalized size = 0.6

$$\frac{(2\,ax+2)\,(3\,ax-7)}{15\,(ax-1)\,a}(-acx+c)^{\frac{3}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a*x-7)*(-a*c*x+c)^(3/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.11792, size = 61, normalized size = 0.79

$$-\frac{2\left(3\,a^{2}\sqrt{-c}cx^{2}-4\,a\sqrt{-c}cx-7\,\sqrt{-c}c\right)\sqrt{ax+1}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^2*sqrt(-c)*c*x^2 - 4*a*sqrt(-c)*c*x - 7*sqrt(-c)*c)*sqrt(a*x + 1
)/a
```

Fricas [A] time = 1.6449, size = 142, normalized size = 1.84

$$-\frac{2\left(3\,a^{3}cx^{3}-a^{2}cx^{2}-11\,acx-7\,c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - a^2*c*x^2 - 11*a*c*x - 7*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13195, size = 100, normalized size = 1.3

$$\frac{2\left(\frac{8\sqrt{2}\sqrt{-c}c}{\mathrm{sgn}(c)} - \frac{3\,(acx+c)^2\sqrt{-acx-c}+10\,(-acx-c)^{\frac{3}{2}}c}{c\mathrm{sgn}(-acx-c)}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(8*sqrt(2)*sqrt(-c)*c/sgn(c) - (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 10*
(-a*c*x - c)^(3/2)*c)/(c*sgn(-a*c*x - c)))/a
```

3.230 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=29

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rubi [A] time = 0.0341808, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6174}

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A] time = 0.0198929, size = 43, normalized size = 1.48

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.04, size = 35, normalized size = 1.2

$$\frac{2\,ax+2}{3\,a}\sqrt{-acx+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Maxima [A] time = 1.06179, size = 35, normalized size = 1.21

$$\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Fricas [A] time = 1.51021, size = 111, normalized size = 3.83

$$\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] time = 1.16605, size = 66, normalized size = 2.28

$$\frac{2\left(\frac{2\sqrt{2}\sqrt{-c}c}{\operatorname{sgn}(c)}-\frac{(-acx-c)^{\frac{3}{2}}}{\operatorname{sgn}(-acx-c)}\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*sqrt(2)*sqrt(-c)*c/sgn(c) - (-a*c*x - c)^(3/2)/sgn(-a*c*x - c))/(a*c
)
```

3.231 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=118

$$\frac{2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] (2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - a*c*x] - (2*Sqrt[2]*Sqrt
[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.166492, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - a*c*x] - (2*Sqrt[2]*Sqrt
[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Mathematica [A] time = 0.0616523, size = 99, normalized size = 0.84

$$\frac{2x\sqrt{1-\frac{1}{ax}}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}-\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{a}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*x*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[c
 - a*c*x])
```

Maple [A] time = 0.154, size = 83, normalized size = 0.7

$$2\,{\frac {\sqrt {-c \left(ax-1 \right) }}{\sqrt {-c \left(ax+1 \right) }ca} \left(\sqrt {c}\sqrt {2}\arctan \left(1/2\,{\frac {\sqrt {-c \left(ax+1 \right) }\sqrt {2}}{\sqrt {c}}} \right) -\sqrt {-c \left(ax+1 \right) } \right) {\frac {1}{\sqrt {{\frac {ax-1}{ax+1}}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*2^(1/2)*arctan(1/2*(-
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/c/
a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-acx + c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.57809, size = 568, normalized size = 4.81

$$\left[\frac{\sqrt{2}(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}, -\frac{2\left(\sqrt{-acx+c}(ax+1)\sqrt{\frac{a}{a}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a
*x + 1)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c), -2*(sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)
*(a*c*x - c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*
x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.20736, size = 123, normalized size = 1.04

$$\frac{2\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-\sqrt{-acx-c}}{\operatorname{sgn}(-acx-c)}-\frac{-i\sqrt{2}\sqrt{-c}\arctan(-i)+\sqrt{2}\sqrt{-c}}{\operatorname{sgn}(c)}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*((sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - sqrt(-a*
c*x - c))/sgn(-a*c*x - c) - (-I*sqrt(2)*sqrt(-c)*arctan(-I) + sqrt(2)*sqrt(
-c))/sgn(c))/(a*c)
```

3.232 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -((a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - a*c*x)^(3/
2))) - (Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]
*Sqrt[1 + 1/(a*x)])])/(Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.184957, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```


```
[Out] -((a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - a*c*x)^(3/
2))) - (Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]
*Sqrt[1 + 1/(a*x)])])/(Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A] time = 0.107615, size = 116, normalized size = 0.91

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}+\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{2\sqrt{a}c(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(-
1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*S
qrt[a]*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] time = 0.141, size = 118, normalized size = 0.9

$$-\frac{1}{(2ax-2)a}\sqrt{-c(ax-1)}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c(ax+1)}\frac{1}{\sqrt{c}}\right)xac-\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c(ax+1)}\frac{1}{\sqrt{c}}\right)c+2\sqrt{-c(ax+1)}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/c
^(5/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.6742, size = 664, normalized size = 5.19

$$\left[-\frac{\sqrt{2}\left(a^2x^2-2ax+1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+4\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{4\left(a^3c^2x^2-2a^2c^2x+ac^2\right)}, -\frac{\sqrt{2}\left(a^2x^2-2ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
```

```
c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/2*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16659, size = 96, normalized size = 0.75

$$-\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{-acx-c}}{acx-c}}{2\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 2*sqrt
(-a*c*x - c)/(a*c*x - c))/(a*c*sgn(-a*c*x - c))
```

3.233 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(8*(a - x^(-1))*(c - a*c*x)
^(5/2)) - (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(4*(a - x^(-1))
^2*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[
x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(8*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x
)^(5/2))
```

Rubi [A] time = 0.201913, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```


```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(8*(a - x^(-1))*(c - a*c*x)
^(5/2)) - (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(4*(a - x^(-1))
^2*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[
x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(8*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x
)^(5/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x\right.}{16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right.}{8\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] time = 0.149885, size = 123, normalized size = 0.64

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\sqrt{a}\sqrt{\frac{1}{ax}+1}(ax+3)\right)}{16\sqrt{a}c^2(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(3 + a*x) + Sqrt[2]*Sqrt
[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
```

```
*x)])]))/(16*Sqrt[a]*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] time = 0.141, size = 165, normalized size = 0.9

$$-\frac{1}{16\,(ax-1)^2\,a}\sqrt{-c\,(ax-1)}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c\,(ax+1)}\frac{1}{\sqrt{c}}\right)x^2a^2c-2\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xac-2\,xa\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)
*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-2*2^(1/2)*arctan(
1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(-c*(a*x+1))^(1/2)*c^(1
/2)+2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-6*(-c*(a*x+1))
^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.69129, size = 775, normalized size = 4.02

$$\left[-\frac{\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\left(a^2x^2+4\,ax+3\right)\sqrt{-acx+c}\sqrt{\frac{ax-}{ax+}}}{32\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan
(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) -
2*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^
3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19808, size = 120, normalized size = 0.62

$$-\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left((-acx-c)^{\frac{3}{2}}-2\sqrt{-acx-c}c\right)}{(acx-c)^{2}c}}{16\,ac\text{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 2*((-
a*c*x - c)^(3/2) - 2*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c))/(a*c*sgn(-a*c*x
 - c))
```

3.234 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=250

$$\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -(a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^2)/(6*(a - x^(-1))^3*(c - a
*c*x)^(7/2)) - (a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(32*(a - x^(
-1))*(c - a*c*x)^(7/2)) + (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^3)
/(16*(a - x^(-1))^2*(c - a*c*x)^(7/2)) - (a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(32*Sqrt[2]*(x^(-1)
)^(7/2)*(c - a*c*x)^(7/2))
```

Rubi [A] time = 0.22085, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] -(a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^2)/(6*(a - x^(-1))^3*(c - a
*c*x)^(7/2)) - (a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(32*(a - x^(
-1))*(c - a*c*x)^(7/2)) + (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^3)
/(16*(a - x^(-1))^2*(c - a*c*x)^(7/2)) - (a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(32*Sqrt[2]*(x^(-1)
)^(7/2)*(c - a*c*x)^(7/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^4}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,d\right.}{32\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\right.\right.}{}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\right.\right.}{}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{a^{5/2}\left(1-\right.}{32\surd}$$

Mathematica [A] time = 0.15543, size = 139, normalized size = 0.56

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(-3a^2x^2+10ax+25\right)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(ax-1)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{192\sqrt{a}c^3\sqrt{\frac{1}{x}}(ax-1)^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(25 + 10*a*x - 3*a^2*x^2))
/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])]))/(192*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 + a*x)^3*Sqrt[c
 - a*c*x])
```

Maple [A] time = 0.141, size = 219, normalized size = 0.9

$$-\frac{1}{192\,(ax-1)^3\,a}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^3a^3c-9\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2c-6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/192/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1
/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-9*2^(1/2)*arct
an(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*x^2*a^2*(-c*(a*x+1))
^(1/2)*c^(1/2)+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a
*c+20*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2
)*2^(1/2)/c^(1/2))*c+50*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.63357, size = 902, normalized size = 3.61

$$\left[-\frac{3\,\sqrt{2}\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)-4\left(3\,a^{3}x^{3}-7\,a^{2}x^{2}-35\,a\right.}{384\left(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-4\,a^{2}c^{4}x+ac^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 - 7*a^
2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x
^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/192*(3*sqrt(2
)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt
(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3
- 7*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5
*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21493, size = 157, normalized size = 0.63

$$-\frac{\frac{3\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2\left(3\,(acx+c)^{2}\sqrt{-acx-c}+16\,(-acx-c)^{\frac{3}{2}}c-12\,\sqrt{-acx-c}c^{2}\right)}{(acx-c)^{3}c^{2}}}{192\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*
x - c)*c^2)/((a*c*x - c)^3*c^2))/(a*c*sgn(-a*c*x - c))
```

3.235 $\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rubi [A] time = 0.0876759, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx\\
&= -\int \frac{(1+ax)(c-acx)^{7/2}}{1-ax}\,dx\\
&= -\left(c\int (1+ax)(c-acx)^{5/2}\,dx\right)\\
&= -\left(c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)dx\right)\\
&= \frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}
\end{aligned}$$

Mathematica [A] time = 0.0405777, size = 34, normalized size = 0.85

$$-\frac{2c^3(ax-1)^3(7ax+11)\sqrt{c-acx}}{63a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

Maple [A] time = 0.041, size = 21, normalized size = 0.5

$$\frac{14\,ax+22}{63\,a}\,(-acx+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

Maxima [A] time = 1.01005, size = 43, normalized size = 1.08

$$-\frac{2\left(7\,(-acx+c)^{\frac{9}{2}}-18\,(-acx+c)^{\frac{7}{2}}c\right)}{63\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

Fricas [A] time = 1.49894, size = 132, normalized size = 3.3

$$-\frac{2\left(7\,a^4c^3x^4-10\,a^3c^3x^3-12\,a^2c^3x^2+26\,ac^3x-11\,c^3\right)\sqrt{-acx+c}}{63\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c^
3)*sqrt(-a*c*x + c)/a
```

Sympy [A] time = 26.0202, size = 172, normalized size = 4.3

$$\begin{cases} -\frac{c^3\left(\begin{cases} 0 & \text{for}\ c=0 \\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right)+\frac{2c^2(-acx+c)^{\frac{3}{2}}}{3}-2c\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{2\left(-\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{3c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{3c(-acx+c)^{\frac{7}{2}}}{7}+\cdots\right.}{c}}{a} \\ -c^{\frac{7}{2}}x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Piecewise((-(c**3*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), T
rue)) + 2*c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-c*(-a*c*x + c)**(3/2)/3 + (-a*
```

```
c*x + c)**(5/2)/5) - 4*c*(-a*c*x + c)**(5/2)/5 + 2*(-a*c*x + c)**(7/2)/7 +
2*(-c**3*(-a*c*x + c)**(3/2)/3 + 3*c**2*(-a*c*x + c)**(5/2)/5 - 3*c*(-a*c*x
 + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/c)/a, Ne(a, 0)), (-c**(7/2)*x, True
))
```

Giac [B] time = 1.155, size = 234, normalized size = 5.85

$$\frac{2\left(45\,(acx-c)^3\sqrt{-acx+c}+126\,(acx-c)^2\sqrt{-acx+c}c+21\left(3\,(acx-c)^2\sqrt{-acx+c}-5\,(-acx+c)^{\frac{3}{2}}c\right)c-\frac{35\,(acx-c)^4\sqrt{-acx+c}}{}\right.}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/315*(45*(a*c*x - c)^3*sqrt(-a*c*x + c) + 126*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c + 21*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 5*(-a*c*x + c)^(3/2)*c)*c - (
35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 135*(a*c*x - c)^3*sqrt(-a*c*x + c)*c +
189*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 105*(-a*c*x + c)^(3/2)*c^3)/c)/a
```

3.236 $\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rubi [A] time = 0.0876669, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx\\ &= -\int \frac{(1+ax)(c-acx)^{5/2}}{1-ax}\,dx\\ &= -\left(c\int (1+ax)(c-acx)^{3/2}\,dx\right)\\ &= -\left(c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)dx\right)\\ &= \frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}\end{aligned}$$

Mathematica [A] time = 0.039523, size = 34, normalized size = 0.85

$$\frac{2c^2(ax-1)^2(5ax+9)\sqrt{c-acx}}{35a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

Maple [A] time = 0.04, size = 21, normalized size = 0.5

$$\frac{10\,ax+18}{35\,a}\left(-acx+c\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

Maxima [A] time = 1.00252, size = 43, normalized size = 1.08

$$-\frac{2\left(5\left(-acx+c\right)^{\frac{7}{2}}-14\left(-acx+c\right)^{\frac{5}{2}}c\right)}{35\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

Fricas [A] time = 1.60886, size = 103, normalized size = 2.58

$$\frac{2\left(5\,a^3c^2x^3-a^2c^2x^2-13\,ac^2x+9\,c^2\right)\sqrt{-acx+c}}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

Sympy [A] time = 12.5798, size = 80, normalized size = 2.

$$\begin{cases}\frac{-c^2\left(\begin{cases}0 & \text{for}\ c=0\\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)-\frac{2\left(\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{c}}{a} & \text{for}\ a\neq 0\\ -c^{\frac{5}{2}}x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Piecewise((((-c**2*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), T
rue)) - 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x
```

```
+ c)**(7/2)/7)/c)/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

Giac [B] time = 1.12242, size = 108, normalized size = 2.7

$$\frac{2\left(35\,(-acx+c)^{\frac{3}{2}}c+\frac{15\,(acx-c)^{3}\sqrt{-acx+c}+42\,(acx-c)^{2}\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^{2}}{c}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*(-a*c*x + c)^(3/2)*c + (15*(a*c*x - c)^3*sqrt(-a*c*x + c) + 42*(a
*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2)/c)/a
```

3.237 $\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rubi [A] time = 0.0891647, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx\\ &= -\int \frac{(1+ax)(c-acx)^{3/2}}{1-ax}\,dx\\ &= -\left(c\int (1+ax)\sqrt{c-acx}\,dx\right)\\ &= -\left(c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx\right)\\ &= \frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}\end{aligned}$$

Mathematica [A] time = 0.0301593, size = 30, normalized size = 0.75

$$-\frac{2c(ax-1)(3ax+7)\sqrt{c-acx}}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

Maple [A] time = 0.04, size = 21, normalized size = 0.5

$$\frac{6\,ax+14}{15\,a}\,(-acx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

Maxima [A] time = 1.01969, size = 43, normalized size = 1.08

$$-\frac{2\left(3\left(-acx+c\right)^{\frac{5}{2}}-10\left(-acx+c\right)^{\frac{3}{2}}c\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

Fricas [A] time = 1.50783, size = 76, normalized size = 1.9

$$-\frac{2\left(3\,a^{2}cx^{2}+4\,acx-7\,c\right)\sqrt{-acx+c}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

Sympy [A] time = 10.8728, size = 61, normalized size = 1.52

$$\begin{cases} -\frac{c\left(\begin{cases} 0 & \text{for}\: c=0 \\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right)+\frac{2\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{c}}{a} & \text{for}\: a\neq 0 \\ -c^{\frac{3}{2}}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Piecewise((-(c*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), True
)) + 2*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/c)/a, Ne(a, 0)),
```

```
(-c**(3/2)*x, True))
```

Giac [A] time = 1.10256, size = 76, normalized size = 1.9

$$\frac{2\left(5\left(-acx+c\right)^{\frac{3}{2}}-\frac{3\left(acx-c\right)^{2}\sqrt{-acx+c}-5\left(-acx+c\right)^{\frac{3}{2}}c}{c}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*(-a*c*x + c)^(3/2) - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 5*(-a*c*x
+ c)^(3/2)*c)/c)/a
```

3.238 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rubi [A] time = 0.0781572, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\ &= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] time = 0.0248467, size = 23, normalized size = 0.61

$$\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A] time = 0.046, size = 20, normalized size = 0.5

$$\frac{2\,ax+10}{3\,a}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Maxima [A] time = 0.994603, size = 41, normalized size = 1.08

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Fricas [A] time = 1.58634, size = 46, normalized size = 1.21

$$\frac{2\sqrt{-acx+c}(ax+5)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] time = 4.98112, size = 31, normalized size = 0.82

$$-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.1624, size = 41, normalized size = 1.08

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

3.239 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\sqrt{c-acx}}{ac}-\frac{4}{a\sqrt{c-acx}}$$

```
[Out] -4/(a*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x])/(a*c)
```

Rubi [A] time = 0.0839535, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$-\frac{2\sqrt{c-acx}}{ac}-\frac{4}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] -4/(a*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x])/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx\\ &= -\int \frac{1+ax}{(1-ax)\sqrt{c-acx}}\,dx\\ &= -\left(c\int \frac{1+ax}{(c-acx)^{3/2}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx\right)\\ &= -\frac{4}{a\sqrt{c-acx}}-\frac{2\sqrt{c-acx}}{ac}\end{aligned}$$

Mathematica [A] time = 0.0251678, size = 21, normalized size = 0.58

$$\frac{2ax-6}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]

[Out] (-6 + 2*a*x)/(a*Sqrt[c - a*c*x])
```

Maple [A] time = 0.041, size = 20, normalized size = 0.6

$$2\,\frac{ax-3}{a\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(a*x-3)/a/(-a*c*x+c)^(1/2)
```

Maxima [A] time = 1.00103, size = 41, normalized size = 1.14

$$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

Fricas [A] time = 1.5983, size = 63, normalized size = 1.75

$$-\frac{2\sqrt{-acx+c}(ax-3)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

Sympy [A] time = 14.7232, size = 51, normalized size = 1.42

$$\begin{cases} -\frac{\frac{2}{\sqrt{-acx+c}}-\frac{2\left(-\frac{c}{\sqrt{-acx+c}}-\sqrt{-acx+c}\right)}{c}}{a} & \text{for}\ a \neq 0 \\ -\frac{x}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise((-(2/sqrt(-a*c*x + c) - 2*(-c/sqrt(-a*c*x + c) - sqrt(-a*c*x + c)
)/c)/a, Ne(a, 0)), (-x/sqrt(c), True))
```

Giac [A] time = 1.16585, size = 43, normalized size = 1.19

$$-\frac{4}{\sqrt{-acx+c}a}-\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -4/(sqrt(-a*c*x + c)*a) - 2*sqrt(-a*c*x + c)/(a*c)
```

3.240 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2}{ac\sqrt{c-acx}}-\frac{4}{3a(c-acx)^{3/2}}$$

```
[Out] -4/(3*a*(c - a*c*x)^(3/2)) + 2/(a*c*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0859509, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{2}{ac\sqrt{c-acx}}-\frac{4}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] -4/(3*a*(c - a*c*x)^(3/2)) + 2/(a*c*Sqrt[c - a*c*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx\\ &= -\int \frac{1+ax}{(1-ax)(c-acx)^{3/2}}\,dx\\ &= -\left(c\int \frac{1+ax}{(c-acx)^{5/2}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx\right)\\ &= -\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}\end{aligned}$$

Mathematica [A] time = 0.0448822, size = 34, normalized size = 0.89

$$-\frac{2(3ax-1)\sqrt{c-acx}}{3ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*(-1 + 3*a*x)*Sqrt[c - a*c*x])/(3*a*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.042, size = 21, normalized size = 0.6

$$-\frac{6\,ax-2}{3\,a}\left(-acx+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -2/3*(3*a*x-1)/a/(-a*c*x+c)^(3/2)
```

Maxima [A] time = 1.02367, size = 35, normalized size = 0.92

$$-\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

Fricas [A] time = 1.56212, size = 96, normalized size = 2.53

$$-\frac{2\,\sqrt{-acx+c}(3\,ax-1)}{3\,\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 24.2837, size = 29, normalized size = 0.76

$$-\frac{4}{3a\,(-acx+c)^{\frac{3}{2}}}+\frac{2}{ac\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] -4/(3*a*(-a*c*x + c)**(3/2)) + 2/(a*c*sqrt(-a*c*x + c))
```

Giac [A] time = 1.1544, size = 49, normalized size = 1.29

$$\frac{2\,(3\,acx - c)}{3\,(acx - c)\sqrt{-acx + c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

3.241 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3ac(c-acx)^{3/2}}-\frac{4}{5a(c-acx)^{5/2}}$$

```
[Out] -4/(5*a*(c - a*c*x)^(5/2)) + 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0886908, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{2}{3ac(c-acx)^{3/2}}-\frac{4}{5a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -4/(5*a*(c - a*c*x)^(5/2)) + 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx \\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{5/2}}\,dx \\
&= -\left(c\int \frac{1+ax}{(c-acx)^{7/2}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)dx\right) \\
&= -\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0585227, size = 34, normalized size = 0.85

$$\frac{2(5ax+1)\sqrt{c-acx}}{15ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (2*(1 + 5*a*x)*Sqrt[c - a*c*x])/(15*a*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.045, size = 21, normalized size = 0.5

$$-\frac{10\,ax+2}{15\,a}(-acx+c)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -2/15*(5*a*x+1)/a/(-a*c*x+c)^(5/2)
```

Maxima [A] time = 1.0351, size = 32, normalized size = 0.8

$$-\frac{2 (5 acx + c)}{15 (-acx + c)^{\frac{5}{2}} ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

Fricas [A] time = 1.48353, size = 117, normalized size = 2.92

$$\frac{2 \sqrt{-acx + c}(5 ax + 1)}{15 \left(a^4c^3x^3 - 3 a^3c^3x^2 + 3 a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)
```

Sympy [A] time = 25.0109, size = 31, normalized size = 0.78

$$-\frac{4}{5a (-acx + c)^{\frac{5}{2}}} + \frac{2}{3ac (-acx + c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] -4/(5*a*(-a*c*x + c)**(5/2)) + 2/(3*a*c*(-a*c*x + c)**(3/2))
```

Giac [A] time = 1.12408, size = 46, normalized size = 1.15

$$-\frac{2(5acx+c)}{15(acx-c)^2\sqrt{-acx+c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

3.242 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{5ac(c-acx)^{5/2}}-\frac{4}{7a(c-acx)^{7/2}}$$

```
[Out] -4/(7*a*(c - a*c*x)^(7/2)) + 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.0863602, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{2}{5ac(c-acx)^{5/2}}-\frac{4}{7a(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -4/(7*a*(c - a*c*x)^(7/2)) + 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx \\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{7/2}}\,dx \\
&= -\left(c\int \frac{1+ax}{(c-acx)^{9/2}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx\right) \\
&= -\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0574687, size = 34, normalized size = 0.85

$$-\frac{2(7ax+3)\sqrt{c-acx}}{35ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*(3 + 7*a*x)*Sqrt[c - a*c*x])/(35*a*c^4*(-1 + a*x)^4)
```

Maple [A] time = 0.039, size = 21, normalized size = 0.5

$$-\frac{14\,ax+6}{35\,a}\left(-acx+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -2/35*(7*a*x+3)/a/(-a*c*x+c)^(7/2)
```

Maxima [A] time = 0.999995, size = 35, normalized size = 0.88

$$-\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

Fricas [B] time = 1.47522, size = 140, normalized size = 3.5

$$-\frac{2\,\sqrt{-acx+c}(7\,ax+3)}{35\,\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4
*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 41.2503, size = 31, normalized size = 0.78

$$-\frac{4}{7a\,(-acx+c)^{\frac{7}{2}}}+\frac{2}{5ac\,(-acx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] -4/(7*a*(-a*c*x + c)**(7/2)) + 2/(5*a*c*(-a*c*x + c)**(5/2))
```

Giac [A] time = 1.11295, size = 49, normalized size = 1.22

$$\frac{2(7acx+3c)}{35(acx-c)^3\sqrt{-acx+cac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

3.243 $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=197

$$-\frac{856\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{1155a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{21a^2x\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] (-8*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(33*a*(1 - 1/(a*x))^(9/2)) - (85
6*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(1155*a^3*(1 - 1/(a*x))^(9/2)*x^2)
 + (16*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(21*a^2*(1 - 1/(a*x))^(9/2)*x
) + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(9/2))/(11*a^3*(1 -
 1/(a*x))^(9/2))
```

Rubi [A] time = 0.192679, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{856\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{1155a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{21a^2x\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```


```
[Out] (-8*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(33*a*(1 - 1/(a*x))^(9/2)) - (85
6*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(1155*a^3*(1 - 1/(a*x))^(9/2)*x^2)
 + (16*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(21*a^2*(1 - 1/(a*x))^(9/2)*x
) + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(9/2))/(11*a^3*(1 -
 1/(a*x))^(9/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{(c-acx)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\left(1+\frac{x}{a}\right)^{3/2}}{x^{13/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(12\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}dx,x\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(8\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)}{3}$$

$$=-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2}{}$$

Mathematica [A] time = 0.0457692, size = 77, normalized size = 0.39

$$\frac{2c^4\sqrt{\frac{1}{ax}+1}(ax+1)^2\left(105a^3x^3-455a^2x^2+755ax-533\right)\sqrt{c-acx}}{1155a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455*
a^2*x^2 + 105*a^3*x^3))/(1155*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.118, size = 64, normalized size = 0.3

$$\frac{(2\,ax+2)\left(105\,x^3a^3-455\,a^2x^2+755\,ax-533\right)}{1155\,(ax-1)^3\,a}(-acx+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/1155*(a*x+1)*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(a*
x-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.0703, size = 143, normalized size = 0.73

$$\frac{2\left(105\,a^5\sqrt{-c}c^4x^5-455\,a^4\sqrt{-c}c^4x^4+650\,a^3\sqrt{-c}c^4x^3-78\,a^2\sqrt{-c}c^4x^2-755\,a\sqrt{-c}c^4x+533\,\sqrt{-c}c^4\right)(ax+1)^{\frac{3}{2}}}{1155\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*a^5*sqrt(-c)*c^4*x^5 - 455*a^4*sqrt(-c)*c^4*x^4 + 650*a^3*sqrt(
-c)*c^4*x^3 - 78*a^2*sqrt(-c)*c^4*x^2 - 755*a*sqrt(-c)*c^4*x + 533*sqrt(-c)
*c^4)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Fricas [A] time = 1.6092, size = 239, normalized size = 1.21

$$\frac{2\left(105\,a^6c^4x^6-140\,a^5c^4x^5-295\,a^4c^4x^4+472\,a^3c^4x^3+211\,a^2c^4x^2-844\,ac^4x-533\,c^4\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{1155\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^6*c^4*x^6 - 140*a^5*c^4*x^5 - 295*a^4*c^4*x^4 + 472*a^3*c^4*x
^3 + 211*a^2*c^4*x^2 - 844*a*c^4*x - 533*c^4)*sqrt(-a*c*x + c)*sqrt((a*x -
```

```
1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29634, size = 317, normalized size = 1.61

$$-\frac{2\left(\frac{1536\sqrt{2}\sqrt{-c}c^4}{\mathrm{sgn}(c)}-\frac{315\,(acx+c)^5\sqrt{-acx-c}-3080\,(acx+c)^4\sqrt{-acx-c}c+11880\,(acx+c)^3\sqrt{-acx-c}c^2-22176\,(acx+c)^2\sqrt{-acx-c}c^3-18480\,(-acx-c)^{\frac{3}{2}}c^4+22\left(35\,(a\right.}{c\mathrm{sgn}(-acx-c)}}{3465\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -2/3465*(1536*sqrt(2)*sqrt(-c)*c^4/sgn(c) - (315*(a*c*x + c)^5*sqrt(-a*c*x
- c) - 3080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*
c*x - c)*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c
)^(3/2)*c^4 + 22*(35*(a*c*x + c)^4*sqrt(-a*c*x - c) - 270*(a*c*x + c)^3*sqr
t(-a*c*x - c)*c + 756*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^2 + 840*(-a*c*x - c)
^(3/2)*c^3)*c)/(c*sgn(-a*c*x - c)))/a
```

3.244 $\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{214\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{315a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{44\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] (-44*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(63*a*(1 - 1/(a*x))^(7/2)) + (2
14*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(315*a^2*(1 - 1/(a*x))^(7/2)*x) +
 (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(7/2))/(9*(1 - 1/(a*x))^(7/2))
```

Rubi [A] time = 0.172731, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{214\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{315a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{44\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-44*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(63*a*(1 - 1/(a*x))^(7/2)) + (2
14*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(315*a^2*(1 - 1/(a*x))^(7/2)*x) +
 (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(7/2))/(9*(1 - 1/(a*x))^(7/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{(c-acx)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(2\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{11}{a}+\frac{9x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}dx,x,\frac{1}{x}\right)}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(107\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}}{63a^2\left(1-\frac{1}{ax}\right)^{7}}$$

$$=-\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] time = 0.0393406, size = 69, normalized size = 0.5

$$-\frac{2c^3\sqrt{\frac{1}{ax}+1}(ax+1)^2\left(35a^2x^2-110ax+107\right)\sqrt{c-acx}}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*a
^2*x^2))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.117, size = 56, normalized size = 0.4

$$\frac{(2\,ax+2)\left(35\,a^2x^2-110\,ax+107\right)}{315\,(ax-1)^2\,a}(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/((a*x-1
)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.06239, size = 122, normalized size = 0.89

$$-\frac{2\left(35\,a^{4}\sqrt{-c}c^{3}x^{4}-110\,a^{3}\sqrt{-c}c^{3}x^{3}+72\,a^{2}\sqrt{-c}c^{3}x^{2}+110\,a\sqrt{-c}c^{3}x-107\,\sqrt{-c}c^{3}\right)(ax+1)^{\frac{3}{2}}}{315\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 110*a^3*sqrt(-c)*c^3*x^3 + 72*a^2*sqrt(-c
)*c^3*x^2 + 110*a*sqrt(-c)*c^3*x - 107*sqrt(-c)*c^3)*(a*x + 1)^(3/2)/((a*x
- 1)*a)
```

Fricas [A] time = 1.55505, size = 209, normalized size = 1.53

$$-\frac{2\left(35\,a^{5}c^{3}x^{5}-5\,a^{4}c^{3}x^{4}-118\,a^{3}c^{3}x^{3}+26\,a^{2}c^{3}x^{2}+211\,ac^{3}x+107\,c^{3}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 118*a^3*c^3*x^3 + 26*a^2*c^3*x^2 +
 211*a*c^3*x + 107*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x -
 a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28112, size = 251, normalized size = 1.83

$$-\frac{2\left(\frac{128\sqrt{2}\sqrt{-c}c^{3}}{\operatorname{sgn}(c)}+\frac{35\,(acx+c)^{4}\sqrt{-acx-c}-270\,(acx+c)^{3}\sqrt{-acx-c}c+756\,(acx+c)^{2}\sqrt{-acx-c}c^{2}+840\,(-acx-c)^{\frac{3}{2}}c^{3}+6\left(15\,(acx+c)^{3}\sqrt{-acx-c}-84\,(acx+c)^{2}\sqrt{-acx-c}c\right.}{c\operatorname{sgn}(-acx-c)}}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/315*(128*sqrt(2)*sqrt(-c)*c^3/sgn(c) + (35*(a*c*x + c)^4*sqrt(-a*c*x - c
) - 270*(a*c*x + c)^3*sqrt(-a*c*x - c)*c + 756*(a*c*x + c)^2*sqrt(-a*c*x -
c)*c^2 + 840*(-a*c*x - c)^(3/2)*c^3 + 6*(15*(a*c*x + c)^3*sqrt(-a*c*x - c)
- 84*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)*c)/(c*s
gn(-a*c*x - c)))/a
```

3.245 $\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{18\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] (-18*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (2
*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rubi [A] time = 0.158009, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{18\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-18*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (2
*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= \frac{(c-acx)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}} \\
&= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(9\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}} \\
&= -\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0388098, size = 59, normalized size = 0.66

$$\frac{2\sqrt{\frac{1}{ax}+1}(5ax-9)\sqrt{c-acx}(acx+c)^2}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(-9 + 5*a*x)*Sqrt[c - a*c*x]*(c + a*c*x)^2)/(35*a*Sqrt
[1 - 1/(a*x)])
```

Maple [A] time = 0.117, size = 48, normalized size = 0.5

$$\frac{(2\,ax+2)\,(5\,ax-9)}{35\,(ax-1)\,a}(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/35*(a*x+1)*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.08365, size = 100, normalized size = 1.12

$$\frac{2\left(5\,a^3\sqrt{-c}c^2x^3-9\,a^2\sqrt{-c}c^2x^2-5\,a\sqrt{-c}c^2x+9\,\sqrt{-c}c^2\right)(ax+1)^{\frac{3}{2}}}{35\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^3*sqrt(-c)*c^2*x^3 - 9*a^2*sqrt(-c)*c^2*x^2 - 5*a*sqrt(-c)*c^2*x
+ 9*sqrt(-c)*c^2)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Fricas [A] time = 1.50827, size = 177, normalized size = 1.99

$$\frac{2\left(5\,a^4c^2x^4+6\,a^3c^2x^3-12\,a^2c^2x^2-22\,ac^2x-9\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 22*a*c^2*x - 9*c^2)*
sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18758, size = 188, normalized size = 2.11

$$-\frac{2\left(\frac{48\sqrt{2}\sqrt{-c}c^2}{\mathrm{sgn}(c)}-\frac{15\,(acx+c)^3\sqrt{-acx-c}-84\,(acx+c)^2\sqrt{-acx-c}c-140\,(-acx-c)^{\frac{3}{2}}c^2+14\left(3\,(acx+c)^2\sqrt{-acx-c}+10\,(-acx-c)^{\frac{3}{2}}c\right)c}{c\,\mathrm{sgn}(-acx-c)}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(48*sqrt(2)*sqrt(-c)*c^2/sgn(c) - (15*(a*c*x + c)^3*sqrt(-a*c*x - c)
 - 84*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2 + 14*(3
*(a*c*x + c)^2*sqrt(-a*c*x - c) + 10*(-a*c*x - c)^(3/2)*c)*c)/(c*sgn(-a*c*x
 - c)))/a
```

3.246 $\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

```
[Out] (2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

Rubi [A] time = 0.0365437, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {6174}

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

Mathematica [A] time = 0.0311314, size = 43, normalized size = 1.39

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Maple [A] time = 0.115, size = 35, normalized size = 1.1

$$\frac{2\,ax+2}{5\,a}\left(-acx+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

Maxima [A] time = 1.06897, size = 55, normalized size = 1.77

$$-\frac{2\left(a^{2}\sqrt{-c}cx^{2}-\sqrt{-c}c\right)(ax+1)^{\frac{3}{2}}}{5\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^2*sqrt(-c)*c*x^2 - sqrt(-c)*c)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Fricas [A] time = 1.47355, size = 136, normalized size = 4.39

$$-\frac{2\left(a^{3}cx^{3}+3\,a^{2}cx^{2}+3\,acx+c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^3*c*x^3 + 3*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1924, size = 76, normalized size = 2.45

$$-\frac{2\left(\frac{4\sqrt{2}\sqrt{-c}c}{\operatorname{sgn}(c)}+\frac{(acx+c)^2\sqrt{-acx-c}}{c\operatorname{sgn}(-acx-c)}\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/5*(4*sqrt(2)*sqrt(-c)*c/sgn(c) + (a*c*x + c)^2*sqrt(-a*c*x - c)/(c*sgn(-
a*c*x - c)))/a
```

3.247 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=163

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.176982, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0689989, size = 105, normalized size = 0.64

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}(ax+7)-6\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.188, size = 107, normalized size = 0.7

$$-\frac{2\,ax-2}{(3\,ax+3)\,a}\sqrt{-c\,(ax-1)}\left(6\,\sqrt{c}\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-xa\sqrt{-c\,(ax+1)}-7\,\sqrt{-c\,(ax+1)}\right)\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\frac{}{\sqrt{-c\,(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(-c*(a*x+1))^(1/
2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.57295, size = 598, normalized size = 3.67

$$\left[\frac{2\left(3\,\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(a^2x^2+8\,ax+7\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\left(a^2x-a\right)},-\frac{2\left(6\,\sqrt{2}(ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.1703, size = 144, normalized size = 0.88

$$-\frac{12i\sqrt{2}\sqrt{-c}\arctan\left(-i\right)-16\sqrt{2}\sqrt{-c}}{3\,a\mathrm{sgn}\left(c\right)}-\frac{2\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)+\left(-acx-c\right)^{\frac{3}{2}}-6\sqrt{-acx-c}c\right)}{3\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(12*I*sqrt(2)*sqrt(-c)*arctan(-I) - 16*sqrt(2)*sqrt(-c))/(a*sgn(c)) -
2/3*(6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) + (-a*c
*x - c)^(3/2) - 6*sqrt(-a*c*x - c)*c)/(a*c*sgn(-a*c*x - c))
```

3.248 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=177

$$\frac{2ax\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] (-6*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - a*c*x]) + (
2*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x)/((a - x^(-1))*Sqrt[c - a*c*x])
 - (3*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqr
t[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.180234, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{2ax\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```


```
[Out] (-6*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - a*c*x]) + (
2*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x)/((a - x^(-1))*Sqrt[c - a*c*x])
 - (3*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqr
t[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Mathematica [A] time = 0.143338, size = 116, normalized size = 0.66

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}(ax-2)-3\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{a}(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 3*Sqrt[2]*Sq
rt[x^(-1)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(Sqrt[a]*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] time = 0.178, size = 135, normalized size = 0.8

$$\frac{1}{a\left(ax+1\right)}\sqrt{-c\left(ax-1\right)}\left(3\sqrt{2}\arctan\left(1/2\frac{\sqrt{-c\left(ax+1\right)}\sqrt{2}}{\sqrt{c}}\right)xac-2xa\sqrt{-c\left(ax+1\right)}\sqrt{c}-3\sqrt{2}\arctan\left(1/2\frac{\sqrt{-c\left(ax+1\right)}\sqrt{2}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(-c*(a*x-1))^(1/2)*(3*2^(1/2)*arctan(1/2*
(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-
3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+4*(-c*(a*x+1))^(
1/2)*c^(1/2))/c^(3/2)/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.63113, size = 682, normalized size = 3.85

$$\left[\frac{3\sqrt{2}\left(a^{2}cx^{2}-2\,acx+c\right)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^{2}x^{2}-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^{2}x^{2}-2\,ax+1}\right)-4\left(a^{2}x^{2}-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{2\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)},-\frac{2\left(a^{2}x^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt
(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x
 - 3)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c), -(2*(a^2*x^2 - a*x - 2
)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)) - 3*sqrt(2)*(a^2*c*x^2 - 2*a*c
*x + c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1
)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20042, size = 115, normalized size = 0.65

$$\frac{3\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-2\sqrt{-acx-c}+\frac{2\sqrt{-acx-c}c}{acx-c}}{ac\operatorname{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] (3*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 2*sqrt(-a
*c*x - c) + 2*sqrt(-a*c*x - c)*c/(a*c*x - c))/(a*c*sgn(-a*c*x - c))
```

3.249 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3ax\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] (-3*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))*(c - a*c*x)^
(3/2)) - (a^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x)/(2*(a - x^(-1))^2*
(c - a*c*x)^(3/2)) - (3*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)
^(3/2))
```

Rubi [A] time = 0.193826, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3ax\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```


```
[Out] (-3*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))*(c - a*c*x)^
(3/2)) - (a^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x)/(2*(a - x^(-1))^2*
(c - a*c*x)^(3/2)) - (3*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)
^(3/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\bigg(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x}{8\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\bigg(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A] time = 0.14897, size = 125, normalized size = 0.67

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}(5ax-1)+3\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{8\sqrt{a}c(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 + 5*a*x) + 3*Sqrt[2]*
Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 +
```

```
1/(a*x)])]))/(8*Sqrt[a]*c*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] time = 0.213, size = 174, normalized size = 0.9

$$-\frac{1}{(8\,ax-8)\,(ax+1)\,a}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2c-6\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+10*x*a*(-c*(a*x+1))^(1
/2)*c^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(-
c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.65276, size = 784, normalized size = 4.19

$$\left[-\frac{3\,\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\left(5\,a^2x^2+4\,ax-1\right)\sqrt{-acx+c}\sqrt{}}{16\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*
c^2*x - a*c^2), -1/8*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) + 2*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20051, size = 120, normalized size = 0.64

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\,(-acx-c)^{\frac{3}{2}}+6\,\sqrt{-acx-c}c\right)}{(acx-c)^{2}}}{8\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - 2*(5
*(-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/(a*c*x - c)^2)/(a*c*sgn(-a*c*x
- c))
```

3.250 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(16*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(24*(a - x^(-1
))^2*(c - a*c*x)^(5/2)) - (a^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^2)
/(6*(a - x^(-1))^3*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))
^(5/2)*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.218019, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(16*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(24*(a - x^(-1
))^2*(c - a*c*x)^(5/2)) - (a^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^2)
/(6*(a - x^(-1))^3*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))
^(5/2)*(c - a*c*x)^(5/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^4}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx,x\right.}{16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right.\right.}{}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right.\right.}{}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right.}{16\sqrt{2}}$$

Mathematica [A] time = 0.138125, size = 142, normalized size = 0.57

$$-\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+22ax+7\right)-\frac{3\sqrt{2}(ax-1)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{x}\right)}{96\sqrt{a}c^2\left(\frac{1}{x}\right)^{3/2}(ax-1)^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a*x)]*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(7 + 22*a*x +
 3*a^2*x^2) - (3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[
a]*Sqrt[1 + 1/(a*x)])])/x))/(96*Sqrt[a]*c^2*(x^(-1))^(3/2)*(-1 + a*x)^3*Sqr
t[c - a*c*x])
```

Maple [A] time = 0.197, size = 226, normalized size = 0.9

$$-\frac{1}{96\,(ax-1)^2\,(ax+1)\,a}\sqrt{-c\,(ax-1)}\left(3\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^3a^3c-9\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/96/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)*
(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-9*2^(1/
2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*x^2*a^2*(-c*(
a*x+1))^(1/2)*c^(1/2)+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/
2))*x*a*c-44*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1
))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/
2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.68572, size = 900, normalized size = 3.6

$$\left[-\frac{3\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^3x^3+25a^2x^2+29\right.}{192\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 + 25*a
^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x
^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/96*(3*sqrt(2)
*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3 +
 25*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*
c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20468, size = 157, normalized size = 0.63

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{2\left(3(acx+c)^2\sqrt{-acx-c}-16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c}}{96\,ac\,\mathrm{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - 2*(
3*(a*c*x + c)^2*sqrt(-a*c*x - c) - 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*x
 - c)*c^2)/((a*c*x - c)^3*c))/(a*c*sgn(-a*c*x - c))
```

3.251 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=307

$$\frac{a^5x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^5x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}-\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\cdots$$

```
[Out] -(a^5*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^2)/(8*(a - x^(-1))^4*(c - a
*c*x)^(7/2)) - (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(256*(a -
x^(-1))*(c - a*c*x)^(7/2)) - (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x
^3)/(128*(a - x^(-1))^2*(c - a*c*x)^(7/2)) + (a^5*(1 - 1/(a*x))^(7/2)*(1 +
1/(a*x))^(5/2)*x^3)/(32*(a - x^(-1))^3*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 -
 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]
)/(256*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rubi [A] time = 0.238829, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{a^5x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^5x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}-\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```


```
[Out] -(a^5*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^2)/(8*(a - x^(-1))^4*(c - a
*c*x)^(7/2)) - (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(256*(a -
x^(-1))*(c - a*c*x)^(7/2)) - (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x
^3)/(128*(a - x^(-1))^2*(c - a*c*x)^(7/2)) + (a^5*(1 - 1/(a*x))^(7/2)*(1 +
1/(a*x))^(5/2)*x^3)/(32*(a - x^(-1))^3*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 -
 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]
)/(256*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{5}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{4}}\,dx, x, \frac{1}{x}\right)}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^{3}}\,dx \ldots\right.}{64\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{\left(3a^2\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\ldots\right.}{32\left(a\ldots\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\ldots\right.}{32\left(a\ldots\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\ldots\right.}{32\left(a\ldots\right.}$$

Mathematica [A] time = 0.188333, size = 147, normalized size = 0.48

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(-3a^3x^3+13a^2x^2+79ax+39\right)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(ax-1)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{512\sqrt{a}c^3\sqrt{\frac{1}{x}}(ax-1)^4\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(39 + 79*a*x + 13*a^2*x^2
- 3*a^3*x^3))/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^4*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(512*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 +
a*x)^4*Sqrt[c - a*c*x])
```

Maple [A] time = 0.25, size = 278, normalized size = 0.9

$$-\frac{1}{512\,(ax-1)^3\,(ax+1)\,a}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^4a^4c-12\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/512/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(9/2)
*(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^4*a^4*c-12*2^(
1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-6*x^3*a^3*(-c
*(a*x+1))^(1/2)*c^(1/2)+18*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^
(1/2))*x^2*a^2*c+26*x^2*a^2*(-c*(a*x+1))^(1/2)*c^(1/2)-12*2^(1/2)*arctan(1/
2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+158*x*a*(-c*(a*x+1))^(1/2)*c^(1
/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+78*(-c*(a*x+
1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.68711, size = 1030, normalized size = 3.36

$$\left[-\frac{3\sqrt{2}\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^4x^4-10\right.}{1024\left(a^6c^4x^5-5a^5c^4x^4+10a^4c^4x^3-10a^3c^4x^2+5a^2c^4x-ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x
- 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x +
 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3
*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4), -1/512*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10
*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^4*x^4 - 10*a^3*x^3
- 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a
^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x
- a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23487, size = 189, normalized size = 0.62

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2\left(3\,(acx+c)^3\sqrt{-acx-c}-22\,(acx+c)^2\sqrt{-acx-c}c+44\,(-acx-c)^{\frac{3}{2}}c^2+24\,\sqrt{-acx-c}c^3\right)}{(acx-c)^4c^2}}{512\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^3*sqrt(-a*c*x - c) - 22*(a*c*x + c)^2*sqrt(-a*c*x - c)*c + 4
4*(-a*c*x - c)^(3/2)*c^2 + 24*sqrt(-a*c*x - c)*c^3)/((a*c*x - c)^4*c^2))/(a
*c*sgn(-a*c*x - c))
```

3.252 $\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=194

$$\frac{16384c^5x\sqrt{1-\frac{1}{a^2x^2}}}{693\sqrt{c-acx}}+\frac{4096}{693}c^4x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{512}{231}c^3x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}+\frac{640}{693}c^2x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{5/2}+$$

```
[Out] (16384*c^5*Sqrt[1 - 1/(a^2*x^2)]*x)/(693*Sqrt[c - a*c*x]) + (4096*c^4*Sqrt[
1 - 1/(a^2*x^2)]*x*Sqrt[c - a*c*x])/693 + (512*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*
(c - a*c*x)^(3/2))/231 + (640*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(5/2)
)/693 + (40*c*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(7/2))/99 + (2*Sqrt[1 - 1
/(a^2*x^2)]*x*(c - a*c*x)^(9/2))/11
```

Rubi [A] time = 0.225662, antiderivative size = 311, normalized size of antiderivative = 1.6, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{512\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{231a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{1024\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{99a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{22016\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{693a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-40*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^5*(1 - 1/(a*
x))^(9/2)) - (22016*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(693*a^5*(1 - 1/(a
*x))^(9/2)*x^4) + (1024*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1
/(a*x))^(9/2)*x^3) - (512*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(231*a^3*(1
- 1/(a*x))^(9/2)*x^2) + (640*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(
9/2))/(693*a^5*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^5*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^(9/2))/(11*a^5*(1 - 1/(a*x))^(9/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{(c-acx)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{13/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(20\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(320\left(\frac{1}{x}\right)^{9/2}(c-a\right.}{}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+}}{11a^5\left(1-\right.}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{512\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{1024\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{512\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{22016\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x^4}+\frac{1024\sqrt{1+\frac{1}{ax}}(c-acx)^{9/}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}$$

Mathematica [A] time = 0.0514603, size = 86, normalized size = 0.44

$$\frac{2c^4\sqrt{\frac{1}{ax}+1}\left(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531\right)\sqrt{c-acx}}{693a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-11531 + 5419*a*x - 3198*a^2*x^2
+ 1510*a^3*x^3 - 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.045, size = 80, normalized size = 0.4

$$\frac{(2ax+2)\left(63x^5a^5-455x^4a^4+1510x^3a^3-3198a^2x^2+5419ax-11531\right)}{693a(ax-1)^5}(-acx+c)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/693*(a*x+1)*(63*a^5*x^5-455*a^4*x^4+1510*a^3*x^3-3198*a^2*x^2+5419*a*x-11
531)*(-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2)/a/(a*x-1)^5
```

Maxima [A] time = 1.08039, size = 173, normalized size = 0.89

$$\frac{2\left(63a^6\sqrt{-c}c^4x^6-392a^5\sqrt{-c}c^4x^5+1055a^4\sqrt{-c}c^4x^4-1688a^3\sqrt{-c}c^4x^3+2221a^2\sqrt{-c}c^4x^2-6112a\sqrt{-c}c^4x-11531\sqrt{-c}\right.}{693\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*a^6*sqrt(-c)*c^4*x^6 - 392*a^5*sqrt(-c)*c^4*x^5 + 1055*a^4*sqrt(-
c)*c^4*x^4 - 1688*a^3*sqrt(-c)*c^4*x^3 + 2221*a^2*sqrt(-c)*c^4*x^2 - 6112*a
*sqrt(-c)*c^4*x - 11531*sqrt(-c)*c^4)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Fricas [A] time = 1.87519, size = 244, normalized size = 1.26

$$\frac{2\left(63\,a^{6}c^{4}x^{6}-392\,a^{5}c^{4}x^{5}+1055\,a^{4}c^{4}x^{4}-1688\,a^{3}c^{4}x^{3}+2221\,a^{2}c^{4}x^{2}-6112\,ac^{4}x-11531\,c^{4}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{693\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^6*c^4*x^6 - 392*a^5*c^4*x^5 + 1055*a^4*c^4*x^4 - 1688*a^3*c^4*x
^3 + 2221*a^2*c^4*x^2 - 6112*a*c^4*x - 11531*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28056, size = 181, normalized size = 0.93

$$-\frac{2\left(63\,(acx+c)^{5}\sqrt{-acx-c}-770\,(acx+c)^{4}\sqrt{-acx-c}c+3960\,(acx+c)^{3}\sqrt{-acx-c}c^{2}-11088\,(acx+c)^{2}\sqrt{-acx-c}c^{3}\right.}{693\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2/693*(63*(a*c*x + c)^5*sqrt(-a*c*x - c) - 770*(a*c*x + c)^4*sqrt(-a*c*x -
 c)*c + 3960*(a*c*x + c)^3*sqrt(-a*c*x - c)*c^2 - 11088*(a*c*x + c)^2*sqrt(
-a*c*x - c)*c^3 - 18480*(-a*c*x - c)^(3/2)*c^4 - 22176*sqrt(-a*c*x - c)*c^5
)*abs(c)/(a*c^2)
```

3.253 $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=161

$$\frac{4096c^4x\sqrt{1-\frac{1}{a^2x^2}}}{315\sqrt{c-acx}}+\frac{1024}{315}c^3x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{128}{105}c^2x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}+\frac{32}{63}cx\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{5/2}+\frac{2}{9}x\sqrt{}$$

```
[Out] (4096*c^4*Sqrt[1 - 1/(a^2*x^2)]*x)/(315*Sqrt[c - a*c*x]) + (1024*c^3*Sqrt[1
 - 1/(a^2*x^2)]*x*Sqrt[c - a*c*x])/315 + (128*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(
c - a*c*x)^(3/2))/105 + (32*c*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(5/2))/63
 + (2*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(7/2))/9
```

Rubi [A] time = 0.222049, antiderivative size = 254, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{256\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{45a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{5504\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{315a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{32\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3(c-acx)^{7/}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-32*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*
x))^(7/2)) + (5504*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(315*a^4*(1 - 1/(a*
x))^(7/2)*x^3) - (256*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(45*a^3*(1 - 1/(
a*x))^(7/2)*x^2) + (128*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(105*a^2*(1 -
1/(a*x))^(7/2)*x) + (2*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(7/2)
)/(9*a^4*(1 - 1/(a*x))^(7/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(64\left(\frac{1}{x}\right)^{7/2}(c-acx)\right.}{21}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x(c-ac}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{256\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{45a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + \frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + $$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{5504\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{315a^4\left(1-\frac{1}{ax}\right)^{7/2}x^3} - \frac{256\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{45a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + $$

Mathematica [A] time = 0.045066, size = 78, normalized size = 0.48

$$-\frac{2c^3\sqrt{\frac{1}{ax}+1}\left(35a^4x^4-220a^3x^3+642a^2x^2-1276ax+2867\right)\sqrt{c-acx}}{315a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(2867 - 1276*a*x + 642*a^2*x^2 -
220*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.044, size = 72, normalized size = 0.5

$$\frac{(2ax+2)\left(35x^4a^4-220x^3a^3+642a^2x^2-1276ax+2867\right)}{315a(ax-1)^4}(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/315*(a*x+1)*(35*a^4*x^4-220*a^3*x^3+642*a^2*x^2-1276*a*x+2867)*(-a*c*x+c)
^(7/2)*((a*x-1)/(a*x+1))^(1/2)/a/(a*x-1)^4
```

Maxima [A] time = 1.10925, size = 151, normalized size = 0.94

$$-\frac{2\left(35a^5\sqrt{-c}c^3x^5-185a^4\sqrt{-c}c^3x^4+422a^3\sqrt{-c}c^3x^3-634a^2\sqrt{-c}c^3x^2+1591a\sqrt{-c}c^3x+2867\sqrt{-c}c^3\right)(ax-1)}{315\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^5*sqrt(-c)*c^3*x^5 - 185*a^4*sqrt(-c)*c^3*x^4 + 422*a^3*sqrt(-
c)*c^3*x^3 - 634*a^2*sqrt(-c)*c^3*x^2 + 1591*a*sqrt(-c)*c^3*x + 2867*sqrt(-
c)*c^3)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Fricas [A] time = 1.87376, size = 216, normalized size = 1.34

$$-\frac{2\left(35a^5c^3x^5-185a^4c^3x^4+422a^3c^3x^3-634a^2c^3x^2+1591ac^3x+2867c^3\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 185*a^4*c^3*x^4 + 422*a^3*c^3*x^3 - 634*a^2*c^3*x^
2 + 1591*a*c^3*x + 2867*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23105, size = 149, normalized size = 0.93

$$\frac{2\left(35\,(acx+c)^4\sqrt{-acx-c}-360\,(acx+c)^3\sqrt{-acx-c}c+1512\,(acx+c)^2\sqrt{-acx-c}c^2+3360\,(-acx-c)^{\frac{3}{2}}c^3+5040\,\sqrt{-acx}\right.}{315\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(a*c*x + c)^4*sqrt(-a*c*x - c) - 360*(a*c*x + c)^3*sqrt(-a*c*x -
c)*c + 1512*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^2 + 3360*(-a*c*x - c)^(3/2)*c^
3 + 5040*sqrt(-a*c*x - c)*c^4)*abs(c)/(a*c^2)
```

3.254 $\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=128

$$\frac{256c^3x\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-acx}}+\frac{64}{35}c^2x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{24}{35}cx\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}+\frac{2}{7}x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{5/2}$$

```
[Out] (256*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)/(35*Sqrt[c - a*c*x]) + (64*c^2*Sqrt[1 - 1
/(a^2*x^2)]*x*Sqrt[c - a*c*x])/35 + (24*c*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*
x)^(3/2))/35 + (2*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(5/2))/7
```

Rubi [A] time = 0.202914, antiderivative size = 197, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{344\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}{35a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{16\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}{5a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-24*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) - (344
*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a^3*(1 - 1/(a*x))^(5/2)*x^2) + (1
6*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(5*a^2*(1 - 1/(a*x))^(5/2)*x) + (2*(
a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(
5/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{(c-acx)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{24\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(24\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Sul}}{35a\left(1-\right.}$$

$$=-\frac{24\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{16\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\cdots$$

$$=-\frac{24\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{344\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a^3\left(1-\frac{1}{ax}\right)^{5/2}x^2}+\frac{16\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3}{7}$$

Mathematica [A] time = 0.0416979, size = 70, normalized size = 0.55

$$\frac{2c^2\sqrt{\frac{1}{ax}+1}\left(5a^3x^3-27a^2x^2+71ax-177\right)\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a^
3*x^3))/(35*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.043, size = 64, normalized size = 0.5

$$\frac{(2ax+2)\left(5x^3a^3-27a^2x^2+71ax-177\right)}{35a\left(ax-1\right)^3}\left(-acx+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/35*(a*x+1)*(5*a^3*x^3-27*a^2*x^2+71*a*x-177)*(-a*c*x+c)^(5/2)*((a*x-1)/(a
*x+1))^(1/2)/a/(a*x-1)^3
```

Maxima [A] time = 1.11759, size = 130, normalized size = 1.02

$$\frac{2\left(5a^4\sqrt{-c}c^2x^4-22a^3\sqrt{-c}c^2x^3+44a^2\sqrt{-c}c^2x^2-106a\sqrt{-c}c^2x-177\sqrt{-c}c^2\right)(ax-1)}{35\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^4*sqrt(-c)*c^2*x^4 - 22*a^3*sqrt(-c)*c^2*x^3 + 44*a^2*sqrt(-c)*c^
2*x^2 - 106*a*sqrt(-c)*c^2*x - 177*sqrt(-c)*c^2)*(a*x - 1)/((a^2*x - a)*sqr
t(a*x + 1))
```

Fricas [A] time = 1.88813, size = 182, normalized size = 1.42

$$\frac{2\left(5a^4c^2x^4-22a^3c^2x^3+44a^2c^2x^2-106ac^2x-177c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 22*a^3*c^2*x^3 + 44*a^2*c^2*x^2 - 106*a*c^2*x - 177*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19111, size = 116, normalized size = 0.91

$$-\frac{2\left(5\,(acx+c)^{3}\sqrt{-acx-c}-42\,(acx+c)^{2}\sqrt{-acx-c}c-140\,(-acx-c)^{\frac{3}{2}}c^{2}-280\,\sqrt{-acx-c}c^{3}\right)|c|}{35\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(a*c*x + c)^3*sqrt(-a*c*x - c) - 42*(a*c*x + c)^2*sqrt(-a*c*x - c)
*c - 140*(-a*c*x - c)^(3/2)*c^2 - 280*sqrt(-a*c*x - c)*c^3)*abs(c)/(a*c^2)
```

3.255 $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{64c^2x\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-acx}}+\frac{16}{15}cx\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{2}{5}x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}$$

```
[Out] (64*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/(15*Sqrt[c - a*c*x]) + (16*c*Sqrt[1 - 1/(a
^2*x^2)]*x*Sqrt[c - a*c*x])/15 + (2*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(3/
2))/5
```

Rubi [A] time = 0.185946, antiderivative size = 137, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{86\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{15a^2x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{28\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-28*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (86*
Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a^2*(1 - 1/(a*x))^(3/2)*x) + (2*Sq
rt[1 + 1/(a*x)]*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{2\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{\ldots}}\right.}{15a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{86\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a^2\left(1-\frac{1}{ax}\right)^{3/2}x} + \frac{2\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.0335499, size = 60, normalized size = 0.63

$$-\frac{2c\sqrt{\frac{1}{ax}+1}\left(3a^2x^2-14ax+43\right)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*Sq
rt[1 - 1/(a*x)])
```

Maple [A] time = 0.046, size = 56, normalized size = 0.6

$$\frac{(2\,ax+2)\left(3\,a^2x^2-14\,ax+43\right)}{15\,a\,(ax-1)^2}(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a^2*x^2-14*a*x+43)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2)
/a/(a*x-1)^2
```

Maxima [A] time = 1.10012, size = 97, normalized size = 1.02

$$-\frac{2\left(3\,a^{3}\sqrt{-c}cx^{3}-11\,a^{2}\sqrt{-c}cx^{2}+29\,a\sqrt{-c}cx+43\,\sqrt{-c}c\right)(ax-1)}{15\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^3*sqrt(-c)*c*x^3 - 11*a^2*sqrt(-c)*c*x^2 + 29*a*sqrt(-c)*c*x + 4
3*sqrt(-c)*c)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Fricas [A] time = 1.8899, size = 147, normalized size = 1.55

$$-\frac{2\left(3\,a^{3}cx^{3}-11\,a^{2}cx^{2}+29\,acx+43\,c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - 11*a^2*c*x^2 + 29*a*c*x + 43*c)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16384, size = 84, normalized size = 0.88

$$\frac{2\left(3\,(acx+c)^{2}\sqrt{-acx-c}+20\,(-acx-c)^{\frac{3}{2}}c+60\,\sqrt{-acx-c}c^{2}\right)|c|}{15\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 20*(-a*c*x - c)^(3/2)*c + 60*sqrt(
-a*c*x - c)*c^2)*abs(c)/(a*c^2)
```

3.256 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-acx}}$$

```
[Out] (8*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1/(a^2*x^2)
]*x*Sqrt[c - a*c*x])/3
```

Rubi [A] time = 0.153406, antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3a\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] time = 0.0257601, size = 50, normalized size = 0.81

$$\frac{2\sqrt{\frac{1}{ax}+1}(ax-5)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.046, size = 47, normalized size = 0.8

$$\frac{(2ax+2)(ax-5)}{(3ax-3)a}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/3*(a*x+1)*(a*x-5)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/a
```

Maxima [A] time = 1.09126, size = 73, normalized size = 1.18

$$\frac{2\left(a^2\sqrt{-c}x^2 - 4a\sqrt{-c}x - 5\sqrt{-c}\right)(ax-1)}{3\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

Fricas [A] time = 1.78196, size = 111, normalized size = 1.79

$$\frac{2\left(a^2x^2 - 4ax - 5\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14229, size = 80, normalized size = 1.29

$$-\frac{2\left(\frac{4\sqrt{2}\sqrt{-c}c}{a}-\frac{(-acx-c)^{\frac{3}{2}}+6\sqrt{-acx-c}c}{a}\right)|c|\mathrm{sgn}\left(ax+1\right)}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(4*sqrt(2)*sqrt(-c)*c/a - ((-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/
a)*abs(c)*sgn(a*x + 1)/c^2
```

3.257 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=29

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

```
[Out] (2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.0378088, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {6174}

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

Mathematica [A] time = 0.0226005, size = 28, normalized size = 0.97

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - a*c*x]
```

Maple [A] time = 0.041, size = 35, normalized size = 1.2

$$2\,\frac{ax+1}{a\sqrt{-acx+c}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Maxima [A] time = 1.10366, size = 39, normalized size = 1.34

$$-\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)}{\sqrt{ax+1}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))/(sqrt(a*x + 1)*a*c)
```

Fricas [A] time = 1.88169, size = 99, normalized size = 3.41

$$-\frac{2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)), x)
```

Giac [A] time = 1.13755, size = 66, normalized size = 2.28

$$-\frac{2\left(\frac{\sqrt{2}\sqrt{-c}}{a}+\frac{(-acx-c)^{\frac{3}{2}}}{(acx+c)a}\right)|c|\operatorname{sgn}(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(2)*sqrt(-c)/a + (-a*c*x - c)^(3/2)/((a*c*x + c)*a))*abs(c)*sgn(a*x
 + 1)/c^2
```

3.258 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Rubi [A] time = 0.180051, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 93, 206}

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```


```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}\\
&= -\frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}\\
&= -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0475276, size = 76, normalized size = 1.

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Maple [A] time = 0.151, size = 78, normalized size = 1.

$$-\frac{(ax+1)\sqrt{2}}{(ax-1)a}\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c(ax+1)}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{-c(ax+1)}}c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)/c^(3/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(3/2), x)
```

Fricas [A] time = 1.92783, size = 352, normalized size = 4.63

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)}{2\,ac}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{ac^{\frac{3}{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/c)*log(-(a^2*x^2 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1))/
(a*c), -sqrt(2)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(
(a*x - 1)*sqrt(c)))/(a*c^(3/2))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17697, size = 86, normalized size = 1.13

$$\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{a\sqrt{c}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)|c|\mathrm{sgn}\left(ax+1\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*sqrt(c)) - sqrt(2)
*arctan(sqrt(-c)/sqrt(c))/(a*sqrt(c)))*abs(c)*sgn(a*x + 1)/c^2
```

3.259 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=136

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[Out] -(a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(2*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/(2*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.192643, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(2*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/(2*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] time = 0.114456, size = 116, normalized size = 0.85

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\sqrt{a}\sqrt{\frac{1}{ax}+1}\right)}{4\sqrt{a}c^2(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(
-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(4*
Sqrt[a]*c^2*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A] time = 0.175, size = 123, normalized size = 0.9

$$\frac{ax+1}{4\,(ax-1)^2\,a}\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\left(-\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c\,(ax+1)}\frac{1}{\sqrt{c}}\right)xac+\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c\,(ax+1)}\frac{1}{\sqrt{c}}\right)c+2\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(7/2)/(a*x-1)^2/(-
c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(5/2), x)
```

Fricas [A] time = 1.89018, size = 664, normalized size = 4.88

$$\left[-\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)},-\frac{\sqrt{2}\left(a^2x^2-2\,ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
```

```
c)/(a^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), -1/4*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21439, size = 99, normalized size = 0.73

$$\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{3}{2}}} - \frac{2\sqrt{-acx-c}}{(acx-c)ac}\right)|c|\operatorname{sgn}(ax+1)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(3/2)) - 2*s
qrt(-a*c*x - c)/((a*c*x - c)*a*c))*abs(c)*sgn(a*x + 1)/c^2
```

3.260 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=193

$$\frac{3a^3x^3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^3x^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -(a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^2)/(4*(a - x^(-1))^2*(c - a*c
*x)^(7/2)) + (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(16*(a - x^(
-1))*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*S
qrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))^(7/2)*(c -
a*c*x)^(7/2))
```

Rubi [A] time = 0.203284, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{3a^3x^3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^3x^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```



```
[Out] -(a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^2)/(4*(a - x^(-1))^2*(c - a*c
*x)^(7/2)) + (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(16*(a - x^(
-1))*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*S
qrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))^(7/2)*(c -
a*c*x)^(7/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^{3}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^{2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{}}{\sqrt{1+}}\right.}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Mathematica [A] time = 0.161128, size = 125, normalized size = 0.65

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}(7-3ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{32\sqrt{a}c^3(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 - 3*a*x) + 3*Sqrt[2]*S
qrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1
```

```
/(a*x)])]))/(32*Sqrt[a]*c^3*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A] time = 0.152, size = 172, normalized size = 0.9

$$-\frac{ax+1}{32\,(ax-1)^3\,a}\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\left(3\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2c-6\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/32*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)
*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*arctan(
1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-6*x*a*(-c*(a*x+1))^(1/2)*c^(1
/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+14*(-c*(a*x+
1))^(1/2)*c^(1/2))/(a*x-1)^3/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(7/2), x)
```

Fricas [A] time = 2.01074, size = 786, normalized size = 4.07

$$\left[-\frac{3\,\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\left(3\,a^2x^2-4\,ax-7\right)\sqrt{-acx+c}\sqrt{}}{64\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*
c^4*x - a*c^4), -1/32*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*
arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x -
c)) - 2*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))
/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22392, size = 123, normalized size = 0.64

$$\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}}+\frac{2\left(3(-acx-c)^{\frac{3}{2}}+10\sqrt{-acx-c}c\right)}{(acx-c)^2ac^2}\right)|c|\operatorname{sgn}(ax+1)}{32c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) +
2*(3*(-a*c*x - c)^(3/2) + 10*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*a*c^2))*abs
(c)*sgn(a*x + 1)/c^2
```

3.261 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{32c^3\sqrt{c-acx}}{a}+\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}-\frac{8c(c-acx)^{5/2}}{5a}$$

```
[Out] (-32*c^3*Sqrt[c - a*c*x])/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c -
a*c*x)^(5/2))/(5*a) - (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(
9*a*c) + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.155718, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{32c^3\sqrt{c-acx}}{a}+\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}-\frac{8c(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-32*c^3*Sqrt[c - a*c*x])/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c -
a*c*x)^(5/2))/(5*a) - (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(
9*a*c) + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$$
$$= -\int \frac{(1-ax)(c-acx)^{7/2}}{1+ax}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{9/2}}{1+ax}\,dx}{c}$$
$$= -\frac{2(c-acx)^{9/2}}{9ac} - 2\int \frac{(c-acx)^{7/2}}{1+ax}\,dx$$
$$= -\frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (4c)\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - \left(8c^2\right)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= -\frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - \left(16c^3\right)\int \frac{\sqrt{c-acx}}{1+ax}$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (3$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} + \frac{(6}{}$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} + \frac{32}{}$$

Mathematica [A] time = 0.0787192, size = 88, normalized size = 0.64

$$\frac{2c^3\left(\left(-35a^4x^4+230a^3x^3-732a^2x^2+1754ax-6257\right)\sqrt{c-acx}+5040\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*c^3*(Sqrt[c - a*c*x]*(-6257 + 1754*a*x - 732*a^2*x^2 + 230*a^3*x^3 - 35*
a^4*x^4) + 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
)/(315*a)
```

Maple [A] time = 0.046, size = 101, normalized size = 0.7

$$-2\,\frac{1}{ac}\left(1/9\,\left(-acx+c\right)^{9/2}+2/7\,\left(-acx+c\right)^{7/2}c+4/5\,\left(-acx+c\right)^{5/2}c^{2}+8/3\,c^{3}\left(-acx+c\right)^{3/2}+16\,\sqrt{-acx+c}c^{4}-16\,c^{9/2}\sqrt{2}\mathrm{A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/9*(-a*c*x+c)^(9/2)+2/7*(-a*c*x+c)^(7/2)*c+4/5*(-a*c*x+c)^(5/2)*c^
2+8/3*c^3*(-a*c*x+c)^(3/2)+16*(-a*c*x+c)^(1/2)*c^4-16*c^(9/2)*2^(1/2)*arcta
nh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87006, size = 518, normalized size = 3.78

$$\left[\frac{2\left(2520\,\sqrt{2}c^{\frac{7}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)-\left(35\,a^{4}c^{3}x^{4}-230\,a^{3}c^{3}x^{3}+732\,a^{2}c^{3}x^{2}-1754\,ac^{3}x+6257\,c^{3}\right)\sqrt{-acx+c}\right)}{315\,a},-\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) - (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 -
 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, -2/315*(5040*sqrt(2)*sqrt(-c
)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4 - 2
30*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c
```

```
))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16467, size = 217, normalized size = 1.58

$$-\frac{32\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^8c^8-90\,(acx-c)^3\sqrt{-acx+c}a^8c^9+252\,(acx-c)^2\sqrt{-acx+c}\right.}{315\,a^9c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
- 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*(-a
*c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

3.262 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{16c^2\sqrt{c-acx}}{a}+\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}-\frac{8c(c-acx)^{3/2}}{3a}$$

```
[Out] (-16*c^2*Sqrt[c - a*c*x])/a - (8*c*(c - a*c*x)^(3/2))/(3*a) - (4*(c - a*c*x
)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c) + (16*Sqrt[2]*c^(5/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.131632, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{16c^2\sqrt{c-acx}}{a}+\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}-\frac{8c(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-16*c^2*Sqrt[c - a*c*x])/a - (8*c*(c - a*c*x)^(3/2))/(3*a) - (4*(c - a*c*x
)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c) + (16*Sqrt[2]*c^(5/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$$
$$= -\int \frac{(1-ax)(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{7/2}}{1+ax}\,dx}{c}$$
$$= -\frac{2(c-acx)^{7/2}}{7ac} - 2\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (4c)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= -\frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - \left(8c^2\right)\int \frac{\sqrt{c-acx}}{1+ax}\,dx$$
$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - \left(16c^3\right)\int \frac{1}{(1+ax)\sqrt{c-}}$$
$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{\left(32c^2\right)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\right.}{a}$$
$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a}}{\sqrt{2}\sqrt{}}\right.}{a}$$

Mathematica [A] time = 0.0592854, size = 80, normalized size = 0.69

$$\frac{2c^2\left(\left(15a^3x^3-87a^2x^2+269ax-1037\right)\sqrt{c-acx}+840\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*S
qrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

Maple [A] time = 0.048, size = 87, normalized size = 0.8

$$-2\,\frac{1}{ac}\left(1/7\,(-acx+c)^{7/2}+2/5\,c\,(-acx+c)^{5/2}+4/3\,(-acx+c)^{3/2}\,c^2+8\,\sqrt{-acx+c}c^3-8\,c^{7/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/7*(-a*c*x+c)^(7/2)+2/5*c*(-a*c*x+c)^(5/2)+4/3*(-a*c*x+c)^(3/2)*c^
2+8*(-a*c*x+c)^(1/2)*c^3-8*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(
1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77967, size = 462, normalized size = 3.98

$$\left[\frac{2\left(420\sqrt{2}c^{\frac{5}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2\right)\sqrt{-acx+c}\right)}{105a}, -\frac{2\left(840\sqrt{2}\sqrt{-c}c^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 1037*
c^2)*sqrt(-a*c*x + c))/a, -2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*
c^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

Sympy [A] time = 84.89, size = 110, normalized size = 0.95

$$-\frac{16\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{16c^2\sqrt{-acx+c}}{a}-\frac{8c\left(-acx+c\right)^{\frac{3}{2}}}{3a}-\frac{4\left(-acx+c\right)^{\frac{5}{2}}}{5a}-\frac{2\left(-acx+c\right)^{\frac{7}{2}}}{7ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -16*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
 16*c**2*sqrt(-a*c*x + c)/a - 8*c*(-a*c*x + c)**(3/2)/(3*a) - 4*(-a*c*x + c
)**(5/2)/(5*a) - 2*(-a*c*x + c)**(7/2)/(7*a*c)
```

Giac [A] time = 1.13706, size = 181, normalized size = 1.56

$$-\frac{16\,\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(15\,(acx-c)^3\sqrt{-acx+c}a^6c^6-42\,(acx-c)^2\sqrt{-acx+c}a^6c^7-140\,(-acx+c)^{\frac{3}{2}}a^6c^8-84\right.}{105\,a^7c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
+ 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqrt(
-a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x + c)
*a^6*c^9)/(a^7*c^7)
```

3.263 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}-\frac{8c\sqrt{c-acx}}{a}$$

```
[Out] (-8*c*Sqrt[c - a*c*x])/a - (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/
2))/(5*a*c) + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a
```

Rubi [A] time = 0.119701, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 50, 63, 206}

$$\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}-\frac{8c\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (-8*c*Sqrt[c - a*c*x])/a - (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/
2))/(5*a*c) + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx \\
&= -\int \frac{(1-ax)(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{5/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{5/2}}{5ac} - 2\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} - (4c)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} - \left(8c^2\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{(16c)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a} \\
&= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0418588, size = 71, normalized size = 0.75

$$
\frac{120\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) - 2c\left(3a^2x^2-16ax+73\right)\sqrt{c-acx}}{15a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) + 120*Sqrt[2]*c^(3/2)*ArcTa
nh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

Maple [A] time = 0.048, size = 73, normalized size = 0.8

$$
-2\,\frac{1}{ac}\left(1/5\,\left(-acx+c\right)^{5/2}+2/3\,c\left(-acx+c\right)^{3/2}+4\,\sqrt{-acx+c}c^2-4\,c^{5/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/5*(-a*c*x+c)^(5/2)+2/3*c*(-a*c*x+c)^(3/2)+4*(-a*c*x+c)^(1/2)*c^2-
4*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84928, size = 381, normalized size = 4.01

$$\left[\frac{2\left(30\,\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)-\left(3\,a^{2}cx^{2}-16\,acx+73\,c\right)\sqrt{-acx+c}\right)}{15\,a},-\frac{2\left(60\,\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)+\right.}{15\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, -2/
15*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) +
 (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

Sympy [A] time = 43.7451, size = 92, normalized size = 0.97

$$-\frac{8\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{8c\sqrt{-acx+c}}{a}-\frac{4\left(-acx+c\right)^{\frac{3}{2}}}{3a}-\frac{2\left(-acx+c\right)^{\frac{5}{2}}}{5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -8*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
8*c*sqrt(-a*c*x + c)/a - 4*(-a*c*x + c)**(3/2)/(3*a) - 2*(-a*c*x + c)**(5/2
)/(5*a*c)
```

Giac [A] time = 1.14778, size = 144, normalized size = 1.52

$$-\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}a^4c^4+10\,(-acx+c)^{\frac{3}{2}}a^4c^5+60\,\sqrt{-acx+c}a^4c^6\right)}{15\,a^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a^4
*c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

3.264 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.102095, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{3/2}}{3ac} - 2\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} - (4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a} \\
&= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0317838, size = 61, normalized size = 0.8

$$
\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A] time = 0.05, size = 59, normalized size = 0.8

$$
-2\,\frac{1}{ac}\left(1/3\,(-acx+c)^{3/2}+2\,c\sqrt{-acx+c}-2\,c^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh
(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84861, size = 313, normalized size = 4.12

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

Sympy [A] time = 5.71487, size = 73, normalized size = 0.96

$$-\frac{2\left(\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.12162, size = 104, normalized size = 1.37

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

3.265 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Rubi [A] time = 0.0985863, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 50, 63, 206}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{c} \\
&= -\frac{2\sqrt{c-acx}}{ac} - 2\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{2\sqrt{c-acx}}{ac} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac} \\
&= -\frac{2\sqrt{c-acx}}{ac} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0263095, size = 58, normalized size = 1.

$$
\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{c-acx}}{ac}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Maple [A] time = 0.043, size = 45, normalized size = 0.8

$$
-2\,\frac{1}{ac}\left(\sqrt{-acx+c} - \mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}\sqrt{c}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)*c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95542, size = 293, normalized size = 5.05

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)-2\sqrt{-acx+c}}{ac}, \frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x +
 1)) - 2*sqrt(-a*c*x + c))/(a*c), 2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*sq
rt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

Sympy [A] time = 20.1008, size = 60, normalized size = 1.03

$$-\frac{2\sqrt{-acx+c}}{ac}-\frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}}{\sqrt{-\frac{1}{c}}\sqrt{-acx+c}}\right)}{ac\sqrt{-\frac{1}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*sqrt(-a*c*x + c)/(a*c) - 2*sqrt(2)*atan(sqrt(2)/(sqrt(-1/c)*sqrt(-a*c*x
+ c)))/(a*c*sqrt(-1/c))
```

Giac [A] time = 1.1126, size = 69, normalized size = 1.19

$$-\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\,\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2*s
qrt(-a*c*x + c)/(a*c)
```

3.266 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rubi [A] time = 0.0927469, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6130, 21, 63, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx\\ &= -\int \frac{1-ax}{(1+ax)(c-acx)^{3/2}}\,dx\\ &= -\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{c}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^2}\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0184084, size = 37, normalized size = 1.

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A] time = 0.052, size = 29, normalized size = 0.8

$$\frac{\sqrt{2}}{a}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-acx+c}\frac{1}{\sqrt{c}}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89117, size = 231, normalized size = 6.24

$$\left[\frac{\sqrt{2}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}}, \frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))/
(a*c^(3/2)), sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c)/
(a*x - 1))/(a*c)]
```

Sympy [A] time = 19.8034, size = 41, normalized size = 1.11

$$-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(-c))
```

Giac [A] time = 1.15827, size = 49, normalized size = 1.32

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

3.267 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}-\frac{1}{ac^2\sqrt{c-acx}}$$

```
[Out] -(1/(a*c^2*Sqrt[c - a*c*x])) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(
Sqrt[2]*a*c^(5/2))
```

Rubi [A] time = 0.103479, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}-\frac{1}{ac^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(1/(a*c^2*Sqrt[c - a*c*x])) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(
Sqrt[2]*a*c^(5/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$
$$= -\int \frac{1-ax}{(1+ax)(c-acx)^{5/2}}\,dx$$
$$= -\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{c}$$
$$= -\frac{1}{ac^2\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{2c^2}$$
$$= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^3}$$
$$= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Mathematica [C] time = 0.022137, size = 37, normalized size = 0.65

$$-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x]))
```

Maple [A] time = 0.049, size = 50, normalized size = 0.9

$$-2\,\frac{1}{ac}\left(1/2\,\frac{1}{c\sqrt{-acx+c}}-1/4\,\frac{\sqrt{2}}{c^{3/2}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -2/c/a*(1/2/c/(-a*c*x+c)^(1/2)-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(
1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82303, size = 362, normalized size = 6.35

$$\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)}, -\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-2\sqrt{-acx+c}}{2\left(a^2c^3x-ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), -1/2*(sqr
t(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2
*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

Sympy [A] time = 15.727, size = 61, normalized size = 1.07

$$-\frac{1}{ac^2\sqrt{-acx+c}}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] -1/(a*c**2*sqrt(-a*c*x + c)) - sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqr
t(-c)))/(2*a*c**2*sqrt(-c))
```

Giac [A] time = 1.16371, size = 73, normalized size = 1.28

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2\,a\sqrt{-c}c^2}-\frac{1}{\sqrt{-acx+c}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
 - 1/(sqrt(-a*c*x + c)*a*c^2)
```

3.268 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{1}{3ac^2(c-acx)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

```
[Out] -1/(3*a*c^2*(c - a*c*x)^(3/2)) - 1/(2*a*c^3*Sqrt[c - a*c*x]) + ArcTanh[Sqrt
[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.115963, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 51, 63, 206}

$$-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{1}{3ac^2(c-acx)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/(3*a*c^2*(c - a*c*x)^(3/2)) - 1/(2*a*c^3*Sqrt[c - a*c*x]) + ArcTanh[Sqrt
[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx\\ &= -\int \frac{1-ax}{(1+ax)(c-acx)^{7/2}}\,dx\\ &= -\frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{c}\\ &= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{2c^2}\\ &= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{4c^3}\\ &= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{2ac^4}\\ &= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.0276531, size = 39, normalized size = 0.47

$$-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(3*a*c^2*(c - a*c*x)^(3/2))
```

Maple [A] time = 0.053, size = 64, normalized size = 0.8

$$-2\,\frac{1}{ac}\left(1/4\,\frac{1}{\sqrt{-acx+c}c^2}+1/6\,\frac{1}{c\,(-acx+c)^{3/2}}-1/8\,\frac{\sqrt{2}}{c^{5/2}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -2/c/a*(1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2)-1/8/c^(5/2)*2^(1/2)
*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90501, size = 478, normalized size = 5.76

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+4\sqrt{-acx+c}(3\,ax-5)}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)},-\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{}\right.}{12\left(a^3c^4x^2-2\,a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(a^
3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)
*(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [A] time = 48.9461, size = 82, normalized size = 0.99

$$-\frac{1}{3ac^2\left(-acx+c\right)^{\frac{3}{2}}}-\frac{1}{2ac^3\sqrt{-acx+c}}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4ac^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] -1/(3*a*c**2*(-a*c*x + c)**(3/2)) - 1/(2*a*c**3*sqrt(-a*c*x + c)) - sqrt(2)
*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(4*a*c**3*sqrt(-c))
```

Giac [A] time = 1.12433, size = 99, normalized size = 1.19

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}c^3}-\frac{3\,acx-5\,c}{6\,(acx-c)\sqrt{-acx+c}ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3)
 - 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

3.269 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{4ac^4\sqrt{c-acx}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{5ac^2(c-acx)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

```
[Out] -1/(5*a*c^2*(c - a*c*x)^(5/2)) - 1/(6*a*c^3*(c - a*c*x)^(3/2)) - 1/(4*a*c^4
*Sqrt[c - a*c*x]) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]*a
*c^(9/2))
```

Rubi [A] time = 0.126211, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 51, 63, 206}

$$-\frac{1}{4ac^4\sqrt{c-acx}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{5ac^2(c-acx)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```


```
[Out] -1/(5*a*c^2*(c - a*c*x)^(5/2)) - 1/(6*a*c^3*(c - a*c*x)^(3/2)) - 1/(4*a*c^4
*Sqrt[c - a*c*x]) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]*a
*c^(9/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$$
$$= -\int \frac{1-ax}{(1+ax)(c-acx)^{9/2}}\,dx$$
$$= -\frac{\int \frac{1}{(1+ax)(c-acx)^{7/2}}\,dx}{c}$$
$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{2c^2}$$
$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{4c^3}$$
$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{8c^4}$$
$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{4ac^5}$$
$$= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Mathematica [C] time = 0.0341427, size = 39, normalized size = 0.38

$$-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(5*a*c^2*(c - a*c*x)^(5/2))
```

Maple [A] time = 0.051, size = 78, normalized size = 0.8

$$-2\,\frac{1}{ac}\left(1/8\,\frac{1}{\sqrt{-acx+c}c^3}+1/12\,\frac{1}{(-acx+c)^{3/2}\,c^2}+1/10\,\frac{1}{c\,(-acx+c)^{5/2}}-1/16\,\frac{\sqrt{2}}{c^{7/2}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(9/2),x)
```

```
[Out] -2/c/a*(1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c
)^(5/2)-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69163, size = 599, normalized size = 5.76

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\left(15a^2x^2-40ax+37\right)\sqrt{-acx+c}}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}, -\frac{15\sqrt{2}\left(a^3x^3-3a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*(15*a^2*x^2 - 40*a*
x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*sqrt(
-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [A] time = 28.0085, size = 100, normalized size = 0.96

$$-\frac{1}{5ac^2\left(-acx+c\right)^{\frac{5}{2}}}-\frac{1}{6ac^3\left(-acx+c\right)^{\frac{3}{2}}}-\frac{1}{4ac^4\sqrt{-acx+c}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8ac^4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(9/2),x)
```

```
[Out] -1/(5*a*c**2*(-a*c*x + c)**(5/2)) - 1/(6*a*c**3*(-a*c*x + c)**(3/2)) - 1/(4
*a*c**4*sqrt(-a*c*x + c)) - sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-
c)))/(8*a*c**4*sqrt(-c))
```

Giac [A] time = 1.14351, size = 126, normalized size = 1.21

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8\,a\sqrt{-c}c^4}-\frac{15\,(acx-c)^2-10\,(acx-c)c+12\,c^2}{60\,(acx-c)^2\sqrt{-acx+c}ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4)
 - 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqrt(
-a*c*x + c)*a*c^4)
```

3.270 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=368

$$-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6x^2\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{4096(c-acx)^{9/2}}{231a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{40960(c-acx)^{9/2}}{231a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{94208(c-acx)^{9/2}}{231a^6x^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}}}$$

```
[Out] (-16*(a - x^(-1))^5*(c - a*c*x)^(9/2))/(33*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 +
 1/(a*x)]) - (94208*(c - a*c*x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1
+ 1/(a*x)]*x^5) - (40960*(c - a*c*x)^(9/2))/(231*a^5*(1 - 1/(a*x))^(9/2)*Sq
rt[1 + 1/(a*x)]*x^4) + (4096*(c - a*c*x)^(9/2))/(231*a^4*(1 - 1/(a*x))^(9/2
)*Sqrt[1 + 1/(a*x)]*x^3) - (1024*(a - x^(-1))^3*(c - a*c*x)^(9/2))/(231*a^6
*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x^2) + (320*(a - x^(-1))^4*(c - a*c*
x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1
))^6*x*(c - a*c*x)^(9/2))/(11*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.266833, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6x^2\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{4096(c-acx)^{9/2}}{231a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{40960(c-acx)^{9/2}}{231a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{94208(c-acx)^{9/2}}{231a^6x^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-16*(a - x^(-1))^5*(c - a*c*x)^(9/2))/(33*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 +
 1/(a*x)]) - (94208*(c - a*c*x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1
+ 1/(a*x)]*x^5) - (40960*(c - a*c*x)^(9/2))/(231*a^5*(1 - 1/(a*x))^(9/2)*Sq
rt[1 + 1/(a*x)]*x^4) + (4096*(c - a*c*x)^(9/2))/(231*a^4*(1 - 1/(a*x))^(9/2
)*Sqrt[1 + 1/(a*x)]*x^3) - (1024*(a - x^(-1))^3*(c - a*c*x)^(9/2))/(231*a^6
*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x^2) + (320*(a - x^(-1))^4*(c - a*c*
x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1
))^6*x*(c - a*c*x)^(9/2))/(11*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
```

```
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^6}{x^{13/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(24\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(160\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\right.}{33a^2\left(1-\frac{1}{ax}\right)}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{(2}{}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^2} + \frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{4096(c-acx)^{9/2}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^3} - \frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^2}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{40960(c-acx)^{9/2}}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^4} + \frac{4096(c-acx)^{9/2}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^3}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{94208(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^5} - \frac{40960(c-acx)^{9/2}}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}x^4}$$

Mathematica [A] time = 0.0511153, size = 84, normalized size = 0.23

$$\frac{2c^4\left(21a^6x^6-182a^5x^5+755a^4x^4-2132a^3x^3+5419a^2x^2-23062ax-46355\right)\sqrt{c-acx}}{231a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^4*Sqrt[c - a*c*x]*(-46355 - 23062*a*x + 5419*a^2*x^2 - 2132*a^3*x^3 +
755*a^4*x^4 - 182*a^5*x^5 + 21*a^6*x^6))/(231*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.051, size = 88, normalized size = 0.2

$$\frac{(2\,ax+2)\left(21\,x^6a^6-182\,x^5a^5+755\,x^4a^4-2132\,x^3a^3+5419\,a^2x^2-23062\,ax-46355\right)}{231\,a\,(ax-1)^6}(-acx+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/231*(a*x+1)*(21*a^6*x^6-182*a^5*x^5+755*a^4*x^4-2132*a^3*x^3+5419*a^2*x^2
-23062*a*x-46355)*(-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^6
```

Maxima [A] time = 1.11471, size = 205, normalized size = 0.56

$$\frac{2\left(21\,a^7\sqrt{-c}c^4x^7-161\,a^6\sqrt{-c}c^4x^6+573\,a^5\sqrt{-c}c^4x^5-1377\,a^4\sqrt{-c}c^4x^4+3287\,a^3\sqrt{-c}c^4x^3-17643\,a^2\sqrt{-c}c^4x^2-69417\,a\right.}{231\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*a^7*sqrt(-c)*c^4*x^7 - 161*a^6*sqrt(-c)*c^4*x^6 + 573*a^5*sqrt(-c
)*c^4*x^5 - 1377*a^4*sqrt(-c)*c^4*x^4 + 3287*a^3*sqrt(-c)*c^4*x^3 - 17643*a
^2*sqrt(-c)*c^4*x^2 - 69417*a*sqrt(-c)*c^4*x - 46355*sqrt(-c)*c^4)*(a*x - 1
```

```
)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A] time = 1.51568, size = 244, normalized size = 0.66

$$\frac{2\left(21\,a^6c^4x^6 - 182\,a^5c^4x^5 + 755\,a^4c^4x^4 - 2132\,a^3c^4x^3 + 5419\,a^2c^4x^2 - 23062\,ac^4x - 46355\,c^4\right)\sqrt{-acx + c}\sqrt{\frac{ax-1}{ax+1}}}{231\left(a^2x - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*a^6*c^4*x^6 - 182*a^5*c^4*x^5 + 755*a^4*c^4*x^4 - 2132*a^3*c^4*x^
3 + 5419*a^2*c^4*x^2 - 23062*a*c^4*x - 46355*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3907, size = 203, normalized size = 0.55

$$-\frac{2\left(21\,(acx + c)^5\sqrt{-acx - c} - 308\,(acx + c)^4\sqrt{-acx - c}c + 1980\,(acx + c)^3\sqrt{-acx - c}c^2 - 7392\,(acx + c)^2\sqrt{-acx - c}c^3 - 18\right.}{231\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -2/231*(21*(a*c*x + c)^5*sqrt(-a*c*x - c) - 308*(a*c*x + c)^4*sqrt(-a*c*x -
 c)*c + 1980*(a*c*x + c)^3*sqrt(-a*c*x - c)*c^2 - 7392*(a*c*x + c)^2*sqrt(-
```

```
a*c*x - c)*c^3 - 18480*(-a*c*x - c)^(3/2)*c^4 - 44352*sqrt(-a*c*x - c)*c^5
+ 14784*c^6/sqrt(-a*c*x - c))*abs(c)/(a*c^2)
```

3.271 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=311

$$-\frac{512(c-acx)^{7/2}}{63a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{5120(c-acx)^{7/2}}{63a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{11776(c-acx)^{7/2}}{63a^5x^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^5(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\cdots$$

```
[Out] (-40*(a - x^(-1))^4*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]) + (11776*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]*x^4) + (5120*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*x))^(7/2)*Sqrt[
1 + 1/(a*x)]*x^3) - (512*(c - a*c*x)^(7/2))/(63*a^3*(1 - 1/(a*x))^(7/2)*Sqr
t[1 + 1/(a*x)]*x^2) + (128*(a - x^(-1))^3*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1
/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1))^5*x*(c - a*c*x)^(7/2))
/(9*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.236138, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{512(c-acx)^{7/2}}{63a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{5120(c-acx)^{7/2}}{63a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{11776(c-acx)^{7/2}}{63a^5x^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^5(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-40*(a - x^(-1))^4*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]) + (11776*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]*x^4) + (5120*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*x))^(7/2)*Sqrt[
1 + 1/(a*x)]*x^3) - (512*(c - a*c*x)^(7/2))/(63*a^3*(1 - 1/(a*x))^(7/2)*Sqr
t[1 + 1/(a*x)]*x^2) + (128*(a - x^(-1))^3*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1
/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1))^5*x*(c - a*c*x)^(7/2))
/(9*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(20\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(320\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\Big(\int}{63a^2\left(1-\frac{1}{ax}\right)^{7}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\Big(25}{}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2} + \frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2}{9}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3} - \frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2} +$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^4} + \frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3} -$$

Mathematica [A] time = 0.0448429, size = 76, normalized size = 0.24

$$-\frac{2c^3\left(7a^5x^5-55a^4x^4+214a^3x^3-638a^2x^2+2867ax+5797\right)\sqrt{c-acx}}{63a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*c^3*Sqrt[c - a*c*x]*(5797 + 2867*a*x - 638*a^2*x^2 + 214*a^3*x^3 - 55*a
^4*x^4 + 7*a^5*x^5))/(63*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.042, size = 80, normalized size = 0.3

$$\frac{(2\,ax+2)\left(7\,x^5a^5-55\,x^4a^4+214\,x^3a^3-638\,a^2x^2+2867\,ax+5797\right)}{63\,a\,(ax-1)^5}(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/63*(a*x+1)*(7*a^5*x^5-55*a^4*x^4+214*a^3*x^3-638*a^2*x^2+2867*a*x+5797)*(
-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^5
```

Maxima [A] time = 1.11219, size = 184, normalized size = 0.59

$$-\frac{2\left(7\,a^6\sqrt{-c}c^3x^6-48\,a^5\sqrt{-c}c^3x^5+159\,a^4\sqrt{-c}c^3x^4-424\,a^3\sqrt{-c}c^3x^3+2229\,a^2\sqrt{-c}c^3x^2+8664\,a\sqrt{-c}c^3x+5797\,\sqrt{-c}c^3}{63\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*a^6*sqrt(-c)*c^3*x^6 - 48*a^5*sqrt(-c)*c^3*x^5 + 159*a^4*sqrt(-c)*
c^3*x^4 - 424*a^3*sqrt(-c)*c^3*x^3 + 2229*a^2*sqrt(-c)*c^3*x^2 + 8664*a*sqr
t(-c)*c^3*x + 5797*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x
```

```
+ 1)^(3/2))
```

Fricas [A] time = 1.60913, size = 212, normalized size = 0.68

$$-\frac{2\left(7\,a^{5}c^{3}x^{5}-55\,a^{4}c^{3}x^{4}+214\,a^{3}c^{3}x^{3}-638\,a^{2}c^{3}x^{2}+2867\,ac^{3}x+5797\,c^{3}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^5*c^3*x^5 - 55*a^4*c^3*x^4 + 214*a^3*c^3*x^3 - 638*a^2*c^3*x^2 +
 2867*a*c^3*x + 5797*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30885, size = 170, normalized size = 0.55

$$\frac{2\left(7\,(acx+c)^{4}\sqrt{-acx-c}-90\,(acx+c)^{3}\sqrt{-acx-c}c+504\,(acx+c)^{2}\sqrt{-acx-c}c^{2}+1680\,(-acx-c)^{\frac{3}{2}}c^{3}+5040\,\sqrt{-acx-}\right.}{63\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 2/63*(7*(a*c*x + c)^4*sqrt(-a*c*x - c) - 90*(a*c*x + c)^3*sqrt(-a*c*x - c)*
c + 504*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^2 + 1680*(-a*c*x - c)^(3/2)*c^3 +
5040*sqrt(-a*c*x - c)*c^4 - 2016*c^5/sqrt(-a*c*x - c))*abs(c)/(a*c^2)
```

3.272 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=254

$$-\frac{256(c-acx)^{5/2}}{7a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{2944(c-acx)^{5/2}}{35a^4x^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^4(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{}{35a^2}$$

```
[Out] (-32*(a - x^(-1))^3*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]) - (2944*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
1/(a*x)]*x^3) - (256*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]*x^2) + (128*(c - a*c*x)^(5/2))/(35*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1
 + 1/(a*x)]*x) + (2*(a - x^(-1))^4*x*(c - a*c*x)^(5/2))/(7*a^4*(1 - 1/(a*x)
)^(5/2)*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.213266, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{256(c-acx)^{5/2}}{7a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{2944(c-acx)^{5/2}}{35a^4x^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^4(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{}{35a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-32*(a - x^(-1))^3*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]) - (2944*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
1/(a*x)]*x^3) - (256*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]*x^2) + (128*(c - a*c*x)^(5/2))/(35*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1
 + 1/(a*x)]*x) + (2*(a - x^(-1))^4*x*(c - a*c*x)^(5/2))/(7*a^4*(1 - 1/(a*x)
)^(5/2)*Sqrt[1 + 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

1]

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(192\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}}{35a^2\left(1-\frac{1}{ax}\right.}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{(1}{}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2} + \frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x} +$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{2944(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3} - \frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^2} +$$

Mathematica [A] time = 0.0381353, size = 68, normalized size = 0.27

$$\frac{2c^2\left(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451\right)\sqrt{c-acx}}{35a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*Sqrt[c - a*c*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4*
x^4))/(35*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.052, size = 72, normalized size = 0.3

$$\frac{(2\,ax+2)\left(5\,x^4a^4-36\,x^3a^3+142\,a^2x^2-708\,ax-1451\right)}{35\,a\,(ax-1)^4}(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/35*(a*x+1)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-708*a*x-1451)*(-a*c*x+c)^(5/
2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^4
```

Maxima [A] time = 1.11333, size = 162, normalized size = 0.64

$$\frac{2\left(5\,a^5\sqrt{-c}c^2x^5-31\,a^4\sqrt{-c}c^2x^4+106\,a^3\sqrt{-c}c^2x^3-566\,a^2\sqrt{-c}c^2x^2-2159\,a\sqrt{-c}c^2x-1451\,\sqrt{-c}c^2\right)(ax-1)^2}{35\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^5*sqrt(-c)*c^2*x^5 - 31*a^4*sqrt(-c)*c^2*x^4 + 106*a^3*sqrt(-c)*c
^2*x^3 - 566*a^2*sqrt(-c)*c^2*x^2 - 2159*a*sqrt(-c)*c^2*x - 1451*sqrt(-c)*c
^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A] time = 1.50044, size = 185, normalized size = 0.73

$$\frac{2\left(5\,a^4c^2x^4-36\,a^3c^2x^3+142\,a^2c^2x^2-708\,ac^2x-1451\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 1451
*c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29361, size = 138, normalized size = 0.54

$$-\frac{2\left(5\,(acx+c)^3\sqrt{-acx-c}-56\,(acx+c)^2\sqrt{-acx-c}c-280\,(-acx-c)^{\frac{3}{2}}c^2-1120\,\sqrt{-acx-c}c^3+\frac{560\,c^4}{\sqrt{-acx-c}}\right)|c|}{35\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(a*c*x + c)^3*sqrt(-a*c*x - c) - 56*(a*c*x + c)^2*sqrt(-a*c*x - c)
*c - 280*(-a*c*x - c)^(3/2)*c^2 - 1120*sqrt(-a*c*x - c)*c^3 + 560*c^4/sqrt(
-a*c*x - c))*abs(c)/(a*c^2)
```

3.273 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{184(c-acx)^{3/2}}{5a^3x^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^3(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{16(c-acx)^{3/2}}{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (-8*(c - a*c*x)^(3/2))/(5*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + (184*(
c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x^2) + (16*(
c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x
^(-1))^3*x*(c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.191599, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6176, 6181, 94, 89, 78, 37}

$$\frac{184(c-acx)^{3/2}}{5a^3x^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^3(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{16(c-acx)^{3/2}}{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-8*(c - a*c*x)^(3/2))/(5*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + (184*(
c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x^2) + (16*(
c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x
^(-1))^3*x*(c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3 x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(12\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^3 x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(8\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{-}{x^{3/2}}\right.}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\frac{1}{x}\right)^3 x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(92\left(\frac{1}{x}\right)^{3}\right.}{}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2} + \frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x} + \frac{2\left(a-\right.}{5a^3\left(1\right.}$$

Mathematica [A] time = 0.0322588, size = 57, normalized size = 0.29

$$-\frac{2c\left(a^3x^3-7a^2x^2+43ax+91\right)\sqrt{c-acx}}{5a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*c*Sqrt[c - a*c*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a^2*Sqrt[1 -
1/(a^2*x^2)]*x)
```

Maple [A] time = 0.043, size = 63, normalized size = 0.3

$$\frac{(2\,ax+2)\left(x^3a^3-7\,a^2x^2+43\,ax+91\right)}{5\,a\,(ax-1)^3}\,(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/5*(a*x+1)*(a^3*x^3-7*a^2*x^2+43*a*x+91)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1)
)^(3/2)/a/(a*x-1)^3
```

Maxima [A] time = 1.07883, size = 126, normalized size = 0.65

$$-\frac{2\left(a^4\sqrt{-c}cx^4-6\,a^3\sqrt{-c}cx^3+36\,a^2\sqrt{-c}cx^2+134\,a\sqrt{-c}cx+91\,\sqrt{-c}c\right)(ax-1)^2}{5\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^4*sqrt(-c)*c*x^4 - 6*a^3*sqrt(-c)*c*x^3 + 36*a^2*sqrt(-c)*c*x^2 + 1
34*a*sqrt(-c)*c*x + 91*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1)^(3/2))
```

Fricas [A] time = 1.56261, size = 142, normalized size = 0.73

$$-\frac{2\left(a^3cx^3-7\,a^2cx^2+43\,acx+91\,c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20292, size = 104, normalized size = 0.53

$$\frac{2\left((acx+c)^2\sqrt{-acx-c}+10\,(-acx-c)^{\frac{3}{2}}c+60\,\sqrt{-acx-c}c^2-\frac{40\,c^3}{\sqrt{-acx-c}}\right)|c|}{5\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*((a*c*x + c)^2*sqrt(-a*c*x - c) + 10*(-a*c*x - c)^(3/2)*c + 60*sqrt(-a*
c*x - c)*c^2 - 40*c^3/sqrt(-a*c*x - c))*abs(c)/(a*c^2)
```

3.274 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=137

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.155477, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 89, 78, 37}

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] time = 0.0268875, size = 48, normalized size = 0.35

$$\frac{2\left(a^2x^2-10ax-23\right)\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

Maple [A] time = 0.049, size = 55, normalized size = 0.4

$$\frac{(2\,ax+2)\left(a^2x^2-10\,ax-23\right)}{3\,a\,(ax-1)^2}\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a/
(a*x-1)^2
```

Maxima [A] time = 1.0863, size = 101, normalized size = 0.74

$$\frac{2\left(a^{3}\sqrt{-c}x^{3}-9\,a^{2}\sqrt{-c}x^{2}-33\,a\sqrt{-c}x-23\,\sqrt{-c}\right)(ax-1)^{2}}{3\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A] time = 1.58946, size = 113, normalized size = 0.82

$$\frac{2\left(a^{2}x^{2}-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19042, size = 70, normalized size = 0.51

$$\frac{2\left((-acx-c)^{\frac{3}{2}}+12\sqrt{-acx-c}\,c-\frac{12\,c^2}{\sqrt{-acx-c}}\right)|c|}{3\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*((-a*c*x - c)^(3/2) + 12*sqrt(-a*c*x - c)*c - 12*c^2/sqrt(-a*c*x - c))*
abs(c)/(a*c^2)
```

3.275 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=85

$$\frac{2x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}+\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1
/(a*x)]*x)/(Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.143442, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}+\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1
/(a*x)]*x)/(Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}\\
&= -\frac{\sqrt{1-\frac{1}{ax}}\,\mathrm{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}\\
&= \frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}\\
&= \frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}
\end{aligned}
$$

Mathematica [A] time = 0.0312703, size = 48, normalized size = 0.56

$$
\frac{2\sqrt{1-\frac{1}{ax}}(ax+3)}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*(3 + a*x))/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.041, size = 47, normalized size = 0.6

$$2\,\frac{(ax+3)\,(ax+1)}{(ax-1)\,a\sqrt{-acx+c}}\left(\frac{ax-1}{ax+1}\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(a*x+1)*(a*x+3)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)/(-a*c*x+c)^(1/2)
```

Maxima [A] time = 1.08734, size = 65, normalized size = 0.76

$$\frac{2\left(a^2x^2+4\,ax+3\right)(ax-1)}{\left(a^2\sqrt{-c}x-a\sqrt{-c}\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(a^2*x^2 + 4*a*x + 3)*(a*x - 1)/((a^2*sqrt(-c)*x - a*sqrt(-c))*(a*x + 1)^
(3/2))
```

Fricas [A] time = 1.53825, size = 99, normalized size = 1.16

$$-\frac{2\sqrt{-acx+c}(ax+3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*c*x + c)*(a*x + 3)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18049, size = 49, normalized size = 0.58

$$\frac{2\left(\sqrt{-acx-c}-\frac{2c}{\sqrt{-acx-c}}\right)|c|}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(-a*c*x - c) - 2*c/sqrt(-a*c*x - c))*abs(c)/(a*c^2)
```

3.276 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

```
[Out] (-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.0380544, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {6174}

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]

[Out] (-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

Mathematica [A] time = 0.0286373, size = 41, normalized size = 1.41

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}\,(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))
```

Maple [A] time = 0.046, size = 35, normalized size = 1.2

$$-2\,{\frac {ax+1}{a \left(-acx+c \right) ^{3/2}} \left({\frac {ax-1}{ax+1}} \right) ^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

Maxima [A] time = 1.09125, size = 61, normalized size = 2.1

$$-\frac{2 \left(a \sqrt{-c} x + \sqrt{-c}\right)(a x - 1)}{\left(a^{2} c^{2} x - a c^{2}\right)(a x + 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))*(a*x - 1)/((a^2*c^2*x - a*c^2)*(a*x + 1)^(3/2)
)
```

Fricas [A] time = 1.54254, size = 90, normalized size = 3.1

$$-\frac{2 \sqrt{-a c x + c} \sqrt{\frac{a x - 1}{a x + 1}}}{a^{2} c^{2} x - a c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15873, size = 55, normalized size = 1.9

$$\frac{\left(\frac{\sqrt{2}}{a\sqrt{-c}} - \frac{2}{\sqrt{-acx-ca}}\right)|c|\mathrm{sgn}\,(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)/(a*sqrt(-c)) - 2/(sqrt(-a*c*x - c)*a))*abs(c)*sgn(a*x + 1)/c^2
```

3.277 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{ax^2\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] (a*(1 - 1/(a*x))^(5/2)*x^2)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2)) - (a^(3/2
)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])])/(Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.179631, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{ax^2\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```


```
[Out] (a*(1 - 1/(a*x))^(5/2)*x^2)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2)) - (a^(3/2
)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])])/(Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2} x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2} x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2} x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}} - \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] time = 0.0779689, size = 122, normalized size = 1.02

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}-\sqrt{2}\sqrt{a}\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{2ac^2\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)] - Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*ArcT
anh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*a*c^2*Sqrt[1 +
 1/(a*x)]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.149, size = 85, normalized size = 0.7

$$-\frac{ax+1}{2\left(ax-1\right)^{2}a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)}\left(\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-c\left(ax+1\right)}\frac{1}{\sqrt{c}}\right)\sqrt{2}\sqrt{-c\left(ax+1\right)}+2\sqrt{c}\right)c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)/c^(7/2)*(
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)+2
*c^(1/2))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-acx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(5/2), x)
```

Fricas [A] time = 1.6623, size = 560, normalized size = 4.67

$$\left[-\frac{\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+4\,\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{4\left(a^{2}c^{3}x-ac^{3}\right)},\ \frac{\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\ldots}}{acx-c\ldots}\right.}{2\left(a^{2}c^{3}x-\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
```

```
- 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x -
a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*sqrt((a*x -
 1)/(a*x + 1)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15681, size = 134, normalized size = 1.12

$$-\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)+\sqrt{c}\right)}{a\sqrt{-c}c^{\frac{3}{2}}}+\frac{2}{\sqrt{-acx-c}ac}\right)|c|\mathrm{sgn}\left(ax+1\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(3/2)) - sq
rt(2)*(sqrt(-c)*arctan(sqrt(-c)/sqrt(c)) + sqrt(c))/(a*sqrt(-c)*c^(3/2)) +
2/(sqrt(-a*c*x - c)*a*c))*abs(c)*sgn(a*x + 1)/c^2
```

3.278 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{3a^2x^3\left(1-\frac{1}{ax}\right)^{7/2}}{4\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}+\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -(a^2*(1 - 1/(a*x))^(7/2)*x^2)/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - a*c*x
)^(7/2)) - (3*a^2*(1 - 1/(a*x))^(7/2)*x^3)/(4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)
^(7/2)) + (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rubi [A] time = 0.197943, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{3a^2x^3\left(1-\frac{1}{ax}\right)^{7/2}}{4\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}+\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```


```
[Out] -(a^2*(1 - 1/(a*x))^(7/2)*x^2)/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - a*c*x
)^(7/2)) - (3*a^2*(1 - 1/(a*x))^(7/2)*x^3)/(4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)
^(7/2)) + (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2} x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,d\right.}{8\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2} x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\surd}{\sqrt{1}}\right.}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2} x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2} x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Mathematica [A] time = 0.141503, size = 140, normalized size = 0.76

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}(3ax-1)-3\sqrt{2}\sqrt{a}\sqrt{\frac{1}{ax}+1}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{8ac^3\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)]*(-1 + 3*a*x) - 3*Sqrt[2]*Sqrt[a]*Sqrt[1
+ 1/(a*x)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
```

```
*x)])]))/(8*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(-1 + a*x)*Sqrt[c - a*c*x]
)
```

Maple [A] time = 0.167, size = 129, normalized size = 0.7

$$-\frac{ax+1}{8\,(ax-1)^3\,a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}\left(3\,\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}xa\sqrt{-c\,(ax+1)}-3\,\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(-c*(a*x-1))^(1/2)/c^(9/2)*(
3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*x*a*(-c*(a*x+1))^(
1/2)-3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^
(1/2)+6*x*a*c^(1/2)-2*c^(1/2))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(7/2), x)
```

Fricas [A] time = 1.58982, size = 675, normalized size = 3.67

$$\left[-\frac{3\,\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}(3\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{16\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)},\ \frac{3\,\sqrt{2}\left(a^2x^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1
)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^2*x^2
- 2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21681, size = 122, normalized size = 0.66

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}}-\frac{2\,(3\,acx-c)}{\left((-acx-c)^{\frac{3}{2}}+2\sqrt{-acx-c}c\right)ac^2}\right)|c|\mathrm{sgn}\,(ax+1)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) -
2*(3*a*c*x - c)/(((-a*c*x - c)^(3/2) + 2*sqrt(-a*c*x - c)*c)*a*c^2))*abs(c)
*sgn(a*x + 1)/c^2
```

3.279 $\int e^{\coth^{-1}(x)}x(1+x)\,dx$

Optimal. Leaf size=99

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{1}{3}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x + ((1 + x^(-1))^(3/2)*Sqrt[(-1 + x)/x]*
x^2)/3 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + ArcTanh[Sqrt[1 + x^(
-1)]*Sqrt[(-1 + x)/x]]
```

Rubi [A] time = 0.0697388, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6175, 6180, 96, 94, 92, 206}

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{1}{3}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x),x]
```


```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x + ((1 + x^(-1))^(3/2)*Sqrt[(-1 + x)/x]*
x^2)/3 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + ArcTanh[Sqrt[1 + x^(
-1)]*Sqrt[(-1 + x)/x]]
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x^2\,dx$$
$$= -\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \frac{2}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x + \frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.$$
$$= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x + \frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 + \text{Subst}\left(\int \frac{1}{1-x^2}\right.$$
$$= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x + \frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right.$$

Mathematica [A] time = 0.0419161, size = 41, normalized size = 0.41

$$\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\left(x^2+3x+5\right)+\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x),x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(5 + 3*x + x^2))/3 + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [A] time = 0.106, size = 67, normalized size = 0.7

$$\frac{-1+x}{3}\left(\left((1+x)(-1+x)\right)^{\frac{3}{2}}+3x\sqrt{x^2-1}+6\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x)
```

```
[Out] 1/3*(-1+x)*(((1+x)*(-1+x))^(3/2)+3*x*(x^2-1)^(1/2)+6*(x^2-1)^(1/2)+3*ln(x+(
x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [A] time = 0.995801, size = 149, normalized size = 1.51

$$-\frac{2\left(3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="maxima")
```

```
[Out] -2/3*(3*((x - 1)/(x + 1))^(5/2) - 8*((x - 1)/(x + 1))^(3/2) + 9*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.73753, size = 158, normalized size = 1.6

$$\frac{1}{3}\left(x^3+4x^2+8x+5\right)\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 4*x^2 + 8*x + 5)*sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1
)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(x+1)}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x),x)
```

```
[Out] Integral(x*(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Giac [A] time = 1.1533, size = 142, normalized size = 1.43

$$\frac{2\left(\frac{8(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\frac{3(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}-9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^3}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="giac")
```

```
[Out] 2/3*(8*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 3*(x - 1)^2*sqrt((x - 1)/(x
+ 1))/(x + 1)^2 - 9*sqrt((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^3 + log(sq
rt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.280 $\int e^{\coth^{-1}(x)}(1+x)\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{3}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] (3*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + ((1 + x^(-1))^(3/2)*Sqrt[(-1 +
x)/x]*x^2)/2 + (3*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rubi [A] time = 0.046782, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6175, 6180, 94, 92, 206}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{3}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x),x]
```

```
[Out] (3*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + ((1 + x^(-1))^(3/2)*Sqrt[(-1 +
x)/x]*x^2)/2 + (3*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1+x)\,dx &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x\,dx \\
&= -\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2 - \frac{3}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2 - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2 + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right) \\
&= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2 + \frac{3}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0240794, size = 40, normalized size = 0.51

$$
\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x(x+4) + \frac{3}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 + x),x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(4 + x))/2 + (3*Log[(1 + Sqrt[1 - x^(-2)])*x])/2
```

Maple [A] time = 0.104, size = 57, normalized size = 0.7

$$\frac{-1+x}{2}\left(x\sqrt{x^2-1}+4\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x),x)
```

```
[Out] 1/2*(-1+x)*(x*(x^2-1)^(1/2)+4*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/((-1+x)/
(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [A] time = 1.04258, size = 117, normalized size = 1.48

$$\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="maxima")
```

```
[Out] (3*((x - 1)/(x + 1))^(3/2) - 5*sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) -
(x - 1)^2/(x + 1)^2 - 1) + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(sqr
t((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.91277, size = 158, normalized size = 2.

$$\frac{1}{2}\left(x^2+5x+4\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 5*x + 4)*sqrt((x - 1)/(x + 1)) + 3/2*log(sqrt((x - 1)/(x + 1)) +
 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x),x)
```

```
[Out] Integral((x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Giac [A] time = 1.11125, size = 113, normalized size = 1.43

$$-\frac{\frac{3\,(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-5\,\sqrt{\frac{x-1}{x+1}}}{\left(\frac{x-1}{x+1}-1\right)^2}+\frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\,\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="giac")
```

```
[Out] -(3*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 5*sqrt((x - 1)/(x + 1)))/((x -
1)/(x + 1) - 1)^2 + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(abs(sqrt((
x - 1)/(x + 1)) - 1))
```

3.281 $\int e^{\coth^{-1}(x)}(1-x)x\,dx$

Optimal. Leaf size=18

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[Out] -((1 - x^(-2))^(3/2)*x^3)/3
```

Rubi [A] time = 0.0525726, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6175, 6178, 264}

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)*x,x]
```

```
[Out] -((1 - x^(-2))^(3/2)*x^3)/3
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x^2\,dx$$
$$= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Mathematica [A] time = 0.0199014, size = 21, normalized size = 1.17

$$-\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)*x,x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x*(-1 + x^2))/3
```

Maple [A] time = 0.06, size = 22, normalized size = 1.2

$$-\frac{(1+x)(-1+x)^2}{3}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x)
```

```
[Out] -1/3*(1+x)*(-1+x)^2/((-1+x)/(1+x))^(1/2)
```

Maxima [B] time = 1.01459, size = 68, normalized size = 3.78

$$\frac{8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="maxima")
```

```
[Out] 8/3*((x - 1)/(x + 1))^(3/2)/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x
 - 1)^3/(x + 1)^3 - 1)
```

Fricas [A] time = 1.83827, size = 65, normalized size = 3.61

$$-\frac{1}{3}\left(x^3+x^2-x-1\right)\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + x^2 - x - 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx-\int \frac{x^2}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)*x,x)
```

```
[Out] -Integral(-x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(x**2/sqrt(x/(x + 1)
 - 1/(x + 1)), x)
```

Giac [B] time = 1.16763, size = 39, normalized size = 2.17

$$\frac{8}{3\left(\sqrt{\frac{x-1}{x+1}}-\frac{1}{\sqrt{\frac{x-1}{x+1}}}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="giac")
```

```
[Out] 8/3/(sqrt((x - 1)/(x + 1)) - 1/sqrt((x - 1)/(x + 1)))^3
```

3.282 $\int e^{\coth^{-1}(x)}(1-x)\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2$$

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + ArcTanh[Sqrt[1 - x^(-2)]]/2
```

Rubi [A] time = 0.0466511, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6175, 6178, 266, 47, 63, 206}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x),x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + ArcTanh[Sqrt[1 - x^(-2)]]/2
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)\,dx &= -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0197955, size = 39, normalized size = 1.11

$$\frac{1}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x),x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Maple [A] time = 0.101, size = 48, normalized size = 1.4

$$-\frac{-1+x}{2}\left(x\sqrt{x^2-1}-\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x),x)
```

```
[Out] -1/2*(-1+x)*(x*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+
x)*(-1+x))^(1/2)
```

Maxima [B] time = 1.0252, size = 112, normalized size = 3.2

$$\frac{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="maxima")
```

```
[Out] (((x - 1)/(x + 1))^(3/2) + sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x
 - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.82398, size = 151, normalized size = 4.31

$$-\frac{1}{2}\left(x^2+x\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + x)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) -
 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx-\int-\frac{1}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x),x)
```

```
[Out] -Integral(x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(-1/sqrt(x/(x + 1) -
1/(x + 1)), x)
```

Giac [B] time = 1.14782, size = 149, normalized size = 4.26

$$-\frac{\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}}{\left(\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}\right)^2-4}+\frac{1}{4}\log\left(\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}+2\right)-\frac{1}{4}\log\left(\left|\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="giac")
```

```
[Out] -(sqrt((x - 1)/(x + 1)) + 1/sqrt((x - 1)/(x + 1)))/((sqrt((x - 1)/(x + 1))
+ 1/sqrt((x - 1)/(x + 1)))^2 - 4) + 1/4*log(sqrt((x - 1)/(x + 1)) + 1/sqrt(
(x - 1)/(x + 1)) + 2) - 1/4*log(abs(sqrt((x - 1)/(x + 1)) + 1/sqrt((x - 1)/
(x + 1)) - 2))
```

3.283 $\int e^{\coth^{-1}(x)}x(1+x)^2\,dx$

Optimal. Leaf size=133

$$\frac{1}{4}\left(\frac{1}{x}+1\right)^{7/2}\sqrt{\frac{x-1}{x}}x^4+\frac{1}{4}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{5}{8}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{15}{8}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\frac{15}{8}\tanh^{-1}\left(\sqrt{\frac{1}{x}+\right.$$

```
[Out] (15*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/8 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
 + x)/x]*x^2)/8 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/4 + ((1 + x^(-1
))^(7/2)*Sqrt[(-1 + x)/x]*x^4)/4 + (15*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]])/8
```

Rubi [A] time = 0.111144, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6175, 6180, 96, 94, 92, 206}

$$\frac{1}{4}\left(\frac{1}{x}+1\right)^{7/2}\sqrt{\frac{x-1}{x}}x^4+\frac{1}{4}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{5}{8}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{15}{8}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\frac{15}{8}\tanh^{-1}\left(\sqrt{\frac{1}{x}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^2,x]
```

```
[Out] (15*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/8 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
 + x)/x]*x^2)/8 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/4 + ((1 + x^(-1
))^(7/2)*Sqrt[(-1 + x)/x]*x^4)/4 + (15*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]])/8
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
```

```
m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}x(1+x)^2\,dx &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^3\,dx \\
&= -\operatorname{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}x^5}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4 - \frac{3}{4}\operatorname{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4 - \frac{5}{4}\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4 - \frac{15}{8}\operatorname{Subst} \\
&= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2} \\
&= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2} \\
&= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}
\end{aligned}
$$

Mathematica [A] time = 0.0441036, size = 52, normalized size = 0.39

$$
\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x\left(2x^3+8x^2+15x+24\right)+\frac{15}{8}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(24 + 15*x + 8*x^2 + 2*x^3))/8 + (15*Log[(1 + Sqrt[1 -
x^(-2)])*x])/8
```

Maple [A] time = 0.107, size = 79, normalized size = 0.6

$$
\frac{-1+x}{8}\left(2\,x\left(x^2-1\right)^{3/2}+8\,\left((1+x)\,(-1+x)\right)^{3/2}+17\,x\sqrt{x^2-1}+32\,\sqrt{x^2-1}+15\,\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)\,(-1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x)
```

```
[Out] 1/8*(-1+x)*(2*x*(x^2-1)^(3/2)+8*((1+x)*(-1+x))^(3/2)+17*x*(x^2-1)^(1/2)+32*
(x^2-1)^(1/2)+15*ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(
1/2)
```

Maxima [A] time = 1.03094, size = 186, normalized size = 1.4

$$\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-55\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+73\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(15*((x - 1)/(x + 1))^(7/2) - 55*((x - 1)/(x + 1))^(5/2) + 73*((x - 1)/
(x + 1))^(3/2) - 49*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 15/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.88403, size = 190, normalized size = 1.43

$$\frac{1}{8}\left(2x^4+10x^3+23x^2+39x+24\right)\sqrt{\frac{x-1}{x+1}}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 + 10*x^3 + 23*x^2 + 39*x + 24)*sqrt((x - 1)/(x + 1)) + 15/8*log(
sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**2,x)
```

```
[Out] Integral(x*(x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [A] time = 1.14158, size = 176, normalized size = 1.32

$$-\frac{\frac{73\,(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\frac{55\,(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}+\frac{15\,(x-1)^3\sqrt{\frac{x-1}{x+1}}}{(x+1)^3}-49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{x-1}{x+1}-1\right)^4}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(73*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 55*(x - 1)^2*sqrt((x - 1)/
(x + 1))/(x + 1)^2 + 15*(x - 1)^3*sqrt((x - 1)/(x + 1))/(x + 1)^3 - 49*sqrt
((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^4 + 15/8*log(sqrt((x - 1)/(x + 1))
 + 1) - 15/8*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.284 $\int e^{\coth^{-1}(x)}(1+x)^2\,dx$

Optimal. Leaf size=106

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{5}{6}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{5}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\frac{5}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] (5*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
+ x)/x]*x^2)/6 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + (5*ArcTanh[S
qrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rubi [A] time = 0.0901089, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6175, 6180, 94, 92, 206}

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}x^3+\frac{5}{6}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}x^2+\frac{5}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x+\frac{5}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (5*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
+ x)/x]*x^2)/6 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + (5*ArcTanh[S
qrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^2\,dx$$

$$= -\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \frac{5}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \frac{5}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 - \frac{5}{2}\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 + \frac{5}{2}\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3 + \frac{5}{2}\tanh^{-1}\left(\sqrt{1}\right.$$

Mathematica [A] time = 0.0315092, size = 47, normalized size = 0.44

$$\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(2x^2+9x+22\right)+\frac{5}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(22 + 9*x + 2*x^2))/6 + (5*Log[(1 + Sqrt[1 - x^(-2)])*x
])/2
```

Maple [A] time = 0.117, size = 69, normalized size = 0.7

$$\frac{-1+x}{6}\left(2\left((1+x)(-1+x)\right)^{3/2}+9x\sqrt{x^2-1}+24\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x)
```

```
[Out] 1/6*(-1+x)*(2*((1+x)*(-1+x))^(3/2)+9*x*(x^2-1)^(1/2)+24*(x^2-1)^(1/2)+15*ln
(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [A] time = 1.03422, size = 151, normalized size = 1.42

$$-\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-40\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*((x - 1)/(x + 1))^(5/2) - 40*((x - 1)/(x + 1))^(3/2) + 33*sqrt((x
- 1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x +
1)^3 - 1) + 5/2*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x +
```

```
1)) - 1)
```

Fricas [A] time = 1.83188, size = 176, normalized size = 1.66

$$\frac{1}{6}\left(2x^3+11x^2+31x+22\right)\sqrt{\frac{x-1}{x+1}}+\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 + 11*x^2 + 31*x + 22)*sqrt((x - 1)/(x + 1)) + 5/2*log(sqrt((x -
1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**2,x)
```

```
[Out] Integral((x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [A] time = 1.16756, size = 144, normalized size = 1.36

$$\frac{\frac{40(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\frac{15(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}-33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{x-1}{x+1}-1\right)^3}+\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/3*(40*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 15*(x - 1)^2*sqrt((x - 1)/(
x + 1))/(x + 1)^2 - 33*sqrt((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^3 + 5/2
*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.285 $\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx$

Optimal. Leaf size=71

$$\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] (Sqrt[1 - x^(-2)]*x^2)/8 - ((1 - x^(-2))^(3/2)*x^3)/3 + ((1 - x^(-2))^(3/2)
*x^4)/4 - ArcTanh[Sqrt[1 - x^(-2)]]/8
```

Rubi [A] time = 0.116344, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6175, 6178, 835, 807, 266, 47, 63, 206}

$$\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x^2)/8 - ((1 - x^(-2))^(3/2)*x^3)/3 + ((1 - x^(-2))^(3/2)
*x^4)/4 - ArcTanh[Sqrt[1 - x^(-2)]]/8
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^2x\,dx &= \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2x^3\,dx\\
&= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^5}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4+\frac{1}{4}\text{Subst}\left(\int \frac{(4-x)\sqrt{1-x^2}}{x^4}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{8}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4+\frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{x^2}}\right)\\
&= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0391362, size = 52, normalized size = 0.73

$$
\frac{1}{24}\sqrt{1-\frac{1}{x^2}}x\left(6x^3-8x^2-3x+8\right)-\frac{1}{8}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(8 - 3*x - 8*x^2 + 6*x^3))/24 - Log[(1 + Sqrt[1 - x^(-2
)])*x]/8
```

Maple [A] time = 0.118, size = 70, normalized size = 1.

$$
\frac{-1+x}{24}\left(6\,x\left(x^2-1\right)^{3/2}-8\,\left((1+x)\,(-1+x)\right)^{3/2}+3\,x\sqrt{x^2-1}-3\,\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)\,(-1+x)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x)
```

```
[Out] 1/24*(-1+x)*(6*x*(x^2-1)^(3/2)-8*((1+x)*(-1+x))^(3/2)+3*x*(x^2-1)^(1/2)-3*l
n(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [B] time = 1.07647, size = 186, normalized size = 2.62

$$-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}+53\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-11\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}-\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="maxima")
```

```
[Out] -1/12*(3*((x - 1)/(x + 1))^(7/2) + 53*((x - 1)/(x + 1))^(5/2) - 11*((x - 1)
/(x + 1))^(3/2) + 3*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 2.00583, size = 185, normalized size = 2.61

$$\frac{1}{24}\left(6x^4-2x^3-11x^2+5x+8\right)\sqrt{\frac{x-1}{x+1}}-\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 - 11*x^2 + 5*x + 8)*sqrt((x - 1)/(x + 1)) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x (x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2*x,x)
```

```
[Out] Integral(x*(x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [B] time = 1.17352, size = 176, normalized size = 2.48

$$-\frac{\frac{11 (x-1)\sqrt{\frac{x-1}{x+1}}}{x+1} - \frac{53 (x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2} - \frac{3 (x-1)^3\sqrt{\frac{x-1}{x+1}}}{(x+1)^3} - 3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{x-1}{x+1}-1\right)^4} - \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + \frac{1}{8}\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="giac")
```

```
[Out] -1/12*(11*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 53*(x - 1)^2*sqrt((x - 1)
/(x + 1))/(x + 1)^2 - 3*(x - 1)^3*sqrt((x - 1)/(x + 1))/(x + 1)^3 - 3*sqrt(
(x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^4 - 1/8*log(sqrt((x - 1)/(x + 1)) +
 1) + 1/8*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.286 $\int e^{\coth^{-1}(x)}(1-x)^2\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + ((1 - x^(-2))^(3/2)*x^3)/3 + ArcTanh[Sqrt[1 - x
^(-2)]]/2
```

Rubi [A] time = 0.0904612, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6175, 6178, 807, 266, 47, 63, 206}

$$\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^2,x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + ((1 - x^(-2))^(3/2)*x^3)/3 + ArcTanh[Sqrt[1 - x
^(-2)]]/2
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^2\,dx &= \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2 x^2\,dx \\
&= -\,\mathrm{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \mathrm{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0318873, size = 47, normalized size = 0.89

$$
\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(2x^2-3x-2\right)+\frac{1}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(-2 - 3*x + 2*x^2))/6 + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Maple [A] time = 0.113, size = 60, normalized size = 1.1

$$
\frac{-1+x}{6}\left(2\left((1+x)(-1+x)\right)^{3/2}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)(-1+x)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x)
```

```
[Out] 1/6*(-1+x)*(2*((1+x)*(-1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))
/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [B] time = 1.12716, size = 151, normalized size = 2.85

$$-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*((x - 1)/(x + 1))^(5/2) + 8*((x - 1)/(x + 1))^(3/2) - 3*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1))
 - 1)
```

Fricas [A] time = 1.57553, size = 169, normalized size = 3.19

$$\frac{1}{6}\left(2x^3-x^2-5x-2\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 - x^2 - 5*x - 2)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x
 + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2,x)
```

```
[Out] Integral((x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [B] time = 1.13403, size = 144, normalized size = 2.72

$$-\frac{\frac{8\,(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}+\frac{3\,(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}-3\,\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{x-1}{x+1}-1\right)^3}+\frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\,\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="giac")
```

```
[Out] -1/3*(8*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) + 3*(x - 1)^2*sqrt((x - 1)/(x
 + 1))/(x + 1)^2 - 3*sqrt((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^3 + 1/2*l
og(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.287 $\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx$

Optimal. Leaf size=22

$$\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x
```

Rubi [A] time = 0.0536301, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6175, 6179, 95}

$$\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}x$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x),x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
```

```
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{1+\frac{1}{x}}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x^2\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x\end{aligned}$$

Mathematica [A] time = 0.0233141, size = 15, normalized size = 0.68

$$x\sqrt{\frac{x^2-1}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x),x]
```

```
[Out] x*Sqrt[(-1 + x^2)/x^2]
```

Maple [A] time = 0.067, size = 16, normalized size = 0.7

$$(-1+x)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x)
```

```
[Out] (-1+x)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.02722, size = 35, normalized size = 1.59

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Fricas [A] time = 1.53454, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)), x)
```

Giac [A] time = 1.16971, size = 39, normalized size = 1.77

$$-\frac{2}{\sqrt{\frac{x-1}{x+1}}-\frac{1}{\sqrt{\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="giac")
```

```
[Out] -2/(sqrt((x - 1)/(x + 1)) - 1/sqrt((x - 1)/(x + 1)))
```

3.288 $\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx$

Optimal. Leaf size=22

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]]
```

Rubi [A] time = 0.0660964, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6175, 6180, 92, 206}

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x),x]
```

```
[Out] ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)x}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right) \\
&= \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0094405, size = 18, normalized size = 0.82

$$\log\left(x\left(\sqrt{\frac{x^2-1}{x^2}}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x),x]
```

```
[Out] Log[x*(1 + Sqrt[(-1 + x^2)/x^2])]
```

Maple [A] time = 0.126, size = 35, normalized size = 1.6

$$(-1+x)\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x),x)
```

```
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)/((1+x)*(-1+x))^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 1.00396, size = 42, normalized size = 1.91

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.57594, size = 88, normalized size = 4.

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [A] time = 20.6437, size = 29, normalized size = 1.32

$$-\log\left(-1+\frac{1}{\sqrt{1-\frac{2}{x+1}}}\right)+\log\left(1+\frac{1}{\sqrt{1-\frac{2}{x+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x),x)
```

```
[Out] -log(-1 + 1/sqrt(1 - 2/(x + 1))) + log(1 + 1/sqrt(1 - 2/(x + 1)))
```

Giac [A] time = 1.14995, size = 43, normalized size = 1.95

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.289 $\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx$

Optimal. Leaf size=47

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}x-2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1 -
 x^(-2)]]
```

Rubi [A] time = 0.135316, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6175, 6177, 852, 1805, 807, 266, 63, 206}

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}x-2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x),x]
```



```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1 -
 x^(-2)]]
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx &= -\int \frac{e^{\coth^{-1}(x)}}{1-\frac{1}{x}}\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x^2\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{-1-2x}{x^2\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x + 2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}x - 2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0521181, size = 41, normalized size = 0.87

$$-\frac{\sqrt{1-\frac{1}{x^2}}(x-3)x}{x-1} - 2\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x),x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*(-3 + x)*x)/(-1 + x)) - 2*Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [B] time = 0.118, size = 106, normalized size = 2.3

$$\frac{1}{-1+x}\left(\left(x^2-1\right)^{\frac{3}{2}}-2x^2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2-1}\right)x^2+4x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x)
```

```
[Out] ((x^2-1)^(3/2)-2*x^2*(x^2-1)^(1/2)-2*ln(x+(x^2-1)^(1/2))*x^2+4*x*(x^2-1)^(1
/2)+4*ln(x+(x^2-1)^(1/2))*x-2*(x^2-1)^(1/2)-2*ln(x+(x^2-1)^(1/2)))/(-1+x)/(
(1+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.03955, size = 100, normalized size = 2.13

$$\frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-\sqrt{\frac{x-1}{x+1}}}-2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+2\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="maxima")
```

```
[Out] 2*(2*(x - 1)/(x + 1) - 1)/(((x - 1)/(x + 1))^(3/2) - sqrt((x - 1)/(x + 1)))
 - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.67737, size = 184, normalized size = 3.91

$$-\frac{2(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-2(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)+\left(x^2-2x-3\right)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="fricas")
```

```
[Out] -(2*(x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 2*(x - 1)*log(sqrt((x - 1)/(x
+ 1)) - 1) + (x^2 - 2*x - 3)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x),x)
```

```
[Out] -Integral(x/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

Giac [B] time = 1.15349, size = 113, normalized size = 2.4

$$\frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\frac{(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\sqrt{\frac{x-1}{x+1}}}-2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+2\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="giac")
```

```
[Out] 2*(2*(x - 1)/(x + 1) - 1)/((x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - sqrt((x
- 1)/(x + 1))) - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(abs(sqrt((x - 1)/
(x + 1)) - 1))
```

3.290 $\int \frac{e^{\coth^{-1}(x)}}{1-x} dx$

Optimal. Leaf size=33

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi [A] time = 0.131617, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6175, 6178, 852, 1805, 266, 63, 206}

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x),x]
```

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx &= -\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)x}\,dx \\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2x}\,dx, x, \frac{1}{x}\right) \\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} + \text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0227887, size = 38, normalized size = 1.15

$$
\frac{2\sqrt{1-\frac{1}{x^2}}x}{x-1} - \log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 - x),x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x)/(-1 + x) - Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [B] time = 0.12, size = 106, normalized size = 3.2

$$
\frac{1}{-1+x}\left(\left(x^2-1\right)^{\frac{3}{2}} - x^2\sqrt{x^2-1} - \ln\left(x+\sqrt{x^2-1}\right)x^2 + 2\,x\sqrt{x^2-1} + 2\,\ln\left(x+\sqrt{x^2-1}\right)x - \sqrt{x^2-1} - \ln\left(x+\sqrt{x^2-1}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x),x)
```

```
[Out] ((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2))*x^2+2*x*(x^2-1)^(1/2)+
2*ln(x+(x^2-1)^(1/2))*x-(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/(-1+x)/((1+x)*(-
1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.00447, size = 59, normalized size = 1.79

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}} - \log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) + \log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="maxima")
```

```
[Out] 2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(sqrt((x - 1)
/(x + 1)) - 1)
```

Fricas [B] time = 1.58125, size = 170, normalized size = 5.15

$$-\frac{(x-1)\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - (x-1)\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right) - 2(x+1)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="fricas")
```

```
[Out] -((x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - (x - 1)*log(sqrt((x - 1)/(x + 1
) - 1) - 2*(x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{\frac{x}{x+1} - \frac{1}{x+1}} - \sqrt{\frac{x}{x+1} - \frac{1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x),x)
```

```
[Out] -Integral(1/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

Giac [A] time = 1.16523, size = 61, normalized size = 1.85

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}} - \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + \log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="giac")
```

```
[Out] 2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(abs(sqrt((x
- 1)/(x + 1)) - 1))
```

3.291 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx$

Optimal. Leaf size=45

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)-\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

```
[Out] -(Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]]
```

Rubi [A] time = 0.0826707, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6175, 6180, 96, 92, 206}

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)-\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

```
[Out] -(Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
```

```
m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} - \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$
$$= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

Mathematica [A] time = 0.0429387, size = 36, normalized size = 0.8

$$\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right) - \frac{\sqrt{1-\frac{1}{x^2}}x}{x+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*x)/(1 + x)) + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [B] time = 0.123, size = 110, normalized size = 2.4

$$\frac{-1+x}{2\,(1+x)^2}\left(\left(x^2-1\right)^{\frac{3}{2}} - x^2\sqrt{x^2-1} + 2\,\ln\left(x+\sqrt{x^2-1}\right)x^2 - 2\,x\sqrt{x^2-1} + 4\,\ln\left(x+\sqrt{x^2-1}\right)x - \sqrt{x^2-1} + 2\,\ln\left(x+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x)
```

```
[Out] 1/2*(-1+x)*((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)+2*ln(x+(x^2-1)^(1/2))*x^2-2*x*(
x^2-1)^(1/2)+4*ln(x+(x^2-1)^(1/2))*x-(x^2-1)^(1/2)+2*ln(x+(x^2-1)^(1/2)))/(
(-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)/(1+x)^2
```

Maxima [A] time = 1.00741, size = 59, normalized size = 1.31

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

Fricas [A] time = 1.62333, size = 122, normalized size = 2.71

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**2), x)
```

Giac [A] time = 1.13298, size = 61, normalized size = 1.36

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="giac")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x -
 1)/(x + 1)) - 1))
```

3.292 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

[Out] Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]

Rubi [A] time = 0.0651539, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6175, 6180, 37}

$$\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[x]/(1 + x)^2,x]

[Out] Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x^2}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}\end{aligned}$$

Mathematica [A] time = 0.0109699, size = 18, normalized size = 0.86

$$\frac{\sqrt{1-\frac{1}{x^2}}x}{x+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x)/(1 + x)
```

Maple [A] time = 0.058, size = 21, normalized size = 1.

$$\frac{-1+x}{1+x}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x)
```

```
[Out] 1/(1+x)*(-1+x)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.03692, size = 15, normalized size = 0.71

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

Fricas [A] time = 1.59365, size = 31, normalized size = 1.48

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

Sympy [A] time = 37.0712, size = 8, normalized size = 0.38

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**2,x)
```

```
[Out] sqrt((x - 1)/(x + 1))
```

Giac [A] time = 1.10328, size = 15, normalized size = 0.71

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

3.293 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{\frac{5}{x}+3}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] (-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) - (3 + 5/x)/(3*Sqrt[1 - x^(-2)]) +
 ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi [A] time = 0.15491, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6175, 6178, 852, 1805, 823, 12, 266, 63, 206}

$$-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{\frac{5}{x}+3}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) - (3 + 5/x)/(3*Sqrt[1 - x^(-2)]) +
 ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x}\,dx$$

$$= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3 x}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{(1+x)^3}{x\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{1}{3}\text{Subst}\left(\int \frac{-3-5x}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \frac{1}{3}\text{Subst}\left(\int -\frac{3}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$

$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A] time = 0.0553172, size = 43, normalized size = 0.78

$$\frac{\sqrt{1-\frac{1}{x^2}}(5-7x)x}{3(x-1)^2} + \log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*(5 - 7*x)*x)/(3*(-1 + x)^2) + Log[(1 + Sqrt[1 - x^(-2)])*
x]
```

Maple [B] time = 0.116, size = 146, normalized size = 2.7

$$-\frac{1}{3\left(-1+x\right)^{2}}\left(3x\left(x^{2}-1\right)^{3/2}-3x^{3}\sqrt{x^{2}-1}-3\ln\left(x+\sqrt{x^{2}-1}\right)x^{3}-2\left(x^{2}-1\right)^{3/2}+9x^{2}\sqrt{x^{2}-1}+9\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x)
```

```
[Out] -1/3*(3*x*(x^2-1)^(3/2)-3*x^3*(x^2-1)^(1/2)-3*ln(x+(x^2-1)^(1/2))*x^3-2*(x^
2-1)^(3/2)+9*x^2*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))*x^2-9*x*(x^2-1)^(1/2)-
9*ln(x+(x^2-1)^(1/2))*x+3*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/(-1+x)^2/((1
+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.00905, size = 76, normalized size = 1.38

$$-\frac{\frac{6\,(x-1)}{x+1}+1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*(x - 1)/(x + 1) + 1)/((x - 1)/(x + 1))^(3/2) + log(sqrt((x - 1)/(x
+ 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.57667, size = 223, normalized size = 4.05

$$\frac{3\left(x^{2}-2x+1\right)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-3\left(x^{2}-2x+1\right)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-\left(7x^{2}+2x-5\right)\sqrt{\frac{x-1}{x+1}}}{3\left(x^{2}-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x^2 - 2*x + 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 3*(x^2 - 2*x + 1)*l
og(sqrt((x - 1)/(x + 1)) - 1) - (7*x^2 + 2*x - 5)*sqrt((x - 1)/(x + 1)))/(x
^2 - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Giac [A] time = 1.14982, size = 88, normalized size = 1.6

$$-\frac{(x+1)\left(\frac{6\,(x-1)}{x+1}+1\right)}{3\,(x-1)\sqrt{\frac{x-1}{x+1}}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x + 1)*(6*(x - 1)/(x + 1) + 1)/((x - 1)*sqrt((x - 1)/(x + 1))) + log(
sqrt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

3.294 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} dx$

Optimal. Leaf size=24

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

```
[Out] -(1 - x^(-2))^(3/2)/(3*(1 - x^(-1))^3)
```

Rubi [A] time = 0.0714459, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6175, 6178, 651}

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x)^2,x]
```

```
[Out] -(1 - x^(-2))^(3/2)/(3*(1 - x^(-1))^3)
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x^2}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.0143856, size = 24, normalized size = 1.

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x(x+1)}{3(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^2,x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x*(1 + x))/(3*(-1 + x)^2)
```

Maple [A] time = 0.061, size = 22, normalized size = 0.9

$$-\frac{1+x}{-3+3\,x}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x)
```

```
[Out] -1/3*(1+x)/(-1+x)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.06845, size = 18, normalized size = 0.75

$$-\frac{1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3/((x - 1)/(x + 1))^(3/2)
```

Fricas [A] time = 1.5881, size = 81, normalized size = 3.38

$$-\frac{\left(x^{2}+2\,x+1\right)\sqrt{\frac{x-1}{x+1}}}{3\left(x^{2}-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 + 2*x + 1)*sqrt((x - 1)/(x + 1))/(x^2 - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Giac [A] time = 1.14127, size = 28, normalized size = 1.17

$$-\frac{x+1}{3(x-1)\sqrt{\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x + 1)/((x - 1)*sqrt((x - 1)/(x + 1)))
```

3.295 $\int e^{\coth^{-1}(ax)} x^m \sqrt{c - acx}\, dx$

Optimal. Leaf size=65

$$\frac{2x^{m+1}\sqrt{c-acx}\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{3}{2},-m-\frac{1}{2},-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.191909, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6176, 6181, 64}

$$\frac{2x^{m+1}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{3}{2};-m-\frac{1}{2};-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```


```
[Out] (2*x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$
$$= -\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int x^{-\frac{5}{2}-m}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$= \frac{2x^{1+m}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-\frac{3}{2}-m;-\frac{1}{2}-m;-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0266707, size = 67, normalized size = 1.03

$$-\frac{x^{m+1}\sqrt{c-acx}\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{3}{2},-m-\frac{1}{2},-\frac{1}{ax}\right)}{\left(-m-\frac{3}{2}\right)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((-3/2 - m)*Sqrt[1 - 1/(a*x)]))
```

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int x^m\sqrt{-acx+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-acx+c}(ax+1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-a*c*x + c)*(a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.296 $\int e^{\coth^{-1}(ax)} x^2 \sqrt{c-acx}\, dx$

Optimal. Leaf size=140

$$\frac{16x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{8x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (16*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (8
*(1 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*(1
+ 1/(a*x))^(3/2)*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.215309, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6176, 6181, 45, 37}

$$\frac{16x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{8x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```


```
[Out] (16*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (8
*(1 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*(1
+ 1/(a*x))^(3/2)*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,d\cdots\right.}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{16\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0331096, size = 64, normalized size = 0.46

$$\frac{2\sqrt{\frac{1}{ax}+1}(ax+1)\left(15a^2x^2-12ax+8\right)\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(8 - 12*a*x + 15*a^2*x^2))/(
105*a^3*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.043, size = 49, normalized size = 0.4

$$\frac{(2\,ax+2)\left(15\,a^2x^2-12\,ax+8\right)}{105\,a^3}\sqrt{-acx+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/((a*x-1)/(a*x+1))^
(1/2)
```

Maxima [A] time = 1.10896, size = 74, normalized size = 0.53

$$\frac{2\left(15\,a^3\sqrt{-c}x^3+3\,a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x+8\,\sqrt{-c}\right)\sqrt{ax+1}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*x^3 + 3*a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x + 8*sqrt(-
c))*sqrt(a*x + 1)/a^3
```

Fricas [A] time = 1.58551, size = 151, normalized size = 1.08

$$\frac{2\left(15\,a^{4}x^{4}+18\,a^{3}x^{3}-a^{2}x^{2}+4\,ax+8\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^{4}x-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/105*(15*a^4*x^4 + 18*a^3*x^3 - a^2*x^2 + 4*a*x + 8)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24733, size = 196, normalized size = 1.4

$$\frac{2\left(\frac{22\sqrt{2}\sqrt{-c}c}{a^{2}\mathrm{sgn}(c)}+\frac{15\,(acx+c)^{3}\sqrt{-acx-c}-84\,(acx+c)^{2}\sqrt{-acx-c}c-175\,(-acx-c)^{\frac{3}{2}}c^{2}+14\left(3\,(acx+c)^{2}\sqrt{-acx-c}+10\,(-acx-c)^{\frac{3}{2}}c\right)c}{a^{2}c^{2}\mathrm{sgn}(-acx-c)}\right)}{105\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] 2/105*(22*sqrt(2)*sqrt(-c)*c/(a^2*sgn(c)) + (15*(a*c*x + c)^3*sqrt(-a*c*x -
 c) - 84*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 175*(-a*c*x - c)^(3/2)*c^2 + 14
```

```
*(3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 10*(-a*c*x - c)^(3/2)*c)*c)/(a^2*c^2*s
gn(-a*c*x - c)))/(a*c)
```

3.297 $\int e^{\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=92

$$\frac{2x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (-4*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]) + (2*(1
 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.172632, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6176, 6181, 45, 37}

$$\frac{2x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```


```
[Out] (-4*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]) + (2*(1
 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)} x\sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx, x, \frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{4\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] time = 0.025395, size = 56, normalized size = 0.61

$$
\frac{2\sqrt{\frac{1}{ax}+1}(ax+1)(3ax-2)\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1
 - 1/(a*x)])
```

Maple [A] time = 0.043, size = 41, normalized size = 0.5

$$\frac{(2\,ax+2)\,(3\,ax-2)}{15\,a^2}\sqrt{-acx+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.0592, size = 55, normalized size = 0.6

$$\frac{2\left(3\,a^2\sqrt{-c}x^2+a\sqrt{-c}x-2\,\sqrt{-c}\right)\sqrt{ax+1}}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/15*(3*a^2*sqrt(-c)*x^2 + a*sqrt(-c)*x - 2*sqrt(-c))*sqrt(a*x + 1)/a^2
```

Fricas [A] time = 1.57287, size = 131, normalized size = 1.42

$$\frac{2\left(3\,a^3x^3+4\,a^2x^2-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(3*a^3*x^3 + 4*a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1))/(a^3*x - a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18634, size = 101, normalized size = 1.1

$$\frac{2\left(\frac{2\sqrt{2}\sqrt{-c}c^2}{\mathrm{sgn}(c)} + \frac{3(acx+c)^2\sqrt{-acx-c}+5(-acx-c)^{\frac{3}{2}}c}{\mathrm{sgn}(-acx-c)}\right)}{15a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*sqrt(2)*sqrt(-c)*c^2/sgn(c) + (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5
*(-a*c*x - c)^(3/2)*c)/sgn(-a*c*x - c))/(a^2*c^2)
```

3.298 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=29

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rubi [A] time = 0.0330856, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6174}

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A] time = 0.0181776, size = 43, normalized size = 1.48

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.039, size = 35, normalized size = 1.2

$$\frac{2\,ax+2}{3\,a}\sqrt{-acx+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Maxima [A] time = 1.04979, size = 35, normalized size = 1.21

$$\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Fricas [A] time = 1.60488, size = 111, normalized size = 3.83

$$\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17996, size = 66, normalized size = 2.28

$$\frac{2\left(\frac{2\sqrt{2}\sqrt{-c}c}{\operatorname{sgn}(c)}-\frac{(-acx-c)^{\frac{3}{2}}}{\operatorname{sgn}(-acx-c)}\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*sqrt(2)*sqrt(-c)*c/sgn(c) - (-a*c*x - c)^(3/2)/sgn(-a*c*x - c))/(a*c
)
```

3.299 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] - (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.194695, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6176, 6181, 47, 54, 215}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] - (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.05537, size = 75, normalized size = 0.8

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{a+\frac{1}{x}}{a}} - \sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[(a + x^(-1))/a] - Sqrt[x^(-1)]*ArcSinh[Sqr
t[x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.135, size = 70, normalized size = 0.7

$$-2\frac{\sqrt{-c(ax-1)}}{\sqrt{-c(ax+1)}}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)-\sqrt{-c(ax+1)}\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] -2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.64905, size = 490, normalized size = 5.21

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},-\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{}}{acx}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.18829, size = 119, normalized size = 1.27

$$-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\operatorname{sgn}(-acx-c)}+\frac{2\left(-i\sqrt{-c}\arctan\left(-i\sqrt{2}\right)+\sqrt{2}\sqrt{-c}\right)}{\operatorname{sgn}(c)}+\frac{2\sqrt{-acx-c}}{\operatorname{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c*x - c) + 2*(-I*sqrt(-c
)*arctan(-I*sqrt(2)) + sqrt(2)*sqrt(-c))/sgn(c) + 2*sqrt(-a*c*x - c)/sgn(-a
*c*x - c)
```

3.300 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -((Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x)) - (Sqrt[a]*Sqr
t[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.194801, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6176, 6181, 50, 54, 215}

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x)) - (Sqrt[a]*Sqr
t[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] time = 0.0387791, size = 76, normalized size = 0.78

$$
-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.144, size = 78, normalized size = 0.8

$$-\frac{1}{x}\left(\arctan\left(\sqrt{-c\,(ax+1)}\frac{1}{\sqrt{c}}\right)xac+\sqrt{-c\,(ax+1)}\sqrt{c}\right)\sqrt{-c\,(ax-1)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{-c\,(ax+1)}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] -(arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+(-c*(a*x+1))^(1/2)*c^(1/2))*(-c*
(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.57579, size = 514, normalized size = 5.3

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)},-\frac{\left(a^2x^2-ax\right)\sqrt{c}\arctan\left(\frac{\sqrt{-acx}\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqrt(
```

```
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -((a^2*x^2 -
a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c
*x - c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.19978, size = 162, normalized size = 1.67

$$-\frac{a^2c^2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}\operatorname{sgn}(-acx-c)}+\frac{\sqrt{-acx-c}}{acx\operatorname{sgn}(-acx-c)}\right)-\frac{-i\,a^2\sqrt{-c}c\arctan\left(-i\sqrt{2}\right)-\sqrt{2}a^2\sqrt{-c}c}{\operatorname{sgn}(c)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] -(a^2*c^2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/(sqrt(c)*sgn(-a*c*x - c)) + sqr
t(-a*c*x - c)/(a*c*x*sgn(-a*c*x - c))) - (-I*a^2*sqrt(-c)*c*arctan(-I*sqrt(
2)) - sqrt(2)*a^2*sqrt(-c)*c)/sgn(c))/(a*c)
```

3.301 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=101

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

```
[Out] (4*Sqrt[c - a*c*x])/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^
(9/2))/(9*a^4*c^4)
```

Rubi [A] time = 0.225181, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6167, 6130, 21, 77}

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^
(9/2))/(9*a^4*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx\\
&= -\int \frac{x^3(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= -\left(c\int \frac{x^3(1+ax)}{\sqrt{c-acx}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{a^3\sqrt{c-acx}}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{(c-acx)^{7/2}}{a^3c^4}\right)dx\right)\\
&= \frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}
\end{aligned}
$$

Mathematica [A] time = 0.0672522, size = 48, normalized size = 0.48

$$\frac{2\left(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272\right)\sqrt{c-acx}}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4))
/(315*a^4)
```

Maple [A] time = 0.046, size = 45, normalized size = 0.5

$$\frac{70\,x^4a^4+170\,x^3a^3+204\,a^2x^2+272\,ax+544}{315\,a^4}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

Maxima [A] time = 0.99861, size = 100, normalized size = 0.99

$$\frac{2\left(35\,(-acx+c)^{\frac{9}{2}}-225\,(-acx+c)^{\frac{7}{2}}c+567\,(-acx+c)^{\frac{5}{2}}c^{2}-735\,(-acx+c)^{\frac{3}{2}}c^{3}+630\,\sqrt{-acx+c}c^{4}\right)}{315\,a^{4}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)^
(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^4
)
```

Fricas [A] time = 1.59629, size = 113, normalized size = 1.12

$$\frac{2\left(35\,a^{4}x^{4}+85\,a^{3}x^{3}+102\,a^{2}x^{2}+136\,ax+272\right)\sqrt{-acx+c}}{315\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x +
 c)/a^4
```

Sympy [A] time = 15.2549, size = 83, normalized size = 0.82

$$\frac{2\left(2c^{4}\sqrt{-acx+c}-\frac{7c^{3}(-acx+c)^{\frac{3}{2}}}{3}+\frac{9c^{2}(-acx+c)^{\frac{5}{2}}}{5}-\frac{5c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^{4}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(2*c**4*sqrt(-a*c*x + c) - 7*c**3*(-a*c*x + c)**(3/2)/3 + 9*c**2*(-a*c*x
+ c)**(5/2)/5 - 5*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c*
*4)
```

Giac [A] time = 1.13925, size = 140, normalized size = 1.39

$$\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}+225\,(acx-c)^3\sqrt{-acx+c}c+567\,(acx-c)^2\sqrt{-acx+c}c^2-735\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx+c}c^4\right)}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 225*(a*c*x - c)^3*sqrt(-a*c*x +
c)*c + 567*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3
+ 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^4)
```

3.302 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=80

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}$$

```
[Out] (4*Sqrt[c - a*c*x])/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*x)
^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rubi [A] time = 0.224154, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6167, 6130, 21, 77}

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*x)
^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx \\
&= -\int \frac{x^2(1+ax)\sqrt{c-acx}}{1-ax}\,dx \\
&= -\left(c\int \frac{x^2(1+ax)}{\sqrt{c-acx}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{a^2\sqrt{c-acx}}-\frac{5\sqrt{c-acx}}{a^2c}+\frac{4(c-acx)^{3/2}}{a^2c^2}-\frac{(c-acx)^{5/2}}{a^2c^3}\right)dx\right) \\
&= \frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}
\end{aligned}
$$

Mathematica [A] time = 0.0533907, size = 40, normalized size = 0.5

$$\frac{2\left(15a^3x^3+39a^2x^2+52ax+104\right)\sqrt{c-acx}}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

Maple [A] time = 0.044, size = 37, normalized size = 0.5

$$\frac{30\,x^3a^3+78\,a^2x^2+104\,ax+208}{105\,a^3}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

Maxima [A] time = 1.02344, size = 81, normalized size = 1.01

$$-\frac{2\left(15\,(-acx+c)^{\frac{7}{2}} - 84\,(-acx+c)^{\frac{5}{2}}c + 175\,(-acx+c)^{\frac{3}{2}}c^2 - 210\,\sqrt{-acx+c}c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)^
(3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Fricas [A] time = 1.53695, size = 93, normalized size = 1.16

$$\frac{2\left(15\,a^3x^3 + 39\,a^2x^2 + 52\,ax + 104\right)\sqrt{-acx+c}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

Sympy [A] time = 11.4617, size = 68, normalized size = 0.85

$$-\frac{2\left(-2c^3\sqrt{-acx+c} + \frac{5c^2(-acx+c)^{\frac{3}{2}}}{3} - \frac{4c(-acx+c)^{\frac{5}{2}}}{5} + \frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(-2*c**3*sqrt(-a*c*x + c) + 5*c**2*(-a*c*x + c)**(3/2)/3 - 4*c*(-a*c*x +
 c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a**3*c**3)
```

Giac [A] time = 1.155, size = 108, normalized size = 1.35

$$\frac{2\left(15\,(acx-c)^3\sqrt{-acx+c}+84\,(acx-c)^2\sqrt{-acx+c}c-175\,(-acx+c)^{\frac{3}{2}}c^2+210\,\sqrt{-acx+c}c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c) + 84*(a*c*x - c)^2*sqrt(-a*c*x + c
)*c - 175*(-a*c*x + c)^(3/2)*c^2 + 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

3.303 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=57

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{4\sqrt{c-acx}}{a^2}$$

```
[Out] (4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(a^2*c) + (2*(c - a*c*x)^(5
/2))/(5*a^2*c^2)
```

Rubi [A] time = 0.152234, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6167, 6130, 21, 77}

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{4\sqrt{c-acx}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(a^2*c) + (2*(c - a*c*x)^(5
/2))/(5*a^2*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx \\
&= -\int \frac{x(1+ax)\sqrt{c-acx}}{1-ax}\,dx \\
&= -\left(c\int \frac{x(1+ax)}{\sqrt{c-acx}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{a\sqrt{c-acx}}-\frac{3\sqrt{c-acx}}{ac}+\frac{(c-acx)^{3/2}}{ac^2}\right)dx\right) \\
&= \frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0429672, size = 31, normalized size = 0.54

$$\frac{2\left(a^2x^2+3ax+6\right)\sqrt{c-acx}}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

Maple [A] time = 0.041, size = 28, normalized size = 0.5

$$\frac{2\,a^2x^2+6\,ax+12}{5\,a^2}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

Maxima [A] time = 1.01281, size = 59, normalized size = 1.04

$$\frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}c^{2}\right)}{5\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2)
/(a^2*c^2)
```

Fricas [A] time = 1.53086, size = 65, normalized size = 1.14

$$\frac{2\left(a^{2}x^{2}+3\,ax+6\right)\sqrt{-acx+c}}{5\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

Sympy [A] time = 6.18236, size = 48, normalized size = 0.84

$$\frac{2\left(2c^{2}\sqrt{-acx+c}-c\,(-acx+c)^{\frac{3}{2}}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(2*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2) + (-a*c*x + c)**(5/2)/5)
/(a**2*c**2)
```

Giac [A] time = 1.17294, size = 74, normalized size = 1.3

$$\frac{2\left((acx-c)^2\sqrt{-acx+c}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}c^2\right)}{5\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*((a*c*x - c)^2*sqrt(-a*c*x + c) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c
*x + c)*c^2)/(a^2*c^2)
```

3.304 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rubi [A] time = 0.0813062, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6130, 21, 43}

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\ &= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] time = 0.0242868, size = 23, normalized size = 0.61

$$\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A] time = 0.049, size = 20, normalized size = 0.5

$$\frac{2\,ax+10}{3\,a}\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Maxima [A] time = 1.00853, size = 41, normalized size = 1.08

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Fricas [A] time = 1.52457, size = 46, normalized size = 1.21

$$\frac{2\sqrt{-acx+c}(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] time = 4.19414, size = 31, normalized size = 0.82

$$-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.11735, size = 41, normalized size = 1.08

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

3.305 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=39

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rubi [A] time = 0.198014, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6167, 6130, 21, 80, 63, 208}

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x\sqrt{c-acx}}\,dx\right) \\
&= 2\sqrt{c-acx} - c\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= 2\sqrt{c-acx} + \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)}{a} \\
&= 2\sqrt{c-acx} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0302798, size = 39, normalized size = 1.

$$2\sqrt{c-acx} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A] time = 0.046, size = 32, normalized size = 0.8

$$2\,\mathrm{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66075, size = 208, normalized size = 5.33

$$\left[\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\sqrt{-acx+c},-2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\sqrt{-acx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x
+ c), -2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x + c)]
```

Sympy [A] time = 7.97232, size = 39, normalized size = 1.

$$-\frac{2c\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] -2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Giac [A] time = 1.14801, size = 49, normalized size = 1.26

$$-\frac{2\,c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\,\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

3.306 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rubi [A] time = 0.203317, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6167, 6130, 21, 78, 63, 208}

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^2(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x^2\sqrt{c-acx}}\,dx\right)\\
&= \frac{\sqrt{c-acx}}{x}-\frac{1}{2}(3ac)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{x}+3\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)\\
&= \frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0310648, size = 42, normalized size = 1.

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A] time = 0.052, size = 45, normalized size = 1.1

$$-2\,ac\left(-1/2\,\frac{\sqrt{-acx+c}}{acx}-3/2\,\frac{1}{\sqrt{c}}\mathrm{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] -2*a*c*(-1/2*(-a*c*x+c)^(1/2)/x/a/c-3/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^
(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58873, size = 235, normalized size = 5.6

$$\left[\frac{3\,a\sqrt{c}x\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}}{2\,x},-\frac{3\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c))/x, -(3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) -
sqrt(-a*c*x + c))/x]
```

Sympy [B] time = 12.1982, size = 119, normalized size = 2.83

$$-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{2ac\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] -a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + a*c**
2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - 2*a*c*atan(s
qrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.12742, size = 53, normalized size = 1.26

$$-\frac{3\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -3*a*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

3.307 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=68

$$\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}$$

```
[Out] Sqrt[c - a*c*x]/(2*x^2) + (7*a*Sqrt[c - a*c*x])/(4*x) + (7*a^2*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rubi [A] time = 0.210414, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 78, 51, 63, 208}

$$\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```


```
[Out] Sqrt[c - a*c*x]/(2*x^2) + (7*a*Sqrt[c - a*c*x])/(4*x) + (7*a^2*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^3(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x^3\sqrt{c-acx}}\,dx\right) \\
&= \frac{\sqrt{c-acx}}{2x^2} - \frac{1}{4}(7ac)\int \frac{1}{x^2\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{2x^2} + \frac{7a\sqrt{c-acx}}{4x} - \frac{1}{8}\left(7a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{2x^2} + \frac{7a\sqrt{c-acx}}{4x} + \frac{1}{4}(7a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{2x^2} + \frac{7a\sqrt{c-acx}}{4x} + \frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0472039, size = 55, normalized size = 0.81

$$\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + \frac{(7ax+2)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] ((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c
*x]/Sqrt[c]])/4
```

Maple [A] time = 0.052, size = 65, normalized size = 1.

$$2\,a^2c^2\left(\frac{1}{a^2x^2c^2}\left(-\frac{7\,(-acx+c)^{3/2}}{8\,c}+\frac{9\,\sqrt{-acx+c}}{8}\right)+\frac{7}{8\,c^{3/2}}\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] 2*a^2*c^2*((-7/8/c*(-a*c*x+c)^(3/2)+9/8*(-a*c*x+c)^(1/2))/x^2/a^2/c^2+7/8/c
^(3/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59502, size = 289, normalized size = 4.25

$$\left[\frac{7\,a^2\sqrt{c}x^2\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}(7\,ax+2)}{8\,x^2}, -\frac{7\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}(7\,ax+2)}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) +
 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, -1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt(
-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

Sympy [B] time = 23.2499, size = 270, normalized size = 3.97

$$\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}-\frac{6a^2c^3\left(-acx+c\right)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] 10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**
2) - 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x
 + c)**2) - 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x
 + c))/8 + 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x +
 c))/8 - a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)
)/2 + a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2
+ a*sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.1418, size = 97, normalized size = 1.43

$$-\frac{7\,a^{2}c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}-\frac{7\,(-acx+c)^{\frac{3}{2}}a^{2}c-9\,\sqrt{-acx+c}a^{2}c^{2}}{4\,a^{2}c^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -7/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/4*(7*(-a*c*x + c)
^(3/2)*a^2*c - 9*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

3.308 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=89

$$\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{\sqrt{c-acx}}{3x^3}$$

```
[Out] Sqrt[c - a*c*x]/(3*x^3) + (11*a*Sqrt[c - a*c*x])/(12*x^2) + (11*a^2*Sqrt[c
- a*c*x])/(8*x) + (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rubi [A] time = 0.219242, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 78, 51, 63, 208}

$$\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```


```
[Out] Sqrt[c - a*c*x]/(3*x^3) + (11*a*Sqrt[c - a*c*x])/(12*x^2) + (11*a^2*Sqrt[c
- a*c*x])/(8*x) + (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^4(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x^4\sqrt{c-acx}}\,dx\right) \\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{1}{6}(11ac)\int \frac{1}{x^3\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}-\frac{1}{8}\left(11a^2c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}-\frac{1}{16}\left(11a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{1}{8}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0581621, size = 63, normalized size = 0.71

$$\frac{\left(33a^2x^2+22ax+8\right)\sqrt{c-acx}}{24x^3}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) + (11*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Maple [A] time = 0.059, size = 80, normalized size = 0.9

$$-2\,c^3a^3\left(-\frac{1}{x^3a^3c^3}\left(\frac{11\,(-acx+c)^{5/2}}{16\,c^2}-\frac{11\,(-acx+c)^{3/2}}{6\,c}+\frac{21\,\sqrt{-acx+c}}{16}\right)-\frac{11}{16\,c^{5/2}}\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] -2*c^3*a^3*(-(11/16/c^2*(-a*c*x+c)^(5/2)-11/6/c*(-a*c*x+c)^(3/2)+21/16*(-a*
c*x+c)^(1/2))/x^3/a^3/c^3-11/16/c^(5/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50698, size = 332, normalized size = 3.73

$$\left[\frac{33\,a^3\sqrt{c}x^3\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\left(33\,a^2x^2+22\,ax+8\right)\sqrt{-acx+c}}{48\,x^3},-\frac{33\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\left(33\,a^2x^2+\right.}{24\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
 + 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, -1/24*(33*a^3*sqrt(-c
)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sqrt(
-a*c*x + c))/x^3]
```

Sympy [B] time = 18.9856, size = 439, normalized size = 4.93

$$-\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] -66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x
+ c)**2 + 48*c**3*(-a*c*x + c)**3) + 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144
*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) -
 30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c
*x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 10*a**3*c**4*sqrt(-a*c*x + c)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 5*a**3*c**4*sqrt(c**(-7))*log(
-c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 5*a**3*c**4*sqrt(c**(-7))*log(
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 6*a**3*c**3*(-a*c*x + c)**(3/2)
/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 3*a**3*c**3*sqrt(c**(-5)
)*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 3*a**3*c**3*sqrt(c**(-5))
*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8
```

Giac [A] time = 1.12548, size = 134, normalized size = 1.51

$$-\frac{11\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}}+\frac{33\,(acx-c)^2\sqrt{-acx+c}a^3c-88\,(-acx+c)^{\frac{3}{2}}a^3c^2+63\,\sqrt{-acx+c}a^3c^3}{24\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -11/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(33*(a*c*x -
c)^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 63*sqrt(-a*c*
x + c)*a^3*c^3)/(a^3*c^3*x^3)
```

3.309 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=110

$$\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{\sqrt{c-acx}}{4x^4}$$

```
[Out] Sqrt[c - a*c*x]/(4*x^4) + (5*a*Sqrt[c - a*c*x])/(8*x^3) + (25*a^2*Sqrt[c -
a*c*x])/(32*x^2) + (75*a^3*Sqrt[c - a*c*x])/(64*x) + (75*a^4*Sqrt[c]*ArcTan
h[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rubi [A] time = 0.234181, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 78, 51, 63, 208}

$$\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{\sqrt{c-acx}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) + (5*a*Sqrt[c - a*c*x])/(8*x^3) + (25*a^2*Sqrt[c -
a*c*x])/(32*x^2) + (75*a^3*Sqrt[c - a*c*x])/(64*x) + (75*a^4*Sqrt[c]*ArcTan
h[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= -\int \frac{(1+ax)\sqrt{c-acx}}{x^5(1-ax)}\,dx$$
$$= -\left(c\int \frac{1+ax}{x^5\sqrt{c-acx}}\,dx\right)$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{1}{8}(15ac)\int \frac{1}{x^4\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} - \frac{1}{16}\left(25a^2c\right)\int \frac{1}{x^3\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} - \frac{1}{64}\left(75a^3c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{75a^3\sqrt{c-acx}}{64x} - \frac{1}{128}\left(75a^4c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{75a^3\sqrt{c-acx}}{64x} + \frac{1}{64}\left(75a^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx\right.$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{75a^3\sqrt{c-acx}}{64x} + \frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.0685087, size = 71, normalized size = 0.65

$$\frac{\left(75a^3x^3+50a^2x^2+40ax+16\right)\sqrt{c-acx}}{64x^4} + \frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) + (75*a^
4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Maple [A] time = 0.051, size = 93, normalized size = 0.9

$$2c^4a^4\left(\frac{1}{x^4a^4c^4}\left(-\frac{75\,(-acx+c)^{7/2}}{128\,c^3}+\frac{275\,(-acx+c)^{5/2}}{128\,c^2}-\frac{365\,(-acx+c)^{3/2}}{128\,c}+\frac{181\sqrt{-acx+c}}{128}\right)+\frac{75}{128\,c^{7/2}}\text{Artanh}\left(\frac{\sqrt{-ac}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] 2*c^4*a^4*((-75/128/c^3*(-a*c*x+c)^(7/2)+275/128/c^2*(-a*c*x+c)^(5/2)-365/1
28/c*(-a*c*x+c)^(3/2)+181/128*(-a*c*x+c)^(1/2))/x^4/a^4/c^4+75/128/c^(7/2)*
arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62456, size = 371, normalized size = 3.37

$$\left[\frac{75\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\left(75\,a^3x^3+50\,a^2x^2+40\,ax+16\right)\sqrt{-acx+c}}{128\,x^4},-\frac{75\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) + 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, -1/64*
(75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 + 50
*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

Sympy [B] time = 27.5307, size = 639, normalized size = 5.81

$$\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}-\frac{102}{1536ac^8x-1152c^8+2304c^6\left(-ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] 558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c
*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 1022*a
**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x
 + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 770*a**4
*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 66*a**4*c**
6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48
*c**3*(-a*c*x + c)**3) - 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x -
 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c*
*4*(-a*c*x + c)**4) + 80*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 96*
c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 35*a**4*c**5*s
qrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 35*a**4*c**5
*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 30*a**4*c**
4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 +
 48*c**3*(-a*c*x + c)**3) - 5*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqrt(c**(-7
)) + sqrt(-a*c*x + c))/16 + 5*a**4*c**4*sqrt(c**(-7))*log(c**4*sqrt(c**(-7)
) + sqrt(-a*c*x + c))/16
```

Giac [A] time = 1.17964, size = 170, normalized size = 1.55

$$-\frac{75\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\,\sqrt{-c}}+\frac{75\,(acx-c)^3\sqrt{-acx+c}a^4c+275\,(acx-c)^2\sqrt{-acx+c}a^4c^2-365\,(-acx+c)^{\frac{3}{2}}a^4c^3+181\,\sqrt{-a}}{64\,a^4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -75/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/64*(75*(a*c*x -
 c)^3*sqrt(-a*c*x + c)*a^4*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2 -
 365*(-a*c*x + c)^(3/2)*a^4*c^3 + 181*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4*x^
4)
```

3.310 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=309

$$\frac{92x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{472x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{1576\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^4}{}$$

```
[Out] (1576*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(315*a^4*Sqrt[1 - 1/(a*x)]) + (472
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(315*a^3*Sqrt[1 - 1/(a*x)]) + (92*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) + (38*Sqrt[
1 + 1/(a*x)]*x^3*Sqrt[c - a*c*x])/(63*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/
(a*x)]*x^4*Sqrt[c - a*c*x])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]
*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.324192, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 98, 152, 12, 93, 206}

$$\frac{92x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{472x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{1576\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```


```
[Out] (1576*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(315*a^4*Sqrt[1 - 1/(a*x)]) + (472
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(315*a^3*Sqrt[1 - 1/(a*x)]) + (92*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) + (38*Sqrt[
1 + 1/(a*x)]*x^3*Sqrt[c - a*c*x])/(63*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/
(a*x)]*x^4*Sqrt[c - a*c*x])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]
*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(a^(9/2)*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{19}{2a}-\frac{17x}{2a^2}}{x^{9/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{9\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{69}{2a^2}+\frac{57x}{2a^3}}{x^{7/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\right.}{63\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{}$$

$$= \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{}}{9\sqrt{1-}}$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3}{63a\sqrt{1-}}$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3}{63a\sqrt{1-}}$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3}{63a\sqrt{1-}}$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}x^3}{63a\sqrt{1-}}$$

Mathematica [A] time = 0.108288, size = 130, normalized size = 0.42

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788\right)-630\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{315a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(788 + 236*a*x + 138*a^2*x^2
+ 95*a^3*x^3 + 35*a^4*x^4) - 630*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt
[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(315*a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.177, size = 161, normalized size = 0.5

$$-\frac{2\,ax-2}{(315\,ax+315)\,a^4}\sqrt{-c\,(ax-1)}\Bigg(-35\,x^4a^4\sqrt{-c\,(ax+1)}-95\,x^3a^3\sqrt{-c\,(ax+1)}-138\,x^2a^2\sqrt{-c\,(ax+1)}+630\,\sqrt{c}\sqrt{2}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/315/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-35*x^4*
a^4*(-c*(a*x+1))^(1/2)-95*x^3*a^3*(-c*(a*x+1))^(1/2)-138*x^2*a^2*(-c*(a*x+1
))^(1/2)+630*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
-236*x*a*(-c*(a*x+1))^(1/2)-788*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.69984, size = 749, normalized size = 2.42

$$\left[\frac{2\left(315\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(35\,a^5x^5+130\,a^4x^4+233\,a^3x^3+374\,a^2x^2+102\right.}{315\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/315*(315*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2
)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (35*a^5*x^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x^2
+ 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x - a^4)
, -2/315*(630*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (35*a^5*x^5 + 130*a^4*x^4 + 2
33*a^3*x^3 + 374*a^2*x^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/(a^5*x - a^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.28071, size = 244, normalized size = 0.79

$$-\frac{1260i\,\sqrt{2}\sqrt{-c}\arctan\left(-i\right)-2584\,\sqrt{2}\sqrt{-c}}{315\,a^4\mathrm{sgn}\left(c\right)}-\frac{2\left(630\,\sqrt{2}c^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-35\,(acx+c)^4\sqrt{-acx-c}+45\,(acx+c)^3\right.}{315\,a^4c^4\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/315*(1260*I*sqrt(2)*sqrt(-c)*arctan(-I) - 2584*sqrt(2)*sqrt(-c))/(a^4*sg
n(c)) - 2/315*(630*sqrt(2)*c^(9/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt
(c)) - 35*(a*c*x + c)^4*sqrt(-a*c*x - c) + 45*(a*c*x + c)^3*sqrt(-a*c*x - c
)*c - 63*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^2 + 105*(-a*c*x - c)^(3/2)*c^3 -
630*sqrt(-a*c*x - c)*c^4)/(a^4*c^4*sgn(-a*c*x - c))
```

3.311 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=261

$$\frac{32x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{104\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+\frac{6x^2\sqrt{\frac{1}{ax}}}{7a}$$

```
[Out] (104*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(21*a^3*Sqrt[1 - 1/(a*x)]) + (32*Sq
rt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(21*a^2*Sqrt[1 - 1/(a*x)]) + (6*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(7*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*x
)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(
a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.299959, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 98, 152, 12, 93, 206}

$$\frac{32x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{104\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+\frac{6x^2\sqrt{\frac{1}{ax}}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (104*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(21*a^3*Sqrt[1 - 1/(a*x)]) + (32*Sq
rt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(21*a^2*Sqrt[1 - 1/(a*x)]) + (6*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(7*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*x
)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(
a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{15}{2a}-\frac{13x}{2a^2}}{x^{7/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{20}{a^2}+\frac{15x}{a^3}}{x^{5/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\right.}{35\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0844121, size = 122, normalized size = 0.47

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(3a^3x^3+9a^2x^2+16ax+52\right)-42\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{21a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(52 + 16*a*x + 9*a^2*x^2 + 3*
a^3*x^3) - 42*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*
Sqrt[1 + 1/(a*x)])]))/(21*a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.191, size = 143, normalized size = 0.6

$$-\frac{2\,ax-2}{(21\,ax+21)\,a^3}\sqrt{-c\,(ax-1)}\left(-3\,x^3a^3\sqrt{-c\,(ax+1)}-9\,x^2a^2\sqrt{-c\,(ax+1)}+42\,\sqrt{c}\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-16\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*x^3*a^
3*(-c*(a*x+1))^(1/2)-9*x^2*a^2*(-c*(a*x+1))^(1/2)+42*c^(1/2)*2^(1/2)*arctan
(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-16*x*a*(-c*(a*x+1))^(1/2)-52*(-c*(
a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+cx^2}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.66304, size = 690, normalized size = 2.64

$$\left[\frac{2\left(21\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2+68\,ax+52\right)\sqrt{-acx}\right.}{21\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/21*(21*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*sqrt
(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3), -2/21*(42*sqrt(2)*(a
*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
 + 1))/(a*c*x - c)) - (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*s
qrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.27086, size = 182, normalized size = 0.7

$$-\frac{84i\,\sqrt{2}\sqrt{-c}\arctan\left(-i\right)-160\,\sqrt{2}\sqrt{-c}}{21\,a^3\mathrm{sgn}\left(c\right)}-\frac{2\left(42\,\sqrt{2}c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-3\left(acx+c\right)^3\sqrt{-acx-c}+7\left(-acx-c\right)^{\frac{3}{2}}c^2-\right.}{21\,a^3c^3\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/21*(84*I*sqrt(2)*sqrt(-c)*arctan(-I) - 160*sqrt(2)*sqrt(-c))/(a^3*sgn(c)
) - 2/21*(42*sqrt(2)*c^(7/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) -
 3*(a*c*x + c)^3*sqrt(-a*c*x - c) + 7*(-a*c*x - c)^(3/2)*c^2 - 42*sqrt(-a*c
*x - c)*c^3)/(a^3*c^3*sgn(-a*c*x - c))
```

3.312 $\int e^{3\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=211

$$\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(
a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*x))^(
5/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])
/(a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.236304, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6176, 6181, 96, 94, 93, 206}

$$\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```


```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(
a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*x))^(
5/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])
/(a^(5/2)*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,d\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-ac\right.}{}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-ac\right.}{}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-}}{a^5}$$

Mathematica [A] time = 0.0740351, size = 114, normalized size = 0.54

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+11ax+38\right)-30\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{15a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(38 + 11*a*x + 3*a^2*x^2) - 3
0*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(
a*x)])]))/(15*a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.178, size = 125, normalized size = 0.6

$$-\frac{2\,ax-2}{(15\,ax+15)\,a^2}\sqrt{-c\,(ax-1)}\left(-3\,x^2a^2\sqrt{-c\,(ax+1)}+30\,\sqrt{c}\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-11\,xa\sqrt{-c\,(ax+1)}-38\,\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*x^2*a^
2*(-c*(a*x+1))^(1/2)+30*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/
2)/c^(1/2))-11*x*a*(-c*(a*x+1))^(1/2)-38*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(
1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+cx}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.63742, size = 655, normalized size = 3.1

$$\left[\frac{2\left(15\,\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^3x^3+14\,a^2x^2+49\,ax+38\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{15\left(a^3x-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] [2/15*(15*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*
sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2), -2/15*(30*sqrt(2)*(a*x - 1)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*x - a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.2015, size = 180, normalized size = 0.85

$$-\frac{60i\sqrt{2}\sqrt{-c}\arctan(-i) - 104\sqrt{2}\sqrt{-c}}{15a^2\mathrm{sgn}(c)} - \frac{2\left(30\sqrt{2}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right) - 3(acx+c)^2\sqrt{-acx-c} + 5(-acx-c)^{\frac{3}{2}}c - 3}{15a^2c^2\mathrm{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(60*I*sqrt(2)*sqrt(-c)*arctan(-I) - 104*sqrt(2)*sqrt(-c))/(a^2*sgn(c)
) - 2/15*(30*sqrt(2)*c^(5/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) -
 3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5*(-a*c*x - c)^(3/2)*c - 30*sqrt(-a*c*x
 - c)*c^2)/(a^2*c^2*sgn(-a*c*x - c))
```

3.313 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=163

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.187324, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0599873, size = 105, normalized size = 0.64

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}(ax+7)-6\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.194, size = 107, normalized size = 0.7

$$-\frac{2ax-2}{(3ax+3)a}\sqrt{-c(ax-1)}\left(6\sqrt{c}\sqrt{2}\arctan\left(1/2\frac{\sqrt{-c(ax+1)}\sqrt{2}}{\sqrt{c}}\right)-xa\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\frac{1}{\sqrt{-c(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(-c*(a*x+1))^(1/
2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.59122, size = 598, normalized size = 3.67

$$\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+\left(a^2x^2+8ax+7\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\left(a^2x-a\right)},-\frac{2\left(6\sqrt{2}(ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.17755, size = 144, normalized size = 0.88

$$-\frac{12i\sqrt{2}\sqrt{-c}\arctan\left(-i\right)-16\sqrt{2}\sqrt{-c}}{3\,a\mathrm{sgn}\left(c\right)}-\frac{2\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)+\left(-acx-c\right)^{\frac{3}{2}}-6\sqrt{-acx-c}c\right)}{3\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(12*I*sqrt(2)*sqrt(-c)*arctan(-I) - 16*sqrt(2)*sqrt(-c))/(a*sgn(c)) -
2/3*(6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) + (-a*c
*x - c)^(3/2) - 6*sqrt(-a*c*x - c)*c)/(a*c*sgn(-a*c*x - c))
```

3.314 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=170

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)]) -
 (4*Sqrt[2]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.240087, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6176, 6181, 98, 157, 54, 215, 93, 206}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```


```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)]) -
 (4*Sqrt[2]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\cdots}{a\sqrt{1\cdots}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Sub}\cdots}{a\sqrt{\cdots}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0627399, size = 120, normalized size = 0.71

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}+\sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-2\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]] - 2*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.182, size = 110, normalized size = 0.7

$$-2\,{\frac {(ax-1)\sqrt {-c(ax-1)}}{(ax+1)\sqrt {-c(ax+1)}}\left(2\,\sqrt {c}\sqrt {2}\arctan \left(1/2\,{\frac {\sqrt {-c(ax+1)}\sqrt {2}}{\sqrt {c}}} \right) -\sqrt {c}\arctan \left({\frac {\sqrt {-c(ax+1)}}{\sqrt {c}}} \right) -\sqrt {-c(ax+1)} \right) \left({\frac {ax-1}{ax+1}} \right) ^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] -2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(2*c^(1/2)*2^
(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-c^(1/2)*arctan((-c*(a*
x+1))^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx + c}}{x \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.71356, size = 855, normalized size = 5.03

$$\left[\frac{2\,\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right.}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2
*a*x + 1)) + (a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt
(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*(2*sqrt(2)*
(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a
*x + 1))/(a*c*x - c)) - (a*x - 1)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*s
qrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*
x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.25051, size = 189, normalized size = 1.11

$$-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\operatorname{sgn}(-acx-c)}+\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\operatorname{sgn}(-acx-c)}+\frac{-4i\sqrt{2}\sqrt{-c}\arctan(-i)+2i\sqrt{-c}\arctan\left(-i\sqrt{2}\right)+2\sqrt{2}\sqrt{-c}}{\operatorname{sgn}(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c*x
- c) + 2*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c*x - c) + (-4*I*s
qrt(2)*sqrt(-c)*arctan(-I) + 2*I*sqrt(-c)*arctan(-I*sqrt(2)) + 2*sqrt(2)*sq
rt(-c))/sgn(c) + 2*sqrt(-a*c*x - c)/sgn(-a*c*x - c)
```

3.315 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (5*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)] -
 (4*Sqrt[2]*Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.251493, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6176, 6181, 102, 157, 54, 215, 93, 206}

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```


```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (5*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)] -
 (4*Sqrt[2]*Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{5x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\Big(}{\sqrt{1-}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\Big(}{\sqrt{1-}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.076673, size = 120, normalized size = 0.7

$$\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}+5\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-4\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + 5*Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]] - 4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/Sqrt[1 - 1/(a*x)]
```

Maple [A] time = 0.184, size = 119, normalized size = 0.7

$$-\frac{ax-1}{(ax+1)x}\sqrt{-c(ax-1)}\left(4\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c(ax+1)}\sqrt{2}}{\sqrt{c}}\right)xac-5\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)xac-\sqrt{-c(ax+1)}\sqrt{c}\right)\left(\frac{ax}{ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] -(a*x-1)*(-c*(a*x-1))^(1/2)*(4*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2
)/c^(1/2))*x*a*c-5*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x*a*c-(-c*(a*x+1))^(1
/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.73131, size = 900, normalized size = 5.23

$$\left[\frac{4\sqrt{2}\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+5\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}}{ax^2-x}\right.}{2\left(ax^2-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt
(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + 5*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x -
 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*
x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
- x), -(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 5*(a^2*x^2 - a*x)*sqrt(c)*a
rctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.28979, size = 211, normalized size = 1.23

$$-ac\left(\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}\operatorname{sgn}\left(-acx-c\right)}-\frac{5\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}\operatorname{sgn}\left(-acx-c\right)}-\frac{\sqrt{-acx-c}}{acx\operatorname{sgn}\left(-acx-c\right)}\right)+\frac{-4i\sqrt{2}a\sqrt{-c}\arctan\left(-i\right)+5i\,a\sqrt{-c}\arctan}{\operatorname{sgn}\left(c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] -a*c*(4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(sqrt(c)*sgn(-
a*c*x - c)) - 5*arctan(sqrt(-a*c*x - c)/sqrt(c))/(sqrt(c)*sgn(-a*c*x - c))
- sqrt(-a*c*x - c)/(a*c*x*sgn(-a*c*x - c))) + (-4*I*sqrt(2)*a*sqrt(-c)*arct
an(-I) + 5*I*a*sqrt(-c)*arctan(-I*sqrt(2)) + sqrt(2)*a*sqrt(-c))/sgn(c)
```

3.316 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=224

$$\frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{2x\sqrt{1-\frac{1}{ax}}}+\frac{7a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (7*a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (a*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x) + (23*a^(3/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(3/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.264502, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6176, 6181, 101, 154, 157, 54, 215, 93, 206}

$$\frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{2x\sqrt{1-\frac{1}{ax}}}+\frac{7a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (7*a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (a*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x) + (23*a^(3/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(3/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
```

```
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}\left(\frac{1}{2}+\frac{7x}{2a}\right)}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{\left(a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{9}{4a}-\frac{23x}{4a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,d\right.}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x\right.}{8\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x,\right.}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{3/}}{}$$

Mathematica [A] time = 0.14794, size = 132, normalized size = 0.59

$$\frac{\sqrt{c-acx}\left(\frac{23a^{3/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}} - \frac{16\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}} + \sqrt{\frac{1}{ax}+1}(9ax+2)\right)}{4x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(2 + 9*a*x) + (23*a^(3/2)*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]])/(x^(-1))^(3/2) - (16*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqr
t[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(3/2)))/(4*Sqrt[1 - 1/(a*
x)]*x^2)
```

Maple [A] time = 0.211, size = 144, normalized size = 0.6

$$\frac{ax-1}{(4\,ax+4)\,x^2}\sqrt{-c\,(ax-1)}\left(-16\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^2a^2c+23\,c\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)x^2a^2+9\,xa\sqrt{-c\,(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-16*2^(1/2)
*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c+23*c*arctan((-c*(
a*x+1))^(1/2)/c^(1/2))*x^2*a^2+9*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)+2*(-c*(a*x+
1))^(1/2)*c^(1/2))/c^(1/2)/(-c*(a*x+1))^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.69091, size = 979, normalized size = 4.37

$$\left[\frac{16\sqrt{2}\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+23\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(}{ax^2-}\right.}{8\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2
 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
- 2*c)/(a*x^2 - x)) + 2*(9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1)))/(a*x^3 - x^2), -1/4*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqr
t((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.31402, size = 248, normalized size = 1.11

$$-\frac{1}{4}a^2c^2\left(\frac{16\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}\operatorname{sgn}\left(-acx-c\right)}-\frac{23\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}\operatorname{sgn}\left(-acx-c\right)}+\frac{9\left(-acx-c\right)^{\frac{3}{2}}+7\sqrt{-acx-c}c}{a^2c^3x^2\operatorname{sgn}\left(-acx-c\right)}\right)-\frac{16i\sqrt{2}a^2\sqrt{-c}\arctan\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] -1/4*a^2*c^2*(16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(c^(3
/2)*sgn(-a*c*x - c)) - 23*arctan(sqrt(-a*c*x - c)/sqrt(c))/(c^(3/2)*sgn(-a*
c*x - c)) + (9*(-a*c*x - c)^(3/2) + 7*sqrt(-a*c*x - c)*c)/(a^2*c^3*x^2*sgn(
-a*c*x - c))) - 1/4*(16*I*sqrt(2)*a^2*sqrt(-c)*arctan(-I) - 23*I*a^2*sqrt(-
c)*arctan(-I*sqrt(2)) - 11*sqrt(2)*a^2*sqrt(-c))/sgn(c)
```

3.317 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=274

$$\frac{3a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}+\frac{13a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^2) + (13*a^2
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (3*a^2*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (45*a^(5/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(5/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.281619, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6176, 6181, 101, 154, 157, 54, 215, 93, 206}

$$\frac{3a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}+\frac{13a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```


```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^2) + (13*a^2
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (3*a^2*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (45*a^(5/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(5/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}\left(\frac{3}{2}+\frac{9x}{2a}\right)}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{9}{4a}-\frac{39x}{4a^2}\right)\sqrt{1\cdots}}{\sqrt{x}\left(1-\frac{x}{a}\right)} \cdots\right.}{6\sqrt{1-\frac{1}{ax}} \cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{\left(a^4\sqrt{\frac{1}{x}}\sqrt{c-ac\cdots}\right.}{\cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{\left(45a^2\sqrt{\frac{1}{x}}\sqrt{c-\cdots}\right.}{\cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{\left(45a^2\sqrt{\frac{1}{x}}\sqrt{c-\cdots}\right.}{\cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-\cdots}}{8\cdots}$$

Mathematica [A] time = 0.203083, size = 140, normalized size = 0.51

$$\frac{\sqrt{c-acx}\left(\sqrt{\frac{1}{ax}+1}\left(57a^2x^2+26ax+8\right)+\frac{135a^{5/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}-\frac{96\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{24x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(8 + 26*a*x + 57*a^2*x^2) + (135*a^(5/2
)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2) - (96*Sqrt[2]*a^(5/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(5/2)))/(2
4*Sqrt[1 - 1/(a*x)]*x^3)
```

Maple [A] time = 0.191, size = 165, normalized size = 0.6

$$\frac{ax-1}{(24\,ax+24)\,x^3}\sqrt{-c\,(ax-1)}\left(-96\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^3a^3c+135\,c\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)x^3a^3+57\,x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] 1/24/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-96*2^(1/2
)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c+135*c*arctan((-c
*(a*x+1))^(1/2)/c^(1/2))*x^3*a^3+57*x^2*a^2*(-c*(a*x+1))^(1/2)*c^(1/2)+26*x
*a*(-c*(a*x+1))^(1/2)*c^(1/2)+8*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(-c*(a*
x+1))^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.74909, size = 1022, normalized size = 3.73

$$\left[\frac{96\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+135\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(\cdots}{ax^2-\cdots}\right.\cdots}{48\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3
*c)/(a^2*x^2 - 2*a*x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x
^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)
) - 2*c)/(a*x^2 - x)) + 2*(57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3), -1/24*(96*sqrt(2)*(a^4*x^4
 - a^3*x^3)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(sqrt(-a*c*
x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (57*a^3*x^3 + 83*a^
2*x^2 + 34*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^
3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

[Out] Timed out

Giac [C] time = 1.325, size = 282, normalized size = 1.03

$$-\frac{1}{24}\,a^3c^3\left(\frac{96\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(-acx-c\right)}-\frac{135\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(-acx-c\right)}-\frac{57\,(acx+c)^2\sqrt{-acx-c}+88\,(-acx-c)^{\frac{3}{2}}c+39\,\sqrt{-acx}}{a^3c^5x^3\mathrm{sgn}\left(-acx-c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] -1/24*a^3*c^3*(96*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(c^(
5/2)*sgn(-a*c*x - c)) - 135*arctan(sqrt(-a*c*x - c)/sqrt(c))/(c^(5/2)*sgn(-
a*c*x - c)) - (57*(a*c*x + c)^2*sqrt(-a*c*x - c) + 88*(-a*c*x - c)^(3/2)*c
+ 39*sqrt(-a*c*x - c)*c^2)/(a^3*c^5*x^3*sgn(-a*c*x - c))) - 1/24*(96*I*sqrt
(2)*a^3*sqrt(-c)*arctan(-I) - 135*I*a^3*sqrt(-c)*arctan(-I*sqrt(2)) - 91*sq
rt(2)*a^3*sqrt(-c))/sgn(c)
```

3.318 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=322

$$\frac{11a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{24x^2\sqrt{1-\frac{1}{ax}}}+\frac{21a^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{32x\sqrt{1-\frac{1}{ax}}}+\frac{107a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}+\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}-\cdots$$

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^3) + (11*a^2
*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^2) + (107*a^3
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) + (21*a^3*(1 +
 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(32*Sqrt[1 - 1/(a*x)]*x) + (363*a^(7/2)*Sq
rt[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a
*x)]) - (4*Sqrt[2]*a^(7/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.30185, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6176, 6181, 101, 154, 157, 54, 215, 93, 206}

$$\frac{11a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{24x^2\sqrt{1-\frac{1}{ax}}}+\frac{21a^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{32x\sqrt{1-\frac{1}{ax}}}+\frac{107a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}+\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```


```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^3) + (11*a^2
*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^2) + (107*a^3
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) + (21*a^3*(1 +
 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(32*Sqrt[1 - 1/(a*x)]*x) + (363*a^(7/2)*Sq
rt[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a
*x)]) - (4*Sqrt[2]*a^(7/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}\left(\frac{5}{2}+\frac{11x}{2a}\right)}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(-\frac{33}{4a}-\frac{63}{4a}\right.}{1-}\right.}{12\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} - \frac{\left(a^5\sqrt{\frac{1}{x}}\right.}{}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\right.}{32}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\right.}{32}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\right.}{32}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\right.}{32}$$

Mathematica [A] time = 0.250251, size = 148, normalized size = 0.46

$$\frac{\sqrt{c-acx}\left(\sqrt{\frac{1}{ax}+1}\left(447a^3x^3+214a^2x^2+136ax+48\right)+\frac{1089a^{7/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}-\frac{768\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{192x^4\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*x
^3) + (1089*a^(7/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2) - (768*Sq
rt[2]*a^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(x^(-1))^(7/2)))/(192*Sqrt[1 - 1/(a*x)]*x^4)
```

Maple [A] time = 0.196, size = 186, normalized size = 0.6

$$-\frac{ax-1}{(192\,ax+192)\,x^4}\sqrt{-c\,(ax-1)}\left(768\,\sqrt{2}\arctan\left(1/2\,\frac{\sqrt{-c\,(ax+1)}\sqrt{2}}{\sqrt{c}}\right)x^4a^4c-1089\,c\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)x^4a^4-447$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/192*(a*x-1)*(-c*(a*x-1))^(1/2)*(768*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2
)*2^(1/2)/c^(1/2))*x^4*a^4*c-1089*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x^4*
a^4-447*x^3*a^3*(-c*(a*x+1))^(1/2)*c^(1/2)-214*x^2*a^2*(-c*(a*x+1))^(1/2)*c
^(1/2)-136*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-48*(-c*(a*x+1))^(1/2)*c^(1/2))/((
a*x-1)/(a*x+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*x+1))^(1/2)/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.71726, size = 1079, normalized size = 3.35

$$\left[\frac{768\sqrt{2}\left(a^5x^5 - a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2 + 2\,acx + 2\sqrt{2}\sqrt{-acx + c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}} - 3\,c}{a^2x^2 - 2\,ax + 1}\right) + 1089\left(a^5x^5 - a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2 + acx - 2\sqrt{-ac}\cdots}{\cdots}\right)\cdots}{384\left(ax^5 - x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*
c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x +
 1)) - 2*c)/(a*x^2 - x)) + 2*(447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184
*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/19
2*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 1089*(a^5*x^5 - a^4*x^4)*s
qrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184*a*x + 48)*sqrt(-a*c*x +
 c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.3485, size = 315, normalized size = 0.98

$$-\frac{1}{192}a^4c^4\left(\frac{768\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}\mathrm{sgn}\left(-acx-c\right)}-\frac{1089\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{7}{2}}\mathrm{sgn}\left(-acx-c\right)}-\frac{447\,(acx+c)^3\sqrt{-acx-c}-1127\,(acx+c)^2\sqrt{-acx-c}}{a^4c^7x^4\mathrm{sgn}\left(-acx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] -1/192*a^4*c^4*(768*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(c
^(7/2)*sgn(-a*c*x - c)) - 1089*arctan(sqrt(-a*c*x - c)/sqrt(c))/(c^(7/2)*sg
n(-a*c*x - c)) - (447*(a*c*x + c)^3*sqrt(-a*c*x - c) - 1127*(a*c*x + c)^2*s
qrt(-a*c*x - c)*c - 1049*(-a*c*x - c)^(3/2)*c^2 - 321*sqrt(-a*c*x - c)*c^3)
/(a^4*c^7*x^4*sgn(-a*c*x - c))) - 1/192*(768*I*sqrt(2)*a^4*sqrt(-c)*arctan(
-I) - 1089*I*a^4*sqrt(-c)*arctan(-I*sqrt(2)) - 845*sqrt(2)*a^4*sqrt(-c))/sg
n(c)
```

3.319 $\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}x^2}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{8\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}x}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{46\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}x}$$

```
[Out] (46*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)) + (92*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)*x) + (8*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(7*(1 + x^(-1))^(3/2)) + (2*Sqrt[-((1 - x)/x)]*x^2*(1 +
x)^(3/2))/(7*(1 + x^(-1))^(3/2))
```

Rubi [A] time = 0.127649, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}x^2}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{8\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}x}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{46\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```


```
[Out] (46*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)) + (92*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)*x) + (8*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(7*(1 + x^(-1))^(3/2)) + (2*Sqrt[-((1 - x)/x)]*x^2*(1 +
x)^(3/2))/(7*(1 + x^(-1))^(3/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{5/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{(1+x)^2}{\sqrt{1-x}x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{10+\frac{7x}{2}}{\sqrt{1-x}x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(23\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x^{5/2}}\,dx,x,\ldots\right.}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{46\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(46\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Su}\ldots}{21\left(1\ldots\right.}$$

$$= \frac{46\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{92\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.0211074, size = 46, normalized size = 0.32

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\left(3x^3+12x^2+23x+46\right)}{21\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(46 + 23*x + 12*x^2 + 3*x^3))/(21*Sqrt[1 + 
x^(-1)])
```

Maple [A] time = 0.06, size = 37, normalized size = 0.3

$$\frac{(-2+2x)\left(3x^3+12x^2+23x+46\right)}{21}\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x)
```

```
[Out] 2/21*(-1+x)*(3*x^3+12*x^2+23*x+46)/(1+x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.07647, size = 36, normalized size = 0.25

$$\frac{2\left(3x^4+9x^3+11x^2+23x-46\right)}{21\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(x - 1)
```

Fricas [A] time = 1.52876, size = 93, normalized size = 0.65

$$\frac{2}{21}\left(3x^3+12x^2+23x+46\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.16036, size = 46, normalized size = 0.32

$$\frac{2}{7}(x-1)^{\frac{7}{2}}+2(x-1)^{\frac{5}{2}}+\frac{16}{3}(x-1)^{\frac{3}{2}}-\frac{64}{21}i\sqrt{2}+8\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*(x - 1)^(7/2) + 2*(x - 1)^(5/2) + 16/3*(x - 1)^(3/2) - 64/21*I*sqrt(2)
+ 8*sqrt(x - 1)
```

3.320 $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{-\frac{1-x}{x}}\,x(x+1)^{3/2}}{5\left(\frac{1}{x}+1\right)^{3/2}}+\frac{28\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}x}$$

```
[Out] (28*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)) + (86*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)*x) + (2*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(5*(1 + x^(-1))^(3/2))
```

Rubi [A] time = 0.107356, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\,x(x+1)^{3/2}}{5\left(\frac{1}{x}+1\right)^{3/2}}+\frac{28\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

```
[Out] (28*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)) + (86*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)*x) + (2*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(5*(1 + x^(-1))^(3/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}$$
$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{(1+x)^2}{\sqrt{1-x}x^{7/2}}\,dx, x, \frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$
$$= \frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{7+\frac{5x}{2}}{\sqrt{1-x}x^{5/2}}\,dx, x, \frac{1}{x}\right)}{5\left(1+\frac{1}{x}\right)^{3/2}}$$
$$= \frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx, x, \frac{1}{x}\right)}{15\left(1+\frac{1}{x}\right)^{3/2}}$$
$$= \frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{86\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{2\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.0158638, size = 41, normalized size = 0.38

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\left(3x^2+14x+43\right)}{15\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(43 + 14*x + 3*x^2))/(15*Sqrt[1 + x^(-1)])
```

Maple [A] time = 0.062, size = 32, normalized size = 0.3

$$\frac{(-2+2x)\left(3x^2+14x+43\right)}{15}\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x)
```

```
[Out] 2/15*(-1+x)*(3*x^2+14*x+43)/(1+x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 1.02345, size = 30, normalized size = 0.28

$$\frac{2\left(3\,x^3+11\,x^2+29\,x-43\right)}{15\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(x - 1)
```

Fricas [A] time = 1.58061, size = 81, normalized size = 0.76

$$\frac{2}{15}\left(3\,x^2+14\,x+43\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 14*x + 43)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.16821, size = 36, normalized size = 0.34

$$\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{8}{3}(x-1)^{\frac{3}{2}}-\frac{64}{15}i\sqrt{2}+8\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(x - 1)^(5/2) + 8/3*(x - 1)^(3/2) - 64/15*I*sqrt(2) + 8*sqrt(x - 1)
```

3.321 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal. Leaf size=104

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{44\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] (44*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(105*(1 - x^(-1))^(3/2)) - (22*(1 + x
^(-1))^(3/2)*(1 - x)^(3/2)*x)/(35*(1 - x^(-1))^(3/2)) + (2*(1 + x^(-1))^(3/
2)*(1 - x)^(3/2)*x^2)/(7*(1 - x^(-1))^(3/2))
```

Rubi [A] time = 0.126104, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{44\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (44*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(105*(1 - x^(-1))^(3/2)) - (22*(1 + x
^(-1))^(3/2)*(1 - x)^(3/2)*x)/(35*(1 - x^(-1))^(3/2)) + (2*(1 + x^(-1))^(3/
2)*(1 - x)^(3/2)*x^2)/(7*(1 - x^(-1))^(3/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{(1-x)^{3/2}\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^{3/2}x^{5/2}\,dx}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{(1-x)\sqrt{1+x}}{x^{9/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}} + \frac{\left(11(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}} - \frac{\left(22(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{5/2}}\right.}{35\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= \frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}} - \frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.0207203, size = 46, normalized size = 0.44

$$-\frac{2\sqrt{\frac{1}{x}+1}\sqrt{1-x}(x+1)\left(15x^2-33x+22\right)}{105\sqrt{\frac{x-1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(22 - 33*x + 15*x^2))/(105*Sqrt[(-
1 + x)/x])
```

Maple [A] time = 0.058, size = 34, normalized size = 0.3

$$-\frac{(2+2x)\left(15x^2-33x+22\right)}{105}\sqrt{1-x}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x)
```

```
[Out] -2/105*(1+x)*(15*x^2-33*x+22)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [C] time = 1.06617, size = 30, normalized size = 0.29

$$-\frac{1}{105}\left(30i\,x^3-36i\,x^2-22i\,x+44i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -1/105*(30*I*x^3 - 36*I*x^2 - 22*I*x + 44*I)*sqrt(x + 1)
```

Fricas [A] time = 1.62142, size = 120, normalized size = 1.15

$$-\frac{2\left(15\,x^4-3\,x^3-29\,x^2+11\,x+22\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -2/105*(15*x^4 - 3*x^3 - 29*x^2 + 11*x + 22)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2)*x,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.2415, size = 78, normalized size = 0.75

$$\frac{1}{105}\left(16i\sqrt{2}+\frac{2\left(15\,(x+1)^3\sqrt{-x-1}-63\,(x+1)^2\sqrt{-x-1}-70\,(-x-1)^{\frac{3}{2}}\right)}{\operatorname{sgn}\left(-x-1\right)}\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/105*(16*I*sqrt(2) + 2*(15*(x + 1)^3*sqrt(-x - 1) - 63*(x + 1)^2*sqrt(-x -
 1) - 70*(-x - 1)^(3/2))/sgn(-x - 1))*sgn(x)
```

3.322 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal. Leaf size=68

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}-\frac{14\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] (-14*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(15*(1 - x^(-1))^(3/2)) + (2*(1 + x^
(-1))^(3/2)*(1 - x)^(3/2)*x)/(5*(1 - x^(-1))^(3/2))
```

Rubi [A] time = 0.0991177, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6176, 6181, 78, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}-\frac{14\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

```
[Out] (-14*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(15*(1 - x^(-1))^(3/2)) + (2*(1 + x^
(-1))^(3/2)*(1 - x)^(3/2)*x)/(5*(1 - x^(-1))^(3/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx &= \frac{(1-x)^{3/2}\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}\\
&= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{(1-x)\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}+\frac{\left(7(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{x}\right)^{3/2}}\\
&= -\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0170106, size = 41, normalized size = 0.6

$$
-\frac{2\sqrt{\frac{1}{x}+1}\sqrt{1-x}(x+1)(3x-7)}{15\sqrt{\frac{x-1}{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(-7 + 3*x))/(15*Sqrt[(-1 + x)/x])
```

Maple [A] time = 0.058, size = 29, normalized size = 0.4

$$-\frac{(2+2x)(3x-7)}{15}\sqrt{1-x}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x)
```

```
[Out] -2/15*(1+x)*(3*x-7)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [C] time = 1.05526, size = 23, normalized size = 0.34

$$-\frac{1}{15}\left(6i\,x^2-8i\,x-14i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(6*I*x^2 - 8*I*x - 14*I)*sqrt(x + 1)
```

Fricas [A] time = 1.62552, size = 101, normalized size = 1.49

$$-\frac{2\left(3x^3-x^2-11x-7\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*x^3 - x^2 - 11*x - 7)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.17059, size = 59, normalized size = 0.87

$$\frac{1}{15}\left(-16i\sqrt{2}+\frac{2\left(3\,(x+1)^2\sqrt{-x-1}+10\,(-x-1)^{\frac{3}{2}}\right)}{\operatorname{sgn}(-x-1)}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(-16*I*sqrt(2) + 2*(3*(x + 1)^2*sqrt(-x - 1) + 10*(-x - 1)^(3/2))/sgn(
-x - 1))*sgn(x)
```

3.323 $\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x^2}{5\sqrt{\frac{1}{x}+1}}+\frac{6\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x}{5\sqrt{\frac{1}{x}+1}}+\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{5\sqrt{\frac{1}{x}+1}}$$

```
[Out] (12*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (6*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x)/x)]*x^2*Sqrt[
1 + x])/(5*Sqrt[1 + x^(-1)])
```

Rubi [A] time = 0.106478, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x^2}{5\sqrt{\frac{1}{x}+1}}+\frac{6\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x}{5\sqrt{\frac{1}{x}+1}}+\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{5\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

```
[Out] (12*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (6*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x)/x)]*x^2*Sqrt[
1 + x])/(5*Sqrt[1 + x^(-1)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}x^{3/2}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$
$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1+x}{\sqrt{1-x}x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$
$$= \frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} - \frac{\left(9\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$
$$= \frac{6\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} - \frac{\left(6\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$
$$= \frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{6\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

Mathematica [A] time = 0.0145349, size = 39, normalized size = 0.36

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\left(x^2+3x+6\right)}{5\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(6 + 3*x + x^2))/(5*Sqrt[1 + x^(-1)])
```

Maple [A] time = 0.058, size = 30, normalized size = 0.3

$$\frac{(-2+2x)\left(x^2+3x+6\right)}{5}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x)
```

```
[Out] 2/5*(-1+x)*(x^2+3*x+6)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.04353, size = 27, normalized size = 0.25

$$\frac{2\left(x^3+2x^2+3x-6\right)}{5\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(x - 1)
```

Fricas [A] time = 1.55453, size = 74, normalized size = 0.69

$$\frac{2}{5}\left(x^2+3x+6\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 + 3*x + 6)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] time = 51.9242, size = 133, normalized size = 1.24

$$-2\left(\begin{cases}\frac{x\sqrt{x-1}}{3}+\frac{5\sqrt{x-1}}{3} & \text{for}\ \frac{|x+1|}{2}>1\\ \frac{ix\sqrt{1-x}}{3}+\frac{5i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)+2\left(\begin{cases}\frac{8x\sqrt{x-1}}{15}+\frac{\sqrt{x-1}(x+1)^2}{5}+\frac{8\sqrt{x-1}}{3} & \text{for}\ \frac{|x+1|}{2}>1\\ \frac{8ix\sqrt{1-x}}{15}+\frac{i\sqrt{1-x}(x+1)^2}{5}+\frac{8i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(1/2),x)
```

```
[Out] -2*Piecewise((x*sqrt(x - 1)/3 + 5*sqrt(x - 1)/3, Abs(x + 1)/2 > 1), (I*x*sq
rt(1 - x)/3 + 5*I*sqrt(1 - x)/3, True)) + 2*Piecewise((8*x*sqrt(x - 1)/15 +
 sqrt(x - 1)*(x + 1)**2/5 + 8*sqrt(x - 1)/3, Abs(x + 1)/2 > 1), (8*I*x*sqrt
(1 - x)/15 + I*sqrt(1 - x)*(x + 1)**2/5 + 8*I*sqrt(1 - x)/3, True))
```

Giac [C] time = 1.14272, size = 36, normalized size = 0.34

$$\frac{2}{5}(x-1)^{\frac{5}{2}} + 2(x-1)^{\frac{3}{2}} - \frac{8}{5}i\sqrt{2} + 4\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x - 1)^(5/2) + 2*(x - 1)^(3/2) - 8/5*I*sqrt(2) + 4*sqrt(x - 1)
```

3.324 $\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$

Optimal. Leaf size=70

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x}{3\sqrt{\frac{1}{x}+1}}+\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{3\sqrt{\frac{1}{x}+1}}$$

```
[Out] (10*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)])
```

Rubi [A] time = 0.0811716, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 78, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}x}{3\sqrt{\frac{1}{x}+1}}+\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{3\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 + x],x]
```

```
[Out] (10*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx &= \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}\sqrt{x}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}\\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\operatorname{Subst}\left(\int\frac{1+x}{\sqrt{1-x}x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}\\
&= \frac{2\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}-\frac{\left(5\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x}x^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1+\frac{1}{x}}}\\
&= \frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}
\end{aligned}$$

Mathematica [A] time = 0.0145598, size = 34, normalized size = 0.49

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}(x+5)}{3\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(5 + x))/(3*Sqrt[1 + x^(-1)])
```

Maple [A] time = 0.058, size = 25, normalized size = 0.4

$$\frac{(-2+2x)(x+5)}{3}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x)
```

```
[Out] 2/3*(-1+x)*(x+5)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.05387, size = 20, normalized size = 0.29

$$\frac{2\left(x^2+4x-5\right)}{3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + 4*x - 5)/sqrt(x - 1)
```

Fricas [A] time = 1.61203, size = 63, normalized size = 0.9

$$\frac{2}{3}(x+5)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 5)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A] time = 23.2935, size = 39, normalized size = 0.56

$$2\left(\begin{cases} 2\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{12}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for } x\geq -1\wedge x<1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(1/2),x)
```

```
[Out] 2*Piecewise((2*sqrt(2)*(sqrt(2)*(x - 1)**(3/2)/12 + sqrt(2)*sqrt(x - 1)/2),
 (x >= -1) & (x < 1)))
```

Giac [C] time = 1.13628, size = 27, normalized size = 0.39

$$\frac{2}{3}\,(x-1)^{\frac{3}{2}}-\frac{8}{3}i\,\sqrt{2}+4\,\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2) - 8/3*I*sqrt(2) + 4*sqrt(x - 1)
```

3.325 $\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx$

Optimal. Leaf size=71

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}-\frac{4\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}$$

```
[Out] (-4*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(15*Sqrt[1 - x^(-1)]) + (2*(1 + x^(-1
))^(3/2)*Sqrt[1 - x]*x^2)/(5*Sqrt[1 - x^(-1)])
```

Rubi [A] time = 0.103072, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6176, 6181, 45, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}-\frac{4\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (-4*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(15*Sqrt[1 - x^(-1)]) + (2*(1 + x^(-1
))^(3/2)*Sqrt[1 - x]*x^2)/(5*Sqrt[1 - x^(-1)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx &= \frac{\sqrt{1-x}\int e^{\coth^{-1}(x)}\sqrt{1-\frac{1}{x}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{x}}\sqrt{x}}\\
&= -\frac{\left(\sqrt{1-x}\sqrt{\frac{1}{x}}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x}}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}+\frac{\left(2\sqrt{1-x}\sqrt{\frac{1}{x}}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+x}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1-\frac{1}{x}}}\\
&= -\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}
\end{aligned}$$

Mathematica [A] time = 0.0150439, size = 41, normalized size = 0.58

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{1-x}(x+1)(3x-2)}{15\sqrt{\frac{x-1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(-2 + 3*x))/(15*Sqrt[(-1 + x)/x])
```

Maple [A] time = 0.058, size = 29, normalized size = 0.4

$$\frac{(2+2x)(3x-2)}{15}\sqrt{1-x}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x)
```

```
[Out] 2/15*(1+x)*(3*x-2)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [C] time = 1.0517, size = 23, normalized size = 0.32

$$\frac{1}{15}\left(6i\,x^2+2i\,x-4i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(6*I*x^2 + 2*I*x - 4*I)*sqrt(x + 1)
```

Fricas [A] time = 1.57844, size = 99, normalized size = 1.39

$$\frac{2\left(3x^3+4x^2-x-2\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^3 + 4*x^2 - x - 2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [C] time = 130.87, size = 46, normalized size = 0.65

$$-\frac{14ix}{15\sqrt{\frac{1}{x+1}}} - \frac{2i(1-x)^2}{5\sqrt{\frac{1}{x+1}}} + \frac{2i}{3\sqrt{\frac{1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1-x)**(1/2),x)
```

```
[Out] -14*I*x/(15*sqrt(1/(x + 1))) - 2*I*(1 - x)**2/(5*sqrt(1/(x + 1))) + 2*I/(3*
sqrt(1/(x + 1)))
```

Giac [C] time = 1.16351, size = 58, normalized size = 0.82

$$-\frac{2}{15}\left(2i\sqrt{2} + \frac{3(x+1)^2\sqrt{-x-1} + 5(-x-1)^{\frac{3}{2}}}{\operatorname{sgn}(-x-1)}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(2*I*sqrt(2) + (3*(x + 1)^2*sqrt(-x - 1) + 5*(-x - 1)^(3/2))/sgn(-x -
 1))*sgn(x)
```

3.326 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx$

Optimal. Leaf size=20

$$\frac{2}{3}\sqrt{1-x}(x+1)e^{\coth^{-1}(x)}$$

```
[Out] (2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

Rubi [A] time = 0.0196056, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6174}

$$\frac{2}{3}\sqrt{1-x}(x+1)e^{\coth^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x],x]
```

```
[Out] (2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}(1+x)$$

Mathematica [A] time = 0.0124854, size = 34, normalized size = 1.7

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}x}{3\sqrt{1-\frac{1}{x}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(3*Sqrt[1 - x^(-1)])
```

Maple [A] time = 0.056, size = 24, normalized size = 1.2

$$\frac{2+2x}{3}\sqrt{1-x}\,\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x)
```

```
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

Maxima [C] time = 1.09669, size = 16, normalized size = 0.8

$$\frac{1}{3}\sqrt{x+1}(2i\,x+2i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x + 1)*(2*I*x + 2*I)
```

Fricas [A] time = 1.59786, size = 86, normalized size = 4.3

$$\frac{2\left(x^2+2\,x+1\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 2*x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [C] time = 48.8617, size = 29, normalized size = 1.45

$$-\frac{2ix}{3\sqrt{\frac{1}{x+1}}}-\frac{2i}{3\sqrt{\frac{1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(1/2),x)
```

```
[Out] -2*I*x/(3*sqrt(1/(x + 1))) - 2*I/(3*sqrt(1/(x + 1)))
```

Giac [C] time = 1.14028, size = 36, normalized size = 1.8

$$-\frac{2}{3}\left(2i\sqrt{2}-\frac{(-x-1)^{\frac{3}{2}}}{\operatorname{sgn}(-x-1)}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*I*sqrt(2) - (-x - 1)^(3/2)/sgn(-x - 1))*sgn(x)
```

3.327 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{x+1}}+\frac{4\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x}{3\sqrt{x+1}}$$

```
[Out] (4*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/(3*Sqrt[1 + x]) + (2*Sqrt[1 + x^(
-1)]*Sqrt[-((1 - x)/x)]*x^2)/(3*Sqrt[1 + x])
```

Rubi [A] time = 0.096523, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6176, 6181, 45, 37}

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{x+1}}+\frac{4\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (4*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/(3*Sqrt[1 + x]) + (2*Sqrt[1 + x^(
-1)]*Sqrt[-((1 - x)/x)]*x^2)/(3*Sqrt[1 + x])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1+\frac{1}{x}}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}\\
&= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{1+x}} - \frac{\left(2\sqrt{1+\frac{1}{x}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{\frac{1}{x}}\sqrt{1+x}}\\
&= \frac{4\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{3\sqrt{1+x}} + \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A] time = 0.0147966, size = 26, normalized size = 0.36

$$
\frac{2\sqrt{1-\frac{1}{x^2}}x(x+2)}{3\sqrt{x+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x*(2 + x))/(3*Sqrt[1 + x])
```

Maple [A] time = 0.06, size = 25, normalized size = 0.3

$$\frac{(-2+2x)(x+2)}{3}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x)
```

```
[Out] 2/3*(-1+x)*(x+2)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.08373, size = 18, normalized size = 0.25

$$\frac{2\left(x^2+x-2\right)}{3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + x - 2)/sqrt(x - 1)
```

Fricas [A] time = 1.55459, size = 63, normalized size = 0.86

$$\frac{2}{3}(x+2)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A] time = 62.8377, size = 48, normalized size = 0.66

$$\begin{cases} \frac{2x\sqrt{x-1}}{3} + \frac{4\sqrt{x-1}}{3} & \text{for } |x| > 1 \\ \frac{2ix\sqrt{1-x}}{3} + \frac{4i\sqrt{1-x}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(x) > 1), (2*I*x*sqrt(1
- x)/3 + 4*I*sqrt(1 - x)/3, True))
```

Giac [C] time = 1.12459, size = 27, normalized size = 0.37

$$\frac{2}{3}(x-1)^{\frac{3}{2}} - \frac{2}{3}i\sqrt{2} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2) - 2/3*I*sqrt(2) + 2*sqrt(x - 1)
```

3.328 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x}{\sqrt{x+1}}$$

```
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/Sqrt[1 + x]
```

Rubi [A] time = 0.0750883, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 37}

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}x}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/Sqrt[1 + x]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1+x}}$$
$$= -\frac{\sqrt{1+\frac{1}{x}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}$$
$$= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{\sqrt{1+x}}$$

Mathematica [A] time = 0.008271, size = 21, normalized size = 0.64

$$\frac{2\sqrt{1-\frac{1}{x^2}}x}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x)/Sqrt[1 + x]
```

Maple [A] time = 0.058, size = 22, normalized size = 0.7

$$2\,\frac{-1+x}{\sqrt{1+x}}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 2*(-1+x)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.11657, size = 9, normalized size = 0.27

$$2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1)
```

Fricas [A] time = 1.5053, size = 50, normalized size = 1.52

$$2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A] time = 56.8934, size = 19, normalized size = 0.58

$$\begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x), True))
```

Giac [C] time = 1.15067, size = 18, normalized size = 0.55

$$-2i\sqrt{2}+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt(2) + 2*sqrt(x - 1)
```

3.329 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{3\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] + (2*Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2)/(3*Sqrt[1 - x]) - (2*Sqrt[2]*Sqrt[1 - x^(-1)]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[x^(-1)])
```

Rubi [A] time = 0.114503, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6176, 6181, 96, 94, 93, 206}

$$\frac{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{3\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] + (2*Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2)/(3*Sqrt[1 - x]) - (2*Sqrt[2]*Sqrt[1 - x^(-1)]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[x^(-1)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1-\frac{1}{x}}}\,dx}{\sqrt{1-x}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Mathematica [A] time = 0.0467435, size = 69, normalized size = 0.55

$$\frac{2\sqrt{\frac{x-1}{x}}x\left(\sqrt{\frac{1}{x}+1}(x+4) - 3\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)\right)}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)]*(4 + x) - 3*Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(3*Sqrt[1 - x])
```

Maple [A] time = 0.118, size = 66, normalized size = 0.5

$$\frac{2}{3}\sqrt{1-x}\left(3\sqrt{2}\arctan\left(1/2\sqrt{-1-x}\sqrt{2}\right)-\sqrt{-1-x}x-4\sqrt{-1-x}\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x)
```

```
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2)*(3*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(
1/2))-(-1-x)^(1/2)*x-4*(-1-x)^(1/2))/(-1-x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A] time = 1.57471, size = 196, normalized size = 1.56

$$\frac{2\left(3\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-\left(x^2+5x+4\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x
 - 1)) - (x^2 + 5*x + 4)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.330 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] - (2*Sqrt[2]*Sqrt[1 - x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[
x^(-1)])
```

Rubi [A] time = 0.0925382, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/Sqrt[1 - x],x]
```


```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] - (2*Sqrt[2]*Sqrt[1 - x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[
x^(-1)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1-x}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Mathematica [A] time = 0.0289919, size = 63, normalized size = 0.7

$$\frac{2\sqrt{\frac{x-1}{x}}x\left(\sqrt{\frac{1}{x}+1}-\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)] - Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[Sqrt
[2]*Sqrt[(1 + x)^(-1)]]))/Sqrt[1 - x]
```

Maple [A] time = 0.121, size = 55, normalized size = 0.6

$$2\,\frac{\sqrt{1-x}\left(\sqrt{2}\arctan\left(1/2\,\sqrt{-1-x}\sqrt{2}\right)-\sqrt{-1-x}\right)}{\sqrt{-1-x}}\frac{1}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x)
```

```
[Out] 2/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2)
)-(-1-x)^(1/2))/(-1-x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A] time = 1.63171, size = 180, normalized size = 2.

$$\frac{2\left(\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1
)) - (x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.331 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(x+1)^{3/2}}+\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[-((1 - x)/x)]*x^2)/(1 + x)^(3/2) + (Sqrt[2]*(1 +
 x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/x)]])/((x^(-1))
^(3/2)*(1 + x)^(3/2))
```

Rubi [A] time = 0.116848, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6176, 6181, 96, 93, 203}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(x+1)^{3/2}}+\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```


```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[-((1 - x)/x)]*x^2)/(1 + x)^(3/2) + (Sqrt[2]*(1 +
 x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/x)]])/((x^(-1))
^(3/2)*(1 + x)^(3/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{\left(\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1+x)^{3/2}}$$

$$= -\frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^{3/2}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(1+x)^{3/2}} + \frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{x}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(1+x)^{3/2}} + \frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}x^2}{(1+x)^{3/2}} + \frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

Mathematica [A] time = 0.0374601, size = 65, normalized size = 0.7

$$\frac{\sqrt{\frac{1}{x}+1}x\left(2\sqrt{\frac{x-1}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\tan^{-1}\left(\frac{\sqrt{\frac{x-1}{x^2}}x}{\sqrt{2}}\right)\right)}{\sqrt{x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 + x^(-1)]*x*(2*Sqrt[(-1 + x)/x] - Sqrt[2]*Sqrt[x^(-1)]*ArcTan[(Sqrt
[(-1 + x)/x^2]*x)/Sqrt[2]]))/Sqrt[1 + x]
```

Maple [A] time = 0.106, size = 47, normalized size = 0.5

$$-\sqrt{-1+x}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1+x}\right)-2\sqrt{-1+x}\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x)
```

```
[Out] -(-1+x)^(1/2)*(2^(1/2)*arctan(1/2*(-1+x)^(1/2)*2^(1/2))-2*(-1+x)^(1/2))/((-
1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A] time = 1.49007, size = 138, normalized size = 1.48

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)+2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))) + 2*sqrt(x +
 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.332 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

```
[Out] -((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/
x)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

Rubi [A] time = 0.0965376, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 93, 203}

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] -((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/
x)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx &= \frac{\left(\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}\,dx}{(1+x)^{3/2}}\\
&= -\frac{\left(1+\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{x}(1+x)}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= -\frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= -\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0191591, size = 41, normalized size = 0.71

$$
\sqrt{2}\sqrt{\frac{1}{x+1}}\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{\frac{x-1}{x^2}}x}{\sqrt{2}}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] Sqrt[2]*Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcTan[(Sqrt[(-1 + x)/x^2]*x)/Sqrt[2
]]
```

Maple [A] time = 0.111, size = 37, normalized size = 0.6

$$\sqrt{2}\sqrt{-1+x}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1+x}\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x)
```

```
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)^(1/2)/(1+x)^(1/2)*2^(1/2)*arctan(1/2*(-1+x)^(
1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A] time = 1.61836, size = 85, normalized size = 1.47

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.333 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{2(1-x)^{3/2}}+\frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{x}+1}x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] (5*(1 - x^(-1))^(3/2)*Sqrt[1 + x^(-1)]*x^2)/(2*(1 - x)^(3/2)) - (Sqrt[1 - x
^(-1)]*(1 + x^(-1))^(3/2)*x^2)/(2*(1 - x)^(3/2)) - (5*(1 - x^(-1))^(3/2)*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*(x^(
-1))^(3/2))
```

Rubi [A] time = 0.13163, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6176, 6181, 96, 94, 93, 206}

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{2(1-x)^{3/2}}+\frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{x}+1}x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```


```
[Out] (5*(1 - x^(-1))^(3/2)*Sqrt[1 + x^(-1)]*x^2)/(2*(1 - x)^(3/2)) - (Sqrt[1 - x
^(-1)]*(1 + x^(-1))^(3/2)*x^2)/(2*(1 - x)^(3/2)) - (5*(1 - x^(-1))^(3/2)*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*(x^(
-1))^(3/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1-x)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2x^{3/2}}\,dx, x, \frac{1}{x}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{4(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{2(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

<u>Mathematica</u> [A] time = 0.0750892, size = 75, normalized size = 0.58

$$-\frac{\sqrt{\frac{x-1}{x}}x\left(2\sqrt{\frac{1}{x}+1}(3-2x)+5\sqrt{2}(x-1)\sqrt{\frac{1}{x}}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)\right)}{2(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] -(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)]*(3 - 2*x) + 5*Sqrt[2]*(-1 + x)*Sqr
t[x^(-1)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(2*(1 - x)^(3/2))
```

Maple [A] time = 0.115, size = 90, normalized size = 0.7

$$\frac{1}{-2+2x}\sqrt{1-x}\left(-5\sqrt{2}\arctan\left(1/2\sqrt{-1-x}\sqrt{2}\right)x+4\sqrt{-1-x}x+5\sqrt{2}\arctan\left(1/2\sqrt{-1-x}\sqrt{2}\right)-6\sqrt{-1-x}\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x)
```

```
[Out] 1/2*(1-x)^(1/2)*(-5*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))*x+4*(-1-x)^(1/
2)*x+5*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))-6*(-1-x)^(1/2))/((-1+x)/(1+
x))^(1/2)/(-1+x)/(-1-x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A] time = 1.46734, size = 221, normalized size = 1.7

$$-\frac{5\sqrt{2}\left(x^2-2x+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-2\left(2x^2-x-3\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(5*sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x
 + 1))/(x - 1)) - 2*(2*x^2 - x - 3)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^
2 - 2*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14619, size = 73, normalized size = 0.56

$$\frac{\left(5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)-4\sqrt{-x-1}+\frac{2\sqrt{-x-1}}{x-1}\right)\mathrm{sgn}\left(x\right)}{2\,\mathrm{sgn}\left(-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(5*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - 4*sqrt(-x - 1) + 2*sqrt(-
x - 1)/(x - 1))*sgn(x)/sgn(-x - 1)
```

3.334 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{\frac{1}{x}+1}x\sqrt{1-\frac{1}{x}}}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -((Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/(1 - x)^(3/2)) - ((1 - x^(-1))^(3/2
)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*
(x^(-1))^(3/2))
```

Rubi [A] time = 0.109552, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{\sqrt{\frac{1}{x}+1}x\sqrt{1-\frac{1}{x}}}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] -((Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/(1 - x)^(3/2)) - ((1 - x^(-1))^(3/2
)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*
(x^(-1))^(3/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}\,dx}{(1-x)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{2(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.0907516, size = 58, normalized size = 0.64

$$\frac{\frac{2}{\sqrt{\frac{1}{x+1}}} + \sqrt{2}(x-1)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)}{2\sqrt{-\frac{(x-1)^2}{x^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] (2/Sqrt[(1 + x)^(-1)] + Sqrt[2]*(-1 + x)*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]
])/(2*Sqrt[-((-1 + x)^2/x^2)]*x)
```

Maple [A] time = 0.11, size = 79, normalized size = 0.9

$$-\frac{1}{-2+2x}\sqrt{1-x}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1-x}\right)x - \sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1-x}\right) + 2\sqrt{-1-x}\right)\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\frac{1}{\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x)
```

```
[Out] -1/2/((-1+x)/(1+x))^(1/2)/(-1+x)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/
2)*2^(1/2))*x-2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))+2*(-1-x)^(1/2))/(-1-
x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A] time = 1.57579, size = 208, normalized size = 2.31

$$-\frac{\sqrt{2}\left(x^2-2\,x+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)+2\,(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)) + 2*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^2 - 2*x +
1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13293, size = 59, normalized size = 0.66

$$\frac{\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)+\frac{2\sqrt{-x-1}}{x-1}\right)\mathrm{sgn}\left(x\right)}{2\,\mathrm{sgn}\left(-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) + 2*sqrt(-x - 1)/(x - 1))*sgn
(x)/sgn(-x - 1)
```

3.335 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c - acx}\, dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{\frac{1}{ax}+1}x^{m+1}\sqrt{c-acx}}{(2m+3)\sqrt{1-\frac{1}{ax}}}-\frac{2(4m+5)x^m\sqrt{c-acx}\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{1}{2}-m,-\frac{1}{ax}\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*Sqrt[1 + 1/(a*x)]*x^(1 + m)*Sqrt[c - a*c*x])/((3 + 2*m)*Sqrt[1 - 1/(a*x)
]) - (2*(5 + 4*m)*x^m*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -1/2 - m, 1/2
- m, -(1/(a*x))])/(a*(1 + 2*m)*(3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.231738, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6176, 6181, 79, 64}

$$\frac{2\sqrt{\frac{1}{ax}+1}x^{m+1}\sqrt{c-acx}}{(2m+3)\sqrt{1-\frac{1}{ax}}}-\frac{2(4m+5)x^m\sqrt{c-acx}\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*x^(1 + m)*Sqrt[c - a*c*x])/((3 + 2*m)*Sqrt[1 - 1/(a*x)
]) - (2*(5 + 4*m)*x^m*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -1/2 - m, 1/2
- m, -(1/(a*x))])/(a*(1 + 2*m)*(3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m} x^m\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{x^{-\frac{5}{2}-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} + \frac{\left((5+4m)\left(\frac{1}{x}\right)^{\frac{1}{2}+m} x^m\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{x^{-\frac{3}{2}-m}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a(3+2m)\sqrt{1-\frac{1}{ax}}}\\
&= \frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} - \frac{2(5+4m)x^m\sqrt{c-acx}\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] time = 0.0705771, size = 102, normalized size = 0.78

$$
\frac{2x^m\sqrt{c-acx}\left(a(2m+1)x\sqrt{\frac{1}{ax}+1}-(4m+5)\mathrm{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{1}{2}-m,-\frac{1}{ax}\right)\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*x^m*Sqrt[c - a*c*x]*(a*(1 + 2*m)*Sqrt[1 + 1/(a*x)]*x - (5 + 4*m)*Hyperge
ometric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))]))/(a*(1 + 2*m)*(3 + 2*m)*Sqr
t[1 - 1/(a*x)])
```

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int x^{m}\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-acx+c}\,x^{m}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-acx+c}\,x^{m}\sqrt{\frac{ax-1}{ax+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-acx + c}x^m \sqrt{\frac{ax - 1}{ax + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.336 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=142

$$-\frac{2x^2\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}}{7ac}+\frac{6x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}}{35a^2c}+\frac{38x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}}{105a^2}+\frac{152cx\sqrt{1-\frac{1}{a^2x^2}}}{105a^2\sqrt{c-acx}}$$

```
[Out] (152*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(105*a^2*Sqrt[c - a*c*x]) + (38*Sqrt[1 - 1/
(a^2*x^2)]*x*Sqrt[c - a*c*x])/(105*a^2) + (6*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a
*c*x)^(3/2))/(35*a^2*c) - (2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(c - a*c*x)^(3/2))/(
7*a*c)
```

Rubi [A] time = 0.236783, antiderivative size = 185, normalized size of antiderivative = 1.3, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{104x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{208\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{26x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (-208*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) + (104
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (26*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 +
1/(a*x)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(13\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{26\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(52\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,d\right.}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{26\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(104\sqrt{\frac{1}{x}}\sqrt{c-\right.}{}$$

$$= -\frac{208\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} + \frac{104\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{26\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^3}{7\sqrt{1-}}$$

Mathematica [A] time = 0.0344716, size = 67, normalized size = 0.47

$$\frac{2\sqrt{\frac{1}{ax}+1}\left(15a^3x^3-39a^2x^2+52ax-104\right)\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*x
^3))/(105*a^3*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.043, size = 64, normalized size = 0.5

$$\frac{(2\,ax+2)\left(15\,x^3a^3-39\,a^2x^2+52\,ax-104\right)}{105\,a^3\,(ax-1)}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^3*x^3-39*a^2*x^2+52*a*x-104)*(-a*c*x+c)^(1/2)*((a*x-1)/
(a*x+1))^(1/2)/a^3/(a*x-1)
```

Maxima [A] time = 1.10097, size = 112, normalized size = 0.79

$$\frac{2\left(15\,a^4\sqrt{-c}x^4-24\,a^3\sqrt{-c}x^3+13\,a^2\sqrt{-c}x^2-52\,a\sqrt{-c}x-104\,\sqrt{-c}\right)(ax-1)}{105\left(a^4x-a^3\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^4*sqrt(-c)*x^4 - 24*a^3*sqrt(-c)*x^3 + 13*a^2*sqrt(-c)*x^2 - 52
*a*sqrt(-c)*x - 104*sqrt(-c))*(a*x - 1)/((a^4*x - a^3)*sqrt(a*x + 1))
```

Fricas [A] time = 1.63865, size = 159, normalized size = 1.12

$$\frac{2\left(15\,a^4x^4-24\,a^3x^3+13\,a^2x^2-52\,ax-104\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 24*a^3*x^3 + 13*a^2*x^2 - 52*a*x - 104)*sqrt(-a*c*x + c
)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17414, size = 150, normalized size = 1.06

$$-\frac{2\left(\frac{76\sqrt{2}\sqrt{-c}c}{a^3}+\frac{15(acx+c)^3\sqrt{-acx-c}-84(acx+c)^2\sqrt{-acx-c}c-175(-acx-c)^{\frac{3}{2}}c^2-210\sqrt{-acx-c}c^3}{a^3c^2}\right)|c|\mathrm{sgn}(ax+1)}{105c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(76*sqrt(2)*sqrt(-c)*c/a^3 + (15*(a*c*x + c)^3*sqrt(-a*c*x - c) - 84
*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 175*(-a*c*x - c)^(3/2)*c^2 - 210*sqrt(-
a*c*x - c)*c^3)/(a^3*c^2))*abs(c)*sgn(a*x + 1)/c^2
```

3.337 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=104

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}}{5ac}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}}{5a}-\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{5a\sqrt{c-acx}}$$

```
[Out] (-8*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(5*a*Sqrt[c - a*c*x]) - (2*Sqrt[1 - 1/(a^2*x
^2)]*x*Sqrt[c - a*c*x])/(5*a) - (2*Sqrt[1 - 1/(a^2*x^2)]*x*(c - a*c*x)^(3/2
))/(5*a*c)
```

Rubi [A] time = 0.193505, antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{12\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{6x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (12*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(5*a^2*Sqrt[1 - 1/(a*x)]) - (6*Sqrt[
1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*
x)]*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{\left(9\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(6\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{12\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}} - \frac{6\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0307322, size = 58, normalized size = 0.56

$$\frac{2\sqrt{\frac{1}{ax}+1}\left(a^2x^2-3ax+6\right)\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[1 -
 1/(a*x)])
```

Maple [A] time = 0.04, size = 55, normalized size = 0.5

$$\frac{(2\,ax+2)\left(a^2x^2-3\,ax+6\right)}{5\,a^2\,(ax-1)}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/5*(a*x+1)*(a^2*x^2-3*a*x+6)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/a^2/
(a*x-1)
```

Maxima [A] time = 1.12581, size = 93, normalized size = 0.89

$$\frac{2\left(a^{3}\sqrt{-c}x^{3}-2\,a^{2}\sqrt{-c}x^{2}+3\,a\sqrt{-c}x+6\,\sqrt{-c}\right)(ax-1)}{5\left(a^{3}x-a^{2}\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a^3*sqrt(-c)*x^3 - 2*a^2*sqrt(-c)*x^2 + 3*a*sqrt(-c)*x + 6*sqrt(-c))*(
a*x - 1)/((a^3*x - a^2)*sqrt(a*x + 1))
```

Fricas [A] time = 1.60257, size = 130, normalized size = 1.25

$$\frac{2\left(a^{3}x^{3}-2\,a^{2}x^{2}+3\,ax+6\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^{3}x-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^3*x^3 - 2*a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1))/(a^3*x - a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16256, size = 120, normalized size = 1.15

$$\frac{2\left(\frac{4\sqrt{2}\sqrt{-c}c^2}{a}-\frac{(acx+c)^2\sqrt{-acx-c}+5(-acx-c)^{\frac{3}{2}}c+10\sqrt{-acx-c}c^2}{a}\right)|c|\mathrm{sgn}(ax+1)}{5ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(4*sqrt(2)*sqrt(-c)*c^2/a - ((a*c*x + c)^2*sqrt(-a*c*x - c) + 5*(-a*c*x
 - c)^(3/2)*c + 10*sqrt(-a*c*x - c)*c^2)/a)*abs(c)*sgn(a*x + 1)/(a*c^3)
```

3.338 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-acx}}$$

```
[Out] (8*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1/(a^2*x^2)
]*x*Sqrt[c - a*c*x])/3
```

Rubi [A] time = 0.153416, antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3a\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A] time = 0.0262458, size = 50, normalized size = 0.81

$$\frac{2\sqrt{\frac{1}{ax}+1}(ax-5)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.039, size = 47, normalized size = 0.8

$$\frac{(2\,ax+2)\,(ax-5)}{(3\,ax-3)\,a}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/3*(a*x+1)*(a*x-5)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/a
```

Maxima [A] time = 1.10216, size = 73, normalized size = 1.18

$$\frac{2\left(a^{2}\sqrt{-c}x^{2}-4\,a\sqrt{-c}x-5\,\sqrt{-c}\right)(ax-1)}{3\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

Fricas [A] time = 1.53216, size = 111, normalized size = 1.79

$$\frac{2\left(a^{2}x^{2}-4\,ax-5\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [C] time = 164.195, size = 66, normalized size = 1.06

$$\frac{4icx\sqrt{\frac{1}{acx+c}}}{3}+\frac{4ic\sqrt{\frac{1}{acx+c}}}{a}-\frac{2i\left(-acx+c\right)^{2}\sqrt{\frac{1}{acx+c}}}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] 4*I*c*x*sqrt(1/(a*c*x + c))/3 + 4*I*c*sqrt(1/(a*c*x + c))/a - 2*I*(-a*c*x +
 c)**2*sqrt(1/(a*c*x + c))/(3*a*c)
```

Giac [A] time = 1.17643, size = 80, normalized size = 1.29

$$-\frac{2\left(\frac{4\sqrt{2}\sqrt{-c}c}{a}-\frac{\left(-acx-c\right)^{\frac{3}{2}}+6\sqrt{-acx-c}c}{a}\right)|c|\mathrm{sgn}\left(ax+1\right)}{3c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(4*sqrt(2)*sqrt(-c)*c/a - ((-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/
a)*abs(c)*sgn(a*x + 1)/c^2
```

3.339 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.208993, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6176, 6181, 78, 54, 215}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0488794, size = 74, normalized size = 0.79

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}+\sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.155, size = 80, normalized size = 0.9

$$2\frac{(ax+1)\sqrt{-c(ax-1)}}{(ax-1)\sqrt{-c(ax+1)}}\sqrt{\frac{ax-1}{ax+1}}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] 2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a
*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [A] time = 1.64288, size = 489, normalized size = 5.2

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, \frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\cdots}}{acx-c}\right.\right.}{ax\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), 2*((a*x - 1)*sqrt(c)*arct
```

```
an(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) + sqrt(-
a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x, x)
```

Giac [A] time = 1.19514, size = 103, normalized size = 1.1

$$\frac{2\left(c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)-\left(\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)+\sqrt{-acx-c}\right)c+\sqrt{2}\sqrt{-c}c\right)|c|\mathrm{sgn}\,(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(c^(3/2)*arctan(sqrt(2)*sqrt(-c)/sqrt(c)) - (sqrt(c)*arctan(sqrt(-a*c*x -
 c)/sqrt(c)) + sqrt(-a*c*x - c))*c + sqrt(2)*sqrt(-c)*c)*abs(c)*sgn(a*x + 1
)/c^2
```

3.340 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) - (3*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.203605, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6176, 6181, 80, 54, 215}

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) - (3*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0391965, size = 78, normalized size = 0.81

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 3*Sqrt
[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.142, size = 90, normalized size = 0.9

$$\frac{ax+1}{(ax-1)\,x}\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\left(-3\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{-c\,(ax+1)}\sqrt{c}\right)\frac{1}{\sqrt{-c\,(ax+1)}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))*x*a*c+(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
/x/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A] time = 1.65831, size = 520, normalized size = 5.42

$$\left[\frac{3\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)},-\frac{3\left(a^2x^2-ax\right)\sqrt{c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(3*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -(3*(a^2*x^
2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/
(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
 - x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16243, size = 136, normalized size = 1.42

$$\frac{\left(ac^2\left(\frac{3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx-c}}{acx}\right)-\frac{3ac^2\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{2}a\sqrt{-c}c^{\frac{3}{2}}}{\sqrt{c}}\right)|c|\text{sgn}(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (a*c^2*(3*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - sqrt(-a*c*x - c)/(a*c*
x)) - (3*a*c^2*arctan(sqrt(2)*sqrt(-c)/sqrt(c)) - sqrt(2)*a*sqrt(-c)*c^(3/2
))/sqrt(c))*abs(c)*sgn(a*x + 1)/c^2
```

3.341 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=139

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] (4*Sqrt[c - a*c*x])/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x)^
(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(9/
2))/(9*a^4*c^4) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[
c])])/a^4
```

Rubi [A] time = 0.264182, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 88, 50, 63, 206}

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x)^
(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(9/
2))/(9*a^4*c^4) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[
c])])/a^4
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= -\int \frac{x^3(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$= -\frac{\int \frac{x^3(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^3} - \frac{(c-acx)^{3/2}}{a^3(1+ax)} - \frac{(c-acx)^{5/2}}{a^3c} + \frac{(c-acx)^{7/2}}{a^3c^2}\right)dx}{c}$$
$$= \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^3c}$$
$$= \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^3}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{(4c)\int \frac{}{(1+a}}{a}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{8\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{4\sqrt{2}\sqrt{c}\tan}{}$$

Mathematica [A] time = 0.126347, size = 85, normalized size = 0.61

$$\frac{2\left(\left(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788\right)\sqrt{c-acx}-630\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4)
 - 630*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(315*a^
4)
```

Maple [A] time = 0.046, size = 101, normalized size = 0.7

$$2\,\frac{1}{c^4a^4}\left(1/9\,\left(-acx+c\right)^{9/2}-1/7\,\left(-acx+c\right)^{7/2}c+1/5\,\left(-acx+c\right)^{5/2}c^2+1/3\,c^3\left(-acx+c\right)^{3/2}+2\,\sqrt{-acx+c}c^4-2\,c^{9/2}\sqrt{2}\mathrm{Art}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 2/c^4/a^4*(1/9*(-a*c*x+c)^(9/2)-1/7*(-a*c*x+c)^(7/2)*c+1/5*(-a*c*x+c)^(5/2)
*c^2+1/3*c^3*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*c^4-2*c^(9/2)*2^(1/2)*arct
anh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59791, size = 452, normalized size = 3.25

$$\left[\frac{2\left(315\,\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+\left(35\,a^4x^4-95\,a^3x^3+138\,a^2x^2-236\,ax+788\right)\sqrt{-acx+c}\right)}{315\,a^4},\frac{2\left(630\,\sqrt{2}\sqrt{-c}\,\mathrm{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 788
)*sqrt(-a*c*x + c))/a^4, 2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*
x + 788)*sqrt(-a*c*x + c))/a^4]
```

Sympy [A] time = 8.78105, size = 126, normalized size = 0.91

$$\frac{2\left(\frac{2\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^4\sqrt{-acx+c}+\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] 2*(2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*
c**4*sqrt(-a*c*x + c) + c**3*(-a*c*x + c)**(3/2)/3 + c**2*(-a*c*x + c)**(5/
2)/5 - c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c**4)
```

Giac [A] time = 1.1452, size = 215, normalized size = 1.55

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}}+\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^{32}c^{32}+45\,(acx-c)^3\sqrt{-acx+c}a^{32}c^{33}+63\,(acx-c)^2\sqrt{-acx+c}\right.}{315\,a^{36}c^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) +
2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 105*(
-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^36)
```

3.342 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c*x)
^(7/2))/(7*a^3*c^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^3
```

Rubi [A] time = 0.248465, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 88, 50, 63, 206}

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c*x)
^(7/2))/(7*a^3*c^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^3
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= -\int \frac{x^2(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$= -\frac{\int \frac{x^2(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(1+ax)} - \frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}$$
$$= -\frac{2(c-acx)^{7/2}}{7a^3c^3} - \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^2c}$$
$$= -\frac{2(c-acx)^{3/2}}{3a^3c} - \frac{2(c-acx)^{7/2}}{7a^3c^3} - \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^2}$$
$$= -\frac{4\sqrt{c-acx}}{a^3} - \frac{2(c-acx)^{3/2}}{3a^3c} - \frac{2(c-acx)^{7/2}}{7a^3c^3} - \frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a^2}$$
$$= -\frac{4\sqrt{c-acx}}{a^3} - \frac{2(c-acx)^{3/2}}{3a^3c} - \frac{2(c-acx)^{7/2}}{7a^3c^3} + \frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a^3}$$
$$= -\frac{4\sqrt{c-acx}}{a^3} - \frac{2(c-acx)^{3/2}}{3a^3c} - \frac{2(c-acx)^{7/2}}{7a^3c^3} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Mathematica [A] time = 0.0899396, size = 78, normalized size = 0.8

$$\frac{2\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{c-acx}+84\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-52 + 16*a*x - 9*a^2*x^2 + 3*a^3*x^3) + 84*Sqrt[2]*Sqrt
[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

Maple [A] time = 0.046, size = 75, normalized size = 0.8

$$-2\,\frac{1}{c^3a^3}\left(1/7\,(-acx+c)^{7/2}+1/3\,(-acx+c)^{3/2}c^2+2\,\sqrt{-acx+c}c^3-2\,c^{7/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c^3/a^3*(1/7*(-a*c*x+c)^(7/2)+1/3*(-a*c*x+c)^(3/2)*c^2+2*(-a*c*x+c)^(1/2
)*c^3-2*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7128, size = 400, normalized size = 4.12

$$\left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{-acx+c}\right)}{21a^3}, -\frac{2\left(42\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\cdots}}{2c}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/
a^3, -2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c
)/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

Sympy [A] time = 5.76929, size = 95, normalized size = 0.98

$$-\frac{2\left(\frac{2\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^3\sqrt{-acx+c}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c**3*sqrt(-a*c*x + c) + c**2*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(7/2)/7
)/(a**3*c**3)
```

Giac [A] time = 1.14079, size = 142, normalized size = 1.46

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}+\frac{2\left(3\,(acx-c)^3\sqrt{-acx+c}a^{18}c^{18}-7\,(-acx+c)^{\frac{3}{2}}a^{18}c^{20}-42\,\sqrt{-acx+c}a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c)) +
 2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*a^
18*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

3.343 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{4\sqrt{c-acx}}{a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] (4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x)^
(5/2))/(5*a^2*c^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^2
```

Rubi [A] time = 0.170344, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6130, 21, 80, 50, 63, 206}

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{4\sqrt{c-acx}}{a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x)^
(5/2))/(5*a^2*c^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^2
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=-\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=-\int\frac{x(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$=-\frac{\int\frac{x(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$=\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{\int\frac{(c-acx)^{3/2}}{1+ax}\,dx}{ac}$$
$$=\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2\int\frac{\sqrt{c-acx}}{1+ax}\,dx}{a}$$
$$=\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{(4c)\int\frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a}$$
$$=\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{8\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a^2}$$
$$=\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Mathematica [A] time = 0.0818633, size = 70, normalized size = 0.72

$$\frac{2\left(3a^2x^2-11ax+38\right)\sqrt{c-acx}-60\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) - 60*Sqrt[2]*Sqrt[c]*ArcTanh[S
qrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

Maple [A] time = 0.043, size = 73, normalized size = 0.8

$$2\,\frac{1}{a^2c^2}\left(1/5\,(-acx+c)^{5/2}+1/3\,c\,(-acx+c)^{3/2}+2\,\sqrt{-acx+c}c^2-2\,c^{5/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 2/c^2/a^2*(1/5*(-a*c*x+c)^(5/2)+1/3*c*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*c
^2-2*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64144, size = 366, normalized size = 3.77

$$\left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^2x^2-11ax+38\right)\sqrt{-acx+c}\right)}{15a^2}, \frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)+\left(3a^2x\right.\right.}{15a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, 2/15*(3
0*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*a^2
*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

Sympy [A] time = 4.81473, size = 92, normalized size = 0.95

$$\frac{2\left(\frac{2\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^2\sqrt{-acx+c}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] 2*(2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*
c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a
**2*c**2)
```

Giac [A] time = 1.12735, size = 142, normalized size = 1.46

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}}+\frac{2\left(3(acx-c)^2\sqrt{-acx+c}a^8c^8+5(-acx+c)^{\frac{3}{2}}a^8c^9+30\sqrt{-acx+c}a^8c^{10}\right)}{15a^{10}c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) +
2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8*c
^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

3.344 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.0992223, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{3/2}}{3ac}-2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}-(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0311752, size = 61, normalized size = 0.8

$$\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A] time = 0.043, size = 59, normalized size = 0.8

$$-2\,\frac{1}{ac}\left(1/3\,(-acx+c)^{3/2}+2\,c\sqrt{-acx+c}-2\,c^{3/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh
(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40241, size = 313, normalized size = 4.12

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

Sympy [A] time = 3.57276, size = 73, normalized size = 0.96

$$-\frac{2\left(\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A] time = 1.18394, size = 104, normalized size = 1.37

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

3.345 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=74

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.232492, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6167, 6130, 21, 84, 156, 63, 208, 206}

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx\\ &= -\int \frac{(1-ax)\sqrt{c-acx}}{x(1+ax)}\,dx\\ &= -\frac{\int \frac{(c-acx)^{3/2}}{x(1+ax)}\,dx}{c}\\ &= 2\sqrt{c-acx}-\frac{\int \frac{ac^2-3a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{ac}\\ &= 2\sqrt{c-acx}-c\int \frac{1}{x\sqrt{c-acx}}\,dx+(4ac)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\ &= 2\sqrt{c-acx}-8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)+\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)}{a}\\ &= 2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\end{aligned}$$

Mathematica [A] time = 0.0325196, size = 74, normalized size = 1.

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.051, size = 58, normalized size = 0.8

$$2\,\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}-4\,\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}\sqrt{c}+2\,\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34585, size = 416, normalized size = 5.62

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}, 4\sqrt{2}\sqrt{-c}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(
a*x + 1)) + sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(-c)/c) - 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x +
 c)]
```

Sympy [A] time = 5.38679, size = 80, normalized size = 1.08

$$-\frac{2c\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}c\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] -2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan(sqrt(2)*sq
rt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Giac [A] time = 1.15313, size = 90, normalized size = 1.22

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\,c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*c*ar
ctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

3.346 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.226114, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6167, 6130, 21, 98, 156, 63, 208, 206}

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^2(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^2(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{x} + \frac{\int \frac{\frac{5ac^2}{2}-\frac{3}{2}a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{x} + \frac{1}{2}(5ac)\int \frac{1}{x\sqrt{c-acx}}\,dx - \left(4a^2c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{x} - 5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) + (8a)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)$$
$$= \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.0411239, size = 78, normalized size = 1.

$$\frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.051, size = 71, normalized size = 0.9

$$-2\,ac\left(-1/2\,\frac{\sqrt{-acx+c}}{acx} + 5/2\,\frac{1}{\sqrt{c}}\text{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right) - 2\,\frac{\sqrt{2}}{\sqrt{c}}\text{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] -2*a*c*(-1/2*(-a*c*x+c)^(1/2)/x/a/c+5/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^
(1/2))-2*2^(1/2)/c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32769, size = 455, normalized size = 5.83

$$\left[\frac{4\sqrt{2}a\sqrt{c}x\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+5a\sqrt{c}x\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}}{2x},-\frac{4\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c)
- 2*c)/x) + 2*sqrt(-a*c*x + c))/x, -(4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt
(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*s
qrt(-c)/c) - sqrt(-a*c*x + c))/x]
```

Sympy [B] time = 6.87575, size = 162, normalized size = 2.08

$$-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{6ac\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}ac\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-ac}}{2\sqrt{-c}}\right.}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] -a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + a*c**
2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + 6*a*c*atan(s
qrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*a*c*atan(sqrt(2)*sqrt(-a*c*x
 + c)/(2*sqrt(-c)))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.15494, size = 96, normalized size = 1.23

$$-\frac{4\sqrt{2}ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 5*a
*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

3.347 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=106

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}$$

```
[Out] Sqrt[c - a*c*x]/(2*x^2) - (9*a*Sqrt[c - a*c*x])/(4*x) + (23*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.259572, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6167, 6130, 21, 98, 151, 156, 63, 208, 206}

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```


```
[Out] Sqrt[c - a*c*x]/(2*x^2) - (9*a*Sqrt[c - a*c*x])/(4*x) + (23*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^3(1+ax)}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^3(1+ax)}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{2x^2}+\frac{\int \frac{\frac{9ac^2}{2}-\frac{7}{2}a^2c^2x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{2c} \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}-\frac{\int \frac{\frac{23a^2c^3}{4}-\frac{9}{4}a^3c^3x}{x(1+ax)\sqrt{c-acx}}\,dx}{2c^2} \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}-\frac{1}{8}\left(23a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx+\left(4a^3c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{1}{4}(23a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)-\left(8a^2\right)\text{Subst}\left(\int \frac{}{2}\right. \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0766494, size = 93, normalized size = 0.88

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{(2-9ax)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] ((2 - 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
```

```
qrt[c])]
```

Maple [A] time = 0.055, size = 95, normalized size = 0.9

$$2\,a^{2}c^{2}\left(-\frac{1}{c}\left(\frac{1}{a^{2}x^{2}c^{2}}\left(-\frac{9\,(-acx+c)^{3/2}}{8}+\frac{7\,c\sqrt{-acx+c}}{8}\right)-\frac{23}{8\,\sqrt{c}}\mathrm{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)-2\,\frac{\sqrt{2}}{c^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-acx+c}\sqrt{2}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] 2*a^2*c^2*(-1/c*((-9/8*(-a*c*x+c)^(3/2)+7/8*c*(-a*c*x+c)^(1/2))/x^2/a^2/c^2
-23/8/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))-2/c^(3/2)*2^(1/2)*arctanh(
1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25115, size = 524, normalized size = 4.94

$$\left[\frac{16\,\sqrt{2}a^{2}\sqrt{c}x^{2}\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+23\,a^{2}\sqrt{c}x^{2}\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\,\sqrt{-acx+c}(9\,ax-2)}{8\,x^{2}},\frac{16\,\sqrt{2}a^{2}\sqrt{-c}x^{2}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c
```

```
)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, 1/4*(16*sqrt(2)*
a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a^2*s
qrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*a*x -
 2))/x^2]
```

Sympy [B] time = 12.8379, size = 352, normalized size = 3.32

$$\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2(-acx+c)^2}-\frac{6a^2c^3(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2(-acx+c)^2}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] 10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**
2) - 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x
 + c)**2) - 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x
 + c))/8 + 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x +
 c))/8 + 3*a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x +
c))/2 - 3*a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)
)/2 - 8*a**2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*a**2*c*
atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 3*a*sqrt(-a*c*x + c)
/x
```

Giac [A] time = 1.14123, size = 143, normalized size = 1.35

$$\frac{4\sqrt{2}a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{9\,(-acx+c)^{\frac{3}{2}}a^2c-7\sqrt{-acx+c}\,a^2c^2}{4\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 23
/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(9*(-a*c*x + c)^(
3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

3.348 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=127

$$\frac{19a^2\sqrt{c-acx}}{8x}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{\sqrt{c-acx}}{3x^3}$$

```
[Out] Sqrt[c - a*c*x]/(3*x^3) - (13*a*Sqrt[c - a*c*x])/(12*x^2) + (19*a^2*Sqrt[c
- a*c*x])/(8*x) - (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.277919, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6167, 6130, 21, 98, 151, 156, 63, 208, 206}

$$\frac{19a^2\sqrt{c-acx}}{8x}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) - (13*a*Sqrt[c - a*c*x])/(12*x^2) + (19*a^2*Sqrt[c
- a*c*x])/(8*x) - (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^4(1+ax)}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^4(1+ax)}\,dx}{c}\\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{\int \frac{\frac{13ac^2}{2}-\frac{11}{2}a^2c^2x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{3c}\\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}-\frac{\int \frac{\frac{57a^2c^3}{4}-\frac{39}{4}a^3c^3x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{6c^2}\\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}+\frac{\int \frac{\frac{135a^3c^4}{8}-\frac{57}{8}a^4c^4x}{x(1+ax)\sqrt{c-acx}}\,dx}{6c^3}\\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}+\frac{1}{16}\left(45a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx-\left(4a^4c\right)\int \frac{}{(1+}\\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}-\frac{1}{8}\left(45a^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)+\\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^3\sqrt{c}\tanh
\end{aligned}$$

Mathematica [A] time = 0.0902037, size = 101, normalized size = 0.8

$$\frac{\left(57a^2x^2-26ax+8\right)\sqrt{c-acx}}{24x^3}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(8 - 26*a*x + 57*a^2*x^2))/(24*x^3) - (45*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.056, size = 110, normalized size = 0.9

$$-2\,c^{3}a^{3}\left(-\frac{1}{c^{2}}\left(-\frac{1}{x^{3}a^{3}c^{3}}\left(-\frac{19\,(-acx+c)^{5/2}}{16}+\frac{11\,c\,(-acx+c)^{3/2}}{6}-\frac{13\,\sqrt{-acx+c}c^{2}}{16}\right)-\frac{45}{16\,\sqrt{c}}\mathrm{Artanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] -2*c^3*a^3*(-1/c^2*(-(-19/16*(-a*c*x+c)^(5/2)+11/6*c*(-a*c*x+c)^(3/2)-13/16
*(-a*c*x+c)^(1/2)*c^2)/x^3/a^3/c^3-45/16/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c
^(1/2)))-2/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37009, size = 568, normalized size = 4.47

$$\left[\frac{96\,\sqrt{2}a^{3}\sqrt{c}x^{3}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+135\,a^{3}\sqrt{c}x^{3}\log\left(\frac{acx+2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\left(57\,a^{2}x^{2}-26\,ax+8\right)\sqrt{-acx+c}}{48\,x^{3}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) + 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3,
-1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt
(-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (57*a^
2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

Sympy [B] time = 16.9645, size = 614, normalized size = 4.83

$$-\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] -66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x
+ c)**2 + 48*c**3*(-a*c*x + c)**3) + 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144
*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) -
 30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c
*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 30*a**3*c**4*sqrt(-a*c*x + c)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 5*a**3*c**4*sqrt(c**(-7))*log(
-c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 5*a**3*c**4*sqrt(c**(-7))*log(
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 18*a**3*c**3*(-a*c*x + c)**(3/2
)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 9*a**3*c**3*sqrt(c**(-5
))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 9*a**3*c**3*sqrt(c**(-5)
)*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 2*a**3*c**2*sqrt(c**(-3))*
log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 2*a**3*c**2*sqrt(c**(-3))*log
(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 8*a**3*c*atan(sqrt(-a*c*x + c)/sq
rt(-c))/sqrt(-c) - 4*sqrt(2)*a**3*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-
c)))/sqrt(-c) + 4*a**2*sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.13983, size = 180, normalized size = 1.42

$$-\frac{4\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{45\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{57\left(acx-c\right)^2\sqrt{-acx+c}a^3c-88\left(-acx+c\right)^{\frac{3}{2}}a^3c^2+39\sqrt{-acx}}{24\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4
5/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(57*(a*c*x - c)
^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c*x
+ c)*a^3*c^3)/(a^3*c^3*x^3)
```

3.349 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=148

$$\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{\sqrt{c-}}{4x}$$

```
[Out] Sqrt[c - a*c*x]/(4*x^4) - (17*a*Sqrt[c - a*c*x])/(24*x^3) + (107*a^2*Sqrt[c
 - a*c*x])/(96*x^2) - (149*a^3*Sqrt[c - a*c*x])/(64*x) + (363*a^4*Sqrt[c]*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
- a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.305208, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6167, 6130, 21, 98, 151, 156, 63, 208, 206}

$$\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{\sqrt{c-}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```


```
[Out] Sqrt[c - a*c*x]/(4*x^4) - (17*a*Sqrt[c - a*c*x])/(24*x^3) + (107*a^2*Sqrt[c
 - a*c*x])/(96*x^2) - (149*a^3*Sqrt[c - a*c*x])/(64*x) + (363*a^4*Sqrt[c]*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
- a*c*x]/(Sqrt[2]*Sqrt[c])]
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^5(1+ax)}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^5(1+ax)}\,dx}{c}\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{\int \frac{\frac{17ac^2}{2}-\frac{15}{2}a^2c^2x}{x^4(1+ax)\sqrt{c-acx}}\,dx}{4c}\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}-\frac{\int \frac{\frac{107a^2c^3}{4}-\frac{85}{4}a^3c^3x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{12c^2}\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}+\frac{\int \frac{\frac{447a^3c^4}{8}-\frac{321}{8}a^4c^4x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{24c^3}\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}-\frac{\int \frac{\frac{1089a^4c^5}{16}-\frac{447}{16}a^5c^5x}{x(1+ax)\sqrt{c-acx}}\,dx}{24c^4}\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}-\frac{1}{128}\left(363a^4c\right)\int \frac{1}{x\sqrt{c-acx}}\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{1}{64}\left(363a^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-}\right.\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ac}}{\sqrt{c}}\right.
\end{aligned}$$

Mathematica [A] time = 0.111842, size = 109, normalized size = 0.74

$$\frac{\left(-447a^3x^3+214a^2x^2-136ax+48\right)\sqrt{c-acx}}{192x^4}+\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a*c*x]*(48 - 136*a*x + 214*a^2*x^2 - 447*a^3*x^3))/(192*x^4) + (3
63*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A] time = 0.055, size = 123, normalized size = 0.8

$$2\,c^4a^4\left(-\frac{1}{c^3}\left(\frac{1}{x^4a^4c^4}\left(-\frac{149\,(-acx+c)^{7/2}}{128}+\frac{1127\,c\,(-acx+c)^{5/2}}{384}-\frac{1049\,(-acx+c)^{3/2}\,c^2}{384}+\frac{107\,\sqrt{-acx+c}c^3}{128}\right)-\frac{363}{128\,\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] 2*c^4*a^4*(-1/c^3*((-149/128*(-a*c*x+c)^(7/2)+1127/384*c*(-a*c*x+c)^(5/2)-1
049/384*(-a*c*x+c)^(3/2)*c^2+107/128*(-a*c*x+c)^(1/2)*c^3)/x^4/a^4/c^4-363/
128/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))-2/c^(7/2)*2^(1/2)*arctanh(1/
2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48473, size = 621, normalized size = 4.2

$$\left[\frac{768\,\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+1089\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\left(447\,a^3x^3-214\,a^2x^2+136\,ax-4\right.}{384\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)
*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*
x + c)*sqrt(c) - 2*c)/x) - 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqr
t(-a*c*x + c))/x^4, 1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*
sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c
)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*x + c)
)/x^4]
```

Sympy [B] time = 44.0773, size = 991, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] 558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c
*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 1022*a
**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x
 + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 770*a**4
*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 198*a**4*c*
*6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 4
8*c**3*(-a*c*x + c)**3) - 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x
- 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c
**4*(-a*c*x + c)**4) - 240*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 9
6*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 35*a**4*c**5
*sqrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 35*a**4*c*
*5*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 90*a**4*c
**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2
 + 48*c**3*(-a*c*x + c)**3) + 40*a**4*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x -
8*c**4 + 8*c**2*(-a*c*x + c)**2) + 15*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqr
t(c**(-7)) + sqrt(-a*c*x + c))/16 - 15*a**4*c**4*sqrt(c**(-7))*log(c**4*sqr
t(c**(-7)) + sqrt(-a*c*x + c))/16 - 24*a**4*c**3*(-a*c*x + c)**(3/2)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 3*a**4*c**3*sqrt(c**(-5))*log(-
c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 + 3*a**4*c**3*sqrt(c**(-5))*log(c*
*3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 + 2*a**4*c**2*sqrt(c**(-3))*log(-c**
2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 2*a**4*c**2*sqrt(c**(-3))*log(c**2*sq
```

```
rt(c**(-3)) + sqrt(-a*c*x + c)) - 8*a**4*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/
sqrt(-c) + 4*sqrt(2)*a**4*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqr
t(-c) - 4*a**3*sqrt(-a*c*x + c)/x
```

Giac [A] time = 1.17103, size = 216, normalized size = 1.46

$$\frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{363\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}}-\frac{447\,(acx-c)^3\sqrt{-acx+c}a^4c+1127\,(acx-c)^2\sqrt{-acx+c}a^4c^2}{192\,a^4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 36
3/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/192*(447*(a*c*x -
 c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2
- 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4*
x^4)
```

3.350 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=281

$$\frac{164x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{656x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}+\frac{1312\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-$$

```
[Out] (1312*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(45*a^4*Sqrt[1 - 1/(a*x)]) - (656*
Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(45*a^3*Sqrt[1 - 1/(a*x)]) - (82*x^2*S
qrt[c - a*c*x])/(9*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (164*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (8*x^3*Sqrt[c -
 a*c*x])/(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^4*Sqrt[c - a*c*x]
)/(9*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.286094, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{164x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{656x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}+\frac{1312\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (1312*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(45*a^4*Sqrt[1 - 1/(a*x)]) - (656*
Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(45*a^3*Sqrt[1 - 1/(a*x)]) - (82*x^2*S
qrt[c - a*c*x])/(9*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (164*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (8*x^3*Sqrt[c -
 a*c*x])/(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^4*Sqrt[c - a*c*x]
)/(9*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{14}{a}+\frac{9x}{2a^2}}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{9\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(41\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\right.}{9a^2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(82\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Su}}{3a^2}$$

$$=-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

$$=-\frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{8x^3\sqrt{c-}}{9a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}x^2}{15a^2\sqrt{1-}}$$

Mathematica [A] time = 0.0414572, size = 73, normalized size = 0.26

$$\frac{2\left(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656\right)\sqrt{c-acx}}{45a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
5*a^5*x^5))/(45*a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.051, size = 80, normalized size = 0.3

$$\frac{(2\,ax+2)\left(5\,x^5a^5-20\,x^4a^4+41\,x^3a^3-82\,a^2x^2+328\,ax+656\right)}{45\,a^4\,(ax-1)^2}\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/45*(a*x+1)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-82*a^2*x^2+328*a*x+656)*(-a*c
*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

Maxima [A] time = 1.12839, size = 158, normalized size = 0.56

$$\frac{2\left(5\,a^6\sqrt{-c}x^6-15\,a^5\sqrt{-c}x^5+21\,a^4\sqrt{-c}x^4-41\,a^3\sqrt{-c}x^3+246\,a^2\sqrt{-c}x^2+984\,a\sqrt{-c}x+656\,\sqrt{-c}\right)(ax-1)^2}{45\left(a^6x^2-2\,a^5x+a^4\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(5*a^6*sqrt(-c)*x^6 - 15*a^5*sqrt(-c)*x^5 + 21*a^4*sqrt(-c)*x^4 - 41*a
^3*sqrt(-c)*x^3 + 246*a^2*sqrt(-c)*x^2 + 984*a*sqrt(-c)*x + 656*sqrt(-c))*(
a*x - 1)^2/((a^6*x^2 - 2*a^5*x + a^4)*(a*x + 1)^(3/2))
```

Fricas [A] time = 1.39528, size = 176, normalized size = 0.63

$$\frac{2\left(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{45\left(a^5x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^5*x - a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30864, size = 184, normalized size = 0.65

$$-\frac{2\left(5\,(acx+c)^4\sqrt{-acx-c}|c| - 45\,(acx+c)^3\sqrt{-acx-c}c|c| + 171\,(acx+c)^2\sqrt{-acx-c}c^2|c| + 375\,(-acx-c)^{\frac{3}{2}}c^3|c| + 720\,\sqrt{-}\right.}{45\,a^4c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -2/45*(5*(a*c*x + c)^4*sqrt(-a*c*x - c)*abs(c) - 45*(a*c*x + c)^3*sqrt(-a*c
*x - c)*c*abs(c) + 171*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^2*abs(c) + 375*(-a*
c*x - c)^(3/2)*c^3*abs(c) + 720*sqrt(-a*c*x - c)*c^4*abs(c) - 180*c^5*abs(c
)/sqrt(-a*c*x - c))/(a^4*c^5)
```

3.351 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=231

$$\frac{1336x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2672\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (-2672*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) - (33
4*x*Sqrt[c - a*c*x])/(35*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1336*S
qrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (44*x^2*S
qrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^3*Sqrt[c
- a*c*x])/(7*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.261385, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{1336x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2672\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-2672*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) - (33
4*x*Sqrt[c - a*c*x])/(35*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1336*S
qrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (44*x^2*S
qrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^3*Sqrt[c
- a*c*x])/(7*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{11}{a}+\frac{7x}{2a^2}}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{7\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(167\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,d\ldots}{35a^2\sqrt{1-\frac{1}{ax}}\ldots}$$

$$= -\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(668\sqrt{\frac{1}{x}}\sqrt{c-ac\ldots}\right.}{\ldots}$$

$$= -\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^3\sqrt{c-\ldots}}{7\sqrt{1-\frac{1}{ax}}\ldots}$$

$$= -\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} - \frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{44x^2\ldots}{35a\sqrt{1-\ldots}}$$

Mathematica [A] time = 0.0340783, size = 65, normalized size = 0.28

$$\frac{2\left(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336\right)\sqrt{c-acx}}{105a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^4
))/(105*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.042, size = 72, normalized size = 0.3

$$\frac{(2ax+2)\left(15x^4a^4-66x^3a^3+167a^2x^2-668ax-1336\right)}{105a^3(ax-1)^2}\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^2-668*a*x-1336)*(-a*c*x+c)^(
1/2)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Maxima [A] time = 1.09701, size = 140, normalized size = 0.61

$$\frac{2\left(15a^5\sqrt{-c}x^5-51a^4\sqrt{-c}x^4+101a^3\sqrt{-c}x^3-501a^2\sqrt{-c}x^2-2004a\sqrt{-c}x-1336\sqrt{-c}\right)(ax-1)^2}{105\left(a^5x^2-2a^4x+a^3\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^5*sqrt(-c)*x^5 - 51*a^4*sqrt(-c)*x^4 + 101*a^3*sqrt(-c)*x^3 - 5
01*a^2*sqrt(-c)*x^2 - 2004*a*sqrt(-c)*x - 1336*sqrt(-c))*(a*x - 1)^2/((a^5*
x^2 - 2*a^4*x + a^3)*(a*x + 1)^(3/2))
```

Fricas [A] time = 1.274, size = 163, normalized size = 0.71

$$\frac{2\left(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25588, size = 149, normalized size = 0.65

$$-\frac{2\left(15\,(acx+c)^3\sqrt{-acx-c}|c|-126\,(acx+c)^2\sqrt{-acx-c}c|c|-455\,(-acx-c)^{\frac{3}{2}}c^2|c|-1260\,\sqrt{-acx-c}c^3|c|+\frac{420\,c^4|c|}{\sqrt{-acx-c}}\right)}{105\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -2/105*(15*(a*c*x + c)^3*sqrt(-a*c*x - c)*abs(c) - 126*(a*c*x + c)^2*sqrt(-
a*c*x - c)*c*abs(c) - 455*(-a*c*x - c)^(3/2)*c^2*abs(c) - 1260*sqrt(-a*c*x
- c)*c^3*abs(c) + 420*c^4*abs(c)/sqrt(-a*c*x - c))/(a^3*c^4)
```

3.352 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=182

$$\frac{316\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (-158*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (316*
Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (32*x*Sqrt[
c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^2*Sqrt[c - a*
c*x])/(5*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.216173, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{316\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-158*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (316*
Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (32*x*Sqrt[
c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^2*Sqrt[c - a*
c*x])/(5*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{8}{a}+\frac{5x}{2a^2}}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(79\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\right.}{15a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(158\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{15}$$

$$= -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

Mathematica [A] time = 0.0302175, size = 57, normalized size = 0.31

$$\frac{2\left(3a^3x^3-16a^2x^2+79ax+158\right)\sqrt{c-acx}}{15a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^3*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.043, size = 64, normalized size = 0.4

$$\frac{(2ax+2)\left(3x^{3}a^{3}-16a^{2}x^{2}+79ax+158\right)}{15a^{2}\left(ax-1\right)^{2}}\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)*(-a*c*x+c)^(1/2)*((a*x-1)/(a
*x+1))^(3/2)/a^2/(a*x-1)^2
```

Maxima [A] time = 1.11408, size = 123, normalized size = 0.68

$$\frac{2\left(3a^{4}\sqrt{-c}x^{4}-13a^{3}\sqrt{-c}x^{3}+63a^{2}\sqrt{-c}x^{2}+237a\sqrt{-c}x+158\sqrt{-c}\right)(ax-1)^{2}}{15\left(a^{4}x^{2}-2a^{3}x+a^{2}\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*a^4*sqrt(-c)*x^4 - 13*a^3*sqrt(-c)*x^3 + 63*a^2*sqrt(-c)*x^2 + 237*
a*sqrt(-c)*x + 158*sqrt(-c))*(a*x - 1)^2/((a^4*x^2 - 2*a^3*x + a^2)*(a*x +
1)^(3/2))
```

Fricas [A] time = 1.39988, size = 139, normalized size = 0.76

$$\frac{2\left(3a^{3}x^{3}-16a^{2}x^{2}+79ax+158\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{3}x-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^3*x - a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19304, size = 113, normalized size = 0.62

$$-\frac{2\left(3\,(acx+c)^2\sqrt{-acx-c}|c|+25\,(-acx-c)^{\frac{3}{2}}c|c|+120\,\sqrt{-acx-c}c^2|c|-\frac{60\,c^3|c|}{\sqrt{-acx-c}}\right)}{15\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -2/15*(3*(a*c*x + c)^2*sqrt(-a*c*x - c)*abs(c) + 25*(-a*c*x - c)^(3/2)*c*ab
s(c) + 120*sqrt(-a*c*x - c)*c^2*abs(c) - 60*c^3*abs(c)/sqrt(-a*c*x - c))/(a
^2*c^3)
```

3.353 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=137

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rubi [A] time = 0.162681, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 89, 78, 37}

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] time = 0.0258916, size = 48, normalized size = 0.35

$$\frac{2\left(a^2x^2-10ax-23\right)\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

Maple [A] time = 0.048, size = 55, normalized size = 0.4

$$\frac{(2\,ax+2)\left(a^2x^2-10\,ax-23\right)}{3\,a\,(ax-1)^2}\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a/
(a*x-1)^2
```

Maxima [A] time = 1.07426, size = 101, normalized size = 0.74

$$\frac{2 \left(a^3 \sqrt{-c} x^3 - 9 \, a^2 \sqrt{-c} x^2 - 33 \, a \sqrt{-c} x - 23 \, \sqrt{-c}\right) (ax-1)^2}{3 \left(a^3 x^2 - 2 \, a^2 x + a\right) (ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A] time = 1.18936, size = 113, normalized size = 0.82

$$\frac{2 \left(a^2 x^2 - 10 \, ax - 23\right) \sqrt{-acx + c} \sqrt{\frac{ax-1}{ax+1}}}{3 \left(a^2 x - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17272, size = 70, normalized size = 0.51

$$\frac{2\left((-acx-c)^{\frac{3}{2}}+12\sqrt{-acx-c}c-\frac{12c^2}{\sqrt{-acx-c}}\right)|c|}{3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*((-a*c*x - c)^(3/2) + 12*sqrt(-a*c*x - c)*c - 12*c^2/sqrt(-a*c*x - c))*
abs(c)/(a*c^2)
```

3.354 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=140

$$\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{10\sqrt{c-acx}}{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (10*Sqrt[c - a*
c*x])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (2*Sqrt[x^(-1)]*Sqrt[c -
a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.22142, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 78, 54, 215}

$$\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{10\sqrt{c-acx}}{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (10*Sqrt[c - a*
c*x])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (2*Sqrt[x^(-1)]*Sqrt[c -
a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{2}{a}+\frac{x}{2a^2}}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.111546, size = 78, normalized size = 0.56

$$\frac{2\sqrt{c-acx}\left(-\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)+a+\frac{5}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(a + 5/x - Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*ArcSin
h[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.138, size = 80, normalized size = 0.6

$$2\,{\frac { \left(ax+1 \right) \sqrt {-c \left(ax-1 \right) }}{c \left(ax-1 \right) ^{2}}\left({\frac {ax-1}{ax+1}} \right) ^{3/2}\left(\sqrt {c}\arctan \left({\frac {\sqrt {-c \left(ax+1 \right) }}{\sqrt {c}}} \right) \sqrt {-c \left(ax+1 \right) }+acx+5\,c \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] 2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(c^(1/2)*arc
tan((-c*(a*x+1))^(1/2)/c^(1/2))*(-c*(a*x+1))^(1/2)+a*c*x+5*c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx + c} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [A] time = 1.38215, size = 490, normalized size = 3.5

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{\ldots}}{acx\ldots}\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
```

```
)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20187, size = 73, normalized size = 0.52

$$2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{4}{\sqrt{-acx-c}}-\frac{\sqrt{-acx-c}}{c}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 4/sqrt(-a*c*x - c) - sqrt(-a*
c*x - c)/c)*abs(c)
```

3.355 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (Sqrt[1 + 1/
(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (7*Sqrt[a]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.222996, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 80, 54, 215}

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (Sqrt[1 + 1/
(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (7*Sqrt[a]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{3}{2a^2}-\frac{x}{2a^3}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x} + \frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.077449, size = 79, normalized size = 0.56

$$\frac{\sqrt{c-acx}\left(\frac{7a^{3/2}\sqrt{\frac{1}{ax}+1}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}-9ax-1\right)}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-1 - 9*a*x + (7*a^(3/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^
(-1)]/Sqrt[a]])/(x^(-1))^(3/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A] time = 0.144, size = 86, normalized size = 0.6

$$-\frac{ax+1}{(ax-1)^2 x}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(7\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)xa\sqrt{-c(ax+1)}+9xa\sqrt{c}+\sqrt{c}\right)\sqrt{-c(ax-1)}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(7*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x*a*
(-c*(a*x+1))^(1/2)+9*x*a*c^(1/2)+c^(1/2))*(-c*(a*x-1))^(1/2)/(a*x-1)^2/c^(1
/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A] time = 1.38317, size = 524, normalized size = 3.74

$$\left[\frac{7\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(9ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, \frac{7\left(a^2x^2-ax\right)\sqrt{c}\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(7*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqr
t(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), (7*(a^2*x
^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))
/(a*c*x - c)) - sqrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^2 - x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21293, size = 85, normalized size = 0.61

$$-a\left(\frac{7\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{9\,acx+c}{(-acx-c)^{\frac{3}{2}}+\sqrt{-acx-c}c}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -a*(7*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - (9*a*c*x + c)/((-a*c*x - c
)^(3/2) + sqrt(-a*c*x - c)*c))*abs(c)
```

3.356 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=190

$$-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{2x^2\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^2) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x^2) + (47*a*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) - (47*a^(3/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.231778, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 89, 80, 50, 54, 215}

$$-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{2x^2\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^2) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x^2) + (47*a*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) - (47*a^(3/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}\left(\frac{11}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{\left(47\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{\left(47a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Su}}{8\sqrt{1}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{\left(47a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Su}}{4\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sin}{4\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0828575, size = 90, normalized size = 0.47

$$-\frac{\sqrt{c-acx}\left(-47a^2x^2+\frac{47a^{5/2}\sqrt{\frac{1}{ax}+1}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}-13ax+2\right)}{4ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -(Sqrt[c - a*c*x]*(2 - 13*a*x - 47*a^2*x^2 + (47*a^(5/2)*Sqrt[1 + 1/(a*x)]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2)))/(4*a*Sqrt[1 - 1/(a^2*x^2)]*
x^3)
```

Maple [A] time = 0.158, size = 103, normalized size = 0.5

$$\frac{ax+1}{4\,(ax-1)^2\,x^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}\left(47\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)x^2a^2\sqrt{-c\,(ax+1)}+47\,x^2a^2\sqrt{c}+13\,xa\sqrt{c}-2\,\sqrt{c}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(47*arctan
((-c*(a*x+1))^(1/2)/c^(1/2))*x^2*a^2*(-c*(a*x+1))^(1/2)+47*x^2*a^2*c^(1/2)+
13*x*a*c^(1/2)-2*c^(1/2))/c^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A] time = 1.71114, size = 587, normalized size = 3.09

$$\left[\frac{47\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\left(47a^2x^2+13ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{8\left(ax^3-x^2\right)},-\frac{47\left(a^3x^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas
")
```

```
[Out] [1/8*(47*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c
*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) +
2*(47*a^2*x^2 + 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^3 - x^2), -1/4*(47*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (47*a^2*x^2 + 13*a*x - 2)*sq
rt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23343, size = 113, normalized size = 0.59

$$\frac{1}{4}a^2c\left(\frac{47\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{32}{\sqrt{-acx-c}c}+\frac{15\left(-acx-c\right)^{\frac{3}{2}}+17\sqrt{-acx-c}c}{a^2c^3x^2}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*a^2*c*(47*arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 32/(sqrt(-a*c*x -
c)*c) + (15*(-a*c*x - c)^(3/2) + 17*sqrt(-a*c*x - c)*c)/(a^2*c^3*x^2))*abs(
c)
```

3.357 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=238

$$-\frac{119a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}+\frac{119a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{12x^2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3x^3\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-}}{x^3\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^3) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^3) + (119*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(12*Sqrt[1 - 1/(a*x)]*x^2) - (119*a^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (119*a^(5/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.246345, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 89, 80, 50, 54, 215}

$$-\frac{119a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}+\frac{119a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{12x^2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3x^3\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-}}{x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```


```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^3) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^3) + (119*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(12*Sqrt[1 - 1/(a*x)]*x^2) - (119*a^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (119*a^(5/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
```

```
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{x^{3/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{x^{3/2}\left(\frac{19}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{\left(119\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{x^{3/2}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2} - \frac{\left(119a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{S}}{8\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2} - \frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{8\sqrt{1-\frac{1}{ax}}x}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2} - \frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{8\sqrt{1-\frac{1}{ax}}x}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2} - \frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{8\sqrt{1-\frac{1}{ax}}x}$$

Mathematica [A] time = 0.0910814, size = 98, normalized size = 0.41

$$\frac{\sqrt{c-acx}\left(-357a^3x^3-119a^2x^2+\frac{357a^{7/2}\sqrt{\frac{1}{ax}+1}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}+38ax-8\right)}{24ax^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-8 + 38*a*x - 119*a^2*x^2 - 357*a^3*x^3 + (357*a^(7/2)*Sq
rt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2)))/(24*a*Sqrt[
1 - 1/(a^2*x^2)]*x^4)
```

Maple [A] time = 0.152, size = 114, normalized size = 0.5

$$-\frac{ax+1}{24\,(ax-1)^2x^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}\left(357\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)x^3a^3\sqrt{-c\,(ax+1)}+357\,x^3a^3\sqrt{c}+119\,x^2a^2\sqrt{c}-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(357*arc
tan((-c*(a*x+1))^(1/2)/c^(1/2))*x^3*a^3*(-c*(a*x+1))^(1/2)+357*x^3*a^3*c^(1
/2)+119*x^2*a^2*c^(1/2)-38*x*a*c^(1/2)+8*c^(1/2))/c^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A] time = 1.63805, size = 632, normalized size = 2.66

$$\left[\frac{357\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\left(357a^3x^3+119a^2x^2-38ax+8\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{48\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas
")
```

```
[Out] [1/48*(357*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a
*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x))
- 2*(357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/(a*x^4 - x^3), 1/24*(357*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(s
qrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (357*a^3*x
^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a*x^4 - x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24309, size = 150, normalized size = 0.63

$$-\frac{1}{24}\,a^{3}c^{2}\left(\frac{357\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{192}{\sqrt{-acx-c}c^{2}}-\frac{165\,(acx+c)^{2}\sqrt{-acx-c}+376\,(-acx-c)^{\frac{3}{2}}c+219\,\sqrt{-acx-c}c^{2}}{a^{3}c^{5}x^{3}}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*a^3*c^2*(357*arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) + 192/(sqrt(-a*
c*x - c)*c^2) - (165*(a*c*x + c)^2*sqrt(-a*c*x - c) + 376*(-a*c*x - c)^(3/2
)*c + 219*sqrt(-a*c*x - c)*c^2)/(a^3*c^5*x^3))*abs(c)
```

3.358 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=286

$$-\frac{1115a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{96x^2\sqrt{1-\frac{1}{ax}}}+\frac{1115a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}-\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}+\frac{223a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{24x^3\sqrt{1-\frac{1}{ax}}}-$$

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^4) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^4) + (223*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^3) - (1115*a^2*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(96*Sqrt[1 - 1/(a*x)]*x^2) + (1115*a^3*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) - (1115*a^(7/2)*Sqrt[x^(-1)]*Sq
rt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.263551, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 89, 80, 50, 54, 215}

$$-\frac{1115a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{96x^2\sqrt{1-\frac{1}{ax}}}+\frac{1115a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}-\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}+\frac{223a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{24x^3\sqrt{1-\frac{1}{ax}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```


```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^4) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^4) + (223*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^3) - (1115*a^2*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(96*Sqrt[1 - 1/(a*x)]*x^2) + (1115*a^3*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) - (1115*a^(7/2)*Sqrt[x^(-1)]*Sq
rt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a*x)])
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(\frac{27}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{\left(223\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{\left(1115a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{48}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{96\sqrt{1-\frac{1}{ax}}x^2}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{96\sqrt{1-\frac{1}{ax}}x^2}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{96\sqrt{1-\frac{1}{ax}}x^2}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3} - \frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{96\sqrt{1-\frac{1}{ax}}x^2}$$

Mathematica [A] time = 0.0917659, size = 106, normalized size = 0.37

$$-\frac{\sqrt{c-acx}\left(-3345a^4x^4-1115a^3x^3+446a^2x^2+\frac{3345a^{9/2}\sqrt{\frac{1}{ax}+1}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{9/2}}-200ax+48\right)}{192ax^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -(Sqrt[c - a*c*x]*(48 - 200*a*x + 446*a^2*x^2 - 1115*a^3*x^3 - 3345*a^4*x^4
 + (3345*a^(9/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^
(9/2)))/(192*a*Sqrt[1 - 1/(a^2*x^2)]*x^5)
```

Maple [A] time = 0.173, size = 125, normalized size = 0.4

$$\frac{ax+1}{192\,(ax-1)^2\,x^4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}\left(3345\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)x^4a^4\sqrt{-c\,(ax+1)}+3345\,x^4a^4\sqrt{c}+1115\,x^3a^3\sqrt{c}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(3345*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*x^4*a^4*(-c*(a*x+1))^(1/2)+3345*x^4*a^4*c^
(1/2)+1115*x^3*a^3*c^(1/2)-446*x^2*a^2*c^(1/2)+200*x*a*c^(1/2)-48*c^(1/2))/
c^(1/2)/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A] time = 1.73291, size = 687, normalized size = 2.4

$$\left[\frac{3345\left(a^5x^5-a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\left(3345a^4x^4+1115a^3x^3-446a^2x^2+200ax-48\right.}{384\left(ax^5-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas
")
```

```
[Out] [1/384*(3345*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)
) + 2*(3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/192*(3345*(a^5*x^5 - a^
4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a
*c*x - c)) - (3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29708, size = 182, normalized size = 0.64

$$\frac{1}{192}\,a^4c^3\left(\frac{3345\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{1536}{\sqrt{-acx-c}c^3}-\frac{1809\,(acx+c)^3\sqrt{-acx-c}-6121\,(acx+c)^2\sqrt{-acx-c}c-7063\,(-acx}{a^4c^7x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/192*a^4*c^3*(3345*arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(7/2) + 1536/(sqrt(-
a*c*x - c)*c^3) - (1809*(a*c*x + c)^3*sqrt(-a*c*x - c) - 6121*(a*c*x + c)^2
*sqrt(-a*c*x - c)*c - 7063*(-a*c*x - c)^(3/2)*c^2 - 2799*sqrt(-a*c*x - c)*c
^3)/(a^4*c^7*x^4))*abs(c)
```

3.359 $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

Optimal. Leaf size=278

$$\frac{2\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a^2(n+6)\left(n^2+6n+8\right)x}-\frac{\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a(n+4)(n+6)}+\frac{(n\ \cdots}{\cdots}$$

```
[Out] -(((56 + 14*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c
- a*c*x)^((4 + n)/2))/(a*(4 + n)*(6 + n))) + (2*(56 + 14*n + n^2)*(1 - 1/(a
*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^((4 + n)/2))/(a^2*(6
+ n)*(8 + 6*n + n^2)*x) + ((8 + n)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/(6 + n) - ((a - x^(-1))*(1 - 1/(a*x))
^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/a
```

Rubi [A] time = 0.267583, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6176, 6181, 90, 79, 45, 37}

$$\frac{2\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a^2(n+6)\left(n^2+6n+8\right)x}-\frac{\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a(n+4)(n+6)}+\frac{(n\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```


```
[Out] -(((56 + 14*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c
- a*c*x)^((4 + n)/2))/(a*(4 + n)*(6 + n))) + (2*(56 + 14*n + n^2)*(1 - 1/(a
*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^((4 + n)/2))/(a^2*(6
+ n)*(8 + 6*n + n^2)*x) + ((8 + n)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/(6 + n) - ((a - x^(-1))*(1 - 1/(a*x))
^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/a
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}x^{-2-\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{2+\frac{n}{2}}x^{2+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-4-\frac{n}{2}}\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a} + \left(a\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right.$$

$$= \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-}{a}$$

$$= -\frac{\left(56+14n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right.}{6+n}$$

$$= -\frac{\left(56+14n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{2\left(56+14n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2}}{a^2(2+n)(4+}$$

Mathematica [A] time = 0.0736152, size = 116, normalized size = 0.42

$$\frac{2c^2(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(2n\left(3a^2x^2-10ax+7\right)+8\left(a^2x^2-4ax+7\right)+n^2(ax-1)^2\right)(c-acx)^{n/2}}{a(n+2)(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(n^2*(-1 + a*x)^2 +
8*(7 - 4*a*x + a^2*x^2) + 2*n*(7 - 10*a*x + 3*a^2*x^2)))/(a*(2 + n)*(4 + n)
*(6 + n)*(1 - 1/(a*x))^(n/2))
```

Maple [A] time = 0.054, size = 104, normalized size = 0.4

$$2\,\frac{\left(a^2n^2x^2+6\,a^2nx^2+8\,a^2x^2-2\,an^2x-20\,anx-32\,ax+n^2+14\,n+56\right)(ax+1)\,e^{n\,\mathrm{arccoth}(ax)}\,(-acx+c)^{2+n/2}}{(ax-1)^2\,a\left(n^3+12\,n^2+44\,n+48\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x)
```

```
[Out] 2*(a*x+1)*(a^2*n^2*x^2+6*a^2*n*x^2+8*a^2*x^2-2*a*n^2*x-20*a*n*x-32*a*x+n^2+
14*n+56)*exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n)/(a*x-1)^2/a/(n^3+12*n^2+4
4*n+48)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n + 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(2+1/2*n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{1}{2}n+2} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.360 $\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

Optimal. Leaf size=127

$$\frac{2x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{n+4}-\frac{2(n+6)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{a(n+2)(n+4)}$$

```
[Out] (-2*(6 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^
((2 + n)/2))/(a*(2 + n)*(4 + n)) + (2*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x)
)^((2 + n)/2)*x*(c - a*c*x)^((2 + n)/2))/(4 + n)
```

Rubi [A] time = 0.157131, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 79, 37}

$$\frac{2x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{n+4}-\frac{2(n+6)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{a(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*(6 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^
((2 + n)/2))/(a*(2 + n)*(4 + n)) + (2*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x)
)^((2 + n)/2)*x*(c - a*c*x)^((2 + n)/2))/(4 + n)
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx=\left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}x^{-1-\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{1+\frac{n}{2}}x^{1+\frac{n}{2}}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-3-\frac{n}{2}}\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}+\frac{\left((6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right.}{a(4+n)}$$

$$=-\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)}+\frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)}{4+n}$$

Mathematica [A] time = 0.0482707, size = 78, normalized size = 0.61

$$-\frac{2c(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(n(ax-1)+2ax-6)(c-acx)^{n/2}}{a(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(-6 + 2*a*x + n*(-1 +
 a*x)))/(a*(2 + n)*(4 + n)*(1 - 1/(a*x))^(n/2))
```

Maple [A] time = 0.04, size = 61, normalized size = 0.5

$$2\,\frac{e^{n\operatorname{arccoth}(ax)}\,(-acx+c)^{1+n/2}\,(anx+2\,ax-n-6)\,(ax+1)}{(ax-1)\,a\left(n^{2}+6\,n+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x)
```

```
[Out] 2*(-a*c*x+c)^(1+1/2*n)*exp(n*arccoth(a*x))*(a*n*x+2*a*x-n-6)*(a*x+1)/(a*x-1
)/a/(n^2+6*n+8)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(-acx+c)^{\frac{1}{2}\,n+1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-acx+c)^{\frac{1}{2}\,n+1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n + 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1+1/2*n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{1}{2}n+1} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.361 $\int e^{n \coth^{-1}(ax)}(c - acx)^{n/2}\, dx$

Optimal. Leaf size=36

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

```
[Out] (2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

Rubi [A] time = 0.0298995, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6174}

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] (2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\, dx = \frac{2e^{n\coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

Mathematica [A] time = 0.0172169, size = 58, normalized size = 1.61

$$-\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+1}(c-acx)^{n/2}}{-\frac{n}{2}-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + n/2)*x*(c - a*c*x)^(n/2))/((-1 - n/2)*(1 - 1/(a*x))^(
n/2)))
```

Maple [A] time = 0.04, size = 34, normalized size = 0.9

$$2\,\frac{e^{n\,\mathrm{arccoth}(ax)}\,(ax+1)\,(-acx+c)^{n/2}}{a\,(2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x)
```

```
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2*n),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{1}{2}n} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.362 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$

Optimal. Leaf size=80

$$\frac{2x\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{n}$$

```
[Out] (2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x*(c - a*c*x)^((-2 + n)/2)*H
ypergeometric2F1[1, -n/2, 1 - n/2, 2/((a + x^(-1))*x)])/n
```

Rubi [A] time = 0.146342, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6176, 6181, 131}

$$\frac{2x\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x*(c - a*c*x)^((-2 + n)/2)*H
ypergeometric2F1[1, -n/2, 1 - n/2, 2/((a + x^(-1))*x)])/n
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}x^{1-\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-1+\frac{n}{2}}x^{-1+\frac{n}{2}}\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{x}\right)^{-1+\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\text{Subst}\left(\int\frac{x^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{1-\frac{x}{a}}\,dx,x,\frac{1}{x}\right)\right)$$
$$= \frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Mathematica [A] time = 0.0256766, size = 78, normalized size = 0.98

$$-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{ax+1}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/
2, 2/(1 + a*x)])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{-1+\frac{n}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{1}{2}n-1} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-acx + c)^{\frac{1}{2}n-1} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n - 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-1+1/2*n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{1}{2}n-1} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.363 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{2-n}$$

```
[Out] (-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x*(c - a*c*x)^((-4 +
 n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, 2/((a + x^(-1))*x)])/(2 - n)
```

Rubi [A] time = 0.147318, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6176, 6181, 131}

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x*(c - a*c*x)^((-4 +
 n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, 2/((a + x^(-1))*x)])/(2 - n)
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx &= \left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}x^{2-\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-2+\frac{n}{2}}x^{-2+\frac{n}{2}}\,dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{x}\right)^{-2+\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\mathrm{Subst}\left(\int\frac{x^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.0390477, size = 89, normalized size = 1.01

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{n/2}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{ax+1}\right)}{ac^2(n-2)(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, 2/(1 + a*x)])/(a*c^2*(-2 + n)*(1 - 1/(a*x))^(n/2)*(1 + a*x))
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-acx+c\right)^{-2+\frac{n}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{1}{2}n-2} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-acx + c)^{\frac{1}{2}n-2} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n - 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-2+1/2*n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{1}{2}n-2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.364 $\int e^{n \coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=104

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-p-1,-p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c -
 a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])/(
(1 + p)*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.127591, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 132}

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}{}_2F_1\left(\frac{1}{2}(n-2p),-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```


```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c -
 a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])/(
(1 + p)*(1 - 1/(a*x))^(n/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,{}_2F_1\left(\frac{1}{2}(n-2p),-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

Mathematica [A] time = 0.0476149, size = 104, normalized size = 1.

$$\frac{(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}(n-2p)}\text{Hypergeometric2F1}\left(-p-1,\frac{n}{2}-p,-p,\frac{2}{ax+1}\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((n - 2*p)/2)*(1 + a*x)*(c - a*
c*x)^p*Hypergeometric2F1[-1 - p, n/2 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*(1 -
 1/(a*x))^(n/2))
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)} \left(-acx+c\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{p} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((-acx+c)^{p} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-c\,(ax-1))^{p}\, e^{n\,\mathrm{acoth}\,(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.365 $\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=81

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] (-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric2
F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rubi [A] time = 0.127304, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 131}

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```


```
[Out] (-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric2
F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx &= -\left(\left(a^3c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)\\ &= \left(a^3c^3\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^5}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\,{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}\end{aligned}$$

Mathematica [B] time = 1.77795, size = 190, normalized size = 2.35

$$-\frac{c^3e^{n\coth^{-1}(ax)}\left((n+2)\left(\left(n^3-12n^2+44n-48\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\coth^{-1}(ax)}\right)+n^2\left(a^2x^2-12ax-1\right)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(-48 + 44*n - 12*n^2 + n^3)*
Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^3
*x + n^2*(-1 - 12*a*x + a^2*x^2) + 2*n*(6 + 21*a*x - 6*a^2*x^2 + a^3*x^3) +
 6*(-7 - 4*a*x + 6*a^2*x^2 - 4*a^3*x^3 + a^4*x^4) + (-48 + 44*n - 12*n^2 +
n^3)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(24*a*(2 + n
))
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}\left(-acx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (acx-c)^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^3c^3x^3 - 3\,a^2c^3x^2 + 3\,ac^3x - c^3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x - 1)/(a*x +
 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int 3axe^{n\,\mathrm{acoth}\,(ax)}\,dx + \int -3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int a^3x^3e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int -e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x*exp(n*acoth(a*x)), x) + Integral(-3*a**2*x**2*exp(n*a
coth(a*x)), x) + Integral(a**3*x**3*exp(n*acoth(a*x)), x) + Integral(-exp(n
*acoth(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(acx-c)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.366 $\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=81

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] (16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rubi [A] time = 0.128074, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 131}

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$
$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Mathematica [A] time = 1.04044, size = 144, normalized size = 1.78

$$\frac{c^2e^{n\coth^{-1}(ax)}\left((n+2)\left(\left(n^2-6n+8\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\coth^{-1}(ax)}\right)+n\left(a^2x^2-6ax-1\right)+2a^3x^3-6a^2x\right.\right.}{6a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(8 - 6*n + n^2)*Hypergeometri
c2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(6 + 6*a*x + a*n^2*
x - 6*a^2*x^2 + 2*a^3*x^3 + n*(-1 - 6*a*x + a^2*x^2) + (8 - 6*n + n^2)*Hype
rgeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(6*a*(2 + n))
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int e^{n\text{arccoth}(ax)}\left(-acx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (acx-c)^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*c*x - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2axe^{n\operatorname{acoth}(ax)}\,dx + \int a^2x^2e^{n\operatorname{acoth}(ax)}\,dx + \int e^{n\operatorname{acoth}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x*exp(n*acoth(a*x)), x) + Integral(a**2*x**2*exp(n*acot
h(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (acx-c)^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*c*x - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.367 $\int e^{n \coth^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=79

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] (-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1[
3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rubi [A] time = 0.075156, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6175, 6180, 131}

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1[
3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\ &= (ac)\,\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}\end{aligned}$$

Mathematica [A] time = 0.494428, size = 104, normalized size = 1.32

$$-\frac{ce^{n\coth^{-1}(ax)}\left((n+2)\left((n-2)\mathrm{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\coth^{-1}(ax)}\right)+a^2x^2+a(n-2)x-1\right)+(n-2)ne^{2\coth^{-1}(}\right.}{2a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -(c*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1[1,
1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*(-2 + n)*x + a^2*x^
2 + (-2 + n)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a
*(2 + n))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}(-acx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (acx - c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(acx - c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int axe^{n\operatorname{acoth}(ax)}\,dx + \int -e^{n\operatorname{acoth}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.368 $\int \frac{e^{n \coth^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=71

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
+ x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.121298, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 131}

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
+ x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx &= -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\ &= \frac{\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx,x,\frac{1}{x}\right)}{ac}\\ &= \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}\end{aligned}$$

Mathematica [A] time = 0.181694, size = 87, normalized size = 1.23

$$-\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+(n+2)\left(\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+\right.\right.\right.}{acn(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + Hypergeometric2F1[1, n/2, 1 + n
/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n)))
```

Maple [F] time = 0.281, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c),x)
```

```
[Out] -Integral(exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

3.369 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

```
[Out] -(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

Rubi [A] time = 0.109206, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 37}

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2x^2}\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^2c^2}\\ &= -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.167723, size = 33, normalized size = 0.69

$$-\frac{(ax+1)e^{n\coth^{-1}(ax)}}{ac^2(n+2)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(1 + a*x))/(a*c^2*(2 + n)*(-1 + a*x)))
```

Maple [A] time = 0.041, size = 33, normalized size = 0.7

$$-\frac{e^{n\,\mathrm{arccoth}(ax)}\,(ax+1)}{(ax-1)\,c^2\,(2+n)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x)
```

```
[Out] -exp(n*arccoth(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Fricas [A] time = 1.67622, size = 122, normalized size = 2.54

$$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^2n-2\,ac^2-\left(a^2c^2n-2\,a^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n - 2*a*c^2 - (a^2*c^2*n -
2*a^2*c^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^2, x)
```

3.370 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=104

$$\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+4)}-\frac{(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+2)(n+4)}$$

```
[Out] ((1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(4 + n)) - ((3
+ n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(2 + n)*(4
+ n))
```

Rubi [A] time = 0.149331, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6175, 6180, 79, 37}

$$\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+4)}-\frac{(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(4 + n)) - ((3
+ n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(2 + n)*(4
+ n))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\,\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^2c^3(4+n)}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}
\end{aligned}
$$

Mathematica [A] time = 0.216626, size = 64, normalized size = 0.62

$$
\frac{(-ax+n+3)e^{n\coth^{-1}(ax)}\left(\cosh\left(3\coth^{-1}(ax)\right)+\sinh\left(3\coth^{-1}(ax)\right)\right)}{a^2c^3(n+2)(n+4)x\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 + n - a*x)*(Cosh[3*ArcCoth[a*x]] + Sinh[3*ArcCoth[a*
x]]))/(a^2*c^3*(2 + n)*(4 + n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.049, size = 46, normalized size = 0.4

$$-\frac{e^{n\operatorname{arccoth}(ax)}\,(ax-n-3)\,(ax+1)}{(ax-1)^{2}\,c^{3}\left(n^{2}+6\,n+8\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x)
```

```
[Out] -exp(n*arccoth(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{(acx-c)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Fricas [A] time = 1.62983, size = 259, normalized size = 2.49

$$-\frac{\left(a^{2}x^{2}+(an-2\,a)x+n-3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^{3}n^{2}-6\,ac^{3}n+8\,ac^{3}+\left(a^{3}c^{3}n^{2}-6\,a^{3}c^{3}n+8\,a^{3}c^{3}\right)x^{2}-2\left(a^{2}c^{3}n^{2}-6\,a^{2}c^{3}n+8\,a^{2}c^{3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 + (a*n - 2*a)*x + n - 3)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^2
 - 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 - 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*(a
^2*c^3*n^2 - 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^3, x)
```

3.371 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=224

$$-\frac{\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{a^2c^4x}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(n\cdots}{}$$

```
[Out] ((5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^4*(6 + n)
) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(
a*c^4*(4 + n)*(6 + n)) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2)) - ((1 - 1/(a*x))^(-3 -
n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a^2*c^4*x)
```

Rubi [A] time = 0.261659, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6180, 90, 79, 45, 37}

$$-\frac{\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{a^2c^4x}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(n\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```


```
[Out] ((5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^4*(6 + n)
) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(
a*c^4*(4 + n)*(6 + n)) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2)) - ((1 - 1/(a*x))^(-3 -
n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a^2*c^4*x)
```


Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x} - \frac{\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{(4+n)x}{a}\right)dx, x, \frac{1}{x}\right)}{a^2c^4}$$

$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x} - \frac{\left(14+8n+n^2\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-3-}\right.}{a^2c^4(6+n)}$$

$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(14+8n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right.}{a^2c^4x}$$

$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(14+8n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{\left(14+8n+n^2\right)\left(1-\right.}{ac^4(2+n)(4}$$

Mathematica [A] time = 0.274697, size = 83, normalized size = 0.37

$$-\frac{e^{n\coth^{-1}(ax)}\left(\cosh\left(4\coth^{-1}(ax)\right)+\sinh\left(4\coth^{-1}(ax)\right)\right)\left((n+4)^2\cosh\left(2\coth^{-1}(ax)\right)-2(n+4)\sinh\left(2\coth^{-1}(ax)\right)\right.}{2ac^4(n+2)(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -(E^(n*ArcCoth[a*x])*(-12 - 8*n - n^2 + (4 + n)^2*Cosh[2*ArcCoth[a*x]] - 2*
(4 + n)*Sinh[2*ArcCoth[a*x]])*(Cosh[4*ArcCoth[a*x]] + Sinh[4*ArcCoth[a*x]])
)/(2*a*c^4*(2 + n)*(4 + n)*(6 + n))
```

Maple [A] time = 0.045, size = 68, normalized size = 0.3

$$-\frac{\left(2a^2x^2-2anx-8ax+n^2+8n+14\right)(ax+1)\,e^{n\,\text{arccoth}(ax)}}{(ax-1)^3c^4a\left(n^2+8n+12\right)(4+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x)
```

```
[Out] -(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arccoth(a*x))/(a*x-1)^3
/c^4/a/(n^2+8*n+12)/(4+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Fricas [A] time = 1.79254, size = 485, normalized size = 2.17

$$-\frac{\left(2\,a^3x^3+2\left(a^2n-3\,a^2\right)x^2+n^2+\left(an^2-6\,an+6\,a\right)x-8\,n+14\right)\left(\frac{a}{a}\right.}{ac^4n^3-12\,ac^4n^2+44\,ac^4n-48\,ac^4-\left(a^4c^4n^3-12\,a^4c^4n^2+44\,a^4c^4n-48\,a^4c^4\right)x^3+3\left(a^3c^4n^3-12\,a^3c^4n^2+44\,a^3c^4n-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -(2*a^3*x^3 + 2*(a^2*n - 3*a^2)*x^2 + n^2 + (a*n^2 - 6*a*n + 6*a)*x - 8*n +
 14)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^4*n^3 - 12*a*c^4*n^2 + 44*a*c^4*n -
 48*a*c^4 - (a^4*c^4*n^3 - 12*a^4*c^4*n^2 + 44*a^4*c^4*n - 48*a^4*c^4)*x^3
+ 3*(a^3*c^4*n^3 - 12*a^3*c^4*n^2 + 44*a^3*c^4*n - 48*a^3*c^4)*x^2 - 3*(a^2
*c^4*n^3 - 12*a^2*c^4*n^2 + 44*a^2*c^4*n - 48*a^2*c^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^4, x)
```

3.372 $\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{n-5}{2},-\frac{5}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)$$

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/((a + x^(-1))*x)
])/(7*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.199031, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6176, 6181, 132}

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{7}{2},\frac{n-5}{2};-\frac{5}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```


```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/((a + x^(-1))*x)
])/(7*(1 - 1/(a*x))^(n/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{(c-acx)^{5/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,{}_2F_1\left(-\frac{7}{2},\frac{1}{2}(-5+n);-\frac{5}{2};\frac{2}{\left(a+\frac{1}{x}\right)}\right.$$

Mathematica [A] time = 0.096458, size = 103, normalized size = 1.05

$$\frac{2c^2(ax+1)^3\sqrt{c-acx}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{n-5}{2},-\frac{5}{2},\frac{2}{ax+1}\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^3*
Sqrt[c - a*c*x]*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/(1 + a*x)])/(7*
a*(1 - 1/(a*x))^(n/2))
```

Maple [F] time = 0.404, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n \operatorname{arccoth}(ax)} (-acx + c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{5}{2}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)\sqrt{-acx + c}\left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x - 1)/(a*x +
 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx + c)^{\frac{5}{2}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.373 $\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{n-3}{2},-\frac{3}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)$$

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/((a + x^(-1))*x)
])/(5*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.199218, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6176, 6181, 132}

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{5}{2},\frac{n-3}{2};-\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```



```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/((a + x^(-1))*x)
])/(5*(1 - 1/(a*x))^(n/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-3+n);-\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)\ldots}\right.$$

Mathematica [A] time = 0.0705067, size = 101, normalized size = 1.03

$$-\frac{2c(ax+1)^2\sqrt{c-acx}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{n-3}{2},-\frac{3}{2},\frac{2}{ax+1}\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^2*S
qrt[c - a*c*x]*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/(1 + a*x)])/(5*a
*(1 - 1/(a*x))^(n/2))
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)} (-acx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-(acx-c)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-acx+c)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.374 $\int e^{n \coth^{-1}(ax)} \sqrt{c - acx} \, dx$

Optimal. Leaf size=98

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n-1}{2},-\frac{1}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)$$

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqr
t[c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/((a + x^(-1))*x)])
/(3*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.177348, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6176, 6181, 132}

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{3}{2},\frac{n-1}{2};-\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```



```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqr
t[c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/((a + x^(-1))*x)])
/(3*(1 - 1/(a*x))^(n/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-1+n);-\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Mathematica [A] time = 0.0567682, size = 98, normalized size = 1.

$$\frac{2(ax+1)\sqrt{c-acx}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n-1}{2},-\frac{1}{2},\frac{2}{ax+1}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)*Sqrt[c
 - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/(1 + a*x)])/(3*a*(1 -
 1/(a*x))^(n/2))
```

Maple [F] time = 0.367, size = 0, normalized size = 0.

$$\int e^{n \operatorname{arccoth}(ax)} \sqrt{-acx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-acx + c} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-acx + c} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.375 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=96

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+1}{2},\frac{1}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{\sqrt{c-acx}}$$

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hype
rgeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*Sqrt[c - a*c*x])
```

Rubi [A] time = 0.179679, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6176, 6181, 132}

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hype
rgeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*Sqrt[c - a*c*x])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

Mathematica [A] time = 0.0534178, size = 96, normalized size = 1.

$$\frac{2(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+1}{2},\frac{1}{2},\frac{2}{ax+1}\right)}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*(1 + a*x)*Hyperge
ometric2F1[-1/2, (1 + n)/2, 1/2, 2/(1 + a*x)])/(a*(1 - 1/(a*x))^(n/2)*Sqrt[
```

```
c - a*c*x])
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)} \frac{1}{\sqrt{-acx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

3.376 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{3/2}} \, dx$

Optimal. Leaf size=96

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{(c-acx)^{3/2}}$$

```
[Out] (-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hyp
ergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*(c - a*c*x)^(3/2))
```

Rubi [A] time = 0.200264, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6176, 6181, 132}

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```



```
[Out] (-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hyp
ergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*(c - a*c*x)^(3/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Mathematica [A] time = 0.0401119, size = 94, normalized size = 0.98

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{ax+1}\right)}{ac\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1
[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)])/(a*c*(1 - 1/(a*x))^(n/2)*Sqrt[c - a*c*x
```

```
])
```

Maple [F] time = 0.367, size = 0, normalized size = 0.

$$\int e^{n \operatorname{arccoth}(ax)} (-acx + c)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-acx + c} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2c^2x^2 - 2\,ac^2x + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*
c^2*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

3.377 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{5/2}} dx$

Optimal. Leaf size=167

$$\frac{ax^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{(n+3)(c-acx)^{5/2}}-\frac{ax^2\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+3)(c-acx)^{5/2}}$$

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((3 + n)*(c -
 a*c*x)^(5/2))) + (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^
((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2*Hypergeometric2F1[1/2, (3 + n)/2,
 3/2, 2/((a + x^(-1))*x)])/((3 + n)*(c - a*c*x)^(5/2))
```

Rubi [A] time = 0.228236, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 94, 132}

$$\frac{ax^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(n+3)(c-acx)^{5/2}}-\frac{ax^2\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+3)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```



```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((3 + n)*(c -
 a*c*x)^(5/2))) + (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^
((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2*Hypergeometric2F1[1/2, (3 + n)/2,
 3/2, 2/((a + x^(-1))*x)])/((3 + n)*(c - a*c*x)^(5/2))
```


Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}dx, x, \frac{1}{x}\right)}{2(3+n)\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(3+n)(c-acx)^{5/2}}$$

Mathematica [A] time = 0.107925, size = 117, normalized size = 0.7

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left((ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{ax+1}\right)-ax-1\right)}{ac^2(n+3)(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(-1 - a*x + (-1 + a*x)*((-1 + a*x)/(1 + a*x))^((1 + n)
/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)]))/(a*c^2*(3 + n)*(1
 - 1/(a*x))^(n/2)*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int e^{n\text{arccoth}(ax)}(-acx+c)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^3*c^3*x^3 - 3*a
^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

3.378 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}} dx$

Optimal. Leaf size=245

$$-\frac{3a^2x^3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}+\frac{3a^2x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}-$$

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((5 + n)*(c -
 a*c*x)^(7/2))) + (3*a^2*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2
)*x^3)/(2*(15 + 8*n + n^2)*(c - a*c*x)^(7/2)) - (3*a^2*((a - x^(-1))/(a + x
^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3
*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(2*(15 + 8*n +
 n^2)*(c - a*c*x)^(7/2))
```

Rubi [A] time = 0.270551, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6176, 6181, 94, 132}

$$-\frac{3a^2x^3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}+\frac{3a^2x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}-\frac{ax^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\right.}{(n+5)(c-acx}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((5 + n)*(c -
 a*c*x)^(7/2))) + (3*a^2*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2
)*x^3)/(2*(15 + 8*n + n^2)*(c - a*c*x)^(7/2)) - (3*a^2*((a - x^(-1))/(a + x
^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3
*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(2*(15 + 8*n +
 n^2)*(c - a*c*x)^(7/2))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int x^{3/2}\left(1-\frac{x}{a}\right)^{-\frac{7}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{2(5+n)\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-}}{}\right.}{4(3+n)(5+n)\left(\frac{1}{x}\right)^{7/2}(c}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right.}{2\left(15+8n+n^2\right)}$$

Mathematica [A] time = 0.157539, size = 138, normalized size = 0.56

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(3(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{ax+1}\right)+(ax+1)(-3ax+2n+9)\right)}{2ac^3(n+3)(n+5)(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((9 + 2*n - 3*a*x)*(1 + a*x) + 3*(-1 + a*x)^2*((-1 + a
*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*
x)]))/(2*a*c^3*(3 + n)*(5 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x)^2*Sqrt[c - a*
c*x])
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)} \left(-acx+c\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-acx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{a^{4}c^{4}x^{4}-4\,a^{3}c^{4}x^{3}+6\,a^{2}c^{4}x^{2}-4\,ac^{4}x+c^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a^
3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

3.379 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=114

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\csc^{-1}(ax)}{2a}$$

```
[Out] -(c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a) + (c^4*Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^
(-1)))/(2*a^2) + c^4*(1 - 1/(a^2*x^2))^(3/2)*x - (c^4*ArcCsc[a*x])/(2*a) -
(3*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.266202, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6177, 1807, 1809, 815, 844, 216, 266, 63, 208}

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] -(c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a) + (c^4*Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^
(-1)))/(2*a^2) + c^4*(1 - 1/(a^2*x^2))^(3/2)*x - (c^4*ArcCsc[a*x])/(2*a) -
(3*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\left(c\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(\frac{3c^3}{a}-\frac{c^3x}{a^2}+\frac{c^3x^2}{a^3}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{1}{3}\left(a^2c\right)\text{Subst}\left(\int \frac{\left(-\frac{9c^3}{a^3}+\frac{3c^3x}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{1}{6}\left(a^4c\right)\text{Subst}\left(\int \frac{\frac{18c^3}{a^5}-\frac{3c^3}{a^6}\cdots}{x\sqrt{1-\frac{x^2}{a^2}}}\cdots\right.$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\cdots$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} + \frac{\left(3c^4\right)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} - \left(3ac^4\right)\text{Subst}\cdots$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} - \frac{3c^4\tanh^{-1}\left(\cdots\right.}{a}\cdots$$

Mathematica [A] time = 0.180316, size = 175, normalized size = 1.54

$$\frac{c^4\left(6a^5x^5+16a^4x^4-15a^3x^3-14a^2x^2+24a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+9a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-18a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right.}{6a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(-2 + 9*a*x - 14*a^2*x^2 - 15*a^3*x^3 + 16*a^4*x^4 + 6*a^5*x^5 + 24*a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 9*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 18*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [B] time = 0.131, size = 224, normalized size = 2.

$$-\frac{(ax-1)\,c^4}{6\,a^4x^3}\left(-18\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+18\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+3\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+18\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x)
```

```
[Out] -1/6*(a*x-1)*c^4*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+18*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*x^2*a^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+18*ln((a^
2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+3*a^3*x^3*(a^2)^(1/
2)*arctan(1/(a^2*x^2-1)^(1/2))-9*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^
4/x^3/(a^2)^(1/2)
```

Maxima [B] time = 1.56253, size = 302, normalized size = 2.65

$$\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-37\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^2 + 9*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21*c
^4*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 37*c^
4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x +
1)^4 + a^2))*a
```

Fricas [A] time = 1.75557, size = 359, normalized size = 3.15

$$\frac{6\,a^{3}c^{4}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^{3}c^{4}x^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^{3}c^{4}x^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^{4}c^{4}x^{4}+22\,a^{3}c^{4}x^{3}+7\,a^{2}c^{4}x^{2}-7\right.}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^4*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 7*a^2*c^4*x^2 - 7*a*c^4*x + 2*c^
4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^{4}\left(\int\frac{a^{4}}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{4a}{x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{6a^{2}}{x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{4a^{3}}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a/(x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(6*a**2/(x**2*sqrt(a*x/(a*x + 1) - 1/(
```

```
a*x + 1))), x) + Integral(-4*a**3/(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)
)/a**4
```

Giac [B] time = 1.20757, size = 309, normalized size = 2.71

$$\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{6\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{28\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{27\,(ax-1)^2c^4\cdots}{(ax+1)^2}}{a^2\left(\frac{ax-1}{ax+1}+1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^2 + 9*c^4*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 -
6*c^4*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (28*(a*x
- 1)*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 27*(a*x - 1)^2*c^4*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1)^2 + 9*c^4*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x
- 1)/(a*x + 1) + 1)^3))*a
```

3.380 $\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=88

$$\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^3\csc^{-1}(ax)}{2a}$$

```
[Out] (c^3*Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1)))/(2*a^2) + c^3*(1 - 1/(a^2*x^2))^
(3/2)*x + (c^3*ArcCsc[a*x])/(2*a) - (2*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/
a
```

Rubi [A] time = 0.190626, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6177, 1807, 815, 844, 216, 266, 63, 208}

$$\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```


```
[Out] (c^3*Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1)))/(2*a^2) + c^3*(1 - 1/(a^2*x^2))^
(3/2)*x + (c^3*ArcCsc[a*x])/(2*a) - (2*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/
a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(c\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + c\,\text{Subst}\left(\int\frac{\left(\frac{2c^2}{a}+\frac{c^2x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int\frac{-\frac{4c^2}{a^3}-\frac{c^2x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2a^2} + \frac{\left(2c^3\right)\text{Subst}\left(\int\right.}{a}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} + \frac{c^3\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{a}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} - \left(2ac^3\right)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}dx, x\right.$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} - \frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.141962, size = 167, normalized size = 1.9

$$\frac{c^3\left(2a^4x^4+4a^3x^3-3a^2x^2+2a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+2a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-4a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-\right.\right.}{2a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(1 - 4*a*x - 3*a^2*x^2 + 4*a^3*x^3 + 2*a^4*x^4 + 2*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 2*a^3*Sqrt[1 - 1/(a^2*x^2)]*
```

```
x^3*ArcSin[1/(a*x)] - 4*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Maple [B] time = 0.131, size = 200, normalized size = 2.3

$$-\frac{(ax-1)c^3}{2x^2a^3}\left(-4\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+4\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+4\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x)
```

```
[Out] -1/2*(a*x-1)*c^3*(-4*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+4*(a^2)^(1/2)*(a
^2*x^2-1)^(3/2)*x*a-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+4*ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-a^2*x^2*(a^2)^(1/2)*arctan(1
/(a^2*x^2-1)^(1/2))-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/
((a*x-1)*(a*x+1))^(1/2)/a^3/x^2/(a^2)^(1/2)
```

Maxima [B] time = 1.63146, size = 271, normalized size = 3.08

$$-\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 2*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (3*c^3*((a
*x - 1)/(a*x + 1))^(5/2) - 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqrt((
a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2
 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Fricas [A] time = 1.6649, size = 332, normalized size = 3.77

$$-\frac{2\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^3x^3+6\,a^2c^3x^2+3\,ac^3x-c^3\right)\sqrt{}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 4*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (2*a^3*c^3*x^3 + 6*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^3\left(\int\frac{a^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{1}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{3a}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{3a^2}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(a**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a/(x**2*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**2/(x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))), x))/a**3
```

Giac [B] time = 1.21314, size = 263, normalized size = 2.99

$$-\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}-\frac{\frac{5\,(ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+3\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 2*c^3*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 2*c^3
*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) - (5*(a*x - 1)*c
^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 3*c^3*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*((a*x - 1)/(a*x + 1) + 1)^2))*a
```

3.381 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=62

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.0960085, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6177, 813, 844, 216, 266, 63, 208}

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
```

```
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(c\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{\frac{2c}{a}+\frac{2cx}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2a}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

<u>Mathematica</u> [B] time = 0.162609, size = 158, normalized size = 2.55

$$\frac{c^2\left(2a^3x^3+2a^2x^2-2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2a\right.}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*(-2 - 2*a*x + 2*a^2*x^2 + 2*a^3*x^3 - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*
ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/
(a*x)] - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(
2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [B] time = 0.126, size = 166, normalized size = 2.7

$$\frac{c^2 (ax-1)}{a^2 x}\left(\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 + ax\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) - \left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2} + \sqrt{a^2}\sqrt{a^2x^2-1}xa - \ln\left(\left(a^2x + \sqrt{a^2x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x)
```

```
[Out] (a*x-1)*c^2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+a*x*(a^2)^(1/2)*arctan(1
/(a^2*x^2-1)^(1/2))-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+(a^2)^(1/2)*(a^2*x^2-1)^(
1/2)*x*a-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x
-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [B] time = 1.52721, size = 169, normalized size = 2.73

$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2} - a^2} + \frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2} + \frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [B] time = 1.63475, size = 278, normalized size = 4.48

$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) - \left(a^2c^2x^2 + 2\,ac^2x + c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 + 2*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{2a}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a/(x*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x))/a**2
```

Giac [B] time = 1.20774, size = 165, normalized size = 2.66

$$-a\left(\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c^2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -a*(2*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^2 - c^2*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4*c^2
*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

3.382 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=27

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x + (c*ArcCsc[a*x])/a
```

Rubi [A] time = 0.0290766, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 277, 216}

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x + (c*ArcCsc[a*x])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0367674, size = 31, normalized size = 1.15

$$
\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}+\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcSin[1/(a*x)]))/a
```

Maple [B] time = 0.116, size = 63, normalized size = 2.3

$$
\frac{c\,(ax-1)}{a}\left(\sqrt{a^2x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax-1)\,(ax+1)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/((a*x-1)*(a*x+1))^(1/2)*c/a*((a^2*x^2-1)^
(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))
```

Maxima [B] time = 1.57694, size = 89, normalized size = 3.3

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2)
```

Fricas [A] time = 1.58948, size = 113, normalized size = 4.19

$$-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*c*x + c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int\frac{a}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{1}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))), x))/a
```

Giac [B] time = 1.28138, size = 117, normalized size = 4.33

$$-\frac{1}{2}\,a\left(\frac{\left(\pi+2\,\arctan\left(\frac{\frac{ax-1}{ax+1}-1}{2\,\sqrt{\frac{ax-1}{ax+1}}}\right)\right)c}{a^2}+\frac{4\,c}{a^2\left(\sqrt{\frac{ax-1}{ax+1}}-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -1/2*a*((pi + 2*arctan(1/2*((a*x - 1)/(a*x + 1) - 1)/sqrt((a*x - 1)/(a*x +
1))))*c/a^2 + 4*c/(a^2*(sqrt((a*x - 1)/(a*x + 1)) - 1/sqrt((a*x - 1)/(a*x +
 1)))))
```

3.383 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=70

$$-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] (-2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)
/c + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rubi [A] time = 0.195578, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (-2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)
/c + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^2-\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.0896123, size = 63, normalized size = 0.9

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(ax-3)+2(ax-1)\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac(ax-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3 + a*x) + 2*(-1 + a*x)*Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c*(-1 + a*x))
```

Maple [B] time = 0.135, size = 250, normalized size = 3.6

$$\frac{1}{(ax-1)\,ac}\left(2\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^2a^3+2\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^2a^2-4\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x)
```

```
[Out] (2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+2*(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-4*(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(1/2))/(a^2)^(1/2))+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(
1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.00315, size = 157, normalized size = 2.24

$$-2\,a\left(\frac{\frac{2\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*((2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Fricas [A] time = 1.53907, size = 223, normalized size = 3.19

$$\frac{2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-2\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 2*a*x - 3)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c*x - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{x}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c
```

Giac [B] time = 1.15889, size = 173, normalized size = 2.47

$$2a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{\frac{2(ax-1)}{ax+1}-1}{a^2c\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\sqrt{\frac{ax-1}{ax+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(abs(sqrt((a*x - 1)/(a
*x + 1)) - 1))/(a^2*c) - (2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1) - sqrt((a*x - 1)/(a*x + 1)))))
```

3.384 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] (-4*(a + x^(-1)))/(3*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (9*a + 11/x)/(3*a^2
*c^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (3*ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(a*c^2)
```

Rubi [A] time = 0.292632, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (-4*(a + x^(-1)))/(3*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (9*a + 11/x)/(3*a^2
*c^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (3*ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(a*c^2)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^3}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^3-\frac{9c^3x}{a}-\frac{8c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^3+\frac{9c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.0625943, size = 94, normalized size = 0.9

$$\frac{3a^3x^3-16a^2x^2+9ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-5ax+14}{3a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Maple [B] time = 0.139, size = 339, normalized size = 3.2

$$\frac{1}{3\,a\,(ax-1)^2\,c^2}\left(9\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^3a^4+9\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^3a^3-27\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(\right.}{\sqrt{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/3*(9*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+
9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-27*ln((a^2*x+(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
3/2)*x*a-27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+27*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+5*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)+27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-9*a*ln((a^2*x+(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-9*(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+
1))^(1/2)
```

Maxima [A] time = 1.02613, size = 185, normalized size = 1.76

$$\frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c^2))
```

Fricas [A] time = 1.60619, size = 309, normalized size = 2.94

$$\frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax+14\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int\frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**
2
```

Giac [A] time = 1.16618, size = 200, normalized size = 1.9

$$\frac{1}{3}a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)\left(\frac{12\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/3*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(abs(sqrt((a*x
 - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)*(12*(a*x - 1)/(a*x + 1) + 1)/(
(a*x - 1)*a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - 6*sqrt((a*x - 1)/(a*x + 1))/
(a^2*c^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.385 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=138

$$-\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] (-8*(a + x^(-1)))/(5*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 8/x))/(15
*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (60*a + 79/x)/(15*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]])/(a*c^3)
```

Rubi [A] time = 0.393037, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-8*(a + x^(-1)))/(5*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 8/x))/(15
*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (60*a + 79/x)/(15*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]])/(a*c^3)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^4-\frac{20c^4x}{a}-\frac{27c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^4+\frac{60c^4x}{a}+\frac{64c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^4-\frac{60c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,\right.}{ac^3}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,\right.}{ac^3}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\right.}{c^3}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A] time = 0.0719576, size = 104, normalized size = 0.75

$$\frac{15a^4x^4 - 134a^3x^3 + 73a^2x^2 + 60ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) + 128ax - 94}{15a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-94 + 128*a*x + 73*a^2*x^2 - 134*a^3*x^3 + 15*a^4*x^4 + 60*a*Sqrt[1 - 1/(a
^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^3*Sqrt[1
- 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] time = 0.135, size = 431, normalized size = 3.1

$$\frac{1}{15\,a\,(ax-1)^3\,c^3}\left(60\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^4a^5 + 60\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^4a^4 - 240\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax\cdot}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/15*(60*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^
5+60*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-240*ln((a^2*x+(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-45*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^2*a^2-240*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+360*ln((a
^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+76*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(3/2)*x*a+360*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*
a^2-240*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-3
4*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-240*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2)*x*a+60*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+60*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)^3/c^3/((a*x-1)*(
a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.02889, size = 207, normalized size = 1.5

$$\frac{1}{30}\,a\left(\frac{\frac{22\,(ax-1)}{ax+1}+\frac{155\,(ax-1)^2}{(ax+1)^2}-\frac{240\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*a*((22*(a*x - 1)/(a*x + 1) + 155*(a*x - 1)^2/(a*x + 1)^2 - 240*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
- 1)/(a*x + 1))^(5/2)) + 120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) -
 120*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A] time = 1.7155, size = 390, normalized size = 2.83

$$\frac{60\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-60\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^4x^4-134\,a^3x^3+73\right.}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 134*a^3*x^3 + 73*a^2*x^2 + 128*a*x - 94)*sqrt((a*x - 1)/(
a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\int\frac{x^3}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Giac [A] time = 1.16823, size = 224, normalized size = 1.62

$$\frac{1}{30}\,a\left(\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^2\left(\frac{25\,(ax-1)}{ax+1}+\frac{180\,(ax-1)^2}{(ax+1)^2}+3\right)}{(ax-1)^2a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{60\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] 1/30*a*(120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 120*log(abs(sqrt
((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^2*(25*(a*x - 1)/(a*x + 1)
 + 180*(a*x - 1)^2/(a*x + 1)^2 + 3)/((a*x - 1)^2*a^2*c^3*sqrt((a*x - 1)/(a*
x + 1))) - 60*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*((a*x - 1)/(a*x + 1) - 1))
)
```

3.386 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=171

$$-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{5\tanh^{-1}\left(\sqrt{1-\right.}{ac^4}$$

```
[Out] (-16*(a + x^(-1)))/(7*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (4*(7*a + 17/x))/(
35*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (175*a + 307/x)/(105*a^2*c^4*(1 - 1/(
a^2*x^2))^(3/2)) - (525*a + 719/x)/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (S
qrt[1 - 1/(a^2*x^2)]*x)/c^4 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rubi [A] time = 0.500726, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{5\tanh^{-1}\left(\sqrt{1-\right.}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```


```
[Out] (-16*(a + x^(-1)))/(7*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (4*(7*a + 17/x))/(
35*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (175*a + 307/x)/(105*a^2*c^4*(1 - 1/(
a^2*x^2))^(3/2)) - (525*a + 719/x)/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (S
qrt[1 - 1/(a^2*x^2)]*x)/c^4 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^5-\frac{35c^5x}{a}-\frac{61c^5x^2}{a^2}+\frac{7c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{35c^5+\frac{175c^5x}{a}+\frac{272c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-105c^5-\frac{525c^5x}{a}-\frac{614c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,\right.}{105c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{10}{}\right.}{}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}$$

Mathematica [A] time = 0.0852049, size = 112, normalized size = 0.65

$$\frac{105a^5x^5 - 1339a^4x^4 + 1812a^3x^3 + 485a^2x^2 + 525ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) - 1947ax + 824}{105a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (824 - 1947*a*x + 485*a^2*x^2 + 1812*a^3*x^3 - 1339*a^4*x^4 + 105*a^5*x^5 +
 525*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [B] time = 0.138, size = 523, normalized size = 3.1

$$\frac{1}{105\,a\,(ax-1)^4\,c^4}\left(525\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^5a^6+525\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^5a^5-2625\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] 1/105*(525*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*
a^6+525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-2625*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-420*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^3*a^3-2625*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+525
0*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+1076*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+5250*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2)*x^3*a^3-5250*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)
^(1/2))*x^2*a^3-970*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-5250*(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+2625*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2))/(a^2)^(1/2))*x*a^2+299*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+2625*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-525*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2))/(a^2)^(1/2))-525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(
a^2)^(1/2)/(a*x-1)^4/c^4/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.01998, size = 228, normalized size = 1.33

$$\frac{1}{420}\,a\left(\frac{\frac{111\,(ax-1)}{ax+1}+\frac{469\,(ax-1)^2}{(ax+1)^2}+\frac{2765\,(ax-1)^3}{(ax+1)^3}-\frac{4200\,(ax-1)^4}{(ax+1)^4}+15}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/420*a*((111*(a*x - 1)/(a*x + 1) + 469*(a*x - 1)^2/(a*x + 1)^2 + 2765*(a*x
 - 1)^3/(a*x + 1)^3 - 4200*(a*x - 1)^4/(a*x + 1)^4 + 15)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 2100*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c^4))
```

Fricas [A] time = 1.63672, size = 475, normalized size = 2.78

$$\frac{525\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-525\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(105}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - 525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sq
rt((a*x - 1)/(a*x + 1)) - 1) + (105*a^5*x^5 - 1339*a^4*x^4 + 1812*a^3*x^3 +
 485*a^2*x^2 - 1947*a*x + 824)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int\frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c**4
```

Giac [A] time = 1.15662, size = 246, normalized size = 1.44

$$\frac{1}{420}a\left(\frac{2100\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{(ax+1)^3\left(\frac{126(ax-1)}{ax+1}+\frac{595(ax-1)^2}{(ax+1)^2}+\frac{3360(ax-1)^3}{(ax+1)^3}+15\right)}{(ax-1)^3a^2c^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{840\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/420*a*(2100*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(abs(s
qrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - (a*x + 1)^3*(126*(a*x - 1)/(a*x
+ 1) + 595*(a*x - 1)^2/(a*x + 1)^2 + 3360*(a*x - 1)^3/(a*x + 1)^3 + 15)/((a
*x - 1)^3*a^2*c^4*sqrt((a*x - 1)/(a*x + 1))) - 840*sqrt((a*x - 1)/(a*x + 1)
)/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1)))
```

3.387 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx$

Optimal. Leaf size=61

$$-\frac{c^5}{a^3x^2}+\frac{c^5}{a^4x^3}-\frac{c^5}{4a^5x^4}-\frac{2c^5}{a^2x}-\frac{3c^5\log(x)}{a}+c^5x$$

```
[Out] -c^5/(4*a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*x
- (3*c^5*Log[x])/a
```

Rubi [A] time = 0.13671, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$-\frac{c^5}{a^3x^2}+\frac{c^5}{a^4x^3}-\frac{c^5}{4a^5x^4}-\frac{2c^5}{a^2x}-\frac{3c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -c^5/(4*a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*x
- (3*c^5*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx\\ &= \frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^5}{x^5}\,dx}{a^5}\\ &= \frac{c^5\int \frac{(1-ax)^4(1+ax)}{x^5}\,dx}{a^5}\\ &= \frac{c^5\int \left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5}\\ &= -\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.219055, size = 63, normalized size = 1.03

$$-\frac{c^5}{a^3x^2}+\frac{c^5}{a^4x^3}-\frac{c^5}{4a^5x^4}-\frac{2c^5}{a^2x}-\frac{3c^5\log(ax)}{a}+c^5x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -c^5/(4*a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*x
- (3*c^5*Log[a*x])/a
```

Maple [A] time = 0.046, size = 60, normalized size = 1.

$$-\frac{c^5}{4\,a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{x^2a^3}-2\,\frac{c^5}{a^2x}+c^5x-3\,\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^5,x)
```

```
[Out] -1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/x^2/a^3-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

Maxima [A] time = 1.01436, size = 77, normalized size = 1.26

$$c^5x - \frac{3\,c^5\log\left(x\right)}{a} - \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - 3*c^5*log(x)/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x + c
^5)/(a^5*x^4)
```

Fricas [A] time = 1.57395, size = 142, normalized size = 2.33

$$\frac{4\,a^5c^5x^5 - 12\,a^4c^5x^4\log\left(x\right) - 8\,a^3c^5x^3 - 4\,a^2c^5x^2 + 4\,ac^5x - c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
+ 4*a*c^5*x - c^5)/(a^5*x^4)
```

Sympy [A] time = 0.46256, size = 63, normalized size = 1.03

$$\frac{a^5c^5x - 3a^4c^5\log\left(x\right) - \frac{8a^3c^5x^3 + 4a^2c^5x^2 - 4ac^5x + c^5}{4x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - 3*a**4*c**5*log(x) - (8*a**3*c**5*x**3 + 4*a**2*c**5*x**2 -
4*a*c**5*x + c**5)/(4*x**4))/a**5
```

Giac [A] time = 1.11383, size = 78, normalized size = 1.28

$$c^5x - \frac{3\,c^5\log\left(|x|\right)}{a} - \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*x - 3*c^5*log(abs(x))/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*
x + c^5)/(a^5*x^4)
```

3.388 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=40

$$-\frac{c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}-\frac{2c^4\log(x)}{a}+c^4x$$

```
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[x])/a
```

Rubi [A] time = 0.130371, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$-\frac{c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx \\
&= -\frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4} \\
&= -\frac{c^4\int \frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4} \\
&= -\frac{c^4\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4} \\
&= \frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.154748, size = 42, normalized size = 1.05

$$-\frac{c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}-\frac{2c^4\log(ax)}{a}+c^4x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[a*x])/a
```

Maple [A] time = 0.043, size = 39, normalized size = 1.

$$\frac{c^4}{3\,a^4x^3}-\frac{c^4}{x^2a^3}+c^4x-2\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^4,x)
```

```
[Out] 1/3*c^4/a^4/x^3-c^4/x^2/a^3+c^4*x-2*c^4*ln(x)/a
```

Maxima [A] time = 1.03881, size = 50, normalized size = 1.25

$$c^4x - \frac{2\,c^4\log(x)}{a} - \frac{3\,ac^4x - c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Fricas [A] time = 1.4932, size = 97, normalized size = 2.42

$$\frac{3\,a^4c^4x^4 - 6\,a^3c^4x^3\log(x) - 3\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] time = 0.36418, size = 39, normalized size = 0.98

$$\frac{a^4c^4x - 2a^3c^4\log(x) - \frac{3ac^4x - c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x - 2*a**3*c**4*log(x) - (3*a*c**4*x - c**4)/(3*x**3))/a**4
```

Giac [A] time = 1.15485, size = 51, normalized size = 1.27

$$c^4x - \frac{2\,c^4\log\left(|x|\right)}{a} - \frac{3\,ac^4x - c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] c^4*x - 2*c^4*log(abs(x))/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

3.389 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=39

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(x)}{a}+c^3x$$

```
[Out] -c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[x])/a
```

Rubi [A] time = 0.128253, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] -c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx\\ &= \frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}\\ &= \frac{c^3\int \frac{(1-ax)^2(1+ax)}{x^3}dx}{a^3}\\ &= \frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}\\ &= -\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.120522, size = 41, normalized size = 1.05

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(ax)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] -c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[a*x])/a
```

Maple [A] time = 0.043, size = 38, normalized size = 1.

$$-\frac{c^3}{2\,x^2a^3}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^3,x)
```

```
[Out] -1/2*c^3/x^2/a^3+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

Maxima [A] time = 1.04247, size = 50, normalized size = 1.28

$$c^3x - \frac{c^3 \log(x)}{a} + \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] c^3*x - c^3*log(x)/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Fricas [A] time = 1.46807, size = 97, normalized size = 2.49

$$\frac{2\,a^3c^3x^3 - 2\,a^2c^3x^2 \log(x) + 2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] time = 0.338769, size = 37, normalized size = 0.95

$$\frac{a^3c^3x - a^2c^3 \log(x) + \frac{2ac^3x - c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**3,x)
```

```
[Out] (a**3*c**3*x - a**2*c**3*log(x) + (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

Giac [A] time = 1.12182, size = 51, normalized size = 1.31

$$c^3x - \frac{c^3 \log\left(|x|\right)}{a} + \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] c^3*x - c^3*log(abs(x))/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

3.390 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=16

$$\frac{c^2}{a^2x}+c^2x$$

```
[Out] c^2/(a^2*x) + c^2*x
```

Rubi [A] time = 0.121349, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6167, 6131, 6129, 73, 14}

$$\frac{c^2}{a^2x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] c^2/(a^2*x) + c^2*x
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx\\
&= -\frac{c^2\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\frac{(1-ax)(1+ax)}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\frac{1-a^2x^2}{x^2}\,dx}{a^2}\\
&= -\frac{c^2\int\left(-a^2+\frac{1}{x^2}\right)\,dx}{a^2}\\
&= \frac{c^2}{a^2x}+c^2x
\end{aligned}
$$

Mathematica [A] time = 0.0886441, size = 16, normalized size = 1.

$$\frac{c^2}{a^2x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] c^2/(a^2*x) + c^2*x
```

Maple [A] time = 0.043, size = 17, normalized size = 1.1

$$\frac{c^2\left(a^2x+x^{-1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^2,x)
```

```
[Out] c^2/a^2*(a^2*x+1/x)
```

Maxima [A] time = 1.00241, size = 22, normalized size = 1.38

$$c^2x+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + c^2/(a^2*x)
```

Fricas [A] time = 1.42953, size = 39, normalized size = 2.44

$$\frac{a^2c^2x^2+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^2*c^2*x^2 + c^2)/(a^2*x)
```

Sympy [A] time = 0.259576, size = 15, normalized size = 0.94

$$\frac{a^2c^2x+\frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**2,x)
```

```
[Out] (a**2*c**2*x + c**2/x)/a**2
```

Giac [A] time = 1.14569, size = 22, normalized size = 1.38

$$c^2x + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] c^2*x + c^2/(a^2*x)
```

3.391 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=11

$$\frac{c\log(x)}{a}+cx$$

```
[Out] c*x + (c*Log[x])/a
```

Rubi [A] time = 0.0746432, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6131, 6129, 43}

$$\frac{c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x + (c*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx \\
&= \frac{c\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a} \\
&= \frac{c\int \frac{1+ax}{x}\,dx}{a} \\
&= \frac{c\int \left(a+\frac{1}{x}\right)dx}{a} \\
&= cx+\frac{c\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.036415, size = 11, normalized size = 1.

$$\frac{c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x + (c*Log[x])/a
```

Maple [A] time = 0.039, size = 12, normalized size = 1.1

$$cx+\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x),x)
```

[Out] c*x+c*ln(x)/a

Maxima [A] time = 1.04582, size = 15, normalized size = 1.36

$$cx + \frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*x + c*log(x)/a
```

Fricas [A] time = 1.40232, size = 30, normalized size = 2.73

$$\frac{acx + c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*c*x + c*log(x))/a
```

Sympy [A] time = 0.091203, size = 10, normalized size = 0.91

$$\frac{acx + c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
```

```
[Out] (a*c*x + c*log(x))/a
```

Giac [A] time = 1.11006, size = 16, normalized size = 1.45

$$cx + \frac{c\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] c*x + c*log(abs(x))/a
```

3.392 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c + 2/(a*c*(1 - a*x)) + (3*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.124074, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c + 2/(a*c*(1 - a*x)) + (3*Log[1 - a*x])/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx \\
&= \frac{a\int \frac{e^{2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c} \\
&= \frac{a\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{c} \\
&= \frac{a\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.0283811, size = 30, normalized size = 0.81

$$
\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c)
```

Maple [A] time = 0.044, size = 36, normalized size = 1.

$$
\frac{x}{c}+3\,\frac{\ln(ax-1)}{ac}-2\,\frac{1}{ac\,(ax-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x),x)
```

```
[Out] x/c+3/a/c*ln(a*x-1)-2/a/c/(a*x-1)
```

Maxima [A] time = 1.02992, size = 47, normalized size = 1.27

$$\frac{x}{c} - \frac{2}{a^2cx - ac} + \frac{3 \log(ax - 1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] x/c - 2/(a^2*c*x - a*c) + 3*log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.54618, size = 86, normalized size = 2.32

$$\frac{a^2x^2 - ax + 3(ax - 1)\log(ax - 1) - 2}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] time = 0.362151, size = 26, normalized size = 0.7

$$-\frac{2}{a^2cx - ac} + \frac{x}{c} + \frac{3\log(ax - 1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
```

```
[Out] -2/(a**2*c*x - a*c) + x/c + 3*log(a*x - 1)/(a*c)
```

Giac [A] time = 1.14996, size = 49, normalized size = 1.32

$$\frac{x}{c} + \frac{3 \log(|ax - 1|)}{ac} - \frac{2}{(ax - 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] x/c + 3*log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

3.393 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=53

$$\frac{5}{ac^2(1-ax)}-\frac{1}{ac^2(1-ax)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 - 1/(a*c^2*(1 - a*x)^2) + 5/(a*c^2*(1 - a*x)) + (4*Log[1 - a*x])/(a*c
^2)
```

Rubi [A] time = 0.15467, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{5}{ac^2(1-ax)}-\frac{1}{ac^2(1-ax)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 - 1/(a*c^2*(1 - a*x)^2) + 5/(a*c^2*(1 - a*x)) + (4*Log[1 - a*x])/(a*c
^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx\\
&= -\frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c^2}\\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.09914, size = 51, normalized size = 0.96

$$-\frac{5}{ac^2(ax-1)}-\frac{1}{ac^2(ax-1)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 - 1/(a*c^2*(-1 + a*x)^2) - 5/(a*c^2*(-1 + a*x)) + (4*Log[1 - a*x])/(a
*c^2)
```

Maple [A] time = 0.043, size = 51, normalized size = 1.

$$\frac{x}{c^{2}}-\frac{1}{ac^{2}\left(ax-1\right)^{2}}+4\,\frac{\ln\left(ax-1\right)}{ac^{2}}-5\,\frac{1}{ac^{2}\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^2,x)
```

```
[Out] x/c^2-1/a/c^2/(a*x-1)^2+4/a/c^2*ln(a*x-1)-5/a/c^2/(a*x-1)
```

Maxima [A] time = 1.04903, size = 74, normalized size = 1.4

$$-\frac{5\,ax-4}{a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2}}+\frac{x}{c^{2}}+\frac{4\,\log\left(ax-1\right)}{ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 + 4*log(a*x - 1)/(
a*c^2)
```

Fricas [A] time = 1.5058, size = 149, normalized size = 2.81

$$\frac{a^{3}x^{3}-2\,a^{2}x^{2}-4\,ax+4\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax-1\right)+4}{a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 0.435774, size = 49, normalized size = 0.92

$$-\frac{5ax-4}{a^{3}c^{2}x^{2}-2a^{2}c^{2}x+ac^{2}}+\frac{x}{c^{2}}+\frac{4\log\left(ax-1\right)}{ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**2,x)
```

```
[Out] -(5*a*x - 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) + x/c**2 + 4*log(a*x
 - 1)/(a*c**2)
```

Giac [A] time = 1.14232, size = 57, normalized size = 1.08

$$\frac{x}{c^2} + \frac{4\,\log\left(|ax - 1|\right)}{ac^2} - \frac{5\,ax - 4}{(ax - 1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

3.394 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=73

$$\frac{9}{ac^3(1-ax)}-\frac{7}{2ac^3(1-ax)^2}+\frac{2}{3ac^3(1-ax)^3}+\frac{5\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 + 2/(3*a*c^3*(1 - a*x)^3) - 7/(2*a*c^3*(1 - a*x)^2) + 9/(a*c^3*(1 - a
*x)) + (5*Log[1 - a*x])/(a*c^3)
```

Rubi [A] time = 0.164493, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{9}{ac^3(1-ax)}-\frac{7}{2ac^3(1-ax)^2}+\frac{2}{3ac^3(1-ax)^3}+\frac{5\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 + 2/(3*a*c^3*(1 - a*x)^3) - 7/(2*a*c^3*(1 - a*x)^2) + 9/(a*c^3*(1 - a
*x)) + (5*Log[1 - a*x])/(a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx\\
&= \frac{a^3\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\
&= \frac{a^3\int \frac{x^3(1+ax)}{(1-ax)^4}\,dx}{c^3}\\
&= \frac{a^3\int \left(\frac{1}{a^3}+\frac{2}{a^3(-1+ax)^4}+\frac{7}{a^3(-1+ax)^3}+\frac{9}{a^3(-1+ax)^2}+\frac{5}{a^3(-1+ax)}\right)dx}{c^3}\\
&= \frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}
\end{aligned}
$$

Mathematica [A] time = 0.120448, size = 63, normalized size = 0.86

$$
\frac{6a^4x^4-18a^3x^3-36a^2x^2+81ax+30(ax-1)^3\log(1-ax)-37}{6ac^3(ax-1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (-37 + 81*a*x - 36*a^2*x^2 - 18*a^3*x^3 + 6*a^4*x^4 + 30*(-1 + a*x)^3*Log[1
 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A] time = 0.043, size = 66, normalized size = 0.9

$$\frac{x}{c^3} - \frac{7}{2\,ac^3\,(ax-1)^2} - \frac{2}{3\,ac^3\,(ax-1)^3} + 5\,\frac{\ln(ax-1)}{ac^3} - 9\,\frac{1}{ac^3\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^3,x)
```

```
[Out] x/c^3-7/2/a/c^3/(a*x-1)^2-2/3/a/c^3/(a*x-1)^3+5/a/c^3*ln(a*x-1)-9/a/c^3/(a*
x-1)
```

Maxima [A] time = 1.04371, size = 101, normalized size = 1.38

$$-\frac{54\,a^2x^2 - 87\,ax + 37}{6\left(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3\right)} + \frac{x}{c^3} + \frac{5\,\log(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
- a*c^3) + x/c^3 + 5*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.50469, size = 217, normalized size = 2.97

$$\frac{6\,a^4x^4 - 18\,a^3x^3 - 36\,a^2x^2 + 81\,ax + 30\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log(ax-1) - 37}{6\left(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
 - a*c^3)
```

Sympy [A] time = 0.534018, size = 73, normalized size = 1.

$$-\frac{54a^2x^2 - 87ax + 37}{6a^4c^3x^3 - 18a^3c^3x^2 + 18a^2c^3x - 6ac^3} + \frac{x}{c^3} + \frac{5\log(ax - 1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] -(54*a**2*x**2 - 87*a*x + 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a*
*2*c**3*x - 6*a*c**3) + x/c**3 + 5*log(a*x - 1)/(a*c**3)
```

Giac [A] time = 1.12201, size = 68, normalized size = 0.93

$$\frac{x}{c^3} + \frac{5\log(|ax - 1|)}{ac^3} - \frac{54a^2x^2 - 87ax + 37}{6(ax - 1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 + 5*log(abs(a*x - 1))/(a*c^3) - 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
- 1)^3*a*c^3)
```

3.395 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=87

$$\frac{14}{ac^4(1-ax)} - \frac{8}{ac^4(1-ax)^2} + \frac{3}{ac^4(1-ax)^3} - \frac{1}{2ac^4(1-ax)^4} + \frac{6\log(1-ax)}{ac^4} + \frac{x}{c^4}$$

```
[Out] x/c^4 - 1/(2*a*c^4*(1 - a*x)^4) + 3/(a*c^4*(1 - a*x)^3) - 8/(a*c^4*(1 - a*x
)^2) + 14/(a*c^4*(1 - a*x)) + (6*Log[1 - a*x])/(a*c^4)
```

Rubi [A] time = 0.172946, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{14}{ac^4(1-ax)} - \frac{8}{ac^4(1-ax)^2} + \frac{3}{ac^4(1-ax)^3} - \frac{1}{2ac^4(1-ax)^4} + \frac{6\log(1-ax)}{ac^4} + \frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 - 1/(2*a*c^4*(1 - a*x)^4) + 3/(a*c^4*(1 - a*x)^3) - 8/(a*c^4*(1 - a*x
)^2) + 14/(a*c^4*(1 - a*x)) + (6*Log[1 - a*x])/(a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx\\
&= -\frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= -\frac{a^4\int \frac{x^4(1+ax)}{(1-ax)^5}\,dx}{c^4}\\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{2}{a^4(-1+ax)^5}-\frac{9}{a^4(-1+ax)^4}-\frac{16}{a^4(-1+ax)^3}-\frac{14}{a^4(-1+ax)^2}-\frac{6}{a^4(-1+ax)}\right)dx}{c^4}\\
&= \frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3}-\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}
\end{aligned}$$

Mathematica [A] time = 0.149446, size = 71, normalized size = 0.82

$$\frac{2a^5x^5-8a^4x^4-16a^3x^3+60a^2x^2-56ax+12(ax-1)^4\log(1-ax)+17}{2ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (17 - 56*a*x + 60*a^2*x^2 - 16*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 12*(-1 + a
*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

Maple [A] time = 0.044, size = 81, normalized size = 0.9

$$\frac{x}{c^4} - 8\frac{1}{ac^4(ax-1)^2} - 3\frac{1}{ac^4(ax-1)^3} + 6\frac{\ln(ax-1)}{ac^4} - \frac{1}{2ac^4(ax-1)^4} - 14\frac{1}{ac^4(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^4,x)
```

```
[Out] x/c^4-8/a/c^4/(a*x-1)^2-3/a/c^4/(a*x-1)^3+6/a/c^4*ln(a*x-1)-1/2/a/c^4/(a*x-
1)^4-14/a/c^4/(a*x-1)
```

Maxima [A] time = 1.02608, size = 126, normalized size = 1.45

$$-\frac{28\,a^3x^3 - 68\,a^2x^2 + 58\,ax - 17}{2\left(a^5c^4x^4 - 4\,a^4c^4x^3 + 6\,a^3c^4x^2 - 4\,a^2c^4x + ac^4\right)} + \frac{x}{c^4} + \frac{6\,\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) + x/c^4 + 6*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.55305, size = 271, normalized size = 3.11

$$\frac{2\,a^5x^5 - 8\,a^4x^4 - 16\,a^3x^3 + 60\,a^2x^2 - 56\,ax + 12\left(a^4x^4 - 4\,a^3x^3 + 6\,a^2x^2 - 4\,ax + 1\right)\log(ax-1) + 17}{2\left(a^5c^4x^4 - 4\,a^4c^4x^3 + 6\,a^3c^4x^2 - 4\,a^2c^4x + ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^4
 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4*a
^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.648879, size = 94, normalized size = 1.08

$$-\frac{28a^3x^3 - 68a^2x^2 + 58ax - 17}{2a^5c^4x^4 - 8a^4c^4x^3 + 12a^3c^4x^2 - 8a^2c^4x + 2ac^4} + \frac{x}{c^4} + \frac{6\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] -(28*a**3*x**3 - 68*a**2*x**2 + 58*a*x - 17)/(2*a**5*c**4*x**4 - 8*a**4*c**
4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) + x/c**4 + 6*log(a*x
 - 1)/(a*c**4)
```

Giac [A] time = 1.12609, size = 78, normalized size = 0.9

$$\frac{x}{c^4} + \frac{6\log(|ax-1|)}{ac^4} - \frac{28\,a^3x^3 - 68\,a^2x^2 + 58\,ax - 17}{2\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 + 6*log(abs(a*x - 1))/(a*c^4) - 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x
 - 17)/((a*x - 1)^4*a*c^4)
```

3.396 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=103

$$\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)}{3a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{3c^4\csc^{-1}(ax)}{2a}$$

```
[Out] (c^4*Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/(2*a^2) + (c^4*(1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/(3*a) + (3*c^4*ArcCsc[a*x])/(2*a) - (c^4*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.13326, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 813, 815, 844, 216, 266, 63, 208}

$$\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)}{3a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{3c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```


```
[Out] (c^4*Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/(2*a^2) + (c^4*(1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/(3*a) + (3*c^4*ArcCsc[a*x])/(2*a) - (c^4*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{1}{2}c^3\,\text{Subst}\left(\int \frac{\left(\frac{2c}{a}+\frac{6cx}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}-\frac{1}{4}\left(a^2c^3\right)\text{Subst}\left(\int \frac{-\frac{4c}{a^3}-\frac{6cx}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right.$$

$$=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{\left(3c^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a^2}+\frac{c^4\ldots}{\ldots}$$

$$=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}+\frac{c^4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\ldots\right.}{2a}$$

$$=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\left(ac^4\right)\text{Subst}\left(\int \frac{\ldots}{a^2-\ldots}\right.$$

$$=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\ldots\right.}{a}$$

Mathematica [A] time = 0.21715, size = 175, normalized size = 1.7

$$-\frac{c^4\left(-24a^5x^5-32a^4x^4+12a^3x^3+40a^2x^2+42a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-15a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)+24a^4x^4\sqrt{1-\ldots}\right.}{24a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -(c^4*(-8 + 12*a*x + 40*a^2*x^2 + 12*a^3*x^3 - 32*a^4*x^4 - 24*a^5*x^5 + 42
*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 15*a^4*S
qrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] + 24*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(24*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [B] time = 0.168, size = 233, normalized size = 2.3

$$-\frac{(ax-1)^2 c^4}{(6\,ax+6)\,a^4x^3}\left(-6\,\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+6\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-9\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+6\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x)
```

```
[Out] -1/6*(a*x-1)^2*c^4*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*x^2*a^2-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+6*ln((a^2
*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-9*a^3*x^3*(a^2)^(1/2
)*arctan(1/(a^2*x^2-1)^(1/2))+3*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-2*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(
1/2)/a^4/x^3/(a^2)^(1/2)
```

Maxima [B] time = 1.59112, size = 301, normalized size = 2.92

$$-\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+29\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3*c
^4*((a*x - 1)/(a*x + 1))^(7/2) + c^4*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^4*(
(a*x - 1)/(a*x + 1))^(3/2) + 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)
*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^
4 + a^2))*a
```

Fricas [A] time = 1.72577, size = 358, normalized size = 3.48

$$-\frac{18\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+14\,a^3c^4x^3+11\,a^2c^4x^2+\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^4*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^4*x^4 + 14*a^3*c^4*x^3 + 11*a^2*c^4*x^2 + a*c^4*x - 2*c^4
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int -\frac{4a}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{6a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{4a^3}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax-}}}{ax+1}}\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(-4*a/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2/(a*x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**3/(a*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(a**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1)), x))/a**4
```

Giac [B] time = 1.20557, size = 311, normalized size = 3.02

$$-\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{6\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}-\frac{\frac{20\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{3\,(ax-1)^2c^4}{(ax+1}}{a^2\left(\frac{ax-1}{ax+1}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 +
 6*c^4*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) - (20*(a*x
 - 1)*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 3*(a*x - 1)^2*c^4*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1)^2 + 9*c^4*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x
- 1)/(a*x + 1) + 1)^3))*a
```

3.397 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=61

$$c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{3c^3\csc^{-1}(ax)}{2a}$$

```
[Out] (3*c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*(1 - 1/(a^2*x^2))^(3/2)*x + (
3*c^3*ArcCsc[a*x])/(2*a)
```

Rubi [A] time = 0.0557759, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6177, 277, 195, 216}

$$c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{3c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (3*c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*(1 - 1/(a^2*x^2))^(3/2)*x + (
3*c^3*ArcCsc[a*x])/(2*a)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx &= -\left(c^3\,\mathrm{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} x + \frac{\left(3c^3\right)\mathrm{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} x + \frac{\left(3c^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} x + \frac{3c^3\csc^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0697137, size = 51, normalized size = 0.84

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+1\right)+3ax\sin^{-1}\left(\frac{1}{ax}\right)\right)}{2a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a^2*x^2) + 3*a*x*ArcSin[1/(a*x)]))/(2*a^
2*x)
```

Maple [A] time = 0.164, size = 105, normalized size = 1.7

$$-\frac{c^{3}\left(ax-1\right)^{2}}{\left(2\,ax+2\right)a^{3}x^{2}}\left(-3\,a^{2}x^{2}\sqrt{a^{2}x^{2}-1}-3\,a^{2}x^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\right)\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\frac{1}{\sqrt{\left(ax-1\right)\left(ax+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x)
```

```
[Out] -1/2*(a*x-1)^2*c^3*(-3*a^2*x^2*(a^2*x^2-1)^(1/2)-3*a^2*x^2*arctan(1/(a^2*x^
2-1)^(1/2))+(a^2*x^2-1)^(3/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*
x+1))^(1/2)/a^3/x^2
```

Maxima [B] time = 1.52451, size = 204, normalized size = 3.34

$$-\left(\frac{3\,c^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^{2}}-\frac{3\,c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^{3}\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^{2}}{ax+1}-\frac{(ax-1)^{2}a^{2}}{(ax+1)^{2}}-\frac{(ax-1)^{3}a^{2}}{(ax+1)^{3}}+a^{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(3*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - (3*c^3*((a*x - 1)/(a*x + 1)
)^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((a*x - 1)/(a*x + 1
)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^
2/(a*x + 1)^3 + a^2))*a
```

Fricas [A] time = 1.6236, size = 192, normalized size = 3.15

$$-\frac{6\,a^{2}c^{3}x^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(2\,a^{3}c^{3}x^{3}+2\,a^{2}c^{3}x^{2}+ac^{3}x+c^{3}\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^3*c^3*x^3 + 2*
a^2*c^3*x^2 + a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^3\left(\int \frac{3a}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{3a^2}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{a^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax-}}}{ax+1}}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(3*a/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2/(a*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(a**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**3*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**3
```

Giac [B] time = 1.20123, size = 232, normalized size = 3.8

$$-\frac{1}{4}\left(\frac{3\left(\pi+2\,\arctan\left(\frac{\frac{ax-1}{ax+1}-1}{2\sqrt{\frac{ax-1}{ax+1}}}\right)\right)c^3}{a^2}+\frac{4\left(3\,c^3\left(\sqrt{\frac{ax-1}{ax+1}}-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2+8\,c^3\right)}{\left(\left(\sqrt{\frac{ax-1}{ax+1}}-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^3+4\sqrt{\frac{ax-1}{ax+1}}-\frac{4}{\sqrt{\frac{ax-1}{ax+1}}}\right)a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*(pi + 2*arctan(1/2*((a*x - 1)/(a*x + 1) - 1)/sqrt((a*x - 1)/(a*x +
1))))*c^3/a^2 + 4*(3*c^3*(sqrt((a*x - 1)/(a*x + 1)) - 1/sqrt((a*x - 1)/(a*x
 + 1)))^2 + 8*c^3)/(((sqrt((a*x - 1)/(a*x + 1)) - 1/sqrt((a*x - 1)/(a*x + 1
)))^3 + 4*sqrt((a*x - 1)/(a*x + 1)) - 4/sqrt((a*x - 1)/(a*x + 1)))*a^2))*a
```

3.398 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.131311, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 850, 813, 844, 216, 266, 63, 208}

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\left(c^3\,\text{Subst}\left(\int\frac{\left(\frac{1}{c}+\frac{x}{ac}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{1}{2}c^3\,\text{Subst}\left(\int\frac{-\frac{2}{ac}+\frac{2x}{a^2c}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}-\frac{c^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\frac{c^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\left(ac^2\right)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

<u>Mathematica</u> [B] time = 0.151151, size = 154, normalized size = 2.44

$$-\frac{c^2\left(-a^3x^3+a^2x^2+4a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+ax\right.}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(-1 + a*x + a^2*x^2 - a^3*x^3 + 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcS
in[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/(a*x
)] - a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2))
```

Maple [B] time = 0.164, size = 174, normalized size = 2.8

$$\frac{c^2 (ax-1)^2}{(ax+1) a^2 x} \left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2 + \left(a^2x^2-1\right)^{\frac{3}{2}} \sqrt{a^2} + \sqrt{a^2}\sqrt{a^2x^2-1}xa + \ln\left(\left(a^2x + \sqrt{a^2x^2-1}\sqrt{a^2}\right) \frac{1}{\sqrt{a^2}}\right) xa^2 + ax\sqrt{a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x)
```

```
[Out] (a*x-1)^2*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^
2)^(1/2)+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*x*a^2+a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)))/((
a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [B] time = 1.64333, size = 169, normalized size = 2.68

$$-\left(\frac{4\,c^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2 a^2}{(ax+1)^2} - a^2} + \frac{2\,c^2 \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2} - \frac{c^2 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2} + \frac{c^2 \log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{a^2}\right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A] time = 1.59294, size = 262, normalized size = 4.16

$$-\frac{2\,ac^2x \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - ac^2x \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) + ac^2x \log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right) - \left(a^2c^2x^2 - c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) + a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int -\frac{2a}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(-2*a/(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2/(a*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1)), x) + Integral(1/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**2
```

Giac [B] time = 1.20726, size = 181, normalized size = 2.87

$$-a\left(\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c^2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{4\,(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -a*(2*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^2 + c^2*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4*(a*
x - 1)*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x + 1)*a^2*((a*x - 1)^2/(a*x + 1)^
2 - 1)))
```

3.399 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=49

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a + (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rubi [A] time = 0.173808, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6177, 852, 1807, 844, 216, 266, 63, 208}

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a + (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^2}dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int\frac{\left(c+\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int\frac{-\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} - \frac{(2c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} - \frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} + (2ac)\,\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} + \frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.11269, size = 73, normalized size = 1.49

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}+2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] time = 0.162, size = 145, normalized size = 3.

$$-\frac{(ax-1)^{2} c}{a(ax+1)}\left(\sqrt{a^{2}x^{2}-1}\sqrt{a^{2}}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{a^{2}}-2a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)-2\sqrt{a^{2}}\sqrt{(ax-1)(ax+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x)
```

```
[Out] -(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^
2)^(1/2)-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-2*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-
1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.52358, size = 154, normalized size = 3.14

$$-2a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^{2}}{ax+1}-a^{2}}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^{2}}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.63803, size = 223, normalized size = 4.55

$$\frac{2c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+2c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int \frac{a}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{1}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**2*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1)), x))/a
```

Giac [B] time = 1.21422, size = 150, normalized size = 3.06

$$2\,a\left(\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{c\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*a*(c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 - c*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - c*sqrt((a*x
- 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.400 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=105

$$-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] (-8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) - (4*(3*a + 4/x))/(3*a^
2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c + (4*ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]])/(a*c)
```

Rubi [A] time = 0.301814, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```



```
[Out] (-8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) - (4*(3*a + 4/x))/(3*a^
2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c + (4*ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]])/(a*c)
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^4-\frac{12c^4x}{a}-\frac{13c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^4+\frac{12c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.122189, size = 70, normalized size = 0.67

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(3a^2x^2-26ax+19)}{(ax-1)^2}+12\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{3ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 26*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + 12*Lo
g[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(3*a*c)
```

Maple [B] time = 0.173, size = 346, normalized size = 3.3

$$\frac{1}{3\,(ax-1)\,ac\,(ax+1)}\left(12\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^3a^4+12\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^3a^3-36\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x)
```

```
[Out] 1/3*(12*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4
+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-36*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(3/2)*x*a-36*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+36*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+7*((a*x-1)*(a*x+1))^(3/
2)*(a^2)^(1/2)+36*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-12*a*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-12*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-
1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.05289, size = 180, normalized size = 1.71

$$\frac{2}{3}\,a\left(\frac{\frac{8\,(ax-1)}{ax+1}-\frac{12\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] 2/3*a*((8*(a*x - 1)/(a*x + 1) - 12*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a*
x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 6*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^
2*c))
```

Fricas [A] time = 1.53813, size = 304, normalized size = 2.9

$$\frac{12\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^{3}x^{3}-23\,a^{2}x^{2}-7\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] 1/3*(12*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*(a^2*
x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 23*a^2*x
^2 - 7*a*x + 19)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{x}{\frac{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x)/c
```

Giac [A] time = 1.22507, size = 200, normalized size = 1.9

$$\frac{2}{3}a\left(\frac{6\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{(ax+1)\left(\frac{9(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2/3*a*(6*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(abs(sqrt((a*x -
 1)/(a*x + 1)) - 1))/(a^2*c) - (a*x + 1)*(9*(a*x - 1)/(a*x + 1) + 1)/((a*x
- 1)*a^2*c*sqrt((a*x - 1)/(a*x + 1))) - 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*
((a*x - 1)/(a*x + 1) - 1)))
```

3.401 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] (-16*(a + x^(-1)))/(5*a^2*c^2*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 11/x))/(
15*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (75*a + 103/x)/(15*a^2*c^2*Sqrt[1 - 1
/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]])/(a*c^2)
```

Rubi [A] time = 0.401889, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```


```
[Out] (-16*(a + x^(-1)))/(5*a^2*c^2*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 11/x))/(
15*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (75*a + 103/x)/(15*a^2*c^2*Sqrt[1 - 1
/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]])/(a*c^2)
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^5-\frac{25c^5x}{a}-\frac{39c^5x^2}{a^2}+\frac{5c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^5+\frac{75c^5x}{a}+\frac{88c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^5-\frac{75c^5x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.}{ac^2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx\right.}{2ac^2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\right.}{c^2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.075933, size = 104, normalized size = 0.75

$$\frac{15a^4x^4-173a^3x^3+91a^2x^2+75ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+161ax-118}{15a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-118 + 161*a*x + 91*a^2*x^2 - 173*a^3*x^3 + 15*a^4*x^4 + 75*a*Sqrt[1 - 1/(
a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^2*Sqrt[1
 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] time = 0.172, size = 438, normalized size = 3.2

$$\frac{1}{15\,a\,(ax-1)^2\,c^2\,(ax+1)}\left(75\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^4a^5+75\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^4a^4-300\,\ln\left(\frac{a^2x+\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/15*(75*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^
5+75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-300*ln((a^2*x+(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-60*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^2*a^2-300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+450*ln((a
^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+97*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(3/2)*x*a+450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*
a^2-300*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-4
3*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2)*x*a+75*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+75*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a*x-1)*(
a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.08625, size = 207, normalized size = 1.5

$$\frac{1}{15}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}+\frac{100\,(ax-1)^2}{(ax+1)^2}-\frac{150\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a*((17*(a*x - 1)/(a*x + 1) + 100*(a*x - 1)^2/(a*x + 1)^2 - 150*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x
- 1)/(a*x + 1))^(5/2)) + 75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) -
75*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A] time = 1.54351, size = 392, normalized size = 2.84

$$\frac{75\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-75\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^4x^4-173\,a^3x^3+91\right.}{15\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 173*a^3*x^3 + 91*a^2*x^2 + 161*a*x - 118)*sqrt((a*x - 1)/
(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int\frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1)), x)/c**2
```

Giac [A] time = 1.23729, size = 224, normalized size = 1.62

$$\frac{1}{15}a\left(\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{75\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)^2\left(\frac{20(ax-1)}{ax+1}+\frac{120(ax-1)^2}{(ax+1)^2}+3\right)}{(ax-1)^2a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{30\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/15*a*(75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 75*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)^2*(20*(a*x - 1)/(a*x + 1) +
 120*(a*x - 1)^2/(a*x + 1)^2 + 3)/((a*x - 1)^2*a^2*c^2*sqrt((a*x - 1)/(a*x
+ 1))) - 30*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.402 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=165

$$-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}-\frac{16}{7a^2c^3x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{6\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] (-32*(a + x^(-1)))/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(7/2)) - (2*(7*a + 13/x))/(
7*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (42*a + 59/x)/(7*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) - 16/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)*x) + (Sqrt[1 - 1/(a^2*x^2
)]*x)/c^3 + (6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rubi [A] time = 0.521118, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}-\frac{16}{7a^2c^3x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{6\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```



```
[Out] (-32*(a + x^(-1)))/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(7/2)) - (2*(7*a + 13/x))/(
7*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (42*a + 59/x)/(7*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) - 16/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)*x) + (Sqrt[1 - 1/(a^2*x^2
)]*x)/c^3 + (6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^3)
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^6-\frac{42c^6x}{a}-\frac{80c^6x^2}{a^2}+\frac{42c^6x^3}{a^3}+\frac{7c^6x^4}{a^4}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Subst}\left(\int \frac{35c^6+\frac{210c^6x}{a}+\frac{355c^6x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\text{Subst}\left(\int \frac{-105c^6-\frac{630c^6x}{a}-\frac{780c^6x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, \ldots\right)}{105c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Subst}\left(\int \frac{105c^6\ldots}{x^2\sqrt{\ldots}}\ldots\right)}{105\ldots}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \ldots$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \ldots$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \ldots$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \ldots$$

Mathematica [A] time = 0.0827369, size = 112, normalized size = 0.68

$$\frac{7a^5x^5-109a^4x^4+145a^3x^3+39a^2x^2+42ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-156ax+66}{7a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (66 - 156*a*x + 39*a^2*x^2 + 145*a^3*x^3 - 109*a^4*x^4 + 7*a^5*x^5 + 42*a*S
qrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(7*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [B] time = 0.174, size = 530, normalized size = 3.2

$$-\frac{1}{7a(ax-1)^3c^3(ax+1)}\left(-42\sqrt{a^2}\sqrt{(ax-1)(ax+1)}x^5a^5-42\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^5a^6+35\sqrt{a^2}((ax-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x)
```

```
[Out] -1/7*(-42*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-42*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+35*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(3/2)*x^3*a^3+210*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+210*l
n((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-87*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-420*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x^3*a^3-420*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))
*x^3*a^4+78*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+420*(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2)*x^2*a^2+420*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))
/(a^2)^(1/2))*x^2*a^3-24*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-210*(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(1/2)*x*a-210*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/(a^2)^(1/2))*x*a^2+42*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+42*a*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1
)^3/c^3/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.07271, size = 228, normalized size = 1.38

$$\frac{1}{14}\,a\left(\frac{\frac{6\,(ax-1)}{ax+1}+\frac{21\,(ax-1)^2}{(ax+1)^2}+\frac{112\,(ax-1)^3}{(ax+1)^3}-\frac{168\,(ax-1)^4}{(ax+1)^4}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/14*a*((6*(a*x - 1)/(a*x + 1) + 21*(a*x - 1)^2/(a*x + 1)^2 + 112*(a*x - 1)
^3/(a*x + 1)^3 - 168*(a*x - 1)^4/(a*x + 1)^4 + 1)/(a^2*c^3*((a*x - 1)/(a*x
+ 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 84*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/(a^2*c^3) - 84*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3)
)
```

Fricas [A] time = 1.55957, size = 460, normalized size = 2.79

$$\frac{42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(7\,a^5x^5\right.}{7\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*(42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((
a*x - 1)/(a*x + 1)) - 1) + (7*a^5*x^5 - 109*a^4*x^4 + 145*a^3*x^3 + 39*a^2*
x^2 - 156*a*x + 66)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3
 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\int\frac{x^3}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

Giac [A] time = 1.3338, size = 246, normalized size = 1.49

$$\frac{1}{14}a\left(\frac{84\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^3\left(\frac{7(ax-1)}{ax+1}+\frac{28(ax-1)^2}{(ax+1)^2}+\frac{140(ax-1)^3}{(ax+1)^3}+1\right)}{(ax-1)^3a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{28\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] 1/14*a*(84*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 84*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^3*(7*(a*x - 1)/(a*x + 1) +
28*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 + 1)/((a*x - 1)^3*
a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - 28*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*
((a*x - 1)/(a*x + 1) - 1)))
```

3.403 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=204

$$\frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}$$

```
[Out] (16*(9*a - 5/x))/(63*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (64*(a + x^(-1)))/(
9*a^2*c^4*(1 - 1/(a^2*x^2))^(9/2)) - (8*(21*a + 41/x))/(105*a^2*c^4*(1 - 1/
(a^2*x^2))^(5/2)) - (735*a + 1417/x)/(315*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2))
- (2205*a + 3149/x)/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*
x^2)]*x)/c^4 + (7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rubi [A] time = 0.639914, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$\frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (16*(9*a - 5/x))/(63*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (64*(a + x^(-1)))/(
9*a^2*c^4*(1 - 1/(a^2*x^2))^(9/2)) - (8*(21*a + 41/x))/(105*a^2*c^4*(1 - 1/
(a^2*x^2))^(5/2)) - (735*a + 1417/x)/(315*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2))
- (2205*a + 3149/x)/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*
x^2)]*x)/c^4 + (7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^7}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^{11}}$$

$$= -\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{-9c^7-\frac{63c^7x}{a}-\frac{134c^7x^2}{a^2}+\frac{198c^7x^3}{a^3}+\frac{63c^7x^4}{a^4}+\frac{9c^7x^5}{a^5}}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{\text{Subst}\left(\int \frac{63c^7+\frac{441c^7x}{a}+\frac{921c^7x^2}{a^2}+\frac{63c^7x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-315c^7-\frac{2205c^7x}{a}-\frac{3936c^7x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\right.}{315c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a}{315a^2c^4}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a}{315a^2c^4}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{2205a}{315a^2c^4}$$

$$\frac{16\left(9a-\frac{5}{x}\right)}{} \quad \frac{64\left(a+\frac{1}{x}\right)}{} \quad \frac{8\left(21a+\frac{41}{x}\right)}{} \quad \frac{735a+\frac{1417}{x}}{} \quad \frac{2205a}{}$$

Mathematica [A] time = 0.0936146, size = 120, normalized size = 0.59

$$\frac{315a^6x^6-6224a^5x^5+13241a^4x^4-5567a^3x^3-10232a^2x^2+2205ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+11651ax-}{315a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-3464 + 11651*a*x - 10232*a^2*x^2 - 5567*a^3*x^3 + 13241*a^4*x^4 - 6224*a^
5*x^5 + 315*a^6*x^6 + 2205*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4)
```

Maple [B] time = 0.18, size = 622, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] 1/315*(2205*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6
*a^7+2205*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a^6-13230*ln((a^2*x+(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6-1890*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(3/2)*x^4*a^4-13230*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^
5+33075*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5
+6376*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+33075*(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2)*x^4*a^4-44100*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
))/(a^2)^(1/2))*x^3*a^4-8646*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-44
100*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+33075*ln((a^2*x+(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+5349*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(3/2)*x*a+33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-13230*ln
((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-1259*((a*x-
1)*(a*x+1))^(3/2)*(a^2)^(1/2)-13230*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a
+2205*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+2205*(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)^4/c^4/((a*x-1)*(a*
x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.17138, size = 250, normalized size = 1.23

$$\frac{1}{1260}a\left(\frac{\frac{235(ax-1)}{ax+1}+\frac{801(ax-1)^2}{(ax+1)^2}+\frac{2289(ax-1)^3}{(ax+1)^3}+\frac{11760(ax-1)^4}{(ax+1)^4}-\frac{17640(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{8820\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{8820\log\left(\sqrt{\frac{ax-}{ax-}}\right.}{a^2c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*a*((235*(a*x - 1)/(a*x + 1) + 801*(a*x - 1)^2/(a*x + 1)^2 + 2289*(a*
x - 1)^3/(a*x + 1)^3 + 11760*(a*x - 1)^4/(a*x + 1)^4 - 17640*(a*x - 1)^5/(a
*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)) + 8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 882
0*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A] time = 1.52512, size = 567, normalized size = 2.78

$$\frac{2205\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2205\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-\right.}{315\left(a^6c^4x^5-5a^5c^4x^4+10a^4c^4x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 -
10*a^2*x^2 + 5*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^6*x^6 -
 6224*a^5*x^5 + 13241*a^4*x^4 - 5567*a^3*x^3 - 10232*a^2*x^2 + 11651*a*x -
3464)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*
x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41816, size = 267, normalized size = 1.31

$$\frac{1}{1260}a\left(\frac{8820\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{8820\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{(ax+1)^4\left(\frac{270(ax-1)}{ax+1}+\frac{1071(ax-1)^2}{(ax+1)^2}+\frac{3360(ax-1)^3}{(ax+1)^3}+\frac{15120(ax-1)^4}{(ax+1)^4}+\right.}{(ax-1)^4a^2c^4\sqrt{\frac{ax-1}{ax+1}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/1260*a*(8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 8820*log(abs(
sqrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - (a*x + 1)^4*(270*(a*x - 1)/(a*x
 + 1) + 1071*(a*x - 1)^2/(a*x + 1)^2 + 3360*(a*x - 1)^3/(a*x + 1)^3 + 15120
*(a*x - 1)^4/(a*x + 1)^4 + 35)/((a*x - 1)^4*a^2*c^4*sqrt((a*x - 1)/(a*x + 1
))) - 2520*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1)))
```

3.404 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=64

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rubi [A] time = 0.137443, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx\\ &= -\frac{c^5\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^5}{x^5}\,dx}{a^5}\\ &= -\frac{c^5\int \frac{(1-ax)^3(1+ax)^2}{x^5}\,dx}{a^5}\\ &= -\frac{c^5\int\left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5}\\ &= \frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.277991, size = 66, normalized size = 1.03

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(ax)}{a}+c^5x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[a*x])/a
```

Maple [A] time = 0.045, size = 61, normalized size = 1.

$$\frac{c^5}{4\,a^5x^4}-\frac{c^5}{3\,a^4x^3}-\frac{c^5}{x^2a^3}+2\,\frac{c^5}{a^2x}+c^5x-\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x)
```

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/x^2/a^3+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Maxima [A] time = 1.03626, size = 80, normalized size = 1.25

$$c^5x - \frac{c^5 \log(x)}{a} + \frac{24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
3*c^5)/(a^5*x^4)
```

Fricas [A] time = 1.45563, size = 150, normalized size = 2.34

$$\frac{12\,a^5c^5x^5 - 12\,a^4c^5x^4\log(x) + 24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5*
x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

Sympy [A] time = 0.477993, size = 63, normalized size = 0.98

$$\frac{a^5c^5x - a^4c^5\log(x) + \frac{24a^3c^5x^3 - 12a^2c^5x^2 - 4ac^5x + 3c^5}{12x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

Giac [B] time = 1.14656, size = 166, normalized size = 2.59

$$\frac{c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(12\,c^5+\frac{37\,c^5}{ax-1}+\frac{52\,c^5}{(ax-1)^2}+\frac{42\,c^5}{(ax-1)^3}+\frac{12\,c^5}{(ax-1)^4}\right)(ax-1)}{12\,a\left(\frac{1}{ax-1}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c^5*log(abs(-1/(a*x - 1) - 1
))/a + 1/12*(12*c^5 + 37*c^5/(a*x - 1) + 52*c^5/(a*x - 1)^2 + 42*c^5/(a*x -
 1)^3 + 12*c^5/(a*x - 1)^4)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^4)
```

3.405 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=30

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rubi [A] time = 0.126472, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6167, 6131, 6129, 73, 270}

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx\\ &= \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\ &= \frac{c^4\int \frac{(1-ax)^2(1+ax)^2}{x^4}\,dx}{a^4}\\ &= \frac{c^4\int \frac{\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\ &= \frac{c^4\int \left(a^4+\frac{1}{x^4}-\frac{2a^2}{x^2}\right)dx}{a^4}\\ &= -\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x\end{aligned}$$

Mathematica [A] time = 0.180698, size = 30, normalized size = 1.

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Maple [A] time = 0.043, size = 27, normalized size = 0.9

$$\frac{c^4}{a^4}\left(xa^4+2\,\frac{a^2}{x}-\frac{1}{3\,x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x)
```

```
[Out] c^4/a^4*(x*a^4+2*a^2/x-1/3/x^3)
```

Maxima [A] time = 1.00567, size = 42, normalized size = 1.4

$$c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Fricas [A] time = 1.54152, size = 72, normalized size = 2.4

$$\frac{3\,a^4c^4x^4+6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Sympy [A] time = 0.327616, size = 31, normalized size = 1.03

$$\frac{a^4c^4x+\frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

Giac [B] time = 1.13582, size = 80, normalized size = 2.67

$$\frac{(ax-1)c^4}{a} - \frac{5\,c^4 + \frac{9\,c^4}{ax-1} + \frac{3\,c^4}{(ax-1)^2}}{3\,a\left(\frac{1}{ax-1} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)*c^4/a - 1/3*(5*c^4 + 9*c^4/(a*x - 1) + 3*c^4/(a*x - 1)^2)/(a*(1/(
a*x - 1) + 1)^3)
```

3.406 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rubi [A] time = 0.127107, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx\\
&= -\frac{c^3\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int\frac{(1-ax)(1+ax)^2}{x^3}\,dx}{a^3}\\
&= -\frac{c^3\int\left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}\\
&= \frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.132784, size = 40, normalized size = 1.05

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(ax)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[a*x])/a
```

Maple [A] time = 0.043, size = 37, normalized size = 1.

$$\frac{c^3}{2\,x^2a^3}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x)
```

```
[Out] 1/2*c^3/x^2/a^3+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

Maxima [A] time = 1.06445, size = 46, normalized size = 1.21

$$c^3x + \frac{c^3 \log(x)}{a} + \frac{2\,ac^3x + c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

Fricas [A] time = 1.61701, size = 97, normalized size = 2.55

$$\frac{2\,a^3c^3x^3 + 2\,a^2c^3x^2\log(x) + 2\,ac^3x + c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

Sympy [A] time = 0.348838, size = 37, normalized size = 0.97

$$\frac{a^3c^3x + a^2c^3\log(x) + \frac{2ac^3x+c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**3,x)
```

```
[Out] (a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

Giac [B] time = 1.17856, size = 132, normalized size = 3.47

$$-\frac{c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(2\,c^3+\frac{c^3}{ax-1}-\frac{2\,c^3}{(ax-1)^2}\right)(ax-1)}{2\,a\left(\frac{1}{ax-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + c^3*log(abs(-1/(a*x - 1) -
1))/a + 1/2*(2*c^3 + c^3/(a*x - 1) - 2*c^3/(a*x - 1)^2)*(a*x - 1)/(a*(1/(a*
x - 1) + 1)^2)
```

3.407 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=27

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rubi [A] time = 0.121187, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 43}

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx\\ &= \frac{c^2\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\ &= \frac{c^2\int\frac{(1+ax)^2}{x^2}\,dx}{a^2}\\ &= \frac{c^2\int\left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}\\ &= -\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.103305, size = 29, normalized size = 1.07

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(ax)}{a}+c^2x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[a*x])/a
```

Maple [A] time = 0.045, size = 28, normalized size = 1.

$$-\frac{c^2}{a^2x}+xc^2+2\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x)
```

```
[Out] -c^2/a^2/x+x*c^2+2*c^2*ln(x)/a
```

Maxima [A] time = 1.0413, size = 36, normalized size = 1.33

$$c^2x + \frac{2\,c^2\log\left(x\right)}{a} - \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

Fricas [A] time = 1.56775, size = 65, normalized size = 2.41

$$\frac{a^2c^2x^2 + 2\,ac^2x\log\left(x\right) - c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

Sympy [A] time = 0.292655, size = 26, normalized size = 0.96

$$\frac{a^2c^2x + 2ac^2\log\left(x\right) - \frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**2,x)
```

```
[Out] (a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

Giac [B] time = 1.12239, size = 127, normalized size = 4.7

$$-\frac{2\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{c^2+\frac{2\,c^2}{ax-1}}{a^2\left(\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x - 1
) - 1))/a + (c^2 + 2*c^2/(a*x - 1))/(a^2*(1/((a*x - 1)*a) + 1/((a*x - 1)^2*
a)))
```

3.408 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0816088, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6131, 6129, 72}

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx\\ &= -\frac{c\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\ &= -\frac{c\int \frac{(1+ax)^2}{x(1-ax)}\,dx}{a}\\ &= -\frac{c\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{a}\\ &= cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0385445, size = 25, normalized size = 1.

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Maple [A] time = 0.043, size = 25, normalized size = 1.

$$cx+4\,\frac{c\ln(ax-1)}{a}-\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x)
```

```
[Out] c*x+4*c/a*ln(a*x-1)-c*ln(x)/a
```

Maxima [A] time = 1.12228, size = 32, normalized size = 1.28

$$cx + \frac{4\,c\log\left(ax - 1\right)}{a} - \frac{c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

Fricas [A] time = 1.54833, size = 55, normalized size = 2.2

$$\frac{acx + 4\,c\log\left(ax - 1\right) - c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

Sympy [A] time = 0.389204, size = 17, normalized size = 0.68

$$cx + \frac{c\left(-\log\left(x\right) + 4\log\left(x - \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x),x)
```

```
[Out] c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

Giac [B] time = 1.15558, size = 74, normalized size = 2.96

$$\frac{(ax-1)c}{a} - \frac{3\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="giac")
```

```
[Out] (a*x - 1)*c/a - 3*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c*log(abs(-1
/(a*x - 1) - 1))/a
```

3.409 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=53

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.133421, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx\\ &= -\frac{a\int \frac{e^{4\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\ &= -\frac{a\int \frac{x(1+ax)^2}{(1-ax)^3}\,dx}{c}\\ &= -\frac{a\int \left(-\frac{1}{a}-\frac{4}{a(-1+ax)^3}-\frac{8}{a(-1+ax)^2}-\frac{5}{a(-1+ax)}\right)dx}{c}\\ &= \frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.0304802, size = 51, normalized size = 0.96

$$-\frac{a\left(-\frac{8}{a^2(1-ax)}+\frac{2}{a^2(1-ax)^2}-\frac{5\log(1-ax)}{a^2}-\frac{x}{a}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/a
^2))/c)
```

Maple [A] time = 0.045, size = 51, normalized size = 1.

$$\frac{x}{c}-2\,\frac{1}{ac\,(ax-1)^2}+5\,\frac{\ln(ax-1)}{ac}-8\,\frac{1}{ac\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x)
```

```
[Out] x/c-2/a/c/(a*x-1)^2+5/a/c*ln(a*x-1)-8/a/c/(a*x-1)
```

Maxima [A] time = 1.03053, size = 66, normalized size = 1.25

$$-\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.43967, size = 140, normalized size = 2.64

$$\frac{a^3x^3-2\,a^2x^2-7\,ax+5\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+6}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] time = 0.429769, size = 41, normalized size = 0.77

$$-\frac{8ax-6}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{5\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x),x)
```

```
[Out] -(8*a*x - 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Giac [A] time = 1.11963, size = 100, normalized size = 1.89

$$\frac{ax-1}{ac}-\frac{5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{2\left(\frac{4a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}\right)}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - 2*(4*a^3
*c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

3.410 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=71

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rubi [A] time = 0.158327, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx \\
&= \frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2} \\
&= \frac{a^2\int \frac{x^2(1+ax)^2}{(1-ax)^4}\,dx}{c^2} \\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{4}{a^2(-1+ax)^4}+\frac{12}{a^2(-1+ax)^3}+\frac{13}{a^2(-1+ax)^2}+\frac{6}{a^2(-1+ax)}\right)dx}{c^2} \\
&= \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.105849, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-30a^2x^2+57ax+18(ax-1)^3\log(1-ax)-25}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.046, size = 66, normalized size = 0.9

$$\frac{x}{c^2}-6\,\frac{1}{ac^2\,(ax-1)^2}-\frac{4}{3\,ac^2\,(ax-1)^3}+6\,\frac{\ln(ax-1)}{ac^2}-13\,\frac{1}{ac^2\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x)
```

```
[Out] x/c^2-6/a/c^2/(a*x-1)^2-4/3/a/c^2/(a*x-1)^3+6/a/c^2*ln(a*x-1)-13/a/c^2/(a*x
-1)
```

Maxima [A] time = 1.01575, size = 101, normalized size = 1.42

$$-\frac{39\,a^2x^2-60\,ax+25}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}+\frac{x}{c^2}+\frac{6\,\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.71908, size = 216, normalized size = 3.04

$$\frac{3\,a^4x^4-9\,a^3x^3-30\,a^2x^2+57\,ax+18\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-25}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

Sympy [A] time = 0.542358, size = 73, normalized size = 1.03

$$-\frac{39a^2x^2-60ax+25}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}+\frac{x}{c^2}+\frac{6\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**2,x)
```

```
[Out] -(39*a**2*x**2 - 60*a*x + 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2
*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

Giac [A] time = 1.1339, size = 127, normalized size = 1.79

$$\frac{ax-1}{ac^2} - \frac{6\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2} - \frac{\frac{39\,a^5c^4}{ax-1} + \frac{18\,a^5c^4}{(ax-1)^2} + \frac{4\,a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 6*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(39*a^5*c^4/(a*x - 1) + 18*a^5*c^4/(a*x - 1)^2 + 4*a^5*c^4/(a*x - 1)^3)/(a^
6*c^6)
```

3.411 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rubi [A] time = 0.172051, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx \\
&= -\frac{a^3\int \frac{e^{4\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3} \\
&= -\frac{a^3\int \frac{x^3(1+ax)^2}{(1-ax)^5}\,dx}{c^3} \\
&= -\frac{a^3\int \left(-\frac{1}{a^3}-\frac{4}{a^3(-1+ax)^5}-\frac{16}{a^3(-1+ax)^4}-\frac{25}{a^3(-1+ax)^3}-\frac{19}{a^3(-1+ax)^2}-\frac{7}{a^3(-1+ax)}\right)dx}{c^3} \\
&= \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}
\end{aligned}$$

Mathematica [A] time = 0.139355, size = 71, normalized size = 0.8

$$\frac{6a^5x^5-24a^4x^4-78a^3x^3+243a^2x^2-218ax+42(ax-1)^4\log(1-ax)+65}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
+ a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.046, size = 81, normalized size = 0.9

$$\frac{x}{c^3}-\frac{25}{2\,ac^3\,(ax-1)^2}-\frac{16}{3\,ac^3\,(ax-1)^3}+7\,\frac{\ln(ax-1)}{ac^3}-\frac{1}{ac^3\,(ax-1)^4}-19\,\frac{1}{ac^3\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x)
```

```
[Out] x/c^3-25/2/a/c^3/(a*x-1)^2-16/3/a/c^3/(a*x-1)^3+7/a/c^3*ln(a*x-1)-1/a/c^3/(
a*x-1)^4-19/a/c^3/(a*x-1)
```

Maxima [A] time = 1.04203, size = 126, normalized size = 1.42

$$-\frac{114\,a^{3}x^{3}-267\,a^{2}x^{2}+224\,ax-65}{6\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}+\frac{x}{c^{3}}+\frac{7\,\log\left(ax-1\right)}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^
3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.84076, size = 275, normalized size = 3.09

$$\frac{6\,a^{5}x^{5}-24\,a^{4}x^{4}-78\,a^{3}x^{3}+243\,a^{2}x^{2}-218\,ax+42\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(ax-1\right)+65}{6\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4*
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4 -
4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] time = 0.663173, size = 94, normalized size = 1.06

$$-\frac{114a^{3}x^{3}-267a^{2}x^{2}+224ax-65}{6a^{5}c^{3}x^{4}-24a^{4}c^{3}x^{3}+36a^{3}c^{3}x^{2}-24a^{2}c^{3}x+6ac^{3}}+\frac{x}{c^{3}}+\frac{7\log\left(ax-1\right)}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**3,x)
```

```
[Out] -(114*a**3*x**3 - 267*a**2*x**2 + 224*a*x - 65)/(6*a**5*c**3*x**4 - 24*a**4
*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*lo
g(a*x - 1)/(a*c**3)
```

Giac [A] time = 1.15071, size = 147, normalized size = 1.65

$$\frac{ax-1}{ac^3} - \frac{7\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3} - \frac{\frac{114\,a^7c^9}{ax-1} + \frac{75\,a^7c^9}{(ax-1)^2} + \frac{32\,a^7c^9}{(ax-1)^3} + \frac{6\,a^7c^9}{(ax-1)^4}}{6\,a^8c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 7*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/6*
(114*a^7*c^9/(a*x - 1) + 75*a^7*c^9/(a*x - 1)^2 + 32*a^7*c^9/(a*x - 1)^3 +
6*a^7*c^9/(a*x - 1)^4)/(a^8*c^12)
```

3.412 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=105

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rubi [A] time = 0.18284, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx\\
&= \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= \frac{a^4\int \frac{x^4(1+ax)^2}{(1-ax)^6}\,dx}{c^4}\\
&= \frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(-1+ax)^6}+\frac{20}{a^4(-1+ax)^5}+\frac{41}{a^4(-1+ax)^4}+\frac{44}{a^4(-1+ax)^3}+\frac{26}{a^4(-1+ax)^2}+\frac{8}{a^4(-1+ax)}\right)dx}{c^4}\\
&= \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}
\end{aligned}$$

Mathematica [A] time = 0.172816, size = 79, normalized size = 0.75

$$\frac{15a^6x^6-75a^5x^5-240a^4x^4+1080a^3x^3-1480a^2x^2+890ax+120(ax-1)^5\log(1-ax)-202}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

Maple [A] time = 0.047, size = 96, normalized size = 0.9

$$\frac{x}{c^4}-22\,\frac{1}{ac^4\,(ax-1)^2}-\frac{41}{3\,ac^4\,(ax-1)^3}-\frac{4}{5\,ac^4\,(ax-1)^5}+8\,\frac{\ln(ax-1)}{ac^4}-5\,\frac{1}{ac^4\,(ax-1)^4}-26\,\frac{1}{ac^4\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x)
```

```
[Out] x/c^4-22/a/c^4/(a*x-1)^2-41/3/a/c^4/(a*x-1)^3-4/5/a/c^4/(a*x-1)^5+8/a/c^4*l
n(a*x-1)-5/a/c^4/(a*x-1)^4-26/a/c^4/(a*x-1)
```

Maxima [A] time = 1.01582, size = 153, normalized size = 1.46

$$-\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}+\frac{x}{c^4}+\frac{8\,\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4*
x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4
) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.82345, size = 347, normalized size = 3.3

$$\frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\log}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2 +
 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*
log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] time = 0.797477, size = 114, normalized size = 1.09

$$-\frac{390a^4x^4 - 1230a^3x^3 + 1555a^2x^2 - 905ax + 202}{15a^6c^4x^5 - 75a^5c^4x^4 + 150a^4c^4x^3 - 150a^3c^4x^2 + 75a^2c^4x - 15ac^4} + \frac{x}{c^4} + \frac{8\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**4,x)
```

```
[Out] -(390*a**4*x**4 - 1230*a**3*x**3 + 1555*a**2*x**2 - 905*a*x + 202)/(15*a**6
*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2 +
75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

Giac [A] time = 1.15734, size = 167, normalized size = 1.59

$$\frac{ax-1}{ac^4} - \frac{8\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{\frac{390\,a^9c^{16}}{ax-1} + \frac{330\,a^9c^{16}}{(ax-1)^2} + \frac{205\,a^9c^{16}}{(ax-1)^3} + \frac{75\,a^9c^{16}}{(ax-1)^4} + \frac{12\,a^9c^{16}}{(ax-1)^5}}{15\,a^{10}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^4) - 8*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 1/15
*(390*a^9*c^16/(a*x - 1) + 330*a^9*c^16/(a*x - 1)^2 + 205*a^9*c^16/(a*x - 1
)^3 + 75*a^9*c^16/(a*x - 1)^4 + 12*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

3.413 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=135

$$c^4x\sqrt{1-\frac{1}{a^2x^2}}-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{5c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{25c^4\csc^{-1}(ax)}{2a}$$

```
[Out] (-32*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) - (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*
x^2) + (5*c^4*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*
x - (25*c^4*ArcCsc[a*x])/(2*a) - (5*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.437769, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 1807, 1809, 844, 216, 266, 63, 208}

$$c^4x\sqrt{1-\frac{1}{a^2x^2}}-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{5c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{25c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```


```
[Out] (-32*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) - (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*
x^2) + (5*c^4*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*
x - (25*c^4*ArcCsc[a*x])/(2*a) - (5*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{\frac{5c^5}{a}-\frac{10c^5x}{a^2}+\frac{10c^5x^2}{a^3}-\frac{5c^5x^3}{a^4}+\frac{c^5x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{15c^5}{a^3}+\frac{30c^5x}{a^4}-\frac{32c^5x^2}{a^5}+\frac{15c^5x^3}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^4\,\text{Subst}\left(\int \frac{\frac{30c^5}{a^5}-\frac{75c^5x}{a^6}+\frac{64c^5x^2}{a^7}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{6c}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^6\,\text{Subst}\left(\int \frac{-\frac{30c^5}{a^7}+\frac{75c}{a}\cdots}{x\sqrt{1-\frac{x^2}{a^2}}}\cdots\right)}{6c}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{(25c^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\cdots}}\cdots\right)}{2a^2}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{25c^4\csc^{-1}(ax)}{2a} + \frac{(5c\cdots}{\cdots}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{25c^4\csc^{-1}(ax)}{2a} - (5a\cdots$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{25c^4\csc^{-1}(ax)}{2a} - \frac{5c^4\cdots}{\cdots}$$

Mathematica [A] time = 0.145488, size = 175, normalized size = 1.3

$$\frac{c^4\left(6a^5x^5-64a^4x^4+9a^3x^3+62a^2x^2+90a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-30a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-30a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\cdots\right.}{6a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(2 - 15*a*x + 62*a^2*x^2 + 9*a^3*x^3 - 64*a^4*x^4 + 6*a^5*x^5 + 90*a^4
*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 30*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 30*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [B] time = 0.136, size = 290, normalized size = 2.2

$$\frac{(ax+1)c^4}{6a^4x^3}\sqrt{\frac{ax-1}{ax+1}}\left(-66\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+66\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-75\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+66\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4*(-66*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x^4*a^4+66*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-75*(a^2*x^2-1)^(1/2)*(a^2)
^(1/2)*x^3*a^3+66*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3
*a^4-75*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+96*(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2)*x^3*a^3-96*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
))/(a^2)^(1/2))*x^3*a^4-15*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^4/x^3/(a^2)^(1/2)
```

Maxima [A] time = 1.58953, size = 301, normalized size = 2.23

$$\frac{1}{3}\left(\frac{75c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{15c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{87c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+61c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55c^4\left(\frac{ax-1}{ax+1}\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(75*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 15*c^4*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (8
```

```
7*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 61*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 55
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 45*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
 + 1)^4 + a^2))*a
```

Fricas [A] time = 1.82614, size = 365, normalized size = 2.7

$$\frac{150\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^4x^4-58\,a^3c^4x^3-49\,a^2c^4x^2\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(150*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 30*a^3*c^4*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) + 30*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^4*x^4 - 58*a^3*c^4*x^3 - 49*a^2*c^4*x^2 + 13*a*c^4*x -
2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\,dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-4*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**3, x) + Integral(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x*
*2, x) + Integral(-4*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**4
```

Giac [B] time = 1.21072, size = 358, normalized size = 2.65

$$\frac{25\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{5\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^4\operatorname{sgn}(ax+1)}{a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 25*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 5*c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
4*sgn(a*x + 1)/a - 1/3*(15*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(
a*x + 1) + 60*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 132*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 15*(x*abs(a) - sqrt(a^2*
x^2 - 1))*c^4*abs(a)*sgn(a*x + 1) + 64*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

3.414 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=106

$$c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{13c^3\csc^{-1}(ax)}{2a}$$

```
[Out] (-4*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) +
c^3*Sqrt[1 - 1/(a^2*x^2)]*x - (13*c^3*ArcCsc[a*x])/(2*a) - (4*c^3*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.3312, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 1807, 1809, 844, 216, 266, 63, 208}

$$c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{13c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```



```
[Out] (-4*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) +
c^3*Sqrt[1 - 1/(a^2*x^2)]*x - (13*c^3*ArcCsc[a*x])/(2*a) - (4*c^3*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{\frac{4c^4}{a}-\frac{6c^4x}{a^2}+\frac{4c^4x^2}{a^3}-\frac{c^4x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{8c^4}{a^3}+\frac{13c^4x}{a^4}-\frac{8c^4x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^4\,\text{Subst}\left(\int \frac{\frac{8c^4}{a^5}-\frac{13c^4x}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{(13c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2a^2} + \frac{(4c \ldots}{\ldots}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{13c^3\csc^{-1}(ax)}{2a} + \frac{(2c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\ldots}}\ldots\right)}{a}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{13c^3\csc^{-1}(ax)}{2a} - (4ac^3)\,\text{Subst}\left(\int \frac{\ldots}{a^2 \ldots}\ldots\right)$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{13c^3\csc^{-1}(ax)}{2a} - \frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2 \ldots}}\ldots\right)}{a}$$

Mathematica [A] time = 0.121667, size = 167, normalized size = 1.58

$$\frac{c^3\left(2a^4x^4-8a^3x^3-a^2x^2+10a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-\ldots}\right.\ldots\right)}{2a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(-1 + 8*a*x - a^2*x^2 - 8*a^3*x^3 + 2*a^4*x^4 + 10*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Maple [B] time = 0.135, size = 266, normalized size = 2.5

$$\frac{(ax+1)\,c^3}{2\,x^2a^3}\sqrt{\frac{ax-1}{ax+1}}\left(-8\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+8\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa-13\,\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+8\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3*(-8*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x
^3*a^3+8*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-13*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)
*x^2*a^2+8*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-13
*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+16*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(1/2)*x^2*a^2-16*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2
)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^3
/x^2/(a^2)^(1/2)
```

Maxima [B] time = 1.57952, size = 271, normalized size = 2.56

$$\left(\frac{13\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] (13*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 4*c^3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^2 + 4*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (11*c^3*
((a*x - 1)/(a*x + 1))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqr
t((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1
)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Fricas [A] time = 2.02462, size = 332, normalized size = 3.13

$$\frac{26\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^3x^3-6\,a^2c^3x^2-7\,ac^3x+c^3\right)\cdot}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(26*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 8*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) + (2*a^3*c^3*x^3 - 6*a^2*c^3*x^2 - 7*a*c^3*x + c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^3\left(\int a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\,dx+\int \frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int -\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**3*(Integral(a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**3, x) + Integral(3*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**2, x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x
, x))/a**3
```

Giac [B] time = 1.23225, size = 313, normalized size = 2.95

$$\frac{13\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{4\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 13*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 4*c^3*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
3*sgn(a*x + 1)/a - ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn(a*x + 1
) + 8*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - (x*abs(a) - sqr
t(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 8*a*c^3*sgn(a*x + 1))/(((x*abs(a)
 - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a))
```

3.415 $\int e^{-\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^2 dx$

Optimal. Leaf size=77

$$c^2 x \sqrt{1 - \frac{1}{a^2 x^2}} - \frac{c^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{a} - \frac{3c^2 \tanh^{-1}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{a} - \frac{3c^2 \csc^{-1}(ax)}{a}$$

```
[Out] -((c^2*Sqrt[1 - 1/(a^2*x^2)])/a) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x - (3*c^2*Arc
Csc[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.238034, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 1807, 1809, 844, 216, 266, 63, 208}

$$c^2 x \sqrt{1 - \frac{1}{a^2 x^2}} - \frac{c^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{a} - \frac{3c^2 \tanh^{-1}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{a} - \frac{3c^2 \csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] -((c^2*Sqrt[1 - 1/(a^2*x^2)])/a) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x - (3*c^2*Arc
Csc[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2dx = -\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^3}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{c}$$

$$= c^2\sqrt{1-\frac{1}{a^2x^2}}x+\frac{\text{Subst}\left(\int\frac{\frac{3c^3}{a}-\frac{3c^3x}{a^2}+\frac{c^3x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{a^2\,\text{Subst}\left(\int\frac{-\frac{3c^3}{a^3}+\frac{3c^3x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{(3c^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{a^2}+\frac{(3c^2)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\ldots}}\right.}{a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c^2\csc^{-1}(ax)}{a}+\frac{(3c^2)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx,x,\frac{1}{x^2}\right)}{2a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c^2\csc^{-1}(ax)}{a}-(3ac^2)\,\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}dx,x,\sqrt{1-\ldots}\right.$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c^2\csc^{-1}(ax)}{a}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.197717, size = 55, normalized size = 0.71

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(ax-1)-3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-3\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] time = 0.131, size = 227, normalized size = 3.

$$\frac{(ax+1)\,c^2}{a^2x}\sqrt{\frac{ax-1}{ax+1}}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\sqrt{a^2x^2-1}xa+\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)xa^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2
+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*
x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-3*a*x*(a^2)^(1/2)*arcta
n(1/(a^2*x^2-1)^(1/2))+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-4*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-1)*(a*x+1)
)^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A] time = 1.56537, size = 170, normalized size = 2.21

$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A] time = 1.91077, size = 266, normalized size = 3.45

$$\frac{6\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2c^2x^2-c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) + 3*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c^2*
x^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-2*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x, x))/a**2
```

Giac [A] time = 1.20112, size = 176, normalized size = 2.29

$$\frac{6\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\,(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\mathrm{sgn}\,(ax+1)}{a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 6*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2
*sgn(a*x + 1)/a - 2*c^2*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1
)*abs(a))
```

3.416 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=49

$$cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a - (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rubi [A] time = 0.145407, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6177, 1807, 844, 216, 266, 63, 208}

$$cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a - (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\text{Subst}\left(\int \frac{\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} - (2ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= c\sqrt{1-\frac{1}{a^2x^2}}x - \frac{c\csc^{-1}(ax)}{a} - \frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.101654, size = 73, normalized size = 1.49

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}} - 2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) - 2\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right) - 2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] time = 0.128, size = 136, normalized size = 2.8

$$-\frac{c(ax+1)}{a}\sqrt{\frac{ax-1}{ax+1}}\left(\sqrt{a^2x^2-1}\sqrt{a^2} + \arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2} + 2a\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right) - 2\sqrt{a^2}\sqrt{(ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/
(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2))/(a^2)^(1/2))-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1)
)^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] time = 1.59328, size = 154, normalized size = 3.14

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 2.00251, size = 223, normalized size = 4.55

$$\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/x, x))/a
```

Giac [A] time = 1.18115, size = 115, normalized size = 2.35

$$\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}\left(ax+1\right)}{a}+\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 2*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a
```

3.417 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=19

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Rubi [A] time = 0.0397154, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6177, 264}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

Mathematica [A] time = 0.0669128, size = 19, normalized size = 1.

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Maple [A] time = 0.04, size = 28, normalized size = 1.5

$$\frac{ax+1}{ac}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x)
```

```
[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c
```

Maxima [B] time = 1.02521, size = 59, normalized size = 3.11

$$-\frac{2\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c)
```

Fricas [A] time = 1.83666, size = 58, normalized size = 3.05

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Giac [A] time = 1.17935, size = 32, normalized size = 1.68

$$\frac{\sqrt{a^2x^2-1}\,\mathrm{sgn}\,(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

3.418 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=73

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - (a*Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(a - x^(
-1))) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rubi [A] time = 0.11113, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 857, 807, 266, 63, 208}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - (a*Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(a - x^(
-1))) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
```

```
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.0348603, size = 69, normalized size = 0.95

$$\frac{a^2x^2 + ax\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) - ax - 2}{a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (-2 - a*x + a^2*x^2 + a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [B] time = 0.137, size = 256, normalized size = 3.5

$$\frac{ax+1}{2\,ac^{2}\left(ax-1\right)^{2}}\sqrt{\frac{ax-1}{ax+1}}\left(2\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}}{\sqrt{a^{2}}}\right)x^{2}a^{3}+3\,\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}x^{2}a^{2}-4\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{(ax}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2
*a^2-4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-((
a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a
+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+3*(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c^2/(a*x-1)^2/(a^2)^(1
/2)
```

Maxima [A] time = 1.04304, size = 162, normalized size = 2.22

$$-a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^{2}c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^{2}c^{2}\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c^{2}}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c^2*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^
2) + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A] time = 1.90645, size = 220, normalized size = 3.01

$$\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^{2}x^{2}-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{2}x-ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^2
*x - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \int \frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1
), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.419 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=105

$$-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] (-2*(a + x^(-1)))/(3*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (6*a + 7/x)/(3*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (2*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rubi [A] time = 0.29333, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (-2*(a + x^(-1)))/(3*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (6*a + 7/x)/(3*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (2*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^2-\frac{6c^2x}{a}-\frac{4c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^2+\frac{6c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A] time = 0.0636371, size = 94, normalized size = 0.9

$$\frac{3a^3x^3-11a^2x^2+6ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax+10}{3a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Maple [B] time = 0.138, size = 344, normalized size = 3.3

$$\frac{ax+1}{12\,ac^3\,(ax-1)^3}\sqrt{\frac{ax-1}{ax+1}}\left(24\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^3a^4+27\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^3a^3-72\,\ln\left(\frac{a^2x+\sqrt{a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/12*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*
x^3*a^3-72*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*
a^3-15*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-81*(a^2)^(1/2)*((a*x-1)*(a*x
+1))^(1/2)*x^2*a^2+72*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^
(1/2))*x*a^2+13*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+81*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x*a-24*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^
2)^(1/2))-27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c
^3/(a*x-1)^3/(a^2)^(1/2)
```

Maxima [A] time = 0.993697, size = 185, normalized size = 1.76

$$\frac{1}{6}\,a\left(\frac{\frac{14\,(ax-1)}{ax+1}-\frac{27\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*a*((14*(a*x - 1)/(a*x + 1) - 27*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 12*log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^3))
```

Fricas [A] time = 1.89322, size = 309, normalized size = 2.94

$$\frac{6\left(a^{2} x^{2}-2 a x+1\right) \log \left(\sqrt{\frac{a x-1}{a x+1}}+1\right)-6\left(a^{2} x^{2}-2 a x+1\right) \log \left(\sqrt{\frac{a x-1}{a x+1}}-1\right)+\left(3 a^{3} x^{3}-11 a^{2} x^{2}-4 a x+10\right) \sqrt{\frac{a x-1}{a x+1}}}{3\left(a^{3} c^{3} x^{2}-2 a^{2} c^{3} x+a c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 11*a^2*x^2
 - 4*a*x + 10)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^
3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{3} \int \frac{x^{3} \sqrt{\frac{a x}{a x+1}-\frac{1}{a x+1}}}{a^{3} x^{3}-3 a^{2} x^{2}+3 a x-1}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x*
*2 + 3*a*x - 1), x)/c**3
```

Giac [A] time = 1.19961, size = 80, normalized size = 0.76

$$-\frac{2 \log \left(\left|-x|a| + \sqrt{a^2x^2 - 1}\right|\right) \operatorname{sgn}(ax + 1)}{c^3|a|} + \frac{\sqrt{a^2x^2 - 1}\operatorname{sgn}(ax + 1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^3*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^3)
```

3.420 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=138

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (5*a + 7/x)/(5*a^2*
c^4*(1 - 1/(a^2*x^2))^(3/2)) - (15*a + 19/x)/(5*a^2*c^4*Sqrt[1 - 1/(a^2*x^2
)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^4 + (3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a
*c^4)
```

Rubi [A] time = 0.390016, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```


```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (5*a + 7/x)/(5*a^2*
c^4*(1 - 1/(a^2*x^2))^(3/2)) - (15*a + 19/x)/(5*a^2*c^4*Sqrt[1 - 1/(a^2*x^2
)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^4 + (3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a
*c^4)
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^3-\frac{15c^3x}{a}-\frac{16c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{5c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^3+\frac{45c^3x}{a}+\frac{42c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{15c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^3-\frac{45c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{15c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,\right.}{ac^4}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,\right.}{2ac^4}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,d\right.}{c^4}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] time = 0.0778986, size = 104, normalized size = 0.75

$$\frac{5a^4x^4 - 34a^3x^3 + 18a^2x^2 + 15ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) + 33ax - 24}{5a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-24 + 33*a*x + 18*a^2*x^2 - 34*a^3*x^3 + 5*a^4*x^4 + 15*a*Sqrt[1 - 1/(a^2*
x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(5*a^2*c^4*Sqrt[1 - 1/
(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] time = 0.148, size = 436, normalized size = 3.2

$$\frac{ax+1}{40\,ac^4\,(ax-1)^4}\sqrt{\frac{ax-1}{ax+1}}\left(120\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^4a^5+125\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^4a^4-480\,\ln\left(\frac{a^2x+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] 1/40*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(120*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^4*a^4-480*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x
^3*a^4-85*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-500*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2)*x^3*a^3+720*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
))/(a^2)^(1/2))*x^2*a^3+148*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+750*(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-480*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-67*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-
500*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+120*a*ln((a^2*x+(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))
/((a*x-1)*(a*x+1))^(1/2)/c^4/(a*x-1)^4/(a^2)^(1/2)
```

Maxima [A] time = 1.09118, size = 207, normalized size = 1.5

$$\frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A] time = 1.93405, size = 385, normalized size = 2.79

$$\frac{15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(5\,a^4x^4-34\,a^3x^3+18\,a^2\right.}{5\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x*
*3 + 6*a**2*x**2 - 4*a*x + 1), x)/c**4
```

Giac [A] time = 1.19832, size = 80, normalized size = 0.58

$$-\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^4|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^4*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^4)
```

3.421 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=65

$$-\frac{3c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}+\frac{16c^4}{a^2x}+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[x])/a - (32*c^4*Log[1 + a*x])/a
```

Rubi [A] time = 0.145399, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$-\frac{3c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}+\frac{16c^4}{a^2x}+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[x])/a - (32*c^4*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx\\ &= -\frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\ &= -\frac{c^4\int \frac{(1-ax)^5}{x^4(1+ax)}\,dx}{a^4}\\ &= -\frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{6a}{x^3}+\frac{16a^2}{x^2}-\frac{26a^3}{x}+\frac{32a^4}{1+ax}\right)dx}{a^4}\\ &= \frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.17904, size = 67, normalized size = 1.03

$$-\frac{3c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}+\frac{16c^4}{a^2x}+\frac{26c^4\log(ax)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[a*x])/a - (32*c^4*Log[1 + a*x])/a
```

Maple [A] time = 0.048, size = 64, normalized size = 1.

$$\frac{c^4}{3\,a^4x^3}-3\,\frac{c^4}{x^2a^3}+16\,\frac{c^4}{a^2x}+c^4x+26\,\frac{c^4\ln(x)}{a}-32\,\frac{c^4\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*c^4/a^4/x^3-3*c^4/x^2/a^3+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(a
*x+1)/a
```

Maxima [A] time = 1.06117, size = 81, normalized size = 1.25

$$c^4x - \frac{32\,c^4\log\left(ax+1\right)}{a} + \frac{26\,c^4\log\left(x\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 32*c^4*log(a*x + 1)/a + 26*c^4*log(x)/a + 1/3*(48*a^2*c^4*x^2 - 9*a
*c^4*x + c^4)/(a^4*x^3)
```

Fricas [A] time = 1.85412, size = 162, normalized size = 2.49

$$\frac{3\,a^4c^4x^4 - 96\,a^3c^4x^3\log\left(ax+1\right) + 78\,a^3c^4x^3\log\left(x\right) + 48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] time = 0.661916, size = 56, normalized size = 0.86

$$c^4x + \frac{2c^4\left(13\log\left(x\right) - 16\log\left(x + \frac{1}{a}\right)\right)}{a} + \frac{48a^2c^4x^2 - 9ac^4x + c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] c**4*x + 2*c**4*(13*log(x) - 16*log(x + 1/a))/a + (48*a**2*c**4*x**2 - 9*a*
c**4*x + c**4)/(3*a**4*x**3)
```

Giac [A] time = 1.13839, size = 84, normalized size = 1.29

$$c^4x - \frac{32\,c^4\log\left(|ax+1|\right)}{a} + \frac{26\,c^4\log\left(|x|\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 32*c^4*log(abs(a*x + 1))/a + 26*c^4*log(abs(x))/a + 1/3*(48*a^2*c^4
*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

3.422 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=54

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

```
[Out] -c^3/(2*a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[x])/a - (16*c^3*Lo
g[1 + a*x])/a
```

Rubi [A] time = 0.141078, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -c^3/(2*a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[x])/a - (16*c^3*Lo
g[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx\\ &= \frac{c^3\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\ &= \frac{c^3\int\frac{(1-ax)^4}{x^3(1+ax)}\,dx}{a^3}\\ &= \frac{c^3\int\left(a^3+\frac{1}{x^3}-\frac{5a}{x^2}+\frac{11a^2}{x}-\frac{16a^3}{1+ax}\right)dx}{a^3}\\ &= -\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.128238, size = 56, normalized size = 1.04

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(ax)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -c^3/(2*a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[a*x])/a - (16*c^3*
Log[1 + a*x])/a
```

Maple [A] time = 0.046, size = 53, normalized size = 1.

$$-\frac{c^3}{2\,x^2a^3}+5\,\frac{c^3}{a^2x}+c^3x+11\,\frac{c^3\ln(x)}{a}-16\,\frac{c^3\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*c^3/x^2/a^3+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

Maxima [A] time = 1.09595, size = 69, normalized size = 1.28

$$c^3x - \frac{16\,c^3\log\left(ax+1\right)}{a} + \frac{11\,c^3\log\left(x\right)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 16*c^3*log(a*x + 1)/a + 11*c^3*log(x)/a + 1/2*(10*a*c^3*x - c^3)/(a
^3*x^2)
```

Fricas [A] time = 1.84189, size = 140, normalized size = 2.59

$$\frac{2\,a^3c^3x^3 - 32\,a^2c^3x^2\log\left(ax+1\right) + 22\,a^2c^3x^2\log\left(x\right) + 10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
10*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] time = 0.56804, size = 42, normalized size = 0.78

$$c^3x + \frac{c^3\left(11\log\left(x\right) - 16\log\left(x + \frac{1}{a}\right)\right)}{a} + \frac{10ac^3x - c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*x + c**3*(11*log(x) - 16*log(x + 1/a))/a + (10*a*c**3*x - c**3)/(2*a**
3*x**2)
```

Giac [A] time = 1.17912, size = 72, normalized size = 1.33

$$c^3x - \frac{16\,c^3\log\left(|ax+1|\right)}{a} + \frac{11\,c^3\log\left(|x|\right)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 16*c^3*log(abs(a*x + 1))/a + 11*c^3*log(abs(x))/a + 1/2*(10*a*c^3*x
 - c^3)/(a^3*x^2)
```

3.423 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=40

$$\frac{c^2}{a^2x}+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

```
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[x])/a - (8*c^2*Log[1 + a*x])/a
```

Rubi [A] time = 0.134509, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{c^2}{a^2x}+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[x])/a - (8*c^2*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx\\ &= -\frac{c^2\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}\\ &= -\frac{c^2\int\frac{(1-ax)^3}{x^2(1+ax)}dx}{a^2}\\ &= -\frac{c^2\int\left(-a^2+\frac{1}{x^2}-\frac{4a}{x}+\frac{8a^2}{1+ax}\right)dx}{a^2}\\ &= \frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0998416, size = 42, normalized size = 1.05

$$\frac{c^2}{a^2x}+\frac{4c^2\log(ax)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[a*x])/a - (8*c^2*Log[1 + a*x])/a
```

Maple [A] time = 0.046, size = 41, normalized size = 1.

$$\frac{c^2}{a^2x}+xc^2+4\,\frac{c^2\ln(x)}{a}-8\,\frac{c^2\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)*(a*x-1),x)
```

```
[Out] c^2/a^2/x+x*c^2+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

Maxima [A] time = 1.04258, size = 54, normalized size = 1.35

$$c^2x - \frac{8\,c^2\log(ax+1)}{a} + \frac{4\,c^2\log(x)}{a} + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^2*x - 8*c^2*log(a*x + 1)/a + 4*c^2*log(x)/a + c^2/(a^2*x)
```

Fricas [A] time = 1.77615, size = 99, normalized size = 2.48

$$\frac{a^2c^2x^2 - 8\,ac^2x\log(ax+1) + 4\,ac^2x\log(x) + c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] (a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

Sympy [A] time = 0.470257, size = 31, normalized size = 0.78

$$c^2x + \frac{4c^2\left(\log(x) - 2\log\left(x + \frac{1}{a}\right)\right)}{a} + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*(a*x-1)/(a*x+1),x)
```

```
[Out] c**2*x + 4*c**2*(log(x) - 2*log(x + 1/a))/a + c**2/(a**2*x)
```

Giac [A] time = 1.19342, size = 57, normalized size = 1.42

$$c^2x - \frac{8\,c^2\log\left(|ax+1|\right)}{a} + \frac{4\,c^2\log\left(|x|\right)}{a} + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^2*x - 8*c^2*log(abs(a*x + 1))/a + 4*c^2*log(abs(x))/a + c^2/(a^2*x)
```

3.424 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=23

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

```
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Rubi [A] time = 0.0822374, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6131, 6129, 72}

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx\\ &= \frac{c\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\ &= \frac{c\int\frac{(1-ax)^2}{x(1+ax)}\,dx}{a}\\ &= \frac{c\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a}\\ &= cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0388513, size = 23, normalized size = 1.

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Maple [A] time = 0.045, size = 24, normalized size = 1.

$$cx+\frac{c\ln(x)}{a}-4\,\frac{c\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)*(a*x-1),x)
```

```
[Out] c*x+c*ln(x)/a-4*c*ln(a*x+1)/a
```

Maxima [A] time = 1.02825, size = 31, normalized size = 1.35

$$cx - \frac{4\,c\log\left(ax+1\right)}{a} + \frac{c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c*x - 4*c*log(a*x + 1)/a + c*log(x)/a
```

Fricas [A] time = 1.85841, size = 55, normalized size = 2.39

$$\frac{acx - 4\,c\log\left(ax+1\right) + c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] (a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

Sympy [A] time = 0.381285, size = 17, normalized size = 0.74

$$cx + \frac{c\left(\log\left(x\right) - 4\log\left(x + \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x)
```

```
[Out] c*x + c*(log(x) - 4*log(x + 1/a))/a
```

Giac [A] time = 1.13548, size = 34, normalized size = 1.48

$$cx - \frac{4\,c\log\left(|ax+1|\right)}{a} + \frac{c\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c*x - 4*c*log(abs(a*x + 1))/a + c*log(abs(x))/a
```

3.425 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=20

$$\frac{x}{c} - \frac{\log(ax+1)}{ac}$$

```
[Out] x/c - Log[1 + a*x]/(a*c)
```

Rubi [A] time = 0.11509, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 43}

$$\frac{x}{c} - \frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] x/c - Log[1 + a*x]/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx \\
&= \frac{a\int \frac{e^{-2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c} \\
&= \frac{a\int \frac{x}{1+ax}\,dx}{c} \\
&= \frac{a\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{c} \\
&= \frac{x}{c}-\frac{\log(1+ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0175695, size = 22, normalized size = 1.1

$$\frac{a\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (a*(x/a - Log[1 + a*x]/a^2))/c
```

Maple [A] time = 0.037, size = 21, normalized size = 1.1

$$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x),x)
```

```
[Out] x/c-ln(a*x+1)/a/c
```

Maxima [A] time = 1.02752, size = 27, normalized size = 1.35

$$\frac{x}{c} - \frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] x/c - log(a*x + 1)/(a*c)
```

Fricas [A] time = 1.76688, size = 38, normalized size = 1.9

$$\frac{ax - \log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*x - log(a*x + 1))/(a*c)
```

Sympy [A] time = 0.269226, size = 17, normalized size = 0.85

$$a\left(\frac{x}{ac} - \frac{\log(ax+1)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x)
```

```
[Out] a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

Giac [A] time = 1.13968, size = 28, normalized size = 1.4

$$\frac{x}{c} - \frac{\log(|ax+1|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] x/c - log(abs(a*x + 1))/(a*c)
```

3.426 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Rubi [A] time = 0.136995, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6167, 6131, 6129, 72, 207}

$$\frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]

[Out] x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx\\
&= -\frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= -\frac{a^2\int \frac{x^2}{(1-ax)(1+ax)}\,dx}{c^2}\\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{1}{a^2\left(-1+a^2x^2\right)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2}\\
&= \frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}
\end{aligned}
$$

Mathematica [B] time = 0.0825677, size = 39, normalized size = 2.17

$$\frac{\log(1-ax)}{2ac^2}-\frac{\log(ax+1)}{2ac^2}+\frac{x}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] x/c^2 + Log[1 - a*x]/(2*a*c^2) - Log[1 + a*x]/(2*a*c^2)
```

Maple [A] time = 0.046, size = 35, normalized size = 1.9

$$\frac{x}{c^2} - \frac{\ln(ax+1)}{2ac^2} + \frac{\ln(ax-1)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^2,x)
```

```
[Out] x/c^2-1/2*ln(a*x+1)/a/c^2+1/2/a/c^2*ln(a*x-1)
```

Maxima [A] time = 1.01405, size = 46, normalized size = 2.56

$$\frac{x}{c^2} - \frac{\log(ax+1)}{2ac^2} + \frac{\log(ax-1)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] x/c^2 - 1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.80369, size = 69, normalized size = 3.83

$$\frac{2ax - \log(ax+1) + \log(ax-1)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

Sympy [B] time = 0.328685, size = 34, normalized size = 1.89

$$a^2\left(\frac{x}{a^2c^2} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{2} - \frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

Giac [B] time = 1.16819, size = 49, normalized size = 2.72

$$\frac{x}{c^2} - \frac{\log(|ax+1|)}{2\,ac^2} + \frac{\log(|ax-1|)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 1/2*log(abs(a*x + 1))/(a*c^2) + 1/2*log(abs(a*x - 1))/(a*c^2)
```

3.427 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=57

$$\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/(
4*a*c^3)
```

Rubi [A] time = 0.150284, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/(
4*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx \\
&= \frac{a^3\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3} \\
&= \frac{a^3\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c^3} \\
&= \frac{a^3\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c^3} \\
&= \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] time = 0.110389, size = 56, normalized size = 0.98

$$-\frac{1}{2ac^3(ax-1)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] x/c^3 - 1/(2*a*c^3*(-1 + a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/
(4*a*c^3)
```

Maple [A] time = 0.044, size = 50, normalized size = 0.9

$$\frac{x}{c^3}-\frac{\ln(ax+1)}{4\,ac^3}-\frac{1}{2\,ac^3\,(ax-1)}+\frac{5\,\ln(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^3,x)
```

```
[Out] x/c^3-1/4*ln(a*x+1)/a/c^3-1/2/a/c^3/(a*x-1)+5/4/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.03942, size = 72, normalized size = 1.26

$$-\frac{1}{2\left(a^{2}c^{3}x-ac^{3}\right)}+\frac{x}{c^{3}}-\frac{\log\left(ax+1\right)}{4\,ac^{3}}+\frac{5\,\log\left(ax-1\right)}{4\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 5/4*log(a*x -
 1)/(a*c^3)
```

Fricas [A] time = 1.80426, size = 136, normalized size = 2.39

$$\frac{4\,a^{2}x^{2}-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2}{4\left(a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)/(a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.505827, size = 56, normalized size = 0.98

$$a^{3}\left(-\frac{1}{2a^{5}c^{3}x-2a^{4}c^{3}}+\frac{x}{a^{3}c^{3}}+\frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{4}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
- log(x + 1/a)/4)/(a**4*c**3))
```

Giac [A] time = 1.15894, size = 69, normalized size = 1.21

$$\frac{x}{c^3} - \frac{\log(|ax+1|)}{4\,ac^3} + \frac{5\,\log(|ax-1|)}{4\,ac^3} - \frac{1}{2\,(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 5/4*log(abs(a*x - 1))/(a*c^3) - 1/2
/((a*x - 1)*a*c^3)
```

3.428 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=75

$$\frac{7}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4 - 1/(4*a*c^4*(1 - a*x)^2) + 7/(4*a*c^4*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Rubi [A] time = 0.162013, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{7}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] x/c^4 - 1/(4*a*c^4*(1 - a*x)^2) + 7/(4*a*c^4*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx\\
&= -\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^4}\\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^4}\\
&= \frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}\end{aligned}$$

Mathematica [A] time = 0.146722, size = 73, normalized size = 0.97

$$-\frac{7}{4ac^4(ax-1)}-\frac{1}{4ac^4(ax-1)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] x/c^4 - 1/(4*a*c^4*(-1 + a*x)^2) - 7/(4*a*c^4*(-1 + a*x)) + (17*Log[1 - a*x
])/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Maple [A] time = 0.046, size = 65, normalized size = 0.9

$$\frac{x}{c^4}-\frac{\ln(ax+1)}{8\,ac^4}-\frac{1}{4\,ac^4\,(ax-1)^2}-\frac{7}{4\,ac^4\,(ax-1)}+\frac{17\,\ln(ax-1)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^4,x)
```

```
[Out] x/c^4-1/8*ln(a*x+1)/a/c^4-1/4/a/c^4/(a*x-1)^2-7/4/a/c^4/(a*x-1)+17/8/a/c^4*
ln(a*x-1)
```

Maxima [A] time = 1.03637, size = 93, normalized size = 1.24

$$-\frac{7\,ax-6}{4\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{\log\left(ax+1\right)}{8\,ac^4}+\frac{17\,\log\left(ax-1\right)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 1/8*log(a*x
+ 1)/(a*c^4) + 17/8*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.67704, size = 211, normalized size = 2.81

$$\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*
c^4)
```

Sympy [A] time = 0.614067, size = 73, normalized size = 0.97

$$a^4\left(-\frac{7ax-6}{4a^7c^4x^2-8a^6c^4x+4a^5c^4}+\frac{x}{a^4c^4}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(-(7*a*x - 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a*
*4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

Giac [A] time = 1.15018, size = 77, normalized size = 1.03

$$\frac{x}{c^4} - \frac{\log\left(|ax+1|\right)}{8\,ac^4} + \frac{17\,\log\left(|ax-1|\right)}{8\,ac^4} - \frac{7\,ax-6}{4\,(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 1/8*log(abs(a*x + 1))/(a*c^4) + 17/8*log(abs(a*x - 1))/(a*c^4) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^4)
```

3.429 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=164

$$\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^4x\sqrt{1-\frac{1}{a^2x^2}}+\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{91c^4\csc^{-1}(a$$

```
[Out] (68*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) + (64*c^4*(a - x^(-1)))/(a^2*Sqrt[1 -
1/(a^2*x^2)]) + (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*x^2) - (7*c^4*Sqrt[1 - 1
/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*x + (91*c^4*ArcCsc[a*x])
/(2*a) - (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.552419, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6177, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^4x\sqrt{1-\frac{1}{a^2x^2}}+\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{91c^4\csc^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (68*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) + (64*c^4*(a - x^(-1)))/(a^2*Sqrt[1 -
1/(a^2*x^2)]) + (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*x^2) - (7*c^4*Sqrt[1 - 1
/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*x + (91*c^4*ArcCsc[a*x])
/(2*a) - (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^7+\frac{7c^7x}{a}+\frac{42c^7x^2}{a^2}-\frac{22c^7x^3}{a^3}+\frac{7c^7x^4}{a^4}-\frac{c^7x^5}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{\text{Subst}\left(\int \frac{-\frac{7c^7}{a}-\frac{42c^7x}{a^2}+\frac{22c^7x^2}{a^3}-\frac{7c^7x^3}{a^4}+\frac{c^7x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^2\,\text{Subst}\left(\int \frac{\frac{21c^7}{a^3}+\frac{126c^7x}{a^4}-\frac{68c^7x^2}{a^5}+\frac{21c^7x^3}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x\right.}{3c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^4\,\text{Subst}\left(\int \frac{-\frac{42c^7}{a^5}-\frac{273c^7x}{a^6}}{x\sqrt{1-}}\right.}{6c^3}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^6\,\text{Su}}{}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{(91c^4)}{}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{91c^4}{}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{91c^4}{}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}x + \frac{91c^4}{}$$

Mathematica [C] time = 1.09876, size = 567, normalized size = 3.46

$$c^4\left(1980\sqrt{2}a^2x^2(ax+1)(ax-1)^4\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{2},\frac{9}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+2772\sqrt{2}a^3x^3(ax+1)(ax-1)^3\text{Hypergeom}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(2772*Sqrt[2]*a^3*x^3*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/
2, 7/2, (1 - 1/(a*x))/2] + 1980*Sqrt[2]*a^2*x^2*(-1 + a*x)^4*(1 + a*x)*Hype
rgeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] + 35*(-198*a^2*Sqrt[1 + 1/(a*
x)]*x^2 + 1716*a^3*Sqrt[1 + 1/(a*x)]*x^3 - 7425*a^4*Sqrt[1 + 1/(a*x)]*x^4 +
 26268*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 29403*a^6*Sqrt[1 + 1/(a*x)]*x^6 - 50160*
a^7*Sqrt[1 + 1/(a*x)]*x^7 + 396*a^8*Sqrt[1 + 1/(a*x)]*x^8 + 66726*a^6*Sqrt[
1 - 1/(a*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 66726*a^7*Sqrt[1 - 1/(
a*x)]*x^7*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1980*a^6*Sqrt[1 - 1/(a*x)]*x^
6*ArcSin[1/(a*x)] - 1980*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[1/(a*x)] - 2772*a
^7*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]] + 44*Sqrt[2]*a*x*(-1 + a*x)^5*(1 + a*x)*Hypergeometric2F1[3/2, 9/2, 11/2
, (1 - 1/(a*x))/2] + 36*Sqrt[2]*(-1 + a*x)^6*(1 + a*x)*Hypergeometric2F1[3/
2, 11/2, 13/2, (1 - 1/(a*x))/2])))/(13860*a^7*Sqrt[1 - 1/(a*x)]*x^6*(1 + a*
x))
```

Maple [B] time = 0.139, size = 672, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(-138*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x^6*a^6+138*(a^2)^(1/2)*(a^2*x^2-1
)^(3/2)*x^4*a^4-549*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+138*ln((a^2*x+(a^
2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-273*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2))*x^5*a^5+96*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-96
*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+255*(a
^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-684*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a
^4+276*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-546*a^
```

```
4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+192*(a^2)^(1/2)*((a*x-1)*(a*x
+1))^(3/2)*x^3*a^3+192*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-192*ln((
a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+98*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-273*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+138
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-273*a^3*x^3*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+96*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x^3*a^3-96*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*
x^3*a^4-17*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
))/a^4*c^4*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x^3/((a*x-1)*(a*x+1))^(1/2)/
(a*x-1)
```

Maxima [A] time = 1.56589, size = 332, normalized size = 2.02

$$-\frac{1}{3}\left(\frac{273\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{192\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{153\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+91\,c^4}{\frac{2\,(ax-1)a^2}{ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(273*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 21*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 21*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
192*c^4*sqrt((a*x - 1)/(a*x + 1))/a^2 + (153*c^4*((a*x - 1)/(a*x + 1))^(7/2
) + 91*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 169*c^4*((a*x - 1)/(a*x + 1))^(3/2
) - 123*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x
- 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a
```

Fricas [A] time = 1.95801, size = 369, normalized size = 2.25

$$-\frac{546\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+526\,a^3c^4x^3+115\,\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(546*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 42*a^3*c^4*x^3*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
```

```
1)) - 1) - (6*a^4*c^4*x^4 + 526*a^3*c^4*x^3 + 115*a^2*c^4*x^2 - 19*a*c^4*x
 + 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] undef
```

3.430 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=135

$$\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{6c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{33c^3\csc^{-1}(ax)}{2a}$$

```
[Out] (6*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^
2*x^2)]) - (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*Sqrt[1 - 1/(a^2*x^2)
]*x + (33*c^3*ArcCsc[a*x])/(2*a) - (6*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.429622, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6177, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{6c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{33c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (6*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^
2*x^2)]) - (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*Sqrt[1 - 1/(a^2*x^2)
]*x + (33*c^3*ArcCsc[a*x])/(2*a) - (6*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^6+\frac{6c^6x}{a}+\frac{16c^6x^2}{a^2}-\frac{6c^6x^3}{a^3}+\frac{c^6x^4}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{\text{Subst}\left(\int \frac{-\frac{6c^6}{a}-\frac{16c^6x}{a^2}+\frac{6c^6x^2}{a^3}-\frac{c^6x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{a^2\,\text{Subst}\left(\int \frac{\frac{12c^6}{a^3}+\frac{33c^6x}{a^4}-\frac{12c^6x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x - \frac{a^4\,\text{Subst}\left(\int \frac{-\frac{12c^6}{a^5}-\frac{33c^6x}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.}{2c^3}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{\left(33c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,d\right.}{2a^2}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{33c^3\csc^{-1}(ax)}{2a} + \frac{\left(3c^3\right)\text{S}}{}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{33c^3\csc^{-1}(ax)}{2a} - \left(6ac^3\right)\text{S}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}x + \frac{33c^3\csc^{-1}(ax)}{2a} - \frac{6c^3\tan}{}$$

Mathematica [C] time = 0.462017, size = 663, normalized size = 4.91

$$c^3\left(70\sqrt{2}a^6x^6\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{2},\frac{11}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)-280\sqrt{2}a^5x^5\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{2},\frac{11}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+35\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(420*a^2*Sqrt[1 + 1/(a*x)]*x^2 - 3465*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 1680
0*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 17955*a^5*Sqrt[1 + 1/(a*x)]*x^5 - 32340*a^6*S
qrt[1 + 1/(a*x)]*x^6 + 630*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 44730*a^5*Sqrt[1 - 1
/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 44730*a^6*Sqrt[1 - 1/(a*x)]
*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2520*a^5*Sqrt[1 - 1/(a*x)]*x^5*Arc
Sin[1/(a*x)] - 2520*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 3780*a^6*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] +
126*Sqrt[2]*a^2*x^2*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/2, 7/2,
 (1 - 1/(a*x))/2] + 90*Sqrt[2]*a*x*(-1 + a*x)^4*(1 + a*x)*Hypergeometric2F1
[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] - 70*Sqrt[2]*Hypergeometric2F1[3/2, 9/2, 1
1/2, (1 - 1/(a*x))/2] + 280*Sqrt[2]*a*x*Hypergeometric2F1[3/2, 9/2, 11/2, (
1 - 1/(a*x))/2] - 350*Sqrt[2]*a^2*x^2*Hypergeometric2F1[3/2, 9/2, 11/2, (1
- 1/(a*x))/2] + 350*Sqrt[2]*a^4*x^4*Hypergeometric2F1[3/2, 9/2, 11/2, (1 -
1/(a*x))/2] - 280*Sqrt[2]*a^5*x^5*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/
(a*x))/2] + 70*Sqrt[2]*a^6*x^6*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/(a*
x))/2]))/(630*a^6*Sqrt[1 - 1/(a*x)]*x^5*(1 + a*x))
```

Maple [B] time = 0.134, size = 450, normalized size = 3.3

$$-\frac{c^3}{2x^2a^3(ax-1)}\left(-12\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5+12\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3-57\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+12\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*(-12*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+12*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)*x^3*a^3-57*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+12*ln((a^2*x+(a^2*x^
2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-33*a^4*x^4*(a^2)^(1/2)*arctan(
1/(a^2*x^2-1)^(1/2))+23*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-78*(a^2*x^2-1
```

```
)^(1/2)*(a^2)^(1/2)*x^3*a^3+24*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^
2)^(1/2))*x^3*a^4-66*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+32*(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+10*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x
*a-33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+12*ln((a^2*x+(a^2*x^2-1)^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-33*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-
1)^(1/2))-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/a^3*c^3*((a*x-1)/(a*x+1))^(3/2)/(a
^2)^(1/2)/x^2/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.58562, size = 304, normalized size = 2.25

$$-\left(\frac{33\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{32\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-1}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3}{(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(33*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 6*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 - 6*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 32*c^3*
sqrt((a*x - 1)/(a*x + 1))/a^2 + (11*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 6*c^3
*((a*x - 1)/(a*x + 1))^(3/2) - 13*c^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)
*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3
+ a^2))*a
```

Fricas [A] time = 1.95096, size = 339, normalized size = 2.51

$$-\frac{66\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^3x^3+78\,a^2c^3x^2+11\,ac^3x-\right.}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(66*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2*c^3*x^2*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c^3)*sqrt((a*x -
```

```
1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.431 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=105

$$\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{5c^2\csc^{-1}(ax)}{a}$$

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/a + (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*
x^2)]) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x + (5*c^2*ArcCsc[a*x])/a - (5*c^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.325673, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6177, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{5c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/a + (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*
x^2)]) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x + (5*c^2*ArcCsc[a*x])/a - (5*c^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^5+\frac{5c^5x}{a}+\frac{5c^5x^2}{a^2}-\frac{c^5x^3}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{5c^5}{a}-\frac{5c^5x}{a^2}+\frac{c^5x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\,\text{Subst}\left(\int \frac{\frac{5c^5}{a^3}+\frac{5c^5x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{\left(5c^2\right)\cdots}{\cdots}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\cdots\right.}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} - \left(5ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a\cdots}\right.$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} - \frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] time = 0.388522, size = 424, normalized size = 4.04

$$c^2\left(7\sqrt{2}ax(ax-1)^3(ax+1)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+5\sqrt{2}(ax-1)^4(ax+1)\text{Hypergeometric2F1}\left(\frac{3}{2},\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-35*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 315*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 280*a
^4*Sqrt[1 + 1/(a*x)]*x^4 - 595*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 35*a^6*Sqrt[1 +
1/(a*x)]*x^6 + 910*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[
2]] + 910*a^5*Sqrt[1 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 105
*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[1/(a*x)] - 105*a^5*Sqrt[1 - 1/(a*x)]*x^5*
ArcSin[1/(a*x)] - 175*a^5*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^5*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]] + 7*Sqrt[2]*a*x*(-1 + a*x)^3*(1 + a*x)*Hypergeome
tric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 5*Sqrt[2]*(-1 + a*x)^4*(1 + a*x)*
Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2]))/(35*a^5*Sqrt[1 - 1/(a*x
)]*x^4*(1 + a*x))
```

Maple [B] time = 0.137, size = 600, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*
a^2-7*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2))/(a^2)^(1/2))*x^3*a^4-5*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(
1/2))-4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+4*ln((a^2*x+(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+2*(a^2)^(1/2)*(a^2*x^2-1)^(3
/2)*x*a-11*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+2*ln((a^2*x+(a^2*x^2-1)^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-10*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*
x^2-1)^(1/2))+8*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-8*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2)*x^2*a^2+8*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))
/(a^2)^(1/2))*x^2*a^3+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-5*(a^2)^(1/2)*(a^2*x^2-
1)^(1/2)*x*a+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-5*
```

```
a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-4*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(1/2)*x*a+4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*
a^2)/a^2*c^2*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x/((a*x-1)*(a*x+1))^(1/2)/
(a*x-1)
```

Maxima [A] time = 1.54951, size = 201, normalized size = 1.91

$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{10\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{16\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 10*
c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 5*c^2*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 - 5*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 16*c^2*sqrt(
(a*x - 1)/(a*x + 1))/a^2)*a
```

Fricas [A] time = 1.86976, size = 286, normalized size = 2.72

$$-\frac{10\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2c^2x^2+18\,ac^2x+c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(10*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 5*a*c^2*x*log(sqrt((a*x -
1)/(a*x + 1)) + 1) - 5*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^
2*x^2 + 18*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.432 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=75

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + c*Sqrt[1 - 1/(a^2*x^2)]*x
+ (c*ArcCsc[a*x])/a - (4*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rubi [A] time = 0.224189, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6177, 1805, 1807, 844, 216, 266, 63, 208}

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + c*Sqrt[1 - 1/(a^2*x^2)]*x
+ (c*ArcCsc[a*x])/a - (4*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx=-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{c^3}$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int\frac{-c^4+\frac{4c^4x}{a}+\frac{c^4x^2}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{c^3}$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{\text{Subst}\left(\int\frac{-\frac{4c^4}{a}-\frac{c^4x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{c^3}$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{a^2}+\frac{(4c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x\right.}{a}$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}+\frac{(2c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx,x,\frac{1}{x^2}\right)}{a}$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}-(4ac)\,\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right.$$

$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] time = 0.461815, size = 234, normalized size = 3.12

$$\frac{\sqrt{2}c(ax+1)(ax-1)^3\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+5a^2cx^2\left((ax+1)\left(\sqrt{\frac{1}{ax}+1}\left(a^2x^2-3ax+2\right)+6ax\sqrt{1\ldots}\right.\right.}{5a^4x^3\sqrt{1-\frac{1}{ax}}(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*a^2*c*x^2*((1 + a*x)*(Sqrt[1 + 1/(a*x)]*(2 - 3*a*x + a^2*x^2) + 6*a*Sqrt
[1 - 1/(a*x)]*x*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*a*Sqrt[1 - 1/(a*x)]*x
*ArcSin[1/(a*x)]) - 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]]) + Sqrt[2]*c*(-1 + a*x)^3*(1 + a*x)*Hypergeometri
c2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2])/(5*a^4*Sqrt[1 - 1/(a*x)]*x^3*(1 + a*x
))
```

Maple [B] time = 0.127, size = 376, normalized size = 5.

$$-\frac{c}{(ax-1)\,a}\left(4\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^2a^3-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2-a^2x^2\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-4\sqrt{a^2}\sqrt{(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -(4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^
2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)
^(1/2))-4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+8*ln((a^2*x+(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(1
/2)*x*a-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+4*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)-8*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+4*a*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2))/a*c*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x
-1)
```

Maxima [A] time = 1.55299, size = 182, normalized size = 2.43

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
```

```
^2 - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 4*c*sqrt((a*x - 1)/(a*x +
 1))/a^2)
```

Fricas [A] time = 1.98663, size = 227, normalized size = 3.03

$$-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(acx+9\,c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 4*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a*c*x + 9*c)*sqrt((a*x -
1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{a^2x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-
2*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.433 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=72

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}-\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/
c - (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rubi [A] time = 0.173347, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 1805, 807, 266, 63, 208}

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}-\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/
c - (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^2+\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.0934256, size = 61, normalized size = 0.85

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(ax+3)-2(ax+1)\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a(acx+c)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(3 + a*x) - 2*(1 + a*x)*Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x])/(a*(c + a*c*x))
```

Maple [B] time = 0.132, size = 250, normalized size = 3.5

$$-\frac{1}{ac\left(ax-1\right)}\left(2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}}{\sqrt{a^{2}}}\right)x^{2}a^{3}-2\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}x^{2}a^{2}+4\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+\right.}}{\sqrt{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x)
```

```
[Out] -(2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-2*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-4*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2))/(a^2)^(1/2))-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-
1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.03693, size = 162, normalized size = 2.25

$$-2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^{2}c}{ax+1}-a^{2}c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Fricas [A] time = 1.86504, size = 166, normalized size = 2.31

$$\frac{(ax+3)\sqrt{\frac{ax-1}{ax+1}}-2\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] ((a*x + 3)*sqrt((a*x - 1)/(a*x + 1)) - 2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 2*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int -\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx+\int \frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(-x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Inte
gral(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] undef
```

3.434 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=74

$$-\frac{x\left(a-\frac{1}{x}\right)}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - ((a - x^(-1))*x)/(a*c^2*Sqrt[1 - 1/(a^2*x
^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rubi [A] time = 0.101428, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 823, 807, 266, 63, 208}

$$-\frac{x\left(a-\frac{1}{x}\right)}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - ((a - x^(-1))*x)/(a*c^2*Sqrt[1 - 1/(a^2*x
^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{c-\frac{cx}{a}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= -\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a^2\,\text{Subst}\left(\int \frac{\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.0372498, size = 69, normalized size = 0.93

$$\frac{a^2x^2 - ax\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) + ax - 2}{a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (-2 + a*x + a^2*x^2 - a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [B] time = 0.133, size = 250, normalized size = 3.4

$$-\frac{1}{2\,ac^{2}\left(ax-1\right)}\left(2\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}}{\sqrt{a^{2}}}\right)x^{2}a^{3}-3\,\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}x^{2}a^{2}+4\,\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(a\right.}}{\sqrt{a^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x)
```

```
[Out] -1/2*(2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3
-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+4*ln((a^2*x+(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((
a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c^2/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.03599, size = 169, normalized size = 2.28

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^{2}c^{2}}{ax+1}-a^{2}c^{2}}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c^{2}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c^{2}}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^2) + l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^2) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Fricas [A] time = 1.93056, size = 163, normalized size = 2.2

$$\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.435 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=45

$$\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -2/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x) + x/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.0494055, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6177, 271, 191}

$$\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -2/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x) + x/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^2c^3}$$

$$= -\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0220276, size = 33, normalized size = 0.73

$$\frac{a^2x^2-2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] (-2 + a^2*x^2)/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.04, size = 44, normalized size = 1.

$$\frac{\left(a^2x^2-2\right)(ax+1)}{a\,(ax-1)^2\,c^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/a*((a*x-1)/(a*x+1))^(3/2)*(a^2*x^2-2)*(a*x+1)/(a*x-1)^2/c^3
```

Maxima [B] time = 1.02256, size = 124, normalized size = 2.76

$$-\frac{1}{2}a\left(\frac{\frac{5(ax-1)}{ax+1}-1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((5*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Fricas [A] time = 1.77781, size = 82, normalized size = 1.82

$$\frac{(a^2x^2-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3x-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*x - a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22081, size = 51, normalized size = 1.13

$$\frac{\left(\sqrt{a^2x^2-1}-\frac{1}{\sqrt{a^2x^2-1}}\right)\operatorname{sgn}(ax+1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] (sqrt(a^2*x^2 - 1) - 1/sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/(a*c^3)
```

3.436 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=111

$$-\frac{x\left(4a+\frac{3}{x}\right)}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] (8*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*c^4) - (a*x)/(3*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a
 - x^(-1))) - ((4*a + 3/x)*x)/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) + ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]/(a*c^4)
```

Rubi [A] time = 0.150347, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 857, 823, 807, 266, 63, 208}

$$-\frac{x\left(4a+\frac{3}{x}\right)}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*c^4) - (a*x)/(3*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a
 - x^(-1))) - ((4*a + 3/x)*x)/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) + ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]/(a*c^4)
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
```

```
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{a^2\,\text{Subst}\left(\int \frac{-\frac{4c}{a^2}-\frac{3cx}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a^4\,\text{Subst}\left(\int \frac{-\frac{8c}{a^4}-\frac{3cx}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] time = 0.0665271, size = 94, normalized size = 0.85

$$\frac{3a^3x^3-7a^2x^2+3ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-5ax+8}{3a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x))
```

Maple [B] time = 0.141, size = 523, normalized size = 4.7

$$\frac{1}{24\,ac^4\,(ax-1)^4}\left(24\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^5a^6+45\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^5a^5-24\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] 1/24*(24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^
6+45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-24*ln((a^2*x+(a^2)^(1/2)*(
(a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-21*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(3/2)*x^3*a^3-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-48*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-11*(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*
a^3+48*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+
5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2)*x^2*a^2+24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)
)*x*a^2+19*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+45*(a^2)^(1/2)*((a*x-1)*(a*x
+1))^(1/2)*x*a-24*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1
/2))-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2
)^(1/2)/c^4/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^4
```

Maxima [A] time = 1.02483, size = 216, normalized size = 1.95

$$\frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^4*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^4) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4))
```

Fricas [A] time = 1.82364, size = 306, normalized size = 2.76

$$\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4, x)
```

3.437 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

Optimal. Leaf size=138

$$-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^5}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

```
[Out] (-2*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(5/2)) - (10*a + 13/x)/(15*a
^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) - (30*a + 41/x)/(15*a^2*c^5*Sqrt[1 - 1/(a^2
*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^5 + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
)/(a*c^5)
```

Rubi [A] time = 0.389577, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^5}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```


```
[Out] (-2*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(5/2)) - (10*a + 13/x)/(15*a
^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) - (30*a + 41/x)/(15*a^2*c^5*Sqrt[1 - 1/(a^2
*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^5 + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
)/(a*c^5)
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^2-\frac{10c^2x}{a}-\frac{8c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^2+\frac{30c^2x}{a}+\frac{26c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^2-\frac{30c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.}{ac^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\right.}{ac^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a}\right.}{}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} + \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2}}\right.}{ac^5}$$

Mathematica [A] time = 0.0808256, size = 104, normalized size = 0.75

$$\frac{15a^4x^4 - 76a^3x^3 + 32a^2x^2 + 30ax\sqrt{1 - \frac{1}{a^2x^2}}(ax-1)^2\tanh^{-1}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right) + 82ax - 56}{15a^2c^5x\sqrt{1 - \frac{1}{a^2x^2}}(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*a*Sqrt[1 - 1/(a^2
*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^5*Sqrt[1 -
1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] time = 0.144, size = 615, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x)
```

```
[Out] 1/30*(60*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^
7+75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a^6-120*ln((a^2*x+(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6-45*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^4*a^4-150*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-60*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-2*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3-75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4
*a^4+240*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^
4+64*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+300*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2)*x^3*a^3-60*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^
2)^(1/2))*x^2*a^3+14*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-75*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-120*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2))*x*a^2-37*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-150*(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+60*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))+75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1
)/(a*x+1))^(3/2)/(a^2)^(1/2)/c^5/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^5
```

Maxima [A] time = 1.05843, size = 238, normalized size = 1.72

$$\frac{1}{120}\,a\left(\frac{\frac{32\,(ax-1)}{ax+1}+\frac{310\,(ax-1)^2}{(ax+1)^2}-\frac{585\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^5}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^5}+\frac{15\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] 1/120*a*((32*(a*x - 1)/(a*x + 1) + 310*(a*x - 1)^2/(a*x + 1)^2 - 585*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^5*((a*x
 - 1)/(a*x + 1))^(5/2)) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^5)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^5) + 15*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*c^5))
```

Fricas [A] time = 1.84735, size = 387, normalized size = 2.8

$$\frac{30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^4x^4-76\,a^3x^3+32\right.}{15\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 76*a^3*x^3 + 32*a^2*x^2 + 82*a*x - 56)*sqrt((a*x - 1)/(a*
x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**5,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^5, x)
```

3.438 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=235

$$\frac{173c^5\sqrt{1-\frac{1}{a^2x^2}}}{105a\sqrt{c-\frac{c}{ax}}}+\frac{227c^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{105a}+\frac{59c^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35a}+\frac{9c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7a}-\frac{7c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{}}{\sqrt{}}\right)}{a}$$

```
[Out] (173*c^5*Sqrt[1 - 1/(a^2*x^2)])/(105*a*Sqrt[c - c/(a*x)]) + (227*c^4*Sqrt[1
 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/(105*a) + (59*c^3*Sqrt[1 - 1/(a^2*x^2)]*
(c - c/(a*x))^(3/2))/(35*a) + (9*c^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5
/2))/(7*a) + c*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(7/2)*x - (7*c^(9/2)*Arc
Tanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.163788, antiderivative size = 279, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 97, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^4\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(400a-\frac{227}{x}\right)\sqrt{\frac{1}{ax}}}{105a^2\left(1-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] ((400*a - 227/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(105*a^2*(1 - 1/(a*
x))^(9/2)) + (59*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(35*
a^3*(1 - 1/(a*x))^(9/2)) + (9*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x)
)^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c
 - c/(a*x))^(9/2)*x)/(a^4*(1 - 1/(a*x))^(9/2)) - (7*(c - c/(a*x))^(9/2)*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(-\frac{7}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\right.}{7\left(1-\right.}$$

$$=\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\right.}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(\right.}{7a^4\left(1-\frac{1}{ax}\right)^{9}}$$

$$=\frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(\right.}{7a^4\left(1-\frac{1}{ax}\right)^{9}}$$

$$=\frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(\right.}{7a^4\left(1-\frac{1}{ax}\right)^{9}}$$

Mathematica [A] time = 0.114718, size = 109, normalized size = 0.46

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(105a^4x^4+292a^3x^3-356a^2x^2+162ax-30\right)-735a^3x^3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{105a^4x^3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-30 + 162*a*x - 356*a^2*x^2 + 29
2*a^3*x^3 + 105*a^4*x^4) - 735*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(105*a^
4*Sqrt[1 - 1/(a*x)]*x^3)
```

Maple [A] time = 0.178, size = 166, normalized size = 0.7

$$-\frac{c^4}{210\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-210\,a^{9/2}\sqrt{(ax+1)\,x}x^4+735\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)x^4a^4-584\,a^{7/2}x^3\sqrt{(ax+1)\,x}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] -1/210/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-210*
a^(9/2)*((a*x+1)*x)^(1/2)*x^4+735*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*x^4*a^4-584*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+712*a^(5/2)*x^2*((a*
x+1)*x)^(1/2)-324*a^(3/2)*x*((a*x+1)*x)^(1/2)+60*((a*x+1)*x)^(1/2)*a^(1/2))
/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.19396, size = 926, normalized size = 3.94

$$\left[\frac{735\left(a^4c^4x^4 - a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right) + 4\left(105a^5c^4x^5 + 397a^4c^4x^4 - 64a^3c^4x^3 - \right.}{420\left(a^5x^4 - a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(105*a^5*c^4*x^5 + 397*a^4*c^4*x^4 -
64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x - 30*c^4)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/210*(735*(a^4*c^4*x
^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(105*a^5
*c^4*x^5 + 397*a^4*c^4*x^4 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x
 - 30*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^
4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")

[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.439 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=196

$$\frac{49c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}}+\frac{31c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a}+\frac{7c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}-\frac{5c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+cx\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right.$$

```
[Out] (49*c^4*Sqrt[1 - 1/(a^2*x^2)])/(15*a*Sqrt[c - c/(a*x)]) + (31*c^3*Sqrt[1 -
1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/(15*a) + (7*c^2*Sqrt[1 - 1/(a^2*x^2)]*(c -
c/(a*x))^(3/2))/(5*a) + c*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5/2)*x - (5*
c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.148126, antiderivative size = 221, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 97, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}\left(\right.}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] ((80*a - 31/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2*(1 - 1/(a*x))
^(7/2)) + (7*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(
1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)
*x)/(a^3*(1 - 1/(a*x))^(7/2)) - (5*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{Subst}\left(\int\frac{\left(-\frac{5}{2a}-\frac{7x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int\right.}{5\left(1-\right.}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{a}\right.}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{a}\right.}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{a}\right.}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] time = 0.0818589, size = 101, normalized size = 0.52

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(15a^3x^3+56a^2x^2-28ax+6\right)-75a^2x^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{15a^3x^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^3
*x^3) - 75*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

Maple [A] time = 0.18, size = 149, normalized size = 0.8

$$-\frac{c^3}{30\,x^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-30\,a^{7/2}x^3\sqrt{(ax+1)\,x}+75\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)x^3a^3-112\,a^{5/2}x^2\sqrt{(ax+1)\,x}+56$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] -1/30/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-30*a^
(7/2)*x^3*((a*x+1)*x)^(1/2)+75*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)
/a^(1/2))*x^3*a^3-112*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+56*a^(3/2)*x*((a*x+1)*x
)^(1/2)-12*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.2213, size = 861, normalized size = 4.39

$$\left[\frac{75\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15a^4c^3x^4+71a^3c^3x^3+28a^2c^3x^2-22a\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 28*a
^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 +
 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.440 $\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{5/2} dx$

Optimal. Leaf size=157

$$\frac{c^4 x \left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}-\frac{3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (-2*c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a*(c - c/(a*x))^(3/2)) + (3*c^3*Sqrt[1
- 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^4*(1 - 1/(a^2*x^2))^(3/2)*x)/(c
- c/(a*x))^(3/2) - (3*c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[
c - c/(a*x)]])/a
```

Rubi [A] time = 0.132817, antiderivative size = 189, normalized size of antiderivative = 1.2, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 89, 80, 50, 63, 208}

$$-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(a*(1 - 1/(a*x))^(5/2)) - (2*(1 +
 1/(a*x))^(3/2)*(c - c/(a*x))^(5/2))/(3*a*(1 - 1/(a*x))^(5/2)) + ((1 + 1/(a
*x))^(3/2)*(c - c/(a*x))^(5/2)*x)/(1 - 1/(a*x))^(5/2) - (3*(c - c/(a*x))^(5
/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(-\frac{3}{2a}+\frac{x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x}dx,x\right.}{2a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right.}{2}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right.}{}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{5/2}\text{t}}{a\left(1\right.}$$

Mathematica [A] time = 0.0736338, size = 89, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+10ax-2\right)-9ax\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{3a^2x\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A] time = 0.18, size = 132, normalized size = 0.8

$$\frac{c^2}{6x}\sqrt{\frac{c(ax-1)}{ax}}\left(6a^{5/2}x^2\sqrt{(ax+1)x}+20a^{3/2}x\sqrt{(ax+1)x}-9\ln\left(1/2\,\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right)x^2a^2-4\sqrt{(ax+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^2*((a*x+
1)*x)^(1/2)+20*a^(3/2)*x*((a*x+1)*x)^(1/2)-9*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2-4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(5/2)/((a*
x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.18703, size = 790, normalized size = 5.03

$$\left[\frac{9\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3+13a^2c^2x^2+8ac^2x-2c^2\right)\sqrt{\frac{ax-}{ax+}}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x
- 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*
x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 2*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x - 2*c^2)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.441 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2))^
(3/2)*x)/(c - c/(a*x))^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.223204, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6177, 879, 865, 875, 208}

$$\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2))^
(3/2)*x)/(c - c/(a*x))^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{c^3\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.0518382, size = 70, normalized size = 0.6

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}(ax+2)-\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + a*x) - ArcTanh[Sqrt[1 + 1/(a*x
)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.178, size = 105, normalized size = 0.9

$$-\frac{c}{2}\sqrt{\frac{c(ax-1)}{ax}}\left(-2a^{3/2}x\sqrt{(ax+1)x}+\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)x}\sqrt{a}+2ax+1\right)\frac{1}{\sqrt{a}}\right)xa-4\sqrt{(ax+1)x}\sqrt{a}\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}a^{-\frac{3}{2}}\frac{1}{\sqrt{(ax+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-2*a^(3/2)*x*
((a*x+1)*x)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a
-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.21834, size = 667, normalized size = 5.7

$$\left[\frac{(acx-c)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2+3acx+2c\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\frac{(acx-c)\sqrt{-c}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*((a*c*x - c)*sqrt(-c)*arctan(2*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*
c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.442 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=78

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.155213, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6177, 863, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```


```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
```

```
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{c^2\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0436105, size = 66, normalized size = 0.85

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax+\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+1\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.17, size = 87, normalized size = 1.1

$$\frac{x}{2}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax+1)x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)x}\sqrt{a}+2ax+1\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax+1)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [B] time = 2.16779, size = 630, normalized size = 8.08

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.443 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - c/(a*x)] + (3*Sqrt[1 - 1/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) - (2*Sqrt[2]*Sqrt[1
 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c - c/(a*x)])
```

Rubi [A] time = 0.134815, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 99, 156, 63, 208, 206}

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```


```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - c/(a*x)] + (3*Sqrt[1 - 1/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) - (2*Sqrt[2]*Sqrt[1
 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c - c/(a*x)])
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-\frac{1}{ax}}\,\mathrm{Subst}\left(\int \frac{\frac{3}{2a}+\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x,\right.}{2a\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x,\right.}{a\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A] time = 0.0647422, size = 93, normalized size = 0.61

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}+3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 3*ArcTanh[Sqrt[1 + 1/(a*x)]] -
2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[c - c/(a*x)])
```

Maple [A] time = 0.171, size = 151, normalized size = 1.

$$\frac{x}{2c}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax+1)x}a^{3/2}\sqrt{a^{-1}}-2\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)x}a+3ax+1}{ax-1}\right)\sqrt{a}+3\ln\left(1/2\frac{2\sqrt{(ax+1)x}\sqrt{a}+2}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(3/2)*(1/a)^(1/2)-2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3
*a*x+1)/(a*x-1))*a^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a*(1/a)^(1/2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 2.70694, size = 1137, normalized size = 7.48

$$\left[\frac{3(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\frac{2\sqrt{2}(acx-c)\log\left(-\frac{17a^3x^3-3a^2x^2\cdots}{\cdots}\right.}{\cdots}}{4\left(a^2cx-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)) + 2*sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sq
rt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c*
x - a*c), 1/2*(2*sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 +
 a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x
^2 - 2*a*x - 1)) - 3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
 + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*c*x - a*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.444 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx$

Optimal. Leaf size=215

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*(c - c/(a*x))^(3/2
)) + (a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*x)
)^(3/2)) + (5*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*
x))^(3/2)) - (7*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sq
rt[2]*a*(c - c/(a*x))^(3/2))
```

Rubi [A] time = 0.1548, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6182, 6179, 99, 151, 156, 63, 208, 206}

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```


```
[Out] (-2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*(c - c/(a*x))^(3/2
)) + (a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*x)
)^(3/2)) + (5*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*
x))^(3/2)) - (7*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sq
rt[2]*a*(c - c/(a*x))^(3/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{-\frac{5}{a^2}-\frac{2x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}} - \cdots$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}} \cdots$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\left(1-\frac{1}{ax}\right)^{3/2} \text{t}\cdots}{\sqrt{2}a\left(c\cdots\right.}$$

Mathematica [A] time = 0.09067, size = 122, normalized size = 0.57

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}(ax-2)+10(ax-1)\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-7\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{2ac(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-2 + a*x) + 10*(-1 + a*x)*ArcT
anh[Sqrt[1 + 1/(a*x)]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqr
t[2]]))/(2*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] time = 0.187, size = 259, normalized size = 1.2

$$\frac{x}{(4\,ax-4)\,c^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x-7\,a^{3/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x+10\,\ln\left(1/2\,\frac{2\,\sqrt{(a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5/2)*(1/a
)^(1/2)*((a*x+1)*x)^(1/2)*x-7*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-8*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)-10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+7*
2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(
1/2))/a^(3/2)/c^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.445 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{79\left(1-\frac{1}{ax}\right.}{8\sqrt{}}$$

```
[Out] (-3*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))^2*(c - c/(a*x)
)^(5/2)) - (23*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(8*(a - x^(-1))*(c -
c/(a*x))^(5/2)) + (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1
))^2*(c - c/(a*x))^(5/2)) + (7*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)
]])/(a*(c - c/(a*x))^(5/2)) - (79*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]/Sqrt[2]])/(8*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rubi [A] time = 0.171197, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6182, 6179, 99, 151, 156, 63, 208, 206}

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{79\left(1-\frac{1}{ax}\right.}{8\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))^2*(c - c/(a*x)
)^(5/2)) - (23*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(8*(a - x^(-1))*(c -
c/(a*x))^(5/2)) + (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1
))^2*(c - c/(a*x))^(5/2)) + (7*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)
]])/(a*(c - c/(a*x))^(5/2)) - (79*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]/Sqrt[2]])/(8*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{-\frac{14}{a^2}-\frac{9x}{a^3}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Sub}}{8\,(c}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(79\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Sub}}{16a^2\,(}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subs}}{(c}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}}{a\left(c-\frac{c}{ax}\right)^{5/}}$$

Mathematica [A] time = 0.120052, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(8a^2x^2-35ax+23\right)+112(ax-1)^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-79\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{16ac^2(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(23 - 35*a*x + 8*a^2*x^2) + 112
*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 79*Sqrt[2]*(-1 + a*x)^2*ArcTanh[
Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(16*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A] time = 0.183, size = 366, normalized size = 1.3

$$\frac{x}{32\,(ax-1)^2 c^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(32\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^2-79\,a^{5/2}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x^2-140\,a^{5/2}\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/32/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(32*a^(7/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-79*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1
/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-140*a^(5/2)*(1/a)^(1/2)*((a*x
+1)*x)^(1/2)*x+112*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^
3*(1/a)^(1/2)*x^2+158*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)
^(1/2)*a+3*a*x+1)/(a*x-1))*x-224*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+
1)/a^(1/2))*a^2*(1/a)^(1/2)*x+92*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+112*
ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-79*2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)
)/a^(3/2)/c^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.446 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=143

$$\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+x\left(c-\frac{c}{ax}\right)^{9/2}$$

```
[Out] (5*c^4*Sqrt[c - c/(a*x)])/a + (5*c^3*(c - c/(a*x))^(3/2))/(3*a) + (c^2*(c -
 c/(a*x))^(5/2))/a + (5*c*(c - c/(a*x))^(7/2))/(7*a) + (c - c/(a*x))^(9/2)*
x - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.23846, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+x\left(c-\frac{c}{ax}\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (5*c^4*Sqrt[c - c/(a*x)])/a + (5*c^3*(c - c/(a*x))^(3/2))/(3*a) + (c^2*(c -
 c/(a*x))^(5/2))/a + (5*c*(c - c/(a*x))^(7/2))/(7*a) + (c - c/(a*x))^(9/2)*
x - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$$

$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}{1-ax}dx$$

$$= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{x}dx}{a}$$

$$= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}dx}{a}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}{x^2}dx, x, \frac{1}{x}\right)}{a}$$

$$= \left(c-\frac{c}{ax}\right)^{9/2}x + \frac{(5c)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2}x + \frac{(5c^2)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2}x + \frac{(5c^3)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2}x + \frac{(5c^4)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\right.}{2a}$$

$$= \frac{5c^4\sqrt{c-\frac{c}{ax}}}{a} + \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2}x + \frac{(5c^5)\,\text{S}}{}$$

$$= \frac{5c^4\sqrt{c-\frac{c}{ax}}}{a} + \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2}x - (5c^4)\,\text{Su}$$

$$= \frac{5c^4\sqrt{c-\frac{c}{ax}}}{a} + \frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a} + \frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a} + \left(c-\frac{c}{ax}\right)^{9/2}x - \frac{5c^{9/2}\,\text{ta}}{}$$

Mathematica [A] time = 0.150846, size = 91, normalized size = 0.64

$$\frac{c^4\left(21a^4x^4+92a^3x^3+4a^2x^2-18ax+6\right)\sqrt{c-\frac{c}{ax}}}{21a^4x^3}-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))/
(21*a^4*x^3) - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A] time = 0.164, size = 163, normalized size = 1.1

$$-\frac{c^4}{42\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-210\,a^{9/2}\sqrt{ax^2-x}x^5+105\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2-x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^5a^4+168\,a^{7/2}\left(ax^2-x\right)^{3/2}x^3-16\,a^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(9/2),x)
```

```
[Out] -1/42*(c*(a*x-1)/a/x)^(1/2)/x^4*c^4*(-210*a^(9/2)*(a*x^2-x)^(1/2)*x^5+105*l
n(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^4+168*a^(7/2)*(a*x
^2-x)^(3/2)*x^3-16*a^(5/2)*(a*x^2-x)^(3/2)*x^2-24*a^(3/2)*(a*x^2-x)^(3/2)*x
+12*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/(a*x - 1), x)
```

Fricas [A] time = 1.93089, size = 516, normalized size = 3.61

$$\left[\frac{105\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(21\,a^4c^4x^4+92\,a^3c^4x^3+4\,a^2c^4x^2-18\,ac^4x+6\,c^4\right)\sqrt{\frac{acx-c}{ax}}}{42\,a^4x^3}, \frac{105\,a^3\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x
 + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), 1/21*(105*a^3*sqrt(-c)*c^4*x^
3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3*c^4
*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^
3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.447 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2} dx$

Optimal. Leaf size=118

$$\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] (3*c^3*Sqrt[c - c/(a*x)])/a + (c^2*(c - c/(a*x))^(3/2))/a + (3*c*(c - c/(a*
x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a
*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.225019, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (3*c^3*Sqrt[c - c/(a*x)])/a + (c^2*(c - c/(a*x))^(3/2))/a + (3*c*(c - c/(a*
x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a
*x)]/Sqrt[c]])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx \\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{1-ax}dx \\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{x}dx}{a} \\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a} \\
&= -\frac{c\,\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)}{a} \\
&= \left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a} \\
&= \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a} \\
&= \frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(3c^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a} \\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(3c^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}d\right.}{2a} \\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\left(3c^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\right. \\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.116848, size = 83, normalized size = 0.7

$$
\frac{c^3\left(5a^3x^3+8a^2x^2+4ax-2\right)\sqrt{c-\frac{c}{ax}}}{5a^3x^2}-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) -
(3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A] time = 0.164, size = 144, normalized size = 1.2

$$-\frac{c^3}{10\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-30\,\sqrt{ax^2-x}a^{7/2}x^4+20\,a^{5/2}\left(ax^2-x\right)^{3/2}x^2+15\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2-x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^4a^3+4\,a^{3/2}\left(ax^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(7/2),x)
```

```
[Out] -1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^(
5/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*x^4*a^3+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1
)*x)^(1/2)/a^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/(a*x - 1), x)
```

Fricas [A] time = 1.87326, size = 460, normalized size = 3.9

$$\left[\frac{15\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^3c^3x^3+8\,a^2c^3x^2+4\,ac^3x-2\,c^3\right)\sqrt{\frac{acx-c}{ax}}}{10\,a^3x^2},\ \frac{15\,a^2\sqrt{-c}c^3x^2\arctan\left(\frac{\sqrt{-c}\sqrt{\ }}{\ }\right.}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*
x - c)/(a*x)))/(a^3*x^2), 1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3
)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

Sympy [C] time = 19.9582, size = 729, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqrt
(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (I
*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a)
, True)) + 2*c**4*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) + 2*c**3*sq
rt(c - c/(a*x))/a - c**3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)
/(3*c), True))/a**2 + c**3*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**
(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a*
*(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)
/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a
*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sq
rt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*
sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1),
 (-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2
)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/
2)) + 4*I*a**5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5
/2)*x**(5/2)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2)
- 15*a**(5/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x*
*(7/2) - 15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/
2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), True))/a**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.448 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] (c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x))^(
5/2)*x - (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.20145, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x))^(
5/2)*x - (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^2\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-c^2\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0648904, size = 75, normalized size = 0.79

$$\frac{c^2\left(3a^2x^2-2ax+2\right)\sqrt{c-\frac{c}{ax}}-3ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 2*a*x + 3*a^2*x^2) - 3*a*c^(5/2)*x*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

Maple [A] time = 0.164, size = 108, normalized size = 1.1

$$-\frac{c^2}{6x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(-6\sqrt{ax^2-x}a^{5/2}x^3+3\ln\left(1/2\frac{2\sqrt{ax^2-x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)x^3a^2+4\left(ax^2-x\right)^{3/2}\sqrt{a}\right)\frac{1}{\sqrt{(ax-1)x}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(5/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/2
*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^2+4*(a*x^2-x)^(3/2)*a^(
1/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/(a*x - 1), x)
```

Fricas [A] time = 1.86239, size = 397, normalized size = 4.18

$$\left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^2c^2x^2-2\,ac^2x+2\,c^2\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^2x},\ \frac{3\,a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+\left(3\,a^2c^2\right.}{3\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c) + 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)
, 1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (3*a
^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.449 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x + (c^(3/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.177587, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x + (c^(3/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{(a+x)\sqrt{c-\frac{cx}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+c\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.047223, size = 55, normalized size = 0.79

$$
\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+c(ax-2)\sqrt{c-\frac{c}{ax}}}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) + c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/a
```

Maple [A] time = 0.163, size = 103, normalized size = 1.5

$$\frac{c}{2x}\sqrt{\frac{c(ax-1)}{ax}}\left(-2\sqrt{ax^2-x}a^{3/2}x^2+4\left(ax^2-x\right)^{3/2}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{ax^2-x}\sqrt{a}+2ax-1\right)\frac{1}{\sqrt{a}}\right)x^2a\right)\frac{1}{\sqrt{(ax-1)x}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^(
3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((a
*x-1)*x)^(1/2)/a^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/(a*x - 1), x)
```

Fricas [A] time = 1.80755, size = 298, normalized size = 4.26

$$\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},-\frac{\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2
*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(sqrt(-c)*c*arctan(sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.450 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=50

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.158676, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 375, 78, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx \\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{1-ax}\,dx \\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a} \\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a} \\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \sqrt{c-\frac{c}{ax}}x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \sqrt{c-\frac{c}{ax}}x + 3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) \\
&= \sqrt{c-\frac{c}{ax}}x + \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0298758, size = 50, normalized size = 1.

$$
x\sqrt{c-\frac{c}{ax}} + \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] time = 0.166, size = 118, normalized size = 2.4

$$\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\sqrt{(ax-1)\,x}\sqrt{a}-2\sqrt{ax^{2}-x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{ax^{2}-x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)+2\,\ln\left(1/2\,\frac{2\sqrt{(ax-1)\,x}\sqrt{a}+2}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x-1)*x)^(1/2)*a^(1/2)-2*(a*x^2-x)^(1/2)*
a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))+2*ln(1/2*(2*((a
*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Fricas [A] time = 1.94038, size = 273, normalized size = 5.46

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c
```

```
)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [B] time = 1.29285, size = 131, normalized size = 2.62

$$\frac{3\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\operatorname{sgn}(x)}{2a}-\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)|a|+a\sqrt{c}\right|\right)}{2a\operatorname{sgn}(x)}+\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a*sgn(x)) + sqr
t(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

3.451 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=70

$$\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -5/(a*Sqrt[c - c/(a*x)]) + x/Sqrt[c - c/(a*x)] + (5*ArcTanh[Sqrt[c - c/(a*x
)]/Sqrt[c]])/(a*Sqrt[c])
```

Rubi [A] time = 0.179245, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```


```
[Out] -5/(a*Sqrt[c - c/(a*x)]) + x/Sqrt[c - c/(a*x)] + (5*ArcTanh[Sqrt[c - c/(a*x
)]/Sqrt[c]])/(a*Sqrt[c])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx\\
&= -\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}x}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{(5c)\,\mathrm{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}\\
&= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [C] time = 0.0303269, size = 43, normalized size = 0.61

$$\frac{ax - 5\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (a*x - 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*x)]
)
```

Maple [B] time = 0.177, size = 194, normalized size = 2.8

$$\frac{x}{2c(ax-1)^2}\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{5/2}\sqrt{(ax-1)x}x^2+5\ln\left(1/2\,\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)x^2a^2-8a^{3/2}((ax-1)x)^{3/2}-20\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(10*a^(5/2)*((a*x-1)*x)^(1/2)*x^2+5*ln(1/2*(2*(
(a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2-8*a^(3/2)*((a*x-1)*x)^(3
/2)-20*a^(3/2)*((a*x-1)*x)^(1/2)*x-10*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*x*a+10*((a*x-1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x-1)*x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c/(a*x-1)^2/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 1.91305, size = 377, normalized size = 5.39

$$\left[\frac{5\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(a^2x^2-5\,ax\right)\sqrt{\frac{acx-c}{ax}}}{2\left(a^2cx-ac\right)},-\frac{5\,(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-\left(a^2x^2-5\right.}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c), -(
5*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x^2
- 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)), x)
```

Giac [B] time = 1.30974, size = 167, normalized size = 2.39

$$-ac\left(\frac{5\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c}+\frac{4\,c-\frac{5\,(acx-c)}{ax}}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -a*c*(5*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c) + (4*c -
5*(a*c*x - c)/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)*sqrt((a*c*x
- c)/(a*x))/(a*x))*a^2*c))
```

3.452 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -7/(3*a*(c - c/(a*x))^(3/2)) - 7/(a*c*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^
(3/2) + (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rubi [A] time = 0.200235, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -7/(3*a*(c - c/(a*x))^(3/2)) - 7/(a*c*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^
(3/2) + (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{a}$$

$$= -\frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(7c)\,\mathrm{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [C] time = 0.0320579, size = 55, normalized size = 0.58

$$\frac{x\left(3ax-7\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{1}{ax}\right)\right)}{3c(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (x*(3*a*x - 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c -
 c/(a*x)]*(-1 + a*x))
```

Maple [B] time = 0.171, size = 260, normalized size = 2.7

$$-\frac{x}{6c^2(ax-1)^3}\sqrt{\frac{c(ax-1)}{ax}}\left(-42\sqrt{(ax-1)x}a^{7/2}x^3+36((ax-1)x)^{3/2}a^{5/2}x-21\ln\left(1/2\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)x^3a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)*x*(-42*((a*x-1)*x)^(1/2)*a^(7/2)*x^3+36*((a*x-1)
*x)^(3/2)*a^(5/2)*x-21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*x^3*a^3+126*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-28*a^(3/2)*((a*x-1)*x)^(3/2)+63
*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2-126*a^(3/2)*
((a*x-1)*x)^(1/2)*x-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*x*a+42*((a*x-1)*x)^(1/2)*a^(1/2)+21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^2/(a*x-1)^3/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 1.77956, size = 514, normalized size = 5.41

$$\left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},-\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c
)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(21*(a^2*x^2 - 2*a*x +
1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 28*a^
2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Giac [A] time = 1.31482, size = 193, normalized size = 2.03

$$-\frac{1}{3}\,ac\left(\frac{2\left(2\,c+\frac{9\,(acx-c)}{ax}\right)x}{(acx-c)ac^2\sqrt{\frac{acx-c}{ax}}}+\frac{21\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}-\frac{3\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*a*c*(2*(2*c + 9*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*c^2*sqrt((a*c*x -
c)/(a*x))) + 21*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2)
 - 3*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))
```

3.453 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] -9/(5*a*(c - c/(a*x))^(5/2)) - 3/(a*c*(c - c/(a*x))^(3/2)) - 9/(a*c^2*Sqrt[
c - c/(a*x)]) + x/(c - c/(a*x))^(5/2) + (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/(a*c^(5/2))
```

Rubi [A] time = 0.215616, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```


```
[Out] -9/(5*a*(c - c/(a*x))^(5/2)) - 3/(a*c*(c - c/(a*x))^(3/2)) - 9/(a*c^2*Sqrt[
c - c/(a*x)]) + x/(c - c/(a*x))^(5/2) + (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/(a*c^(5/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{a}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{(9c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Mathematica [C] time = 0.0353326, size = 58, normalized size = 0.49

$$\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] x/(c - c/(a*x))^(5/2) - (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)])/(
5*a*(c - c/(a*x))^(5/2))
```

Maple [B] time = 0.173, size = 328, normalized size = 2.8

$$-\frac{x}{10\,c^3\,(ax-1)^4}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-90\,\sqrt{(ax-1)\,x}a^{9/2}x^4+80\,((ax-1)\,x)^{3/2}\,a^{7/2}x^2-45\,\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/10*(c*(a*x-1)/a/x)^(1/2)*x*(-90*((a*x-1)*x)^(1/2)*a^(9/2)*x^4+80*((a*x-1
)*x)^(3/2)*a^(7/2)*x^2-45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*x^4*a^4+360*((a*x-1)*x)^(1/2)*a^(7/2)*x^3-132*((a*x-1)*x)^(3/2)*a^(5/2
)*x+180*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-540*a
^(5/2)*((a*x-1)*x)^(1/2)*x^2+60*a^(3/2)*((a*x-1)*x)^(3/2)-270*ln(1/2*(2*((a
*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2+360*a^(3/2)*((a*x-1)*x)^(1
/2)*x+180*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a-90*((a*
x-1)*x)^(1/2)*a^(1/2)-45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2)))/((a*x-1)*x)^(1/2)/c^3/(a*x-1)^4/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [A] time = 1.93625, size = 629, normalized size = 5.33

$$\left[\frac{45\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\sqrt{c}\log\left(-2\,acx - 2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}} + c\right) + 2\left(5\,a^4x^4 - 69\,a^3x^3 + 105\,a^2x^2 - 45\,ax\right)\sqrt{\frac{acx-c}{ax}}}{10\left(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3\right)}, -\frac{45 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^
2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c
^3*x - a*c^3), -1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^2
- 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax + 1}{\left(-c\left(-1 + \frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

Giac [A] time = 1.25614, size = 220, normalized size = 1.86

$$-\frac{1}{5}\,ac\left(\frac{2\left(2\,c^2+\frac{5\,(acx-c)c}{ax}+\frac{20\,(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c^3\sqrt{\frac{acx-c}{ax}}}+\frac{45\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}-\frac{5\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/5*a*c*(2*(2*c^2 + 5*(a*c*x - c)*c/(a*x) + 20*(a*c*x - c)^2/(a^2*x^2))*x^
2/((a*c*x - c)^2*c^3*sqrt((a*c*x - c)/(a*x))) + 45*arctan(sqrt((a*c*x - c)/
(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^3) - 5*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (
a*c*x - c)/(a*x))*c^3))
```

3.454 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] -11/(7*a*(c - c/(a*x))^(7/2)) - 11/(5*a*c*(c - c/(a*x))^(5/2)) - 11/(3*a*c^
2*(c - c/(a*x))^(3/2)) - 11/(a*c^3*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^(7/
2) + (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rubi [A] time = 0.229653, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] -11/(7*a*(c - c/(a*x))^(7/2)) - 11/(5*a*c*(c - c/(a*x))^(5/2)) - 11/(3*a*c^
2*(c - c/(a*x))^(3/2)) - 11/(a*c^3*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^(7/
2) + (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{9/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx}{a}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{(11c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\cdots}}\cdots\right)}{2ac^3}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{a-\frac{ax\cdots}{c}}\cdots\right)}{c\cdots}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\cdots\right.}{ac^{7/2}}$$

Mathematica [C] time = 0.0408748, size = 46, normalized size = 0.32

$$\frac{7x - \frac{11\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\frac{1}{ax}\right)}{a}}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] (7*x - (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*x
))^(7/2))
```

Maple [B] time = 0.174, size = 396, normalized size = 2.7

$$\frac{x}{210\,c^4\,(ax-1)^5}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2310\,a^{11/2}\sqrt{(ax-1)\,x}x^5+1155\,\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)x^5a^5-2100\,a^{9/2}\,((ax-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/210*(c*(a*x-1)/a/x)^(1/2)*x*(2310*a^(11/2)*((a*x-1)*x)^(1/2)*x^5+1155*ln(
1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^5-2100*a^(9/2)*((a
*x-1)*x)^(3/2)*x^3-11550*((a*x-1)*x)^(1/2)*a^(9/2)*x^4-5775*ln(1/2*(2*((a*x
-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^4+5368*((a*x-1)*x)^(3/2)*a^(7/
2)*x^2+23100*((a*x-1)*x)^(1/2)*a^(7/2)*x^3+11550*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-4928*((a*x-1)*x)^(3/2)*a^(5/2)*x-23100*
a^(5/2)*((a*x-1)*x)^(1/2)*x^2-11550*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a
*x-1)/a^(1/2))*x^2*a^2+1540*a^(3/2)*((a*x-1)*x)^(3/2)+11550*a^(3/2)*((a*x-1
)*x)^(1/2)*x+5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a
-2310*((a*x-1)*x)^(1/2)*a^(1/2)-1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^5/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(7/2)), x)
```

Fricas [A] time = 1.80492, size = 774, normalized size = 5.34

$$\left[\frac{1155\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(105\,a^5x^5-1936\,a^4x^4+4466\,a^3x^3-\right.}{210\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*a
*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(105*a^5*x^5 - 1936*a
^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(
a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/105*
(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sqrt(-c
)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3*x^3 -
 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*a^4*c^4
*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31902, size = 250, normalized size = 1.72

$$-\frac{1}{105}\,ac\left(\frac{2\left(30\,c^3+\frac{63\,(acx-c)c^2}{ax}+\frac{140\,(acx-c)^2c}{a^2x^2}+\frac{525\,(acx-c)^3}{a^3x^3}\right)ax^3}{(acx-c)^3c^4\sqrt{\frac{acx-c}{ax}}}+\frac{1155\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^4}-\frac{105\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/105*a*c*(2*(30*c^3 + 63*(a*c*x - c)*c^2/(a*x) + 140*(a*c*x - c)^2*c/(a^2
*x^2) + 525*(a*c*x - c)^3/(a^3*x^3))*a*x^3/((a*c*x - c)^3*c^4*sqrt((a*c*x -
 c)/(a*x))) + 1155*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c
^4) - 105*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^4))
```

3.455 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=268

$$\frac{9\left(a-\frac{1}{x}\right)^2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\,(}{a\left(1-\right.}$$

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(a*(1 - 1/(a*x))^(9/2)) + (3*(28*
a - 17/x)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(35*a^2*(1 - 1/(a*x))^(9
/2)) + (9*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(7*a^3*(1
 - 1/(a*x))^(9/2)) + ((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2
)*x)/(a^3*(1 - 1/(a*x))^(9/2)) - (3*(c - c/(a*x))^(9/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rubi [A] time = 0.163274, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 97, 153, 147, 50, 63, 208}

$$\frac{9\left(a-\frac{1}{x}\right)^2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\,(}{a\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```


```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(a*(1 - 1/(a*x))^(9/2)) + (3*(28*
a - 17/x)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(35*a^2*(1 - 1/(a*x))^(9
/2)) + (9*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(7*a^3*(1
 - 1/(a*x))^(9/2)) + ((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2
)*x)/(a^3*(1 - 1/(a*x))^(9/2)) - (3*(c - c/(a*x))^(9/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(-\frac{3}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Su}}{}$$

$$=\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right.}{a^3\left(1-\right.}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{a}\right.}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{a}\right.}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{a}\right.}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [A] time = 0.127988, size = 109, normalized size = 0.41

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(35a^4x^4+164a^3x^3-12a^2x^2-26ax+10\right)-105a^3x^3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{35a^4x^4\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(10 - 26*a*x - 12*a^2*x^2 + 164*a
^3*x^3 + 35*a^4*x^4) - 105*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(35*a^4*Sqr
t[1 - 1/(a*x)]*x^3)
```

Maple [A] time = 0.181, size = 178, normalized size = 0.7

$$-\frac{(ax-1)\,c^4}{(70\,ax+70)\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-70\,a^{9/2}\sqrt{(ax+1)\,x}x^4+105\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)x^4a^4-328\,a^{7/2}x^3\sqrt{(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] -1/70/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4
/a^(9/2)*(-70*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+105*ln(1/2*(2*((a*x+1)*x)^(1/2)
*a^(1/2)+2*a*x+1)/a^(1/2))*x^4*a^4-328*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+24*a^(
5/2)*x^2*((a*x+1)*x)^(1/2)+52*a^(3/2)*x*((a*x+1)*x)^(1/2)-20*((a*x+1)*x)^(1
/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.32026, size = 919, normalized size = 3.43

$$\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(35a^5c^4x^5+199a^4c^4x^4+152a^3c^4x^3-\right.}{140\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^5*c^4*x^5 + 199*a^4*c^4*x^4 + 1
52*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10*c^4)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/70*(105*(a^4*c^4*x^4
- a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(35*a^5*c^4
*x^5 + 199*a^4*c^4*x^4 + 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10
*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.456 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=237

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}(}{a\left(1-\frac{1}{ax}\right)^{7}}$$

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(
a*x))^(3/2)*(c - c/(a*x))^(7/2))/(3*a*(1 - 1/(a*x))^(7/2)) - (2*(1 + 1/(a*x
))^(5/2)*(c - c/(a*x))^(7/2))/(5*a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(a*x))^(5
/2)*(c - c/(a*x))^(7/2)*x)/(1 - 1/(a*x))^(7/2) - ((c - c/(a*x))^(7/2)*ArcTa
nh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rubi [A] time = 0.145512, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 89, 80, 50, 63, 208}

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}(}{a\left(1-\frac{1}{ax}\right)^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(
a*x))^(3/2)*(c - c/(a*x))^(7/2))/(3*a*(1 - 1/(a*x))^(7/2)) - (2*(1 + 1/(a*x
))^(5/2)*(c - c/(a*x))^(7/2))/(5*a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(a*x))^(5
/2)*(c - c/(a*x))^(7/2)*x)/(1 - 1/(a*x))^(7/2) - ((c - c/(a*x))^(7/2)*ArcTa
nh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(-\frac{1}{2a}+\frac{x}{a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\right.}{2a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}}{}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\right.}{\left(1-\frac{1}{ax}\right)^{7}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\right.}{\left(1-\frac{1}{ax}\right)^{7}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\right.}{\left(1-\frac{1}{ax}\right)^{7}}$$

Mathematica [A] time = 0.0885057, size = 101, normalized size = 0.43

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(15a^3x^3+44a^2x^2+8ax-6\right)-15a^2x^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{15a^3x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-6 + 8*a*x + 44*a^2*x^2 + 15*a^3
*x^3) - 15*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

Maple [A] time = 0.181, size = 161, normalized size = 0.7

$$-\frac{(ax-1)\,c^3}{(30\,ax+30)\,x^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-30\,a^{7/2}x^3\sqrt{(ax+1)\,x}+15\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)x^3a^3-88\,a^{5/2}x^2\sqrt{(ax+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] -1/30/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3
/a^(7/2)*(-30*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+15*ln(1/2*(2*((a*x+1)*x)^(1/2)*
a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3-88*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-16*a^(3/
2)*x*((a*x+1)*x)^(1/2)+12*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.228, size = 859, normalized size = 3.62

$$\left[\frac{15\left(a^3c^3x^3 - a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8\,a^3cx^3 - 7\,acx - 4\left(2\,a^3x^3 + 3\,a^2x^2 + ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\left(15\,a^4c^3x^4 + 59\,a^3c^3x^3 + 52\,a^2c^3x^2 + 2\,ac\right.}{60\left(a^4x^3 - a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 + 52*a
^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 +
52*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.457 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{c^5x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -(c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a*(c - c/(a*x))^(3/2)) - (c^3*Sqrt[1 - 1/
(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^5*(1 - 1/(a^2*x^2))^(5/2)*x)/(c - c/
(a*x))^(5/2) + (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/
(a*x)]])/a
```

Rubi [A] time = 0.268086, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 879, 865, 875, 208}

$$\frac{c^5x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -(c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a*(c - c/(a*x))^(3/2)) - (c^3*Sqrt[1 - 1/
(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^5*(1 - 1/(a^2*x^2))^(5/2)*x)/(c - c/
(a*x))^(5/2) + (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/
(a*x)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\left(c^3\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^3\,\mathrm{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^2\,\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\,\mathrm{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.0658166, size = 89, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+2ax+2\right)+3ax\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{3a^2x\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + 2*a*x + 3*a^2*x^2) + 3*a*x*A
rcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A] time = 0.18, size = 144, normalized size = 0.9

$$\frac{(ax-1)\,c^2}{(6\,ax+6)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\left(6\,a^{5/2}x^2\sqrt{(ax+1)\,x}+3\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)x^2a^2+4\,a^{3/2}x\sqrt{(ax+1)\,x}+4\,\sqrt{(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(
5/2)*(6*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*x^2*a^2+4*a^(3/2)*x*((a*x+1)*x)^(1/2)+4*((a*x+1)*x)^(1/2
)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.13802, size = 788, normalized size = 5.05

$$\left[\frac{3\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3\,a^3c^2x^3+5\,a^2c^2x^2+4\,ac^2x+2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x +
 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x
), -1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x + 2*c^2)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.458 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (-3*c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2
))^(3/2)*x)/(c - c/(a*x))^(3/2) + (3*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.214601, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 863, 865, 875, 208}

$$\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2
))^(3/2)*x)/(c - c/(a*x))^(3/2) + (3*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(3c^2)\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(3c)\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(3c^3)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.0440087, size = 66, normalized size = 0.56

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{ax}+1\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(1 + 1/(a*x))^(5/2)*(c - c/(a*x))^(3/2)*Hypergeometric2F1[2, 5/2, 7/2,
1 + 1/(a*x)])/(5*a*(1 - 1/(a*x))^(3/2))
```

Maple [A] time = 0.181, size = 118, normalized size = 1.

$$\frac{c(ax-1)}{2ax+2}\sqrt{\frac{c(ax-1)}{ax}}\left(2a^{3/2}x\sqrt{(ax+1)x}+3\ln\left(1/2\,\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right)xa-4\sqrt{(ax+1)x}\sqrt{a}\right)\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)
*(2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*x*a-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.24657, size = 668, normalized size = 5.66

$$\left[\frac{3(acx-c)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2-acx-2c\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},-\frac{3(acx-c)\sqrt{-c}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), -1/2*(3*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) - 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.459 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.133691, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 156, 63, 208, 206}

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\cdots}}\right.}{2a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{2-\cdots}\right.}{a\sqrt{1-\cdots}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0726192, size = 93, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}+5\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 5*ArcTanh[Sqrt[1 + 1/(a*x)]] -
4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.18, size = 160, normalized size = 1.1

$$\frac{(ax-1)\,x}{2\,ax+2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{(ax+1)\,x}a^{3/2}\sqrt{a^{-1}}-4\,\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)\sqrt{a}+5\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*
x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x
+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1
/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.460 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=215

$$\frac{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - c/(a*x)]) +
 (a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*Sqrt[c - c/(a*x)])
 + (7*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) -
 (5*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c
 - c/(a*x)])
```

Rubi [A] time = 0.150193, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 151, 156, 63, 208, 206}

$$\frac{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```


```
[Out] (-3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - c/(a*x)]) +
 (a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*Sqrt[c - c/(a*x)])
 + (7*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) -
 (5*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c
 - c/(a*x)])
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\mathrm{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,\mathrm{Subst}\left(\int \frac{-\frac{7}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(a\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{3x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(5\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} - \frac{\left(7\sqrt{1-\frac{1}{ax}}\right)\cdots}{\cdots}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(7\sqrt{1-\frac{1}{ax}}\right)\mathrm{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(10\sqrt{1-\cdots}\right.}{\cdots}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{1\cdots}}{\cdots}\right.}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A] time = 0.099784, size = 115, normalized size = 0.53

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}(ax-3)+7(ax-1)\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-5\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x*(-3 + a*x) + 7*(-1 + a*x)*ArcTanh
[Sqrt[1 + 1/(a*x)]] - 5*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2
]]))/(a*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] time = 0.181, size = 259, normalized size = 1.2

$$\frac{x}{(2\,ax+2)\,c}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x-5\,a^{3/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x+7\,\ln\left(1/2\,\frac{2\,\sqrt{(ax}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(5/2)*(1/a
)^(1/2)*((a*x+1)*x)^(1/2)*x-5*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+
2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-6*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)
-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+5*2^
(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/
2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c - \frac{c}{ax}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.461 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{51\left(1-\right.}{4}$$

```
[Out] (-2*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))^2*(c - c/(a*x))^
(3/2)) - (15*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))*(c - c/
(a*x))^(3/2)) + (a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))
^2*(c - c/(a*x))^(3/2)) + (9*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]]
)/(a*(c - c/(a*x))^(3/2)) - (51*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]/Sqrt[2]])/(4*Sqrt[2]*a*(c - c/(a*x))^(3/2))
```

Rubi [A] time = 0.171335, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 151, 156, 63, 208, 206}

$$\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{51\left(1-\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```


```
[Out] (-2*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))^2*(c - c/(a*x))^
(3/2)) - (15*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))*(c - c/
(a*x))^(3/2)) + (a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))
^2*(c - c/(a*x))^(3/2)) + (9*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]]
)/(a*(c - c/(a*x))^(3/2)) - (51*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]/Sqrt[2]])/(4*Sqrt[2]*a*(c - c/(a*x))^(3/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{\frac{18}{a^2}+\frac{12x}{a^3}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subs}\cdots}{8\left(c\cdots\right.}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(51\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subs}\cdots}{8a^2\left(c\cdots\right.}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(9\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\cdots}{\left(c\cdots\right.}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\cdots\right.}{a\left(c-\frac{c}{ax}\right)^{3/2}\cdots}$$

Mathematica [A] time = 0.12598, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(4a^2x^2-23ax+15\right)+72(ax-1)^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-51\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{8ac(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(15 - 23*a*x + 4*a^2*x^2) + 72*
(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh[S
qrt[1 + 1/(a*x)]/Sqrt[2]]))/(8*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A] time = 0.186, size = 373, normalized size = 1.4

$$\frac{x}{(16\,ax-16)\,(ax+1)\,c^{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(16\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^{2}-51\,a^{5/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x^{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^
(7/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-51*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1
/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-92*a^(5/2)*(1/a)^(1/2)*
((a*x+1)*x)^(1/2)*x+72*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2+102*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-144*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*
a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x+60*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+
72*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-51*2
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1
/2))/a^(3/2)/c^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 2.69249, size = 1455, normalized size = 5.29

$$\left[\frac{51\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+72\left(a^3x^3-3a^2x^2+3\right.}{32\left(a^4c^2x^3-3a^3c^2x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*a
^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^4*x^4 - 19*a^3
*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/16*(51*sqrt(2)*(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*
a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 4*(4*a^4*x^4 - 19*a^3*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2
 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.462 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$\frac{a^3x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\frac{1}{ax}\right)^{5/2}\text{ta}}{a\left(c-\right.}$$

```
[Out] (-5*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(3*(a - x^(-1))^3*(c - c/(a*
x))^(5/2)) - (29*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(12*(a - x^(-1))^
2*(c - c/(a*x))^(5/2)) - (73*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(16*(a
- x^(-1))*(c - c/(a*x))^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]
*x)/((a - x^(-1))^3*(c - c/(a*x))^(5/2)) + (11*(1 - 1/(a*x))^(5/2)*ArcTanh[
Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - (249*(1 - 1/(a*x))^(5/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rubi [A] time = 0.191238, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 151, 156, 63, 208, 206}

$$\frac{a^3x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\frac{1}{ax}\right)^{5/2}\text{ta}}{a\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```



```
[Out] (-5*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(3*(a - x^(-1))^3*(c - c/(a*
x))^(5/2)) - (29*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(12*(a - x^(-1))^
2*(c - c/(a*x))^(5/2)) - (73*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(16*(a
- x^(-1))*(c - c/(a*x))^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]
*x)/((a - x^(-1))^3*(c - c/(a*x))^(5/2)) + (11*(1 - 1/(a*x))^(5/2)*ArcTanh[
Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - (249*(1 - 1/(a*x))^(5/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{-\frac{11}{2a}-\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\frac{33}{a^2}+\frac{25x}{a^3}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right.}{6\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{S}}{2}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)}$$

Mathematica [A] time = 0.144821, size = 143, normalized size = 0.43

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(48a^3x^3-415a^2x^2+554ax-219\right)+1056(ax-1)^3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-747\sqrt{2}(ax-1)^3\tanh^{-1}\left(\right.\right.}{96ac^2(ax-1)^3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-219 + 554*a*x - 415*a^2*x^2 +
 48*a^3*x^3) + 1056*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]] - 747*Sqrt[2]*(
-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(96*a*c^2*Sqrt[c - c/(a*x)
]*(-1 + a*x)^3)
```

Maple [A] time = 0.191, size = 480, normalized size = 1.4

$$\frac{x}{192\,(ax-1)^2\,(ax+1)\,c^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(192\,a^{9/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^3-747\,a^{7/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(19
2*a^(9/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^3-747*a^(7/2)*2^(1/2)*ln((2*2^(1/
2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^3-1660*a^(7/2)*(1/a)
^(1/2)*((a*x+1)*x)^(1/2)*x^2+1056*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*a^4*(1/a)^(1/2)*x^3+2241*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+2216*a^(5/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*x-3168*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*a^3*(1/a)^(1/2)*x^2-2241*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+3168*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-876*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)-1056*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+
747*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2))/a^(3/2)/c^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 3.19411, size = 1625, normalized size = 4.85

$$\left[\frac{747\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+1056\left(a^4x\right.}{384\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*l
og(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x
^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 -
4*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 8*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219*a*x)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), 1/192*(747*sqrt(2)*(a^4*x^4 - 4
*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^
3*x^3 + 335*a^2*x^2 - 219*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.463 $\int e^{-\coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{7/2} dx$

Optimal. Leaf size=221

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}(}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -((80*a - 7/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^2*(1 - 1/(a*x))^
(7/2)) + (3*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(1
 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)*
x)/(a^3*(1 - 1/(a*x))^(7/2)) - (9*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rubi [A] time = 0.144436, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}(}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -((80*a - 7/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^2*(1 - 1/(a*x))^
(7/2)) + (3*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(1
 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)*
x)/(a^3*(1 - 1/(a*x))^(7/2)) - (9*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subs}}{5(1}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c}{a^3\left(1-\frac{1}{ax}\right)^{7}}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c}{a^3\left(1-\frac{1}{ax}\right)^{7}}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c}{a^3\left(1-\frac{1}{ax}\right)^{7}}$$

Mathematica [A] time = 0.102816, size = 101, normalized size = 0.46

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(5a^3x^3-92a^2x^2+16ax-2\right)-45a^2x^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{5a^3x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^3
*x^3) - 45*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(5*a^3*Sqrt[1 - 1/(a*x)]*x^
2)
```

Maple [A] time = 0.181, size = 161, normalized size = 0.7

$$\frac{(ax+1)c^3}{10x^2(ax-1)}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{7/2}x^3\sqrt{(ax+1)x}-184a^{5/2}x^2\sqrt{(ax+1)x}-45\ln\left(1/2\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/10*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(10*a^(7/2)*
x^3*((a*x+1)*x)^(1/2)-184*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-45*ln(1/2*(2*((a*x+
1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3+32*a^(3/2)*x*((a*x+1)*x)^(1/2
)-4*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.63215, size = 859, normalized size = 3.89

$$\left[\frac{45\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(5a^4c^3x^4-87a^3c^3x^3-76a^2c^3x^2+14a\right.}{20\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 76*a^
2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/10*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 7
6*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.464 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=161

$$\frac{x\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -((16*a + x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1 - 1/(a*x
))^(5/2)) + ((a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)*x)/(a^2*(
1 - 1/(a*x))^(5/2)) - (7*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a
*(1 - 1/(a*x))^(5/2))
```

Rubi [A] time = 0.124572, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6182, 6179, 98, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```



```
[Out] -((16*a + x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1 - 1/(a*x
))^(5/2)) + ((a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)*x)/(a^2*(
1 - 1/(a*x))^(5/2)) - (7*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a
*(1 - 1/(a*x))^(5/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{7}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(7\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subs}}{2a\left(1-\right.}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(7\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subs}}{(1}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}}{a\left(1-\frac{1}{ax}\right)^{5}}$$

Mathematica [A] time = 0.0643764, size = 89, normalized size = 0.55

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(3a^2x^2-22ax+2\right)-21ax\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{3a^2x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 - 22*a*x + 3*a^2*x^2) - 21*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A] time = 0.179, size = 144, normalized size = 0.9

$$\frac{(ax+1)\,c^2}{6\,(ax-1)\,x}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(6\,a^{5/2}x^2\sqrt{(ax+1)\,x}-44\,a^{3/2}x\sqrt{(ax+1)\,x}-21\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)-21*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2+4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(
5/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.52613, size = 795, normalized size = 4.94

$$\left[\frac{21\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3\,a^3c^2x^3-19\,a^2c^2x^2-20\,ac^2x+2\,c^2\right)\sqrt{\frac{ax-}{ax-}}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*
x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^
2*x), 1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*x + 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^{\frac{5}{2}} \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.465 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))/(a*(1 - 1/(a*x))^(3/2)) + (Sqrt[
1 + 1/(a*x)]*(c - c/(a*x))^(3/2)*x)/(1 - 1/(a*x))^(3/2) - (5*(c - c/(a*x))^
(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(3/2))
```

Rubi [A] time = 0.125543, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6182, 6179, 89, 80, 63, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))/(a*(1 - 1/(a*x))^(3/2)) + (Sqrt[
1 + 1/(a*x)]*(c - c/(a*x))^(3/2)*x)/(1 - 1/(a*x))^(3/2) - (5*(c - c/(a*x))^
(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(3/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
```

```
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{-\frac{5}{2a}+\frac{x}{a^2}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right.}{2a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{\right.}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.0514815, size = 70, normalized size = 0.5

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}(ax-2)-5\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 5*ArcTanh[Sqrt[1 + 1/(
a*x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.184, size = 118, normalized size = 0.8

$$\frac{c\,(ax+1)}{2\,ax-2}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{3/2}x\sqrt{(ax+1)\,x}-5\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)xa-4\,\sqrt{(ax+1)\,x}\sqrt{a}\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^(3/2)*x*((
a*x+1)*x)^(1/2)-5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a
-4*((a*x+1)*x)^(1/2)*a^(1/2))/a^(3/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.64371, size = 667, normalized size = 4.76

$$\left[\frac{5\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2-acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{5\,(acx-c)\sqrt{-c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
```

```
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(5*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^
2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.466 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=79

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.156439, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 879, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```


```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
```

```
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0379987, size = 65, normalized size = 0.82

$$\frac{\sqrt{c-\frac{c}{ax}}\left(x\sqrt{\frac{1}{ax}+1}-\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

Maple [A] time = 0.176, size = 101, normalized size = 1.3

$$\frac{(ax+1)x}{2ax-2}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}\left(2\sqrt{(ax+1)x}\sqrt{a}-3\ln\left(1/2\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{(ax+1)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [B] time = 2.56923, size = 635, normalized size = 8.04

$$\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, \frac{3\,(ax-1)\sqrt{-c}\arctan\left(\frac{2(\right.}{}\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.467 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=78

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])
```

Rubi [A] time = 0.156713, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 873, 875, 208}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
```

```
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.0461144, size = 66, normalized size = 0.85

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}-\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x - ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a
*Sqrt[c - c/(a*x)])
```

Maple [A] time = 0.17, size = 104, normalized size = 1.3

$$\frac{(ax+1)x}{2c(ax-1)}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax+1)x}\sqrt{a}-\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)x}\sqrt{a}+2ax+1\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{(ax+1)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2
)/c/(a*x-1)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Fricas [B] time = 2.61423, size = 640, normalized size = 8.21

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2cx-ac\right)},\ \frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\left(a^2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*c*x - a*c), 1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*c*x - a*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

3.468 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] ((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(c - c/(a*x))^(3/2) + ((1 - 1/(a*
x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(3/2)) - (Sqrt[2]*(1
 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*(c - c/(a*x))^(3/2
))
```

Rubi [A] time = 0.139191, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 103, 21, 83, 63, 208, 206}

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```


```
[Out] ((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(c - c/(a*x))^(3/2) + ((1 - 1/(a*
x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(3/2)) - (Sqrt[2]*(1
 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*(c - c/(a*x))^(3/2
))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, \right.}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{2-x^2} \right.}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.0680978, size = 91, normalized size = 0.6

$$\frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(ax\sqrt{\frac{1}{ax}+1}+\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] ((1 - 1/(a*x))^(3/2)*(a*Sqrt[1 + 1/(a*x)]*x + ArcTanh[Sqrt[1 + 1/(a*x)]] -
Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*(c - c/(a*x))^(3/2))
```

Maple [A] time = 0.18, size = 162, normalized size = 1.1

$$\frac{(ax+1)\,x}{2\,c^2\,(ax-1)}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{(ax+1)\,x}a^{3/2}\sqrt{a^{-1}}+\ln\left(\frac{1}{2}\left(2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}}-\sqrt{2}\ln\left(\frac{1}{ax\cdot}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(2*
((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(a*x-1)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

3.469 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{2\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] (-3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))*(c - c/(a*x))^(5
/2)) + (a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*
x))^(5/2)) + (3*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(
a*x))^(5/2)) - (9*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(
2*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rubi [A] time = 0.14896, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6182, 6179, 103, 21, 99, 156, 63, 208, 206}

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{2\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```


```
[Out] (-3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))*(c - c/(a*x))^(5
/2)) + (a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*
x))^(5/2)) + (3*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(
a*x))^(5/2)) - (9*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(
2*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{-1-\frac{x}{2a}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(9\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4a^2\left(c-\frac{c}{ax}\right)^{5/2}} - $$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\left(1-\frac{1}{ax}\right)^{5/2}}{2\sqrt{2}a\left(\right.}$$

Mathematica [A] time = 0.0994173, size = 123, normalized size = 0.56

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}(2ax-3)+12(ax-1)\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-9\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{4ac^2(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-3 + 2*a*x) + 12*(-1 + a*x)*Ar
cTanh[Sqrt[1 + 1/(a*x)]] - 9*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/S
qrt[2]]))/(4*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] time = 0.186, size = 264, normalized size = 1.2

$$\frac{(ax+1)\,x}{8\,c^3\,(ax-1)^2}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(8\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x-9\,a^{3/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x+12\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(8*a^(5/2)*(1/a
)^(1/2)*((a*x+1)*x)^(1/2)*x-9*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-12*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/
2)-12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+9
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^
(1/2))/a^(3/2)/c^3/(a*x-1)^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

Fricas [A] time = 3.04721, size = 1300, normalized size = 5.94

$$\left[\frac{9\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+12\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-\right.}{16\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
- 1)) + 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), 1
/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2
- 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 4*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

3.470 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{115\left(1-\right.}{16}$$

```
[Out] (-5*a*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))^2*(c - c/(a*x)
)^(7/2)) - (35*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(16*(a - x^(-1))*(c -
 c/(a*x))^(7/2)) + (a^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-
1))^2*(c - c/(a*x))^(7/2)) + (5*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]])/(a*(c - c/(a*x))^(7/2)) - (115*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(7/2))
```

Rubi [A] time = 0.169433, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6182, 6179, 103, 21, 99, 151, 156, 63, 208, 206}

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{115\left(1-\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```


```
[Out] (-5*a*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))^2*(c - c/(a*x)
)^(7/2)) - (35*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(16*(a - x^(-1))*(c -
 c/(a*x))^(7/2)) + (a^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-
1))^2*(c - c/(a*x))^(7/2)) + (5*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]])/(a*(c - c/(a*x))^(7/2)) - (115*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(7/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{-\frac{5}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{-2-\frac{3x}{2a}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}}{8\left(c-\right.}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(115\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Sub}}{32a^2\left(\right.}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}}{\left(c-\right.}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\right.}{a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Mathematica [A] time = 0.155664, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(16a^2x^2-55ax+35\right)+160(ax-1)^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-115\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{32ac^3(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(35 - 55*a*x + 16*a^2*x^2) + 16
0*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcTan
h[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(32*a*c^3*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A] time = 0.186, size = 371, normalized size = 1.3

$$\frac{(ax+1)\,x}{64\,c^4\,(ax-1)^3}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(64\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^2-115\,a^{5/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/64*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(64*a^(7/2)*(1
/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-115*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/
2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-220*a^(5/2)*(1/a)^(1/2)*((a*x+
1)*x)^(1/2)*x+160*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3
*(1/a)^(1/2)*x^2+230*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^
(1/2)*a+3*a*x+1)/(a*x-1))*x-320*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1
)/a^(1/2))*a^2*(1/a)^(1/2)*x+140*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+160*
ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-115*2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2
))/a^(3/2)/c^4/(a*x-1)^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Fricas [A] time = 3.156, size = 1467, normalized size = 5.3

$$\left[\frac{115\sqrt{2}\left(a^3x^3 - 3a^2x^2 + 3ax - 1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3 - 3a^2cx^2 - 13acx - 4\sqrt{2}\left(3a^3x^3 + 4a^2x^2 + ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}} - c}{a^3x^3 - 3a^2x^2 + 3ax - 1}\right) + 160\left(a^3x^3 - 3a^2x^2 + \right.}{128\left(a^4c^4x^3 - 3a^3c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a
^2*x^2 + 3*a*x - 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(
8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(16*a^4*x^4 - 39
*a^3*x^3 - 20*a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/64*(115*sqrt
(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) + 4*(16*a^4*x^4 - 39*a^3*x^3 - 20*a^2*x^2 + 35*
a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^
3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

3.471 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=163

$$-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] (-21*c^3*Sqrt[c - c/(a*x)])/a - (5*c^2*(c - c/(a*x))^(3/2))/(3*a) + (3*c*(c
 - c/(a*x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (11*c^(7/2)*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.280299, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (-21*c^3*Sqrt[c - c/(a*x)])/a - (5*c^2*(c - c/(a*x))^(3/2))/(3*a) + (3*c*(c
 - c/(a*x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (11*c^(7/2)*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$

$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{1+ax}dx$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{1+ax}dx}{c}$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$

$$=-\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{9/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{11c^2}{2}+\frac{3c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{2\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{55c^3}{4}-\frac{25c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{5c}$$

$$=-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{4\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{165c^4}{8}-\frac{315c^4x}{8a}\right)}{x(a+x)}dx,x,\ldots\right)}{15c}$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{8\,\text{Subst}\left(\int\frac{\frac{165c^5}{16}-\frac{795\ldots}{16\ldots}}{x(a+x)\sqrt{c\ldots}}\ldots\right)}{15c}$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(11c^4\right)\text{Subst}\left(\int\frac{\ldots}{x\sqrt{\ldots}}\ldots\right)}{2a}$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\left(11c^3\right)\text{Subst}\left(\int\frac{\ldots}{a-\ldots}\ldots\right)$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\ldots\right)}{a}$$

Mathematica [A] time = 0.260302, size = 125, normalized size = 0.77

$$\frac{c^3\left(15a^3x^3-376a^2x^2+52ax-6\right)\sqrt{c-\frac{c}{ax}}}{15a^3x^2}-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-6 + 52*a*x - 376*a^2*x^2 + 15*a^3*x^3))/(15*a^3*x^
2) - (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2
)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.168, size = 281, normalized size = 1.7

$$-\frac{c^3}{30\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(1110\,a^{7/2}\sqrt{a^{-1}}\sqrt{ax^2-x}x^4-480\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^4+480\,a^{5/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}a-3\,a}{ax+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3/a^(7/2)*(1110*a^(7/2)*(1/a)^(1/2)*(a*x^
2-x)^(1/2)*x^4-480*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+480*a^(5/2)*2^
(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4-6
60*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x^2-555*(1/a)^(1/2)*ln(1/2*(2*(a*x^2
-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^3+720*(1/a)^(1/2)*ln(1/2*(2*((a*x
-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^3+92*a^(3/2)*(a*x^2-x)^(3/2)*x
*(1/a)^(1/2)-12*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/(1/a
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

Fricas [A] time = 2.2818, size = 744, normalized size = 4.56

$$\left[\frac{480\sqrt{2}a^2c^{\frac{7}{2}}x^2\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+165\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(15\,a^3c^3x^3-376\,a^2c^3x^2\right.}{30\,a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x +
2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(15*a^3*c^3*x^3 - 376*a^2*c^
3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/15*(480*
sqrt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))
/c) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x -
 c)/(a*x)))/(a^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.472 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=138

$$-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] (-7*c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x)
)^(5/2)*x - (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (16*Sqrt[2]*
c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.254173, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```



```
[Out] (-7*c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x)
)^(5/2)*x - (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (16*Sqrt[2]*
c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
```

```
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$

$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{1+ax}dx$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{1+ax}dx}{c}$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$

$$=-\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{9c^2}{2}+\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{2\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{27c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{3c}$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{4\,\text{Subst}\left(\int\frac{\frac{27c^4}{8}-\frac{69c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{3c}$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{(9c^3)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}-\frac{(16c^3)}{}$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-(9c^2)\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\tanh^{-1}}{a}$$

Mathematica [A] time = 0.101251, size = 116, normalized size = 0.84

$$\frac{c^2\left(3a^2x^2-26ax+2\right)\sqrt{c-\frac{c}{ax}}-27ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+48\sqrt{2}ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 26*a*x + 3*a^2*x^2) - 27*a*c^(5/2)*x*ArcTanh[Sq
rt[c - c/(a*x)]/Sqrt[c]] + 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x)]
/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

Maple [B] time = 0.164, size = 257, normalized size = 1.9

$$\frac{c^2}{6x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(-90\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}x^3+48a^{5/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^3+48a^{3/2}\left(ax^2-x\right)^{3/2}x\sqrt{a^{-1}}+45\ln\left(1/2\frac{2\sqrt{ax^2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2/a^(5/2)*(-90*(a*x^2-x)^(1/2)*a^(5/2)*(1/a
)^(1/2)*x^3+48*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+48*a^(3/2)*(a*x^2-
x)^(3/2)*x*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2
))*(1/a)^(1/2)*x^3*a^2-48*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-72*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^3*a^2-4*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))
/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

Fricas [A] time = 2.2649, size = 657, normalized size = 4.76

$$\left[\frac{48\sqrt{2}ac^{\frac{5}{2}}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+27\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^2c^2x^2-26\,ac^2x+2\,c^2\right)\sqrt{\frac{acx-}{ax}}}{6\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(48*sqrt(2)*a*c^(5/2)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + 3*a*c*x - c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x), -1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*arctan(
sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*
sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.473 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x - (7*c^(3/2)*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/a + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.238425, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x - (7*c^(3/2)*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/a + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
```

```
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$

$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{1+ax}dx$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{1+ax}dx}{c}$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx}{c}$$

$$=-\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{7c^2}{2}-\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{2\,\text{Subst}\left(\int\frac{\frac{7c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{(7c^2)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}-\frac{(8c^2)\,\text{Subst}\left(\int\frac{1}{(a+x)\sqrt{c-\cdots}}\right.}{a}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-(7c)\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)+(16c)\,\text{Subst}\left(\int\cdots\right.$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0829177, size = 95, normalized size = 0.84

$$\frac{-7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+8\sqrt{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+c(ax-2)\sqrt{c-\frac{c}{ax}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) - 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[
c]] + 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.163, size = 229, normalized size = 2.

$$\frac{c}{2x}\sqrt{\frac{c(ax-1)}{ax}}\left(-10\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}x^2+8a^{3/2}\sqrt{a^{-1}}\sqrt{(ax-1)x}x^2+4\left(ax^2-x\right)^{3/2}\sqrt{a}\sqrt{a^{-1}}+5\ln\left(1/2\frac{2\sqrt{ax^2-x}\sqrt{}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c/a^(3/2)*(-10*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1
/2)*x^2+8*a^(3/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2+4*(a*x^2-x)^(3/2)*a^(1/
2)*(1/a)^(1/2)+5*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^
(1/2)*x^2*a-8*a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a
-3*a*x+1)/(a*x+1))*x^2-12*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*(1/a)^(1/2)*x^2*a)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

Fricas [A] time = 2.20556, size = 541, normalized size = 4.79

$$\left[\frac{8\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a}, -\frac{8\sqrt{2}\sqrt{-c}c\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(8*sqrt(2)*c^(3/2)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 + 3*a*c*x - c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(8*sqr
t(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 7*
sqrt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt(
(a*c*x - c)/(a*x)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.474 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=92

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.209188, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 98, 156, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx \\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx \\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c} \\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c} \\
&= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c} \\
&= \sqrt{c-\frac{c}{ax}}x+\frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c} \\
&= \sqrt{c-\frac{c}{ax}}x+\frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}-\frac{(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a} \\
&= \sqrt{c-\frac{c}{ax}}x-5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)+8\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right) \\
&= \sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0510048, size = 92, normalized size = 1.

$$
x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.162, size = 190, normalized size = 2.1

$$-\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}-4\,\sqrt{(ax-1)\,x}a^{3/2}\sqrt{a^{-1}}-\ln\left(\frac{1}{2}\left(2\,\sqrt{ax^2-x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}}+4\,\sqrt{2}\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x
-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*a-3*a*x+1)/(a*x+1))*a^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Fricas [A] time = 1.9768, size = 508, normalized size = 5.52

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\sqrt{-c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.475 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=95

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rubi [A] time = 0.206442, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 99, 156, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```


```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx\\
&= -\int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}(1+ax)}\,dx\\
&= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}x}{1+ax}\,dx}{c}\\
&= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}\,dx}{c}\\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{3c}{2}+\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}\\
&= \frac{\sqrt{c-\frac{c}{ax}}x}{c} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0499849, size = 95, normalized size = 1.

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Maple [A] time = 0.161, size = 136, normalized size = 1.4

$$-\frac{x}{2c}\sqrt{\frac{c(ax-1)}{ax}}\left(-2\sqrt{(ax-1)x}a^{3/2}\sqrt{a^{-1}}+3\ln\left(1/2\,\frac{2\sqrt{(ax-1)x}\sqrt{a}+2ax-1}{\sqrt{a}}\right)a\sqrt{a^{-1}}+2\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax-1}}{ax+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+3*ln
(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2*2^(1/2)
*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/(
(a*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

Fricas [A] time = 1.9522, size = 536, normalized size = 5.64

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+2\,\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\,\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+3\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac},-\frac{2\,\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{}}{ax-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x
*sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + 3*sqrt(c)*log(-2
*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), -(2*sqrt(2)*c*s
qrt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1))
- a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)
/(a*x))/c))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

Giac [A] time = 1.26935, size = 177, normalized size = 1.86

$$-ac\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}c}-\frac{3\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c}-\frac{\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -a*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*s
qrt(-c)*c) - 3*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c) -
sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c))
```

3.476 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rubi [A] time = 0.21546, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 21, 83, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\sqrt{c-\frac{c}{ax}}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{\frac{c}{2}-\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [A] time = 0.0517848, size = 94, normalized size = 1.

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A] time = 0.17, size = 134, normalized size = 1.4

$$-\frac{x}{2\,c^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-2\,\sqrt{(ax-1)\,x}a^{3/2}\sqrt{a^{-1}}+\ln\left(\frac{1}{2}\left(2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}}+\sqrt{2}\ln\left(\frac{1}{ax+1}\left(2\,\sqrt{2}\sqrt{a^{-1}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1
/2)+ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2
))/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A] time = 1.98137, size = 531, normalized size = 5.65

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac^2},-\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x*s
qrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + sqrt(c)*log(-2*a*c
*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), -(sqrt(2)*c*sqrt(
-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) - a*
x*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x)
)/c))/(a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Giac [A] time = 1.25617, size = 176, normalized size = 1.87

$$-ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}-\frac{\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}-\frac{\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] -a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqr
t(-c)*c^2) - arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2) -
sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))
```

3.477 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

```
[Out] -2/(a*c^2*Sqrt[c - c/(a*x)]) + x/(c^2*Sqrt[c - c/(a*x)]) + ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c]
)]/(Sqrt[2]*a*c^(5/2))
```

Rubi [A] time = 0.23544, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```


```
[Out] -2/(a*c^2*Sqrt[c - c/(a*x)]) + x/(c^2*Sqrt[c - c/(a*x)]) + ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c]
)]/(Sqrt[2]*a*c^(5/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$=-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx$$

$$=\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx}{c}$$

$$=\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{c}$$

$$=-\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int \frac{-\frac{c}{2}-\frac{3cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int \frac{\frac{c^2}{2}+\frac{c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c^4}$$

$$=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac^2}-\frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac^2}$$

$$=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{c^3}+\frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{c^3}$$

$$=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

Mathematica [C] time = 0.0547699, size = 70, normalized size = 0.6

$$\frac{-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)+ax}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (a*x - Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] - Hypergeometric
2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

Maple [B] time = 0.17, size = 368, normalized size = 3.2

$$-\frac{x}{4\,c^3\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-8\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^2+\ln\left(\frac{1}{ax+1}\left(2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}a-3\,ax+1\right)\right)a^{\frac{5}{2}}\sqrt{2}x^2-2\,\mathrm{ln}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-8*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1
/2)*x^2+ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(
5/2)*2^(1/2)*x^2-2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1
/a)^(1/2)*x^2*a^3+4*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)+16*a^(5/2)*(1/a)^
(1/2)*((a*x-1)*x)^(1/2)*x-2*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3
*a*x+1)/(a*x+1))*a^(3/2)*2^(1/2)*x+4*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2-8*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+2
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1
/2)-2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2))/
((a*x-1)*x)^(1/2)/c^3/(1/a)^(1/2)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [A] time = 2.00711, size = 652, normalized size = 5.62

$$\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+4\left(a^2x^2-2\,ax\right)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2c^3x-ac^3\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a
*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c^3*x - a*c^3), -1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*arctan
(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*x - c)
/(a*x)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Giac [A] time = 1.25974, size = 224, normalized size = 1.93

$$-\frac{1}{2}\,ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}+\frac{2\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^3}+\frac{2\left(c-\frac{2\,(acx-c)}{ax}\right)}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2
*sqrt(-c)*c^3) + 2*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c
^3) + 2*(c - 2*(a*c*x - c)/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)
*sqrt((a*c*x - c)/(a*x))/(a*x))*a^2*c^3))
```

3.478 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

```
[Out] -4/(3*a*c^2*(c - c/(a*x))^(3/2)) - 7/(2*a*c^3*Sqrt[c - c/(a*x)]) + x/(c^2*(
c - c/(a*x))^(3/2)) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) +
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rubi [A] time = 0.264219, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```


```
[Out] -4/(3*a*c^2*(c - c/(a*x))^(3/2)) - 7/(2*a*c^3*Sqrt[c - c/(a*x)]) + x/(c^2*(
c - c/(a*x))^(3/2)) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) +
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx \\
&= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx \\
&= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx}{c} \\
&= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{c} \\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}-\frac{5cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2} \\
&= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}+\frac{6c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^4} \\
&= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{9c^3}{2}-\frac{21c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^6} \\
&= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4ac^3} - \frac{3\,\text{Subst}\left(\int \frac{\ldots}{x\sqrt{\ldots}} \ldots\right)}{2a\ldots} \\
&= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{2c^4} + \frac{3\,\text{Subst}\left(\int \ldots\right)}{\ldots} \\
&= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0560752, size = 79, normalized size = 0.54

$$\frac{x\left(-\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)-3\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{1}{ax}\right)+3ax\right)}{3c^3(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (x*(3*a*x - Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] - 3*Hyperg
eometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 + a*
x))
```

Maple [B] time = 0.179, size = 497, normalized size = 3.4

$$\frac{x}{24\,c^4\,(ax-1)^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(84\,a^{9/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^3+36\,\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)\sqrt{a^{-1}}x^3a^4-3\,a^{7/2}\sqrt{2}\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/24*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(84*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1
/2)*x^3+36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2
)*x^3*a^4-3*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3
*a*x+1)/(a*x+1))*x^3-60*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)*x-252*a^(7/2)
*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2-108*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a^3+9*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*
x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*2^(1/2)*x^2+52*a^(5/2)*(1/a)^(1/2)*((a
*x-1)*x)^(3/2)+252*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x+108*ln(1/2*(2*((
a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2-9*ln((2*2^(1/2)
*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(3/2)*2^(1/2)*x-84*((a
*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)
*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^3/(1/a
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

Fricas [A] time = 1.99427, size = 814, normalized size = 5.54

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+36\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+4\,(6}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x + 1
)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(6*
a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 29*a^2*x
^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Giac [A] time = 1.27634, size = 252, normalized size = 1.71

$$-\frac{1}{12}\,ac\left(\frac{2\left(2\,c+\frac{15\,(acx-c)}{ax}\right)x}{(acx-c)ac^4\sqrt{\frac{acx-c}{ax}}}+\frac{3\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}c^4}+\frac{36\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^4}-\frac{12\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/12*a*c*(2*(2*c + 15*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*c^4*sqrt((a*c*x
- c)/(a*x))) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c
))/(a^2*sqrt(-c)*c^4) + 36*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sq
rt(-c)*c^4) - 12*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^4))
```

3.479 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

```
[Out] -6/(5*a*c^2*(c - c/(a*x))^(5/2)) - 11/(6*a*c^3*(c - c/(a*x))^(3/2)) - 21/(4
*a*c^4*Sqrt[c - c/(a*x)]) + x/(c^2*(c - c/(a*x))^(5/2)) + (5*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt
[c])]/(4*Sqrt[2]*a*c^(9/2))
```

Rubi [A] time = 0.293287, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```


```
[Out] -6/(5*a*c^2*(c - c/(a*x))^(5/2)) - 11/(6*a*c^3*(c - c/(a*x))^(3/2)) - 21/(4
*a*c^4*Sqrt[c - c/(a*x)]) + x/(c^2*(c - c/(a*x))^(5/2)) + (5*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt
[c])]/(4*Sqrt[2]*a*c^(9/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{c}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{5c}{2}-\frac{7cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\mathrm{Subst}\left(\int \frac{\frac{25c^2}{2}+\frac{15c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{75c^3}{2}-\frac{165c^3x}{4a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^6}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\mathrm{Subst}\left(\int \frac{\frac{75c^4}{2}+\frac{315c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{15c^8}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8ac^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\ldots}\right)}{4c^5}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}} + \frac{\tanh^{-1}\left(\ldots\right)}{4\sqrt{2}\ldots}$$

Mathematica [C] time = 0.0649847, size = 82, normalized size = 0.48

$$\frac{ax^2\left(-\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{a-\frac{1}{x}}{2a}\right)-5\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\frac{1}{ax}\right)+5ax\right)}{5c^4(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] (a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*Hy
pergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-1
+ a*x)^2)
```

Maple [B] time = 0.174, size = 626, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(9/2),x)
```

```
[Out] -1/240*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-1260*((a*x-1)*x)^(1/2)*a^(11/2)*(1
/a)^(1/2)*x^4+1020*((a*x-1)*x)^(3/2)*a^(9/2)*(1/a)^(1/2)*x^2-600*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^4*a^5+15*a^(9/2)*
2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4
+5040*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3-1792*a^(7/2)*(1/a)^(1/2)*((
a*x-1)*x)^(3/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*x^3*a^4-60*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)
*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-7560*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
)*x^2+820*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)-3600*ln(1/2*(2*((a*x-1)*x)^
(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a^3+90*ln((2*2^(1/2)*(1/a)^
(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*2^(1/2)*x^2+5040*a^(5/2
)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2-60*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x
)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(3/2)*2^(1/2)*x-1260*((a*x-1)*x)^(1/2)*a^(3/2
)*(1/a)^(1/2)-600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(
1/a)^(1/2)+15*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1
)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^5/(a*x-1)^4/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(9/2)), x)
```

Fricas [A] time = 2.00998, size = 976, normalized size = 5.67

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+600\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\right.}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(2*sqrt(2
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 600*(a^3*
x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c
*x - c)/(a*x)) + c) + 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*
sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5
), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*x^2 + 3
*a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(60*a^4*x
^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5
*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34761, size = 279, normalized size = 1.62

$$-\frac{1}{120}\,ac\left(\frac{2\left(12\,c^2+\frac{50\,(acx-c)c}{ax}+\frac{255\,(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c^5\sqrt{\frac{acx-c}{ax}}}+\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}c^5}+\frac{600\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^5}-\frac{120\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/120*a*c*(2*(12*c^2 + 50*(a*c*x - c)*c/(a*x) + 255*(a*c*x - c)^2/(a^2*x^2
))*x^2/((a*c*x - c)^2*c^5*sqrt((a*c*x - c)/(a*x))) + 15*sqrt(2)*arctan(1/2*
sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^5) + 600*arctan(s
qrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^5) - 120*sqrt((a*c*x - c)/
(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^5))
```

3.480 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=335

$$\frac{x\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{65\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{5\cdots}{\cdots}+\cdots$$

```
[Out] (10*(a - x^(-1))^4*(c - c/(a*x))^(9/2))/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1
/(a*x)]) + (5*(304*a - 65/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(7*a^2*
(1 - 1/(a*x))^(9/2)) + (135*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(9/2))/(7*a^3*(1 - 1/(a*x))^(9/2)) + (65*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(
c - c/(a*x))^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^5*(c - c/(a
*x))^(9/2)*x)/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]) - (15*(c - c/(a*x
))^(9/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rubi [A] time = 0.200309, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 150, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{65\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{5\cdots}{\cdots}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^4*(c - c/(a*x))^(9/2))/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1
/(a*x)]) + (5*(304*a - 65/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(7*a^2*
(1 - 1/(a*x))^(9/2)) + (135*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(9/2))/(7*a^3*(1 - 1/(a*x))^(9/2)) + (65*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(
c - c/(a*x))^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^5*(c - c/(a
*x))^(9/2)*x)/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]) - (15*(c - c/(a*x
))^(9/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^6}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{9/2}\mathrm{Subst}\left(\int\frac{\left(\frac{15}{2a}+\frac{5x}{2a^2}\right)\left(1-\frac{x}{a}\right)^4}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\mathrm{Subst}\left(\int\frac{\left(-\frac{15}{4a^2}-\frac{65x}{4a^3}\right)(}{x\sqrt{1+\frac{}{}}}\right.}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{(4}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\right.}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c\right.}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c\right.}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c\right.}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

Mathematica [C] time = 0.114218, size = 140, normalized size = 0.42

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(70a^4x^4\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+7a^5x^5+1685a^4x^4+720a^3x^3-110a^2x^2-35a^4x^4\sqrt{\frac{1}{ax}+1}\ \text{ta}\right.}{7a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(-2 + 20*a*x - 110*a^2*x^2 + 720*a^3*x^3 + 1685*a^4*
x^4 + 7*a^5*x^5 - 35*a^4*Sqrt[1 + 1/(a*x)]*x^4*ArcTanh[Sqrt[1 + 1/(a*x)]] +
 70*a^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(7*a^5*Sqrt[1 -
1/(a^2*x^2)]*x^4)
```

Maple [A] time = 0.184, size = 229, normalized size = 0.7

$$\frac{(ax+1)\,c^4}{14\,(ax-1)^2\,x^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(14\sqrt{(ax+1)\,x}a^{11/2}x^5+3510\,a^{9/2}\sqrt{(ax+1)\,x}x^4+1440\,a^{7/2}x^3\sqrt{(ax+1)\,x}-105\ \text{ln}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/14*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^4*(1
4*((a*x+1)*x)^(1/2)*a^(11/2)*x^5+3510*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+1440*a^
(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1
)/a^(1/2))*x^5*a^5-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*x^4*a^4-220*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+40*a^(3/2)*x*((a*x+1)*x)^(1/2)-
4*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.21413, size = 918, normalized size = 2.74

$$\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(7\,a^5c^4x^5+1755\,a^4c^4x^4+720\,a^3c^4x^3-\right.}{28\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/28*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 72
0*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/14*(105*(a^4*c^4*x^4 -
 a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(7*a^5*c^4*x
^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*
c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^{\frac{9}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.481 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=277

$$\frac{x\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] (10*(a - x^(-1))^3*(c - c/(a*x))^(7/2))/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1
/(a*x)]) + ((1360*a - 311/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2
*(1 - 1/(a*x))^(7/2)) + (47*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(7/2))/(5*a^3*(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^4*(c - c/(a*x))^(7/2)*x)
/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - (13*(c - c/(a*x))^(7/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rubi [A] time = 0.170589, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 150, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (10*(a - x^(-1))^3*(c - c/(a*x))^(7/2))/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1
/(a*x)]) + ((1360*a - 311/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2
*(1 - 1/(a*x))^(7/2)) + (47*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(7/2))/(5*a^3*(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^4*(c - c/(a*x))^(7/2)*x)
/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - (13*(c - c/(a*x))^(7/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
```

```
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{13}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{13}{4a^2}-\frac{47x}{4a^3}\right)(1-}{x\sqrt{1+\frac{x}{a}}}\right.}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{(4a}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\right.}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\right.}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\right.}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [C] time = 0.104313, size = 132, normalized size = 0.48

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(150a^3x^3\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+15a^4x^4+1441a^3x^3+548a^2x^2-45a^3x^3\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\right.\right.}{15a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(6 - 62*a*x + 548*a^2*x^2 + 1441*a^3*x^3 + 15*a^4*x^
4 - 45*a^3*Sqrt[1 + 1/(a*x)]*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]] + 150*a^3*x^3*H
ypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(15*a^4*Sqrt[1 - 1/(a^2*x^2)]
*x^3)
```

Maple [A] time = 0.212, size = 212, normalized size = 0.8

$$\frac{(ax+1)c^3}{30(ax-1)^2x^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c(ax-1)}{ax}}\left(30a^{9/2}\sqrt{(ax+1)x}x^4+3182a^{7/2}x^3\sqrt{(ax+1)x}-195\ln\left(1/2\frac{2\sqrt{(ax+1)x}\sqrt{a}+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/30*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^3*(3
0*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+3182*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-195*ln(1
/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^4*a^4+1096*a^(5/2)*x^2*
((a*x+1)*x)^(1/2)-195*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*x^3*a^3-124*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/
a^(7/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.30256, size = 872, normalized size = 3.15

$$\left[\frac{195\left(a^{3}c^{3}x^{3}-a^{2}c^{3}x^{2}\right)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15\,a^{4}c^{3}x^{4}+1591\,a^{3}c^{3}x^{3}+548\,a^{2}c^{3}x^{2}-\right.}{60\left(a^{4}x^{3}-a^{3}x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 5
48*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqr
t(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 1591*a^3*c^
3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.482 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=219

$$\frac{x\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] (10*(a - x^(-1))^2*(c - c/(a*x))^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1
/(a*x)]) + ((112*a - 29/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1
 - 1/(a*x))^(5/2)) + ((a - x^(-1))^3*(c - c/(a*x))^(5/2)*x)/(a^3*(1 - 1/(a*
x))^(5/2)*Sqrt[1 + 1/(a*x)]) - (11*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rubi [A] time = 0.15001, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 150, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (10*(a - x^(-1))^2*(c - c/(a*x))^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1
/(a*x)]) + ((112*a - 29/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1
 - 1/(a*x))^(5/2)) + ((a - x^(-1))^3*(c - c/(a*x))^(5/2)*x)/(a^3*(1 - 1/(a*
x))^(5/2)*Sqrt[1 + 1/(a*x)]) - (11*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{11}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{11}{4a^2}-\frac{29x}{4a^3}\right)(1\cdots}{x\sqrt{1+\frac{x}{a}}}\cdots\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{(11\cdots}{\cdots}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{(11\cdots}{\cdots}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{11\cdots}{\cdots}$$

Mathematica [C] time = 0.0937006, size = 124, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(30a^2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+3a^3x^3+103a^2x^2-3a^2x^2\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+32a\right.}{3a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 103*a^2*x^2 + 3*a^3*x^3 - 3*a^2*Sqrt[
1 + 1/(a*x)]*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 30*a^2*x^2*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + 1/(a*x)]))/(3*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A] time = 0.191, size = 195, normalized size = 0.9

$$\frac{(ax+1)c^2}{6(ax-1)^2x}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c(ax-1)}{ax}}\left(6a^{7/2}x^3\sqrt{(ax+1)x}+266a^{5/2}x^2\sqrt{(ax+1)x}-33\ln\left(1/2\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)+266*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-33*ln(1/2*(
2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3+64*a^(3/2)*x*((a*x+1)
*x)^(1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2-
4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(5/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

[Out] integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A] time = 2.076, size = 798, normalized size = 3.64

$$\left[\frac{33\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3+133a^2c^2x^2+32ac^2x-2c^2\right)\sqrt{\frac{a}{a}}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/12*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), 1/6*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.483 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=158

$$\frac{x\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] ((21*a + x^(-1))*(c - c/(a*x))^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(
a*x)]) + ((a - x^(-1))^2*(c - c/(a*x))^(3/2)*x)/(a^2*(1 - 1/(a*x))^(3/2)*Sq
rt[1 + 1/(a*x)]) - (9*(c - c/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1
 - 1/(a*x))^(3/2))
```

Rubi [A] time = 0.133294, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6182, 6179, 98, 146, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] ((21*a + x^(-1))*(c - c/(a*x))^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(
a*x)]) + ((a - x^(-1))^2*(c - c/(a*x))^(3/2)*x)/(a^2*(1 - 1/(a*x))^(3/2)*Sq
rt[1 + 1/(a*x)]) - (9*(c - c/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1
 - 1/(a*x))^(3/2))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$
$$=-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$
$$=\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}-\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$
$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}dx,x,\right.}{2a\left(1-\frac{1}{ax}\right)^{3/2}}$$
$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}dx,x,\right.}{\left(1-\frac{1}{ax}\right)^{3/2}}$$
$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [C] time = 0.0411834, size = 71, normalized size = 0.45

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(9ax\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+a^2x^2+10ax+2\right)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 + 10*a*x + a^2*x^2 + 9*a*x*Hypergeometric2F1[-1/2,
1, 1/2, 1 + 1/(a*x)]))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.189, size = 169, normalized size = 1.1

$$\frac{c(ax+1)}{2(ax-1)^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c(ax-1)}{ax}}\left(2a^{5/2}x^2\sqrt{(ax+1)x}+38a^{3/2}x\sqrt{(ax+1)x}-9\ln\left(1/2\,\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^
(5/2)*x^2*((a*x+1)*x)^(1/2)+38*a^(3/2)*x*((a*x+1)*x)^(1/2)-9*ln(1/2*(2*((a*
x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2-9*ln(1/2*(2*((a*x+1)*x)^(1/
2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a+4*((a*x+1)*x)^(1/2)*a^(1/2))/a^(3/2)/((a*x
+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.12392, size = 675, normalized size = 4.27

$$\left[\frac{9(acx-c)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2+19acx+2c\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{9(acx-c)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(9*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(9*(a*c*x - c)*sqrt(-c)*arctan(2*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.484 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=140

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.117883, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6182, 6179, 89, 78, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
```

```
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0462443, size = 67, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax-7\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+9\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.184, size = 146, normalized size = 1.

$$-\frac{(ax+1)\,x}{2\,(ax-1)^{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-2\,a^{3/2}x\sqrt{(ax+1)\,x}+7\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)xa-18\,\sqrt{(ax+1)\,x}\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(-2*
a^(3/2)*x*((a*x+1)*x)^(1/2)+7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*x*a-18*((a*x+1)*x)^(1/2)*a^(1/2)+7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(
1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.10432, size = 640, normalized size = 4.57

$$\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{2}x^{2}+9\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^{2}x-a\right)},\ \frac{7\,(ax-1)\sqrt{-c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
```

```
/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.485 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=118

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] (5*Sqrt[c - c/(a*x)])/(a*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a*x)]*x)/(
c*Sqrt[1 - 1/(a^2*x^2)]) - (5*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[
c - c/(a*x)]])/(a*Sqrt[c])
```

Rubi [A] time = 0.221027, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 879, 869, 875, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (5*Sqrt[c - c/(a*x)])/(a*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a*x)]*x)/(
c*Sqrt[1 - 1/(a^2*x^2)]) - (5*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[
c - c/(a*x)]])/(a*Sqrt[c])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 869

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*(p + 1)*(e*f + d*g)), x] + Dist[(e^2*g*(m - n - 2))/(c*(p +
 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(5c)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [C] time = 0.040337, size = 69, normalized size = 0.58

$$\frac{\sqrt{1-\frac{1}{ax}}\left(5\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+ax\right)}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/
(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])
```

Maple [A] time = 0.184, size = 149, normalized size = 1.3

$$\frac{(ax+1)\,x}{2\,(ax-1)^2\,c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,a^{3/2}x\sqrt{(ax+1)\,x}-5\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)xa+10\,\sqrt{(ax+1)\,x}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*((a*x+1)*x)^(1/2)-5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*x*a+10*((a*x+1)*x)^(1/2)*a^(1/2)-5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/c/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

Fricas [A] time = 2.2879, size = 651, normalized size = 5.52

$$\left[\frac{5\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+5\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2cx-ac\right)},\frac{5\,(ax-1)\sqrt{-c}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c*x - a*c), 1/2*(5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 2*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.486 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2x\sqrt{c-\frac{c}{ax}}}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (3*Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a*x)]) - (2*Sqrt[c - c/(a*x)]*x)
/(c^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/S
qrt[c - c/(a*x)]])/(a*c^(3/2))
```

Rubi [A] time = 0.222819, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 869, 873, 875, 208}

$$-\frac{2x\sqrt{c-\frac{c}{ax}}}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (3*Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a*x)]) - (2*Sqrt[c - c/(a*x)]*x)
/(c^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/S
qrt[c - c/(a*x)]])/(a*c^(3/2))
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 869

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*(p + 1)*(e*f + d*g)), x] + Dist[(e^2*g*(m - n - 2))/(c*(p +
 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

Mathematica [C] time = 0.0446935, size = 64, normalized size = 0.55

$$\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{ax}+1\right)}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (2*(1 - 1/(a*x))^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, 1 + 1/(a*x)])/(a*Sqr
t[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))
```

Maple [A] time = 0.182, size = 149, normalized size = 1.3

$$-\frac{(ax+1)x}{2(ax-1)^2c^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c(ax-1)}{ax}}\left(-2a^{3/2}x\sqrt{(ax+1)x}+3\ln\left(1/2\,\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right)xa-6\sqrt{(ax+1)x}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1
/2)/c^2*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*x*a-6*((a*x+1)*x)^(1/2)*a^(1/2)+3*ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Fricas [A] time = 2.23823, size = 662, normalized size = 5.66

$$\left[\frac{3(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+3ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2c^2x-ac^2\right)},\ \frac{3(ax-1)\sqrt{-c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c^2*x - a*c^2), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2
+ a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*
x^2 - a*c*x - c)) + 2*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.487 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] (2*(1 - 1/(a*x))^(5/2))/(a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) + ((1 - 1
/(a*x))^(5/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rubi [A] time = 0.167721, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 103, 152, 156, 63, 208, 206}

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (2*(1 - 1/(a*x))^(5/2))/(a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) + ((1 - 1
/(a*x))^(5/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{1}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\frac{1}{2a^2}-\frac{x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}}{}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right.}{}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\cdots\right.}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [C] time = 0.0624847, size = 90, normalized size = 0.45

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+ax\right)}{ac^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + Hypergeometric2F1[-1/2, 1, 1/2, (a + x^(-1))/(2*a
)] + Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(a*c^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - c/(a*x)])
```

Maple [A] time = 0.187, size = 262, normalized size = 1.3

$$-\frac{(ax+1)\,x}{4\,(ax-1)^2\,c^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-4\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x+2\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)a^2\sqrt{a^{-1}}x+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3
/2)/c^3*(-4*a^(5/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x+2*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x+a^(3/2)*2^(1/2)*ln((2*2^
(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-8*((a*x+1)*x)^(1/
2)*a^(3/2)*(1/a)^(1/2)+2*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/
2))*a*(1/a)^(1/2)+2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a
*x+1)/(a*x-1))*a^(1/2))/(1/a)^(1/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.488 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=267

$$\frac{ax\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-$$

```
[Out] (7*(1 - 1/(a*x))^(7/2))/(4*a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) - (3*(1
 - 1/(a*x))^(7/2))/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) +
 (a*(1 - 1/(a*x))^(7/2)*x)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7
/2)) + ((1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(7
/2)) - (11*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(4*Sqrt[
2]*a*(c - c/(a*x))^(7/2))
```

Rubi [A] time = 0.171721, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6182, 6179, 103, 151, 152, 156, 63, 208, 206}

$$\frac{ax\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (7*(1 - 1/(a*x))^(7/2))/(4*a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) - (3*(1
 - 1/(a*x))^(7/2))/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) +
 (a*(1 - 1/(a*x))^(7/2)*x)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7
/2)) + ((1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(7
/2)) - (11*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(4*Sqrt[
2]*a*(c - c/(a*x))^(7/2))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
```

```
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\frac{1}{a^2}+\frac{9x}{2a^3}}{x\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)}\right.}{2\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7}\right.}{}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(11\left(1-\frac{1}{ax}\right)^{7}\right.}{}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{S}}{}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{ta}}{a\left(c\right.}$$

Mathematica [C] time = 0.0808455, size = 121, normalized size = 0.45

$$\frac{\sqrt{1-\frac{1}{ax}}\left(11(ax-1)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+(4-4ax)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{ax}+1\right)+2ax(2ax\right.}{4ac^3\sqrt{\frac{1}{ax}+1}(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*x*(-3 + 2*a*x) + 11*(-1 + a*x)*Hypergeometric2F1[-1
/2, 1, 1/2, (a + x^(-1))/(2*a)] + (4 - 4*a*x)*Hypergeometric2F1[-1/2, 1, 1/
2, 1 + 1/(a*x)]))/(4*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A] time = 0.191, size = 290, normalized size = 1.1

$$\frac{(ax+1)\,x}{16\,(ax-1)^3\,c^4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(16\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^2-11\,a^{5/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/16*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(c*(a*x-1)/a/x)^(1/2)*x*(16*
a^(7/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-11*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+4*a^(5/2)*(1/a)^(1/2)
*((a*x+1)*x)^(1/2)*x+8*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2-28*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-8*ln(1/2*(2*
((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+11*2^(1/2)*ln((2*
2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/
c^4/(1/a)^(1/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Fricas [A] time = 2.69819, size = 1301, normalized size = 4.87

$$\left[\frac{11\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+8\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-\frac{8}{}\right.}{32\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/32*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 8*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1
/16*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 8*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 4*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.489 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

Optimal. Leaf size=60

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-1,-m,-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.221682, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6182, 6181, 64}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},-m-1;-m;-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^m\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\,{}_2F_1\left(-\frac{1}{2}, -1-m; -m; -\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0311424, size = 60, normalized size = 1.

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\text{Hypergeometric2F1}\left(-\frac{1}{2}, -m-1, -m, -\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int x^m\sqrt{c-\frac{c}{ax}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ax+1)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a
*x - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***m*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.490 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=164

$$\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] -(c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*Sqr
t[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c -
 c/(a*x)]])/(8*a^3)
```

Rubi [A] time = 0.342206, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6178, 863, 873, 875, 208}

$$\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -(c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*Sqr
t[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c -
 c/(a*x)]])/(8*a^3)
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{6a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{16a^3}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^5}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Mathematica [A] time = 0.463492, size = 147, normalized size = 0.9

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^2x^2+2ax-3\right)\sqrt{c-\frac{c}{ax}}}{ax-1} + 3\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) - 3\sqrt{c}\log(1-ax)}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3 + 2*a*x + 8*a^2*x^2
))/(-1 + a*x) - 3*Sqrt[c]*Log[1 - a*x] + 3*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1
```

```
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a^3)
```

Maple [A] time = 0.175, size = 121, normalized size = 0.7

$$\frac{x}{48}\sqrt{\frac{c(ax-1)}{ax}}\left(16\,a^{5/2}x^2\sqrt{(ax+1)\,x}+4\,a^{3/2}x\sqrt{(ax+1)\,x}-6\,\sqrt{(ax+1)\,x}\sqrt{a}+3\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(5/2)*(16*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)+4*a^(3/2)*x*((a*x+1)*x)^(1/2)-6*((a*x+1)*x)^(1/2)*a^(1/
2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.22936, size = 717, normalized size = 4.37

$$\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}\quad 3}{96\left(a^4x-a^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(3*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x
^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.491 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=124

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^
2)]*x^2)/(2*Sqrt[c - c/(a*x)]) - (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rubi [A] time = 0.239776, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6178, 863, 873, 875, 208}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```


```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^
2)]*x^2)/(2*Sqrt[c - c/(a*x)]) - (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{c^2\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.324908, size = 148, normalized size = 1.19

$$\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}(2ax+1)\sqrt{c-\frac{c}{ax}}+\sqrt{c}(1-ax)\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+\sqrt{c}(ax-1)\log(1}{8a^2(ax-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(1 + 2*a*x) + Sqrt[c]*(-
1 + a*x)*Log[1 - a*x] + Sqrt[c]*(1 - a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2*(-1 + a*x)
)
```

Maple [A] time = 0.17, size = 102, normalized size = 0.8

$$-\frac{x}{8}\sqrt{\frac{c(ax-1)}{ax}}\left(-4a^{3/2}x\sqrt{(ax+1)x}-2\sqrt{(ax+1)x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)x}\sqrt{a}+2ax+1\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}a^{-\frac{3}{2}}\frac{1}{\sqrt{(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-4*a^(3/2)*x*
((a*x+1)*x)^(1/2)-2*((a*x+1)*x)^(1/2)*a^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a
^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 2.20489, size = 674, normalized size = 5.44

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)},\frac{(ax-1)\sqrt{-c}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.492 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=78

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.153558, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6177, 863, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
```

```
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{c^2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0473212, size = 66, normalized size = 0.85

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax+\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+1\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.173, size = 87, normalized size = 1.1

$$\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{(ax+1)\,x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax+1)\,x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [B] time = 2.24803, size = 630, normalized size = 8.08

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.493 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=76

$$2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rubi [A] time = 0.227336, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6178, 865, 875, 208}

$$2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
```

```
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.217604, size = 132, normalized size = 1.74

$$
\frac{-2ax\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\sqrt{c}(ax-1)\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+\sqrt{c}(1-ax)\log(1-ax)}{ax-1}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
 a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
 c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Maple [A] time = 0.178, size = 88, normalized size = 1.2

$$\sqrt{\frac{c\left(ax-1\right)}{ax}}\left(\ln\left(\frac{1}{2}\left(2\sqrt{\left(ax+1\right)x}\sqrt{a}+2\,ax+1\right)\frac{1}{\sqrt{a}}\right)xa-2\sqrt{\left(ax+1\right)x}\sqrt{a}\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{\left(ax+1\right)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x)^(1/2
)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a-2*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [B] time = 1.96849, size = 603, normalized size = 7.93

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+a\right.}{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.494 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))
```

Rubi [A] time = 0.147761, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6178, 649}

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.0872397, size = 45, normalized size = 1.22

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}(ax+1)\sqrt{c-\frac{c}{ax}}}{3ax-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + a*x))/(-3 + 3*a*x)
```

Maple [A] time = 0.117, size = 41, normalized size = 1.1

$$-\frac{2\,ax+2}{3\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] -2/3*(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.63512, size = 122, normalized size = 3.3

$$-\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
)/(a*x^2 - x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.495 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=77

$$\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (-2*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) + (2*a^2*c*(1
 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])
```

Rubi [A] time = 0.188534, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6178, 795, 649}

$$\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) + (2*a^2*c*(1
 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
```

```
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\left(c\,\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}} - \frac{1}{5}(ac)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A] time = 0.102677, size = 58, normalized size = 0.75

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-ax-3\right)\sqrt{c-\frac{c}{ax}}}{15x(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-3 - a*x + 2*a^2*x^2))/(15*x*
(-1 + a*x))
```

Maple [A] time = 0.118, size = 47, normalized size = 0.6

$$\frac{(2\,ax+2)(2\,ax-3)}{15\,x^2}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 2/15*(a*x+1)*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^3 \sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.62791, size = 140, normalized size = 1.82

$$\frac{2\left(2\,a^3x^3 + a^2x^2 - 4\,ax - 3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3 - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] 2/15*(2*a^3*x^3 + a^2*x^2 - 4*a*x - 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.496 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=117

$$\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (8*a^3*c^2*(1 - 1/(a^2*x^2))^(3/2))/(105*(c - c/(a*x))^(3/2)) - (8*a^3*c*(1
 - 1/(a^2*x^2))^(3/2))/(35*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^2))^
(3/2))/(7*(c - c/(a*x))^(3/2)*x^2)
```

Rubi [A] time = 0.246082, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6178, 871, 795, 649}

$$\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (8*a^3*c^2*(1 - 1/(a^2*x^2))^(3/2))/(105*(c - c/(a*x))^(3/2)) - (8*a^3*c*(1
 - 1/(a^2*x^2))^(3/2))/(35*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^2))^
(3/2))/(7*(c - c/(a*x))^(3/2)*x^2)
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
```

```
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2} + \frac{1}{7}(4ac)\,\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2} + \frac{1}{35}\left(4a^2c\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}
\end{aligned}$$

Mathematica [A] time = 0.108857, size = 66, normalized size = 0.56

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(8a^3x^3-4a^2x^2+3ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^2(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(15 + 3*a*x - 4*a^2*x^2 + 8*a
^3*x^3))/(105*x^2*(-1 + a*x))
```

Maple [A] time = 0.119, size = 55, normalized size = 0.5

$$-\frac{(2\,ax+2)\left(8\,a^{2}x^{2}-12\,ax+15\right)}{105\,x^{3}}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+1)*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/((a*x-1)/(a*
x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^{4}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.58065, size = 162, normalized size = 1.38

$$-\frac{2\left(8\,a^{4}x^{4}+4\,a^{3}x^{3}-a^{2}x^{2}+18\,ax+15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^{4}-x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] -2/105*(8*a^4*x^4 + 4*a^3*x^3 - a^2*x^2 + 18*a*x + 15)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.497 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=159

$$-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9x^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-16*a^4*c^2*(1 - 1/(a^2*x^2))^(3/2))/(315*(c - c/(a*x))^(3/2)) + (16*a^4*c
*(1 - 1/(a^2*x^2))^(3/2))/(105*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^
2))^(3/2))/(9*(c - c/(a*x))^(3/2)*x^3) + (4*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)
)/(21*(c - c/(a*x))^(3/2)*x^2)
```

Rubi [A] time = 0.306008, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6178, 871, 795, 649}

$$-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9x^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```


```
[Out] (-16*a^4*c^2*(1 - 1/(a^2*x^2))^(3/2))/(315*(c - c/(a*x))^(3/2)) + (16*a^4*c
*(1 - 1/(a^2*x^2))^(3/2))/(105*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^
2))^(3/2))/(9*(c - c/(a*x))^(3/2)*x^3) + (4*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)
)/(21*(c - c/(a*x))^(3/2)*x^2)
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{1}{3}(2ac)\,\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{21}\left(8a^2c\right)\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{105}\left(8a^3c\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{\sqrt{c-}}\right.\\
&= -\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2}
\end{aligned}$$

Mathematica [A] time = 0.116209, size = 74, normalized size = 0.47

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(16a^4x^4-8a^3x^3+6a^2x^2-5ax-35\right)\sqrt{c-\frac{c}{ax}}}{315x^3(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-35 - 5*a*x + 6*a^2*x^2 - 8*a
^3*x^3 + 16*a^4*x^4))/(315*x^3*(-1 + a*x))
```

Maple [A] time = 0.117, size = 63, normalized size = 0.4

$$\frac{(2\,ax+2)\left(16\,x^3a^3-24\,a^2x^2+30\,ax-35\right)}{315\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(a*x+1)*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4/(
(a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.56027, size = 178, normalized size = 1.12

$$\frac{2\left(16\,a^{5}x^{5}+8\,a^{4}x^{4}-2\,a^{3}x^{3}+a^{2}x^{2}-40\,ax-35\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^{5}-x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] 2/315*(16*a^5*x^5 + 8*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 40*a*x - 35)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^{5}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.498 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=130

$$\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}+\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

```
[Out] (75*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) + (
5*Sqrt[c - c/(a*x)]*x^3)/(8*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (75*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rubi [A] time = 0.377355, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 78, 51, 63, 208}

$$\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}+\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (75*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) + (
5*Sqrt[c - c/(a*x)]*x^3)/(8*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (75*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$=-\int\frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)}{1-ax}\,dx$$

$$=\frac{c\int\frac{x^2(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=\frac{c\int\frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=-\frac{c\,\mathrm{Subst}\left(\int\frac{a+x}{x^5\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{(15c)\,\mathrm{Subst}\left(\int\frac{1}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a}$$

$$=\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{(25c)\,\mathrm{Subst}\left(\int\frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2}$$

$$=\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{(75c)\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{64a^3}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4-\frac{(75c)\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,\right.}{128a^4}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{75\,\mathrm{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{}\right.}{64a^3}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

Mathematica [C] time = 0.0380654, size = 50, normalized size = 0.38

$$\frac{\sqrt{c-\frac{c}{ax}}\left(15\mathrm{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\frac{1}{ax}\right)+a^4x^4\right)}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x)
]))/(4*a^4)
```

Maple [A] time = 0.166, size = 172, normalized size = 1.3

$$\frac{x}{128}\sqrt{\frac{c\,(ax-1)}{ax}}\left(32\,x\left(ax^2-x\right)^{3/2}a^{7/2}+112\,\left(ax^2-x\right)^{3/2}a^{5/2}+212\,\sqrt{ax^2-x}a^{5/2}x-106\,\sqrt{ax^2-x}a^{3/2}+256\,a^{3/2}\sqrt{(ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/128*(c*(a*x-1)/a/x)^(1/2)*x*(32*x*(a*x^2-x)^(3/2)*a^(7/2)+112*(a*x^2-x)^(
3/2)*a^(5/2)+212*(a*x^2-x)^(1/2)*a^(5/2)*x-106*(a*x^2-x)^(1/2)*a^(3/2)+256*
a^(3/2)*((a*x-1)*x)^(1/2)+128*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))-53*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-
1)*x)^(1/2)/a^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^3}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^3/(a*x - 1), x)
```

Fricas [A] time = 1.59547, size = 409, normalized size = 3.15

$$\left[\frac{2\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}+75\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},\frac{\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)/
(a*x)) + 75*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c))/a^4, 1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x
- c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.2295, size = 193, normalized size = 1.48

$$\frac{1}{64}\sqrt{a^2cx^2-acx}\left(2\left(4\,x\left(\frac{2\,x|a|}{a^2\mathrm{sgn}\left(x\right)}+\frac{5\,|a|}{a^3\mathrm{sgn}\left(x\right)}\right)+\frac{25\,|a|}{a^4\mathrm{sgn}\left(x\right)}\right)x+\frac{75\,|a|}{a^5\mathrm{sgn}\left(x\right)}\right)+\frac{75\,\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}\left(x\right)}{128\,a^4}-\frac{75\,\sqrt{c}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(a
^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) + 75/128*
sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a^4 - 75/128*sqrt(c)*log(abs(-2*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a^4*sgn(x))
```

3.499 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=105

$$\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] (11*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 + (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3)
```

Rubi [A] time = 0.352273, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 78, 51, 63, 208}

$$\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (11*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 + (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{x(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{(11c)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{6a}\\
&= \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{(11c)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}\\
&= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{(11c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^3}\\
&= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{11\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{8a^2}\\
&= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}
\end{aligned}$$

Mathematica [C] time = 0.0360501, size = 50, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(11\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{1}{ax}\right)+a^3x^3\right)}{3a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x)
]))/(3*a^3)
```

Maple [A] time = 0.168, size = 155, normalized size = 1.5

$$\frac{x}{48}\sqrt{\frac{c(ax-1)}{ax}}\left(16\left(ax^2-x\right)^{3/2}a^{5/2}+60\sqrt{ax^2-x}a^{5/2}x+96a^{3/2}\sqrt{(ax-1)x}-30\sqrt{ax^2-x}a^{3/2}+48a\ln\left(1/2\frac{2\sqrt{(ax-1)}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(5/2)+60*(a*x^2-x)^(1/2)
*a^(5/2)*x+96*a^(3/2)*((a*x-1)*x)^(1/2)-30*(a*x^2-x)^(1/2)*a^(3/2)+48*a*ln(
1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-15*ln(1/2*(2*(a*x^2-x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-1)*x)^(1/2)/a^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^2}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/(a*x - 1), x)
```

Fricas [A] time = 1.62967, size = 370, normalized size = 3.52

$$\left[\frac{2\left(8a^3x^3+22a^2x^2+33ax\right)\sqrt{\frac{acx-c}{ax}}+33\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48a^3},\ \frac{\left(8a^3x^3+22a^2x^2+33ax\right)\sqrt{\frac{acx-c}{ax}}-33\cdots}{24a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) + 33*sqr
t(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, 1/24*(
(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*arc
tan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.25853, size = 173, normalized size = 1.65

$$\frac{1}{24}\sqrt{a^2cx^2-acx}\left(2x\left(\frac{4x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{11|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{33|a|}{a^4\mathrm{sgn}\,(x)}\right)+\frac{11\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}\,(x)}{16a^3}-\frac{11\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2cx}-\sqrt{a^2\cdots}\right.\right.\right.}{16a^3\mathrm{sgn}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^3
*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) + 11/16*sqrt(c)*log(abs(a)*sqrt(abs(c))
)*sgn(x)/a^3 - 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a
*c*x))*abs(a) + a*sqrt(c)))/(a^3*sgn(x))
```

3.500 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=80

$$\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] (7*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (7*Sqrt[c]*ArcT
anh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rubi [A] time = 0.240854, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {6167, 6133, 25, 434, 446, 78, 51, 63, 208}

$$\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```


```
[Out] (7*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (7*Sqrt[c]*ArcT
anh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}x(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{(7c)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}\\
&= \frac{7\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2-\frac{(7c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}\\
&= \frac{7\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{7\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{4a}\\
&= \frac{7\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.0713571, size = 77, normalized size = 0.96

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{1-\frac{1}{ax}}(2ax+7)+7\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Maple [B] time = 0.161, size = 139, normalized size = 1.7

$$\frac{x}{8}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\,\sqrt{ax^2-x}a^{5/2}x-2\,\sqrt{ax^2-x}a^{3/2}+16\,a^{3/2}\sqrt{(ax-1)\,x}+8\,a\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1}{\sqrt{a}}\right)-\ln\left(\frac{1}{2}\left(2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(5/2)*x-2*(a*x^2-x)^(1/2)*
a^(3/2)+16*a^(3/2)*((a*x-1)*x)^(1/2)+8*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/
((a*x-1)*x)^(1/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/(a*x - 1), x)
```

Fricas [A] time = 1.54527, size = 327, normalized size = 4.09

$$\left[\frac{2\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}+7\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},\ \frac{\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{4\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) + 7*sqrt(c)*log(-2*a*c*
x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, 1/4*((2*a^2*x^2 + 7*a*
x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a
*x))/c))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.24579, size = 153, normalized size = 1.91

$$\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2\,x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{7\,|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{7\,\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}\,(x)}{8\,a^2}-\frac{7\,\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)|a|+a\sqrt{c}\right|\right)}{8\,a^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x)
)) + 7/8*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a^2 - 7/8*sqrt(c)*log(abs(
-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a^2*sgn(
x))
```

3.501 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=50

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] time = 0.146272, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 375, 78, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```


```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{1-ax}\,dx$$
$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a}$$
$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \sqrt{c-\frac{c}{ax}}x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \sqrt{c-\frac{c}{ax}}x + 3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$
$$= \sqrt{c-\frac{c}{ax}}x + \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.0391691, size = 50, normalized size = 1.

$$x\sqrt{c-\frac{c}{ax}} + \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] time = 0.165, size = 118, normalized size = 2.4

$$\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\,\sqrt{(ax-1)\,x}\sqrt{a}-2\,\sqrt{ax^{2}-x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\,\sqrt{ax^{2}-x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)+2\,\ln\left(1/2\,\frac{2\,\sqrt{(ax-1)\,x}\sqrt{a}+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x-1)*x)^(1/2)*a^(1/2)-2*(a*x^2-x)^(1/2)*
a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))+2*ln(1/2*(2*((a
*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Fricas [A] time = 1.64124, size = 273, normalized size = 5.46

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c
```

```
)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [B] time = 1.24782, size = 131, normalized size = 2.62

$$\frac{3\,\sqrt{c}\log\left(|a|\sqrt{|c|}\right)\mathrm{sgn}\left(x\right)}{2\,a}-\frac{3\,\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)|a|+a\sqrt{c}\right|\right)}{2\,a\mathrm{sgn}\left(x\right)}+\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(c)*log(abs(a)*sqrt(abs(c)))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*abs(a) + a*sqrt(c)))/(a*sgn(x)) + sqr
t(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

3.502 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=47

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rubi [A] time = 0.329759, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6133, 25, 514, 446, 80, 63, 208}

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^2}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}\,dx}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= 2\sqrt{c-\frac{c}{ax}} - c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= 2\sqrt{c-\frac{c}{ax}} + (2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0326874, size = 47, normalized size = 1.

$$
2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Maple [B] time = 0.17, size = 99, normalized size = 2.1

$$-\frac{1}{x}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-2\,a^{3/2}\sqrt{(ax-1)\,x}x^{2}+2\,\left(ax^{2}-x\right)^{3/2}\sqrt{a}-\ln\left(\frac{1}{2}\left(2\,\sqrt{(ax-1)\,x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)x^{2}a\right)\frac{1}{\sqrt{(ax-1)\,x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(-2*a^(3/2)*((a*x-1)*x)^(1/2)*x^2+2*(a*x^2-x)^(3/2
)*a^(1/2)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((a*
x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x), x)
```

Fricas [A] time = 1.62098, size = 247, normalized size = 5.26

$$\left[\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,\sqrt{\frac{acx-c}{ax}},-2\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+2\,\sqrt{\frac{acx-c}{ax}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*sqrt
((a*c*x - c)/(a*x)), -2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c)
 + 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [A] time = 9.74763, size = 39, normalized size = 0.83

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] -2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 2*sqrt(c - c/(a*x))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.503 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=42

$$4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

```
[Out] 4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c)
```

Rubi [A] time = 0.32396, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 444, 43}

$$4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```


```
[Out] 4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
```

```
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^2(1-ax)}\,dx$$
$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^3}\,dx}{a}$$
$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}\,dx}{a}$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{c\,\text{Subst}\left(\int \left(\frac{2a}{\sqrt{c-\frac{cx}{a}}}-\frac{a\sqrt{c-\frac{cx}{a}}}{c}\right)dx, x, \frac{1}{x}\right)}{a}$$
$$= 4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

Mathematica [A] time = 0.0385772, size = 28, normalized size = 0.67

$$\frac{2(5ax+1)\sqrt{c-\frac{c}{ax}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

Maple [A] time = 0.122, size = 27, normalized size = 0.6

$$\frac{10\,ax+2}{3\,x}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] 2/3*(5*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^2), x)
```

Fricas [A] time = 1.54706, size = 58, normalized size = 1.38

$$\frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.504 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+4a^2\sqrt{c-\frac{c}{ax}}$$

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/c + (2*a^2*(c - c/(a*
x))^(5/2))/(5*c^2)
```

Rubi [A] time = 0.332806, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 446, 77}

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+4a^2\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```


```
[Out] 4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/c + (2*a^2*(c - c/(a*
x))^(5/2))/(5*c^2)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^3(1-ax)}\,dx$$
$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^4}\,dx}{a}$$
$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}\,dx}{a}$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{x(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{c\,\text{Subst}\left(\int \left(\frac{2a^2}{\sqrt{c-\frac{cx}{a}}} - \frac{3a^2\sqrt{c-\frac{cx}{a}}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{a}$$
$$= 4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

Mathematica [A] time = 0.0457306, size = 36, normalized size = 0.52

$$\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

Maple [A] time = 0.125, size = 35, normalized size = 0.5

$$\frac{12\,a^2x^2+6\,ax+2}{5\,x^2}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 2/5*(6*a^2*x^2+3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^3), x)
```

Fricas [A] time = 1.57498, size = 77, normalized size = 1.12

$$\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.505 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}$$

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] - (10*a^3*(c - c/(a*x))^(3/2))/(3*c) + (8*a^3*(c -
c/(a*x))^(5/2))/(5*c^2) - (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rubi [A] time = 0.351501, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 446, 77}

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] - (10*a^3*(c - c/(a*x))^(3/2))/(3*c) + (8*a^3*(c -
c/(a*x))^(5/2))/(5*c^2) - (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^4(1-ax)}\,dx$$
$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^5}\,dx}{a}$$
$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^4}\,dx}{a}$$
$$= -\frac{c\,\mathrm{Subst}\left(\int \frac{x^2(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a^3}{\sqrt{c-\frac{cx}{a}}} - \frac{5a^3\sqrt{c-\frac{cx}{a}}}{c} + \frac{4a^3\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}\right)dx, x, \frac{1}{x}\right)}{a}$$
$$= 4a^3\sqrt{c-\frac{c}{ax}} - \frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

Mathematica [A] time = 0.0510289, size = 44, normalized size = 0.46

$$\frac{2\left(104a^3x^3+52a^2x^2+39ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

Maple [A] time = 0.116, size = 43, normalized size = 0.5

$$\frac{208\,x^3a^3+104\,a^2x^2+78\,ax+30}{105\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] 2/105*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^4), x)
```

Fricas [A] time = 1.55228, size = 103, normalized size = 1.07

$$\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.506 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=121

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}$$

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] - (14*a^4*(c - c/(a*x))^(3/2))/(3*c) + (18*a^4*(c -
 c/(a*x))^(5/2))/(5*c^2) - (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c
 - c/(a*x))^(9/2))/(9*c^4)
```

Rubi [A] time = 0.377641, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 446, 77}

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] - (14*a^4*(c - c/(a*x))^(3/2))/(3*c) + (18*a^4*(c -
 c/(a*x))^(5/2))/(5*c^2) - (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c
 - c/(a*x))^(9/2))/(9*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^5(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}x^6}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^5}\,dx}{a}$$

$$= -\frac{c\,\mathrm{Subst}\left(\int \frac{x^3(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a^4}{\sqrt{c-\frac{cx}{a}}} - \frac{7a^4\sqrt{c-\frac{cx}{a}}}{c} + \frac{9a^4\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{5a^4\left(c-\frac{cx}{a}\right)^{5/2}}{c^3} + \frac{a^4\left(c-\frac{cx}{a}\right)^{7/2}}{c^4}\right)dx, x, \frac{1}{x}\right)}{a}$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} - \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

Mathematica [A] time = 0.0608266, size = 52, normalized size = 0.43

$$\frac{2\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)\sqrt{c-\frac{c}{ax}}}{315x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^4
))/(315*x^4)
```

Maple [A] time = 0.119, size = 51, normalized size = 0.4

$$\frac{544\,x^4a^4+272\,x^3a^3+204\,a^2x^2+170\,ax+70}{315\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)*(c*(a*x-1)/a/x)^(1/2)
/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^5), x)
```

Fricas [A] time = 1.49116, size = 123, normalized size = 1.02

$$\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x -
 c)/(a*x))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.507 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=313

$$\frac{107x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{149x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{363\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}+\frac{x^4\sqrt{\cdots}}{\cdots}$$

```
[Out] (149*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) + (1
07*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) + (1
7*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]) + (Sqrt
[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]) + (363*Sqrt[c -
c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2
]*Sqrt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^4*Sqrt[1 - 1/(a*
x)])
```

Rubi [A] time = 0.349143, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6182, 6180, 98, 151, 156, 63, 208, 206}

$$\frac{107x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{149x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{363\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}+\frac{x^4\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (149*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) + (1
07*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) + (1
7*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]) + (Sqrt
[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]) + (363*Sqrt[c -
c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2
]*Sqrt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^4*Sqrt[1 - 1/(a*
x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
```

```
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^5\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{17}{2a}-\frac{15x}{2a^2}}{x^4\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\frac{107}{4a^2}+\frac{85x}{4a^3}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{12\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}}{24}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{4\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{4\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{4\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{4\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.915673, size = 252, normalized size = 0.81

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}(48a^3x^3+136a^2x^2+214ax+447)\sqrt{c-\frac{c}{ax}}}{ax-1}+1089\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-768\sqrt{2}\sqrt{c}\,\mathrm{lo}}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(447 + 214*a*x + 136*a^
2*x^2 + 48*a^3*x^3))/(-1 + a*x) - 1089*Sqrt[c]*Log[1 - a*x] + 768*Sqrt[2]*S
qrt[c]*Log[(-1 + a*x)^2] + 1089*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 768*Sqrt[2]*Sqrt[c
]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c
*(-1 - 2*a*x + 3*a^2*x^2)])/(384*a^4)
```

Maple [A] time = 0.179, size = 224, normalized size = 0.7

$$\frac{(ax-1)\,x}{384\,ax+384}\sqrt{\frac{c\,(ax-1)}{ax}}\left(96\,a^{9/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^3+272\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^2+428\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x+894\,\sqrt{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/384/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(96*a
^(9/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^3+272*a^(7/2)*(1/a)^(1/2)*((a*x+1)*x
)^(1/2)*x^2+428*a^(5/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x+894*((a*x+1)*x)^(1/
2)*a^(3/2)*(1/a)^(1/2)-768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1
/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+1089*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*
a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(9/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.35615, size = 1285, normalized size = 4.11

$$\left[\frac{768\,\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+1089\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,\cdots}{\cdots}\right.}{768\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/768*(768*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13
*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1
089*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^
2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*
x - 1)) + 4*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x
)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4), 1/384*(
768*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)
) - 1089*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(48*a^5
*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.508 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=261

$$\frac{19x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{45\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{13x^2\sqrt{\frac{1}{ax}}}{12a}$$

```
[Out] (19*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) + (13*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]) + (45*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^3*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.316886, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6182, 6180, 98, 151, 156, 63, 208, 206}

$$\frac{19x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{45\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{13x^2\sqrt{\frac{1}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (19*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) + (13*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]) + (45*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^3*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{-\frac{13}{2a}-\frac{11x}{2a^2}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\frac{57}{4a^2}+\frac{39x}{4a^3}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{6\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int -\right.}{6\sqrt{1-}}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\right.}{a^4\sqrt{}}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\right.}{a^3\sqrt{}}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{45\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\right.}{8a^3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.719492, size = 244, normalized size = 0.93

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^2x^2+26ax+57\right)\sqrt{c-\frac{c}{ax}}}{ax-1} + 135\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) - 96\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x\right.}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(57 + 26*a*x + 8*a^2*x^
2))/(-1 + a*x) - 135*Sqrt[c]*Log[1 - a*x] + 96*Sqrt[2]*Sqrt[c]*Log[(-1 + a*
x)^2] + 135*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x
)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 96*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*S
qrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*
x^2)])/(48*a^3)
```

Maple [A] time = 0.174, size = 202, normalized size = 0.8

$$\frac{(ax-1)\,x}{48\,ax+48}\sqrt{\frac{c\,(ax-1)}{ax}}\left(16\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x^{2}+52\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x+114\,\sqrt{(ax+1)\,x}a^{3/2}\sqrt{a^{-1}}-96\,\sqrt{2}\ln\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^
(7/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2+52*a^(5/2)*(1/a)^(1/2)*((a*x+1)*x)^
(1/2)*x+114*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-96*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+135*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(7/2)/((a*x+1)*x)
^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}x^{2}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.509 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=209

$$\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}}+\frac{9x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (9*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]) + (23*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^2*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.21892, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {6182, 6180, 98, 151, 156, 63, 208, 206}

$$\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}}+\frac{9x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```


```
[Out] (9*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]) + (23*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^2*Sqrt[1 - 1/(a*x)])
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\frac{23}{4a^2}+\frac{9x}{4a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^3\sqrt{1-\frac{1}{ax}}} - ($$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{a^2\sqrt{1-\frac{1}{ax}}} -$$

$$= \frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}} + \frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}}{a^2}$$

Mathematica [A] time = 0.532059, size = 236, normalized size = 1.13

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}(2ax+9)\sqrt{c-\frac{c}{ax}}}{ax-1}+23\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-16\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\right.}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(9 + 2*a*x))/(-1 + a*x)
 - 23*Sqrt[c]*Log[1 - a*x] + 16*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + 23*Sqrt
[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 -
 a*x + 2*a^2*x^2)] - 16*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 -
1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]])/(8*a^2)
```

Maple [A] time = 0.176, size = 180, normalized size = 0.9

$$\frac{(ax-1)\,x}{8\,ax+8}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\,a^{5/2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}x+18\,\sqrt{(ax+1)\,x}a^{3/2}\sqrt{a^{-1}}-16\,\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/8/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5
/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x+18*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)-16*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1)
)*a^(1/2)+23*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^
(1/2))/a^(5/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 2.20549, size = 1187, normalized size = 5.68

$$\left[\frac{16\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+23(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3\right.}{}\right.}{16\left(a^3x-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(16*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 23*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 4*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2
*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.510 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.126063, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 156, 63, 208, 206}

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1}}\right.}{2a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\right.}{a\sqrt{1-}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{ax}}} + \frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0792743, size = 93, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}+5\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 5*ArcTanh[Sqrt[1 + 1/(a*x)]] -
4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A] time = 0.181, size = 160, normalized size = 1.1

$$\frac{(ax-1)x}{2ax+2}\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax+1)x}a^{3/2}\sqrt{a^{-1}}-4\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)x}a+3ax+1}{ax-1}\right)\sqrt{a}+5\ln\left(1/2\frac{2\sqrt{(ax+1)x}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*
x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x
+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1
/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.511 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=146

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)] - (4*Sqrt[2]*Sqrt[c - c
/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.272623, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6182, 6180, 84, 156, 63, 208, 206}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)] - (4*Sqrt[2]*Sqrt[c - c
/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
```

```
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx \cdots\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+\cdots}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.369214, size = 218, normalized size = 1.49

$$\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{ax-1} + \sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) - 2\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x)/(-1 + a*x) - Sqrt[c]*Log[1
- a*x] + 2*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 2*S
qrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(
a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] time = 0.18, size = 161, normalized size = 1.1

$$-\frac{ax-1}{ax+1}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{a}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x-\ln\left(\frac{1}{2}\left(2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1\right)\frac{1}{\sqrt{a}}\right)\sqrt{a^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] -1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(2*a^(1/2)
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-
ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*(1/a)^(1/2)*x*a-2*((a
*x+1)*x)^(1/2)*a^(1/2)*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.512 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=125

$$\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] (2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a*c*Sqrt[1 -
 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rubi [A] time = 0.239222, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 665, 661, 208}

$$\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```


```
[Out] (2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a*c*Sqrt[1 -
 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
```

```
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\left(c^3\operatorname{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}-\left(2c^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-(4c)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\left(8c^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}\\
&= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.247701, size = 155, normalized size = 1.24

$$
\frac{2a\left(\sqrt{1-\frac{1}{a^2x^2}}(7ax+1)\sqrt{c-\frac{c}{ax}}-3\sqrt{2}\sqrt{c}(ax-1)\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(3a^2x^2-2ax-1\right)\right)+3\sqrt{2}\sqrt{c}(ax}{3ax-3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + 7*a*x) + 3*Sqrt[2]*Sqrt[
c]*(-1 + a*x)*Log[(-1 + a*x)^2] - 3*Sqrt[2]*Sqrt[c]*(-1 + a*x)*Log[2*Sqrt[2
]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x +
 3*a^2*x^2)]))/(-3 + 3*a*x)
```

Maple [A] time = 0.181, size = 141, normalized size = 1.1

$$-\frac{2\,ax-2}{(3\,ax+3)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\left(3\,a\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)\,x}a+3\,ax+1}{ax-1}\right)x^2-7\,a\sqrt{a^{-1}}x\sqrt{(ax+1)\,x}-\sqrt{a^{-1}}\sqrt{(ax+1)\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*(3*a*2
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-
7*a*(1/a)^(1/2)*x*((a*x+1)*x)^(1/2)-(1/a)^(1/2)*((a*x+1)*x)^(1/2))/((a*x+1)
*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.95061, size = 770, normalized size = 6.16

$$\left[\frac{3\sqrt{2}\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\left(7\,a^2x^2+8\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] [1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
 2*(7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x^2 - x), 2/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(2*sqrt(2)*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3
*a^2*c*x^2 - 2*a*c*x - c)) + (7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.513 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=170

$$\frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] (2*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + (2*a^2*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a^2*c*Sqrt[1 - 1/(a^2*x
^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rubi [A] time = 0.299706, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6178, 795, 665, 661, 208}

$$\frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```


```
[Out] (2*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + (2*a^2*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a^2*c*Sqrt[1 - 1/(a^2*x
^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} - \left(ac^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \left(8c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x\right.$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right.$$

<u>Mathematica</u> [A] time = 0.269863, size = 162, normalized size = 0.95

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(38a^2x^2+11ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x(ax-1)} - 2\sqrt{2}a^2\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(3a^2x^2-2ax-1\right)\right) + 2\sqrt{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 11*a*x + 38*a^2*x^2))/(15
*x*(-1 + a*x)) + 2*Sqrt[2]*a^2*Sqrt[c]*Log[(-1 + a*x)^2] - 2*Sqrt[2]*a^2*Sq
rt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2
 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] time = 0.214, size = 165, normalized size = 1.

$$-\frac{2ax-2}{(15ax+15)x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(15a^2\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)x}a+3ax+1}{ax-1}\right)x^3-38a^2\sqrt{a^{-1}}x^2\sqrt{(ax+1)x}-11a\sqrt{a^{-1}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2*(15
*a^2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1)
)*x^3-38*a^2*(1/a)^(1/2)*x^2*((a*x+1)*x)^(1/2)-11*a*(1/a)^(1/2)*x*((a*x+1)*
x)^(1/2)-3*(1/a)^(1/2)*((a*x+1)*x)^(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.91223, size = 832, normalized size = 4.89

$$\left[\frac{15\sqrt{2}\left(a^3x^3-a^2x^2\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(38a^3x^3+49a^2x^2+14ax+3\right.}{15\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(38*a^3*x^3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2), 2/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)
*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (38*a^3*x^3 + 49*
a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a
*x^3 - x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.514 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=209

$$-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] (4*a^3*c^3*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(5/2)) + (2*a^3*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) - (2*a^3*c^2*(1 - 1/(a^2*x^2
))^(5/2))/(7*(c - c/(a*x))^(3/2)) + (4*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
- c/(a*x)] - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rubi [A] time = 0.476001, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6178, 1639, 795, 665, 661, 208}

$$-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```


```
[Out] (4*a^3*c^3*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(5/2)) + (2*a^3*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) - (2*a^3*c^2*(1 - 1/(a^2*x^2
))^(5/2))/(7*(c - c/(a*x))^(3/2)) + (4*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
- c/(a*x)] - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
(Sqrt[2]*Sqrt[c - c/(a*x)])]
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{1}{7}\left(2a^4c\right)\text{Subst}\left(\int \frac{\left(\frac{3c^2}{2a^2}-\frac{5c^2x}{a^3}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - \left(a^2c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2a^2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{\left(c-\frac{cx}{a}\right)}\right.$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \left(4a^2c\right)\text{Su}$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \left(8ac^2\right)\text{Su}$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a^3\sqrt{c}$$

Mathematica [A] time = 0.283286, size = 170, normalized size = 0.81

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(52a^3x^3+16a^2x^2+9ax+3\right)\sqrt{c-\frac{c}{ax}}}{21x^2(ax-1)} - 2\sqrt{2}a^3\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(3a^2x^2-2ax-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 9*a*x + 16*a^2*x^2 + 52*a
^3*x^3))/(21*x^2*(-1 + a*x)) + 2*Sqrt[2]*a^3*Sqrt[c]*Log[(-1 + a*x)^2] - 2*
Sqrt[2]*a^3*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
```

```
- c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] time = 0.187, size = 187, normalized size = 0.9

$$-\frac{2\,ax-2}{\left(21\,ax+21\right)x^{3}}\sqrt{\frac{c\left(ax-1\right)}{ax}}\left(21\,a^{3}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{\left(ax+1\right)x}a+3\,ax+1}{ax-1}\right)x^{4}-52\,a^{3}\sqrt{a^{-1}}x^{3}\sqrt{\left(ax+1\right)x}-16\,a^{2}\sqrt{a^{-1}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^3*(21
*a^3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1)
)*x^4-52*a^3*(1/a)^(1/2)*x^3*((a*x+1)*x)^(1/2)-16*a^2*(1/a)^(1/2)*x^2*((a*x
+1)*x)^(1/2)-9*a*(1/a)^(1/2)*x*((a*x+1)*x)^(1/2)-3*(1/a)^(1/2)*((a*x+1)*x)^
(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 2.01236, size = 867, normalized size = 4.15

$$\left[\frac{21\,\sqrt{2}\left(a^{4}x^{4}-a^{3}x^{3}\right)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}\left(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+2\left(52\,a^{4}x^{4}+68\,a^{3}x^{3}+25\,a^{2}x^{2}+\right.}{21\left(ax^{4}-x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] [1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3), 2/21*(21*sqrt(2)*(a^4*x
^4 - a^3*x^3)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (52*
a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.515 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=303

$$\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4a^4\sqrt{\frac{1}{ax}+1}\sqrt{c}}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (4*a^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*a^4*(1 +
 1/(a*x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a
*x))^(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) - (2*a^4*(1 + 1/(a*x))^
(7/2)*Sqrt[c - c/(a*x)])/(7*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(9/2)
*Sqrt[c - c/(a*x)])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*a^4*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.320336, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6182, 6180, 88, 50, 63, 206}

$$\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4a^4\sqrt{\frac{1}{ax}+1}\sqrt{c}}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```


```
[Out] (4*a^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*a^4*(1 +
 1/(a*x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a
*x))^(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) - (2*a^4*(1 + 1/(a*x))^
(7/2)*Sqrt[c - c/(a*x)])/(7*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(9/2)
*Sqrt[c - c/(a*x)])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*a^4*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-a^3\left(1+\frac{x}{a}\right)^{3/2} + \frac{a^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}} + a^3\left(1+\frac{x}{a}\right)^{5/2} - a^3\left(1+\frac{x}{a}\right)^{7/2}\right)dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(a^3\sqrt{c-\frac{c}{ax}}\right)\text{S}}{}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9}}{9\sqrt{1-}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.297134, size = 178, normalized size = 0.59

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(788a^4x^4+236a^3x^3+138a^2x^2+95ax+35\right)\sqrt{c-\frac{c}{ax}}}{315x^3(ax-1)} - 2\sqrt{2}a^4\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(35 + 95*a*x + 138*a^2*x^2 + 2
36*a^3*x^3 + 788*a^4*x^4))/(315*x^3*(-1 + a*x)) + 2*Sqrt[2]*a^4*Sqrt[c]*Log
[(-1 + a*x)^2] - 2*Sqrt[2]*a^4*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A] time = 0.195, size = 209, normalized size = 0.7

$$-\frac{2ax-2}{(315ax+315)x^4}\sqrt{\frac{c(ax-1)}{ax}}\left(315a^4\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax+1)x}a+3ax+1}{ax-1}\right)x^5-788a^4\sqrt{a^{-1}}x^4\sqrt{(ax+1)x}-236a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] -2/315/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^4*(3
15*a^4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-
1))*x^5-788*a^4*(1/a)^(1/2)*x^4*((a*x+1)*x)^(1/2)-236*a^3*(1/a)^(1/2)*x^3*(
(a*x+1)*x)^(1/2)-138*a^2*(1/a)^(1/2)*x^2*((a*x+1)*x)^(1/2)-95*a*(1/a)^(1/2)
*x*((a*x+1)*x)^(1/2)-35*(1/a)^(1/2)*((a*x+1)*x)^(1/2))/((a*x+1)*x)^(1/2)/(1
/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.94215, size = 926, normalized size = 3.06

$$\left[\frac{315\sqrt{2}\left(a^5x^5-a^4x^4\right)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\left(788\,a^5x^5+1024\,a^4x^4+374\,a\right.}{315\left(ax^5-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 2*(788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2 + 130*a*
x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^4), 2
/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) + (788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2
 + 130*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5
- x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.516 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{\frac{1}{ax}+1}x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)\sqrt{1-\frac{1}{ax}}}-\frac{(4m+3)x^m\sqrt{c-\frac{c}{ax}}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a*x)])
 - ((3 + 4*m)*Sqrt[c - c/(a*x)]*x^m*Hypergeometric2F1[1/2, -m, 1 - m, -(1/(
a*x))])/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.266844, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6182, 6181, 79, 64}

$$\frac{\sqrt{\frac{1}{ax}+1}x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)\sqrt{1-\frac{1}{ax}}}-\frac{(4m+3)x^m\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a*x)])
 - ((3 + 4*m)*Sqrt[c - c/(a*x)]*x^m*Hypergeometric2F1[1/2, -m, 1 - m, -(1/(
a*x))])/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx &= \frac{\sqrt{c-\frac{c}{ax}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^m\,dx}{\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int\frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}+\frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a(1+m)\sqrt{1-\frac{1}{ax}}}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}-\frac{(3+4m)\sqrt{c-\frac{c}{ax}}x^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] time = 0.0660519, size = 93, normalized size = 0.74

$$\frac{x^m\sqrt{c-\frac{c}{ax}}\left(2amx\sqrt{\frac{1}{ax}+1}-(4m+3)\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^m*(2*a*m*Sqrt[1 + 1/(a*x)]*x - (3 + 4*m)*Hypergeometri
c2F1[1/2, -m, 1 - m, -(1/(a*x))]))/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int x^m\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.517 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=164

$$\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{11cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}+\frac{11cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] (11*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) - (11*c*Sqrt[1 - 1
/(a^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(
3*Sqrt[c - c/(a*x)]) - (11*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
Sqrt[c - c/(a*x)]])/(8*a^3)
```

Rubi [A] time = 0.360375, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6178, 879, 873, 875, 208}

$$\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{11cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}+\frac{11cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```


```
[Out] (11*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) - (11*c*Sqrt[1 - 1
/(a^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(
3*Sqrt[c - c/(a*x)]) - (11*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
Sqrt[c - c/(a*x)]])/(8*a^3)
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{11\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= -\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{11\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} - \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{11\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$

$$= \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} - \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{(11c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^5}$$

$$= \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}} - \frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Mathematica [A] time = 0.470569, size = 147, normalized size = 0.9

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^2x^2-22ax+33\right)\sqrt{c-\frac{c}{ax}}}{ax-1} - 33\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) + 33\sqrt{c}\log(1-ax)}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(33 - 22*a*x + 8*a^2*x^
2))/(-1 + a*x) + 33*Sqrt[c]*Log[1 - a*x] - 33*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a
^3)
```

Maple [A] time = 0.185, size = 133, normalized size = 0.8

$$\frac{(ax+1)\,x}{48\,ax-48}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(16\,a^{5/2}x^2\sqrt{(ax+1)\,x}-44\,a^{3/2}x\sqrt{(ax+1)\,x}+66\,\sqrt{(ax+1)\,x}\sqrt{a}-33\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)+66*((a*x+1)*x)^(1/2)*a^(
1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(a*x
-1)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.86933, size = 729, normalized size = 4.45

$$\left[\frac{33\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^4x^4-14\,a^3x^3+11\,a^2x^2+33\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(33*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(8*a^4*x^4 - 14*a^3*x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(33*(a*x - 1)*
sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 14*a^3*x^3 +
11*a^2*x^2 + 33*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
4*x - a^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}} x^2 \sqrt{\frac{ax - 1}{ax + 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.518 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=124

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}-\frac{7cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] (-7*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(2*Sqrt[c - c/(a*x)]) + (7*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rubi [A] time = 0.254909, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6178, 879, 873, 875, 208}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}-\frac{7cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```


```
[Out] (-7*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(2*Sqrt[c - c/(a*x)]) + (7*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{4a}$$

$$=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{7\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{(7c^2)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}$$

$$=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.494314, size = 139, normalized size = 1.12

$$\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}(2ax-7)\sqrt{c-\frac{c}{ax}}}{4ax-4}+\frac{7\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)}{8a^2}-\frac{7\sqrt{c}\log(1-ax)}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-7 + 2*a*x))/(-4 + 4*a*x) - (
7*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (7*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2)
```

Maple [A] time = 0.181, size = 116, normalized size = 0.9

$$\frac{(ax+1)\,x}{8\,ax-8}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}\left(4\,a^{3/2}x\sqrt{(ax+1)\,x}-14\,\sqrt{(ax+1)\,x}\sqrt{a}+7\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)\right)a^{-\frac{3}{2}}\frac{}{\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(4*a^(3
/2)*x*((a*x+1)*x)^(1/2)-14*((a*x+1)*x)^(1/2)*a^(1/2)+7*ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.79098, size = 686, normalized size = 5.53

$$\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2\,a^3x^3-5\,a^2x^2-7\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)},-\frac{7\,(ax-1)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*
```

```
sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(7*(a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.519 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=79

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rubi [A] time = 0.158053, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 879, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```


```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```


Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
```

```
1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.0494984, size = 65, normalized size = 0.82

$$\frac{\sqrt{c-\frac{c}{ax}}\left(x\sqrt{\frac{1}{ax}+1}-\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

Maple [A] time = 0.188, size = 101, normalized size = 1.3

$$\frac{(ax+1)x}{2ax-2}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}\left(2\sqrt{(ax+1)x}\sqrt{a}-3\ln\left(1/2\,\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{(ax+1)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [B] time = 1.76575, size = 635, normalized size = 8.04

$$\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{3\,(ax-1)\sqrt{-c}\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.520 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=76

$$\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] (2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rubi [A] time = 0.244024, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 881, 875, 208}

$$\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}\\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.232706, size = 132, normalized size = 1.74

$$\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \sqrt{c}(ax-1)\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) + \sqrt{c}(1-ax)\log(1-ax)}{ax-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Maple [A] time = 0.196, size = 100, normalized size = 1.3

$$\frac{ax+1}{ax-1}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)x}\sqrt{a}+2ax+1\right)\frac{1}{\sqrt{a}}\right)xa+2\sqrt{(ax+1)x}\sqrt{a}\right)\frac{1}{\sqrt{(ax+1)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a+2*((a*x+1)*x)^(1/2)*a^(1/2))/(a*x-1)/
((a*x+1)*x)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [B] time = 1.73686, size = 603, normalized size = 7.93

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)},-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.521 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-8*a*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (2*a*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)])/3
```

Rubi [A] time = 0.184962, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6178, 657, 649}

$$-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```



```
[Out] (-8*a*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (2*a*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)])/3
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
```

```
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\ &= -\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{4}{3}\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\end{aligned}$$

Mathematica [A] time = 0.0815907, size = 46, normalized size = 0.66

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}(5ax-1)\sqrt{c-\frac{c}{ax}}}{3ax-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 5*a*x))/(-3 + 3*a*x)
```

Maple [A] time = 0.125, size = 54, normalized size = 0.8

$$-\frac{(2\,ax+2)\,(5\,ax-1)}{(3\,ax-3)\,x}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] -2/3*(a*x+1)*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A] time = 1.55286, size = 124, normalized size = 1.77

$$-\frac{2\left(5\,a^2x^2 + 4\,ax - 1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2 - x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x))/(a*x^2 - x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.522 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (8*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(5*Sqrt[c - c/(a*x)]) + (2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)])/5 + (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))
^(3/2))/(5*c)
```

Rubi [A] time = 0.223833, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 795, 657, 649}

$$\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (8*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(5*Sqrt[c - c/(a*x)]) + (2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)])/5 + (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))
^(3/2))/(5*c)
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
```

```
x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{(3a)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{1}{5}(4a)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}} + \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

Mathematica [A] time = 0.105657, size = 58, normalized size = 0.51

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(6a^2x^2-3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 - 3*a*x + 6*a^2*x^2))/(5*x*
(-1 + a*x))
```

Maple [A] time = 0.118, size = 62, normalized size = 0.6

$$\frac{(2\,ax+2)\left(6\,a^2x^2-3\,ax+1\right)}{5\,x^2\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

```
[Out] 2/5*(a*x+1)*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/
2)/x^2/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Fricas [A] time = 1.5235, size = 142, normalized size = 1.26

$$\frac{2\left(6\,a^3x^3+3\,a^2x^2-2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{5\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] 2/5*(6*a^3*x^3 + 3*a^2*x^2 - 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

3.523 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35x^2\sqrt{c-\frac{c}{ax}}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-104*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) - (104*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (2*c*Sqrt[1 - 1/(a^2*x^2)])/(7*Sq
rt[c - c/(a*x)]*x^3) - (26*a*c*Sqrt[1 - 1/(a^2*x^2)])/(35*Sqrt[c - c/(a*x)]
*x^2)
```

Rubi [A] time = 0.302801, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6178, 881, 871, 795, 649}

$$-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35x^2\sqrt{c-\frac{c}{ax}}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```


```
[Out] (-104*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) - (104*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (2*c*Sqrt[1 - 1/(a^2*x^2)])/(7*Sq
rt[c - c/(a*x)]*x^3) - (26*a*c*Sqrt[1 - 1/(a^2*x^2)])/(35*Sqrt[c - c/(a*x)]
*x^2)
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{13}{7}\,\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2} + \frac{1}{35}(52a)\,\text{Subst}\left(\int \frac{x\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2} - \frac{1}{105}\left(52a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-}}{\sqrt{1-}}\right.$$

$$= -\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} - \frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2}$$

Mathematica [A] time = 0.123479, size = 66, normalized size = 0.44

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(104a^3x^3-52a^2x^2+39ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^2(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 39*a*x - 52*a^2*x^2 +
104*a^3*x^3))/(105*x^2*(-1 + a*x))
```

Maple [A] time = 0.112, size = 70, normalized size = 0.5

$$-\frac{(2\,ax+2)\left(104\,x^3a^3-52\,a^2x^2+39\,ax-15\right)}{105\,x^3\,(ax-1)}\sqrt{\frac{c\,(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+1)*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*((a
*x-1)/(a*x+1))^(1/2)/x^3/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Fricas [A] time = 1.55549, size = 170, normalized size = 1.14

$$-\frac{2\left(104\,a^4x^4 + 52\,a^3x^3 - 13\,a^2x^2 + 24\,ax - 15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4 - x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] -2/105*(104*a^4*x^4 + 52*a^3*x^3 - 13*a^2*x^2 + 24*a*x - 15)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

3.524 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=172

$$\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}-\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

```
[Out] (-149*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2)
- (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (363*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) - (4*Sqrt[2]*Sqrt[c]*ArcTa
nh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```

Rubi [A] time = 0.469543, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {6167, 6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}-\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (-149*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2)
- (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (363*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) - (4*Sqrt[2]*Sqrt[c]*ArcTa
nh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^3(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^5(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{\mathrm{Subst}\left(\int \frac{\frac{17c^2}{2}-\frac{15c^2x}{2a}}{x^4(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= -\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{\mathrm{Subst}\left(\int \frac{\frac{107c^3}{4}-\frac{85c^3x}{4a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{12ac^2}$$

$$= \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{\mathrm{Subst}\left(\int \frac{\frac{447c^4}{8}-\frac{321c^4x}{8a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{24a^2c^3}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{\mathrm{Subst}\left(\int \frac{\frac{1089c^5}{16}-\frac{447c^5}{16a}}{x(a+x)\sqrt{c-\frac{c}{a}}}\right.}{24a^3c^4}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 - \frac{(363c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c}}\right.}{128a^4}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{363\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\right.}{64a^3}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4 + \frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{c}}\right.}{64a^4}$$

Mathematica [A] time = 0.154629, size = 116, normalized size = 0.67

$$\frac{ax\left(48a^3x^3-136a^2x^2+214ax-447\right)\sqrt{c-\frac{c}{ax}}+1089\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-768\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-447 + 214*a*x - 136*a^2*x^2 + 48*a^3*x^3) + 1089*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 768*Sqrt[2]*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

Maple [A] time = 0.161, size = 259, normalized size = 1.5

$$-\frac{x}{384}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-96\,x\left(ax^2-x\right)^{3/2}a^{9/2}\sqrt{a^{-1}}+176\,\left(ax^2-x\right)^{3/2}a^{7/2}\sqrt{a^{-1}}-252\,\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x+768\,\sqrt{(ax-1)\,x}a^{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(-96*x*(a*x^2-x)^(3/2)*a^(9/2)*(1/a)^(1/2)+1
76*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)-252*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1
/2)*x+768*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)+126*(a*x^2-x)^(1/2)*a^(5/2)
*(1/a)^(1/2)-768*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
)*a-3*a*x+1)/(a*x+1))-1152*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*(1/a)^(1/2)+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(11/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1), x)
```

Fricas [A] time = 1.6554, size = 663, normalized size = 3.85

$$\left[\frac{768\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+2\left(48a^4x^4-136a^3x^3+214a^2x^2-447ax\right)\sqrt{\frac{acx-c}{ax}}+1089\sqrt{c}\log\left(-2acx-2a\right.}{384a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2 -
 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, 1/192*(768*sqrt(2)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*x^3 +
 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arctan(sqrt
(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.525 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=147

$$\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] (19*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 - (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^3
```

Rubi [A] time = 0.451411, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {6167, 6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (19*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 - (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^3
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{\text{Subst}\left(\int \frac{\frac{13c^2}{2}-\frac{11c^2x}{2a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{\text{Subst}\left(\int \frac{\frac{57c^3}{4}-\frac{39c^3x}{4a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6ac^2}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{\text{Subst}\left(\int \frac{\frac{135c^4}{8}-\frac{57c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a^2c^3}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{(45c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3} - \frac{(4c)\,\text{Su}}{}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{45\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{8a^2} + \frac{8\,\text{Su}}{}$$

$$= \frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\right.}{a^3}$$

Mathematica [A] time = 0.107813, size = 108, normalized size = 0.73

$$\frac{ax\left(8a^2x^2-26ax+57\right)\sqrt{c-\frac{c}{ax}}-135\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+96\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(57 - 26*a*x + 8*a^2*x^2) - 135*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]] + 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt
[2]*Sqrt[c])])/(24*a^3)
```

Maple [A] time = 0.158, size = 237, normalized size = 1.6

$$\frac{x}{48}\sqrt{\frac{c\,(ax-1)}{ax}}\Bigg(16\,\left(ax^2-x\right)^{3/2}a^{7/2}\sqrt{a^{-1}}-36\,\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x+18\,\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}+96\,\sqrt{(ax-1)\,x}a^{5/2}\sqrt{a^{-1}}-9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)-36*(a*
x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x+18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)+9
6*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)-96*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1
/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))-144*a^2*ln(1/2*(2*((a*x-1)*
x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)*
a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(9/2)/(1/a)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

Fricas [A] time = 1.64023, size = 612, normalized size = 4.16

$$\left[\frac{96\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
)) + 3*a*c*x - c)/(a*x + 1)) + 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*
c*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)
/(a*x)) + c))/a^3, -1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c*x -
 c)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.526 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=122

$$\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] (-9*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (23*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rubi [A] time = 0.323909, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6167, 6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (-9*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (23*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{\mathrm{Subst}\left(\int \frac{\frac{9c^2}{2}-\frac{7c^2x}{2a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 - \frac{\mathrm{Subst}\left(\int \frac{\frac{23c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 - \frac{(23c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\right.}{a^2}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{23\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{4a} - \frac{8\,\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,\right.}{a}$$

$$= -\frac{9\sqrt{c-\frac{c}{ax}}x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2 + \frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Mathematica [A] time = 0.0943168, size = 100, normalized size = 0.82

$$\frac{ax(2ax-9)\sqrt{c-\frac{c}{ax}} + 23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 16\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-9 + 2*a*x) + 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]] - 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])
/(4*a^2)
```

Maple [B] time = 0.157, size = 216, normalized size = 1.8

$$\frac{x}{8}\sqrt{\frac{c\,(ax-1)}{ax}}\left(4\,\sqrt{ax^2-x}a^{7/2}\sqrt{a^{-1}}x-2\,\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}-16\,\sqrt{(ax-1)\,x}a^{5/2}\sqrt{a^{-1}}+16\,a^{3/2}\sqrt{2}\ln\left(\frac{2\,\sqrt{2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}a-3\,ax+1}{ax+1}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x-2*(a*x
^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-16*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)+16
*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*
x+1))+24*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1
/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((
a*x-1)*x)^(1/2)/a^(7/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

Fricas [A] time = 1.72295, size = 566, normalized size = 4.64

$$\left[\frac{16\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\left(2\,a^2x^2-9\,ax\right)\sqrt{\frac{acx-c}{ax}}+23\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2}, \frac{16\sqrt{2}\sqrt{-c}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 - 3*a*c*x + c)/(a*x + 1)) + 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x))
+ 23*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2
, 1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.527 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=92

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] time = 0.198132, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 98, 156, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}\\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \sqrt{c-\frac{c}{ax}}x+\frac{\mathrm{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \sqrt{c-\frac{c}{ax}}x+\frac{(5c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}-\frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \sqrt{c-\frac{c}{ax}}x-5\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)+8\,\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)\\
&= \sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0480419, size = 92, normalized size = 1.

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] time = 0.165, size = 190, normalized size = 2.1

$$-\frac{x}{2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(2\,\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}-4\,\sqrt{(ax-1)\,x}a^{3/2}\sqrt{a^{-1}}-\ln\left(\frac{1}{2}\left(2\,\sqrt{ax^2-x}\sqrt{a}+2\,ax-1\right)\frac{1}{\sqrt{a}}\right)a\sqrt{a^{-1}}+4\,\sqrt{2}\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x
-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*a-3*a*x+1)/(a*x+1))*a^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Fricas [A] time = 1.62031, size = 508, normalized size = 5.52

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\sqrt{-c}\,\mathrm{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.528 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.373219, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 434, 446, 84, 156, 63, 208}

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```



```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= 2\sqrt{c-\frac{c}{ax}} - \frac{a\,\text{Subst}\left(\int \frac{c^2-\frac{3c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= 2\sqrt{c-\frac{c}{ax}} - c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right) + (4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 2\sqrt{c-\frac{c}{ax}} + (2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) - (8a)\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\right.$$

$$= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.0423367, size = 86, normalized size = 1.

$$2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.174, size = 228, normalized size = 2.7

$$-\frac{1}{x}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-4\sqrt{ax^2-x}a^{3/2}\sqrt{a^{-1}}x^2+2\,a^{3/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^2+2\,\left(ax^2-x\right)^{3/2}\sqrt{a}\sqrt{a^{-1}}+2\,\ln\left(1/2\,\frac{2\sqrt{ax^2-x}\sqrt{a}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(-4*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)*x^2+2*a^(3
/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2+2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)
+2*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a-2*
a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x
+1))*x^2-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2
)*x^2*a)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

Fricas [A] time = 1.64372, size = 479, normalized size = 5.57

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\sqrt{\frac{acx-c}{ax}},4\sqrt{2}\sqrt{-c}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a
*c*x + c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sq
rt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*sqrt(-c)*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [A] time = 11.3001, size = 80, normalized size = 0.93

$$-\frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}c\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] -2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan(sqrt(2)*s
qrt(c - c/(a*x))/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(c - c/(a*x))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.529 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c) + 4*Sqrt[2]*a*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.37594, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6133, 25, 514, 444, 50, 63, 208}

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```


```
[Out] -4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c) + 4*Sqrt[2]*a*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^2(1+ax)}\,dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x(1+ax)}\,dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}\,dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - (2a)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$
$$= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - (4ac)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \left(8a^2\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$
$$= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + 4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.0702351, size = 69, normalized size = 0.84

$$\frac{2(1-7ax)\sqrt{c-\frac{c}{ax}}}{3x} + 4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 - 7*a*x))/(3*x) + 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.176, size = 254, normalized size = 3.1

$$\frac{1}{3x^2}\sqrt{\frac{c(ax-1)}{ax}}\left(6\sqrt{(ax-1)x}a^{5/2}\sqrt{a^{-1}}x^3-18\sqrt{ax^2-x}a^{5/2}\sqrt{a^{-1}}x^3+12a^{3/2}\left(ax^2-x\right)^{3/2}x\sqrt{a^{-1}}+9\ln\left(1/2\,\frac{2\sqrt{ax^2-x}\sqrt{}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] 1/3*(c*(a*x-1)/a/x)^(1/2)/x^2*(6*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x^3-
18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x^3+12*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/
a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*
x^3*a^2-6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a
*x+1)/(a*x+1))*x^3-9*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*
(1/a)^(1/2)*x^3*a^2-2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2
)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

Fricas [A] time = 1.59768, size = 374, normalized size = 4.56

$$\left[\frac{2\left(3\sqrt{2}a\sqrt{c}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x},-\frac{2\left(6\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a*sqrt(c)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + 3*a*c*x - c)/(a*x + 1)) - (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x, -2
/3*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.530 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^2\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/(3*c) + (2*a^2*(c - c
/(a*x))^(5/2))/(5*c^2) - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.393706, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 80, 50, 63, 208}

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^2\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/(3*c) + (2*a^2*(c - c
/(a*x))^(5/2))/(5*c^2) - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^3(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^2(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}\,dx}{c}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(2a^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(4a^2c\right)\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(8a^3\right)\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.0844854, size = 79, normalized size = 0.7

$$\frac{2\left(38a^2x^2-11ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x^2}-4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) - 4*Sqrt[2]*a^2*Sq
rt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.17, size = 278, normalized size = 2.5

$$-\frac{1}{15\,x^3}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-90\,a^{7/2}\sqrt{a^{-1}}\sqrt{ax^2-x}x^4+30\,a^{7/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^4+60\,a^{5/2}\sqrt{a^{-1}}\left(ax^2-x\right)^{3/2}x^2+45\,\sqrt{a^{-1}}\ln\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] -1/15*(c*(a*x-1)/a/x)^(1/2)/x^3*(-90*a^(7/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^
4+30*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+60*a^(5/2)*(1/a)^(1/2)*(a*x^
2-x)^(3/2)*x^2+45*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^
(1/2))*x^4*a^3-30*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*a-3*a*x+1)/(a*x+1))*x^4-45*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/
2)+2*a*x-1)/a^(1/2))*x^4*a^3-16*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)+6*(a*
x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

Fricas [A] time = 1.61618, size = 431, normalized size = 3.81

$$\left[\frac{2\left(15\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}, \frac{2\left(30\sqrt{2}a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+\left(\right.\right.}{15\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x
 - c)/(a*x)))/x^2, 2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x -
c)/(a*x)))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Giac [B] time = 1.97729, size = 375, normalized size = 3.32

$$\frac{4\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)}+\frac{2\left(60\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3a^4c^{\frac{3}{2}}|a|\right.}{15\left(\sqrt{a^2c}x-\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/15*(60*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^4*c
^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a^2
*abs(a)*sgn(x))
```

3.531 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=113

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/(3*c) - (2*a^3*(c -
c/(a*x))^(7/2))/(7*c^3) + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.416017, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 88, 50, 63, 208}

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/(3*c) - (2*a^3*(c -
c/(a*x))^(7/2))/(7*c^3) + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^4(1+ax)}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^3(1+ax)}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}\,dx}{c}\\
&= -\frac{a\,\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{a\,\text{Subst}\left(\int \left(\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}-\frac{a\left(c-\frac{cx}{a}\right)^{5/2}}{c}\right)dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{a^3\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\left(2a^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)\\
&= -4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\left(4a^3c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\left(8a^4\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= -4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.134165, size = 87, normalized size = 0.77

$$\frac{2\left(-52a^3x^3+16a^2x^2-9ax+3\right)\sqrt{c-\frac{c}{ax}}}{21x^3}+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 9*a*x + 16*a^2*x^2 - 52*a^3*x^3))/(21*x^3) + 4*Sq
rt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] time = 0.168, size = 302, normalized size = 2.7

$$\frac{1}{21\,x^4}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-126\,\sqrt{ax^2-x}a^{9/2}\sqrt{a^{-1}}x^5+42\,a^{9/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^5+84\,\left(ax^2-x\right)^{3/2}a^{7/2}\sqrt{a^{-1}}x^3+63\,\ln\left(1/2\,\frac{2\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 1/21*(c*(a*x-1)/a/x)^(1/2)/x^4*(-126*(a*x^2-x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^
5+42*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^5+84*(a*x^2-x)^(3/2)*a^(7/2)*(
1/a)^(1/2)*x^3+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)
^(1/2)*x^5*a^4-42*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*a-3*a*x+1)/(a*x+1))*x^5-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*(1/a)^(1/2)*x^5*a^4-20*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x^2+12*
a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)-6*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)
)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

Fricas [A] time = 1.7059, size = 466, normalized size = 4.12

$$\left[\frac{2\left(21\sqrt{2}a^3\sqrt{c}x^3\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(52\,a^3x^3-16\,a^2x^2+9\,ax-3\right)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}, -\frac{2\left(42\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}\sqrt{-}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x - 3
)*sqrt((a*c*x - c)/(a*x)))/x^3, -2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1
/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x^2 +
 9*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Giac [B] time = 2.2953, size = 481, normalized size = 4.26

$$-\frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)}-\frac{2\left(84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^6c^{\frac{3}{2}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/21*(84*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*
a^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^7*c^3
 - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) + 3*a^7*
c^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x))
```

3.532 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=163

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] + (2*a^4*(c - c/(a*x))^(3/2))/(3*c) + (2*a^4*(c - c
/(a*x))^(5/2))/(5*c^2) - (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c -
c/(a*x))^(9/2))/(9*c^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rubi [A] time = 0.458131, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 88, 50, 63, 208}

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}-4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] + (2*a^4*(c - c/(a*x))^(3/2))/(3*c) + (2*a^4*(c - c
/(a*x))^(5/2))/(5*c^2) - (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c -
c/(a*x))^(9/2))/(9*c^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^5(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^4(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(a^2\left(c-\frac{cx}{a}\right)^{3/2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a^2\left(c-\frac{cx}{a}\right)^{5/2}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{7/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \frac{a^4\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \left(2a^4\right)\text{Subst}\left(\int \frac{\sqrt{c}}{a}\right.$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \left(4a^4\right.$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \left(8a^5\right.$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - 4\sqrt{2}$$

Mathematica [A] time = 0.139376, size = 95, normalized size = 0.58

$$\frac{2\left(788a^4x^4 - 236a^3x^3 + 138a^2x^2 - 95ax + 35\right)\sqrt{c-\frac{c}{ax}}}{315x^4} - 4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^4
))/(315*x^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqr
t[c])]
```

Maple [B] time = 0.186, size = 326, normalized size = 2.

$$-\frac{1}{315\,x^5}\sqrt{\frac{c\,(ax-1)}{ax}}\left(-1890\,\sqrt{ax^2-x}a^{11/2}\sqrt{a^{-1}}x^6+630\,a^{11/2}\sqrt{a^{-1}}\sqrt{(ax-1)\,x}x^6+1260\,\left(ax^2-x\right)^{3/2}a^{9/2}\sqrt{a^{-1}}x^4+945\,\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] -1/315*(c*(a*x-1)/a/x)^(1/2)/x^5*(-1890*(a*x^2-x)^(1/2)*a^(11/2)*(1/a)^(1/2
)*x^6+630*a^(11/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^6+1260*(a*x^2-x)^(3/2)*a
^(9/2)*(1/a)^(1/2)*x^4+945*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2))*(1/a)^(1/2)*x^6*a^5-630*a^(9/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x
-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^6-945*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^6*a^5-316*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(
1/2)*x^3+156*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x^2-120*a^(3/2)*(a*x^2-x)^
(3/2)*x*(1/a)^(1/2)+70*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/
2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^5), x)
```

Fricas [A] time = 1.65622, size = 517, normalized size = 3.17

$$\left[\frac{2\left(315\,\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4},\ \frac{2\left(630\,\sqrt{2}a^4\sqrt{-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*a^
2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, 2/315*(630*sqrt(2)*a^4*s
qrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (788*a
^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/
x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Giac [B] time = 2.70279, size = 586, normalized size = 3.6

$$\frac{4\,\sqrt{2}a^5c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)}+\frac{2\left(1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^5*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/315*(1260
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4
*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^(7/2)*abs
(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 - 315*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*c^5)/((sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

3.533 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal. Leaf size=303

$$\frac{223x^2\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{1115x\sqrt{c-\frac{c}{ax}}}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{1115\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}+\frac{x^4\sqrt{c}}{4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (-1115*Sqrt[c - c/(a*x)])/(64*a^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (1
115*Sqrt[c - c/(a*x)]*x)/(192*a^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2
23*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2
5*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt
[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1115*Sqrt[c -
 c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.318334, antiderivative size = 306, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6182, 6180, 89, 78, 51, 63, 208}

$$\frac{1115x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}}-\frac{223x^2\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{1115x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{1115\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}+\frac{x^4}{4\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-1115*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) -
(223*Sqrt[c - c/(a*x)]*x^2)/(24*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) +
(1115*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) -
 (25*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (S
qrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1115*Sqrt[
c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{25}{2a}+\frac{4x}{a^2}}{x^4\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(223\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{48a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(1115\sqrt{c-\frac{c}{ax}}\right)\text{Su}}{48a}$$

$$= -\frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} - \frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x}{4\sqrt{1-\frac{1}{ax}}\sqrt{1}}$$

$$= -\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} - \frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} - \frac{25\sqrt{c-}}{24a\sqrt{1-}}$$

$$= -\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} - \frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} - \frac{25\sqrt{c-}}{24a\sqrt{1-}}$$

$$= -\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} - \frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} - \frac{25\sqrt{c-}}{24a\sqrt{1-}}$$

Mathematica [A] time = 0.585685, size = 167, normalized size = 0.55

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(48a^4x^4-200a^3x^3+446a^2x^2-1115ax-3345\right)\sqrt{c-\frac{c}{ax}}}{a^2x^2-1} + 3345\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) - 334}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3345 - 1115*a*x + 446
*a^2*x^2 - 200*a^3*x^3 + 48*a^4*x^4))/(-1 + a^2*x^2) - 3345*Sqrt[c]*Log[1 -
 a*x] + 3345*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*
x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(384*a^4)
```

Maple [A] time = 0.188, size = 197, normalized size = 0.7

$$\frac{(ax+1)\,x}{384\,(ax-1)^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(96\,a^{9/2}\sqrt{(ax+1)\,x}x^4-400\,a^{7/2}x^3\sqrt{(ax+1)\,x}+892\,a^{5/2}x^2\sqrt{(ax+1)\,x}-2230\,a^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/384*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(96
*a^(9/2)*((a*x+1)*x)^(1/2)*x^4-400*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+892*a^(5/2
)*x^2*((a*x+1)*x)^(1/2)-2230*a^(3/2)*x*((a*x+1)*x)^(1/2)+3345*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a-6690*((a*x+1)*x)^(1/2)*a^(1/2)
+3345*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(7/2)/((a*x+
1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.9376, size = 792, normalized size = 2.61

$$\left[\frac{3345\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2x^2-33\right.}{768\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3345*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 +
 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2
 - 3345*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^
4), -1/384*(3345*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2
*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.534 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal. Leaf size=251

$$\frac{119x\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{119\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{19x^2\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (119*Sqrt[c - c/(a*x)])/(8*a^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (119*
Sqrt[c - c/(a*x)]*x)/(24*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (19*Sqr
t[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c -
c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (119*Sqrt[c - c/(a*
x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.292824, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6182, 6180, 89, 78, 51, 63, 208}

$$\frac{119x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{119x\sqrt{c-\frac{c}{ax}}}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{119\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{19x^2\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-119*Sqrt[c - c/(a*x)]*x)/(12*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (
119*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) - (19*
Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c
 - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (119*Sqrt[c - c/
(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int\frac{-\frac{19}{2a}+\frac{3x}{a^2}}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\operatorname{Subst}\left(\int\frac{1}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{24a^2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\operatorname{Sub}}{8a^2}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

Mathematica [A] time = 0.490683, size = 159, normalized size = 0.63

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^3x^3-38a^2x^2+119ax+357\right)\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}-357\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+357\sqrt{c}\log(1-}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(357 + 119*a*x - 38*a^2
*x^2 + 8*a^3*x^3))/(-1 + a^2*x^2) + 357*Sqrt[c]*Log[1 - a*x] - 357*Sqrt[c]*
Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x
 + 2*a^2*x^2)])/(48*a^3)
```

Maple [A] time = 0.187, size = 180, normalized size = 0.7

$$\frac{(ax+1)\,x}{48\,(ax-1)^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c\,(ax-1)}{ax}}\left(16\,a^{7/2}x^3\sqrt{(ax+1)\,x}-76\,a^{5/2}x^2\sqrt{(ax+1)\,x}+238\,a^{3/2}x\sqrt{(ax+1)\,x}-357\,\ln\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(16*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)-76*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+238*a^(3/2)*
x*((a*x+1)*x)^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2
))*x*a+714*((a*x+1)*x)^(1/2)*a^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2
)+2*a*x+1)/a^(1/2)))/a^(5/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

[Out] integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A] time = 1.94247, size = 737, normalized size = 2.94

$$\left[\frac{357\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2+357\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(357*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(357*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 38*a^3*x^
3 + 119*a^2*x^2 + 357*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*x - a^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.535 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal. Leaf size=199

$$-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{13x\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] (-47*Sqrt[c - c/(a*x)])/(4*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (13*S
qrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (47*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.199565, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6182, 6180, 89, 78, 51, 63, 208}

$$-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{13x\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-47*Sqrt[c - c/(a*x)])/(4*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (13*S
qrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (47*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)])
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
```

```
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{-\frac{13}{2a}+\frac{2x}{a^2}}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\int\frac{1}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\right.}{8a^2\sqrt{1-}}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\right.}{4a\sqrt{}}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{47\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{}\right.}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.492999, size = 151, normalized size = 0.76

$$\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-13ax-47\right)\sqrt{c-\frac{c}{ax}}}{4a^2x^2-4}+\frac{47\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)}{8a^2}-\frac{47\sqrt{c}\log(1-}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-47 - 13*a*x + 2*a^2*x^2))/(-
4 + 4*a^2*x^2) - (47*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (47*Sqrt[c]*Log[2*a^2*
Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x
^2)])/(8*a^2)
```

Maple [A] time = 0.182, size = 163, normalized size = 0.8

$$\frac{(ax+1)\,x}{8\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(4\,a^{5/2}x^2\sqrt{(ax+1)\,x}-26\,a^{3/2}x\sqrt{(ax+1)\,x}+47\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^
(5/2)*x^2*((a*x+1)*x)^(1/2)-26*a^(3/2)*x*((a*x+1)*x)^(1/2)+47*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a-94*((a*x+1)*x)^(1/2)*a^(1/2)+4
7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(3/2)/((a*x+1)*x
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}\,x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.91449, size = 694, normalized size = 3.49

$$\left[\frac{47\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2\,a^3x^3-13\,a^2x^2-47\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)},-\frac{47\,(ax}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(47*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(47*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}}\,x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.536 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=140

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.113666, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6182, 6179, 89, 78, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
```

```
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.0481412, size = 67, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax-7\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+9\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.175, size = 146, normalized size = 1.

$$\frac{(ax+1)\,x}{2\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2\,a^{3/2}x\sqrt{(ax+1)\,x}-7\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1}{\sqrt{a}}\right)xa+18\,\sqrt{(ax+1)\,x}\sqrt{a}-7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*x*a+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.90996, size = 640, normalized size = 4.57

$$\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+9\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{7\,(ax-1)\sqrt{-c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
```

```
/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.537 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=134

$$-\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (-8*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2*Sqrt[1 +
1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(a*x)]*ArcTan
h[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)]
```

Rubi [A] time = 0.261511, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6182, 6180, 87, 63, 208}

$$-\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-8*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2*Sqrt[1 +
1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(a*x)]*ArcTan
h[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-\frac{4}{a\left(1+\frac{x}{a}\right)^{3/2}}+\frac{1}{a\sqrt{1+\frac{x}{a}}}+\frac{1}{x\sqrt{1+\frac{x}{a}}}\right)dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.373846, size = 131, normalized size = 0.98

$$-\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}(5ax+1)\sqrt{c-\frac{c}{ax}}}{a^2x^2-1} + \sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) - \sqrt{c}\log(1-ax)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x*(1 + 5*a*x))/(-1 + a^2*x^2)
 - Sqrt[c]*Log[1 - a*x] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*S
qrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]
```

Maple [A] time = 0.182, size = 151, normalized size = 1.1

$$-\frac{ax+1}{(ax-1)^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{\frac{c(ax-1)}{ax}}\left(10\,a^{3/2}x\sqrt{(ax+1)\,x}-\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1\right)\frac{1}{\sqrt{a}}\right)x^2a^2-\ln\left(\frac{1}{2}\left(2\sqrt{(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*(10*a^(3/2
)*x*((a*x+1)*x)^(1/2)-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*x^2*a^2-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a+2*((a*x+
1)*x)^(1/2)*a^(1/2))/a^(1/2)/((a*x+1)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [A] time = 1.81085, size = 609, normalized size = 4.54

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\left(a^2x\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
)/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(
5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.538 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (64*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a^2*x^2)]) - (16*a*(c - c/(a*x))^(3
/2))/(3*c*Sqrt[1 - 1/(a^2*x^2)]) - (2*a*(c - c/(a*x))^(5/2))/(3*c^2*Sqrt[1
- 1/(a^2*x^2)])
```

Rubi [A] time = 0.216214, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6178, 657, 649}

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```


```
[Out] (64*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a^2*x^2)]) - (16*a*(c - c/(a*x))^(3
/2))/(3*c*Sqrt[1 - 1/(a^2*x^2)]) - (2*a*(c - c/(a*x))^(5/2))/(3*c^2*Sqrt[1
- 1/(a^2*x^2)])
```


Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
```

```
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{8\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{32\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0955544, size = 58, normalized size = 0.53

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(23a^2x^2+10ax-1\right)\sqrt{c-\frac{c}{ax}}}{3a^2x^2-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(-
3 + 3*a^2*x^2)
```

Maple [A] time = 0.122, size = 62, normalized size = 0.6

$$\frac{(2\,ax+2)\left(23\,a^2x^2+10\,ax-1\right)}{3\,x\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] 2/3*(a*x+1)*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(
3/2)/x/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A] time = 1.53207, size = 126, normalized size = 1.16

$$\frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] 2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(a*x^2 - x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.539 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=150

$$-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (-224*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(15*Sqrt[c - c/(a*x)]) - (56*a^2*Sqrt[1
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/15 - (7*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/
(a*x))^(3/2))/(5*c) - (a^2*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.266011, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 789, 657, 649}

$$-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-224*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(15*Sqrt[c - c/(a*x)]) - (56*a^2*Sqrt[1
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/15 - (7*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/
(a*x))^(3/2))/(5*c) - (a^2*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
```

```
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(7a)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(28a)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= -\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{15}(112a)\,\text{Subst}\Bigg(\int$$

$$= -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}} - \frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.113466, size = 70, normalized size = 0.47

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(158a^3x^3+79a^2x^2-16ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 15
8*a^3*x^3))/(15*x*(-1 + a^2*x^2))
```

Maple [A] time = 0.128, size = 70, normalized size = 0.5

$$-\frac{(2\,ax+2)\left(158\,x^3a^3+79\,a^2x^2-16\,ax+3\right)}{15\,x^2\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] -2/15*(a*x+1)*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*((a*x
-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A] time = 1.65063, size = 150, normalized size = 1.

$$-\frac{2\left(158\,a^3x^3+79\,a^2x^2-16\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] -2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.540 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=188

$$\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (1888*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) + (472*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (59*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c
- c/(a*x))^(3/2))/(35*c) + (2*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5/2)
)/(7*c^2) + (a^3*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.413827, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6178, 1635, 795, 657, 649}

$$\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (1888*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) + (472*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (59*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c
- c/(a*x))^(3/2))/(35*c) + (2*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5/2)
)/(7*c^2) + (a^3*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{7a^2}{2}-ax\right)\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\left(59a^2\right)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{14c^2}$$

$$= \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\left(236a^2\right)\text{Subst}\left(\int \right.}{35}$$

$$= \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(\right.}{c^3}$$

$$= \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} + \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-}}{}$$

Mathematica [A] time = 0.123501, size = 78, normalized size = 0.41

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^2\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 +
668*a^3*x^3 + 1336*a^4*x^4))/(105*x^2*(-1 + a^2*x^2))
```

Maple [A] time = 0.115, size = 78, normalized size = 0.4

$$\frac{(2\,ax+2)\left(1336\,x^4a^4+668\,x^3a^3-167\,a^2x^2+66\,ax-15\right)}{105\,x^3\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] 2/105*(a*x+1)*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/a
/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A] time = 1.70569, size = 173, normalized size = 0.92

$$\frac{2\left(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] 2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.541 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=289

$$-\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{32a^4\sqrt{\frac{1}{ax}+}}{\sqrt{1-}}$$

```
[Out] (-8*a^4*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (32*a^4*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (50*a^4*(1 + 1/(a*
x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) - (38*a^4*(1 + 1/(a*x))^
(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(7/2)
*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] - (2*a^4*(1 + 1/(a*x))^(9/2)*Sqrt[c -
 c/(a*x)])/(9*Sqrt[1 - 1/(a*x)])
```

Rubi [A] time = 0.293352, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6182, 6180, 88}

$$-\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{32a^4\sqrt{\frac{1}{ax}+}}{\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*a^4*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (32*a^4*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (50*a^4*(1 + 1/(a*
x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) - (38*a^4*(1 + 1/(a*x))^
(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(7/2)
*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] - (2*a^4*(1 + 1/(a*x))^(9/2)*Sqrt[c -
 c/(a*x)])/(9*Sqrt[1 - 1/(a*x)])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-\frac{4a^3}{\left(1+\frac{x}{a}\right)^{3/2}} + \frac{16a^3}{\sqrt{1+\frac{x}{a}}} - 25a^3\sqrt{1+\frac{x}{a}} + 19a^3\left(1+\frac{x}{a}\right)^{3/2} - 7a^3\left(1+\frac{x}{a}\right)^{5/2} + \right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} - \frac{38a^4\left(1+\frac{1}{ax}\right)^{5/2}}{5\sqrt{1-\frac{1}{}}}$$

Mathematica [A] time = 0.167155, size = 86, normalized size = 0.3

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(656a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{c-\frac{c}{ax}}}{45x^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 82
*a^3*x^3 + 328*a^4*x^4 + 656*a^5*x^5))/(45*x^3*(-1 + a^2*x^2))
```

Maple [A] time = 0.118, size = 86, normalized size = 0.3

$$-\frac{(2\,ax+2)\left(656\,x^5a^5+328\,x^4a^4-82\,x^3a^3+41\,a^2x^2-20\,ax+5\right)}{45\,x^4\,(ax-1)^2}\sqrt{\frac{c\,(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] -2/45*(a*x+1)*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*(
a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A] time = 1.65585, size = 186, normalized size = 0.64

$$-\frac{2\left(656\,a^5x^5+328\,a^4x^4-82\,a^3x^3+41\,a^2x^2-20\,ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{45\left(ax^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] -2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{ax}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.542 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=185

$$-\frac{c2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}-\frac{2c(1-n)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometri}}{an}$$

```
[Out] c*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x - (2*c*(1 - n)*(1 + 1/(a*x)
)^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a
*n*(1 - 1/(a*x))^(n/2)) - (2^(n/2)*c*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric
2F1[1 - n/2, 1 - n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(a*(2 - n))
```

Rubi [C] time = 0.0645511, antiderivative size = 81, normalized size of antiderivative = 0.44, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6179, 136}

$$-\frac{c2^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-2}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] -((2^(2 - n/2)*c*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 + n)/2, (-2 + n)/2,
2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)))
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx=-\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=-\frac{2^{2-\frac{n}{2}}c\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1}{2}(-2+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}$$

Mathematica [A] time = 0.350406, size = 155, normalized size = 0.84

$$\frac{ce^{n\coth^{-1}(ax)}\left(n\left(-e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,-e^{2\coth^{-1}(ax)}\right)+(n-1)ne^{2\coth^{-1}(ax)}\text{Hypergeome}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2,
 2 + n/2, -E^(2*ArcCoth[a*x])]) + E^(2*ArcCoth[a*x])*(-1 + n)*n*Hypergeomet
ric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n*x + Hypergeo
metric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])] + (-1 + n)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*n*(2 + n))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int e^{n\,\text{arccoth}(ax)}\left(c-\frac{c}{ax}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral((a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int ae^{n\operatorname{acoth}(ax)}\,dx + \int -\frac{e^{n\operatorname{acoth}(ax)}}{x}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x),x)
```

```
[Out] c*(Integral(a*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x, x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.543 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=113

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}-\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] ((1 + 1/(a*x))^((2 + n)/2)*x)/(c*(1 - 1/(a*x))^(n/2)) - (2*(1 + n)*(1 + 1/(
a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a + x^(-1))])
/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.0849104, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6179, 96, 131}

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}-\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((1 + 1/(a*x))^((2 + n)/2)*x)/(c*(1 - 1/(a*x))^(n/2)) - (2*(1 + n)*(1 + 1/(
a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a + x^(-1))])
/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{(1+n)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Mathematica [A] time = 0.401441, size = 97, normalized size = 0.86

$$\frac{e^{n\coth^{-1}(ax)}\left(n(n+1)e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+(n+2)\left((n+1)\text{Hypergeometric}\right.\right.}{acn(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(1 + n)*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + (1 + n)*Hypergeom
etric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{ax}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c - \frac{c}{ax}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ax\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{xe^{n\operatorname{acoth}(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x),x)
```

```
[Out] a*Integral(x*exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x)), x)
```

3.544 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{2(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}-\frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+\right.}{}$$

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 +
n))) + ((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x)/c^2 - (2*(2 +
 n)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
 + x^(-1))])/(a*c^2*n*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.115471, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6179, 129, 155, 12, 131}

$$-\frac{2(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}-\frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 +
n))) + ((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x)/c^2 - (2*(2 +
 n)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
 + x^(-1))])/(a*c^2*n*(1 - 1/(a*x))^(n/2))
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{2+n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{a\,\text{Subst}\left(\int \frac{(2+n)^2\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{a^2x}\right.}{c^2(2+n)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{(2+n)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\right.}{ac^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\ 2}{ac^2n}$$

Mathematica [A] time = 0.0725854, size = 113, normalized size = 0.68

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(n(ax+1)(n(ax-1)+2ax-3)-2(n+2)^2(ax-1)\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{ax-1}{ax+1}\right)\right)}{ac^2n(n+2)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(n*(1 + a*x)*(-3 + 2*a*x + n*(-1 + a*x)) - 2*(2 + n)^2
*(-1 + a*x)*Hypergeometric2F1[1, -n/2, 1 - n/2, (-1 + a*x)/(1 + a*x)]))/(a*
c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int e^{n\,\text{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x + c
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int\frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

3.545 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
+ n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(1 - 1/(a*x))^(3/2)))
```

Rubi [A] time = 0.152319, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
+ n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(1 - 1/(a*x))^(3/2)))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}F_1\left(\frac{2+n}{2};\frac{1}{2}(-3+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [F] time = 180.005, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)} \left(c-\frac{c}{ax}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\sqrt{\frac{acx-c}{ax}}}{ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/
(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.546 $\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}}\, dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}} \sqrt{c-\frac{c}{ax}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} F_1\left(\frac{n+2}{2};\frac{n-1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 +
n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n
)*Sqrt[1 - 1/(a*x)]))
```

Rubi [A] time = 0.143544, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}} \sqrt{c-\frac{c}{ax}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} F_1\left(\frac{n+2}{2};\frac{n-1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 +
n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n
)*Sqrt[1 - 1/(a*x)]))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\sqrt{c-\frac{c}{ax}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}F_1\left(\frac{2+n}{2};\frac{1}{2}(-1+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

Mathematica [F] time = 180.008, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]

[Out] $Aborted
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\operatorname{arccoth}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\left(-1+\frac{1}{ax}\right)}e^{n\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{ax}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.547 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} F_1\left(\frac{n+2}{2};\frac{n+1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)
*Sqrt[c - c/(a*x)]))
```

Rubi [A] time = 0.149866, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} F_1\left(\frac{n+2}{2};\frac{n+1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```


```
[Out] -((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)
*Sqrt[c - c/(a*x)]))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}$$

Mathematica [F] time = 180.006, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)} \frac{1}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ax\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x -
 c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

3.548 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2
 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(c - c/(a*x))^(3/2)))
```

Rubi [A] time = 0.155392, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2
 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(c - c/(a*x))^(3/2)))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{3+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [F] time = 180.013, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)} \left(c-\frac{c}{ax}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\sqrt{\frac{acx-c}{ax}}}{a^2c^2x^2-2\,ac^2x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^2
*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

3.549 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=110

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{n+2}{2};\frac{1}{2}(n-2p),2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

```
[Out] -((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*(1 - 1/(a*x))^p))
```

Rubi [A] time = 0.106145, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6182, 6179, 136}

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{n+2}{2};\frac{1}{2}(n-2p),2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```


```
[Out] -((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*(1 - 1/(a*x))^p))
```


Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
```

```
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{2+n}{2};\frac{1}{2}(n-2p),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}\end{aligned}$$

Mathematica [F] time = 0.877213, size = 0, normalized size = 0.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p, x]
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{ax}\right)^{p} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} \left(\frac{acx-c}{ax}\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{n \operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.550 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=67

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{1}{ax}+1\right)}{a(p+1)}$$

```
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Rubi [A] time = 0.0769245, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6182, 6179, 65}

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(2,p+1;p+2;1+\frac{1}{ax}\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^p}{x^2}dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(2,1+p;2+p;1+\frac{1}{ax}\right)}{a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0278952, size = 67, normalized size = 1.

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{1}{ax}+1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Maple [F] time = 0.182, size = 0, normalized size = 0.

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

```
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^p \left(\frac{ax-1}{ax+1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^p \left(\frac{acx-c}{ax}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^p*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1 + \frac{1}{ax}\right)\right)^p e^{2p \operatorname{acoth}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^p \left(\frac{ax - 1}{ax + 1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^p, x)
```

3.551 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=93

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

```
[Out] -((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 - p
, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

Rubi [A] time = 0.0995908, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6182, 6179, 136}

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] -((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 - p
, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\
&= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}\left(1+\frac{x}{a}\right)^{-p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}
\end{aligned}$$

Mathematica [F] time = 0.607364, size = 0, normalized size = 0.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]), x]
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int\frac{1}{e^{2p\,\mathrm{arccoth}(ax)}}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

```
[Out] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{acx-c}{ax}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*c*x - c)/(a*x))^p/((a*x - 1)/(a*x + 1))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^p, x)
```

3.552 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=57

$$\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)}{ap}+x\left(c-\frac{c}{ax}\right)^p$$

```
[Out] (c - c/(a*x))^p*x + ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p,
 1 - 1/(a*x)])/(a*p)
```

Rubi [A] time = 0.106547, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6167, 6133, 25, 514, 375, 78, 65}

$$\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}+x\left(c-\frac{c}{ax}\right)^p$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] (c - c/(a*x))^p*x + ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p,
 1 - 1/(a*x)])/(a*p)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
```

```
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=-\int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)}{1-ax}\,dx$$
$$=\frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{-1+p}(1+ax)}{x}\,dx}{a}$$
$$=\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{-1+p}\,dx}{a}$$
$$=-\frac{c\,\mathrm{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{-1+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=\left(c-\frac{c}{ax}\right)^p x-\frac{(c(2-p))\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=\left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}$$

Mathematica [A] time = 0.0235685, size = 46, normalized size = 0.81

$$\frac{\left(c-\frac{c}{ax}\right)^p\left(apx-(p-2)\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*(a*p*x - (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*
x)]))/(a*p)
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int \frac{ax+1}{ax-1}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^p,x)
```

```
[Out] int((a*x+1)/(a*x-1)*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ax+1)\left(\frac{acx-c}{ax}\right)^p}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

Sympy [C] time = 7.64406, size = 272, normalized size = 4.77

$$a\left(\left(\begin{cases} \frac{0^p x}{a} + \frac{0^p \log(ax-1)}{a^2} - \frac{a^{-p}c^p p x^2 x^{-p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p, 2-p\\ 3-p\end{matrix}\middle| ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1 \\ \frac{0^p x}{a} + \frac{0^p \log(-ax+1)}{a^2} - \frac{a^{-p}c^p p x^2 x^{-p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p, 2-p\\ 3-p\end{matrix}\middle| ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right) + \left\{\begin{matrix} \frac{0^p \log(ax-1)}{a} - \frac{a^{-p}c^p p x x^{-p} e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1}{\Gamma(2-p)\Gamma(p+1)} \\ \frac{0^p \log(-ax+1)}{a} - \frac{a^{-p}c^p p x x^{-p} e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1}{\Gamma(2-p)\Gamma(p+} \end{matrix}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**p,x)
```

```
[Out] a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - a**(-p)*c**p*p*x**2*x**(-p
)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(g
amma(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a*
*2 - a**(-p)*c**p*p*x**2*x**(-p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1
 - p, 2 - p), (3 - p,), a*x)/(gamma(3 - p)*gamma(p + 1)), True)) + Piecewis
e((0**p*log(a*x - 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p)*gamma(p)*gamm
a(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*gamma(p + 1)),
Abs(a*x) > 1), (0**p*log(-a*x + 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p)
*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*g
amma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

3.553 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=90

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

```
[Out] -(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 - p, 2,
 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(3*a*(1 - 1/(a*x))^p)
```

Rubi [A] time = 0.0795841, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6182, 6179, 136}

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] -(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 - p, 2,
 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(3*a*(1 - 1/(a*x))^p)
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}\end{aligned}$$

Mathematica [F] time = 0.817151, size = 0, normalized size = 0.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p, x]
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^p\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ax+1)\left(\frac{acx-c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1 + \frac{1}{ax}\right)\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.554 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{1}{2};-p-\frac{1}{2},2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

```
[Out] -((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2,
 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

Rubi [A] time = 0.0853336, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6182, 6179, 136}

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{1}{2};-p-\frac{1}{2},2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

```
[Out] -((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2,
 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx=\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$=-\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=-\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{1}{2};-\frac{1}{2}-p,2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}$$

Mathematica [F] time = 1.08491, size = 0, normalized size = 0.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

```
[Out] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x], x]
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^p \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{acx - c}{ax}\right)^p \sqrt{\frac{ax - 1}{ax + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{ax}\right)^p \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.555 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=114

$$\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{ac^2}+\frac{x}{}$$

```
[Out] ((c - c/(a*x))^(2 + p)*x)/c^2 + ((c - c/(a*x))^(2 + p)*Hypergeometric2F1[1,
 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) - ((c - c/(a*x))^(2 +
 p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rubi [A] time = 0.152468, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6167, 6133, 25, 514, 375, 103, 156, 65, 68}

$$\frac{\left(c-\frac{c}{ax}\right)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\,{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}+\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^(2 + p)*x)/c^2 + ((c - c/(a*x))^(2 + p)*Hypergeometric2F1[1,
 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) - ((c - c/(a*x))^(2 +
 p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx \\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{1+ax}dx \\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}x}{1+ax}dx}{c} \\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}}{a+\frac{1}{x}}dx}{c} \\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c} \\
&= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}\left(c(2+p)+\frac{c(1+p)x}{a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c^2} \\
&= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}-\frac{\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{a+x}dx,x,\frac{1}{x}\right)}{ac}+\frac{(2+p)\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x}dx,x,\frac{1}{x}\right)}{ac} \\
&= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}-\frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3+p;1\cdot\right.}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0506705, size = 87, normalized size = 0.76

$$\frac{(ax-1)^2\left(c-\frac{c}{ax}\right)^p\left(\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)+2(p+2)\left(ax-\text{Hypergeometric2F1}\left(1,p+2,p+3,1\right.\right.\right.}{2a^3(p+2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(-
1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x
)])))/(2*a^3*(2 + p)*x^2)
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{ax-1}{ax+1}\left(c-\frac{c}{ax}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/(a*x+1)*(a*x-1),x)
```

```
[Out] int((c-c/a/x)^p/(a*x+1)*(a*x-1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ax-1)\left(\frac{acx-c}{ax}\right)^{p}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p (ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x - 1)/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

3.556 $\int e^{\coth^{-1}(ax)} \left(c-a^2cx^2\right)^4 dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{7}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{5}{72}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{5}{144}a^5c^4x^6$$

```
[Out] (35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (35*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(5/2)*x^3)/192 + (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/6
4 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/144 - (5*a^5*c^4*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/144 + (5*a^6*c^4*(1 - 1/(a*x))^(3
/2)*(1 + 1/(a*x))^(11/2)*x^7)/72 - (7*a^7*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a
*x))^(11/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(11/2)*x^9
)/9 + (35*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rubi [A] time = 0.33667, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{7}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{5}{72}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{5}{144}a^5c^4x^6$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```


```
[Out] (35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (35*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(5/2)*x^3)/192 + (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/6
4 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/144 - (5*a^5*c^4*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/144 + (5*a^6*c^4*(1 - 1/(a*x))^(3
/2)*(1 + 1/(a*x))^(11/2)*x^7)/72 - (7*a^7*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a
*x))^(11/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(11/2)*x^9
)/9 + (35*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9+\frac{1}{9}\left(7a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^9}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9-\frac{1}{72}\left(35a^6c^4\right)\text{S}$$

$$=\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right.$$

$$=-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4\left(1-\right.$$

$$=\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3}$$

$$=\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(\right.$$

$$=\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(\right.$$

$$=\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1\right.$$

$$=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.$$

$$=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.$$

$$=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.$$

Mathematica [A] time = 0.179308, size = 111, normalized size = 0.28

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-512a^2x^2-837ax+128\right)+315\log\left(x\left(\right.\right.\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3
+ 768*a^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) + 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

Maple [A] time = 0.149, size = 279, normalized size = 0.7

$$\frac{(ax-1)\,c^4}{1152\,a}\left(128\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6+144\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^5a^5-384\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4-456\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/1152*(a*x-1)*c^4/a*(128*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+144*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5-384*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-45
6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+384*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x
^2*a^2+522*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+256*(a^2*x^2-1)^(3/2)*(a^2)^(1
/2)-315*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-384*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)+315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1
)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.06928, size = 560, normalized size = 1.42

$$\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-23\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-1)^2a^2}{(ax+1)^2}+\frac{84\,(ax-1)^3a^2}{(ax+1)^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 27
30*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/2
) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 32768*c^4*((a*x - 1)/(a*x + 1)
)^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*x
 + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x - 1
)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2 +
 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a*
x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7*
a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1)
^9 - a^2))*a
```

Fricas [A] time = 1.39674, size = 404, normalized size = 1.03

$$\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(128\,a^9c^4x^9+272\,a^8c^4x^8-368\,a^7c^4x^7-1112\,a^6c^4x^6+168\,a^5c^4x^5+\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (128*a^9*c^4*x^9 + 272*a^8*c^4*x^8 - 368*a^7*c^4*x^7
 - 1112*a^6*c^4*x^6 + 168*a^5*c^4*x^5 + 1746*a^4*c^4*x^4 + 466*a^3*c^4*x^3
- 1349*a^2*c^4*x^2 - 709*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17999, size = 500, normalized size = 1.27

$$\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{2730\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{10458\,(ax-1)^2c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{23202\,(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(abs(sq
rt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(2730*(a*x - 1)*c^4*sqrt((a*x - 1)/(a
*x + 1))/(a*x + 1) - 10458*(a*x - 1)^2*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x +
 1)^2 + 23202*(a*x - 1)^3*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 32768
*(a*x - 1)^4*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 23202*(a*x - 1)^5*
c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^5 + 10458*(a*x - 1)^6*c^4*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1)^6 - 2730*(a*x - 1)^7*c^4*sqrt((a*x - 1)/(a*x + 1
))/(a*x + 1)^7 + 315*(a*x - 1)^8*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^8
- 315*c^4*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^9))*a
```

3.557 $\int e^{\coth^{-1}(ax)} \left(c - a^2 c x^2\right)^3 \, dx$

Optimal. Leaf size=313

$$-\frac{1}{7} a^6 c^3 x^7 \left(1-\frac{1}{ax}\right)^{5/2} \left(\frac{1}{ax}+1\right)^{9/2} + \frac{5}{42} a^5 c^3 x^6 \left(1-\frac{1}{ax}\right)^{3/2} \left(\frac{1}{ax}+1\right)^{9/2} - \frac{1}{14} a^4 c^3 x^5 \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{9/2} + \frac{1}{56} a^3 c^3 x^4 \sqrt{1-}$$

```
[Out] (5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (5*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/48 + (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/56 - (a^
4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/14 + (5*a^5*c^3*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/42 - (a^6*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1
/(a*x))^(9/2)*x^7)/7 + (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])
/(16*a)
```

Rubi [A] time = 0.250656, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7} a^6 c^3 x^7 \left(1-\frac{1}{ax}\right)^{5/2} \left(\frac{1}{ax}+1\right)^{9/2} + \frac{5}{42} a^5 c^3 x^6 \left(1-\frac{1}{ax}\right)^{3/2} \left(\frac{1}{ax}+1\right)^{9/2} - \frac{1}{14} a^4 c^3 x^5 \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{9/2} + \frac{1}{56} a^3 c^3 x^4 \sqrt{1-}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```


```
[Out] (5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (5*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/48 + (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/56 - (a^
4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/14 + (5*a^5*c^3*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/42 - (a^6*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1
/(a*x))^(9/2)*x^7)/7 + (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])
/(16*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
```

```
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\left(\left(a^6c^3\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{7}\left(5a^5c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^7}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 + \frac{1}{14}\left(5a^4c^3\right)\text{Subs}$$

$$= -\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/}$$

$$= \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}($$

$$= \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$= \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

Mathematica [A] time = 0.187864, size = 95, normalized size = 0.3

$$\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-48a^6x^6-56a^5x^5+144a^4x^4+182a^3x^3-144a^2x^2-231ax+48\right)+105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 - 231*a*x - 144*a^2*x^2 + 182*a^3*x^3 +
 144*a^4*x^4 - 56*a^5*x^5 - 48*a^6*x^6) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

Maple [A] time = 0.138, size = 231, normalized size = 0.7

$$-\frac{(ax-1)\,c^3}{336\,a}\left(48\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4+56\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3-96\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-126\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/336*(a*x-1)*c^3/a*(48*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4+56*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-126*(
a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+105*(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)*x*a+112*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-105*ln
((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1
/2)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.05407, size = 455, normalized size = 1.45

$$\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-3072\,c\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(ax-1)^3a^2}{(ax+1)^3}-\frac{35\,(ax-1)^4}{(ax+1)^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 700
```

```
*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) -
3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2)
 + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1)))
/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1)
^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(a
*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 - a
^2))*a
```

Fricas [A] time = 1.3204, size = 344, normalized size = 1.1

$$\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(48\,a^7c^3x^7+104\,a^6c^3x^6-88\,a^5c^3x^5-326\,a^4c^3x^4-38\,a^3c^3x^3+375\,a\right.}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (48*a^7*c^3*x^7 + 104*a^6*c^3*x^6 - 88*a^5*c^3*x^5 -
326*a^4*c^3*x^4 - 38*a^3*c^3*x^3 + 375*a^2*c^3*x^2 + 183*a*c^3*x - 48*c^3)*
sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{3a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6/s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)), x))
```

Giac [A] time = 1.19224, size = 408, normalized size = 1.3

$$\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{700\,(ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{1981\,(ax-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{3072\,(ax-1)^3c^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(abs(sqr
t((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(700*(a*x - 1)*c^3*sqrt((a*x - 1)/(a*x
 + 1))/(a*x + 1) - 1981*(a*x - 1)^2*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)
^2 - 3072*(a*x - 1)^3*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 1981*(a*x
 - 1)^4*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 700*(a*x - 1)^5*c^3*sqr
t((a*x - 1)/(a*x + 1))/(a*x + 1)^5 + 105*(a*x - 1)^6*c^3*sqrt((a*x - 1)/(a*
x + 1))/(a*x + 1)^6 - 105*c^3*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a
*x + 1) - 1)^7))*a
```

3.558 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{20}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\right.$$

```
[Out] (3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 + (a*c^2*Sqrt[1 - 1/(a*x)]*
(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)
*x^3)/20 - (3*a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (a^4*
c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/5 + (3*c^2*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rubi [A] time = 0.193515, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{20}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```


```
[Out] (3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 + (a*c^2*Sqrt[1 - 1/(a*x)]*
(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)
*x^3)/20 - (3*a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (a^4*
c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/5 + (3*c^2*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{5}\left(3a^3c^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}{x^5}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{1}{20}\left(3a^2c^2\right)\text{Subst}$$

$$=\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1\right.$$

$$=\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x$$

Mathematica [A] time = 0.0970448, size = 79, normalized size = 0.34

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8\right)+15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*a
^4*x^4) + 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A] time = 0.132, size = 183, normalized size = 0.8

$$\frac{(ax-1)\,c^2}{120\,a}\left(24\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+30\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+16\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}-45\,\sqrt{a^2}\sqrt{a^2x^2-1}xa-40\,((ax-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/120*(a*x-1)*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+30*(a^2)^(1/2
)*(a^2*x^2-1)^(3/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-45*(a^2)^(1/2)*(a^
2*x^2-1)^(1/2)*x*a-40*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+45*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)
*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.05823, size = 350, normalized size = 1.5

$$\frac{1}{40}a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+128\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^2
*((a*x - 1)/(a*x + 1))^(7/2) + 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^2
*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
 + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2
))
```

Fricas [A] time = 1.32178, size = 285, normalized size = 1.22

$$\frac{15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(8\,a^{5}c^{2}x^{5}+18\,a^{4}c^{2}x^{4}-6\,a^{3}c^{2}x^{3}-41\,a^{2}c^{2}x^{2}-17\,ac^{2}x+8\,c^{2}\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) + (8*a^5*c^2*x^5 + 18*a^4*c^2*x^4 - 6*a^3*c^2*x^3 - 41*a^2
*c^2*x^2 - 17*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int -\frac{2a^{2}x^{2}}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{a^{4}x^{4}}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x))
```

Giac [A] time = 1.16965, size = 316, normalized size = 1.36

$$\frac{1}{40}\,a\left(\frac{15\,c^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}}-\frac{15\,c^{2}\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}}-\frac{2\left(\frac{70\,(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{128\,(ax-1)^{2}c^{2}\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^{2}}-\frac{70\,(ax-1)^{3}c^{2}\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^{3}}+\frac{15\,(ax-\cdots}{\cdots}\right.}{a^{2}\left(\frac{ax-1}{ax+1}-1\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(abs(sqrt
((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(70*(a*x - 1)*c^2*sqrt((a*x - 1)/(a*x +
 1))/(a*x + 1) + 128*(a*x - 1)^2*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2
- 70*(a*x - 1)^3*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 15*(a*x - 1)^4
*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 15*c^2*sqrt((a*x - 1)/(a*x + 1
)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^5))
```

3.559 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

```
[Out] (c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 + 1
/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^3)/3
+ (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rubi [A] time = 0.116332, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 + 1
/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^3)/3
+ (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
```

```
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\left(\left(a^2c\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^4}dx,x,\frac{1}{x}\right)$$
$$=-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{3}(ac)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{2}c\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}dx\right.$$
$$=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{c}{}$$
$$=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{c}{}$$
$$=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{c}{}$$

Mathematica [A] time = 0.0914075, size = 61, normalized size = 0.42

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^2x^2-3ax+2\right)+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] time = 0.137, size = 119, normalized size = 0.8

$$-\frac{c\,(ax-1)}{6\,a}\left(3\sqrt{a^2}\sqrt{a^2x^2-1}xa+2\,((ax-1)\,(ax+1))^{3/2}\sqrt{a^2}-3\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{(ax-1)\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x)
```

```
[Out] -1/6*(a*x-1)*c*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^(3/
2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/(
(a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A] time = 1.12305, size = 231, normalized size = 1.59

$$-\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) - 8*c*((a*x - 1)/(a*x + 1))^(3/2
) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)
^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.37761, size = 215, normalized size = 1.48

$$\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3+5\,a^2cx^2+acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x +
 1)) - 1) - (2*a^3*c*x^3 + 5*a^2*c*x^2 + a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int -\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/
sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A] time = 1.19318, size = 211, normalized size = 1.46

$$\frac{1}{6}\,a\left(\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{8\,(ax-1)c\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{3\,(ax-1)^2c\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/6*a*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(abs(sqrt((a*x -
 1)/(a*x + 1)) - 1))/a^2 + 2*(8*(a*x - 1)*c*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1) - 3*(a*x - 1)^2*c*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 3*c*sqrt((a*
x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.560 $\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

```
[Out] E^ArcCoth[a*x]/(a*c)
```

Rubi [A] time = 0.0288307, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {6183}

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]

[Out] E^ArcCoth[a*x]/(a*c)
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\coth^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.0438475, size = 13, normalized size = 1.

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcCoth[a*x]/(a*c)
```

Maple [A] time = 0.043, size = 23, normalized size = 1.8

$$\frac{1}{ac}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

Maxima [A] time = 1.08499, size = 30, normalized size = 2.31

$$\frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/(a*c*sqrt((a*x - 1)/(a*x + 1)))
```

Fricas [A] time = 1.54037, size = 72, normalized size = 5.54

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")

[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)

[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c

---

Giac [A] time = 1.12888, size = 30, normalized size = 2.31

$$\frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")

[Out] 1/(a*c*sqrt((a*x - 1)/(a*x + 1)))

3.561 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}$$

```
[Out] (2*E^ArcCoth[a*x])/(3*a*c^2) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*c^2*(1 - a
^2*x^2))
```

Rubi [A] time = 0.0604559, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6185, 6183}

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcCoth[a*x])/(3*a*c^2) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*c^2*(1 - a
^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2\left(1-a^2x^2\right)} + \frac{2\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}$$
$$= \frac{2e^{\coth^{-1}(ax)}}{3ac^2} - \frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.143481, size = 50, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-2ax-1\right)}{3c^2(ax-1)^2(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^2*(-1 + a*x)^2*(1 +
 a*x))
```

Maple [A] time = 0.045, size = 49, normalized size = 1.

$$\frac{2\,a^2x^2-2\,ax-1}{\left(3\,a^2x^2-3\right)c^2a}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/3*(2*a^2*x^2-2*a*x-1)/(a^2*x^2-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

Maxima [A] time = 1.06119, size = 88, normalized size = 1.73

$$\frac{1}{12}a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A] time = 1.57265, size = 123, normalized size = 2.41

$$\frac{\left(2\,a^2x^2-2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*
c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x**2*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Giac [A] time = 1.10668, size = 104, normalized size = 2.04

$$\frac{1}{12}\,a\left(\frac{(ax+1)\left(\frac{6\,(ax-1)}{ax+1}-1\right)}{(ax-1)a^2c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*a*((a*x + 1)*(6*(a*x - 1)/(a*x + 1) - 1)/((a*x - 1)*a^2*c^2*sqrt((a*x
- 1)/(a*x + 1))) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

3.562 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=85

$$-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4(1-2ax)e^{\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)}+\frac{8e^{\coth^{-1}(ax)}}{15ac^3}$$

```
[Out] (8*E^ArcCoth[a*x])/(15*a*c^3) - (E^ArcCoth[a*x]*(1 - 4*a*x))/(15*a*c^3*(1 -
 a^2*x^2)^2) - (4*E^ArcCoth[a*x]*(1 - 2*a*x))/(15*a*c^3*(1 - a^2*x^2))
```

Rubi [A] time = 0.0929698, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6185, 6183}

$$-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4(1-2ax)e^{\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)}+\frac{8e^{\coth^{-1}(ax)}}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8*E^ArcCoth[a*x])/(15*a*c^3) - (E^ArcCoth[a*x]*(1 - 4*a*x))/(15*a*c^3*(1 -
 a^2*x^2)^2) - (4*E^ArcCoth[a*x]*(1 - 2*a*x))/(15*a*c^3*(1 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{5c}$$

$$= -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3\left(1-a^2x^2\right)}+\frac{8\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2}$$

$$= \frac{8e^{\coth^{-1}(ax)}}{15ac^3}-\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.166856, size = 66, normalized size = 0.78

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-8a^3x^3-12a^2x^2+12ax+3\right)}{15c^3(ax-1)^3(ax+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4))
/(15*c^3*(-1 + a*x)^3*(1 + a*x)^2)
```

Maple [A] time = 0.044, size = 65, normalized size = 0.8

$$\frac{8\,x^4a^4-8\,x^3a^3-12\,a^2x^2+12\,ax+3}{15\,c^3\left(a^2x^2-1\right)^2a}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/(a^2*x^2-1)^2/c^3/((a*x-1)/(
a*x+1))^(1/2)/a
```

Maxima [A] time = 0.996608, size = 134, normalized size = 1.58

$$-\frac{1}{240}\,a\left(\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-12\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/240*a*(5*(((a*x - 1)/(a*x + 1))^(3/2) - 12*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c^3) + (20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a^2*c^
3*((a*x - 1)/(a*x + 1))^(5/2)))
```

Fricas [A] time = 1.63693, size = 181, normalized size = 2.13

$$\frac{\left(8\,a^4x^4-8\,a^3x^3-12\,a^2x^2+12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*a^4*x^4 - 8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
+ 1))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

[Out] Timed out

Giac [A] time = 1.13373, size = 186, normalized size = 2.19

$$-\frac{1}{240}\,a\left(\frac{(ax+1)^2\left(\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3\right)}{(ax-1)^2a^2c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{5\left(\frac{(ax-1)a^4c^6\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-12\,a^4c^6\sqrt{\frac{ax-1}{ax+1}}\right)}{a^6c^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/240*a*((a*x + 1)^2*(20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2
- 3)/((a*x - 1)^2*a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) + 5*((a*x - 1)*a^4*c^6
*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 12*a^4*c^6*sqrt((a*x - 1)/(a*x + 1))
)/(a^6*c^9))
```

3.563 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=119

$$-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}-\frac{8(1-2ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)}-\frac{2(1-4ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^2}+\frac{16e^{\coth^{-1}(ax)}}{35ac^4}$$

```
[Out] (16*E^ArcCoth[a*x])/(35*a*c^4) - (E^ArcCoth[a*x]*(1 - 6*a*x))/(35*a*c^4*(1
- a^2*x^2)^3) - (2*E^ArcCoth[a*x]*(1 - 4*a*x))/(35*a*c^4*(1 - a^2*x^2)^2) -
 (8*E^ArcCoth[a*x]*(1 - 2*a*x))/(35*a*c^4*(1 - a^2*x^2))
```

Rubi [A] time = 0.127988, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6185, 6183}

$$-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}-\frac{8(1-2ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)}-\frac{2(1-4ax)e^{\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^2}+\frac{16e^{\coth^{-1}(ax)}}{35ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```



```
[Out] (16*E^ArcCoth[a*x])/(35*a*c^4) - (E^ArcCoth[a*x]*(1 - 6*a*x))/(35*a*c^4*(1
- a^2*x^2)^3) - (2*E^ArcCoth[a*x]*(1 - 4*a*x))/(35*a*c^4*(1 - a^2*x^2)^2) -
 (8*E^ArcCoth[a*x]*(1 - 2*a*x))/(35*a*c^4*(1 - a^2*x^2))
```


Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{6\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{7c}$$

$$= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}+\frac{24\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{35c^2}$$

$$= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)}+\frac{16\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^3}$$

$$= \frac{16e^{\coth^{-1}(ax)}}{35ac^4}-\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.247455, size = 82, normalized size = 0.69

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5\right)}{35c^4(ax-1)^4(ax+1)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 - 30*a*x + 30*a^2*x^2 + 40*a^3*x^3 - 40*a^4*x^
4 - 16*a^5*x^5 + 16*a^6*x^6))/(35*c^4*(-1 + a*x)^4*(1 + a*x)^3)
```

Maple [A] time = 0.044, size = 81, normalized size = 0.7

$$\frac{16\,x^6a^6-16\,x^5a^5-40\,x^4a^4+40\,x^3a^3+30\,a^2x^2-30\,ax-5}{35\,c^4\left(a^2x^2-1\right)^3a}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/(a^2
*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/a
```

Maxima [A] time = 1.10038, size = 178, normalized size = 1.5

$$\frac{1}{2240}\,a\left(\frac{7\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+75\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{42\,(ax-1)}{ax+1}-\frac{175\,(ax-1)^2}{(ax+1)^2}+\frac{700\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*(7*(((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) +
 75*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + (42*(a*x - 1)/(a*x + 1) - 175*(a
*x - 1)^2/(a*x + 1)^2 + 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(7/2)))
```

Fricas [A] time = 1.55682, size = 278, normalized size = 2.34

$$\frac{\left(16\,a^6x^6-16\,a^5x^5-40\,a^4x^4+40\,a^3x^3+30\,a^2x^2-30\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*x^2 - 30*a
*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^
4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12541, size = 259, normalized size = 2.18

$$\frac{1}{2240}a\left(\frac{(ax+1)^3\left(\frac{42\,(ax-1)}{ax+1}-\frac{175\,(ax-1)^2}{(ax+1)^2}+\frac{700\,(ax-1)^3}{(ax+1)^3}-5\right)}{(ax-1)^3a^2c^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{7\left(\frac{10\,(ax-1)a^8c^{16}\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{(ax-1)^2a^8c^{16}\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-75\,a^8c^{16}\sqrt{\frac{ax-1}{ax+1}}\right)}{a^{10}c^{20}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/2240*a*((a*x + 1)^3*(42*(a*x - 1)/(a*x + 1) - 175*(a*x - 1)^2/(a*x + 1)^2
 + 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/((a*x - 1)^3*a^2*c^4*sqrt((a*x - 1)/(a*
x + 1))) - 7*(10*(a*x - 1)*a^8*c^16*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) - (
a*x - 1)^2*a^8*c^16*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 75*a^8*c^16*sqr
t((a*x - 1)/(a*x + 1)))/(a^10*c^20))
```

3.564 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

Optimal. Leaf size=84

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{4c^5(ax+1)^{10}}{5a}-\frac{8c^5(ax+1)^9}{3a}+\frac{4c^5(ax+1)^8}{a}-\frac{16c^5(ax+1)^7}{7a}$$

```
[Out] (-16*c^5*(1 + a*x)^7)/(7*a) + (4*c^5*(1 + a*x)^8)/a - (8*c^5*(1 + a*x)^9)/(
3*a) + (4*c^5*(1 + a*x)^10)/(5*a) - (c^5*(1 + a*x)^11)/(11*a)
```

Rubi [A] time = 0.0988688, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{4c^5(ax+1)^{10}}{5a}-\frac{8c^5(ax+1)^9}{3a}+\frac{4c^5(ax+1)^8}{a}-\frac{16c^5(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (-16*c^5*(1 + a*x)^7)/(7*a) + (4*c^5*(1 + a*x)^8)/a - (8*c^5*(1 + a*x)^9)/(
3*a) + (4*c^5*(1 + a*x)^10)/(5*a) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx\\
&= -\left(c^5\int (1-ax)^4(1+ax)^6\,dx\right)\\
&= -\left(c^5\int \left(16(1+ax)^6-32(1+ax)^7+24(1+ax)^8-8(1+ax)^9+(1+ax)^{10}\right)dx\right)\\
&= -\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}
\end{aligned}$$

Mathematica [A] time = 0.0411871, size = 47, normalized size = 0.56

$$-\frac{c^5(ax+1)^7\left(105a^4x^4-504a^3x^3+938a^2x^2-812ax+281\right)}{1155a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -(c^5*(1 + a*x)^7*(281 - 812*a*x + 938*a^2*x^2 - 504*a^3*x^3 + 105*a^4*x^4)
)/(1155*a)
```

Maple [A] time = 0.037, size = 85, normalized size = 1.

$$c^5\left(-\frac{a^{10}x^{11}}{11}-\frac{a^9x^{10}}{5}+\frac{x^9a^8}{3}+a^7x^8-\frac{2\,x^7a^6}{7}-2\,x^6a^5-\frac{2\,x^5a^4}{5}+2\,x^4a^3+x^3a^2-ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^5,x)
```

```
[Out] c^5*(-1/11*a^10*x^11-1/5*a^9*x^10+1/3*x^9*a^8+a^7*x^8-2/7*x^7*a^6-2*x^6*a^5
-2/5*x^5*a^4+2*x^4*a^3+x^3*a^2-a*x^2-x)
```

Maxima [A] time = 1.12469, size = 153, normalized size = 1.82

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Fricas [A] time = 1.37059, size = 235, normalized size = 2.8

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Sympy [A] time = 0.111206, size = 119, normalized size = 1.42

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{a^9c^5x^{10}}{5}+\frac{a^8c^5x^9}{3}+a^7c^5x^8-\frac{2a^6c^5x^7}{7}-2a^5c^5x^6-\frac{2a^4c^5x^5}{5}+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - a**9*c**5*x**10/5 + a**8*c**5*x**9/3 + a**7*c**5*x**
8 - 2*a**6*c**5*x**7/7 - 2*a**5*c**5*x**6 - 2*a**4*c**5*x**5/5 + 2*a**3*c**
5*x**4 + a**2*c**5*x**3 - a*c**5*x**2 - c**5*x
```

Giac [A] time = 1.15025, size = 153, normalized size = 1.82

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

3.565 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=69

$$\frac{c^4(ax+1)^9}{9a}-\frac{3c^4(ax+1)^8}{4a}+\frac{12c^4(ax+1)^7}{7a}-\frac{4c^4(ax+1)^6}{3a}$$

```
[Out] (-4*c^4*(1 + a*x)^6)/(3*a) + (12*c^4*(1 + a*x)^7)/(7*a) - (3*c^4*(1 + a*x)^
8)/(4*a) + (c^4*(1 + a*x)^9)/(9*a)
```

Rubi [A] time = 0.0850346, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^4(ax+1)^9}{9a}-\frac{3c^4(ax+1)^8}{4a}+\frac{12c^4(ax+1)^7}{7a}-\frac{4c^4(ax+1)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-4*c^4*(1 + a*x)^6)/(3*a) + (12*c^4*(1 + a*x)^7)/(7*a) - (3*c^4*(1 + a*x)^
8)/(4*a) + (c^4*(1 + a*x)^9)/(9*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx\\
&= -\left(c^4\int(1-ax)^3(1+ax)^5\,dx\right)\\
&= -\left(c^4\int\left(8(1+ax)^5-12(1+ax)^6+6(1+ax)^7-(1+ax)^8\right)dx\right)\\
&= -\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}
\end{aligned}
$$

Mathematica [A] time = 0.0315883, size = 39, normalized size = 0.57

$$\frac{c^4(ax+1)^6\left(28a^3x^3-105a^2x^2+138ax-65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A] time = 0.038, size = 63, normalized size = 0.9

$$c^4\left(\frac{x^9a^8}{9}+\frac{a^7x^8}{4}-\frac{2\,x^7a^6}{7}-x^6a^5+\frac{3\,x^4a^3}{2}+\frac{2\,x^3a^2}{3}-ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(1/9*x^9*a^8+1/4*a^7*x^8-2/7*x^7*a^6-x^6*a^5+3/2*x^4*a^3+2/3*x^3*a^2-a*
x^2-x)
```

Maxima [A] time = 1.21455, size = 111, normalized size = 1.61

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Fricas [A] time = 1.55152, size = 166, normalized size = 2.41

$$\frac{1}{9}a^8c^4x^9+\frac{1}{4}a^7c^4x^8-\frac{2}{7}a^6c^4x^7-a^5c^4x^6+\frac{3}{2}a^3c^4x^4+\frac{2}{3}a^2c^4x^3-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Sympy [A] time = 0.100179, size = 87, normalized size = 1.26

$$\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}-a^5c^4x^6+\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 - a**5*c**4*x**6 +
 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 - a*c**4*x**2 - c**4*x
```

Giac [A] time = 1.1178, size = 111, normalized size = 1.61

$$\frac{1}{9}a^8c^4x^9+\frac{1}{4}a^7c^4x^8-\frac{2}{7}a^6c^4x^7-a^5c^4x^6+\frac{3}{2}a^3c^4x^4+\frac{2}{3}a^2c^4x^3-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

3.566 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=52

$$-\frac{c^3(ax+1)^7}{7a}+\frac{2c^3(ax+1)^6}{3a}-\frac{4c^3(ax+1)^5}{5a}$$

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a) + (2*c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/
(7*a)
```

Rubi [A] time = 0.076907, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^3(ax+1)^7}{7a}+\frac{2c^3(ax+1)^6}{3a}-\frac{4c^3(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a) + (2*c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/
(7*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\ &= -\left(c^3\int (1-ax)^2(1+ax)^4\,dx\right)\\ &= -\left(c^3\int \left(4(1+ax)^4-4(1+ax)^5+(1+ax)^6\right)dx\right)\\ &= -\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.0226959, size = 31, normalized size = 0.6

$$-\frac{c^3(ax+1)^5\left(15a^2x^2-40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -(c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/(105*a)
```

Maple [A] time = 0.039, size = 54, normalized size = 1.

$$c^3\left(-\frac{x^7a^6}{7}-\frac{x^6a^5}{3}+\frac{x^5a^4}{5}+x^4a^3+\frac{x^3a^2}{3}-ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(-1/7*x^7*a^6-1/3*x^6*a^5+1/5*x^5*a^4+x^4*a^3+1/3*x^3*a^2-a*x^2-x)
```

Maxima [A] time = 1.07407, size = 95, normalized size = 1.83

$$-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Fricas [A] time = 1.53196, size = 143, normalized size = 2.75

$$-\frac{1}{7}a^6c^3x^7 - \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 + a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Sympy [A] time = 0.092369, size = 70, normalized size = 1.35

$$-\frac{a^6c^3x^7}{7} - \frac{a^5c^3x^6}{3} + \frac{a^4c^3x^5}{5} + a^3c^3x^4 + \frac{a^2c^3x^3}{3} - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - a**5*c**3*x**6/3 + a**4*c**3*x**5/5 + a**3*c**3*x**4 +
a**2*c**3*x**3/3 - a*c**3*x**2 - c**3*x
```

Giac [A] time = 1.12104, size = 95, normalized size = 1.83

$$-\frac{1}{7}a^6c^3x^7 - \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 + a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

3.567 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=35

$$\frac{c^2(ax+1)^5}{5a}-\frac{c^2(ax+1)^4}{2a}$$

```
[Out] -(c^2*(1 + a*x)^4)/(2*a) + (c^2*(1 + a*x)^5)/(5*a)
```

Rubi [A] time = 0.0668577, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^2(ax+1)^5}{5a}-\frac{c^2(ax+1)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*(1 + a*x)^4)/(2*a) + (c^2*(1 + a*x)^5)/(5*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\
&= -\left(c^2\int(1-ax)(1+ax)^3\,dx\right)\\
&= -\left(c^2\int\left(2(1+ax)^3-(1+ax)^4\right)dx\right)\\
&= -\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}
\end{aligned}
$$

Mathematica [A] time = 0.0162947, size = 23, normalized size = 0.66

$$\frac{c^2(ax+1)^4(2ax-3)}{10a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

Maple [A] time = 0.039, size = 31, normalized size = 0.9

$$c^2\left(\frac{x^5a^4}{5}+\frac{x^4a^3}{2}-ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(1/5*x^5*a^4+1/2*x^4*a^3-a*x^2-x)
```

Maxima [A] time = 1.04745, size = 51, normalized size = 1.46

$$\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Fricas [A] time = 1.44836, size = 74, normalized size = 2.11

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Sympy [A] time = 0.081442, size = 36, normalized size = 1.03

$$\frac{a^4c^2x^5}{5} + \frac{a^3c^2x^4}{2} - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 - c**2*x
```

Giac [A] time = 1.12924, size = 51, normalized size = 1.46

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

3.568 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=15

$$-\frac{c(ax+1)^3}{3a}$$

```
[Out] -(c*(1 + a*x)^3)/(3*a)
```

Rubi [A] time = 0.033691, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6167, 6140, 32}

$$-\frac{c(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*(1 + a*x)^3)/(3*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx = -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$$
$$= -\left(c\int (1+ax)^2\,dx\right)$$
$$= -\frac{c(1+ax)^3}{3a}$$

Mathematica [A] time = 0.0121771, size = 20, normalized size = 1.33

$$-c\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*(x + a*x^2 + (a^2*x^3)/3))
```

Maple [A] time = 0.037, size = 14, normalized size = 0.9

$$-\frac{c\,(ax+1)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c),x)
```

```
[Out] -1/3*c*(a*x+1)^3/a
```

Maxima [A] time = 1.05957, size = 28, normalized size = 1.87

$$-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Fricas [A] time = 1.47445, size = 43, normalized size = 2.87

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Sympy [A] time = 0.06979, size = 20, normalized size = 1.33

$$-\frac{a^2cx^3}{3} - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c),x)
```

```
[Out] -a**2*c*x**3/3 - a*c*x**2 - c*x
```

Giac [A] time = 1.09652, size = 28, normalized size = 1.87

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

3.569 $\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{ac(1-ax)}$$

```
[Out] -(1/(a*c*(1 - a*x)))
```

Rubi [A] time = 0.0653081, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$-\frac{1}{ac(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] -(1/(a*c*(1 - a*x)))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^2}\,dx}{c}$$
$$= -\frac{1}{ac(1-ax)}$$

Mathematica [C] time = 0.0145529, size = 18, normalized size = 1.12

$$\frac{e^{2\coth^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(2*ArcCoth[a*x])/(2*a*c)
```

Maple [A] time = 0.039, size = 15, normalized size = 0.9

$$\frac{1}{ac\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/c/a/(a*x-1)
```

Maxima [A] time = 1.02187, size = 18, normalized size = 1.12

$$\frac{1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/(a^2*c*x - a*c)
```

Fricas [A] time = 1.49786, size = 26, normalized size = 1.62

$$\frac{1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/(a^2*c*x - a*c)
```

Sympy [A] time = 0.292063, size = 10, normalized size = 0.62

$$\frac{1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c),x)
```

```
[Out] 1/(a**2*c*x - a*c)
```

Giac [A] time = 1.12216, size = 19, normalized size = 1.19

$$\frac{1}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/((a*x - 1)*a*c)
```

3.570 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] -1/(4*a*c^2*(1 - a*x)^2) - 1/(4*a*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rubi [A] time = 0.0787354, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{1}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/(4*a*c^2*(1 - a*x)^2) - 1/(4*a*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^2} \\
&= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2} \\
&= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2} \\
&= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.0245763, size = 35, normalized size = 0.69

$$
\frac{ax+(ax-1)^2\left(-\tanh^{-1}(ax)\right)-2}{4ac^2(ax-1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

Maple [A] time = 0.048, size = 60, normalized size = 1.2

$$
-\frac{\ln(ax+1)}{8\,ac^2}-\frac{1}{4\,ac^2\,(ax-1)^2}+\frac{1}{4\,ac^2\,(ax-1)}+\frac{\ln(ax-1)}{8\,ac^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/8*ln(a*x+1)/a/c^2-1/4/c^2/a/(a*x-1)^2+1/4/c^2/a/(a*x-1)+1/8/c^2/a*ln(a*x
-1)
```

Maxima [A] time = 1.0614, size = 85, normalized size = 1.67

$$\frac{ax-2}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.57383, size = 171, normalized size = 3.35

$$\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 0.464657, size = 54, normalized size = 1.06

$$\frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

Giac [A] time = 1.11966, size = 69, normalized size = 1.35

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax-2}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^2)
```

3.571 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}-\frac{1}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] -1/(12*a*c^3*(1 - a*x)^3) - 1/(8*a*c^3*(1 - a*x)^2) - 3/(16*a*c^3*(1 - a*x)
) + 1/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rubi [A] time = 0.100349, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}-\frac{1}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/(12*a*c^3*(1 - a*x)^3) - 1/(8*a*c^3*(1 - a*x)^2) - 3/(16*a*c^3*(1 - a*x)
) + 1/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{4(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}\\
&= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}\\
&= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}
\end{aligned}
$$

Mathematica [A] time = 0.0367401, size = 63, normalized size = 0.73

$$\frac{3a^3x^3-6a^2x^2+ax-3(ax-1)^3(ax+1)\tanh^{-1}(ax)+4}{12ac^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/(
12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A] time = 0.05, size = 90, normalized size = 1.1

$$\frac{1}{16\,ac^3\,(ax+1)}-\frac{\ln(ax+1)}{8\,ac^3}+\frac{1}{12\,ac^3\,(ax-1)^3}-\frac{1}{8\,ac^3\,(ax-1)^2}+\frac{3}{16\,ac^3\,(ax-1)}+\frac{\ln(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16/a/c^3/(a*x+1)-1/8*ln(a*x+1)/a/c^3+1/12/a/c^3/(a*x-1)^3-1/8/a/c^3/(a*x-
1)^2+3/16/a/c^3/(a*x-1)+1/8/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.06068, size = 123, normalized size = 1.43

$$\frac{3\,a^3x^3 - 6\,a^2x^2 + ax + 4}{12\left(a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^2c^3x - ac^3\right)} - \frac{\log\left(ax+1\right)}{8\,ac^3} + \frac{\log\left(ax-1\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.5362, size = 266, normalized size = 3.09

$$\frac{6\,a^3x^3 - 12\,a^2x^2 + 2\,ax - 3\left(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1\right)\log\left(ax+1\right) + 3\left(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1\right)\log\left(ax-1\right) + 8}{24\left(a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5*c
^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.675573, size = 83, normalized size = 0.97

$$\frac{3a^3x^3 - 6a^2x^2 + ax + 4}{12a^5c^3x^4 - 24a^4c^3x^3 + 24a^2c^3x - 12ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8} + \frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (3*a**3*x**3 - 6*a**2*x**2 + a*x + 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*x**
3 + 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c**
3)
```

Giac [A] time = 1.11658, size = 100, normalized size = 1.16

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^3}+\frac{\log\left(|ax-1|\right)}{8\,ac^3}+\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

3.572 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=121

$$-\frac{5}{32ac^4(1-ax)}+\frac{5}{64ac^4(ax+1)}-\frac{3}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}-\frac{1}{16ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

```
[Out] -1/(32*a*c^4*(1 - a*x)^4) - 1/(16*a*c^4*(1 - a*x)^3) - 3/(32*a*c^4*(1 - a*x
)^2) - 5/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) + 5/(64*a*c^4*(1 +
 a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rubi [A] time = 0.123069, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{5}{32ac^4(1-ax)}+\frac{5}{64ac^4(ax+1)}-\frac{3}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}-\frac{1}{16ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/(32*a*c^4*(1 - a*x)^4) - 1/(16*a*c^4*(1 - a*x)^3) - 3/(32*a*c^4*(1 - a*x
)^2) - 5/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) + 5/(64*a*c^4*(1 +
 a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$$

$$= -\frac{\int \frac{1}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= -\frac{\int \left(-\frac{1}{8(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{5}{32(-1+ax)^2}+\frac{1}{32(1+ax)^3}+\frac{5}{64(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}$$

$$= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1}$$

$$= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1}$$

Mathematica [A] time = 0.0656299, size = 82, normalized size = 0.68

$$-\frac{-15a^5x^5+30a^4x^4+10a^3x^3-50a^2x^2+17ax+15(ax-1)^4(ax+1)^2\tanh^{-1}(ax)+16}{64ac^4(ax-1)^4(ax+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -(16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(64*a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

Maple [A] time = 0.05, size = 120, normalized size = 1.

$$\frac{1}{64\,ac^4\,(ax+1)^2}+\frac{5}{64\,ac^4\,(ax+1)}-\frac{15\,\ln(ax+1)}{128\,ac^4}-\frac{1}{32\,ac^4\,(ax-1)^4}+\frac{1}{16\,ac^4\,(ax-1)^3}-\frac{3}{32\,ac^4\,(ax-1)^2}+\frac{5}{32\,ac^4\,(ax-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/128*ln(a*x+1)/a/c^4-1/32/c^4/a/(
a*x-1)^4+1/16/c^4/a/(a*x-1)^3-3/32/c^4/a/(a*x-1)^2+5/32/c^4/a/(a*x-1)+15/12
8/c^4/a*ln(a*x-1)
```

Maxima [A] time = 1.03077, size = 189, normalized size = 1.56

$$\frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}-\frac{15\,\log(ax+1)}{128\,ac^4}+\frac{15\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^7
*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

Fricas [B] time = 1.52844, size = 456, normalized size = 3.77

$$\frac{30\,a^5x^5-60\,a^4x^4-20\,a^3x^3+100\,a^2x^2-34\,ax-15\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log(ax+1)+15\left(a\right.}{128\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1)
```

```
 + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3
- a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.98543, size = 141, normalized size = 1.17

$$\frac{15a^5x^5 - 30a^4x^4 - 10a^3x^3 + 50a^2x^2 - 17ax - 16}{64a^7c^4x^6 - 128a^6c^4x^5 - 64a^5c^4x^4 + 256a^4c^4x^3 - 64a^3c^4x^2 - 128a^2c^4x + 64ac^4} + \frac{\frac{15\log\left(x - \frac{1}{a}\right)}{128} - \frac{15\log\left(x + \frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/(
64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Giac [A] time = 1.11439, size = 123, normalized size = 1.02

$$-\frac{15\log\left(|ax + 1|\right)}{128\,ac^4} + \frac{15\log\left(|ax - 1|\right)}{128\,ac^4} + \frac{15\,a^5x^5 - 30\,a^4x^4 - 10\,a^3x^3 + 50\,a^2x^2 - 17\,ax - 16}{64\,(ax + 1)^2(ax - 1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x +
1)^2*(a*x - 1)^4*a*c^4)
```

3.573 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{13/2}+\frac{5}{168}a^6c^4x^7\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5a^5c^4x^6\sqrt{1-}}{10}$$

```
[Out] (-55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (55*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x
^4)/448 - (11*a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/1008 - (5*
a^5*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/1008 + (5*a^6*c^4*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(13/2)*x^7)/168 - (5*a^7*c^4*(1 - 1/(a*x))^(3/2)
*(1 + 1/(a*x))^(13/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(13/2)*x^9)/9 - (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*
a)
```

Rubi [A] time = 0.332593, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{13/2}+\frac{5}{168}a^6c^4x^7\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5a^5c^4x^6\sqrt{1-}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```


```
[Out] (-55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (55*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x
^4)/448 - (11*a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/1008 - (5*
a^5*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/1008 + (5*a^6*c^4*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(13/2)*x^7)/168 - (5*a^7*c^4*(1 - 1/(a*x))^(3/2)
*(1 + 1/(a*x))^(13/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(13/2)*x^9)/9 - (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*
a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{1}{9}\left(5a^7c^4\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^9}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9-\frac{1}{24}\left(5a^6c^4\right)\mathrm{Su}$$

$$=\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)$$

$$=-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}$$

$$=-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.}{1008}$$

$$=-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1\right.}{1008}$$

$$=-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

Mathematica [A] time = 0.191668, size = 111, normalized size = 0.28

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-3712\right)\cdots\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a^
3*x^3 - 8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a^8
*x^8) - 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

Maple [A] time = 0.207, size = 288, normalized size = 0.7

$$\frac{(ax-1)^2c^4}{8064\,a\,(ax+1)}\left(896\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6+3024\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^5a^5+1920\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4-4200\left(a^2x^2-\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/8064*(a*x-1)^2*c^4/a*(896*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+3024*(a^2
*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5+1920*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^
4-4200*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-6528*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^2*a^2-1134*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-4352*(a^2*x^2-1)^(3/2)*
(a^2)^(1/2)+3465*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+8064*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)-3465*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*a)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.04752, size = 560, normalized size = 1.42

$$-\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+115038\,c^4\left(\cdots\right.\right.}{\cdots\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-1)^2a^2}{(ax+1)^2}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2)
- 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))
^(13/2) - 255222*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 360448*c^4*((a*x - 1)/(
a*x + 1))^(9/2) - 334602*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
 - 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*s
qrt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(
a*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)
^4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*
(a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a
^2/(a*x + 1)^9 - a^2))*a
```

Fricas [A] time = 1.62773, size = 419, normalized size = 1.07

$$-\frac{3465\, c^4 \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - 3465\, c^4 \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) - \left(896\, a^9 c^4 x^9 + 3920\, a^8 c^4 x^8 + 4048\, a^7 c^4 x^7 - 6200\, a^6 c^4 x^6 - 15672\, a\right.}{8064\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1) - (896*a^9*c^4*x^9 + 3920*a^8*c^4*x^8 + 4048*a^7*c^
4*x^7 - 6200*a^6*c^4*x^6 - 15672*a^5*c^4*x^5 - 5382*a^4*c^4*x^4 + 13306*a^3
*c^4*x^3 + 14839*a^2*c^4*x^2 + 887*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20879, size = 500, normalized size = 1.27

$$-\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{30030\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{115038\,(ax-1)^2c^4\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{334602\,(ax-1}{(ax+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(abs
(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(30030*(a*x - 1)*c^4*sqrt((a*x - 1
)/(a*x + 1))/(a*x + 1) - 115038*(a*x - 1)^2*c^4*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^2 - 334602*(a*x - 1)^3*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 +
 360448*(a*x - 1)^4*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 255222*(a*x
 - 1)^5*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^5 + 115038*(a*x - 1)^6*c^4*
sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^6 - 30030*(a*x - 1)^7*c^4*sqrt((a*x - 1
)/(a*x + 1))/(a*x + 1)^7 + 3465*(a*x - 1)^8*c^4*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^8 - 3465*c^4*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1)
- 1)^9))*a
```

3.574 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{14}a^5c^3x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{1}{70}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{9}{280}a^3c^3x^4\sqrt{1-}$$

```
[Out] (-9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (3*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(5/2)*x^3)/40 - (9*a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/280
 - (a^4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/70 + (a^5*c^3*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(3/2)*(1
+ 1/(a*x))^(11/2)*x^7)/7 - (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]])/(16*a)
```

Rubi [A] time = 0.253584, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{14}a^5c^3x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{1}{70}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{9}{280}a^3c^3x^4\sqrt{1-}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```


```
[Out] (-9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (3*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(5/2)*x^3)/40 - (9*a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/280
 - (a^4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/70 + (a^5*c^3*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(3/2)*(1
+ 1/(a*x))^(11/2)*x^7)/7 - (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]])/(16*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
```

```
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{1}{7}\left(3a^5c^3\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}{x^7}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7+\frac{1}{14}\left(a^4c^3\right)\text{Subst}\Bigg($$

$$=-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\Bigg($$

$$=-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\Bigg($$

$$=-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\Bigg($$

$$=-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\Bigg(1$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/}$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/}$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/}$$

Mathematica [A] time = 0.135254, size = 95, normalized size = 0.3

$$-\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax+368\right)+315\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 - 245*a*x - 656*a^2*x^2 - 350*a^3*x^3
 + 208*a^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 315*Log[(1 + Sqrt[1 - 1/(a^2*x
^2)])*x]))/(560*a)
```

Maple [A] time = 0.19, size = 240, normalized size = 0.8

$$\frac{(ax-1)^2 c^3}{560\, a\,(ax+1)}\left(-80\sqrt{a^2}\left(a^2x^2-1\right)^{3/2} x^4a^4-280\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3-288\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+70\sqrt{a^2}\left(a^2x^2-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/560*(a*x-1)^2*c^3/a*(-80*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-280*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-288*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+7
0*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+560*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-192*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+315*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-31
5*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1)
)^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.0872, size = 455, normalized size = 1.45

$$-\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+5943\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-92\ldots\right.}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}-\frac{35(\ldots}{(\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 21
```

```
00*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 5943*c^3*((a*x - 1)/(a*x + 1))^(9/2)
- 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 8393*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2
/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7
- a^2))*a
```

Fricas [A] time = 1.70373, size = 351, normalized size = 1.12

$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(80\,a^7c^3x^7+360\,a^6c^3x^6+488\,a^5c^3x^5-142\,a^4c^3x^4-1006\,a^3c^3x^3-9\right.}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (80*a^7*c^3*x^7 + 360*a^6*c^3*x^6 + 488*a^5*c^3*x^5
- 142*a^4*c^3*x^4 - 1006*a^3*c^3*x^3 - 901*a^2*c^3*x^2 + 123*a*c^3*x + 368*
c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int-\frac{3a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4*x**
4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1)), x) + Integral(a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*
```

```
x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [A] time = 1.205, size = 408, normalized size = 1.3

$$-\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{2100\,(ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{8393\,(ax-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{9216\,(ax-1)^3c^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(abs(sq
rt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(2100*(a*x - 1)*c^3*sqrt((a*x - 1)/(a
*x + 1))/(a*x + 1) + 8393*(a*x - 1)^2*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x +
1)^2 - 9216*(a*x - 1)^3*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 5943*(a
*x - 1)^4*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 2100*(a*x - 1)^5*c^3*
sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^5 + 315*(a*x - 1)^6*c^3*sqrt((a*x - 1)/
(a*x + 1))/(a*x + 1)^6 - 315*c^3*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)
/(a*x + 1) - 1)^7))*a
```

3.575 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{7}{60}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{7}{24}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.$$

```
[Out] (-7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/24 - (7*a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/60 - (a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (
a^4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/5 - (7*c^2*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rubi [A] time = 0.212644, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{7}{60}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{7}{24}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```


```
[Out] (-7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/24 - (7*a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/60 - (a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (
a^4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/5 - (7*c^2*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\mathrm{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{5}\left(a^3c^2\right)\mathrm{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{7/2}}{x^5\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{20}\left(7a^2c^2\right)\mathrm{Subst}\Bigg(\int$$

$$=-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\Bigg(1+$$

$$=-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\Bigg(1+$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

Mathematica [A] time = 0.102832, size = 79, normalized size = 0.34

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136\right)-105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3 +
 24*a^4*x^4) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A] time = 0.184, size = 192, normalized size = 0.8

$$\frac{(ax-1)^2 c^2}{120\,a\,(ax+1)}\Bigg(24\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+90\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+16\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}+105\sqrt{a^2}\sqrt{a^2x^2-1}xa+120$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/120*(a*x-1)^2*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+90*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+105*(a^2)^(1/2)*
(a^2*x^2-1)^(1/2)*x*a+120*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-105*ln((a^2*x
+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(3/2)/(a*
x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.08232, size = 350, normalized size = 1.5

$$-\frac{1}{120}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-490\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+896\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-790\,c\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 4
90*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) -
790*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5
*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*
a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)
^5 - a^2))
```

Fricas [A] time = 1.67084, size = 301, normalized size = 1.29

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(24\,a^5c^2x^5+114\,a^4c^2x^4+202\,a^3c^2x^3+127\,a^2c^2x^2-121\,ac^2x-136\,c\right.}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) - (24*a^5*c^2*x^5 + 114*a^4*c^2*x^4 + 202*a^3*c^2*x^3
+ 127*a^2*c^2*x^2 - 121*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -\frac{2a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4*x**4/(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a
*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [A] time = 1.18678, size = 316, normalized size = 1.36

$$-\frac{1}{120}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{790\,(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{896\,(ax-1)^2c^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{490\,(ax-1)^3c^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}-\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(abs(
sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 2*(790*(a*x - 1)*c^2*sqrt((a*x - 1)/(
a*x + 1))/(a*x + 1) - 896*(a*x - 1)^2*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x +
1)^2 + 490*(a*x - 1)^3*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 - 105*(a*x
 - 1)^4*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 + 105*c^2*sqrt((a*x - 1)/
(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^5))
```

3.576 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{5}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 - (5*a*c*Sqrt[1 - 1/(a*x)]*(
1 + 1/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^
3)/3 - (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rubi [A] time = 0.119083, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{5}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```


```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 - (5*a*c*Sqrt[1 - 1/(a*x)]*(
1 + 1/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^
3)/3 - (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
```

```
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\left(\left(a^2c\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$
$$=-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{3}(5ac)\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$
$$=-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{2}(5c)\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{}{}}}\right.$$
$$=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+$$
$$=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-$$
$$=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-$$

Mathematica [A] time = 0.0755309, size = 61, normalized size = 0.42

$$-\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+9ax+22\right)+15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 + 9*a*x + 2*a^2*x^2) + 15*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] time = 0.175, size = 183, normalized size = 1.3

$$-\frac{(ax-1)^2c}{(6ax+6)a}\left(2\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}+9\sqrt{a^2}\sqrt{a^2x^2-1}xa+24\sqrt{a^2}\sqrt{(ax-1)(ax+1)}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x)
```

```
[Out] -1/6*(a*x-1)^2*c*(2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+9*(a^2)^(1/2)*(a^2*
x^2-1)^(1/2)*x*a+24*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)-9*ln((a^2*x+(a^2*x^
2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+24*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x
+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A] time = 1.04643, size = 231, normalized size = 1.59

$$\frac{1}{6}\,a\left(\frac{2\left(15\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+33\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(15*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 33*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.51622, size = 225, normalized size = 1.55

$$-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3+11\,a^2cx^2+31\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (2*a^3*c*x^3 + 11*a^2*c*x^2 + 31*a*c*x + 22*c)*sqrt((a*x - 1
```

```
)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int \frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
x + 1)), x))
```

Giac [A] time = 1.18765, size = 211, normalized size = 1.46

$$-\frac{1}{6}a\left(\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{40\,(ax-1)c\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{15\,(ax-1)^2c\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-33\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/6*a*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c*log(abs(sqrt((a*
x - 1)/(a*x + 1)) - 1))/a^2 + 2*(40*(a*x - 1)*c*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1) - 15*(a*x - 1)^2*c*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 33*c*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.577 $\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

```
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Rubi [A] time = 0.0313877, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.0464494, size = 18, normalized size = 1.

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Maple [A] time = 0.123, size = 24, normalized size = 1.3

$$\frac{1}{3\,ac}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

Maxima [A] time = 1.0772, size = 31, normalized size = 1.72

$$\frac{1}{3\,ac\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3/(a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

Fricas [B] time = 1.58278, size = 112, normalized size = 6.22

$$\frac{\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c*x^2 - 2*a^2*c*x
+ a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c
```

Giac [A] time = 1.17372, size = 47, normalized size = 2.61

$$\frac{ax+1}{3\,(ax-1)ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + 1)/((a*x - 1)*a*c*sqrt((a*x - 1)/(a*x + 1)))
```

3.578 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2\left(1-a^2x^2\right)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

```
[Out] (-2*E^(3*ArcCoth[a*x]))/(15*a*c^2) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*
c^2*(1 - a^2*x^2))
```

Rubi [A] time = 0.0650561, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2\left(1-a^2x^2\right)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2*E^(3*ArcCoth[a*x]))/(15*a*c^2) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*
c^2*(1 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)} - \frac{2\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c}$$
$$= -\frac{2e^{3\coth^{-1}(ax)}}{15ac^2} + \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}$$

Mathematica [A] time = 0.149389, size = 43, normalized size = 0.78

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-6ax+7\right)}{15c^2(ax-1)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(15*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.131, size = 49, normalized size = 0.9

$$-\frac{2\,a^2x^2-6\,ax+7}{\left(15\,a^2x^2-15\right)c^2a}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/15*(2*a^2*x^2-6*a*x+7)/(a^2*x^2-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

Maxima [A] time = 1.31571, size = 74, normalized size = 1.35

$$\frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3}{60\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^2*((a*x
 - 1)/(a*x + 1))^(5/2))
```

Fricas [A] time = 1.49653, size = 161, normalized size = 2.93

$$-\frac{\left(2\,a^3x^3-4\,a^2x^2+ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*
x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14126, size = 93, normalized size = 1.69

$$\frac{(ax+1)^2\left(\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3\right)}{60\,(ax-1)^2ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/60*(a*x + 1)^2*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/
((a*x - 1)^2*a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

3.579 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=91

$$-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12(3-2ax)e^{3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}$$

```
[Out] (-8*E^(3*ArcCoth[a*x]))/(35*a*c^3) - (E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(7*a*
c^3*(1 - a^2*x^2)^2) + (12*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(35*a*c^3*(1 - a
^2*x^2))
```

Rubi [A] time = 0.100469, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12(3-2ax)e^{3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-8*E^(3*ArcCoth[a*x]))/(35*a*c^3) - (E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(7*a*
c^3*(1 - a^2*x^2)^2) + (12*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(35*a*c^3*(1 - a
^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{7c}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}-\frac{24\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}$$

$$= -\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}-\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.204774, size = 66, normalized size = 0.73

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13\right)}{35c^3(ax-1)^4(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^
4))/(35*c^3*(-1 + a*x)^4*(1 + a*x))
```

Maple [A] time = 0.122, size = 65, normalized size = 0.7

$$-\frac{8\,x^4a^4-24\,x^3a^3+20\,a^2x^2+4\,ax-13}{35\,c^3\left(a^2x^2-1\right)^2a}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/35*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a^2*x^2-1)^2/c^3/((a*x-1)
/(a*x+1))^(3/2)/a
```

Maxima [A] time = 1.2443, size = 131, normalized size = 1.44

$$-\frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + (28*(a*x - 1)/(a*x + 1)
- 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^3*((
a*x - 1)/(a*x + 1))^(7/2)))
```

Fricas [A] time = 1.60854, size = 205, normalized size = 2.25

$$-\frac{\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17203, size = 150, normalized size = 1.65

$$-\frac{1}{560}\,a\left(\frac{(ax+1)^3\left(\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5\right)}{(ax-1)^3a^2c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/560*a*((a*x + 1)^3*(28*(a*x - 1)/(a*x + 1) - 70*(a*x - 1)^2/(a*x + 1)^2
+ 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/((a*x - 1)^3*a^2*c^3*sqrt((a*x - 1)/(a*x
 + 1))) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

3.580 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$-\frac{10(3-4ax)e^{3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8(3-2ax)e^{3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}$$

```
[Out] (-16*E^(3*ArcCoth[a*x]))/(63*a*c^4) - (E^(3*ArcCoth[a*x])*(1 - 2*a*x))/(9*a
*c^4*(1 - a^2*x^2)^3) - (10*E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(63*a*c^4*(1 -
a^2*x^2)^2) + (8*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(21*a*c^4*(1 - a^2*x^2))
```

Rubi [A] time = 0.138726, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{10(3-4ax)e^{3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8(3-2ax)e^{3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-16*E^(3*ArcCoth[a*x]))/(63*a*c^4) - (E^(3*ArcCoth[a*x])*(1 - 2*a*x))/(9*a
*c^4*(1 - a^2*x^2)^3) - (10*E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(63*a*c^4*(1 -
a^2*x^2)^2) + (8*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(21*a*c^4*(1 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{9c}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{40\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{21c^2}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4\left(1-a^2x^2\right)}-\frac{16\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{21c^3}$$

$$= -\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}-\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.306899, size = 82, normalized size = 0.65

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19\right)}{63c^4(ax-1)^5(ax+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^
4 - 48*a^5*x^5 + 16*a^6*x^6))/(63*c^4*(-1 + a*x)^5*(1 + a*x)^2)
```

Maple [A] time = 0.135, size = 81, normalized size = 0.6

$$-\frac{16\,x^6a^6-48\,x^5a^5+24\,x^4a^4+56\,x^3a^3-66\,a^2x^2+6\,ax+19}{63\,c^4\left(a^2x^2-1\right)^3a}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/(a^
2*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/a
```

Maxima [A] time = 1.21075, size = 177, normalized size = 1.39

$$\frac{1}{4032}\,a\left(\frac{21\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-18\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{54\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{420\,(ax-1)^3}{(ax+1)^3}-\frac{945\,(ax-1)^4}{(ax+1)^4}-7}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*(21*(((a*x - 1)/(a*x + 1))^(3/2) - 18*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*c^4) + (54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 420*(a*x
 - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/(a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)))
```

Fricas [A] time = 1.53683, size = 263, normalized size = 2.07

$$-\frac{\left(16\,a^6x^6-48\,a^5x^5+24\,a^4x^4+56\,a^3x^3-66\,a^2x^2+6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/63*(16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a^2*x^2 + 6*a
*x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4
*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22066, size = 230, normalized size = 1.81

$$\frac{1}{4032}\,a\left(\frac{(ax+1)^4\left(\frac{54\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{420\,(ax-1)^3}{(ax+1)^3}-\frac{945\,(ax-1)^4}{(ax+1)^4}-7\right)}{(ax-1)^4a^2c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{21\left(\frac{(ax-1)a^4c^8\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-18\,a^4c^8\sqrt{\frac{ax-1}{ax+1}}\right)}{a^6c^{12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/4032*a*((a*x + 1)^4*(54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2
 + 420*(a*x - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/((a*x - 1
)^4*a^2*c^4*sqrt((a*x - 1)/(a*x + 1))) + 21*((a*x - 1)*a^4*c^8*sqrt((a*x -
1)/(a*x + 1))/(a*x + 1) - 18*a^4*c^8*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^12))
```

3.581 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

Optimal. Leaf size=66

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rubi [A] time = 0.0849376, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx\\ &= c^5\int (1-ax)^3(1+ax)^7\,dx\\ &= c^5\int \left(8(1+ax)^7-12(1+ax)^8+6(1+ax)^9-(1+ax)^{10}\right)dx\\ &= \frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}\end{aligned}$$

Mathematica [A] time = 0.0343649, size = 39, normalized size = 0.59

$$-\frac{c^5(ax+1)^8\left(15a^3x^3-54a^2x^2+67ax-29\right)}{165a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/(165*a)
```

Maple [A] time = 0.038, size = 75, normalized size = 1.1

$$c^5\left(-\frac{a^{10}x^{11}}{11}-\frac{2\,a^9x^{10}}{5}-\frac{x^9a^8}{3}+a^7x^8+2\,x^7a^6-\frac{14\,x^5a^4}{5}-2\,x^4a^3+x^3a^2+2\,ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x)
```

```
[Out] c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*x^9*a^8+a^7*x^8+2*x^7*a^6-14/5*x^5*a^
4-2*x^4*a^3+x^3*a^2+2*a*x^2+x)
```

Maxima [A] time = 1.07198, size = 136, normalized size = 2.06

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Fricas [A] time = 1.39913, size = 215, normalized size = 3.26

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Sympy [B] time = 0.116605, size = 109, normalized size = 1.65

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*x
**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c**5
*x**3 + 2*a*c**5*x**2 + c**5*x
```

Giac [A] time = 1.14754, size = 138, normalized size = 2.09

$$-\frac{\left(15\,c^5+\frac{231\,c^5}{ax-1}+\frac{1540\,c^5}{(ax-1)^2}+\frac{5775\,c^5}{(ax-1)^3}+\frac{13200\,c^5}{(ax-1)^4}+\frac{18480\,c^5}{(ax-1)^5}+\frac{14784\,c^5}{(ax-1)^6}+\frac{5280\,c^5}{(ax-1)^7}\right)(ax-1)^{11}}{165\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/165*(15*c^5 + 231*c^5/(a*x - 1) + 1540*c^5/(a*x - 1)^2 + 5775*c^5/(a*x -
 1)^3 + 13200*c^5/(a*x - 1)^4 + 18480*c^5/(a*x - 1)^5 + 14784*c^5/(a*x - 1)
^6 + 5280*c^5/(a*x - 1)^7)*(a*x - 1)^11/a
```

3.582 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=52

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rubi [A] time = 0.0788417, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=c^4\int(1-ax)^2(1+ax)^6\,dx$$
$$=c^4\int\left(4(1+ax)^6-4(1+ax)^7+(1+ax)^8\right)dx$$
$$=\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}$$

Mathematica [A] time = 0.0300276, size = 31, normalized size = 0.6

$$\frac{c^4(ax+1)^7\left(14a^2x^2-35ax+23\right)}{126a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

Maple [A] time = 0.037, size = 69, normalized size = 1.3

$$c^4\left(\frac{x^9a^8}{9}+\frac{a^7x^8}{2}+\frac{4\,x^7a^6}{7}-\frac{2\,x^6a^5}{3}-2\,x^5a^4-x^4a^3+\frac{4\,x^3a^2}{3}+2\,ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(1/9*x^9*a^8+1/2*a^7*x^8+4/7*x^7*a^6-2/3*x^6*a^5-2*x^5*a^4-x^4*a^3+4/3*
x^3*a^2+2*a*x^2+x)
```

Maxima [A] time = 1.11268, size = 124, normalized size = 2.38

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Fricas [A] time = 1.47327, size = 190, normalized size = 3.65

$$\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{2}\,a^{7}c^{4}x^{8}+\frac{4}{7}\,a^{6}c^{4}x^{7}-\frac{2}{3}\,a^{5}c^{4}x^{6}-2\,a^{4}c^{4}x^{5}-a^{3}c^{4}x^{4}+\frac{4}{3}\,a^{2}c^{4}x^{3}+2\,ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Sympy [B] time = 0.111963, size = 100, normalized size = 1.92

$$\frac{a^{8}c^{4}x^{9}}{9}+\frac{a^{7}c^{4}x^{8}}{2}+\frac{4a^{6}c^{4}x^{7}}{7}-\frac{2a^{5}c^{4}x^{6}}{3}-2a^{4}c^{4}x^{5}-a^{3}c^{4}x^{4}+\frac{4a^{2}c^{4}x^{3}}{3}+2ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**6
/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x**2
 + c**4*x
```

Giac [A] time = 1.16992, size = 122, normalized size = 2.35

$$\frac{\left(14\,c^{4}+\frac{189\,c^{4}}{ax-1}+\frac{1080\,c^{4}}{(ax-1)^{2}}+\frac{3360\,c^{4}}{(ax-1)^{3}}+\frac{6048\,c^{4}}{(ax-1)^{4}}+\frac{6048\,c^{4}}{(ax-1)^{5}}+\frac{2688\,c^{4}}{(ax-1)^{6}}\right)(ax-1)^{9}}{126\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/126*(14*c^4 + 189*c^4/(a*x - 1) + 1080*c^4/(a*x - 1)^2 + 3360*c^4/(a*x -
1)^3 + 6048*c^4/(a*x - 1)^4 + 6048*c^4/(a*x - 1)^5 + 2688*c^4/(a*x - 1)^6)*
(a*x - 1)^9/a
```

3.583 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=35

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rubi [A] time = 0.0628281, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\ &= c^3\int (1-ax)(1+ax)^5\,dx\\ &= c^3\int \left(2(1+ax)^5-(1+ax)^6\right)dx\\ &= \frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.0206693, size = 23, normalized size = 0.66

$$-\frac{c^3(ax+1)^6(3ax-4)}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -(c^3*(1 + a*x)^6*(-4 + 3*a*x))/(21*a)
```

Maple [A] time = 0.041, size = 45, normalized size = 1.3

$$c^3\left(-\frac{x^7a^6}{7}-\frac{2\,x^6a^5}{3}-x^5a^4+\frac{5\,x^3a^2}{3}+2\,ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(-1/7*x^7*a^6-2/3*x^6*a^5-x^5*a^4+5/3*x^3*a^2+2*a*x^2+x)
```

Maxima [A] time = 0.980798, size = 80, normalized size = 2.29

$$-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Fricas [A] time = 1.51495, size = 122, normalized size = 3.49

$$-\frac{1}{7}\,a^{6}c^{3}x^{7}-\frac{2}{3}\,a^{5}c^{3}x^{6}-a^{4}c^{3}x^{5}+\frac{5}{3}\,a^{2}c^{3}x^{3}+2\,ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Sympy [B] time = 0.096159, size = 63, normalized size = 1.8

$$-\frac{a^{6}c^{3}x^{7}}{7}-\frac{2a^{5}c^{3}x^{6}}{3}-a^{4}c^{3}x^{5}+\frac{5a^{2}c^{3}x^{3}}{3}+2ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3/
3 + 2*a*c**3*x**2 + c**3*x
```

Giac [B] time = 1.15696, size = 105, normalized size = 3.

$$-\frac{\left(3\,c^{3}+\frac{35\,c^{3}}{ax-1}+\frac{168\,c^{3}}{(ax-1)^{2}}+\frac{420\,c^{3}}{(ax-1)^{3}}+\frac{560\,c^{3}}{(ax-1)^{4}}+\frac{336\,c^{3}}{(ax-1)^{5}}\right)(ax-1)^{7}}{21\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/21*(3*c^3 + 35*c^3/(a*x - 1) + 168*c^3/(a*x - 1)^2 + 420*c^3/(a*x - 1)^3
 + 560*c^3/(a*x - 1)^4 + 336*c^3/(a*x - 1)^5)*(a*x - 1)^7/a
```

3.584 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^5}{5a}$$

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rubi [A] time = 0.0571792, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=c^2\int(1+ax)^4\,dx$$
$$=\frac{c^2(1+ax)^5}{5a}$$

Mathematica [B] time = 0.0283111, size = 37, normalized size = 2.18

$$c^2\left(\frac{a^4x^5}{5}+a^3x^4+2a^2x^3+2ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

Maple [A] time = 0.04, size = 16, normalized size = 0.9

$$\frac{c^2\left(ax+1\right)^5}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/5*c^2*(a*x+1)^5/a
```

Maxima [B] time = 1.08093, size = 63, normalized size = 3.71

$$\frac{1}{5}\,a^4c^2x^5+a^3c^2x^4+2\,a^2c^2x^3+2\,ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Fricas [B] time = 1.53714, size = 93, normalized size = 5.47

$$\frac{1}{5}a^4c^2x^5 + a^3c^2x^4 + 2a^2c^2x^3 + 2ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Sympy [B] time = 0.091581, size = 48, normalized size = 2.82

$$\frac{a^4c^2x^5}{5} + a^3c^2x^4 + 2a^2c^2x^3 + 2ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**2,x)
```

```
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**2
*x
```

Giac [B] time = 1.11551, size = 86, normalized size = 5.06

$$\frac{\left(c^2 + \frac{10\,c^2}{ax-1} + \frac{40\,c^2}{(ax-1)^2} + \frac{80\,c^2}{(ax-1)^3} + \frac{80\,c^2}{(ax-1)^4}\right)(ax-1)^5}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*(c^2 + 10*c^2/(a*x - 1) + 40*c^2/(a*x - 1)^2 + 80*c^2/(a*x - 1)^3 + 80*
c^2/(a*x - 1)^4)*(a*x - 1)^5/a
```

3.585 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=46

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0477622, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6167, 6140, 43}

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$$
$$=c\int\frac{(1+ax)^3}{1-ax}\,dx$$
$$=c\int\left(-4+\frac{8}{1-ax}-2(1+ax)-(1+ax)^2\right)dx$$
$$=-4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}$$

Mathematica [A] time = 0.0137819, size = 36, normalized size = 0.78

$$-\frac{1}{3}a^2cx^3-2acx^2-\frac{8c\log(1-ax)}{a}-7cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -7*c*x - 2*a*c*x^2 - (a^2*c*x^3)/3 - (8*c*Log[1 - a*x])/a
```

Maple [A] time = 0.042, size = 34, normalized size = 0.7

$$-\frac{cx^3a^2}{3}-2\,acx^2-7\,cx-8\,\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x)
```

```
[Out] -1/3*c*x^3*a^2-2*a*c*x^2-7*c*x-8*c/a*ln(a*x-1)
```

Maxima [A] time = 1.09658, size = 45, normalized size = 0.98

$$-\frac{1}{3}\,a^2cx^3-2\,acx^2-7\,cx-\frac{8\,c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Fricas [A] time = 1.63154, size = 88, normalized size = 1.91

$$-\frac{a^3cx^3 + 6\,a^2cx^2 + 21\,acx + 24\,c\log\left(ax - 1\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

Sympy [A] time = 0.293501, size = 36, normalized size = 0.78

$$-\frac{a^2cx^3}{3} - 2acx^2 - 7cx - \frac{8c\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c),x)
```

```
[Out] -a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Giac [A] time = 1.12558, size = 81, normalized size = 1.76

$$-\frac{(ax-1)^3\left(c + \frac{9\,c}{ax-1} + \frac{36\,c}{(ax-1)^2}\right)}{3\,a} + \frac{8\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/3*(a*x - 1)^3*(c + 9*c/(a*x - 1) + 36*c/(a*x - 1)^2)/a + 8*c*log(abs(a*x
 - 1)/((a*x - 1)^2*abs(a)))/a
```

3.586 $\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c(1-ax)^2}$$

```
[Out] x/(c*(1 - a*x)^2)
```

Rubi [A] time = 0.0625637, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 34}

$$\frac{x}{c(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] x/(c*(1 - a*x)^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$$
$$= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c}$$
$$= \frac{x}{c(1-ax)^2}$$

Mathematica [A] time = 0.0096785, size = 25, normalized size = 1.92

$$\frac{(ax+1)^2}{4ac(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

Maple [B] time = 0.046, size = 28, normalized size = 2.2

$$\frac{1}{c}\left(\frac{1}{a\,(ax-1)^2}+\frac{1}{a\,(ax-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x)
```

```
[Out] 1/c*(1/a/(a*x-1)^2+1/a/(a*x-1))
```

Maxima [A] time = 1.09274, size = 26, normalized size = 2.

$$\frac{x}{a^2cx^2-2\,acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Fricas [A] time = 1.48406, size = 39, normalized size = 3.

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Sympy [B] time = 0.343586, size = 17, normalized size = 1.31

$$\frac{x}{a^2cx^2 - 2acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c),x)
```

```
[Out] x/(a**2*c*x**2 - 2*a*c*x + c)
```

Giac [B] time = 1.11978, size = 36, normalized size = 2.77

$$\frac{\frac{1}{(ax-1)a} + \frac{1}{(ax-1)^2a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] (1/((a*x - 1)*a) + 1/((a*x - 1)^2*a))/c
```

3.587 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3ac^2(1-ax)^3}$$

[Out] 1/(3*a*c^2*(1 - a*x)^3)

Rubi [A] time = 0.0597022, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$\frac{1}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]

[Out] 1/(3*a*c^2*(1 - a*x)^3)

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$$
$$=\frac{\int \frac{1}{(1-ax)^4}\,dx}{c^2}$$
$$=\frac{1}{3ac^2(1-ax)^3}$$

Mathematica [A] time = 0.0180535, size = 17, normalized size = 0.94

$$-\frac{1}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.039, size = 16, normalized size = 0.9

$$-\frac{1}{3\,ac^2\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3/c^2/a/(a*x-1)^3
```

Maxima [B] time = 1.01847, size = 55, normalized size = 3.06

$$-\frac{1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Fricas [B] time = 1.39524, size = 78, normalized size = 4.33

$$-\frac{1}{3\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] time = 0.400754, size = 42, normalized size = 2.33

$$-\frac{1}{3a^4c^2x^3 - 9a^3c^2x^2 + 9a^2c^2x - 3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

Giac [A] time = 1.13769, size = 20, normalized size = 1.11

$$-\frac{1}{3\,(ax-1)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/3/((a*x - 1)^3*a*c^2)
```

3.588 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=87

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rubi [A] time = 0.0933569, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx\\ &= \frac{\int \frac{1}{(1-ax)^5(1+ax)}\,dx}{c^3}\\ &= \frac{\int \left(-\frac{1}{2(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{1}{16(-1+ax)^2}-\frac{1}{16\left(-1+a^2x^2\right)}\right)dx}{c^3}\\ &= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{16c^3}\\ &= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{16ac^3}\end{aligned}$$

Mathematica [A] time = 0.0375491, size = 52, normalized size = 0.6

$$\frac{-3a^3x^3+12a^2x^2-19ax+3(ax-1)^4\tanh^{-1}(ax)+16}{48ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a*
c^3*(-1 + a*x)^4)
```

Maple [A] time = 0.049, size = 90, normalized size = 1.

$$\frac{\ln(ax+1)}{32\,ac^3}+\frac{1}{8\,ac^3\,(ax-1)^4}-\frac{1}{12\,ac^3\,(ax-1)^3}+\frac{1}{16\,ac^3\,(ax-1)^2}-\frac{1}{16\,ac^3\,(ax-1)}-\frac{\ln(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/32*ln(a*x+1)/a/c^3+1/8/c^3/a/(a*x-1)^4-1/12/a/c^3/(a*x-1)^3+1/16/a/c^3/(a
*x-1)^2-1/16/a/c^3/(a*x-1)-1/32/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.04561, size = 138, normalized size = 1.59

$$-\frac{3\,a^{3}x^{3}-12\,a^{2}x^{2}+19\,ax-16}{48\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}+\frac{\log\left(ax+1\right)}{32\,ac^{3}}-\frac{\log\left(ax-1\right)}{32\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 +
 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*lo
g(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.58107, size = 324, normalized size = 3.72

$$-\frac{6\,a^{3}x^{3}-24\,a^{2}x^{2}+38\,ax-3\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(ax+1\right)+3\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log (}{96\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2
 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1
)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c
^3*x + a*c^3)
```

Sympy [A] time = 0.681876, size = 99, normalized size = 1.14

$$-\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*c
**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1/a
)/32 - log(x + 1/a)/32)/(a*c**3)
```

Giac [A] time = 1.14152, size = 123, normalized size = 1.41

$$\frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3}-\frac{\frac{3\,a^3c^9}{ax-1}-\frac{3\,a^3c^9}{(ax-1)^2}+\frac{4\,a^3c^9}{(ax-1)^3}-\frac{6\,a^3c^9}{(ax-1)^4}}{48\,a^4c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(3*a^3*c^9/(a*x - 1) - 3*a^3
*c^9/(a*x - 1)^2 + 4*a^3*c^9/(a*x - 1)^3 - 6*a^3*c^9/(a*x - 1)^4)/(a^4*c^12
)
```

3.589 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=122

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rubi [A] time = 0.117967, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx\\ &= \frac{\int \frac{1}{(1-ax)^6(1+ax)^2}\,dx}{c^4}\\ &= \frac{\int \left(\frac{1}{4(-1+ax)^6}-\frac{1}{4(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{64(1+ax)^2}-\frac{3}{32\left(-1+a^2x^2\right)}\right)dx}{c^4}\\ &= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+}\\ &= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+}\end{aligned}$$

Mathematica [A] time = 0.0611854, size = 80, normalized size = 0.66

$$\frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax+15(ax-1)^5(ax+1)\tanh^{-1}(ax)-48}{160ac^4(ax-1)^5(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

Maple [A] time = 0.055, size = 120, normalized size = 1.

$$-\frac{1}{64\,ac^{4}\left(ax+1\right)}+\frac{3\ln\left(ax+1\right)}{64\,ac^{4}}-\frac{1}{20\,ac^{4}\left(ax-1\right)^{5}}+\frac{1}{16\,ac^{4}\left(ax-1\right)^{4}}-\frac{1}{16\,ac^{4}\left(ax-1\right)^{3}}+\frac{1}{16\,ac^{4}\left(ax-1\right)^{2}}-\frac{5}{64\,ac^{4}\left(ax-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/64/a/c^4/(a*x+1)+3/64*ln(a*x+1)/a/c^4-1/20/c^4/a/(a*x-1)^5+1/16/c^4/a/(a
*x-1)^4-1/16/c^4/a/(a*x-1)^3+1/16/c^4/a/(a*x-1)^2-5/64/c^4/a/(a*x-1)-3/64/c
^4/a*ln(a*x-1)
```

Maxima [A] time = 1.08779, size = 176, normalized size = 1.44

$$-\frac{15\,a^{5}x^{5}-60\,a^{4}x^{4}+80\,a^{3}x^{3}-20\,a^{2}x^{2}-47\,ax+48}{160\left(a^{7}c^{4}x^{6}-4\,a^{6}c^{4}x^{5}+5\,a^{5}c^{4}x^{4}-5\,a^{3}c^{4}x^{2}+4\,a^{2}c^{4}x-ac^{4}\right)}+\frac{3\log\left(ax+1\right)}{64\,ac^{4}}-\frac{3\log\left(ax-1\right)}{64\,ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(a
^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x -
a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.48673, size = 421, normalized size = 3.45

$$-\frac{30\,a^{5}x^{5}-120\,a^{4}x^{4}+160\,a^{3}x^{3}-40\,a^{2}x^{2}-94\,ax-15\left(a^{6}x^{6}-4\,a^{5}x^{5}+5\,a^{4}x^{4}-5\,a^{2}x^{2}+4\,ax-1\right)\log\left(ax+1\right)+15\left(a^{6}x^{6}\right.}{320\left(a^{7}c^{4}x^{6}-4\,a^{6}c^{4}x^{5}+5\,a^{5}c^{4}x^{4}-5\,a^{3}c^{4}x^{2}+4\,a^{2}c^{4}x-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*(
a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) + 96
```

```
)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*
x - a*c^4)
```

Sympy [A] time = 0.957356, size = 129, normalized size = 1.06

$$-\frac{15a^5x^5-60a^4x^4+80a^3x^3-20a^2x^2-47ax+48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4}+\frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**4,x)
```

```
[Out] -(15*a**5*x**5 - 60*a**4*x**4 + 80*a**3*x**3 - 20*a**2*x**2 - 47*a*x + 48)/
(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*c*
*4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x + 1
/a)/64)/(a*c**4)
```

Giac [A] time = 1.1502, size = 171, normalized size = 1.4

$$\frac{3\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4}+\frac{1}{128\,ac^4\left(\frac{2}{ax-1}+1\right)}-\frac{\frac{25\,a^9c^{16}}{ax-1}-\frac{20\,a^9c^{16}}{(ax-1)^2}+\frac{20\,a^9c^{16}}{(ax-1)^3}-\frac{20\,a^9c^{16}}{(ax-1)^4}+\frac{16\,a^9c^{16}}{(ax-1)^5}}{320\,a^{10}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 3/64*log(abs(-2/(a*x - 1) - 1))/(a*c^4) + 1/128/(a*c^4*(2/(a*x - 1) + 1)) -
 1/320*(25*a^9*c^16/(a*x - 1) - 20*a^9*c^16/(a*x - 1)^2 + 20*a^9*c^16/(a*x
- 1)^3 - 20*a^9*c^16/(a*x - 1)^4 + 16*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

3.590 $\int e^{-\coth^{-1}(ax)} \left(c-a^2cx^2\right)^4 \, dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{8}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{8}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{5}{48}a^5c^4x^6\left(1-\frac{1}{ax}\right.$$

```
[Out] (-35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (35*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/
64 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/16 - (5*a^5*c^4*(1
 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/48 + (a^6*c^4*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(9/2)*x^7)/8 - (a^7*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
9/2)*x^8)/8 + (a^8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(9/2)*x^9)/9 - (35
*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rubi [A] time = 0.321884, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{8}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{8}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{5}{48}a^5c^4x^6\left(1-\frac{1}{ax}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```


```
[Out] (-35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (35*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/
64 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/16 - (5*a^5*c^4*(1
 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/48 + (a^6*c^4*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(9/2)*x^7)/8 - (a^7*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
9/2)*x^8)/8 + (a^8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(9/2)*x^9)/9 - (35
*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9+\left(a^7c^4\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^9}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9-\frac{1}{8}\left(7a^6c^4\right)\operatorname{Subst}$$

$$=\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}$$

$$=-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)$$

$$=\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}$$

$$=-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}$$

$$=-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(\right.$$

$$=-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(\right.$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{a}\right.$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{a}\right.$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{a}\right.$$

Mathematica [A] time = 0.199277, size = 111, normalized size = 0.28

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128\right)-315\log\left(x\right.\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3
+ 768*a^4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) - 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

Maple [A] time = 0.154, size = 279, normalized size = 0.7

$$\frac{(ax+1)c^4}{1152a}\sqrt{\frac{ax-1}{ax+1}}\left(128\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6-144\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^5a^5-384\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4+456\left(a^2x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/1152*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4/a*(128*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^6*a^6-144*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5-384*(a^2)^(1/2)*(a^2
*x^2-1)^(3/2)*x^4*a^4+456*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+384*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-522*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+256*(a
^2*x^2-1)^(3/2)*(a^2)^(1/2)+315*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-384*((a*x
-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))
/(a^2)^(1/2))*a)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.12695, size = 560, normalized size = 1.42

$$-\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-1)^2a^2}{(ax+1)^2}+\frac{84\,(ax-1}{(ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 2
730*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/
2) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 32768*c^4*((a*x - 1)/(a*x + 1
))^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*
x + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x -
1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2
+ 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a
*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7
*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1
)^9 - a^2))*a
```

Fricas [A] time = 1.65731, size = 401, normalized size = 1.02

$$-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(128\,a^9c^4x^9-16\,a^8c^4x^8-656\,a^7c^4x^7+88\,a^6c^4x^6+1368\,a^5c^4x^5-21\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) - (128*a^9*c^4*x^9 - 16*a^8*c^4*x^8 - 656*a^7*c^4*x^7
 + 88*a^6*c^4*x^6 + 1368*a^5*c^4*x^5 - 210*a^4*c^4*x^4 - 1490*a^3*c^4*x^3 +
 325*a^2*c^4*x^2 + 965*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18317, size = 265, normalized size = 0.67

$$\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|}+\frac{1}{1152}\sqrt{a^2x^2-1}\left(\frac{128\,c^4\operatorname{sgn}(ax+1)}{a}+\left(837\,c^4\operatorname{sgn}(ax+1)-2\left(256\,ac^4\operatorname{sg}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/
1152*sqrt(a^2*x^2 - 1)*(128*c^4*sgn(a*x + 1)/a + (837*c^4*sgn(a*x + 1) - 2*
(256*a*c^4*sgn(a*x + 1) + (489*a^2*c^4*sgn(a*x + 1) - 4*(96*a^3*c^4*sgn(a*x
 + 1) + (75*a^4*c^4*sgn(a*x + 1) - 2*(32*a^5*c^4*sgn(a*x + 1) - (8*a^7*c^4*
x*sgn(a*x + 1) - 9*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

3.591 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{6}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{6}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{8}a^3c^3x^4\sqrt{1-\frac{}{}}$$

```
[Out] (-5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (5*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/48 - (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/8 - (a^
4*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/6 + (a^5*c^3*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/6 - (a^6*c^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(
a*x))^(7/2)*x^7)/7 - (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(
16*a)
```

Rubi [A] time = 0.254862, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{6}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{6}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{8}a^3c^3x^4\sqrt{1-\frac{}{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```


```
[Out] (-5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (5*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/48 - (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/8 - (a^
4*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/6 + (a^5*c^3*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/6 - (a^6*c^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(
a*x))^(7/2)*x^7)/7 - (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(
16*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
```

```
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\left(\left(a^6c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\left(a^5c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^7}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7+\frac{1}{6}\left(5a^4c^3\right)\text{Subst}\Bigg($$

$$= -\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/}$$

$$= \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\Bigg(1$$

$$= -\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\Bigg(1$$

$$= -\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\Bigg(1+$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5}$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5}$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5}$$

Mathematica [A] time = 0.196621, size = 95, normalized size = 0.3

$$\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-48a^6x^6+56a^5x^5+144a^4x^4-182a^3x^3-144a^2x^2+231ax+48\right)-105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 231*a*x - 144*a^2*x^2 - 182*a^3*x^3 +
 144*a^4*x^4 + 56*a^5*x^5 - 48*a^6*x^6) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

Maple [A] time = 0.138, size = 231, normalized size = 0.7

$$-\frac{(ax+1)c^3}{336a}\sqrt{\frac{ax-1}{ax+1}}\left(48\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4-56\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3-96\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+126\sqrt{a^2}\left(a^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/336*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3/a*(48*(a^2)^(1/2)*(a^2*x^2-1)^(3
/2)*x^4*a^4-56*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*x^2*a^2+126*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-64*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)-105*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+112*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)+105*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*a)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.07542, size = 455, normalized size = 1.45

$$-\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+307\right.}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}-\frac{35(ax}{(ax}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 70
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
```

```
3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1
)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(
a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
a^2))*a
```

Fricas [A] time = 2.01974, size = 343, normalized size = 1.1

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(48\,a^7c^3x^7-8\,a^6c^3x^6-200\,a^5c^3x^5+38\,a^4c^3x^4+326\,a^3c^3x^3-87\,a^2c}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (48*a^7*c^3*x^7 - 8*a^6*c^3*x^6 - 200*a^5*c^3*x^5 +
38*a^4*c^3*x^4 + 326*a^3*c^3*x^3 - 87*a^2*c^3*x^2 - 279*a*c^3*x - 48*c^3)*s
qrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17291, size = 217, normalized size = 0.69

$$\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{16\,|a|}+\frac{1}{336}\sqrt{a^2x^2-1}\left(\frac{48\,c^3\mathrm{sgn}\,(ax+1)}{a}+\left(231\,c^3\mathrm{sgn}\,(ax+1)-2\left(72\,ac^3\mathrm{sgn}\,(ax+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/33
6*sqrt(a^2*x^2 - 1)*(48*c^3*sgn(a*x + 1)/a + (231*c^3*sgn(a*x + 1) - 2*(72*
a*c^3*sgn(a*x + 1) + (91*a^2*c^3*sgn(a*x + 1) - 4*(18*a^3*c^3*sgn(a*x + 1)
- (6*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

3.592 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{4}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{4}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{a\ldots}\right.$$

```
[Out] (-3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (a*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
)*x^3)/4 - (a^3*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/4 + (a^4*c
^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/5 - (3*c^2*ArcTanh[Sqrt[1 -
 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rubi [A] time = 0.199331, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{4}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{4}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{a\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```



```
[Out] (-3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (a*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
)*x^3)/4 - (a^3*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/4 + (a^4*c
^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/5 - (3*c^2*ArcTanh[Sqrt[1 -
 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\left(a^3c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^5}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5-\frac{1}{4}\left(3a^2c^2\right)\text{Subst}$$

$$=\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1\right.$$

$$=-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x$$

Mathematica [A] time = 0.10375, size = 79, normalized size = 0.34

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A] time = 0.138, size = 183, normalized size = 0.8

$$-\frac{(ax+1)c^2}{120a}\sqrt{\frac{ax-1}{ax+1}}\left(-24\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+30\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+40\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-16\left(a^2x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/120*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2/a*(-24*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^2*a^2+30*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+40*((a*x-1)*(a*x+1))^(3/2
)*(a^2)^(1/2)-16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-45*(a^2)^(1/2)*(a^2*x^2-1)^(
1/2)*x*a+45*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-
1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.13966, size = 350, normalized size = 1.5

$$-\frac{1}{40}a\left(\frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70c^2\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-\frac{5(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1}{(ax+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^
2*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^
2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*
x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^
2))
```

Fricas [A] time = 2.00816, size = 285, normalized size = 1.22

$$-\frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(8a^5c^2x^5-2a^4c^2x^4-26a^3c^2x^3+9a^2c^2x^2+33ac^2x+8c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^2*x^5 - 2*a^4*c^2*x^4 - 26*a^3*c^2*x^3 + 9*a^2
*c^2*x^2 + 33*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)), x))
```

Giac [A] time = 1.16409, size = 170, normalized size = 0.73

$$\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{8\,|a|}+\frac{1}{40}\sqrt{a^2x^2-1}\left(\left(25\,c^2\mathrm{sgn}\,(ax+1)-2\left(8\,ac^2\mathrm{sgn}\,(ax+1)-\left(4\,a^3c^2x\mathrm{sgn}\,(ax+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/40*
sqrt(a^2*x^2 - 1)*((25*c^2*sgn(a*x + 1) - 2*(8*a*c^2*sgn(a*x + 1) - (4*a^3*
c^2*x*sgn(a*x + 1) - 5*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 8*c^2*sgn(a*x + 1)/a
)
```

3.593 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

```
[Out] -(c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 +
1/(a*x))^(3/2)*x^2)/2 - (a^2*c*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x^3)
/3 - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rubi [A] time = 0.114582, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 +
1/(a*x))^(3/2)*x^2)/2 - (a^2*c*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x^3)
/3 - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
```

```
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx = -\left(\left(a^2c\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^4}dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-(ac)\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}{x^3}dx,x,\frac{1}{x}\right)$$

$$=\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3+\frac{1}{2}c\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^2\sqrt{1-\frac{x}{a}}}\right.$$

$$=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x$$

$$=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x$$

$$=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x$$

Mathematica [A] time = 0.0875979, size = 61, normalized size = 0.42

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^2x^2+3ax+2\right)-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] time = 0.134, size = 119, normalized size = 0.8

$$\frac{c\,(ax+1)}{6\,a}\sqrt{\frac{ax-1}{ax+1}}\left(3\sqrt{a^2}\sqrt{a^2x^2-1}xa-2\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)\frac{1}{\sqrt{a^2}}\frac{1}{\sqrt{(ax-1)(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a-
2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*a)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A] time = 1.07162, size = 231, normalized size = 1.59

$$\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) + 8*c*((a*x - 1)/(a*x + 1))^(3/2)
 - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^
2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.82624, size = 216, normalized size = 1.49

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3-a^2cx^2-5\,acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (2*a^3*c*x^3 - a^2*c*x^2 - 5*a*c*x - 2*c)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sq
rt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A] time = 1.15411, size = 111, normalized size = 0.77

$$\frac{c\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\operatorname{sgn}(ax+1)}{2|a|}-\frac{1}{6}\sqrt{a^{2}x^{2}-1}\left((2acx\operatorname{sgn}(ax+1)-3c\operatorname{sgn}(ax+1))x-\frac{2c\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sqr
t(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 3*c*sgn(a*x + 1))*x - 2*c*sgn(a*x +
 1)/a)
```

3.594 $\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal. Leaf size=16

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

```
[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Rubi [A] time = 0.0301235, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]

[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2} dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.0446563, size = 16, normalized size = 1.

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
```

```
[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Maple [A] time = 0.043, size = 24, normalized size = 1.5

$$-\frac{1}{ac}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.04444, size = 31, normalized size = 1.94

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Fricas [A] time = 1.82908, size = 46, normalized size = 2.88

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] undef
```

3.595 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=55

$$\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

```
[Out] -2/(3*a*c^2*E^ArcCoth[a*x]) + (1 + 2*a*x)/(3*a*c^2*E^ArcCoth[a*x]*(1 - a^2*
x^2))
```

Rubi [A] time = 0.0622484, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -2/(3*a*c^2*E^ArcCoth[a*x]) + (1 + 2*a*x)/(3*a*c^2*E^ArcCoth[a*x]*(1 - a^2*
x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2\left(1-a^2x^2\right)} + \frac{2\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}$$
$$= -\frac{2e^{-\coth^{-1}(ax)}}{3ac^2} + \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.140154, size = 48, normalized size = 0.87

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+2ax-1\right)}{3(ax-1)(acx+c)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x + 2*a^2*x^2))/(3*(-1 + a*x)*(c + a*c*
x)^2)
```

Maple [A] time = 0.046, size = 49, normalized size = 0.9

$$-\frac{2\,a^2x^2+2\,ax-1}{\left(3\,a^2x^2-3\right)ac^2}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2+2*a*x-1)/(a^2*x^2-1)/a/c^2
```

Maxima [A] time = 0.994994, size = 90, normalized size = 1.64

$$\frac{1}{12}a\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-6\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}-\frac{3}{a^2c^2\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((((a*x - 1)/(a*x + 1))^(3/2) - 6*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^
2) - 3/(a^2*c^2*sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A] time = 1.64564, size = 105, normalized size = 1.91

$$-\frac{\left(2\,a^{2}x^{2}+2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}c^{2}x^{2}-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a^2*x^2 + 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - a*c^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{4}x^{4}-2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x**2 + 1), x
)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^2, x)
```

3.596 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=91

$$\frac{4(2ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}$$

```
[Out] -8/(15*a*c^3*E^ArcCoth[a*x]) + (1 + 4*a*x)/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^
2*x^2)^2) + (4*(1 + 2*a*x))/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rubi [A] time = 0.0969696, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{4(2ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```


```
[Out] -8/(15*a*c^3*E^ArcCoth[a*x]) + (1 + 4*a*x)/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^
2*x^2)^2) + (4*(1 + 2*a*x))/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^2*x^2))
```


Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{5c}$$

$$=\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3\left(1-a^2x^2\right)}+\frac{8\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2}$$

$$=-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}+\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.188673, size = 64, normalized size = 0.7

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3\right)}{15(ax-1)^2(acx+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^4*x^4)
)/(15*(-1 + a*x)^2*(c + a*c*x)^3)
```

Maple [A] time = 0.046, size = 65, normalized size = 0.7

$$-\frac{8x^4a^4+8x^3a^3-12a^2x^2-12ax+3}{15\left(a^2x^2-1\right)^2c^3a}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/(a^
2*x^2-1)^2/c^3/a
```

Maxima [A] time = 0.978571, size = 138, normalized size = 1.52

$$-\frac{1}{240}\,a\left(\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-20\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+90\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{5\left(\frac{12\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/240*a*((3*((a*x - 1)/(a*x + 1))^(5/2) - 20*((a*x - 1)/(a*x + 1))^(3/2) +
 90*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 5*(12*(a*x - 1)/(a*x + 1) - 1)/(
a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A] time = 1.51358, size = 163, normalized size = 1.79

$$-\frac{\left(8\,a^4x^4+8\,a^3x^3-12\,a^2x^2-12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^4*x^4 + 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x**4 + 3*a*
*2*x**2 - 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^3, x)
```

3.597 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$\frac{8(2ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)}+\frac{2(4ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^2}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}$$

```
[Out] -16/(35*a*c^4*E^ArcCoth[a*x]) + (1 + 6*a*x)/(35*a*c^4*E^ArcCoth[a*x]*(1 - a
^2*x^2)^3) + (2*(1 + 4*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (8
*(1 + 2*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rubi [A] time = 0.135755, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{8(2ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)}+\frac{2(4ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^2}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -16/(35*a*c^4*E^ArcCoth[a*x]) + (1 + 6*a*x)/(35*a*c^4*E^ArcCoth[a*x]*(1 - a
^2*x^2)^3) + (2*(1 + 4*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (8
*(1 + 2*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] && !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx &= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{6\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{7c}\\ &= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4\left(1-a^2x^2\right)^2}+\frac{24\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{35c^2}\\ &= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4\left(1-a^2x^2\right)}+\frac{16\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^3}\\ &= -\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}+\frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4\left(1-a^2x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.254699, size = 80, normalized size = 0.63

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5\right)}{35(ax-1)^3(acx+c)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40*a^4*x
^4 + 16*a^5*x^5 + 16*a^6*x^6))/(35*(-1 + a*x)^3*(c + a*c*x)^4)
```

Maple [A] time = 0.046, size = 81, normalized size = 0.6

$$-\frac{16\,x^6a^6+16\,x^5a^5-40\,x^4a^4-40\,x^3a^3+30\,a^2x^2+30\,ax-5}{35\,\left(a^2x^2-1\right)^3c^4a}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3+
30*a^2*x^2+30*a*x-5)/(a^2*x^2-1)^3/c^4/a
```

Maxima [A] time = 1.04753, size = 182, normalized size = 1.43

$$\frac{1}{2240}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+175\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-700\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{7\left(\frac{10\,(ax-1)}{ax+1}-\frac{75\,(ax-1)^2}{(ax+1)^2}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 42*((a*x - 1)/(a*x + 1))^(5/2) +
 175*((a*x - 1)/(a*x + 1))^(3/2) - 700*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4)
 + 7*(10*(a*x - 1)/(a*x + 1) - 75*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^4*((a
*x - 1)/(a*x + 1))^(5/2)))
```

Fricas [A] time = 1.59774, size = 223, normalized size = 1.76

$$-\frac{\left(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x^2+30\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2*x^2 + 30*
a*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^4, x)
```

3.598 $\int e^{-2\coth^{-1}(ax)} \left(c - a^2cx^2\right)^4 \, dx$

Optimal. Leaf size=73

$$-\frac{c^4(1-ax)^9}{9a} + \frac{3c^4(1-ax)^8}{4a} - \frac{12c^4(1-ax)^7}{7a} + \frac{4c^4(1-ax)^6}{3a}$$

```
[Out] (4*c^4*(1 - a*x)^6)/(3*a) - (12*c^4*(1 - a*x)^7)/(7*a) + (3*c^4*(1 - a*x)^8
)/(4*a) - (c^4*(1 - a*x)^9)/(9*a)
```

Rubi [A] time = 0.0805593, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^4(1-ax)^9}{9a} + \frac{3c^4(1-ax)^8}{4a} - \frac{12c^4(1-ax)^7}{7a} + \frac{4c^4(1-ax)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^4*(1 - a*x)^6)/(3*a) - (12*c^4*(1 - a*x)^7)/(7*a) + (3*c^4*(1 - a*x)^8
)/(4*a) - (c^4*(1 - a*x)^9)/(9*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx\\
&= -\left(c^4\int(1-ax)^5(1+ax)^3\,dx\right)\\
&= -\left(c^4\int\left(8(1-ax)^5-12(1-ax)^6+6(1-ax)^7-(1-ax)^8\right)dx\right)\\
&= \frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}
\end{aligned}
$$

Mathematica [A] time = 0.0280838, size = 39, normalized size = 0.53

$$\frac{c^4(ax-1)^6\left(28a^3x^3+105a^2x^2+138ax+65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A] time = 0.042, size = 61, normalized size = 0.8

$$c^4\left(\frac{x^9a^8}{9}-\frac{a^7x^8}{4}-\frac{2\,x^7a^6}{7}+x^6a^5-\frac{3\,x^4a^3}{2}+\frac{2\,x^3a^2}{3}+ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] c^4*(1/9*x^9*a^8-1/4*a^7*x^8-2/7*x^7*a^6+x^6*a^5-3/2*x^4*a^3+2/3*x^3*a^2+a*
x^2-x)
```

Maxima [A] time = 1.1169, size = 108, normalized size = 1.48

$$\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Fricas [A] time = 1.52778, size = 166, normalized size = 2.27

$$\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Sympy [A] time = 0.104245, size = 87, normalized size = 1.19

$$\frac{a^8c^4x^9}{9}-\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}+a^5c^4x^6-\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] a**8*c**4*x**9/9 - a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 + a**5*c**4*x**6 -
 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 + a*c**4*x**2 - c**4*x
```

Giac [A] time = 1.11704, size = 108, normalized size = 1.48

$$\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

3.599 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=55

$$\frac{c^3(1-ax)^7}{7a}-\frac{2c^3(1-ax)^6}{3a}+\frac{4c^3(1-ax)^5}{5a}$$

```
[Out] (4*c^3*(1 - a*x)^5)/(5*a) - (2*c^3*(1 - a*x)^6)/(3*a) + (c^3*(1 - a*x)^7)/(
7*a)
```

Rubi [A] time = 0.0754054, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^3(1-ax)^7}{7a}-\frac{2c^3(1-ax)^6}{3a}+\frac{4c^3(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^3*(1 - a*x)^5)/(5*a) - (2*c^3*(1 - a*x)^6)/(3*a) + (c^3*(1 - a*x)^7)/(
7*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\
&= -\left(c^3\int(1-ax)^4(1+ax)^2\,dx\right)\\
&= -\left(c^3\int\left(4(1-ax)^4-4(1-ax)^5+(1-ax)^6\right)dx\right)\\
&= \frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}
\end{aligned}$$

Mathematica [A] time = 0.0252143, size = 31, normalized size = 0.56

$$-\frac{c^3(ax-1)^5\left(15a^2x^2+40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/(105*a)
```

Maple [A] time = 0.042, size = 54, normalized size = 1.

$$c^3\left(-\frac{x^7a^6}{7}+\frac{x^6a^5}{3}+\frac{x^5a^4}{5}-x^4a^3+\frac{x^3a^2}{3}+ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] c^3*(-1/7*x^7*a^6+1/3*x^6*a^5+1/5*x^5*a^4-x^4*a^3+1/3*x^3*a^2+a*x^2-x)
```

Maxima [A] time = 1.00553, size = 95, normalized size = 1.73

$$-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Fricas [A] time = 1.48316, size = 143, normalized size = 2.6

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Sympy [A] time = 0.095268, size = 70, normalized size = 1.27

$$-\frac{a^6c^3x^7}{7} + \frac{a^5c^3x^6}{3} + \frac{a^4c^3x^5}{5} - a^3c^3x^4 + \frac{a^2c^3x^3}{3} + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**6*c**3*x**7/7 + a**5*c**3*x**6/3 + a**4*c**3*x**5/5 - a**3*c**3*x**4 +
a**2*c**3*x**3/3 + a*c**3*x**2 - c**3*x
```

Giac [A] time = 1.11891, size = 95, normalized size = 1.73

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

3.600 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=37

$$\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

```
[Out] (c^2*(1 - a*x)^4)/(2*a) - (c^2*(1 - a*x)^5)/(5*a)
```

Rubi [A] time = 0.063194, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(1 - a*x)^4)/(2*a) - (c^2*(1 - a*x)^5)/(5*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$= -\left(c^2\int(1-ax)^3(1+ax)\,dx\right)$$
$$= -\left(c^2\int\left(2(1-ax)^3-(1-ax)^4\right)\,dx\right)$$
$$= \frac{c^2(1-ax)^4}{2a} - \frac{c^2(1-ax)^5}{5a}$$

Mathematica [A] time = 0.0165207, size = 30, normalized size = 0.81

$$\frac{1}{10}c^2x\left(2a^4x^4-5a^3x^3+10ax-10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Maple [A] time = 0.04, size = 30, normalized size = 0.8

$$c^2\left(\frac{x^5a^4}{5}-\frac{x^4a^3}{2}+ax^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)*(a*x-1),x)
```

```
[Out] c^2*(1/5*x^5*a^4-1/2*x^4*a^3+a*x^2-x)
```

Maxima [A] time = 1.13044, size = 50, normalized size = 1.35

$$\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Fricas [A] time = 1.5016, size = 74, normalized size = 2.

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Sympy [A] time = 0.083979, size = 36, normalized size = 0.97

$$\frac{a^4c^2x^5}{5} - \frac{a^3c^2x^4}{2} + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 - c**2*x
```

Giac [A] time = 1.13428, size = 50, normalized size = 1.35

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

3.601 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=16

$$\frac{c(1-ax)^3}{3a}$$

```
[Out] (c*(1 - a*x)^3)/(3*a)
```

Rubi [A] time = 0.0340112, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6167, 6140, 32}

$$\frac{c(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*(1 - a*x)^3)/(3*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$$
$$=-\left(c\int(1-ax)^2\,dx\right)$$
$$=\frac{c(1-ax)^3}{3a}$$

Mathematica [A] time = 0.013573, size = 21, normalized size = 1.31

$$-c\left(\frac{a^2x^3}{3}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c*(x - a*x^2 + (a^2*x^3)/3))
```

Maple [A] time = 0.04, size = 14, normalized size = 0.9

$$-\frac{c\left(ax-1\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/3*c*(a*x-1)^3/a
```

Maxima [A] time = 1.01997, size = 27, normalized size = 1.69

$$-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Fricas [A] time = 1.52522, size = 43, normalized size = 2.69

$$-\frac{1}{3}\,a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Sympy [A] time = 0.071557, size = 19, normalized size = 1.19

$$-\frac{a^2cx^3}{3} + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**2*c*x**3/3 + a*c*x**2 - c*x
```

Giac [A] time = 1.14692, size = 27, normalized size = 1.69

$$-\frac{1}{3}\,a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

3.602 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=14

$$\frac{1}{ac(ax+1)}$$

```
[Out] 1/(a*c*(1 + a*x))
```

Rubi [A] time = 0.0611401, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$\frac{1}{ac(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] 1/(a*c*(1 + a*x))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$$
$$= -\frac{\int \frac{1}{(1+ax)^2}\,dx}{c}$$
$$= \frac{1}{ac(1+ax)}$$

Mathematica [C] time = 0.0144369, size = 18, normalized size = 1.29

$$-\frac{e^{-2\coth^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(2*a*c*E^(2*ArcCoth[a*x]))
```

Maple [A] time = 0.039, size = 15, normalized size = 1.1

$$\frac{1}{ac\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/a/c/(a*x+1)
```

Maxima [A] time = 1.03499, size = 16, normalized size = 1.14

$$\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/(a^2*c*x + a*c)
```

Fricas [A] time = 1.49258, size = 26, normalized size = 1.86

$$\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/(a^2*c*x + a*c)
```

Sympy [A] time = 0.291627, size = 10, normalized size = 0.71

$$\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c),x)
```

```
[Out] 1/(a**2*c*x + a*c)
```

Giac [A] time = 1.11098, size = 19, normalized size = 1.36

$$\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/((a*x + 1)*a*c)
```

3.603 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] 1/(4*a*c^2*(1 + a*x)^2) + 1/(4*a*c^2*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rubi [A] time = 0.0764191, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$\frac{1}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 1/(4*a*c^2*(1 + a*x)^2) + 1/(4*a*c^2*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)(1+ax)^3}\,dx}{c^2}\\
&= -\frac{\int \left(\frac{1}{2(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2}\\
&= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}\\
&= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.0265241, size = 33, normalized size = 0.67

$$
\frac{ax+(ax+1)^2\left(-\tanh^{-1}(ax)\right)+2}{4a(acx+c)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] (2 + a*x - (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

Maple [A] time = 0.05, size = 60, normalized size = 1.2

$$
\frac{1}{4\,ac^2\,(ax+1)^2}+\frac{1}{4\,ac^2\,(ax+1)}-\frac{\ln(ax+1)}{8\,ac^2}+\frac{\ln(ax-1)}{8\,ac^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/8*ln(a*x+1)/a/c^2+1/8/c^2/a*ln(a*x-
1)
```

Maxima [A] time = 1.11156, size = 85, normalized size = 1.73

$$\frac{ax+2}{4\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.50556, size = 171, normalized size = 3.49

$$\frac{2\,ax-\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)+4}{8\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [A] time = 0.463682, size = 54, normalized size = 1.1

$$\frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

Giac [A] time = 1.10729, size = 69, normalized size = 1.41

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax+2}{4\,(ax+1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x +
 2)/((a*x + 1)^2*a*c^2)
```

3.604 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{16ac^3(1-ax)}+\frac{3}{16ac^3(ax+1)}+\frac{1}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] -1/(16*a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) + 1/(8*a*c^3*(1 + a*x)^2
) + 3/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rubi [A] time = 0.10043, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{1}{16ac^3(1-ax)}+\frac{3}{16ac^3(ax+1)}+\frac{1}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/(16*a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) + 1/(8*a*c^3*(1 + a*x)^2
) + 3/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2(1+ax)^4}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{16(-1+ax)^2}+\frac{1}{4(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}\\
&= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}\\
&= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] time = 0.0398514, size = 61, normalized size = 0.73

$$\frac{3a^3x^3+6a^2x^2+ax-3(ax-1)(ax+1)^3\tanh^{-1}(ax)-4}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])/
(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] time = 0.053, size = 90, normalized size = 1.1

$$\frac{1}{12\,ac^3\,(ax+1)^3}+\frac{1}{8\,ac^3\,(ax+1)^2}+\frac{3}{16\,ac^3\,(ax+1)}-\frac{\ln(ax+1)}{8\,ac^3}+\frac{1}{16\,ac^3\,(ax-1)}+\frac{\ln(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+1)^2+3/16/a/c^3/(a*x+1)-1/8*ln(a*x+1)/a
/c^3+1/16/a/c^3/(a*x-1)+1/8/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.0935, size = 123, normalized size = 1.46

$$\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}-\frac{\log\left(ax+1\right)}{8\,ac^3}+\frac{\log\left(ax-1\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.55461, size = 266, normalized size = 3.17

$$\frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(ax-1\right)-8}{24\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5*c
^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.652583, size = 83, normalized size = 0.99

$$\frac{3a^3x^3+6a^2x^2+ax-4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (3*a**3*x**3 + 6*a**2*x**2 + a*x - 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*x**
3 - 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c**
3)
```

Giac [A] time = 1.1132, size = 100, normalized size = 1.19

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^3}+\frac{\log\left(|ax-1|\right)}{8\,ac^3}+\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(ax+1)^3(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 + 6*a^2*x^2 + a*x - 4)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

3.605 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=119

$$-\frac{5}{64ac^4(1-ax)}+\frac{5}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{3}{32ac^4(ax+1)^2}+\frac{1}{16ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

```
[Out] -1/(64*a*c^4*(1 - a*x)^2) - 5/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1 + a*x)^
4) + 1/(16*a*c^4*(1 + a*x)^3) + 3/(32*a*c^4*(1 + a*x)^2) + 5/(32*a*c^4*(1 +
 a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rubi [A] time = 0.119755, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{5}{64ac^4(1-ax)}+\frac{5}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{3}{32ac^4(ax+1)^2}+\frac{1}{16ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```


```
[Out] -1/(64*a*c^4*(1 - a*x)^2) - 5/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1 + a*x)^
4) + 1/(16*a*c^4*(1 + a*x)^3) + 3/(32*a*c^4*(1 + a*x)^2) + 5/(32*a*c^4*(1 +
 a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx\\ &= -\frac{\int \frac{1}{(1-ax)^3(1+ax)^5}\,dx}{c^4}\\ &= -\frac{\int \left(-\frac{1}{32(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{8(1+ax)^5}+\frac{3}{16(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{5}{32(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}\\ &= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1\\ &= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1\end{aligned}$$

Mathematica [A] time = 0.0617833, size = 80, normalized size = 0.67

$$\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax-15(ax-1)^2(ax+1)^4\tanh^{-1}(ax)+16}{64a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (16 - 17*a*x - 50*a^2*x^2 - 10*a^3*x^3 + 30*a^4*x^4 + 15*a^5*x^5 - 15*(-1 +
 a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

Maple [A] time = 0.052, size = 120, normalized size = 1.

$$\frac{1}{32\,ac^{4}\left(ax+1\right)^{4}}+\frac{1}{16\,ac^{4}\left(ax+1\right)^{3}}+\frac{3}{32\,ac^{4}\left(ax+1\right)^{2}}+\frac{5}{32\,ac^{4}\left(ax+1\right)}-\frac{15\,\ln\left(ax+1\right)}{128\,ac^{4}}-\frac{1}{64\,ac^{4}\left(ax-1\right)^{2}}+\frac{5}{64\,ac^{4}\left(ax-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/32/a/c^4/(a*x+1)^4+1/16/a/c^4/(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(
a*x+1)-15/128*ln(a*x+1)/a/c^4-1/64/c^4/a/(a*x-1)^2+5/64/c^4/a/(a*x-1)+15/12
8/c^4/a*ln(a*x-1)
```

Maxima [A] time = 1.03425, size = 189, normalized size = 1.59

$$\frac{15\,a^{5}x^{5}+30\,a^{4}x^{4}-10\,a^{3}x^{3}-50\,a^{2}x^{2}-17\,ax+16}{64\left(a^{7}c^{4}x^{6}+2\,a^{6}c^{4}x^{5}-a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}-a^{3}c^{4}x^{2}+2\,a^{2}c^{4}x+ac^{4}\right)}-\frac{15\,\log\left(ax+1\right)}{128\,ac^{4}}+\frac{15\,\log\left(ax-1\right)}{128\,ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^7
*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

Fricas [B] time = 1.64474, size = 456, normalized size = 3.83

$$\frac{30\,a^{5}x^{5}+60\,a^{4}x^{4}-20\,a^{3}x^{3}-100\,a^{2}x^{2}-34\,ax-15\left(a^{6}x^{6}+2\,a^{5}x^{5}-a^{4}x^{4}-4\,a^{3}x^{3}-a^{2}x^{2}+2\,ax+1\right)\log\left(ax+1\right)+15\left(a\right.}{128\left(a^{7}c^{4}x^{6}+2\,a^{6}c^{4}x^{5}-a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}-a^{3}c^{4}x^{2}+2\,a^{2}c^{4}x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
```

```
 + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3
- a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 0.974898, size = 141, normalized size = 1.18

$$\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4}+\frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/(
64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Giac [A] time = 1.14136, size = 123, normalized size = 1.03

$$-\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{15\,\log\left(|ax-1|\right)}{128\,ac^4}+\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(ax+1)^4(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/((a*x +
1)^4*(a*x - 1)^2*a*c^4)
```

3.606 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{56}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{48}a^5c^4x^6\left(1-\right.$$

```
[Out] (55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (55*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^
4)/64 + (11*a^4*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/48 - (11*a
^5*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/48 + (11*a^6*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2)*x^7)/56 - (11*a^7*c^4*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(7/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^
(7/2)*x^9)/9 + (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a
)
```

Rubi [A] time = 0.337411, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{56}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{48}a^5c^4x^6\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (55*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^
4)/64 + (11*a^4*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/48 - (11*a
^5*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/48 + (11*a^6*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2)*x^7)/56 - (11*a^7*c^4*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(7/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^
(7/2)*x^9)/9 + (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a
)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4 dx=\left(a^8c^4\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4 x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9+\frac{1}{9}\left(11a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^9}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9-\frac{1}{8}\left(11a^6c^4\right)\text{Su}$$

$$=\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right.$$

$$=-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\right.$$

$$=\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right.$$

$$=-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right.$$

$$=\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}$$

$$=\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1\right.$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{a}\right.$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{a}\right.$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{a}\right.$$

Mathematica [A] time = 0.18618, size = 111, normalized size = 0.28

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(896a^8x^8-3024a^7x^7+1024a^6x^6+7224a^5x^5-8448a^4x^4-3066a^3x^3+10240a^2x^2-4599ax-3712\right)\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a^
3*x^3 - 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a^8
*x^8) + 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

Maple [A] time = 0.15, size = 288, normalized size = 0.7

$$\frac{(ax+1)^2c^4}{8064\,(ax-1)\,a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(896\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6-3024\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^5a^5+1920\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4+42\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8064*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^4/a*(896*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2)*x^6*a^6-3024*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5+1920*(a^2)^(1/2)*
(a^2*x^2-1)^(3/2)*x^4*a^4+4200*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-6528*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+1134*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a
-4352*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3465*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+
8064*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+3465*ln((a^2*x+(a^2*x^2-1)^(1/2)*(
a^2)^(1/2))/(a^2)^(1/2))*a)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.06843, size = 560, normalized size = 1.42

$$\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+115038\,c^4\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{9(ax-1)a^2}{ax+1}-\frac{36(ax-1)^2a^2}{(ax+1)^2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2) -
 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))^
(13/2) + 334602*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 360448*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 255222*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
- 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*sq
rt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a
*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^
4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(
a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^
2/(a*x + 1)^9 - a^2))*a
```

Fricas [A] time = 1.68322, size = 416, normalized size = 1.06

$$\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(896\,a^9c^4x^9-2128\,a^8c^4x^8-2000\,a^7c^4x^7+8248\,a^6c^4x^6-1224\,a^5c^4\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (896*a^9*c^4*x^9 - 2128*a^8*c^4*x^8 - 2000*a^7*c^4
*x^7 + 8248*a^6*c^4*x^6 - 1224*a^5*c^4*x^5 - 11514*a^4*c^4*x^4 + 7174*a^3*c
^4*x^3 + 5641*a^2*c^4*x^2 - 8311*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x +
1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1457, size = 267, normalized size = 0.68

$$-\frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|}-\frac{1}{8064}\sqrt{a^2x^2-1}\left(\frac{3712\,c^4\operatorname{sgn}(ax+1)}{a}+\left(4599\,c^4\operatorname{sgn}(ax+1)-2\left(5120\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1
/8064*sqrt(a^2*x^2 - 1)*(3712*c^4*sgn(a*x + 1)/a + (4599*c^4*sgn(a*x + 1) -
 2*(5120*a*c^4*sgn(a*x + 1) - (1533*a^2*c^4*sgn(a*x + 1) + 4*(1056*a^3*c^4*
sgn(a*x + 1) - (903*a^4*c^4*sgn(a*x + 1) + 2*(64*a^5*c^4*sgn(a*x + 1) + 7*(
8*a^7*c^4*x*sgn(a*x + 1) - 27*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

3.607 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{14}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{10}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{8}a^3c^3x^4\left(1-\right.$$

```
[Out] (9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (3*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/8 + (3*a^3*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/8 -
 (3*a^4*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/10 + (3*a^5*c^3*(1
 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(9/2
)*(1 + 1/(a*x))^(5/2)*x^7)/7 + (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/(16*a)
```

Rubi [A] time = 0.263775, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{14}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{10}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{8}a^3c^3x^4\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (3*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/8 + (3*a^3*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/8 -
 (3*a^4*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/10 + (3*a^5*c^3*(1
 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(9/2
)*(1 + 1/(a*x))^(5/2)*x^7)/7 + (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/(16*a)
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
```

```
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\left(\left(a^6c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^8}dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 - \frac{1}{7}\left(9a^5c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^7}dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 + \frac{1}{2}\left(3a^4c^3\right)\text{Subst}$$

$$= -\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right.$$

$$= \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)$$

$$= -\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}$$

$$= \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

Mathematica [A] time = 0.143684, size = 95, normalized size = 0.3

$$-\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368\right)-315\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 + 245*a*x - 656*a^2*x^2 + 350*a^3*x^3
 + 208*a^4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 315*Log[(1 + Sqrt[1 - 1/(a^2*x
^2)])*x]))/(560*a)
```

Maple [A] time = 0.143, size = 240, normalized size = 0.8

$$\frac{(ax+1)^2 c^3}{560\,(ax-1)\,a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-80\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4+280\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^3a^3-288\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-70\sqrt{a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/560*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^3/a*(-80*(a^2)^(1/2)*(a^2*x^2-1)^
(3/2)*x^4*a^4+280*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-288*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^2*a^2-70*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+560*((a*x-1)*(a
*x+1))^(3/2)*(a^2)^(1/2)-192*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-315*(a^2)^(1/2)*
(a^2*x^2-1)^(1/2)*x*a+315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*a)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.09191, size = 455, normalized size = 1.45

$$\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-8393\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+921\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(ax-1)^3a^2}{(ax+1)^3}-\frac{35\,(ax}{(ax}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 210
```

```
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) - 8393*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 5943*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1)
))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/
(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
 a^2))*a
```

Fricas [A] time = 1.59634, size = 347, normalized size = 1.11

$$\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(80\,a^7c^3x^7-200\,a^6c^3x^6-72\,a^5c^3x^5+558\,a^4c^3x^4-306\,a^3c^3x^3-411\,a\right.}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (80*a^7*c^3*x^7 - 200*a^6*c^3*x^6 - 72*a^5*c^3*x^5 +
558*a^4*c^3*x^4 - 306*a^3*c^3*x^3 - 411*a^2*c^3*x^2 + 613*a*c^3*x + 368*c^3
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18645, size = 219, normalized size = 0.7

$$-\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{16\,|a|}-\frac{1}{560}\sqrt{a^2x^2-1}\left(\frac{368\,c^3\mathrm{sgn}\left(ax+1\right)}{a}+\left(245\,c^3\mathrm{sgn}\left(ax+1\right)-2\left(328\,ac^3\mathrm{sgn}\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/5
60*sqrt(a^2*x^2 - 1)*(368*c^3*sgn(a*x + 1)/a + (245*c^3*sgn(a*x + 1) - 2*(3
28*a*c^3*sgn(a*x + 1) - (175*a^2*c^3*sgn(a*x + 1) + 4*(26*a^3*c^3*sgn(a*x +
 1) + 5*(2*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

3.608 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{20}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{12}a^2c^2x^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{8}ac^2x^2\sqrt{1-\frac{1}{a}}$$

```
[Out] (7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(3/2)*x^2)/8 + (7*a^2*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))
^(3/2)*x^3)/12 - (7*a^3*c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^4)/20
 + (a^4*c^2*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^5)/5 + (7*c^2*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rubi [A] time = 0.203663, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{20}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{12}a^2c^2x^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{8}ac^2x^2\sqrt{1-\frac{1}{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(3/2)*x^2)/8 + (7*a^2*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))
^(3/2)*x^3)/12 - (7*a^3*c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^4)/20
 + (a^4*c^2*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^5)/5 + (7*c^2*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{1}{5}\left(7a^3c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}{x^5}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5-\frac{1}{4}\left(7a^2c^2\right)\text{Sub}$$

$$=\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right.$$

$$=-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/}$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/}$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/}$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/}$$

Mathematica [A] time = 0.105606, size = 79, normalized size = 0.34

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A] time = 0.132, size = 192, normalized size = 0.8

$$\frac{(ax+1)^2 c^2}{120\,(ax-1)\,a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(24\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-90\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}xa+120\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}+16\left(a^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/120*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^2*a^2-90*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+120*((a*x-1)*(a*x+1))^(3/
2)*(a^2)^(1/2)+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-105*(a^2)^(1/2)*(a^2*x^2-1)
^(1/2)*x*a+105*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/(a*
x-1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

Maxima [A] time = 1.0217, size = 350, normalized size = 1.5

$$\frac{1}{120}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+490\,c^2\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) + 79
0*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 4
90*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*
(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a
^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^
5 - a^2))
```

Fricas [A] time = 1.53896, size = 296, normalized size = 1.27

$$\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^2x^5-66\,a^4c^2x^4+22\,a^3c^2x^3+97\,a^2c^2x^2-151\,ac^2x-136\,c^2\right)\sqrt{}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^2*x^5 - 66*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 9
7*a^2*c^2*x^2 - 151*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13636, size = 170, normalized size = 0.73

$$-\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{8\,|a|}-\frac{1}{120}\,\sqrt{a^2x^2-1}\left(\left(15\,c^2\mathrm{sgn}\,(ax+1)-2\left(56\,ac^2\mathrm{sgn}\,(ax+1)+3\left(4\,a^3c^2x\mathrm{sgn}\,(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/12
0*sqrt(a^2*x^2 - 1)*((15*c^2*sgn(a*x + 1) - 2*(56*a*c^2*sgn(a*x + 1) + 3*(4
*a^3*c^2*x*sgn(a*x + 1) - 15*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 136*c^2*sgn(a*
x + 1)/a)
```

3.609 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}+\frac{5}{6}acx^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (5*a*c*(1 - 1/(a*x))^(3/2)
*Sqrt[1 + 1/(a*x)]*x^2)/6 - (a^2*c*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^
3)/3 + (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rubi [A] time = 0.12013, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}+\frac{5}{6}acx^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (5*a*c*(1 - 1/(a*x))^(3/2)
*Sqrt[1 + 1/(a*x)]*x^2)/6 - (a^2*c*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^
3)/3 + (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
```

```
n/2] && IntegerQ[m]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx = -\left(\left(a^2c\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 - \frac{1}{3}(5ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3 + \frac{1}{2}(5c)\,\text{Subst}\left(\int \frac{\sqrt{1-}}{x^2\sqrt{1+}}\right.$$
$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3$$
$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3$$
$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3$$

Mathematica [A] time = 0.100573, size = 61, normalized size = 0.42

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^2x^2+9ax-22\right)+15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-22 + 9*a*x - 2*a^2*x^2) + 15*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A] time = 0.133, size = 183, normalized size = 1.3

$$\frac{(ax+1)^2c}{(6ax-6)a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(9\sqrt{a^2}\sqrt{a^2x^2-1}xa-2\left((ax-1)(ax+1)\right)^{3/2}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\sqrt{(ax\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c*(9*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*
a-2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)
^(1/2))/(a^2)^(1/2))*a-24*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+24*a*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/(a*x-1)/((a*x-1)*(a*x+
1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A] time = 1.11668, size = 231, normalized size = 1.59

$$\frac{1}{6}\,a\left(\frac{2\left(33\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(33*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 15*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) + 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.67929, size = 223, normalized size = 1.54

$$\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3-7\,a^2cx^2+13\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (2*a^3*c*x^3 - 7*a^2*c*x^2 + 13*a*c*x + 22*c)*sqrt((a*x - 1)/
```

```
(a*x + 1)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14056, size = 111, normalized size = 0.77

$$-\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}\left(ax+1\right)}{2\,|a|}-\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,acx\operatorname{sgn}\left(ax+1\right)-9\,c\operatorname{sgn}\left(ax+1\right)\right)x+\frac{22\,c\operatorname{sgn}\left(ax+1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sq
rt(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 9*c*sgn(a*x + 1))*x + 22*c*sgn(a*x
 + 1)/a)
```

3.610 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

```
[Out] -1/(3*a*c*E^(3*ArcCoth[a*x]))
```

Rubi [A] time = 0.03191, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*a*c*E^(3*ArcCoth[a*x]))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.047077, size = 18, normalized size = 1.

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*a*c*E^(3*ArcCoth[a*x]))
```

Maple [A] time = 0.043, size = 24, normalized size = 1.3

$$-\frac{1}{3\,ac}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.04226, size = 31, normalized size = 1.72

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/3*((a*x - 1)/(a*x + 1))^(3/2)/(a*c)
```

Fricas [A] time = 1.51913, size = 78, normalized size = 4.33

$$-\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x
- 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2
*x**2 - a*x - 1), x))/c
```

Giac [B] time = 1.20713, size = 66, normalized size = 3.67

$$\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x + 1)^
3*a*c)
```

3.611 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}$$

```
[Out] 2/(15*a*c^2*E^(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*c^2*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2))
```

Rubi [A] time = 0.0662282, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 2/(15*a*c^2*E^(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*c^2*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2\left(1-a^2x^2\right)} - \frac{2\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c}$$
$$= \frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.141018, size = 43, normalized size = 0.78

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+6ax+7\right)}{15c^2(ax+1)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(7 + 6*a*x + 2*a^2*x^2))/(15*c^2*(1 + a*x)^3)
```

Maple [A] time = 0.043, size = 49, normalized size = 0.9

$$\frac{2\,a^2x^2+6\,ax+7}{\left(15\,a^2x^2-15\right)ac^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/15*((a*x-1)/(a*x+1))^(3/2)*(2*a^2*x^2+6*a*x+7)/(a^2*x^2-1)/a/c^2
```

Maxima [A] time = 1.04872, size = 81, normalized size = 1.47

$$\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\sqrt{\frac{ax-1}{ax+1}}}{60\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 15*s
qrt((a*x - 1)/(a*x + 1)))/(a*c^2)
```

Fricas [A] time = 1.57377, size = 124, normalized size = 2.25

$$\frac{\left(2\,a^2x^2+6\,ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^2*x^2 + 6*a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 + 2*a^2
*c^2*x + a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17797, size = 88, normalized size = 1.6

$$-\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^5ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x + 1)^5*a*c^2)
```

3.612 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=91

$$-\frac{12(2ax+3)e^{-3\coth^{-1}(ax)}}{35ac^3(1-a^2x^2)}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}+\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}$$

```
[Out] 8/(35*a*c^3*E^(3*ArcCoth[a*x])) + (3 + 4*a*x)/(7*a*c^3*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2)^2) - (12*(3 + 2*a*x))/(35*a*c^3*E^(3*ArcCoth[a*x])*(1 - a^2*x^
2))
```

Rubi [A] time = 0.100999, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{12(2ax+3)e^{-3\coth^{-1}(ax)}}{35ac^3(1-a^2x^2)}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}+\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```


```
[Out] 8/(35*a*c^3*E^(3*ArcCoth[a*x])) + (3 + 4*a*x)/(7*a*c^3*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2)^2) - (12*(3 + 2*a*x))/(35*a*c^3*E^(3*ArcCoth[a*x])*(1 - a^2*x^
2))
```


Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{7c}$$

$$=\frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}-\frac{24\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}$$

$$=\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}+\frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.185284, size = 66, normalized size = 0.73

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13\right)}{35c^3(ax-1)(ax+1)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^3*(-1 + a*x)*(1 + a*x)^4)
```

Maple [A] time = 0.047, size = 65, normalized size = 0.7

$$\frac{8\,x^4a^4+24\,x^3a^3+20\,a^2x^2-4\,ax-13}{35\,\left(a^2x^2-1\right)^2c^3a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a^
2*x^2-1)^2/c^3/a
```

Maxima [A] time = 1.03213, size = 139, normalized size = 1.53

$$-\frac{1}{560}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-28\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-140\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{35}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 28*((a*x - 1)/(a*x + 1))^(5/2) +
 70*((a*x - 1)/(a*x + 1))^(3/2) - 140*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3)
- 35/(a^2*c^3*sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A] time = 1.53686, size = 182, normalized size = 2.

$$\frac{\left(8\,a^4x^4+24\,a^3x^3+20\,a^2x^2-4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^3, x)
```

3.613 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{8(2ax+3)e^{-3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}+\frac{10(4ax+3)e^{-3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}$$

```
[Out] 16/(63*a*c^4*E^(3*ArcCoth[a*x])) + (1 + 2*a*x)/(9*a*c^4*E^(3*ArcCoth[a*x])*
(1 - a^2*x^2)^3) + (10*(3 + 4*a*x))/(63*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x
^2)^2) - (8*(3 + 2*a*x))/(21*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))
```

Rubi [A] time = 0.137822, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{8(2ax+3)e^{-3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}+\frac{10(4ax+3)e^{-3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```


```
[Out] 16/(63*a*c^4*E^(3*ArcCoth[a*x])) + (1 + 2*a*x)/(9*a*c^4*E^(3*ArcCoth[a*x])*
(1 - a^2*x^2)^3) + (10*(3 + 4*a*x))/(63*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x
^2)^2) - (8*(3 + 2*a*x))/(21*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))
```


Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] && !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx &= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{9c}\\
&= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{40\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{21c^2}\\
&= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4\left(1-a^2x^2\right)}-\frac{16\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{21c^3}\\
&= \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}+\frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4\left(1-a^2x^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.250705, size = 82, normalized size = 0.65

$$
\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19\right)}{63c^4(ax-1)^2(ax+1)^5}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^4
 + 48*a^5*x^5 + 16*a^6*x^6))/(63*c^4*(-1 + a*x)^2*(1 + a*x)^5)
```

Maple [A] time = 0.046, size = 81, normalized size = 0.6

$$
\frac{16\,x^6a^6+48\,x^5a^5+24\,x^4a^4-56\,x^3a^3-66\,a^2x^2-6\,ax+19}{63\,\left(a^2x^2-1\right)^3c^4a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/63*((a*x-1)/(a*x+1))^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-6
6*a^2*x^2-6*a*x+19)/(a^2*x^2-1)^3/c^4/a
```

Maxima [A] time = 1.05721, size = 184, normalized size = 1.45

$$\frac{1}{4032}\,a\left(\frac{7\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}} - 54\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 189\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 420\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 945\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4} + \frac{21\left(\frac{18\,(ax-1)}{ax+1} - 1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*((7*((a*x - 1)/(a*x + 1))^(9/2) - 54*((a*x - 1)/(a*x + 1))^(7/2) +
 189*((a*x - 1)/(a*x + 1))^(5/2) - 420*((a*x - 1)/(a*x + 1))^(3/2) + 945*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + 21*(18*(a*x - 1)/(a*x + 1) - 1)/(a^2*c
^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A] time = 1.63781, size = 278, normalized size = 2.19

$$\frac{\left(16\,a^6x^6 + 48\,a^5x^5 + 24\,a^4x^4 - 56\,a^3x^3 - 66\,a^2x^2 - 6\,ax + 19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6 + 2\,a^6c^4x^5 - a^5c^4x^4 - 4\,a^4c^4x^3 - a^3c^4x^2 + 2\,a^2c^4x + ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/63*(16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*a^2*x^2 - 6*a*
x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^
4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^4, x)
```

3.614 $\int e^{\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{9/2} \, dx$

Optimal. Leaf size=229

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(ax+1}{3a^{10}x^9}$$

```
[Out] (8*(1 + a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rubi [A] time = 0.200254, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(ax+1}{3a^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (8*(1 + a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^4(1+ax)^5\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(1+ax)^5-32(1+ax)^6+24(1+ax)^7-8(1+ax)^8+(1+ax)^9\right)dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1}{9a}$$

Mathematica [A] time = 0.0641679, size = 79, normalized size = 0.34

$$\frac{c^4(ax+1)^6\left(63a^4x^4-308a^3x^3+588a^2x^2-528ax+193\right)\sqrt{c-a^2cx^2}}{630a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*(1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 - 528*a*x + 588*a^2*x^2 - 308*a^3
*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.046, size = 116, normalized size = 0.5

$$\frac{x\left(63a^9x^9+70x^8a^8-315a^7x^7-360x^6a^6+630x^5a^5+756x^4a^4-630x^3a^3-840a^2x^2+315ax+630\right)}{630\,(ax-1)^4\,(ax+1)^5}\left(-a^2cx^2+c\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/630*x*(63*a^9*x^9+70*a^8*x^8-315*a^7*x^7-360*a^6*x^6+630*a^5*x^5+756*a^4*
x^4-630*a^3*x^3-840*a^2*x^2+315*a*x+630)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^4/(a*
x+1)^5/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.65864, size = 261, normalized size = 1.14

$$\frac{\left(63a^9c^4x^{10}+70a^8c^4x^9-315a^7c^4x^8-360a^6c^4x^7+630a^5c^4x^6+756a^4c^4x^5-630a^3c^4x^4-840a^2c^4x^3+315ac^4x^2+6\right.}{630a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 - 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 + 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 - 840*a^2*c^4*x^3 +
315*a*c^4*x^2 + 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.615 $\int e^{\coth^{-1}(ax)} \left(c - a^2 c x^2\right)^{7/2} \, dx$

Optimal. Leaf size=183

$$\frac{(ax+1)^8 \left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(ax+1)^7 \left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6 \left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(ax+1)^5 \left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] (-8*(1 + a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.182818, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^8 \left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(ax+1)^7 \left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6 \left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(ax+1)^5 \left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```


```
[Out] (-8*(1 + a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx &= \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^3(1+ax)^4\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(-8(1+ax)^4+12(1+ax)^5-6(1+ax)^6+(1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+}{8a}
\end{aligned}$$

Mathematica [A] time = 0.0518518, size = 71, normalized size = 0.39

$$-\frac{c^3(ax+1)^5\left(35a^3x^3-135a^2x^2+185ax-93\right)\sqrt{c-a^2cx^2}}{280a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 + 35*a^3
*x^3))/(280*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.046, size = 100, normalized size = 0.6

$$\frac{x\left(35\,a^{7}x^{7}+40\,x^{6}a^{6}-140\,x^{5}a^{5}-168\,x^{4}a^{4}+210\,x^{3}a^{3}+280\,a^{2}x^{2}-140\,ax-280\right)}{280\,\left(ax-1\right)^{3}\left(ax+1\right)^{4}}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*
x^2-140*a*x-280)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^3/(a*x+1)^4/((a*x-1)/(a*x+1))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.56936, size = 212, normalized size = 1.16

$$-\frac{\left(35\,a^{7}c^{3}x^{8}+40\,a^{6}c^{3}x^{7}-140\,a^{5}c^{3}x^{6}-168\,a^{4}c^{3}x^{5}+210\,a^{3}c^{3}x^{4}+280\,a^{2}c^{3}x^{3}-140\,ac^{3}x^{2}-280\,c^{3}x\right)\sqrt{-a^{2}c}}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.616 $\int e^{\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{5/2} dx$

Optimal. Leaf size=136

$$\frac{(ax+1)^6 \left(c-a^2cx^2\right)^{5/2}}{6a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(ax+1)^5 \left(c-a^2cx^2\right)^{5/2}}{5a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{5/2}}{a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.175529, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^6 \left(c-a^2cx^2\right)^{5/2}}{6a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(ax+1)^5 \left(c-a^2cx^2\right)^{5/2}}{5a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{5/2}}{a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^2(1+ax)^3\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(1+ax)^3-4(1+ax)^4+(1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{(1+ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

Mathematica [A] time = 0.0405933, size = 63, normalized size = 0.46

$$\frac{c^2(ax+1)^4\left(5a^2x^2-14ax+11\right)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.043, size = 84, normalized size = 0.6

$$\frac{x\left(5x^5a^5+6x^4a^4-15x^3a^3-20a^2x^2+15ax+30\right)}{30\,(ax-1)^2\,(ax+1)^3}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*(-a^2*c*x^2+c)
^(5/2)/(a*x-1)^2/(a*x+1)^3/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.60498, size = 153, normalized size = 1.12

$$\frac{\left(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.617 $\int e^{\coth^{-1}(ax)} \left(c - a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=93

$$\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^3 \left(c-a^2cx^2\right)^{3/2}}{3a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] (-2*(1 + a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.166046, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^3 \left(c-a^2cx^2\right)^{3/2}}{3a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)(1+ax)^2\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left(c-a^2cx^2\right)^{3/2}\int\left(-2(1+ax)^2+(1+ax)^3\right)dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}$$

Mathematica [A] time = 0.0314056, size = 53, normalized size = 0.57

$$-\frac{c(ax+1)^3(3ax-5)\sqrt{c-a^2cx^2}}{12a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(c*(1 + a*x)^3*(-5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a^2*Sqrt[1 - 1/(a^2*x
^2)]*x)
```

Maple [A] time = 0.041, size = 68, normalized size = 0.7

$$\frac{x\left(3\,x^{3}a^{3}+4\,a^{2}x^{2}-6\,ax-12\right)}{\left(12\,ax-12\right)\left(ax+1\right)^{2}}\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*(-a^2*c*x^2+c)^(3/2)/(a*x-1)/(a*x+1)^
2/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.56379, size = 96, normalized size = 1.03

$$-\frac{\left(3\,a^{3}cx^{4}+4\,a^{2}cx^{3}-6\,acx^{2}-12\,cx\right)\sqrt{-a^{2}c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.618 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.108736, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0192544, size = 41, normalized size = 0.6

$$\frac{(ax+2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.04, size = 44, normalized size = 0.7

$$\frac{x\,(ax+2)}{2\,ax+2}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.55544, size = 46, normalized size = 0.68

$$\frac{\sqrt{-a^2c}\left(ax^2+2\,x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.619 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=38

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.146069, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 31}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{1}{-1+ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0186762, size = 38, normalized size = 1.

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Maple [A] time = 0.134, size = 51, normalized size = 1.3

$$-\frac{\ln(ax-1)}{ca(ax+1)}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/a/c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2 + c}\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.46157, size = 49, normalized size = 1.29

$$-\frac{\sqrt{-a^2c}\log\left(ax - 1\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] -sqrt(-a^2*c)*log(a*x - 1)/(a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")

[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.620 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.165858, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0369501, size = 56, normalized size = 0.62

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax-1)\tanh^{-1}(ax)-1\right)}{2c(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (-1 + a*x)*ArcTanh[a*x]))/(2*c*(-1 + a*x)*S
qrt[c - a^2*c*x^2])
```

Maple [A] time = 0.146, size = 84, normalized size = 0.9

$$\frac{ax\ln(ax+1)-\ln(ax-1)\,xa-\ln(ax+1)+\ln(ax-1)-2}{\left(4\,a^2x^2-4\right)c^2a}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1)
*x*a-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.65068, size = 177, normalized size = 1.95

$$-\frac{\left(a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.621 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.185466, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/}\right.}{8\left(c-a\right.}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8\left(c-a^2\right.}$$

Mathematica [A] time = 0.0594257, size = 83, normalized size = 0.45

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)
*ArcTanh[a*x]))/(8*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.147, size = 169, normalized size = 0.9

$$\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3-3\ln(ax+1)a^2x^2+3\ln(ax-1)a^2x^2-6a^2x^2-3ax\ln(ax+1)+3\ln(ax-1)}{(16ax-16)\left(a^2x^2-1\right)c^3a(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a
*x+1)-3*ln(a*x-1)*x^3*a^3-3*ln(a*x+1)*a^2*x^2+3*ln(a*x-1)*a^2*x^2-6*a^2*x^2
-3*a*x*ln(a*x+1)+3*ln(a*x-1)*x*a+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-
1)/c^3/a/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.67057, size = 278, normalized size = 1.51

$$-\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(3a^2x^2-3ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.622 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7}{24(1-a}$$

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (
5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```

Rubi [A] time = 0.209838, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7}{24(1-a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```


```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (
5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^4(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{8(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{16\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\right.}{32(1+ax)^2}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\right.}{32(1+ax)^2}$$

Mathematica [A] time = 0.0806764, size = 101, normalized size = 0.36

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4+15a^3x^3+25a^2x^2-25ax+15(ax-1)^3(ax+1)^2\tanh^{-1}(ax)-8\right)}{48c^3(ax-1)^3(ax+1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15*a^4*x
^4 + 15*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(48*c^3*(-1 + a*x)^3*(1 + a
*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.161, size = 241, normalized size = 0.9

$$\frac{15\,\ln(ax+1)\,x^5a^5-15\,\ln(ax-1)\,x^5a^5-15\,\ln(ax+1)\,a^4x^4+15\,\ln(ax-1)\,a^4x^4-30\,x^4a^4-30\,a^3x^3\ln(ax+1)+30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/96/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)
*x^5*a^5-15*ln(a*x-1)*x^5*a^5-15*ln(a*x+1)*a^4*x^4+15*ln(a*x-1)*a^4*x^4-30*
x^4*a^4-30*a^3*x^3*ln(a*x+1)+30*ln(a*x-1)*x^3*a^3+30*x^3*a^3+30*ln(a*x+1)*a
^2*x^2-30*ln(a*x-1)*a^2*x^2+50*a^2*x^2+15*a*x*ln(a*x+1)-15*ln(a*x-1)*x*a-50
*a*x-15*ln(a*x+1)+15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.58036, size = 393, normalized size = 1.42

$$-\frac{15\left(a^6x^5-a^5x^4-2\,a^4x^3+2\,a^3x^2+a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(15\,a^4x^4-15\,a^3x^3-25\,a^2x^2+25\,ax+8\right)\cdots}{96\left(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+a^3c^4x-a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/96*(15*(a^6*x^5 - a^5*x^4 - 2*a^4*x^3 + 2*a^3*x^2 + a^2*x - a)*sqrt(-c)*
log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15*a^4*
x^4 - 15*a^3*x^3 - 25*a^2*x^2 + 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 - a^
6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.623 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=176

$$-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{77c^{9/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{(a}{}$$

```
[Out] (-77*c^4*x*Sqrt[c - a^2*c*x^2])/256 - (77*c^3*x*(c - a^2*c*x^2)^(3/2))/384
- (77*c^2*x*(c - a^2*c*x^2)^(5/2))/480 - (11*c*x*(c - a^2*c*x^2)^(7/2))/80
+ (11*(c - a^2*c*x^2)^(9/2))/(90*a) + ((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(10
*a) - (77*c^(9/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(256*a)
```

Rubi [A] time = 0.165263, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{77c^{9/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```


```
[Out] (-77*c^4*x*Sqrt[c - a^2*c*x^2])/256 - (77*c^3*x*(c - a^2*c*x^2)^(3/2))/384
- (77*c^2*x*(c - a^2*c*x^2)^(5/2))/480 - (11*c*x*(c - a^2*c*x^2)^(7/2))/80
+ (11*(c - a^2*c*x^2)^(9/2))/(90*a) + ((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(10
*a) - (77*c^(9/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(256*a)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$$
$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{7/2}\,dx\right)$$
$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{1}{10}(11c)\int(1+ax)\left(c-a^2cx^2\right)^{7/2}\,dx$$
$$=\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{1}{10}(11c)\int\left(c-a^2cx^2\right)^{7/2}\,dx$$
$$=-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{1}{80}\left(77c^2\right)\int\left(c-a\right.$$
$$=-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)}{10a}$$
$$=-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11\left(c-a^2cx^2\right)^{9}}{90a}$$
$$=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right.$$
$$=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right.$$
$$=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right.$$

Mathematica [A] time = 0.15354, size = 167, normalized size = 0.95

$$\frac{c^4\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(-1152a^{10}x^{10}-1408a^9x^9+5584a^8x^8+7216a^7x^7-10552a^6x^6-15048a^5x^5+9210a^4x^4+16390a\right.\right.}{11520a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(2560 - 10615*a*x - 2185*a^2*x^2 +
16390*a^3*x^3 + 9210*a^4*x^4 - 15048*a^5*x^5 - 10552*a^6*x^6 + 7216*a^7*x^7
 + 5584*a^8*x^8 - 1408*a^9*x^9 - 1152*a^10*x^10) + 6930*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(11520*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.059, size = 350, normalized size = 2.

$$\frac{x}{10}\left(-a^2cx^2+c\right)^{\frac{9}{2}}+\frac{9\,cx}{80}\left(-a^2cx^2+c\right)^{\frac{7}{2}}+\frac{21\,xc^2}{160}\left(-a^2cx^2+c\right)^{\frac{5}{2}}+\frac{21\,c^3x}{128}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{63\,c^4x}{256}\sqrt{-a^2cx^2+c}+\frac{63\,c^5}{256}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/10*x*(-a^2*c*x^2+c)^(9/2)+9/80*c*x*(-a^2*c*x^2+c)^(7/2)+21/160*c^2*x*(-a^
2*c*x^2+c)^(5/2)+21/128*c^3*x*(-a^2*c*x^2+c)^(3/2)+63/256*c^4*x*(-a^2*c*x^2
+c)^(1/2)+63/256*c^5/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1
/2))+2/9/a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(9/2)-1/4*c*(-c*a^2*(x-1/a)^2-2
*a*c*(x-1/a))^(7/2)*x-7/24*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)*x-35/
96*c^3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x-35/64*c^4*(-c*a^2*(x-1/a)^2
-2*a*c*(x-1/a))^(1/2)*x-35/64*c^5/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*
a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91798, size = 790, normalized size = 4.49

$$\left[\frac{3465\sqrt{-c}c^4\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}x-c\right)+2\left(1152\,a^9c^4x^9+2560\,a^8c^4x^8-3024\,a^7c^4x^7-10240\,a^6c^4x^6+3\right.}{23040\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(3465*sqrt(-c)*c^4*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) + 2*(1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 1
0240*a^6*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 -
 10240*a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a, 1/11
520*(3465*c^(9/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c))
+ (1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 10240*a^6*c^4*x
^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 - 10240*a^2*c^4
*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17853, size = 221, normalized size = 1.26

$$\frac{77\,c^5\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{256\,\sqrt{-c}|a|}+\frac{1}{11520}\,\sqrt{-a^2cx^2+c}\left(\frac{2560\,c^4}{a}-\left(8055\,c^4+2\left(5120\,ac^4-\left(3075\,a^2c^4+4\left(1920\,a^3c\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 77/256*c^5*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/11520*sqrt(-a^2*c*x^2 + c)*(2560*c^4/a - (8055*c^4 + 2*(5120*a*c^4 -
 (3075*a^2*c^4 + 4*(1920*a^3*c^4 + (39*a^4*c^4 - 2*(640*a^5*c^4 + (189*a^6*
c^4 - 8*(9*a^8*c^4*x + 20*a^7*c^4)*x)*x)*x)*x)*x)*x)*x)*x)
```

3.624 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=153

$$-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}+$$

```
[Out] (-45*c^3*x*Sqrt[c - a^2*c*x^2])/128 - (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 -
 (3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1
+ a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) - (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(128*a)
```

Rubi [A] time = 0.152873, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-45*c^3*x*Sqrt[c - a^2*c*x^2])/128 - (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 -
 (3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1
+ a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) - (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(128*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx$$

$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{5/2}dx\right)$$

$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int(1+ax)\left(c-a^2cx^2\right)^{5/2}dx$$

$$=\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int\left(c-a^2cx^2\right)^{5/2}dx$$

$$=-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{16}\left(15c^2\right)\int\left(c-a\right.$$

$$=-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7}}{8a}$$

$$=-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+$$

$$=-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+$$

$$=-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+$$

Mathematica [A] time = 0.131442, size = 151, normalized size = 0.99

$$\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(112a^8x^8+144a^7x^7-424a^6x^6-600a^5x^5+558a^4x^4+978a^3x^3-187a^2x^2-837ax+256\right)+63\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978*
a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8*x
^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 - a*x
]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.052, size = 296, normalized size = 1.9

$$\frac{x}{8}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}+\frac{7\,cx}{48}\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}+\frac{35\,xc^{2}}{192}\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}+\frac{35\,c^{3}x}{128}\sqrt{-a^{2}cx^{2}+c}+\frac{35\,c^{4}}{128}\arctan\left(x\sqrt{a^{2}c}\frac{1}{\sqrt{-a^{2}cx^{2}+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/8*x*(-a^2*c*x^2+c)^(7/2)+7/48*c*x*(-a^2*c*x^2+c)^(5/2)+35/192*c^2*x*(-a^2
*c*x^2+c)^(3/2)+35/128*c^3*x*(-a^2*c*x^2+c)^(1/2)+35/128*c^4/(a^2*c)^(1/2)*
arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/7/a*(-c*a^2*(x-1/a)^2-2*a*c*
(x-1/a))^(7/2)-1/3*c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)*x-5/12*c^2*(-c*
a^2*(x-1/a)^2-2*a*c*(x-1/a))^(3/2)*x-5/8*c^3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a
))^(1/2)*x-5/8*c^4/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2
*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68114, size = 653, normalized size = 4.27

$$\left[\frac{315\sqrt{-c}c^{3}\log\left(2\,a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)-2\left(112\,a^{7}c^{3}x^{7}+256\,a^{6}c^{3}x^{6}-168\,a^{5}c^{3}x^{5}-768\,a^{4}c^{3}x^{4}-210\,a^{3}c^{3}x^{3}\right.}{1792\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, 1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(
c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^
5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256
*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] time = 34.3969, size = 1091, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -a**6*c**3*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqr
t(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**
2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(
128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2
*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/
(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) +
5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt
(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) - 2*a**5*c**3*Pi
ecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a*
*2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(
105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + a**4*c**3*Piecewise((I*a**2
*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**
4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1
), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-
a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(
16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) + 4*a**
3*c**3*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 +
c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4
/4, True)) + a**2*c**3*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1
)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*
sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 +
 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3)
, True)) - 2*a*c**3*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)),
 (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) - c**3*Piecewise((I*a**2*sq
```

```
rt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) -
I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2
 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A] time = 1.17002, size = 190, normalized size = 1.24

$$\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}|a|}+\frac{1}{896}\,\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a}-\left(581\,c^3+2\left(384\,ac^3-\left(105\,a^2c^3+4\left(96\,a^3c^3+\left(21\,a^4\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (105
*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)*x)
*x)*x)*x)*x)
```

3.625 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=130

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(
c - a^2*c*x^2)^(5/2))/(30*a) + ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rubi [A] time = 0.137716, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(
c - a^2*c*x^2)^(5/2))/(30*a) + ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)$$
$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int(1+ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}\left(7c^2\right)\int\sqrt{c-a^2}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}$$

Mathematica [A] time = 0.116158, size = 135, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(-40a^6x^6-56a^5x^5+106a^4x^4+182a^3x^3-57a^2x^2-231ax+96\right)+210\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 - 231*a*x - 57*a^2*x^2 + 182*a^
3*x^3 + 106*a^4*x^4 - 56*a^5*x^5 - 40*a^6*x^6) + 210*Sqrt[1 - a*x]*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.049, size = 242, normalized size = 1.9

$$\frac{x}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}+\frac{5\,cx}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{5\,xc^2}{16}\sqrt{-a^2cx^2+c}+\frac{5\,c^3}{16}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+\frac{2}{5\,a}\left(-ca^2\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)+5/24*c*x*(-a^2*c*x^2+c)^(3/2)+5/16*c^2*x*(-a^2*c
*x^2+c)^(1/2)+5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^
(1/2))+2/5/a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(5/2)-1/2*c*(-c*a^2*(x-1/a)^2
-2*a*c*(x-1/a))^(3/2)*x-3/4*c^2*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x-3/
4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71476, size = 544, normalized size = 4.18

$$\left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2 - 2\sqrt{-a^2cx^2+c}a\sqrt{-c}x - c\right) + 2\left(40a^5c^2x^5 + 96a^4c^2x^4 - 10a^3c^2x^3 - 192a^2c^2x^2 - 135ac^2x + 96\right.}{480a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^
2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 +
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] time = 17.1193, size = 478, normalized size = 3.68

$$a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2| > 1 \\ -\frac{a^2\sqrt{c}x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) + 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2cx^2+c}}{5} - \frac{x^2\sqrt{}}{} \\ \frac{\sqrt{c}x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt
(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(1
6*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
+ 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a*
*2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(16*a**5), True)) + 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
- x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4
), Ne(a, 0)), (sqrt(c)*x**4/4, True)) - 2*a*c**2*Piecewise((0, Eq(c, 0)), (
sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)
) - c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)
*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1
), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A] time = 1.18673, size = 157, normalized size = 1.21

$$\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x + \sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}|a|} - \frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2 + 2\left(96\,ac^2 + \left(5\,a^2c^2 - 4\left(5\,a^4c^2x + 12\,a^3c^2\right)x\right)x\right)x\right)x - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5*a
^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

3.626 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=107

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 + a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rubi [A] time = 0.126653, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 + a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\left(c\int(1+ax)^2\sqrt{c-a^2cx^2}\,dx\right)$$
$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int(1+ax)\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int\sqrt{c-a^2cx^2}\,dx$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\int\frac{1}{\sqrt{c-a^2cx^2}}\,d$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\text{Subst}\left(\int\frac{1}{1+a}\right.$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

Mathematica [A] time = 0.0915976, size = 117, normalized size = 1.09

$$\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4+10a^3x^3-7a^2x^2-25ax+16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^3
 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqrt
[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.047, size = 188, normalized size = 1.8

$$\frac{x}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{3\,cx}{8}\sqrt{-a^2cx^2+c}+\frac{3\,c^2}{8}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+\frac{2}{3\,a}\left(-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)+3/8*c*x*(-a^2*c*x^2+c)^(1/2)+3/8*c^2/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a*(-c*a^2*(x-1/a)^2-2*a
*c*(x-1/a))^(3/2)-c*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)*x-c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71642, size = 412, normalized size = 3.85

$$\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(6a^3cx^3+16a^2cx^2+9acx-16c\right)\sqrt{-a^2cx^2+c}}{48a}, \frac{15c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/a
]
```

Sympy [C] time = 20.0646, size = 342, normalized size = 3.2

$$-a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{ia^2 \ldots}{2\sqrt{a \ldots}} \\ \sqrt{cx \ldots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c
)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
 I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4
*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*
x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*
c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
 c)**(3/2)/(3*a**2*c), True)) - c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/
(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asi
n(a*x)/(2*a), True))
```

Giac [A] time = 1.1332, size = 115, normalized size = 1.07

$$-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3a^2cx+8ac\right)x+9c\right)x-\frac{16c}{a}\right)+\frac{5c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.627 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rubi [A] time = 0.109782, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6141, 671, 641, 217, 203}

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0516647, size = 76, normalized size = 0.88

$$\frac{\sqrt{c-a^2cx^2}\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.049, size = 134, normalized size = 1.6

$$\frac{x}{2}\sqrt{-a^2cx^2+c}+\frac{c}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\frac{1}{a}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)}-2\frac{c}{\sqrt{a^2c}}\arctan\left(\sqrt{a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))+2/a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70325, size = 306, normalized size = 3.56

$$\left[\frac{2\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4\,a},\frac{\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2}}{a^2cx}\right.}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.1516, size = 84, normalized size = 0.98

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.628 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] (-2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rubi [A] time = 0.104667, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 217, 203}

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
```

```
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\right)\\ &= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0371664, size = 82, normalized size = 1.39

$$-\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}+\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.045, size = 79, normalized size = 1.3

$$\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\frac{1}{a^2c}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x-1/a
)*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79183, size = 339, normalized size = 5.75

$$\left[-\frac{(ax-1)\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\sqrt{-a^2cx^2+c}}{2\left(a^2cx-ac\right)},-\frac{(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\sqrt{-a^2}}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), -((a*x - 1)*sqrt(c)*ar
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-c(ax-1)(ax+1)}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.629 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (-2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.104968, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6141, 653, 191}

$$-\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
```

```
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\right)\\ &= -\frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= -\frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0349434, size = 64, normalized size = 1.25

$$-\frac{(2-ax)\sqrt{ax+1}\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((2 - a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(3*a*c*(1 - a*x)^(3/2)*Sqrt[c
- a^2*c*x^2])
```

Maple [A] time = 0.04, size = 31, normalized size = 0.6

$$\frac{(ax+1)^2(ax-2)}{3a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(a*x+1)^2*(a*x-2)/a/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65754, size = 97, normalized size = 1.9

$$\frac{\sqrt{-a^2cx^2+c}(ax-2)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x - 1)), x)
```

Giac [B] time = 1.20202, size = 200, normalized size = 3.92

$$\frac{\left(ac-3\sqrt{-a^{2}c}\sqrt{c}\right)\operatorname{sgn}\left(x\right)}{3\left(a^{2}c^{\frac{5}{2}}-\sqrt{-a^{2}c}ac^{2}\right)}-\frac{2\left(2a^{2}c+3a\sqrt{c}\left(\sqrt{-a^{2}c+\frac{c}{x^{2}}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^{2}c+\frac{c}{x^{2}}}-\frac{\sqrt{c}}{x}\right)^{2}\right)}{3\left(a\sqrt{c}+\sqrt{-a^{2}c+\frac{c}{x^{2}}}-\frac{\sqrt{c}}{x}\right)^{3}c\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^2
) - 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqrt
(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sqrt(
c)/x)^3*c*sgn(x))
```

3.630 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (-2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2))
- (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.110384, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 192, 191}

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2))
- (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
```

```
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\right) \\
&= -\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= -\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c} \\
&= -\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} - \frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0381066, size = 53, normalized size = 0.72

$$-\frac{2a^3x^3-4a^2x^2+ax+2}{5ac^2(ax-1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(5*a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*x^2
])
```

Maple [A] time = 0.043, size = 47, normalized size = 0.6

$$-\frac{(ax+1)^2\left(2x^3a^3-4a^2x^2+ax+2\right)}{5a}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/5*(a*x+1)^2*(2*a^3*x^3-4*a^2*x^2+a*x+2)/a/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0755, size = 151, normalized size = 2.04

$$\frac{\left(2a^3x^3-4a^2x^2+ax+2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4-2a^4c^3x^3+2a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 - 2
*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x - 1)), x)
```

3.631 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] (-2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2))
- (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.119516, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 192, 191}

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2))
- (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
```

```
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx\right) \\
&= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx \\
&= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c} \\
&= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2} \\
&= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.053841, size = 96, normalized size = 0.99

$$
-\frac{\sqrt{1-a^2x^2}\left(-8a^5x^5+16a^4x^4+4a^3x^3-24a^2x^2+9ax+6\right)}{21ac^3(1-ax)^{7/2}(ax+1)^{3/2}\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(6 + 9*a*x - 24*a^2*x^2 + 4*a^3*x^3 + 16*a^4*x^4 - 8*a^
5*x^5))/(21*a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 64, normalized size = 0.7

$$\frac{(ax+1)^2\left(8\,x^5a^5-16\,x^4a^4-4\,x^3a^3+24\,a^2x^2-9\,ax-6\right)}{21\,a}\left(-a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/21*(a*x+1)^2*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)/a/(-a^2*
c*x^2+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.02914, size = 250, normalized size = 2.58

$$\frac{\left(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6\right)\sqrt{-a^2cx^2+c}}{21\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a
^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(-c(ax-1)(ax+1))^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x - 1)), x)
```

3.632 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2(ax+1)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

```
[Out] (-2*(1 + a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2))
- (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2
)) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.131384, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 192, 191}

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2(ax+1)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```


```
[Out] (-2*(1 + a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2))
- (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2
)) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx\right) \\
&= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{7}{9}\int \frac{1}{\left(c-a^2cx^2\right)^{9/2}}\,dx \\
&= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx}{3c} \\
&= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{15c^2} \\
&= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{16\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}}{45c^3} \\
&= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0594047, size = 112, normalized size = 0.93

$$\frac{\sqrt{1-a^2x^2}\left(-16a^7x^7+32a^6x^6+24a^5x^5-80a^4x^4+10a^3x^3+60a^2x^2-25ax-10\right)}{45ac^4(1-ax)^{9/2}(ax+1)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 - 25*a*x + 60*a^2*x^2 + 10*a^3*x^3 - 80*a^4*x^4 + 2
4*a^5*x^5 + 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)^(
5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.057, size = 80, normalized size = 0.7

$$-\frac{(ax+1)^2\left(16\,a^7x^7-32\,x^6a^6-24\,x^5a^5+80\,x^4a^4-10\,x^3a^3-60\,a^2x^2+25\,ax+10\right)}{45\,a}\left(-a^2cx^2+c\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] -1/45*(a*x+1)^2*(16*a^7*x^7-32*a^6*x^6-24*a^5*x^5+80*a^4*x^4-10*a^3*x^3-60*
a^2*x^2+25*a*x+10)/a/(-a^2*c*x^2+c)^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.04126, size = 317, normalized size = 2.64

$$\frac{\left(16\,a^{7}x^{7}-32\,a^{6}x^{6}-24\,a^{5}x^{5}+80\,a^{4}x^{4}-10\,a^{3}x^{3}-60\,a^{2}x^{2}+25\,ax+10\right)\sqrt{-a^{2}cx^{2}+c}}{45\left(a^{9}c^{5}x^{8}-2\,a^{8}c^{5}x^{7}-2\,a^{7}c^{5}x^{6}+6\,a^{6}c^{5}x^{5}-6\,a^{4}c^{5}x^{3}+2\,a^{3}c^{5}x^{2}+2\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(16*a^7*x^7 - 32*a^6*x^6 - 24*a^5*x^5 + 80*a^4*x^4 - 10*a^3*x^3 - 60*a
^2*x^2 + 25*a*x + 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*x
- a*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x - 1)), x)
```

3.633 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=185

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[Out] (-8*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rubi [A] time = 0.19981, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```



```
[Out] (-8*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^3(1+ax)^6\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(-8(1+ax)^6+12(1+ax)^7-6(1+ax)^8+(1+ax)^9\right)dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1}{1}$$

Mathematica [A] time = 0.0575649, size = 71, normalized size = 0.38

$$\frac{c^4(ax+1)^7\left(21a^3x^3-77a^2x^2+98ax-44\right)\sqrt{c-a^2cx^2}}{210a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.128, size = 100, normalized size = 0.5

$$\frac{x\left(21\,a^{9}x^{9}+70\,x^{8}a^{8}-240\,x^{6}a^{6}-210\,x^{5}a^{5}+252\,x^{4}a^{4}+420\,x^{3}a^{3}-315\,ax-210\right)}{210\,\left(ax-1\right)^{3}\left(ax+1\right)^{6}}\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3*
x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^3/(a*x+1)^6/((a*x-1)/(a*x+1))
^(3/2)
```

Maxima [A] time = 1.07743, size = 275, normalized size = 1.49

$$\frac{\left(21\,a^{11}\sqrt{-c}c^{4}x^{11}+49\,a^{10}\sqrt{-c}c^{4}x^{10}-70\,a^{9}\sqrt{-c}c^{4}x^{9}-240\,a^{8}\sqrt{-c}c^{4}x^{8}+30\,a^{7}\sqrt{-c}c^{4}x^{7}+462\,a^{6}\sqrt{-c}c^{4}x^{6}+168\,a^{5}\sqrt{-c}c^{4}\cdots}{210\left(a^{3}x^{2}+2\,a^{2}x+a\right)\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 + 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 - 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 + 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 - 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 + 105*a^2*sqrt(-c)*c^4*x^2 + 210*sqrt(-c)*c^4)*(a*x
+ 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Fricas [A] time = 1.50789, size = 212, normalized size = 1.15

$$\frac{\left(21\,a^{9}c^{4}x^{10}+70\,a^{8}c^{4}x^{9}-240\,a^{6}c^{4}x^{7}-210\,a^{5}c^{4}x^{6}+252\,a^{4}c^{4}x^{5}+420\,a^{3}c^{4}x^{4}-315\,ac^{4}x^{2}-210\,c^{4}x\right)\sqrt{-a^{2}c}}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.634 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.19329, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```


```
[Out] (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx &= \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^2(1+ax)^5\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(1+ax)^5-4(1+ax)^6+(1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\
&= \frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}
\end{aligned}$$

Mathematica [A] time = 0.0490649, size = 63, normalized size = 0.45

$$-\frac{c^3(ax+1)^6\left(21a^2x^2-54ax+37\right)\sqrt{c-a^2cx^2}}{168a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(168*a^2*
Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.133, size = 100, normalized size = 0.7

$$\frac{x\left(21\,a^{7}x^{7}+72\,x^{6}a^{6}+28\,x^{5}a^{5}-168\,x^{4}a^{4}-210\,x^{3}a^{3}+56\,a^{2}x^{2}+252\,ax+168\right)}{168\,\left(ax-1\right)^{2}\left(ax+1\right)^{5}}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x^
2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^5/((a*x-1)/(a*x+1))^(
3/2)
```

Maxima [A] time = 1.08362, size = 232, normalized size = 1.67

$$-\frac{\left(21\,a^{9}\sqrt{-c}c^{3}x^{9}+51\,a^{8}\sqrt{-c}c^{3}x^{8}-44\,a^{7}\sqrt{-c}c^{3}x^{7}-196\,a^{6}\sqrt{-c}c^{3}x^{6}-42\,a^{5}\sqrt{-c}c^{3}x^{5}+266\,a^{4}\sqrt{-c}c^{3}x^{4}+196\,a^{3}\sqrt{-c}c^{3}x^{3}}{168\left(a^{3}x^{2}+2\,a^{2}x+a\right)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 + 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 - 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 + 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 - 84*a^2*sqrt(-c)*c^3*x^2 - 168*sq
rt(-c)*c^3)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Fricas [A] time = 1.51507, size = 209, normalized size = 1.5

$$-\frac{\left(21\,a^{7}c^{3}x^{8}+72\,a^{6}c^{3}x^{7}+28\,a^{5}c^{3}x^{6}-168\,a^{4}c^{3}x^{5}-210\,a^{3}c^{3}x^{4}+56\,a^{2}c^{3}x^{3}+252\,ac^{3}x^{2}+168\,c^{3}x\right)\sqrt{-a^{2}c}}{168\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2*c
)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.635 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=93

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] (-2*(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.184599, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] (-2*(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)(1+ax)^4\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(-2(1+ax)^4+(1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= -\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}
\end{aligned}$$

Mathematica [A] time = 0.0410363, size = 55, normalized size = 0.59

$$\frac{c^2(ax+1)^5(5ax-7)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A] time = 0.123, size = 84, normalized size = 0.9

$$\frac{x\left(5\,x^{5}a^{5}+18\,x^{4}a^{4}+15\,x^{3}a^{3}-20\,a^{2}x^{2}-45\,ax-30\right)}{\left(30\,ax-30\right)\left(ax+1\right)^{4}}\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+c
)^(5/2)/(a*x-1)/(a*x+1)^4/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.06843, size = 189, normalized size = 2.03

$$\frac{\left(5\,a^{7}\sqrt{-c}c^{2}x^{7}+13\,a^{6}\sqrt{-c}c^{2}x^{6}-3\,a^{5}\sqrt{-c}c^{2}x^{5}-35\,a^{4}\sqrt{-c}c^{2}x^{4}-25\,a^{3}\sqrt{-c}c^{2}x^{3}+15\,a^{2}\sqrt{-c}c^{2}x^{2}+30\,\sqrt{-c}c^{2}\right)\left(ax+1\right)^{2}}{30\left(a^{3}x^{2}+2\,a^{2}x+a\right)\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 + 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 - 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 + 15*a^2*sqrt(-c)*
c^2*x^2 + 30*sqrt(-c)*c^2)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Fricas [A] time = 1.61048, size = 154, normalized size = 1.66

$$\frac{\left(5\,a^{5}c^{2}x^{6}+18\,a^{4}c^{2}x^{5}+15\,a^{3}c^{2}x^{4}-20\,a^{2}c^{2}x^{3}-45\,ac^{2}x^{2}-30\,c^{2}x\right)\sqrt{-a^{2}c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.636 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=46

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.176097, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int(1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.0313415, size = 58, normalized size = 1.26

$$-\frac{c\left(a^3x^3+4a^2x^2+6ax+4\right)\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(4 + 6*a*x + 4*a^2*x^2 + a^3*x^3))/(4*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.116, size = 60, normalized size = 1.3

$$\frac{x\left(x^3a^3+4\,a^2x^2+6\,ax+4\right)}{4\,(ax+1)^3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3+4*a^2*x^2+6*a*x+4)*(-a^2*c*x^2+c)^(3/2)/(a*x+1)^3/((a*x-1)/(
a*x+1))^(3/2)
```

Maxima [B] time = 1.10909, size = 131, normalized size = 2.85

$$-\frac{\left(a^{5}\sqrt{-c}cx^{5}+3\,a^{4}\sqrt{-c}cx^{4}+2\,a^{3}\sqrt{-c}cx^{3}-2\,a^{2}\sqrt{-c}cx^{2}-4\,\sqrt{-c}c\right)(ax+1)^{2}}{4\left(a^{3}x^{2}+2\,a^{2}x+a\right)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 + 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 - 2*
a^2*sqrt(-c)*c*x^2 - 4*sqrt(-c)*c)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*
x - 1))
```

Fricas [A] time = 1.59019, size = 90, normalized size = 1.96

$$-\frac{\left(a^{3}cx^{4}+4\,a^{2}cx^{3}+6\,acx^{2}+4\,cx\right)\sqrt{-a^{2}c}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.637 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.133094, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.0276691, size = 57, normalized size = 0.5

$$\frac{\sqrt{c-a^2cx^2}(ax(ax+6)+8\log(1-ax))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Maple [A] time = 0.166, size = 67, normalized size = 0.6

$$\frac{\left(a^2x^2+6\,ax+8\,\ln(ax-1)\right)(ax-1)}{2\,a\,(ax+1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.59989, size = 77, normalized size = 0.68

$$\frac{\left(a^2x^2+6\,ax+8\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.638 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.167164, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{1+ax}{(-1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0323132, size = 53, normalized size = 0.67

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}((ax-1)\log(1-ax)-2)}{(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + (-1 + a*x)*Log[1 - a*x]))/((-1 + a*x)*Sqrt[c
 - a^2*c*x^2])
```

Maple [A] time = 0.17, size = 64, normalized size = 0.8

$$-\frac{\ln(ax-1)xa-\ln(ax-1)-2}{ac(ax+1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-ln(a*x-1)-2)/a/c/(a*x+1)^2/((a*x-1)/
(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.89787, size = 84, normalized size = 1.06

$$-\frac{\sqrt{-a^2c}((ax-1)\log(ax-1)-2)}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] -sqrt(-a^2*c)*((a*x - 1)*log(a*x - 1) - 2)/(a^3*c*x - a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.639 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.171715, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$=\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$=-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0447238, size = 51, normalized size = 1.09

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}{2c^2(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(2*c^2*(-1 + a*x)^3*(1 + a*x
))
```

Maple [A] time = 0.114, size = 39, normalized size = 0.8

$$-\frac{ax-1}{2\,a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x-1)/a/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.89955, size = 77, normalized size = 1.64

$$-\frac{\sqrt{-a^2c}}{2\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.640 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 - a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.203737, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 - a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^4(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x\right.}{8\left(c-a^2\right.}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{8\left(c-a^2c\right.}$$

Mathematica [A] time = 0.0610113, size = 71, normalized size = 0.38

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2+9ax+3(ax-1)^3\tanh^{-1}(ax)-10\right)}{24c^2(ax-1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[
a*x]))/(24*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.175, size = 169, normalized size = 0.9

$$-\frac{3\,a^3x^3\ln(ax+1)-3\,\ln(ax-1)\,x^3a^3-9\,\ln(ax+1)\,a^2x^2+9\,\ln(ax-1)\,a^2x^2-6\,a^2x^2+9\,ax\ln(ax+1)-9\,\ln(ax-}{(48\,ax-48)\,(ax+1)\left(a^2x^2-1\right)c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/48/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3
*x^3*ln(a*x+1)-3*ln(a*x-1)*x^3*a^3-9*ln(a*x+1)*a^2*x^2+9*ln(a*x-1)*a^2*x^2-
6*a^2*x^2+9*a*x*ln(a*x+1)-9*ln(a*x-1)*x*a+18*a*x-3*ln(a*x+1)+3*ln(a*x-1)-20
)/(a^2*x^2-1)/c^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.90131, size = 290, normalized size = 1.57

$$-\frac{3\left(a^{4}x^{3}-3\,a^{3}x^{2}+3\,a^{2}x-a\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}c}\sqrt{-c}x+c}{a^{2}x^{2}-1}\right)-2\left(3\,a^{2}x^{2}-9\,ax+10\right)\sqrt{-a^{2}c}}{48\left(a^{5}c^{3}x^{3}-3\,a^{4}c^{3}x^{2}+3\,a^{3}c^{3}x-a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/48*(3*(a^4*x^3 - 3*a^3*x^2 + 3*a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 - 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.641 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=278

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7}{16(1-a}$$

```
[Out] -(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^4*(c - a^2*c*x^2)^(7/2)) -
 (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) -
 (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) +
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) - (5
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rubi [A] time = 0.225189, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7}{16(1-a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^4*(c - a^2*c*x^2)^(7/2)) -
 (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) -
 (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) +
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) - (5
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$
$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^5(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$
$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{4(-1+ax)^5}-\frac{1}{4(-1+ax)^4}+\frac{3}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{32(1+ax)^2}+\frac{5}{32\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$
$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\right.}{8(1-ax)\left(c\right.}$$
$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\right.}{8(1-ax)\left(c\right.}$$

Mathematica [A] time = 0.0875844, size = 99, normalized size = 0.36

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4+45a^3x^3-35a^2x^2-15ax+15(ax-1)^4(ax+1)\tanh^{-1}(ax)+32\right)}{96c^3(ax-1)^4(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)^4*(1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.181, size = 241, normalized size = 0.9

$$-\frac{15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-45\ln(ax+1)a^4x^4+45\ln(ax-1)a^4x^4-30x^4a^4+30a^3x^3\ln(ax+1)-30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(
15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5-45*ln(a*x+1)*a^4*x^4+45*ln(a*x-1)
*a^4*x^4-30*x^4*a^4+30*a^3*x^3*ln(a*x+1)-30*ln(a*x-1)*x^3*a^3+90*x^3*a^3+30
*ln(a*x+1)*a^2*x^2-30*ln(a*x-1)*a^2*x^2-70*a^2*x^2-45*a*x*ln(a*x+1)+45*ln(a
*x-1)*x*a-30*a*x+15*ln(a*x+1)-15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.96986, size = 406, normalized size = 1.46

$$-\frac{15\left(a^6x^5-3\,a^5x^4+2\,a^4x^3+2\,a^3x^2-3\,a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(15\,a^4x^4-45\,a^3x^3+35\,a^2x^2+15\,ax\right.}{192\left(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4x+a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/192*(15*(a^6*x^5 - 3*a^5*x^4 + 2*a^4*x^3 + 2*a^3*x^2 - 3*a^2*x + a)*sqrt
(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15
*a^4*x^4 - 45*a^3*x^3 + 35*a^2*x^2 + 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^
5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.642 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=234

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(1-ax)}{3a^{10}x^9}$$

```
[Out] (8*(1 - a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rubi [A] time = 0.211301, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(1-ax)}{3a^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```


```
[Out] (8*(1 - a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^5(1+ax)^4\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(-1+ax)^5+32(-1+ax)^6+24(-1+ax)^7+8(-1+ax)^8+(-1+a\right.}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1}{9}$$

Mathematica [A] time = 0.0609177, size = 79, normalized size = 0.34

$$\frac{c^4(ax-1)^6\left(63a^4x^4+308a^3x^3+588a^2x^2+528ax+193\right)\sqrt{c-a^2cx^2}}{630a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a^
3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.044, size = 116, normalized size = 0.5

$$\frac{x\left(63\,a^9x^9-70\,x^8a^8-315\,a^7x^7+360\,x^6a^6+630\,x^5a^5-756\,x^4a^4-630\,x^3a^3+840\,a^2x^2+315\,ax-630\right)}{630\,(ax+1)^4\,(ax-1)^5}\left(-a^2cx^2+c\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/630*x*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*
x^4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x
+1))^(1/2)/(a*x+1)^4/(a*x-1)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.8097, size = 261, normalized size = 1.12

$$\frac{\left(63\,a^9c^4x^{10}-70\,a^8c^4x^9-315\,a^7c^4x^8+360\,a^6c^4x^7+630\,a^5c^4x^6-756\,a^4c^4x^5-630\,a^3c^4x^4+840\,a^2c^4x^3+315\,ac^4x^2-6\right.}{630\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3 +
315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.643 $\int e^{-\coth^{-1}(ax)} \left(c - a^2cx^2\right)^{7/2} dx$

Optimal. Leaf size=187

$$\frac{(1-ax)^8 \left(c-a^2cx^2\right)^{7/2}}{8a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{6(1-ax)^7 \left(c-a^2cx^2\right)^{7/2}}{7a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{2(1-ax)^6 \left(c-a^2cx^2\right)^{7/2}}{a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{8(1-ax)^5 \left(c-a^2cx^2\right)^{7/2}}{5a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] (-8*(1 - a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.20121, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^8 \left(c-a^2cx^2\right)^{7/2}}{8a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{6(1-ax)^7 \left(c-a^2cx^2\right)^{7/2}}{7a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{2(1-ax)^6 \left(c-a^2cx^2\right)^{7/2}}{a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{8(1-ax)^5 \left(c-a^2cx^2\right)^{7/2}}{5a^8x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-8*(1 - a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^4(1+ax)^3\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int\left(8(-1+ax)^4+12(-1+ax)^5+6(-1+ax)^6+(-1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

$$=-\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-}{8}$$

Mathematica [A] time = 0.0483062, size = 71, normalized size = 0.38

$$-\frac{c^3(ax-1)^5\left(35a^3x^3+135a^2x^2+185ax+93\right)\sqrt{c-a^2cx^2}}{280a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 + 35*a^3
*x^3))/(280*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.043, size = 100, normalized size = 0.5

$$\frac{x\left(35\,a^{7}x^{7}-40\,x^{6}a^{6}-140\,x^{5}a^{5}+168\,x^{4}a^{4}+210\,x^{3}a^{3}-280\,a^{2}x^{2}-140\,ax+280\right)}{280\,\left(ax+1\right)^{3}\left(ax-1\right)^{4}}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*
x^2-140*a*x+280)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^3/(a*
x-1)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.98201, size = 212, normalized size = 1.13

$$-\frac{\left(35\,a^{7}c^{3}x^{8}-40\,a^{6}c^{3}x^{7}-140\,a^{5}c^{3}x^{6}+168\,a^{4}c^{3}x^{5}+210\,a^{3}c^{3}x^{4}-280\,a^{2}c^{3}x^{3}-140\,ac^{3}x^{2}+280\,c^{3}x\right)\sqrt{-a^{2}c}}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a^2
```

```
*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.644 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=139

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.192872, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^3(1+ax)^2\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(-1+ax)^3+4(-1+ax)^4+(-1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

Mathematica [A] time = 0.0426163, size = 63, normalized size = 0.45

$$\frac{c^2(ax-1)^4\left(5a^2x^2+14ax+11\right)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sq
rt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.046, size = 84, normalized size = 0.6

$$\frac{x\left(5x^5a^5-6x^4a^4-15x^3a^3+20a^2x^2+15ax-30\right)}{30\left(ax+1\right)^2\left(ax-1\right)^3}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*(-a^2*c*x^2+c)
^(5/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2/(a*x-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.85607, size = 153, normalized size = 1.1

$$\frac{\left(5a^5c^2x^6-6a^4c^2x^5-15a^3c^2x^4+20a^2c^2x^3+15ac^2x^2-30c^2x\right)\sqrt{-a^2c}}{30a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.645 $\int e^{-\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=95

$$\frac{(1-ax)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^3 \left(c-a^2cx^2\right)^{3/2}}{3a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] (-2*(1 - a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.180267, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^3 \left(c-a^2cx^2\right)^{3/2}}{3a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*(1 - a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx &= \frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)^2(1+ax)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{\left(c-a^2cx^2\right)^{3/2}\int\left(2(-1+ax)^2+(-1+ax)^3\right)dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= -\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] time = 0.0324301, size = 53, normalized size = 0.56

$$-\frac{c(ax-1)^3(3ax+5)\sqrt{c-a^2cx^2}}{12a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c*(-1 + a*x)^3*(5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a^2*Sqrt[1 - 1/(a^2*x
^2)]*x)
```

Maple [A] time = 0.041, size = 68, normalized size = 0.7

$$\frac{x\left(3x^3a^3-4a^2x^2-6ax+12\right)}{(12ax+12)(ax-1)^2}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*(-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1)
)^(1/2)/(a*x+1)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.88801, size = 96, normalized size = 1.01

$$-\frac{\left(3a^3cx^4-4a^2cx^3-6acx^2+12cx\right)\sqrt{-a^2c}}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.646 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.117439, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0195669, size = 41, normalized size = 0.59

$$\frac{(ax-2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.041, size = 44, normalized size = 0.6

$$\frac{x(ax-2)}{2ax-2}\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.79792, size = 46, normalized size = 0.67

$$\frac{\sqrt{-a^2c}\left(ax^2 - 2x\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.647 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=37

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.152539, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 31}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{1}{1+ax}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0193147, size = 37, normalized size = 1.

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Maple [A] time = 0.134, size = 51, normalized size = 1.4

$$-{\frac{\ln \left(ax+1 \right)}{c \left(ax-1 \right) a}\sqrt{-c \left({a}^{2}{x}^{2}-1 \right)}\sqrt{{\frac{ax-1}{ax+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/a/c/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [A] time = 1.70503, size = 49, normalized size = 1.32

$$-\frac{\sqrt{-a^2c}\log(ax+1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c)*log(a*x + 1)/(a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

3.648 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)*(c - a^2*c*x^2)^(3/2)) - (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.183534, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)*(c - a^2*c*x^2)^(3/2)) - (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(-\frac{1}{2(1+ax)^2}+\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0489857, size = 54, normalized size = 0.6

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax+1)\tanh^{-1}(ax)-1\right)}{2(acx+c)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (1 + a*x)*ArcTanh[a*x]))/(2*(c + a*c*x)*Sqrt
[c - a^2*c*x^2])
```

Maple [A] time = 0.145, size = 84, normalized size = 0.9

$$-\frac{ax\ln(ax+1)-\ln(ax-1)xa+\ln(ax+1)-\ln(ax-1)-2}{(4a^2x^2-4)c^2a}\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(a^2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1
)*x*a+ln(a*x+1)-ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.84255, size = 177, normalized size = 1.97

$$-\frac{(a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\sqrt{-a^2c}}{4(a^3c^2x+a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*((a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) - 2*sqrt(-a^2*c))/(a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

3.649 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) - (a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (3*a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.197224, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```


```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) - (a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (3*a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^2(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{8(-1+ax)^2}+\frac{1}{4(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right.}{8\left(c-a^2\right.}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{8\left(c-a^2c\right.}$$

Mathematica [A] time = 0.0658235, size = 81, normalized size = 0.44

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2-3ax+3(ax-1)(ax+1)^2\tanh^{-1}(ax)+2\right)}{8(ax-1)(acx+c)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2*
ArcTanh[a*x]))/(8*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.146, size = 169, normalized size = 0.9

$$-\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3+3\ln(ax+1)a^2x^2-3\ln(ax-1)a^2x^2-6a^2x^2-3ax\ln(ax+1)+3\ln(ax-1}{(16ax+16)\left(a^2x^2-1\right)c^3a(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(
a*x+1)-3*ln(a*x-1)*x^3*a^3+3*ln(a*x+1)*a^2*x^2-3*ln(a*x-1)*a^2*x^2-6*a^2*x^
2-3*a*x*ln(a*x+1)+3*ln(a*x-1)*x*a-6*a*x-3*ln(a*x+1)+3*ln(a*x-1)+4)/(a^2*x^2
-1)/c^3/a/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.93605, size = 278, normalized size = 1.52

$$-\frac{3\left(a^4x^3+a^3x^2-a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(3a^2x^2+3ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/16*(3*(a^4*x^3 + a^3*x^2 - a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 + 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

3.650 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6}{24(ax}$$

```
[Out] -(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2)) -
 (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) + (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) + (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) -
(5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```

Rubi [A] time = 0.220785, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6}{24(ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```


```
[Out] -(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2)) -
 (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) + (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) + (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) -
(5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^3(1+ax)^4}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{8(1+ax)^4}-\frac{3}{16(1+ax)^3}-\frac{3}{16(1+ax)^2}+\frac{5}{16\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\right.}{32(1+ax)^2}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\right.}{32(1+ax)^2}$$

Mathematica [A] time = 0.0861285, size = 99, normalized size = 0.36

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4-15a^3x^3+25a^2x^2+25ax+15(ax-1)^2(ax+1)^3\tanh^{-1}(ax)-8\right)}{48(ax-1)^2(acx+c)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*(-1 + a*x)^2*(c + a*c*x)
^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.151, size = 241, normalized size = 0.9

$$-\frac{15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+15\ln(ax+1)a^4x^4-15\ln(ax-1)a^4x^4-30x^4a^4-30a^3x^3\ln(ax+1)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/96*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1
)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+15*ln(a*x+1)*a^4*x^4-15*ln(a*x-1)*a^4*x^4-30
*x^4*a^4-30*a^3*x^3*ln(a*x+1)+30*ln(a*x-1)*x^3*a^3-30*x^3*a^3-30*ln(a*x+1)*
a^2*x^2+30*ln(a*x-1)*a^2*x^2+50*a^2*x^2+15*a*x*ln(a*x+1)-15*ln(a*x-1)*x*a+5
0*a*x+15*ln(a*x+1)-15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 1.92782, size = 393, normalized size = 1.42

$$-\frac{15\left(a^6x^5+a^5x^4-2\,a^4x^3-2\,a^3x^2+a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(15\,a^4x^4+15\,a^3x^3-25\,a^2x^2-25\,ax+8\right)}{96\left(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3c^4x+a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/96*(15*(a^6*x^5 + a^5*x^4 - 2*a^4*x^3 - 2*a^3*x^2 + a^2*x + a)*sqrt(-c)*
log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^4*
x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 + a^
6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

3.651 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=131

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(
c - a^2*c*x^2)^(5/2))/(30*a) - ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rubi [A] time = 0.141186, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6142, 671, 641, 195, 217, 203}

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(
c - a^2*c*x^2)^(5/2))/(30*a) - ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$=-\left(c\int(1-ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)$$
$$=-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int(1-ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}\left(7c^2\right)\int\sqrt{c-a^2c}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-$$

Mathematica [A] time = 0.136358, size = 136, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(210\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\sqrt{ax+1}\left(40a^6x^6-136a^5x^5+86a^4x^4+202a^3x^3-327a^2x^2+39ax+96\right)\right)}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202*
a^3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6)) + 210*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] time = 0.05, size = 226, normalized size = 1.7

$$\frac{x}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}+\frac{5\,cx}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{5\,xc^2}{16}\sqrt{-a^2cx^2+c}+\frac{5\,c^3}{16}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-\frac{2}{5\,a}\left(-a^2c\left(x+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)+5/24*c*x*(-a^2*c*x^2+c)^(3/2)+5/16*c^2*x*(-a^2*c
*x^2+c)^(1/2)+5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^
(1/2))-2/5/a*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(5/2)-1/2*c*(-a^2*c*(x+1/a)^2
+2*(x+1/a)*a*c)^(3/2)*x-3/4*c^2*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)*x-3/
4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98265, size = 544, normalized size = 4.15

$$\left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2 - 2\sqrt{-a^2cx^2+c}a\sqrt{-c}x - c\right) + 2\left(40a^5c^2x^5 - 96a^4c^2x^4 - 10a^3c^2x^3 + 192a^2c^2x^2 - 135ac^2x - 9\right.}{480a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^
2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 -
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] time = 14.1213, size = 478, normalized size = 3.65

$$a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,\mathrm{acosh}\,(ax)}{16a^5} & \text{for } |a^2x^2| > 1 \\ -\frac{a^2\sqrt{c}x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\mathrm{asin}\,(ax)}{16a^5} & \text{otherwise} \end{cases}\right) - 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2cx^2+c}}{5} - \frac{x^2\sqrt{-a}}{15} \\ \frac{\sqrt{c}x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt
(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(1
6*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
+ 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a*
*2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(16*a**5), True)) - 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
- x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4
), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a*c**2*Piecewise((0, Eq(c, 0)), (
sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)
) - c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)
*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1
), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A] time = 1.17109, size = 158, normalized size = 1.21

$$\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x + \sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}|a|} - \frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2 - 2\left(96\,ac^2 - \left(5\,a^2c^2 - 4\left(5\,a^4c^2x - 12\,a^3c^2\right)x\right)x\right)x\right)x + \frac{9}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 - 2*(96*a*c^2 - (5*a^2*c^2 - 4*(5*a
^4*c^2*x - 12*a^3*c^2)*x)*x)*x)*x + 96*c^2/a)
```

3.652 $\int e^{-2\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=108

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 - a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rubi [A] time = 0.127054, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6142, 671, 641, 195, 217, 203}

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 - a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\left(c\int(1-ax)^2\sqrt{c-a^2cx^2}\,dx\right)$$
$$=-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int(1-ax)\sqrt{c-a^2cx^2}\,dx$$
$$=-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int\sqrt{c-a^2cx^2}\,dx$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\int\frac{1}{\sqrt{c-a^2cx^2}}$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\text{Subst}\left(\int\frac{}{1+}\right.$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

Mathematica [A] time = 0.08938, size = 117, normalized size = 1.08

$$\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4-22a^3x^3+25a^2x^2+7ax-16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqr
t[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.049, size = 176, normalized size = 1.6

$$\frac{x}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{3\,cx}{8}\sqrt{-a^2cx^2+c}+\frac{3\,c^2}{8}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-\frac{2}{3\,a}\left(-a^2c\left(x+a^{-1}\right)^2+2\left(x+a^{-1}\right)ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)+3/8*c*x*(-a^2*c*x^2+c)^(1/2)+3/8*c^2/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a*(-a^2*c*(x+1/a)^2+2*(
x+1/a)*a*c)^(3/2)-c*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)*x-c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72989, size = 412, normalized size = 3.81

$$\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2 - 2\sqrt{-a^2cx^2+c}a\sqrt{-c}x - c\right) - 2\left(6a^3cx^3 - 16a^2cx^2 + 9acx + 16c\right)\sqrt{-a^2cx^2+c}}{48a}, \frac{15c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/a
]
```

Sympy [C] time = 9.25963, size = 340, normalized size = 3.15

$$-a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for } a^2=0\\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{i}{2}\ldots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c
)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
 I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4
*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*
x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) + 2*a*
c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
 c)**(3/2)/(3*a**2*c), True)) - c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/
(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asi
n(a*x)/(2*a), True))
```

Giac [A] time = 1.14766, size = 115, normalized size = 1.06

$$-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx-8\,ac\right)x+9\,c\right)x+\frac{16\,c}{a}\right)+\frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x - 8*a*c)*x + 9*c)*x + 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.653 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rubi [A] time = 0.11069, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6142, 671, 641, 217, 203}

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int\frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.050338, size = 100, normalized size = 1.15

$$
\frac{\sqrt{c-a^2cx^2}\left(6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\sqrt{ax+1}\left(a^2x^2-5ax+4\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a
*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.05, size = 126, normalized size = 1.5

$$\frac{x}{2}\sqrt{-a^2cx^2+c}+\frac{c}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\frac{\sqrt{-a^2c\left(x+a^{-1}\right)^2+2\left(x+a^{-1}\right)ac}}{a}-2\frac{c}{\sqrt{a^2c}}\arctan\left(\frac{}{\sqrt{-a^2c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))-2/a*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76703, size = 306, normalized size = 3.52

$$\left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}}{a^2cx^2-}\right.}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.17145, size = 84, normalized size = 0.97

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.654 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=60

$$\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] (2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rubi [A] time = 0.103932, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 217, 203}

$$\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
```

```
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\right)\\
&= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0605168, size = 100, normalized size = 1.67

$$-\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}(ax-1)+\sqrt{1-ax}(ax+1)\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}(ax+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*A
rcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*x^
```

```
2])
```

Maple [A] time = 0.05, size = 73, normalized size = 1.2

$$\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\,\frac{\sqrt{-a^2c\left(x+a^{-1}\right)^2+2\,\left(x+a^{-1}\right)ac}}{a^2c\left(x+a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x+1/a
)*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66441, size = 339, normalized size = 5.65

$$\left[-\frac{(ax+1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\left(a^2cx+ac\right)},-\frac{(ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2c}}{a^2cx+ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), -((a*x + 1)*sqrt(c)*ar
```

```
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.655 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] (2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.104115, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6142, 653, 191}

$$\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
```

```
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\right)\\
&= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0351886, size = 63, normalized size = 1.21

$$\frac{\sqrt{1-ax}(ax+2)\sqrt{1-a^2x^2}}{3ac(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c -
 a^2*c*x^2])
```

Maple [A] time = 0.044, size = 31, normalized size = 0.6

$$\frac{(ax-1)^2(ax+2)}{3a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(a*x-1)^2*(a*x+2)/a/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70498, size = 97, normalized size = 1.87

$$\frac{\sqrt{-a^2cx^2+c}(ax+2)}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)), x)
```

Giac [B] time = 1.2031, size = 200, normalized size = 3.85

$$-\frac{\left(ac+3\sqrt{-a^2c}\sqrt{c}\right)\operatorname{sgn}\left(x\right)}{3\left(a^2c^{\frac{5}{2}}+\sqrt{-a^2c}ac^2\right)}+\frac{2\left(2a^2c-3a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}-\sqrt{-a^2c+\frac{c}{x^2}}+\frac{\sqrt{c}}{x}\right)^3c\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) + sqrt(-a^2*c)*a*c^
2) + 2/3*(2*a^2*c - 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqr
t(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) - sqrt(-a^2*c + c/x^2) + sqrt
(c)/x)^3*c*sgn(x))
```

3.656 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2)) -
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.11495, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 192, 191}

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2)) -
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
```

```
1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\right) \\
&= \frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= \frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c} \\
&= \frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} - \frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0534234, size = 79, normalized size = 1.05

$$-\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+4a^2x^2+ax-2\right)}{5ac^2\sqrt{1-ax}(ax+1)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(5*a*c^2*Sqrt[1 - a
*x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 47, normalized size = 0.6

$$-\frac{(ax-1)^2\left(2x^3a^3+4a^2x^2+ax-2\right)}{5a}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/a/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89237, size = 151, normalized size = 2.01

$$\frac{\left(2a^3x^3+4a^2x^2+ax-2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4+2a^4c^3x^3-2a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2
*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(-c(ax-1)(ax+1))^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

3.657 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] (2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2)) -
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.124215, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 192, 191}

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2)) -
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
```

```
1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx\right) \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c} \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2} \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.061376, size = 96, normalized size = 0.98

$$
-\frac{\sqrt{1-a^2x^2}\left(-8a^5x^5-16a^4x^4+4a^3x^3+24a^2x^2+9ax-6\right)}{21ac^3(1-ax)^{3/2}(ax+1)^{7/2}\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(-6 + 9*a*x + 24*a^2*x^2 + 4*a^3*x^3 - 16*a^4*x^4 - 8*a
^5*x^5))/(21*a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 64, normalized size = 0.7

$$\frac{(ax-1)^2\left(8\,x^5a^5+16\,x^4a^4-4\,x^3a^3-24\,a^2x^2-9\,ax+6\right)}{21\,a}\left(-a^2cx^2+c\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/a/(-a^2*
c*x^2+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.03701, size = 250, normalized size = 2.55

$$\frac{\left(8\,a^5x^5+16\,a^4x^4-4\,a^3x^3-24\,a^2x^2-9\,ax+6\right)\sqrt{-a^2cx^2+c}}{21\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a
^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax - 1}{\left(-c\left(ax - 1\right)\left(ax + 1\right)\right)^{\frac{7}{2}}\left(ax + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax - 1}{\left(-a^2cx^2 + c\right)^{\frac{7}{2}}(ax + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

3.658 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}+\frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

```
[Out] (2*(1 - a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2)) -
 (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2)
) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.140998, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 192, 191}

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}+\frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (2*(1 - a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2)) -
 (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2)
) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx\right)\\
&= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{7}{9}\int \frac{1}{\left(c-a^2cx^2\right)^{9/2}}\,dx\\
&= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx}{3c}\\
&= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{15c^2}\\
&= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{16\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}}{45c^3}\\
&= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{16x}{45c^4\sqrt{c-a^2cx}}
\end{aligned}$$

Mathematica [A] time = 0.0720245, size = 112, normalized size = 0.93

$$\frac{\sqrt{1-a^2x^2}\left(-16a^7x^7-32a^6x^6+24a^5x^5+80a^4x^4+10a^3x^3-60a^2x^2-25ax+10\right)}{45ac^4(1-ax)^{5/2}(ax+1)^{9/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(10 - 25*a*x - 60*a^2*x^2 + 10*a^3*x^3 + 80*a^4*x^4 + 24
*a^5*x^5 - 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^(9
/2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.044, size = 80, normalized size = 0.7

$$-\frac{(ax-1)^2\left(16a^7x^7+32x^6a^6-24x^5a^5-80x^4a^4-10x^3a^3+60a^2x^2+25ax-10\right)}{45a}\left(-a^2cx^2+c\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] -1/45*(a*x-1)^2*(16*a^7*x^7+32*a^6*x^6-24*a^5*x^5-80*a^4*x^4-10*a^3*x^3+60*
a^2*x^2+25*a*x-10)/a/(-a^2*c*x^2+c)^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.07198, size = 317, normalized size = 2.62

$$\frac{\left(16\,a^7x^7+32\,a^6x^6-24\,a^5x^5-80\,a^4x^4-10\,a^3x^3+60\,a^2x^2+25\,ax-10\right)\sqrt{-a^2cx^2+c}}{45\left(a^9c^5x^8+2\,a^8c^5x^7-2\,a^7c^5x^6-6\,a^6c^5x^5+6\,a^4c^5x^3+2\,a^3c^5x^2-2\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(16*a^7*x^7 + 32*a^6*x^6 - 24*a^5*x^5 - 80*a^4*x^4 - 10*a^3*x^3 + 60*a
^2*x^2 + 25*a*x - 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 + 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 - 2*a^2*c^5*x
- a*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)), x)
```

3.659 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=189

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[Out] (-8*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rubi [A] time = 0.202343, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-8*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx &= \frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^6(1+ax)^3\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= \frac{\left(c-a^2cx^2\right)^{9/2}\int\left(8(-1+ax)^6+12(-1+ax)^7+6(-1+ax)^8+(-1+ax)^9\right)dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}\\
&= -\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\ldots\end{aligned}$$

Mathematica [A] time = 0.0564303, size = 71, normalized size = 0.38

$$\frac{c^4(ax-1)^7\left(21a^3x^3+77a^2x^2+98ax+44\right)\sqrt{c-a^2cx^2}}{210a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.046, size = 100, normalized size = 0.5

$$\frac{x\left(21\,a^{9}x^{9}-70\,x^{8}a^{8}+240\,x^{6}a^{6}-210\,x^{5}a^{5}-252\,x^{4}a^{4}+420\,x^{3}a^{3}-315\,ax+210\right)}{210\,\left(ax+1\right)^{3}\left(ax-1\right)^{6}}\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3*
x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^3/(a*
x-1)^6
```

Maxima [A] time = 1.09494, size = 275, normalized size = 1.46

$$\frac{\left(21\,a^{11}\sqrt{-c}c^{4}x^{11}-49\,a^{10}\sqrt{-c}c^{4}x^{10}-70\,a^{9}\sqrt{-c}c^{4}x^{9}+240\,a^{8}\sqrt{-c}c^{4}x^{8}+30\,a^{7}\sqrt{-c}c^{4}x^{7}-462\,a^{6}\sqrt{-c}c^{4}x^{6}+168\,a^{5}\sqrt{-c}c^{4}x^{5}}{210\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 - 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 + 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 - 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 + 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 - 105*a^2*sqrt(-c)*c^4*x^2 - 210*sqrt(-c)*c^4)*(a*x
- 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Fricas [A] time = 1.66939, size = 212, normalized size = 1.12

$$\frac{\left(21\,a^{9}c^{4}x^{10}-70\,a^{8}c^{4}x^{9}+240\,a^{6}c^{4}x^{7}-210\,a^{5}c^{4}x^{6}-252\,a^{4}c^{4}x^{5}+420\,a^{3}c^{4}x^{4}-315\,ac^{4}x^{2}+210\,c^{4}x\right)\sqrt{-a^{2}c}}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.660 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=142

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.191245, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx &= \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^5(1+ax)^2\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(-1+ax)^5+4(-1+ax)^6+(-1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\end{aligned}$$

Mathematica [A] time = 0.0493986, size = 63, normalized size = 0.44

$$-\frac{c^3(ax-1)^6\left(21a^2x^2+54ax+37\right)\sqrt{c-a^2cx^2}}{168a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(168*a^2
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.046, size = 100, normalized size = 0.7

$$\frac{x\left(21\,a^{7}x^{7}-72\,x^{6}a^{6}+28\,x^{5}a^{5}+168\,x^{4}a^{4}-210\,x^{3}a^{3}-56\,a^{2}x^{2}+252\,ax-168\right)}{168\,\left(ax+1\right)^{2}\left(ax-1\right)^{5}}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x^
2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-
1)^5
```

Maxima [A] time = 1.12915, size = 232, normalized size = 1.63

$$-\frac{\left(21\,a^{9}\sqrt{-c}c^{3}x^{9}-51\,a^{8}\sqrt{-c}c^{3}x^{8}-44\,a^{7}\sqrt{-c}c^{3}x^{7}+196\,a^{6}\sqrt{-c}c^{3}x^{6}-42\,a^{5}\sqrt{-c}c^{3}x^{5}-266\,a^{4}\sqrt{-c}c^{3}x^{4}+196\,a^{3}\sqrt{-c}c^{3}x^{3}+}{168\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 - 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 + 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 - 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 + 84*a^2*sqrt(-c)*c^3*x^2 + 168*sq
rt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Fricas [A] time = 1.61501, size = 209, normalized size = 1.47

$$-\frac{\left(21\,a^{7}c^{3}x^{8}-72\,a^{6}c^{3}x^{7}+28\,a^{5}c^{3}x^{6}+168\,a^{4}c^{3}x^{5}-210\,a^{3}c^{3}x^{4}-56\,a^{2}c^{3}x^{3}+252\,ac^{3}x^{2}-168\,c^{3}x\right)\sqrt{-a^{2}c}}{168\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2*c
)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.661 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] (-2*(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.179096, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^4(1+ax)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(2(-1+ax)^4+(-1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= -\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

Mathematica [A] time = 0.0392302, size = 55, normalized size = 0.58

$$\frac{c^2(ax-1)^5(5ax+7)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A] time = 0.04, size = 84, normalized size = 0.9

$$\frac{x\left(5x^5a^5-18x^4a^4+15x^3a^3+20a^2x^2-45ax+30\right)}{(30ax+30)(ax-1)^4}\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+c
)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)^4
```

Maxima [A] time = 1.10204, size = 189, normalized size = 1.99

$$\frac{\left(5a^7\sqrt{-c}c^2x^7-13a^6\sqrt{-c}c^2x^6-3a^5\sqrt{-c}c^2x^5+35a^4\sqrt{-c}c^2x^4-25a^3\sqrt{-c}c^2x^3-15a^2\sqrt{-c}c^2x^2-30\sqrt{-c}c^2\right)(ax-1)^2}{30\left(a^3x^2-2a^2x+a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 - 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 + 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 - 15*a^2*sqrt(-c)*
c^2*x^2 - 30*sqrt(-c)*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Fricas [A] time = 1.62129, size = 154, normalized size = 1.62

$$\frac{\left(5a^5c^2x^6-18a^4c^2x^5+15a^3c^2x^4+20a^2c^2x^3-45ac^2x^2+30c^2x\right)\sqrt{-a^2c}}{30a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.662 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=47

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.168354, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] time = 0.0318155, size = 58, normalized size = 1.23

$$-\frac{c\left(a^3x^3-4a^2x^2+6ax-4\right)\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(-4 + 6*a*x - 4*a^2*x^2 + a^3*x^3))/(4*a*Sqrt[1 - 1
/(a^2*x^2)])
```

Maple [A] time = 0.042, size = 60, normalized size = 1.3

$$\frac{x\left(x^3a^3-4\,a^2x^2+6\,ax-4\right)}{4\,(ax-1)^3}\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3-4*a^2*x^2+6*a*x-4)*(-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3
/2)/(a*x-1)^3
```

Maxima [B] time = 1.11477, size = 131, normalized size = 2.79

$$-\frac{\left(a^{5}\sqrt{-c}cx^{5}-3\,a^{4}\sqrt{-c}cx^{4}+2\,a^{3}\sqrt{-c}cx^{3}+2\,a^{2}\sqrt{-c}cx^{2}+4\,\sqrt{-c}c\right)(ax-1)^{2}}{4\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 - 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 + 2*
a^2*sqrt(-c)*c*x^2 + 4*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1))
```

Fricas [A] time = 1.61588, size = 90, normalized size = 1.91

$$-\frac{\left(a^{3}cx^{4}-4\,a^{2}cx^{3}+6\,acx^{2}-4\,cx\right)\sqrt{-a^{2}c}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.663 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.127254, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] time = 0.0259987, size = 56, normalized size = 0.5

$$\frac{\sqrt{c-a^2cx^2}(ax(ax-6)+8\log(ax+1))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*x)
```

Maple [A] time = 0.131, size = 67, normalized size = 0.6

$$\frac{\left(a^2x^2-6\,ax+8\,\ln(ax+1)\right)(ax+1)}{2\,a\,(ax-1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/a/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}\left(\frac{ax - 1}{ax + 1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.65544, size = 77, normalized size = 0.69

$$\frac{\left(a^2x^2 - 6\,ax + 8\,\log\left(ax + 1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.664 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=77

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(ax+1)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] time = 0.163186, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(ax+1)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```


```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \frac{-1+ax}{(1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)\int \left(-\frac{2}{(1+ax)^2}+\frac{1}{1+ax}\right)dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0338258, size = 52, normalized size = 0.68

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}((ax+1)\log(ax+1)+2)}{(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + (1 + a*x)*Log[1 + a*x]))/((1 + a*x)*Sqrt[c -
a^2*c*x^2])
```

Maple [A] time = 0.139, size = 62, normalized size = 0.8

$$-\frac{ax\ln(ax+1)+\ln(ax+1)+2}{ac\,(ax-1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+ln(a*x+1)+2)*((a*x-1)/(a*x+1))^(3/2)
/a/c/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [A] time = 1.59553, size = 84, normalized size = 1.09

$$-\frac{\sqrt{-a^2c}((ax+1)\log(ax+1)+2)}{a^3cx+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c)*((a*x + 1)*log(a*x + 1) + 2)/(a^3*c*x + a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

3.665 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.169029, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0480128, size = 51, normalized size = 1.11

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}{2c^2(ax-1)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(2*c^2*(-1 + a*x)*(1 + a*x)^
3)
```

Maple [A] time = 0.043, size = 39, normalized size = 0.9

$$-\frac{ax+1}{2\,a}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.53842, size = 77, normalized size = 1.67

$$-\frac{\sqrt{-a^2c}}{2\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.666 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(ax+1)^3\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 + a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.198197, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(ax+1)^3\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```


```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 + a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)(1+ax)^4}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{2(1+ax)^4}-\frac{1}{4(1+ax)^3}-\frac{1}{8(1+ax)^2}+\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/\right.}{8\left(c-\right.}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8\left(c-a\right.}$$

Mathematica [A] time = 0.0630013, size = 71, normalized size = 0.39

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2-9ax+3(ax+1)^3\tanh^{-1}(ax)-10\right)}{24c^2(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*ArcTanh[
a*x]))/(24*c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.144, size = 169, normalized size = 0.9

$$\frac{3\,a^3x^3\ln(ax+1)-3\,\ln(ax-1)\,x^3a^3+9\,\ln(ax+1)\,a^2x^2-9\,\ln(ax-1)\,a^2x^2-6\,a^2x^2+9\,ax\ln(ax+1)-9\,\ln(ax-1)\,x}{(48\,ax+48)\,(ax-1)\left(a^2x^2-1\right)c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*
x^3*ln(a*x+1)-3*ln(a*x-1)*x^3*a^3+9*ln(a*x+1)*a^2*x^2-9*ln(a*x-1)*a^2*x^2-6
*a^2*x^2+9*a*x*ln(a*x+1)-9*ln(a*x-1)*x*a-18*a*x+3*ln(a*x+1)-3*ln(a*x-1)-20)
/(a^2*x^2-1)/c^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.60945, size = 290, normalized size = 1.59

$$-\frac{3\left(a^{4}x^{3}+3\,a^{3}x^{2}+3\,a^{2}x+a\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{2}+2\,\sqrt{-a^{2}c}\sqrt{-c}x+c}{a^{2}x^{2}-1}\right)+2\left(3\,a^{2}x^{2}+9\,ax+10\right)\sqrt{-a^{2}c}}{48\left(a^{5}c^{3}x^{3}+3\,a^{4}c^{3}x^{2}+3\,a^{3}c^{3}x+a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/48*(3*(a^4*x^3 + 3*a^3*x^2 + 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 + 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.667 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=275

$$\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(ax+1)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7}{16(ax+}$$

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)^4*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) - (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) -
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (5*
a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rubi [A] time = 0.22187, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(ax+1)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7}{16(ax+}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```


```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)^4*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) - (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) -
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (5*
a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^2(1+ax)^5}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{32(-1+ax)^2}+\frac{1}{4(1+ax)^5}+\frac{1}{4(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{32\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}} - \frac{3a^6\left(1-\right.}{32(1+ax)^2}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}} - \frac{3a^6\left(1-\right.}{32(1+ax)^2}$$

Mathematica [A] time = 0.0939201, size = 99, normalized size = 0.36

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4-45a^3x^3-35a^2x^2+15ax+15(ax-1)(ax+1)^4\tanh^{-1}(ax)+32\right)}{96c^3(ax-1)(ax+1)^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15*a^4*x
^4 + 15*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)*(1 + a*x)^
4*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.151, size = 241, normalized size = 0.9

$$\underline{15\ \ln(ax+1)\,x^5a^5-15\ \ln(ax-1)\,x^5a^5+45\ \ln(ax+1)\,a^4x^4-45\ \ln(ax-1)\,a^4x^4-30\,x^4a^4+30\,a^3x^3\ln(ax+1)-30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(1
5*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+45*ln(a*x+1)*a^4*x^4-45*ln(a*x-1)*
a^4*x^4-30*x^4*a^4+30*a^3*x^3*ln(a*x+1)-30*ln(a*x-1)*x^3*a^3-90*x^3*a^3-30*
ln(a*x+1)*a^2*x^2+30*ln(a*x-1)*a^2*x^2-70*a^2*x^2-45*a*x*ln(a*x+1)+45*ln(a*
x-1)*x*a+30*a*x-15*ln(a*x+1)+15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 1.72071, size = 406, normalized size = 1.48

$$-\frac{15\left(a^6x^5+3a^5x^4+2a^4x^3-2a^3x^2-3a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(15a^4x^4+45a^3x^3+35a^2x^2-15ax-\right.}{192\left(a^7c^4x^5+3a^6c^4x^4+2a^5c^4x^3-2a^4c^4x^2-3a^3c^4x-a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/192*(15*(a^6*x^5 + 3*a^5*x^4 + 2*a^4*x^3 - 2*a^3*x^2 - 3*a^2*x - a)*sqrt
(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15
*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^
```

```
5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.668 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=76

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c
*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.202835, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 43}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c
*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int x^2(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0236939, size = 45, normalized size = 0.59

$$\frac{x^2(3ax+4)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.04, size = 47, normalized size = 0.6

$$\frac{x^3\,(3\,ax+4)}{12\,ax+12}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.59317, size = 53, normalized size = 0.7

$$\frac{\left(3\,ax^4+4\,x^3\right)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/12*(3*a*x^4 + 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.669 $\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x
^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.173718, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6192, 6193, 43}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```



```
[Out] (x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x
^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0205325, size = 43, normalized size = 0.58

$$
\frac{x(2ax+3)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.042, size = 47, normalized size = 0.6

$$
\frac{x^2\,(2\,ax+3)}{6\,ax+6}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+cx}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.57437, size = 51, normalized size = 0.69

$$\frac{\left(2\,ax^3+3\,x^2\right)\sqrt{-a^2c}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.670 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.107973, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0167089, size = 41, normalized size = 0.6

$$\frac{(ax+2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.04, size = 44, normalized size = 0.7

$$\frac{x\,(ax+2)}{2\,ax+2}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.54896, size = 46, normalized size = 0.68

$$\frac{\sqrt{-a^2c}\left(ax^2+2x\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.671 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x} dx$

Optimal. Leaf size=69

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.138636, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 43}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.0203672, size = 42, normalized size = 0.61

$$\frac{\sqrt{c-a^2cx^2}(ax+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.138, size = 44, normalized size = 0.6

$$\frac{ax+\ln(x)}{ax+1}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (a*x+ln(x))*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.72308, size = 42, normalized size = 0.61

$$\frac{\sqrt{-a^2c}(ax+\log(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + log(x))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.672 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=73

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) + (Sqrt[c - a^2*c*x^2]
*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.194528, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 43}

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) + (Sqrt[c - a^2*c*x^2]
*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A] time = 0.0240446, size = 43, normalized size = 0.59

$$
\frac{\sqrt{c-a^2cx^2}(ax\log(x)-1)}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1 + a*x*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A] time = 0.136, size = 48, normalized size = 0.7

$$
\frac{a\ln(x)\,x-1}{(ax+1)\,x}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] (a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.50128, size = 50, normalized size = 0.68

$$\frac{\sqrt{-a^2c}(ax\log(x) - 1)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) - 1)/(a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.673 $\int e^{2\coth^{-1}(ax)} x^3 \sqrt{c-a^2cx^2}\, dx$

Optimal. Leaf size=137

$$\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) - (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rubi [A] time = 0.412657, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6151, 1809, 833, 780, 217, 203}

$$\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) - (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = -\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^3(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x^3(-9a^2c-10a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}$$
$$= \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2(30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x(-72a^4c^3-90a^5c^3x)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} - \frac{(3c)\int \frac{1}{\sqrt{c-a}}}{4a^3}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} - \frac{(3c)\,\text{Subst}}{}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} - \frac{3\sqrt{c}\tan^{-1}}{4a}$$

Mathematica [A] time = 0.121149, size = 96, normalized size = 0.7

$$\frac{(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)\sqrt{c-a^2cx^2}+15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

Maple [A] time = 0.056, size = 210, normalized size = 1.5

$$-\frac{x^2}{5a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{4}{5ca^4}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{x}{2a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{5x}{4a^3}\sqrt{-a^2cx^2+c} + \frac{5c}{4a^3}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c-4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)-1/2/a^3*
x*(-a^2*c*x^2+c)^(3/2)/c+5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)+5/4/a^3*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^4*(-c*a^2*(x-1/a)^2-2
*a*c*(x-1/a))^(1/2)-2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x
-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67638, size = 431, normalized size = 3.15

$$\left[\frac{2\left(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24\right)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{40\,a^4}, \frac{\left(4\,a^4x^4+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c))/a^4, 1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-
a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.17659, size = 124, normalized size = 0.91

$$\frac{1}{20}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(2\,x+\frac{5}{a}\right)x+\frac{6}{a^2}\right)x+\frac{15}{a^3}\right)x+\frac{24}{a^4}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{4\,a^3\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(-a^2*c*x^2 + c)*((2*((2*x + 5/a)*x + 6/a^2)*x + 15/a^3)*x + 24/a^
4) + 3/4*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^3*sqrt(-c)*a
bs(a))
```

3.674 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 + 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rubi [A] time = 0.387016, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6151, 1809, 833, 780, 217, 203}

$$\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 + 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^2(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2(-7a^2c-8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x(16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{8a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,\right.}{8a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Mathematica [A] time = 0.0953186, size = 88, normalized size = 0.79

$$\frac{(6a^3x^3+16a^2x^2+21ax+32)\sqrt{c-a^2cx^2}+21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [B] time = 0.052, size = 186, normalized size = 1.7

$$-\frac{x}{4a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{9x}{8a^2}\sqrt{-a^2cx^2+c} + \frac{9c}{8a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} - \frac{2}{3a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + 2\frac{1}{a^3}\sqrt{-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)+9/8/a^2*c/
(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a^3*(-a^2*c*
x^2+c)^(3/2)/c+2/a^3*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-2/a^2*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68948, size = 396, normalized size = 3.54

$$\left[\frac{2\left(6\,a^3x^3+16\,a^2x^2+21\,ax+32\right)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3}, \frac{\left(6\,a^3x^3+16\,a^2x^2+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.13488, size = 113, normalized size = 1.01

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right)+\frac{7c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

3.675 $\int e^{2\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=85

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) - (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rubi [A] time = 0.250733, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6167, 6151, 1809, 780, 217, 203}

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) - (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
```

```
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int \frac{x\left(-5a^2c-6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0788109, size = 79, normalized size = 0.93

$$\frac{\left(a^2x^2+3ax+5\right)\sqrt{c-a^2cx^2}+3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [B] time = 0.052, size = 162, normalized size = 1.9

$$-\frac{1}{3\,a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}+\frac{x}{a}\sqrt{-a^2cx^2+c}+\frac{c}{a}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\,\frac{1}{a^2}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c+x/a*(-a^2*c*x^2+c)^(1/2)+1/a*c/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2*(-c*a^2*(x-1/a)^2-2*a*
c*(x-1/a))^(1/2)-2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)
^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.703, size = 344, normalized size = 4.05

$$\left[\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax+5\right)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2}, \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax+5\right)+3\cdots}{3\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.14597, size = 97, normalized size = 1.14

$$\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right)+\frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) + c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

3.676 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rubi [A] time = 0.113928, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6141, 671, 641, 217, 203}

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```



```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0532475, size = 76, normalized size = 0.88

$$\frac{\sqrt{c-a^2cx^2}\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.049, size = 134, normalized size = 1.6

$$\frac{x}{2}\sqrt{-a^2cx^2+c}+\frac{c}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\frac{1}{a}\sqrt{-ca^2\left(x-a^{-1}\right)^2-2ac\left(x-a^{-1}\right)}-2\frac{c}{\sqrt{a^2c}}\arctan\left(\sqrt{a^2c}x\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))+2/a*(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64674, size = 306, normalized size = 3.56

$$\left[\frac{2\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c\cdots}}{a^2cx^2-\cdots}\right.}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.15314, size = 84, normalized size = 0.98

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.677 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=75

$$\sqrt{c-a^2cx^2}-2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.344603, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6151, 1809, 844, 217, 203, 266, 63, 208}

$$\sqrt{c-a^2cx^2}-2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \sqrt{c-a^2cx^2} + \frac{\int \frac{-a^2c-2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}$$
$$= \sqrt{c-a^2cx^2} - c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \sqrt{c-a^2cx^2} - \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - (2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-}}\right.$$
$$= \sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a^2}$$
$$= \sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.0767689, size = 97, normalized size = 1.29

$$\sqrt{c-a^2cx^2} + \sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + 2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) - \sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

Maple [B] time = 0.055, size = 129, normalized size = 1.7

$$-\sqrt{-a^2cx^2+c} + \sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right) + 2\,\sqrt{-ca^2\left(x-a^{-1}\right)^2-2\,ac\left(x-a^{-1}\right)} - 2\,\frac{ac}{\sqrt{a^2c}}\arctan\left(\sqrt{a^2c}x\frac{}{\sqrt{-c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66141, size = 441, normalized size = 5.88

$$\left[2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c},\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx}}{a^2cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*s
qrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt(
-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 -
 c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
+ sqrt(-a^2*c*x^2 + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}{x\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*(a*x - 1)), x)
```

Giac [A] time = 1.17935, size = 128, normalized size = 1.71

$$-\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) - 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

3.678 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.347903, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6151, 1807, 844, 217, 203, 266, 63, 208}

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + \int \frac{-2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - (ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - \left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \sqrt{}\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a}$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + 2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.0913365, size = 104, normalized size = 1.27

$$\frac{\sqrt{c-a^2cx^2}}{x} + 2a\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + a\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) - 2a\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

Maple [B] time = 0.058, size = 208, normalized size = 2.5

$$\frac{1}{cx}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + a^2x\sqrt{-a^2cx^2+c} + a^2c\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} + 2\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)a - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] 1/c/x*(-a^2*c*x^2+c)^(3/2)+a^2*x*(-a^2*c*x^2+c)^(1/2)+a^2*c/(a^2*c)^(1/2)*a
rctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a
^2*c*x^2+c)^(1/2))/x)*a-2*(-a^2*c*x^2+c)^(1/2)*a+2*a*(-c*a^2*(x-1/a)^2-2*a*
c*(x-1/a))^(1/2)-2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*a^2*(x-1/
a)^2-2*a*c*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^2), x)
```

Fricas [A] time = 1.60041, size = 477, normalized size = 5.82

$$\left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x}, \frac{4a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+a\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqrt
(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Giac [A] time = 1.19055, size = 181, normalized size = 2.21

$$-\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
 a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a
^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

3.679 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.3395, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6151, 1807, 807, 266, 63, 208}

$$\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{1}{2}\int \frac{-4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.113107, size = 76, normalized size = 0.97

$$
\frac{1}{2}\left(\frac{(4ax+1)\sqrt{c-a^2cx^2}}{x^2} + 3a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - 3a^2\sqrt{c}\log(x)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [B] time = 0.057, size = 239, normalized size = 3.1

$$
2\,\frac{a\left(-a^2cx^2+c\right)^{3/2}}{cx} + 2\,a^3x\sqrt{-a^2cx^2+c} + 2\,\frac{a^3c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) + \frac{3\,a^2}{2}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right) - \frac{3\,a}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] 2*a/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^3*x*(-a^2*c*x^2+c)^(1/2)+2*a^3*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+3/2*c^(1/2)*ln((2*c+2*c^(
1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^2-3/2*(-a^2*c*x^2+c)^(1/2)*a^2+2*a^2*(-c*a^
2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2)-2*a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-c*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))+1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^3), x)
```

Fricas [A] time = 1.58456, size = 338, normalized size = 4.33

$$\left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\sqrt{-a^2cx^2+c}(4\,ax+1)}{4\,x^2},\frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2cx^2+c}(4\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 +
 c)*(4*a*x + 1))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Giac [B] time = 1.12797, size = 270, normalized size = 3.46

$$-\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\right.}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

3.680 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=99

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.37803, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6151, 1807, 835, 807, 266, 63, 208}

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{1}{3}\int \frac{-6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\int \frac{10a^2c^2+6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c} \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.115516, size = 82, normalized size = 0.83

$$
\frac{\left(5a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+a^3\left(-\sqrt{c}\right)\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] ((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
+ a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.065, size = 261, normalized size = 2.6

$$
2\,\frac{a^2\left(-a^2cx^2+c\right)^{3/2}}{cx}+2\,a^4x\sqrt{-a^2cx^2+c}+2\,\frac{a^4c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right)+\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)a^3-\sqrt{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] 2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^4*x*(-a^2*c*x^2+c)^(1/2)+2*a^4*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+c^(1/2)*ln((2*c+2*c^(1/
2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3-(-a^2*c*x^2+c)^(1/2)*a^3+2*a^3*(-c*a^2*(x-1
/a)^2-2*a*c*(x-1/a))^(1/2)-2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c
*a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))+a/c/x^2*(-a^2*c*x^2+c)^(3/2)+1/3/c/x^3
*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^4), x)
```

Fricas [A] time = 1.6548, size = 370, normalized size = 3.74

$$\left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}\left(5\,a^2x^2+3\,ax+1\right)}{6\,x^3},\ \frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2cx^2 \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, 1/3*(3*a^3
*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(
-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Giac [B] time = 1.17871, size = 338, normalized size = 3.41

$$-\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c-3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|+12\,\ldots\right.}{3\left(\left(\sqrt{-a^2c}x-\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*c)
*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-
a^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))^2 - c)^3
```

3.681 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.402853, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6151, 1807, 835, 807, 266, 63, 208}

$$\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{1}{4}\int \frac{-8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{\int \frac{21a^2c^2+16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{\int \frac{-32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}\left(7a^4c\right)\int \frac{1}{x\sqrt{c-a^2cx}}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{16}\left(7a^4c\right)\text{Subst}\left(\int \frac{}{x}\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}\left(7a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-}\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a}}{\sqrt{c}}\right.$$

Mathematica [A] time = 0.137362, size = 95, normalized size = 0.73

$$\frac{\left(32a^3x^3+21a^2x^2+16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4} + \frac{7}{8}a^4\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - \frac{7}{8}a^4\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

Maple [B] time = 0.066, size = 287, normalized size = 2.2

$$\frac{1}{4\,c x^{4}}\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}}+\frac{9\,a^{2}}{8\,c x^{2}}\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}}+\frac{7\,a^{4}}{8}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^{2} c x^{2}+c}\right)\right)-\frac{7\,a^{4}}{8}\sqrt{-a^{2} c x^{2}+c}+2\,\frac{a^{3}\left(-a^{2}cx\right.}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] 1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)+9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)+7/8*a^4*c
^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-7/8*a^4*(-a^2*c*x^2+c)^(1
/2)+2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^5*x*(-a^2*c*x^2+c)^(1/2)+2*a^5*c/(a^
2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a^4*(-c*a^2*(x-1/
a)^2-2*a*c*(x-1/a))^(1/2)-2*a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-c*
a^2*(x-1/a)^2-2*a*c*(x-1/a))^(1/2))+2/3*a/c/x^3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2}+c}(a x+1)}{(a x-1)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^5), x)
```

Fricas [A] time = 1.61686, size = 416, normalized size = 3.2

$$\left[\frac{21\,a^{4}\sqrt{c}x^{4}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\sqrt{c}-2\,c}{x^{2}}\right)+2\left(32\,a^{3}x^{3}+21\,a^{2}x^{2}+16\,ax+6\right)\sqrt{-a^{2}cx^{2}+c}}{48\,x^{4}},\ \frac{21\,a^{4}\sqrt{-c}x^{4}\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+}}{a^{2}cx^{2}-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Giac [B] time = 1.1829, size = 437, normalized size = 3.36

$$-\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96\left(\sqrt{-a^2c}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 + 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

3.682 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=228

$$\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) + (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*
x^2]*Log[1 - a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.27035, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) + (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*
x^2]*Log[1 - a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}+\frac{4x}{a^2}+\frac{4x^2}{a}+3x^3+ax^4+\frac{4}{a^3(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{}}{}
\end{aligned}
$$

Mathematica [A] time = 0.0591102, size = 88, normalized size = 0.39

$$
\frac{\sqrt{c-a^2cx^2}\left(\frac{2x^2}{a^2}+\frac{4x}{a^3}+\frac{4\log(1-ax)}{a^4}+\frac{ax^5}{5}+\frac{4x^3}{3a}+\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 + (2*x^2)/a^2 + (4*x^3)/(3*a) + (3*x^4)/4 +
 (a*x^5)/5 + (4*Log[1 - a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.175, size = 92, normalized size = 0.4

$$\frac{\left(12\,x^{5}a^{5}+45\,x^{4}a^{4}+80\,x^{3}a^{3}+120\,a^{2}x^{2}+240\,ax+240\,\ln\left(ax-1\right)\right)\left(ax-1\right)}{60\,a^{4}\left(ax+1\right)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/60*(12*x^5*a^5+45*x^4*a^4+80*x^3*a^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^{2}cx^{2}+c}x^{3}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.60196, size = 142, normalized size = 0.62

$$\frac{\left(12\,a^{5}x^{5}+45\,a^{4}x^{4}+80\,a^{3}x^{3}+120\,a^{2}x^{2}+240\,ax+240\,\log\left(ax-1\right)\right)\sqrt{-a^{2}c}}{60\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/60*(12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x + 240*lo
g(a*x - 1))*sqrt(-a^2*c)/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + cx^3}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.683 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=186

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/
(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt
[c - a^2*c*x^2]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.252277, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/
(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt
[c - a^2*c*x^2]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-a}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.0440059, size = 74, normalized size = 0.4

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^2}+\frac{4\log(1-ax)}{a^3}+\frac{ax^4}{4}+\frac{2x^2}{a}+x^3\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^2 + (2*x^2)/a + x^3 + (a*x^4)/4 + (4*Log[1 -
a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.181, size = 83, normalized size = 0.5

$$\frac{\left(x^4a^4+4\,x^3a^3+8\,a^2x^2+16\,ax+16\,\ln\left(ax-1\right)\right)\left(ax-1\right)}{4\,a^3\left(ax+1\right)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(x^4*a^4+4*x^3*a^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2cx^2+c}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.61602, size = 112, normalized size = 0.6

$$\frac{\left(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(-a^2*
c)/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.684 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=152

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rubi [A] time = 0.224114, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 77}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.0384857, size = 66, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\left(ax\left(2a^2x^2+9ax+24\right)+24\log(1-ax)\right)}{6a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.178, size = 76, normalized size = 0.5

$$\frac{\left(2 x^{3} a^{3}+9 a^{2} x^{2}+24 a x+24 \ln \left(a x-1\right)\right) \left(a x-1\right)}{6 a^{2} \left(a x+1\right)^{2}}\sqrt{-c \left(a^{2} x^{2}-1\right)}\left(\frac{a x-1}{a x+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(2*x^3*a^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1
)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2}+c}x}{\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.6348, size = 99, normalized size = 0.65

$$\frac{\left(2 a^{3} x^{3}+9 a^{2} x^{2}+24 a x+24 \log \left(a x-1\right)\right)\sqrt{-a^{2} c}}{6 a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(-a^2*c)/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.685 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.133175, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.0244481, size = 57, normalized size = 0.5

$$\frac{\sqrt{c-a^2cx^2}(ax(ax+6)+8\log(1-ax))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Maple [A] time = 0.174, size = 67, normalized size = 0.6

$$\frac{\left(a^2x^2+6\,ax+8\,\ln(ax-1)\right)(ax-1)}{2\,a\,(ax+1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.53231, size = 77, normalized size = 0.68

$$\frac{\left(a^2x^2+6\,ax+8\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.686 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.161223, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 72}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.033081, size = 53, normalized size = 0.46

$$\frac{\sqrt{c-a^2cx^2}(ax+4\log(1-ax)-\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

Maple [A] time = 0.187, size = 59, normalized size = 0.5

$$-\frac{(-ax+\ln(x)-4\ln(ax-1))(ax-1)}{(ax+1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x)-4*ln(a*x-1))*(a*x-1)/(a*x+1)^2/((a*x-1)
/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.67815, size = 65, normalized size = 0.57

$$\frac{\sqrt{-a^2c}\,(ax+4\,\log{(ax-1)}-\log{(x)})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.687 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*Sqrt[c - a^2*c*x^2]*
Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(S
qrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.237844, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*Sqrt[c - a^2*c*x^2]*
Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(S
qrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.0316096, size = 55, normalized size = 0.48

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(1-ax)+\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(x^(-1) - 3*a*Log[x] + 4*a*Log[1 - a*x]))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

Maple [A] time = 0.184, size = 65, normalized size = 0.6

$$-\frac{(3\,a\ln(x)\,x-4\,\ln(ax-1)\,xa-1)\,(ax-1)}{x\,(ax+1)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*ln(a*x-1)*x*a-1)*(a*x-1)/x/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^{2}cx^{2}+c}}{x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.62928, size = 81, normalized size = 0.71

$$\frac{\sqrt{-a^{2}c}(4\,ax\log(ax-1)-3\,ax\log(x)+1)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.688 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=153

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^2
])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 -
 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Rubi [A] time = 0.240446, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^2
])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 -
 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.0376005, size = 69, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^2\log(x)+4a^2\log(1-ax)+\frac{3a}{x}+\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(2*x^2) + (3*a)/x - 4*a^2*Log[x] + 4*a^2*Log[1 - a*
x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.191, size = 77, normalized size = 0.5

$$-\frac{\left(8\,a^{2}\ln\left(x\right)x^{2}-8\,\ln\left(ax-1\right)a^{2}x^{2}-6\,ax-1\right)\left(ax-1\right)}{2\,x^{2}\left(ax+1\right)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*ln(a*x-1)*a^2*x^2-6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)/x^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^{2}cx^{2}+c}}{x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.56174, size = 197, normalized size = 1.29

$$\frac{8\,a^{3}\sqrt{-c}x^{2}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx+\sqrt{-a^{2}c}(2\,ax-1)\sqrt{-c}+ac}{ax^{2}-x}\right)+\sqrt{-a^{2}c}(6\,ax+1)}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
fricas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
- 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + sqrt(-a^2*c)*(6*a*x + 1))/(a*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.689 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=194

$$\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x
) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.247803, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x
) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-}}
\end{aligned}$$

Mathematica [A] time = 0.0430334, size = 79, normalized size = 0.41

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)+\frac{3a}{2x^2}+\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(3*x^3) + (3*a)/(2*x^2) + (4*a^2)/x - 4*a^3*Log[x]
+ 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.195, size = 85, normalized size = 0.4

$$-\frac{\left(24 a^{3} \ln \left(x\right) x^{3}-24 \ln \left(a x-1\right) x^{3} a^{3}-24 a^{2} x^{2}-9 a x-2\right) \left(a x-1\right)}{6 x^{3} \left(a x+1\right)^{2}} \sqrt{-c \left(a^{2} x^{2}-1\right)} \left(\frac{a x-1}{a x+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*ln(a*x-1)*x^3*a^3-24*a^2*x^2-9*a*x-2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x-1)/x^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2} + c}}{x^{4} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.64415, size = 216, normalized size = 1.11

$$\frac{24 a^{4} \sqrt{-c} x^{3} \log \left(\frac{2 a^{3} c x^{2} - 2 a^{2} c x + \sqrt{-a^{2} c} (2 a x - 1) \sqrt{-c} + a c}{a x^{2} - x}\right) + \left(24 a^{2} x^{2} + 9 a x + 2\right) \sqrt{-a^{2} c}}{6 a x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.690 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=228

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/(
Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2
)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a^3*
Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a^3*Sqrt[c - a^2
*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.252988, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/(
Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2
)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a^3*
Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a^3*Sqrt[c - a^2
*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A] time = 0.0566774, size = 84, normalized size = 0.37

$$
\frac{\sqrt{c-a^2cx^2}\left(\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)+\frac{a}{x^3}+\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(4*x^4) + a/x^3 + (2*a^2)/x^2 + (4*a^3)/x - 4*a^4*L
og[x] + 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.2, size = 93, normalized size = 0.4

$$-\frac{\left(16 a^{4} \ln \left(x \right) x^{4}-16 \ln \left(a x -1\right) a^{4} x^{4}-16 x^{3} a^{3}-8 a^{2} x^{2}-4 a x -1\right) \left(a x -1\right)}{4 x^{4} \left(a x +1\right)^{2}} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(\frac{a x -1}{a x +1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4*(16*a^4*ln(x)*x^4-16*ln(a*x-1)*a^4*x^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/x^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2} + c}}{x^{5} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.62436, size = 232, normalized size = 1.02

$$\frac{16 \, a^{5} \sqrt{-c} x^{4} \log\left(\frac{2 \, a^{3} c x^{2} - 2 \, a^{2} c x + \sqrt{-a^{2} c} {\left(2 \, a x - 1\right)} \sqrt{-c} + a c}{a x^{2} - x}\right) + {\left(16 \, a^{3} x^{3} + 8 \, a^{2} x^{2} + 4 \, a x + 1\right)} \sqrt{-a^{2} c}}{4 \, a x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
fricas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*s
qrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}}{x^5 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.691 $\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax}{4a^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(a*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2)
)^(3/2)*x^5)/(2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*a
^2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (7*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
- a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
+ a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.230426, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax}{4a^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```



```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(a*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2)
)^(3/2)*x^5)/(2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*a
^2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (7*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
- a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
+ a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^4}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\end{aligned}$$

Mathematica [A] time = 0.0815126, size = 77, normalized size = 0.36

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2\left(a^2x^2+2ax+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(ax+1)\right)}{4a^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[
1 - a*x] + Log[1 + a*x]))/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

Maple [A] time = 0.16, size = 106, normalized size = 0.5

$$\frac{2x^3a^3+2a^2x^2+ax\ln(ax+1)+7\ln(ax-1)xa-4ax-\ln(ax+1)-7\ln(ax-1)-2}{\left(4a^2x^2-4\right)c^2a^5}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+2*a^2*x^2+a*x
*ln(a*x+1)+7*ln(a*x-1)*x*a-4*a*x-ln(a*x+1)-7*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.73703, size = 173, normalized size = 0.82

$$\frac{\left(2a^3x^3+2a^2x^2-4ax+(ax-1)\log(ax+1)+7(ax-1)\log(ax-1)-2\right)\sqrt{-a^2c}}{4\left(a^7c^2x-a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*l
og(a*x - 1) - 2)*sqrt(-a^2*c)/(a^7*c^2*x - a^6*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.692 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(c - a^2*c*x^2)^(3/2) + ((1 - 1/(a^2*x^2))^(3
/2)*x^3)/(2*a*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (5*(1 - 1/(a^2*x^2))^(3/2)
*x^3*Log[1 - a*x])/(4*a*(c - a^2*c*x^2)^(3/2)) - ((1 - 1/(a^2*x^2))^(3/2)*x
^3*Log[1 + a*x])/(4*a*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.199598, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(c - a^2*c*x^2)^(3/2) + ((1 - 1/(a^2*x^2))^(3
/2)*x^3)/(2*a*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (5*(1 - 1/(a^2*x^2))^(3/2)
*x^3*Log[1 - a*x])/(4*a*(c - a^2*c*x^2)^(3/2)) - ((1 - 1/(a^2*x^2))^(3/2)*x
^3*Log[1 + a*x])/(4*a*(c - a^2*c*x^2)^(3/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1}{4a\left(c-a^2cx^2\right)^{3/}}
\end{aligned}
$$

Mathematica [A] time = 0.0550632, size = 71, normalized size = 0.41

$$
\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)\right)}{4a\left(c-a^2cx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1
+ a*x]))/(4*a*(c - a^2*c*x^2)^(3/2))
```

Maple [A] time = 0.152, size = 98, normalized size = 0.6

$$-\frac{-4a^2x^2+ax\ln(ax+1)-5\ln(ax-1)xa+4ax-\ln(ax+1)+5\ln(ax-1)+2}{(4a^2x^2-4)c^2a^4}\sqrt{-c(a^2x^2-1)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*a^2*x^2+a*x*ln(a*x+
1)-5*ln(a*x-1)*x*a+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.59521, size = 157, normalized size = 0.91

$$\frac{(4a^2x^2-4ax-(ax-1)\log(ax+1)+5(ax-1)\log(ax-1)-2)\sqrt{-a^2c}}{4(a^6c^2x-a^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)*sqrt(-a^2*c)/(a^6*c^2*x - a^5*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.693 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (3*(1 -
 1/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(
a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.25405, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (3*(1 -
 1/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(
a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^2}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.0442195, size = 75, normalized size = 0.58

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(3(ax-1)\log(1-ax)+(ax-1)\log(ax+1)-2)}{4a^2c(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*(-1 + a*x)*Log[1 - a*x] + (-1 + a*x)*Log[
1 + a*x]))/(4*a^2*c*(-1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.166, size = 86, normalized size = 0.7

$$\frac{ax\ln(ax+1)+3\,\ln(ax-1)\,xa-\ln(ax+1)-3\,\ln(ax-1)-2}{(4\,a^2x^2-4)\,c^2a^3}\sqrt{-c\,(a^2x^2-1)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+3*ln(a*x-
1)*x*a-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.62549, size = 130, normalized size = 1.

$$\frac{\sqrt{-a^2c}((ax-1)\log(ax+1)+3\,(ax-1)\log(ax-1)-2)}{4\,(a^5c^2x-a^4c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(a
^5*c^2*x - a^4*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.694 $\int \frac{e^{\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) - (a*(1
 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.210995, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6192, 6193, 77, 207}

$$\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) - (a*(1
 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0412113, size = 59, normalized size = 0.68

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax-1)\tanh^{-1}(ax)+1\right)}{2ac(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(1 + (-1 + a*x)*ArcTanh[a*x]))/(2*a*c*(-1 + a*x)*S
qrt[c - a^2*c*x^2])
```

Maple [A] time = 0.153, size = 84, normalized size = 1.

$$-\frac{ax\ln(ax+1)-\ln(ax-1)\,xa-\ln(ax+1)+\ln(ax-1)+2}{\left(4a^2x^2-4\right)c^2a^2}\sqrt{-c\left(a^2x^2-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1
)*x*a-ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.5346, size = 177, normalized size = 2.03

$$-\frac{\left(a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^4c^2x-a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^4*c^2*x - a^3*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.695 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.170931, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0318519, size = 56, normalized size = 0.62

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax-1)\tanh^{-1}(ax)-1\right)}{2c(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (-1 + a*x)*ArcTanh[a*x]))/(2*c*(-1 + a*x)*S
qrt[c - a^2*c*x^2])
```

Maple [A] time = 0.14, size = 84, normalized size = 0.9

$$\frac{ax\ln\left(ax+1\right)-\ln\left(ax-1\right)xa-\ln\left(ax+1\right)+\ln\left(ax-1\right)-2}{\left(4\,a^{2}x^{2}-4\right)c^{2}a}\sqrt{-c\left(a^{2}x^{2}-1\right)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)-ln(a*x-1)
*x*a-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.62827, size = 177, normalized size = 1.95

$$-\frac{\left(a^{2}x-a\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{2}+2\,\sqrt{-a^{2}c}\sqrt{-c}x+c}{a^{2}x^{2}-1}\right)+2\,\sqrt{-a^{2}c}}{4\left(a^{3}c^{2}x-a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.696 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
3*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (3*a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.252474, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 72}

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
3*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (3*a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}} - \frac{a^3\left(1-\frac{}{a}\right.}{4}$$

Mathematica [A] time = 0.0593408, size = 68, normalized size = 0.38

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2-2ax}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*
x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

Maple [A] time = 0.144, size = 92, normalized size = 0.5

$$\frac{4a\ln(x)x - ax\ln(ax+1) - 3\ln(ax-1)xa - 4\ln(x) + \ln(ax+1) + 3\ln(ax-1) - 2}{(4a^2x^2-4)c^2}\sqrt{-c(a^2x^2-1)}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a*ln(x)*x-a*x*ln(a*x+
1)-3*ln(a*x-1)*x*a-4*ln(x)+ln(a*x+1)+3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.6928, size = 157, normalized size = 0.89

$$-\frac{\sqrt{-a^2c}((ax-1)\log(ax+1) + 3(ax-1)\log(ax-1) - 4(ax-1)\log(x) + 2)}{4(a^2c^2x - ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(a
*x - 1)*log(x) + 2)/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.697 $\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{5a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\cdots\right.}{4\left(c-\cdots\right.}$$

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*
x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (5*a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.265469, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{5a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\cdots\right.}{4\left(c-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*
x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (5*a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^2(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{5a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{4\left(c-a^2cx^2\right.}$$

Mathematica [A] time = 0.0651484, size = 79, normalized size = 0.37

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(ax+1)-\frac{4}{x}\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*a
*Log[1 - a*x] + a*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/2))
```

Maple [A] time = 0.136, size = 118, normalized size = 0.6

$$\frac{4\,a^2\ln(x)\,x^2+\ln(ax+1)\,a^2x^2-5\,\ln(ax-1)\,a^2x^2-4\,a\ln(x)\,x-ax\ln(ax+1)+5\,\ln(ax-1)\,xa-6\,ax+4}{\left(4\,a^2x^2-4\right)c^2x}\sqrt{-c\left(a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a^2*ln(x)*x^2+ln(a*x+
1)*a^2*x^2-5*ln(a*x-1)*a^2*x^2-4*a*ln(x)*x-a*x*ln(a*x+1)+5*ln(a*x-1)*x*a-6*
a*x+4)/(a^2*x^2-1)/c^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.72626, size = 197, normalized size = 0.92

$$-\frac{\sqrt{-a^2c}\left(6\,ax-\left(a^2x^2-ax\right)\log(ax+1)+5\left(a^2x^2-ax\right)\log(ax-1)-4\left(a^2x^2-ax\right)\log(x)-4\right)}{4\left(a^2c^2x^2-ac^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*sqrt(-a^2*c)*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x)
*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a^2*c^2*x^2 - a*c^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.698 $\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=252

$$\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(3/2)*x)/(2*(c - a^2*c*x^2)^(3/2)) - (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2) + (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^
3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (2*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3
*Log[x])/(c - a^2*c*x^2)^(3/2) - (7*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 -
 a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 +
 a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.271751, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(3/2)*x)/(2*(c - a^2*c*x^2)^(3/2)) - (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2) + (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^
3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (2*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3
*Log[x])/(c - a^2*c*x^2)^(3/2) - (7*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 -
 a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 +
 a*x])/(4*(c - a^2*c*x^2)^(3/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^3(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.080199, size = 94, normalized size = 0.37

$$
\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(ax+1)-\frac{4a}{x}-\frac{2}{x^2}\right)}{4\left(c-a^2cx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*
a^2*Log[x] - 7*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/
2))
```

Maple [A] time = 0.143, size = 138, normalized size = 0.6

$$\frac{8\,a^{3}\ln \left(x\right)x^{3}-a^{3}x^{3}\ln \left(ax+1\right)-7\,\ln \left(ax-1\right)x^{3}a^{3}-8\,a^{2}\ln \left(x\right)x^{2}+\ln \left(ax+1\right)a^{2}x^{2}+7\,\ln \left(ax-1\right)a^{2}x^{2}-6\,a^{2}x^{2}+2\,ax}{\left(4\,a^{2}x^{2}-4\right)c^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*a^3*ln(x)*x^3-a^3*x^3
*ln(a*x+1)-7*ln(a*x-1)*x^3*a^3-8*a^2*ln(x)*x^2+ln(a*x+1)*a^2*x^2+7*ln(a*x-1
)*a^2*x^2-6*a^2*x^2+2*a*x+2)/(a^2*x^2-1)/c^2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x^{3}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.72366, size = 232, normalized size = 0.92

$$-\frac{\left(6\,a^{2}x^{2}-2\,ax+\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(ax+1\right)+7\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(ax-1\right)-8\left(a^{3}x^{3}-a^{2}x^{2}\right)\log\left(x\right)-2\right)\sqrt{-a^{2}c}}{4\left(a^{2}c^{2}x^{3}-ac^{2}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 - a
^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)*sqrt(-a^2*c)/(a^2*
c^2*x^3 - a*c^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.699 $\int \frac{e^{\coth^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{x^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{a(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{23x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\mathrm{l}}{16a\left(c-a^2cx\right.}$$

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^6)/(c - a^2*c*x^2)^(5/2) - ((1 - 1/(a^2*x^2))^(5
/2)*x^5)/(8*a*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(a^2*x^2))^(5/2)
*x^5)/(a*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - ((1 - 1/(a^2*x^2))^(5/2)*x^5)/(
8*a*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (23*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[
1 - a*x])/(16*a*(c - a^2*c*x^2)^(5/2)) - (7*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log
[1 + a*x])/(16*a*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.242256, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{x^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{a(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{23x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\mathrm{l}}{16a\left(c-a^2cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^6)/(c - a^2*c*x^2)^(5/2) - ((1 - 1/(a^2*x^2))^(5
/2)*x^5)/(8*a*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(a^2*x^2))^(5/2)
*x^5)/(a*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - ((1 - 1/(a^2*x^2))^(5/2)*x^5)/(
8*a*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (23*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[
1 - a*x])/(16*a*(c - a^2*c*x^2)^(5/2)) - (7*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log
[1 + a*x])/(16*a*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1+ax)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.109289, size = 89, normalized size = 0.34

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(16ax+\frac{16}{1-ax}-\frac{2}{ax+1}-\frac{2}{(ax-1)^2}+23\log(1-ax)-7\log(ax+1)\right)}{16a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*(16*a*x + 16/(1 - a*x) - 2/(-1 + a*x)^2 - 2/(1
 + a*x) + 23*Log[1 - a*x] - 7*Log[1 + a*x]))/(16*a*(c - a^2*c*x^2)^(5/2))
```

Maple [A] time = 0.139, size = 185, normalized size = 0.7

$$\frac{-16\,x^4a^4+7\,a^3x^3\ln\left(ax+1\right)-23\,\ln\left(ax-1\right)x^3a^3+16\,x^3a^3-7\,\ln\left(ax+1\right)a^2x^2+23\,\ln\left(ax-1\right)a^2x^2+34\,a^2x^2-7\,ax\,\ln}{\left(16\,ax-16\right)\left(a^2x^2-1\right)c^3a^6\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(-16*x^4*a^4+7*
a^3*x^3*ln(a*x+1)-23*ln(a*x-1)*x^3*a^3+16*x^3*a^3-7*ln(a*x+1)*a^2*x^2+23*ln
(a*x-1)*a^2*x^2+34*a^2*x^2-7*a*x*ln(a*x+1)+23*ln(a*x-1)*x*a-18*a*x+7*ln(a*x
+1)-23*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.59674, size = 296, normalized size = 1.13

$$-\frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)+}{16\left(a^{10}c^3x^3-a^9c^3x^2-a^8c^3x+a^7c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
12)*sqrt(-a^2*c)/(a^10*c^3*x^3 - a^9*c^3*x^2 - a^8*c^3*x + a^7*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.700 $\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{11x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5x^5\left(1\right.}{1}$$

```
[Out] -((1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (3*(
1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(
a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (11*(1 - 1/(a^2*
x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*(1 - 1/(a^2*x
^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.277597, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$\frac{3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{11x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5x^5\left(1\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (3*(
1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(
a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (11*(1 - 1/(a^2*
x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*(1 - 1/(a^2*x
^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^4}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^4(-1+ax)^3}+\frac{3}{4a^4(-1+ax)^2}+\frac{11}{16a^4(-1+ax)}-\frac{1}{8a^4(1+ax)^2}+\frac{5}{16a^4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{11\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{16\left(c-a^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.094583, size = 84, normalized size = 0.39

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{2\left(5a^2x^2+3ax-6\right)}{(ax-1)^2(ax+1)}+11\log(1-ax)+5\log(ax+1)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*((-2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(
1 + a*x)) + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5/2))
```

Maple [A] time = 0.143, size = 169, normalized size = 0.8

$$-\frac{5 a^3x^3 \ln (ax+1)+11 \ln (ax-1) x^3a^3-5 \ln (ax+1) a^2x^2-11 \ln (ax-1) a^2x^2-10 a^2x^2-5 ax \ln (ax+1)-11 \ln (a}{(16 ax-16) \left(a^2x^2-1\right) c^3a^5 (ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*a^3*x^3*ln(
a*x+1)+11*ln(a*x-1)*x^3*a^3-5*ln(a*x+1)*a^2*x^2-11*ln(a*x-1)*a^2*x^2-10*a^2
*x^2-5*a*x*ln(a*x+1)-11*ln(a*x-1)*x*a-6*a*x+5*ln(a*x+1)+11*ln(a*x-1)+12)/(a
^2*x^2-1)/c^3/a^5/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.54577, size = 257, normalized size = 1.18

$$\frac{\left(10 a^2x^2+6 ax-5 \left(a^3x^3-a^2x^2-ax+1\right) \log (ax+1)-11 \left(a^3x^3-a^2x^2-ax+1\right) \log (ax-1)-12\right) \sqrt{-a^2c}}{16 \left(a^9c^3x^3-a^8c^3x^2-a^7c^3x+a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)*sqrt(-a^2*c)/(a^9*c^3*x^
3 - a^8*c^3*x^2 - a^7*c^3*x + a^6*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.701 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a*(1 -
 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a*(1 - 1/
(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.267924, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6192, 6193, 88, 207}

$$\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a*(1 -
 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a*(1 - 1/
(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^3}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^3(-1+ax)^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{1}{8a^3(1+ax)^2}+\frac{3}{8a^3\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^2\left(1-\frac{1}{a^2x^2}\right)^{5/}\right.}{8\left(c-a\right.}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{8\left(c-a^2c\right.}$$

Mathematica [A] time = 0.0641193, size = 86, normalized size = 0.49

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(5a^2x^2-ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)-2\right)}{8a^3c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*
ArcTanh[a*x]))/(8*a^3*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.142, size = 169, normalized size = 1.

$$\frac{3\,a^3x^3\ln(ax+1)-3\,\ln(ax-1)\,x^3a^3-3\,\ln(ax+1)\,a^2x^2+3\,\ln(ax-1)\,a^2x^2+10\,a^2x^2-3\,ax\ln(ax+1)+3\,\ln(ax-1}{(16\,ax-16)\left(a^2x^2-1\right)c^3a^4\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a
*x+1)-3*ln(a*x-1)*x^3*a^3-3*ln(a*x+1)*a^2*x^2+3*ln(a*x-1)*a^2*x^2+10*a^2*x^
2-3*a*x*ln(a*x+1)+3*ln(a*x-1)*x*a-2*a*x+3*ln(a*x+1)-3*ln(a*x-1)-4)/(a^2*x^2
-1)/c^3/a^4/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.61313, size = 275, normalized size = 1.56

$$-\frac{3\left(a^{4}x^{3}-a^{3}x^{2}-a^{2}x+a\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{2}+2\sqrt{-a^{2}c}\sqrt{-c}x+c}{a^{2}x^{2}-1}\right)-2\left(5a^{2}x^{2}-ax-2\right)\sqrt{-a^{2}c}}{16\left(a^{8}c^{3}x^{3}-a^{7}c^{3}x^{2}-a^{6}c^{3}x+a^{5}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(5*a^2*x^2 - a*x - 2)*sqrt(-a^2*c
))/(a^8*c^3*x^3 - a^7*c^3*x^2 - a^6*c^3*x + a^5*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.702 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) +
(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^2*(
1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.27233, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6192, 6193, 88, 207}

$$\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) +
(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^2*(
1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^2}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^2(-1+ax)^3}+\frac{1}{4a^2(-1+ax)^2}-\frac{1}{8a^2(1+ax)^2}-\frac{1}{8a^2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)\right.}{8\left(c-\right.}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/}}{8\left(c-a\right.}$$

Mathematica [A] time = 0.0596156, size = 85, normalized size = 0.46

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-a^2x^2-3ax+(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8a^2c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - a^2*x^2 + (-1 + a*x)^2*(1 + a*x)*ArcT
anh[a*x]))/(8*a^2*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.144, size = 164, normalized size = 0.9

$$-\frac{a^3x^3\ln(ax+1)-\ln(ax-1)x^3a^3-\ln(ax+1)a^2x^2+\ln(ax-1)a^2x^2-2a^2x^2-ax\ln(ax+1)+\ln(ax-1)xa-6ax+}{(16ax-16)\left(a^2x^2-1\right)c^3a^3(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a^3*x^3*ln(a*
x+1)-ln(a*x-1)*x^3*a^3-ln(a*x+1)*a^2*x^2+ln(a*x-1)*a^2*x^2-2*a^2*x^2-a*x*ln
(a*x+1)+ln(a*x-1)*x*a-6*a*x+ln(a*x+1)-ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a^3/(a*x
+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.50437, size = 273, normalized size = 1.48

$$-\frac{\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(a^2x^2+3ax-2\right)\sqrt{-a^2c}}{16\left(a^7c^3x^3-a^6c^3x^2-a^5c^3x+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 + 3*a*x - 2)*sqrt(-a^2*c))
/(a^7*c^3*x^3 - a^6*c^3*x^2 - a^5*c^3*x + a^4*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.703 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
3*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.221176, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6192, 6193, 77, 207}

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
3*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a(-1+ax)^3}+\frac{1}{8a(1+ax)^2}-\frac{1}{8a\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0677811, size = 60, normalized size = 0.44

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax+1}+\frac{1}{(ax-1)^2}-\tanh^{-1}(ax)\right)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*((-1 + a*x)^(-2) + (1 + a*x)^(-1) - ArcTa
nh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

Maple [A] time = 0.15, size = 164, normalized size = 1.2

$$-\frac{a^3x^3\ln(ax+1)-\ln(ax-1)x^3a^3-\ln(ax+1)a^2x^2+\ln(ax-1)a^2x^2-2a^2x^2-ax\ln(ax+1)+\ln(ax-1)xa+2ax}{(16ax-16)\left(a^2x^2-1\right)c^3a^2(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a^3*x^3*ln(a*
x+1)-ln(a*x-1)*x^3*a^3-ln(a*x+1)*a^2*x^2+ln(a*x-1)*a^2*x^2-2*a^2*x^2-a*x*ln
(a*x+1)+ln(a*x-1)*x*a+2*a*x+ln(a*x+1)-ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^2/(a*x
+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.51764, size = 270, normalized size = 1.97

$$-\frac{\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(a^2x^2-ax+2\right)\sqrt{-a^2c}}{16\left(a^6c^3x^3-a^5c^3x^2-a^4c^3x+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 - a*x + 2)*sqrt(-a^2*c))/(
a^6*c^3*x^3 - a^5*c^3*x^2 - a^4*c^3*x + a^3*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.704 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.196546, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^5\left(1-\frac{1}{a^2x^2}\right)\right.}{8\left(c-\right.}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5}}{8\left(c-a\right.}$$

Mathematica [A] time = 0.0501508, size = 83, normalized size = 0.45

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)
*ArcTanh[a*x]))/(8*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.15, size = 169, normalized size = 0.9

$$\frac{3a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3-3\ln(ax+1)a^2x^2+3\ln(ax-1)a^2x^2-6a^2x^2-3ax\ln(ax+1)+3\ln(ax-1)x}{(16ax-16)\left(a^2x^2-1\right)c^3a(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*x^3*ln(a
*x+1)-3*ln(a*x-1)*x^3*a^3-3*ln(a*x+1)*a^2*x^2+3*ln(a*x-1)*a^2*x^2-6*a^2*x^2
-3*a*x*ln(a*x+1)+3*ln(a*x-1)*x*a+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-
1)/c^3/a/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.57054, size = 278, normalized size = 1.51

$$-\frac{3\left(a^{4}x^{3}-a^{3}x^{2}-a^{2}x+a\right)\sqrt{-c}\log\left(\frac{a^{2}cx^{2}+2\sqrt{-a^{2}c}\sqrt{-c}x+c}{a^{2}x^{2}-1}\right)+2\left(3a^{2}x^{2}-3ax-2\right)\sqrt{-a^{2}c}}{16\left(a^{5}c^{3}x^{3}-a^{4}c^{3}x^{2}-a^{3}c^{3}x+a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.705 $\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{\left(c-a^2cx^2\right)^{5/2}}+\frac{11a^5x^5\left(1-\cdots\right.}{16\left(c\cdots\right.}$$

```
[Out] -(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^5*(
1 - 1/(a^2*x^2))^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (11*a^5*(1 - 1/(
a^2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*a^5*(1 -
1/(a^2*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.2836, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{\left(c-a^2cx^2\right)^{5/2}}+\frac{11a^5x^5\left(1-\cdots\right.}{16\left(c\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^5*(
1 - 1/(a^2*x^2))^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (11*a^5*(1 - 1/(
a^2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*a^5*(1 -
1/(a^2*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x}+\frac{a}{4(-1+ax)^3}-\frac{a}{2(-1+ax)^2}+\frac{11a}{16(-1+ax)}+\frac{a}{8(1+ax)^2}+\frac{5a}{16(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5}}{\left(c-a^2c\right.}
\end{aligned}$$

Mathematica [A] time = 0.0961849, size = 88, normalized size = 0.32

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{8}{ax-1}-\frac{2}{ax+1}-\frac{2}{(ax-1)^2}+11\log(1-ax)+5\log(ax+1)-16\log(x)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-2/(-1 + a*x)^2 + 8/(-1 + a*x) - 2/(1 + a
*x) - 16*Log[x] + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5
/2))
```

Maple [A] time = 0.139, size = 196, normalized size = 0.7

$$\frac{16\,a^3\ln(x)\,x^3-5\,a^3x^3\ln(ax+1)-11\,\ln(ax-1)\,x^3a^3-16\,a^2\ln(x)\,x^2+5\,\ln(ax+1)\,a^2x^2+11\,\ln(ax-1)\,a^2x^2-6\,a^2x^2}{(16\,ax-16)\left(a^2x^2-1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(16*a^3*ln(x)*x
^3-5*a^3*x^3*ln(a*x+1)-11*ln(a*x-1)*x^3*a^3-16*a^2*ln(x)*x^2+5*ln(a*x+1)*a^
2*x^2+11*ln(a*x-1)*a^2*x^2-6*a^2*x^2-16*a*ln(x)*x+5*a*x*ln(a*x+1)+11*ln(a*x
-1)*x*a-2*a*x+16*ln(x)-5*ln(a*x+1)-11*ln(a*x-1)+12)/(a^2*x^2-1)/c^3/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.67067, size = 311, normalized size = 1.15

$$-\frac{\left(6\,a^2x^2+2\,ax+5\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)+11\left(a^3x^3-a^2x^2-ax+1\right)\log(ax-1)-16\left(a^3x^3-a^2x^2-ax+1\right.\right.}{16\left(a^4c^3x^3-a^3c^3x^2-a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*x
+ 1)*log(x) - 12)*sqrt(-a^2*c)/(a^4*c^3*x^3 - a^3*c^3*x^2 - a^2*c^3*x + a*c
^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.706 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{3a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^5x^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{lo}}{\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^4)/(c - a^2*c*x^2)^(5/2) - (a^6*(1 - 1/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) - (3*a^6*(1 - 1/(a^2
*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^6*(1 - 1/(a^2*x^
2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^6*(1 - 1/(a^2*x^2))
^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (23*a^6*(1 - 1/(a^2*x^2))^(5/2)*
x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) - (7*a^6*(1 - 1/(a^2*x^2))^(5/
2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.292859, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 88}

$$-\frac{3a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^5x^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\text{lo}}{\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^4)/(c - a^2*c*x^2)^(5/2) - (a^6*(1 - 1/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) - (3*a^6*(1 - 1/(a^2
*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^6*(1 - 1/(a^2*x^
2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^6*(1 - 1/(a^2*x^2))
^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (23*a^6*(1 - 1/(a^2*x^2))^(5/2)*
x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) - (7*a^6*(1 - 1/(a^2*x^2))^(5/
2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x^2(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x^2}-\frac{a}{x}+\frac{a^2}{4(-1+ax)^3}-\frac{3a^2}{4(-1+ax)^2}+\frac{23a^2}{16(-1+ax)}-\frac{a^2}{8(1+ax)^2}-\frac{7a^2}{16(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{8(1+ax)\left(c-a^2cx^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.0888769, size = 99, normalized size = 0.32

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{12a}{ax-1}+\frac{2a}{ax+1}-\frac{2a}{(ax-1)^2}-16a\log(x)+23a\log(1-ax)-7a\log(ax+1)+\frac{16}{x}\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(16/x - (2*a)/(-1 + a*x)^2 + (12*a)/(-1 +
a*x) + (2*a)/(1 + a*x) - 16*a*Log[x] + 23*a*Log[1 - a*x] - 7*a*Log[1 + a*x]
))/(16*(c - a^2*c*x^2)^(5/2))
```

Maple [A] time = 0.143, size = 225, normalized size = 0.7

$$\frac{16\,a^4\ln(x)\,x^4+7\,\ln(ax+1)\,a^4x^4-23\,\ln(ax-1)\,a^4x^4-16\,a^3\ln(x)\,x^3-7\,a^3x^3\ln(ax+1)+23\,\ln(ax-1)\,x^3a^3-30\,x^3a}{(16\,ax-16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(16*a^4*ln(x)*x
^4+7*ln(a*x+1)*a^4*x^4-23*ln(a*x-1)*a^4*x^4-16*a^3*ln(x)*x^3-7*a^3*x^3*ln(a
*x+1)+23*ln(a*x-1)*x^3*a^3-30*x^3*a^3-16*a^2*ln(x)*x^2-7*ln(a*x+1)*a^2*x^2+
23*ln(a*x-1)*a^2*x^2+22*a^2*x^2+16*a*ln(x)*x+7*a*x*ln(a*x+1)-23*ln(a*x-1)*x
*a+28*a*x-16)/(a^2*x^2-1)/c^3/x/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.76535, size = 360, normalized size = 1.17

$$-\frac{\left(30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax+1\right)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax-1\right)-16\right.}{16\left(a^4c^3x^4-a^3c^3x^3-a^2c^3x^2+ac^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) - 1
6*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)*sqrt(-a^2*c)/(a^4*c^3*x^
4 - a^3*c^3*x^3 - a^2*c^3*x^2 + a*c^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.707 $\int e^{-\coth^{-1}(ax)} x^2 \sqrt{c - a^2 c x^2} \, dx$

Optimal. Leaf size=76

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*
c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.224563, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*
c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int x^2(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(-x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= -\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0269459, size = 45, normalized size = 0.59

$$\frac{x^2(3ax-4)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] (x^2*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.04, size = 47, normalized size = 0.6

$$\frac{x^3\,(3\,ax-4)}{12\,ax-12}\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + cx^2}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.58257, size = 53, normalized size = 0.7

$$\frac{\left(3\,ax^4 - 4\,x^3\right)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*a*x^4 - 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}x^2\sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.708 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*
x^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.19266, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 43}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -(x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*
x^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(-x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= -\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0233199, size = 43, normalized size = 0.58

$$\frac{x(2ax-3)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] (x*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.038, size = 47, normalized size = 0.6

$$\frac{x^2\left(2\,ax-3\right)}{6\,ax-6}\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.58114, size = 51, normalized size = 0.69

$$\frac{\left(2\,ax^3-3\,x^2\right)\sqrt{-a^2c}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + cx}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.709 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.120105, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0195418, size = 41, normalized size = 0.59

$$\frac{(ax-2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.037, size = 44, normalized size = 0.6

$$\frac{x(ax-2)}{2ax-2}\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.59375, size = 46, normalized size = 0.67

$$\frac{\sqrt{-a^2c}\left(ax^2-2\,x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.710 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.146107, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.0193746, size = 44, normalized size = 0.63

$$\frac{\sqrt{c-a^2cx^2}(ax-\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.12, size = 46, normalized size = 0.7

$$-\frac{-ax+\ln(x)}{ax-1}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x))*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [A] time = 1.63677, size = 42, normalized size = 0.6

$$\frac{\sqrt{-a^2c}(ax-\log(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fric
as")
```

```
[Out] sqrt(-a^2*c)*(a*x - log(x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.711 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (Sqrt[c - a^2*c*x^2]*Lo
g[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.215674, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (Sqrt[c - a^2*c*x^2]*Lo
g[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.0224395, size = 44, normalized size = 0.61

$$\frac{\sqrt{c-a^2cx^2}\left(a\log(x)+\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.133, size = 48, normalized size = 0.7

$$\frac{a\ln(x)x+1}{(ax-1)x}\sqrt{-c\left(a^2x^2-1\right)}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] (a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A] time = 1.62168, size = 50, normalized size = 0.69

$$\frac{\sqrt{-a^2c}(ax\log(x)+1)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) + 1)/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.712 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) + (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rubi [A] time = 0.429526, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6152, 1809, 833, 780, 217, 203}

$$\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) + (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x^3\left(-9a^2c+10a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}$$
$$= -\frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2\left(-30a^3c^2+36a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x\left(-72a^4c^3+90a^5c^3x\right)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4} + \frac{(3c)\int \frac{}{\sqrt{c-}}}{4a^3}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4} + \frac{(3c)\,\mathrm{Subs}}{}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4} + \frac{3\sqrt{c}\tan^{-1}}{4}$$

Mathematica [A] time = 0.127231, size = 96, normalized size = 0.7

$$\frac{\left(4a^4x^4-10a^3x^3+12a^2x^2-15ax+24\right)\sqrt{c-a^2cx^2}-15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 10*a^3*x^3 + 4*a^4*x^4) -
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

Maple [A] time = 0.055, size = 202, normalized size = 1.5

$$-\frac{x^2}{5a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{4}{5ca^4}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{x}{2a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - \frac{5x}{4a^3}\sqrt{-a^2cx^2+c} - \frac{5c}{4a^3}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c-4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)+1/2/a^3*
x*(-a^2*c*x^2+c)^(3/2)/c-5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)-5/4/a^3*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^4*(-a^2*c*(x+1/a)^2+2
*(x+1/a)*a*c)^(1/2)+2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x
+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77678, size = 431, normalized size = 3.15

$$\left[\frac{2\left(4\,a^4x^4-10\,a^3x^3+12\,a^2x^2-15\,ax+24\right)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{40\,a^4},\ \frac{\left(4\,a^4x^4-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c))/a^4, 1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-
a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.16753, size = 124, normalized size = 0.91

$$\frac{1}{20}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(2\,x-\frac{5}{a}\right)x+\frac{6}{a^2}\right)x-\frac{15}{a^3}\right)x+\frac{24}{a^4}\right)-\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{4\,a^3\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(-a^2*c*x^2 + c)*((2*((2*x - 5/a)*x + 6/a^2)*x - 15/a^3)*x + 24/a^
4) - 3/4*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^3*sqrt(-c)*a
bs(a))
```

3.713 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 - 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rubi [A] time = 0.396259, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6152, 1809, 833, 780, 217, 203}

$$\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 - 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2(-7a^2c+8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x(-16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{8a^2}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,d\right.}{8a^2}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{cx}}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Mathematica [A] time = 0.0994461, size = 88, normalized size = 0.79

$$\frac{\left(6a^3x^3-16a^2x^2+21ax-32\right)\sqrt{c-a^2cx^2}+21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [A] time = 0.049, size = 178, normalized size = 1.6

$$-\frac{x}{4a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + \frac{9x}{8a^2}\sqrt{-a^2cx^2+c} + \frac{9c}{8a^2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} + \frac{2}{3a^3c}\left(-a^2cx^2+c\right)^{\frac{3}{2}} - 2\frac{\sqrt{-a^2c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)+9/8/a^2*c/
(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a^3*(-a^2*c*
x^2+c)^(3/2)/c-2/a^3*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)-2/a^2*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62629, size = 396, normalized size = 3.54

$$\left[\frac{2\left(6a^3x^3-16a^2x^2+21ax-32\right)\sqrt{-a^2cx^2+c}+21\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48a^3}, \frac{\left(6a^3x^3-16a^2x^2+21\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.13788, size = 113, normalized size = 1.01

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,x-\frac{8}{a}\right)x+\frac{21}{a^2}\right)x-\frac{32}{a^3}\right)+\frac{7\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,a^2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x - 8/a)*x + 21/a^2)*x - 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

3.714 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) + (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rubi [A] time = 0.25367, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6167, 6152, 1809, 780, 217, 203}

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```



```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) + (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
```

```
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int \frac{x\left(-5a^2c+6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0833393, size = 79, normalized size = 0.94

$$\frac{(a^2x^2-3ax+5)\sqrt{c-a^2cx^2}-3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] - 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [B] time = 0.048, size = 156, normalized size = 1.9

$$-\frac{1}{3a^2c}\left(-a^2cx^2+c\right)^{\frac{3}{2}}-\frac{x}{a}\sqrt{-a^2cx^2+c}-\frac{c}{a}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}+2\frac{\sqrt{-a^2c\left(x+a^{-1}\right)^2+2\left(x+a^{-1}\right)ac}}{a^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c-x/a*(-a^2*c*x^2+c)^(1/2)-1/a*c/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2*(-a^2*c*(x+1/a)^2+2*(x
+1/a)*a*c)^(1/2)+2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)
^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73737, size = 344, normalized size = 4.1

$$\left[\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,ax+5\right)+3\sqrt{-c}\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2}, \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,ax+5\right)-3\,\sqrt{}}{3\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.17287, size = 99, normalized size = 1.18

$$\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x-\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x - 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

3.715 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rubi [A] time = 0.113209, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6167, 6142, 671, 641, 217, 203}

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0533999, size = 100, normalized size = 1.15

$$
\frac{\sqrt{c-a^2cx^2}\left(6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\sqrt{ax+1}\left(a^2x^2-5ax+4\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a
*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.048, size = 126, normalized size = 1.5

$$\frac{x}{2}\sqrt{-a^2cx^2+c}+\frac{c}{2}\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}}-2\,\frac{\sqrt{-a^2c\left(x+a^{-1}\right)^2+2\left(x+a^{-1}\right)ac}}{a}-2\,\frac{c}{\sqrt{a^2c}}\arctan\left(\frac{}{\sqrt{-a^2c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))-2/a*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69543, size = 306, normalized size = 3.52

$$\left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4\,a},\frac{\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}}{a^2cx^2-}\right.}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.17382, size = 84, normalized size = 0.97

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.716 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=75

$$\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.349085, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6152, 1809, 844, 217, 203, 266, 63, 208}

$$\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right)\\ &= \sqrt{c-a^2cx^2}+\frac{\int \frac{-a^2c+2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}\\ &= \sqrt{c-a^2cx^2}-c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx+(2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \sqrt{c-a^2cx^2}-\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)+(2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{}{\sqrt{c-}}\right.\\ &= \sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2}\\ &= \sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A] time = 0.0823079, size = 97, normalized size = 1.29

$$\sqrt{c-a^2cx^2}+\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

Maple [A] time = 0.052, size = 121, normalized size = 1.6

$$-\sqrt{-a^2cx^2+c}+\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)+2\,\sqrt{-a^2c\left(x+a^{-1}\right)^2+2\,\left(x+a^{-1}\right)ac}+2\,\frac{ac}{\sqrt{a^2c}}\arctan\left(\frac{}{\sqrt{-a^2c\left(x\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67423, size = 443, normalized size = 5.91

$$\left[-2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c},\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2c}}{a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt
(-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
- c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
 + sqrt(-a^2*c*x^2 + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}(ax-1)}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x*(a*x + 1)), x)
```

Giac [A] time = 1.1369, size = 128, normalized size = 1.71

$$-\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

3.717 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.346569, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6152, 1807, 844, 217, 203, 266, 63, 208}

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + \int \frac{2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + (ac)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - \left(a^2c\right)\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, -$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a}$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.0945642, size = 104, normalized size = 1.27

$$\frac{\sqrt{c-a^2cx^2}}{x} - 2a\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + a\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + 2a\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

Maple [B] time = 0.052, size = 200, normalized size = 2.4

$$\frac{1}{cx}\left(-a^2cx^2+c\right)^{\frac{3}{2}} + a^2x\sqrt{-a^2cx^2+c} + a^2c\arctan\left(x\sqrt{a^2c}\frac{1}{\sqrt{-a^2cx^2+c}}\right)\frac{1}{\sqrt{a^2c}} - 2\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)a + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] 1/c/x*(-a^2*c*x^2+c)^(3/2)+a^2*x*(-a^2*c*x^2+c)^(1/2)+a^2*c/(a^2*c)^(1/2)*a
rctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a
^2*c*x^2+c)^(1/2))/x)*a+2*(-a^2*c*x^2+c)^(1/2)*a-2*a*(-a^2*c*(x+1/a)^2+2*(x
+1/a)*a*c)^(1/2)-2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/
a)^2+2*(x+1/a)*a*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^2), x)
```

Fricas [A] time = 1.6808, size = 478, normalized size = 5.83

$$\left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},-\frac{4a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqr
t(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
 c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Giac [A] time = 1.16683, size = 181, normalized size = 2.21

$$\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] 4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a^
2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

3.718 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.347263, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6152, 1807, 807, 266, 63, 208}

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```


```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{1}{2}\int \frac{4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.113853, size = 76, normalized size = 0.97

$$\frac{1}{2}\left(\frac{(1-4ax)\sqrt{c-a^2cx^2}}{x^2} + 3a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - 3a^2\sqrt{c}\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (((1 - 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [B] time = 0.056, size = 231, normalized size = 3.

$$-2\,\frac{a\left(-a^2cx^2+c\right)^{3/2}}{cx} - 2\,a^3x\sqrt{-a^2cx^2+c} - 2\,\frac{a^3c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) + \frac{3\,a^2}{2}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right) - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] -2*a/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^3*x*(-a^2*c*x^2+c)^(1/2)-2*a^3*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+3/2*c^(1/2)*ln((2*c+2*c^
(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^2-3/2*(-a^2*c*x^2+c)^(1/2)*a^2+2*a^2*(-a^2
*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+2*a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2
)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))+1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^3), x)
```

Fricas [A] time = 1.73074, size = 338, normalized size = 4.33

$$\left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}(4\,ax-1)}{4\,x^2},\ \frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2cx^2+c}(4\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 +
 c)*(4*a*x - 1))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Giac [B] time = 1.13852, size = 270, normalized size = 3.46

$$-\frac{3\,a^{2}c\arctan\left(-\frac{\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}\right)^{3}a^{2}c+4\left(\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}\right)^{2}a\sqrt{-c}c|a|+\left(\sqrt{-a^{2}c}x-\right.}{\left(\left(\sqrt{-a^{2}c}x-\sqrt{-a^{2}cx^{2}+c}\right)^{2}-c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c + 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 - 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

3.719 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=101

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.368385, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6152, 1807, 835, 807, 266, 63, 208}

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```


```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{1}{3}\int \frac{6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{\int \frac{10a^2c^2-6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + \left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + \frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.11434, size = 82, normalized size = 0.81

$$\frac{\left(5a^2x^2-3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3} - a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + a^3\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] ((1 - 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x]
- a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] time = 0.059, size = 254, normalized size = 2.5

$$2\,\frac{a^2\left(-a^2cx^2+c\right)^{3/2}}{cx} + 2\,a^4x\sqrt{-a^2cx^2+c} + 2\,\frac{a^4c}{\sqrt{a^2c}}\arctan\left(\frac{\sqrt{a^2c}x}{\sqrt{-a^2cx^2+c}}\right) - \sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2cx^2+c}\right)\right)a^3 + \sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^4*x*(-a^2*c*x^2+c)^(1/2)+2*a^4*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-c^(1/2)*ln((2*c+2*c^(1/
2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3+(-a^2*c*x^2+c)^(1/2)*a^3-2*a^3*(-a^2*c*(x+1
/a)^2+2*(x+1/a)*a*c)^(1/2)-2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a
^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))-a/c/x^2*(-a^2*c*x^2+c)^(3/2)+1/3/c/x^3
*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^4), x)
```

Fricas [A] time = 1.72065, size = 371, normalized size = 3.67

$$\left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}\left(5\,a^2x^2-3\,ax+1\right)}{6\,x^3},-\frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2c\cdots}}{3\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, -1/3*(3*a^
3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt
(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Giac [B] time = 1.14053, size = 338, normalized size = 3.35

$$\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\left(3\left(\sqrt{-a^2c}x - \sqrt{-a^2cx^2+c}\right)^5 a^3c + 3\left(\sqrt{-a^2c}x - \sqrt{-a^2cx^2+c}\right)^4 a^2\sqrt{-c}c|a| - 12\left(\sqrt{}\right.\right.}{3\left(\left(\sqrt{-a^2c}x - \sqrt{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c + 3*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) - 12*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))*a^3*c^3 + 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^2 - c)^3
```

3.720 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.402421, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6152, 1807, 835, 807, 266, 63, 208}

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{1}{4}\int \frac{8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{\int \frac{21a^2c^2-16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{\int \frac{32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}\left(7a^4c\right)\int \frac{1}{x\sqrt{c-a^2c}}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{16}\left(7a^4c\right)\text{Subst}\left(\int \right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}\left(7a^2\right)\text{Subst}\left(\int \right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.138829, size = 95, normalized size = 0.73

$$\frac{\left(-32a^3x^3+21a^2x^2-16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4} + \frac{7}{8}a^4\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - \frac{7}{8}a^4\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 - 16*a*x + 21*a^2*x^2 - 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

Maple [B] time = 0.063, size = 279, normalized size = 2.2

$$\frac{1}{4\,c x^4}\left(-a^2 c x^2+c\right)^{\frac{3}{2}}+\frac{9\,a^2}{8\,c x^2}\left(-a^2 c x^2+c\right)^{\frac{3}{2}}+\frac{7\,a^4}{8}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{-a^2 c x^2+c}\right)\right)-\frac{7\,a^4}{8}\sqrt{-a^2 c x^2+c}-2\,\frac{a^3\left(-a^2 c x^2\right.}{c x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] 1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)+9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)+7/8*a^4*c
^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-7/8*a^4*(-a^2*c*x^2+c)^(1
/2)-2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^5*x*(-a^2*c*x^2+c)^(1/2)-2*a^5*c/(a^
2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a^4*(-a^2*c*(x+1/
a)^2+2*(x+1/a)*a*c)^(1/2)+2*a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^
2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))-2/3*a/c/x^3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2 c x^2+c}(ax-1)}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^5), x)
```

Fricas [A] time = 1.66766, size = 416, normalized size = 3.2

$$\left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\left(32\,a^3x^3-21\,a^2x^2+16\,ax-6\right)\sqrt{-a^2cx^2+c}}{48\,x^4},\ \frac{21\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-a^2cx^2+c}}{a^2cx^2-c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Giac [B] time = 1.17523, size = 437, normalized size = 3.36

$$-\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2-96\left(\sqrt{-a^2c}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 - 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 + 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 - 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

3.721 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=227

$$\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) - (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) - (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - a^2*c*
x^2]*Log[1 + a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.261407, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) - (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) - (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - a^2*c*
x^2]*Log[1 + a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}-\frac{4x}{a^2}+\frac{4x^2}{a}-3x^3+ax^4-\frac{4}{a^3(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-4\sqrt{}
\end{aligned}$$

Mathematica [A] time = 0.0570865, size = 87, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{2x^2}{a^2}+\frac{4x}{a^3}-\frac{4\log(ax+1)}{a^4}+\frac{ax^5}{5}+\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4 +
 (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.127, size = 92, normalized size = 0.4

$$-\frac{\left(-12\,x^{5}a^{5}+45\,x^{4}a^{4}-80\,x^{3}a^{3}+120\,a^{2}x^{2}-240\,ax+240\,\ln\left(ax+1\right)\right)\left(ax+1\right)}{60\,a^{4}\left(ax-1\right)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/60*(-12*x^5*a^5+45*x^4*a^4-80*x^3*a^3+120*a^2*x^2-240*a*x+240*ln(a*x+1))
*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^{2}cx^{2}+c}x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.64446, size = 142, normalized size = 0.63

$$\frac{\left(12\,a^{5}x^{5}-45\,a^{4}x^{4}+80\,a^{3}x^{3}-120\,a^{2}x^{2}+240\,ax-240\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{60\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x - 240*lo
g(a*x + 1))*sqrt(-a^2*c)/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.722 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=186

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1
/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqr
t[c - a^2*c*x^2]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.257982, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1
/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqr
t[c - a^2*c*x^2]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= -\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.0510912, size = 72, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(ax\left(a^3x^3-4a^2x^2+8ax-16\right)+16\log(ax+1)\right)}{4a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-16 + 8*a*x - 4*a^2*x^2 + a^3*x^3) + 16*Log[1 +
a*x]))/(4*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.132, size = 83, normalized size = 0.5

$$\frac{\left(x^{4}a^{4}-4\,x^{3}a^{3}+8\,a^{2}x^{2}-16\,ax+16\,\ln\left(ax+1\right)\right)\left(ax+1\right)}{4\,a^{3}\left(ax-1\right)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/4*(x^4*a^4-4*x^3*a^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^{2}cx^{2}+c}x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.56866, size = 112, normalized size = 0.6

$$\frac{\left(a^{4}x^{4}-4\,a^{3}x^{3}+8\,a^{2}x^{2}-16\,ax+16\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{4\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(-a^2*
c)/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.723 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=151

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rubi [A] time = 0.21185, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 77}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int\left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] time = 0.039138, size = 65, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\left(ax\left(2a^2x^2-9ax+24\right)-24\log(ax+1)\right)}{6a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.131, size = 76, normalized size = 0.5

$$-\frac{\left(-2\,x^{3}a^{3}+9\,a^{2}x^{2}-24\,ax+24\,\ln\left(ax+1\right)\right)\left(ax+1\right)}{6\,a^{2}\left(ax-1\right)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(-2*x^3*a^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x
+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^{2}cx^{2}+c}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.58555, size = 99, normalized size = 0.66

$$\frac{\left(2\,a^{3}x^{3}-9\,a^{2}x^{2}+24\,ax-24\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(-a^2*c)/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}\,x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.724 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.12924, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}
\end{aligned}$$

Mathematica [A] time = 0.0240492, size = 56, normalized size = 0.5

$$\frac{\sqrt{c-a^2cx^2}(ax(ax-6)+8\log(ax+1))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*x)
```

Maple [A] time = 0.127, size = 67, normalized size = 0.6

$$\frac{\left(a^2x^2-6\,ax+8\,\ln(ax+1)\right)(ax+1)}{2\,a\,(ax-1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/a/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.63843, size = 77, normalized size = 0.69

$$\frac{\left(a^2x^2 - 6\,ax + 8\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.725 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.160221, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 72}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A] time = 0.0255497, size = 50, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}(ax-4\log(ax+1)+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

Maple [A] time = 0.13, size = 57, normalized size = 0.5

$$\frac{(ax+\ln(x)-4\ln(ax+1))(ax+1)}{(ax-1)^2}\sqrt{-c\left(a^2x^2-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] (a*x+ln(x)-4*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3
/2)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [A] time = 1.73554, size = 65, normalized size = 0.58

$$\frac{\sqrt{-a^2c}\left(ax-4\,\log\left(ax+1\right)+\log\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fric
as")
```

```
[Out] sqrt(-a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.726 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) - (3*Sqrt[c - a^2*c*x^
2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.233059, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```


```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) - (3*Sqrt[c - a^2*c*x^
2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(Sqrt[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.0298139, size = 56, normalized size = 0.49

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1 -
 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.132, size = 65, normalized size = 0.6

$$-\frac{(3 a \ln (x) x-4 a x \ln (a x+1)+1)(a x+1)}{x(a x-1)^{2}} \sqrt{-c\left(a^{2} x^{2}-1\right)}\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*a*x*ln(a*x+1)+1)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/x/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2}+c}\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{x^{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A] time = 1.64058, size = 81, normalized size = 0.71

$$\frac{\sqrt{-a^{2} c}(4 a x \log (a x+1)-3 a x \log (x)-1)}{a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.727 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=152

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^
2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Rubi [A] time = 0.23877, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^
2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A] time = 0.0352233, size = 68, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(2*x^2) + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 + a
*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.131, size = 77, normalized size = 0.5

$$\frac{\left(8 a^{2} \ln \left(x\right) x^{2}-8 \ln \left(a x+1\right) a^{2} x^{2}+6 a x-1\right) \left(a x+1\right)}{2 x^{2} \left(a x-1\right)^{2}} \sqrt{-c \left(a^{2} x^{2}-1\right)} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/2*(8*a^2*ln(x)*x^2-8*ln(a*x+1)*a^2*x^2+6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a
*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2}+c} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A] time = 1.639, size = 197, normalized size = 1.3

$$\frac{8 a^{3} \sqrt{-c} x^{2} \log \left(\frac{2 a^{3} c x^{2}+2 a^{2} c x+\sqrt{-a^{2} c}(2 a x+1) \sqrt{-c}+a c}{a x^{2}+x}\right)+\sqrt{-a^{2} c}(6 a x-1)}{2 a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
+ 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + sqrt(-a^2*c)*(6*a*x - 1))/(a*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.728 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=193

$$-\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^
2])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(
a^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*
x) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.245103, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$-\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^
2])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(
a^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*
x) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^2\sqrt{c-a^2 \cdots}}{\sqrt{1 \cdots}}$$

Mathematica [A] time = 0.0422018, size = 78, normalized size = 0.4

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(ax+1)+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(3*x^3) + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x]
 + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.132, size = 85, normalized size = 0.4

$$\frac{\left(24\,a^{3}x^{3}\ln\left(ax+1\right)-24\,a^{3}\ln\left(x\right)x^{3}-24\,a^{2}x^{2}+9\,ax-2\right)\left(ax+1\right)}{6\,x^{3}\left(ax-1\right)^{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] 1/6*(24*a^3*x^3*ln(a*x+1)-24*a^3*ln(x)*x^3-24*a^2*x^2+9*a*x-2)*(-c*(a^2*x^2
-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^{2}cx^{2}+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A] time = 1.67532, size = 216, normalized size = 1.12

$$\frac{24\,a^{4}\sqrt{-c}x^{3}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx-\sqrt{-a^{2}c}(2\,ax+1)\sqrt{-c}+ac}{ax^{2}+x}\right)-\left(24\,a^{2}x^{2}-9\,ax+2\right)\sqrt{-a^{2}c}}{6\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(-a^2*c))
```

```
/(a*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.729 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=227

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/
(Sqrt[1 - 1/(a^2*x^2)]*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^
2)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a^3
*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^3*Sqrt[c - a^
2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.250525, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```


```
[Out] -Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/
(Sqrt[1 - 1/(a^2*x^2)]*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^
2)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a^3
*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^3*Sqrt[c - a^
2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^3\sqrt{c-a^2cx^2}\log(}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] time = 0.0517898, size = 83, normalized size = 0.37

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{2a^2}{x^2}+\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(ax+1)+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/(4*x^4) + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A] time = 0.133, size = 93, normalized size = 0.4

$$\frac{\left(16 a^{4} \ln \left(x \right) x^{4}-16 \ln \left(a x +1\right) a^{4} x^{4}+16 x^{3} a^{3}-8 a^{2} x^{2}+4 a x -1\right) \left(a x +1\right)}{4 x^{4} \left(a x -1\right)^{2}}\sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] 1/4*(16*a^4*ln(x)*x^4-16*ln(a*x+1)*a^4*x^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)*(-
c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2} c x^{2} + c} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{2}}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A] time = 1.55909, size = 232, normalized size = 1.02

$$\frac{16 a^{5} \sqrt{-c} x^{4} \log \left(\frac{2 a^{3} c x^{2} + 2 a^{2} c x + \sqrt{-a^{2} c} (2 a x + 1) \sqrt{-c} + a c}{a x^{2} + x}\right) + \left(16 a^{3} x^{3} - 8 a^{2} x^{2} + 4 a x - 1\right) \sqrt{-a^{2} c}}{4 a x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*s
```

```
qrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.730 $\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$-\frac{4x^m\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) - (4*x^m*Sqrt[c - a^2*
c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(a*(1 + m)*Sqrt[1 - 1/(a^2*
x^2)])
```

Rubi [A] time = 0.242837, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6192, 6193, 88, 64}

$$-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) - (4*x^m*Sqrt[c - a^2*
c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(a*(1 + m)*Sqrt[1 - 1/(a^2*
x^2)])
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(3x^m+ax^{1+m}+\frac{4x^m}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\
&= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0509021, size = 74, normalized size = 0.54

$$\frac{x^m\sqrt{c-a^2cx^2}(-4(m+2)\text{Hypergeometric2F1}(1,m+1,m+2,ax)+m(ax+3)+ax+6)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, a*x]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int x^{m}\sqrt{-a^{2}c\,x^{2}+c}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}\,x^{m}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^{2}cx^{2}+c}\left(a^{2}x^{2}+2\,ax+1\right)x^{m}\sqrt{\frac{ax-1}{ax+1}}}{a^{2}x^{2}-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.731 $\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m\ldots}{2}\right.}{(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.386524, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6151, 1809, 808, 365, 364}

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\int e^{2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^m(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\int \frac{x^m\left(-a^2c(3+2m)-2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - (2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - \frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx}{(2+m)\sqrt{c-a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} - \frac{2acx^{2+m}\sqrt{1-a}}{(2}$$

Mathematica [C] time = 0.209297, size = 129, normalized size = 0.75

$$\frac{x^{m+1}\left(\frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-ax}\sqrt{-c(ax+1)}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;ax,-ax\right)}{\sqrt{ax-1}\sqrt{ax+1}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/2
, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[c - a^2*c*x^2
]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]
))/(1 + m)
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int \frac{(ax+1)\,x^m}{ax-1}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)/(a*x-1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

3.732 $\int e^{\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=82

$$\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*Sq
rt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.212945, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6192, 6193, 43}

$$\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```


```
[Out] (x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*Sq
rt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x^m(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x^m+ax^{1+m}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0330308, size = 56, normalized size = 0.68

$$\frac{x^m\sqrt{c-a^2cx^2}(amx+ax+m+2)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*(2 + m + a*x + a*m*x)*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*(2 + m)*Sqrt[1 -
 1/(a^2*x^2)])
```

Maple [A] time = 0.039, size = 62, normalized size = 0.8

$$\frac{x^{1+m}\left(amx+ax+m+2\right)}{\left(ax+1\right)\left(2+m\right)\left(1+m\right)}\sqrt{-a^2cx^2+c}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)*(-a^2*c*x^2+c)^(1/2)/(2+m)/(1+m)/(a*x+1)/((a*x-1)/(
a*x+1))^(1/2)
```

Maxima [A] time = 1.08742, size = 73, normalized size = 0.89

$$\frac{\left(a\sqrt{-c}(m+1)x^2+\sqrt{-c}(m+2)x\right)(ax+1)x^m}{\left(m^2+3\,m+2\right)ax+m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 + sqrt(-c)*(m + 2)*x)*(a*x + 1)*x^m/((m^2 + 3*m + 2
)*a*x + m^2 + 3*m + 2)
```

Fricas [A] time = 1.96989, size = 166, normalized size = 2.02

$$-\frac{\sqrt{-a^2cx^2+c}\left((am+a)x^2+(m+2)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-\left(am^2+3\,am+2\,a\right)x+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 + (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.733 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=83

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -((x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)])) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.226649, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -((x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)])) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int x^m(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(-x^m+ax^{1+m}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x}\\ &= -\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0338995, size = 58, normalized size = 0.7

$$\frac{x^m\sqrt{c-a^2cx^2}(m(ax-1)+ax-2)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-2 + a*x + m*(-1 + a*x)))/(a*(1 + m)*(2 + m)*Sqrt
[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.04, size = 64, normalized size = 0.8

$$\frac{x^{1+m}\left(amx+ax-m-2\right)}{\left(2+m\right)\left(1+m\right)\left(ax-1\right)}\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x-m-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(2+m)/
(1+m)/(a*x-1)
```

Maxima [A] time = 1.15089, size = 77, normalized size = 0.93

$$\frac{\left(a\sqrt{-c}(m+1)x^2-\sqrt{-c}(m+2)x\right)(ax-1)x^m}{\left(m^2+3\,m+2\right)ax-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 - sqrt(-c)*(m + 2)*x)*(a*x - 1)*x^m/((m^2 + 3*m + 2
)*a*x - m^2 - 3*m - 2)
```

Fricas [A] time = 2.10874, size = 166, normalized size = 2.

$$-\frac{\sqrt{-a^2cx^2+c}\left((am+a)x^2-(m+2)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-\left(am^2+3\,am+2\,a\right)x+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 - (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+cx^m}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.734 $\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2}\right.}{(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.377894, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6152, 1809, 808, 365, 364}

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\int \frac{x^m\left(-a^2c(3+2m)+2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + (2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - \frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx}{(2+m)\sqrt{c-a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} + \frac{2acx^{2+m}\sqrt{1-a}}{(2+}$$

Mathematica [C] time = 0.148672, size = 110, normalized size = 0.64

$$\frac{x^{m+1}\left(\frac{\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}} - \frac{2\sqrt{c-acx}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-ax,ax\right)}{\sqrt{1-ax}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*((-2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x), a
*x])/Sqrt[1 - a*x] + (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2
, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int \frac{x^m(ax-1)}{ax+1}\sqrt{-a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2cx^2 + c}(ax - 1)x^m}{ax + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

3.735 $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$\frac{4x^m\sqrt{c-a^2cx^2}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^m*Sqrt[c - a^2
*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(a*(1 + m)*Sqrt[1 - 1/(
a^2*x^2)])
```

Rubi [A] time = 0.243495, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6192, 6193, 88, 64}

$$\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^m*Sqrt[c - a^2
*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(a*(1 + m)*Sqrt[1 - 1/(
a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m+ax^{1+m}+\frac{4x^m}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}x} \\
&= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;-ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.051405, size = 75, normalized size = 0.55

$$\frac{x^m\sqrt{c-a^2cx^2}(4(m+2)\text{Hypergeometric2F1}(1,m+1,m+2,-ax)+m(ax-3)+ax-6)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hypergeometri
c2F1[1, 1 + m, 2 + m, -(a*x)]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int x^{m}\sqrt{-a^{2}c x^{2}+c}\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^{2}cx^{2}+c}\,x^{m}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^{2}cx^{2}+c}(ax-1)x^{m}\sqrt{\frac{ax-1}{ax+1}}}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2 + c}\,x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.736 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=81

$$-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\text{Hypergeometric2F1}\left(8,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] (-256*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[8, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rubi [A] time = 0.135166, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6191, 6195, 131}

$$-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```


```
[Out] (-256*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[8, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```


Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{3+\frac{n}{2}}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Mathematica [B] time = 2.28098, size = 267, normalized size = 3.3

$$-\frac{c^3e^{n\coth^{-1}(ax)}\left(n\left(n^5-2n^4-52n^3+104n^2+576n-1152\right)e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -(c^3*E^(n*ArcCoth[a*x])*(-912*n + 58*n^3 - n^5 - 5040*a*x + 912*a*n^2*x -
58*a*n^4*x + a*n^6*x + 1368*a^2*n*x^2 - 64*a^2*n^3*x^2 + a^2*n^5*x^2 + 5040
*a^3*x^3 - 152*a^3*n^2*x^3 + 2*a^3*n^4*x^3 - 576*a^4*n*x^4 + 6*a^4*n^3*x^4
- 3024*a^5*x^5 + 24*a^5*n^2*x^5 + 120*a^6*n*x^6 + 720*a^7*x^7 + E^(2*ArcCot
h[a*x])*n*(-1152 + 576*n + 104*n^2 - 52*n^3 - 2*n^4 + n^5)*Hypergeometric2F
1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (-2304 + 784*n^2 - 56*n^4 + n^
6)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/(5040*a)
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2cx^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a^2cx^2 - c\right)^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^6c^3x^6 - 3\,a^4c^3x^4 + 3\,a^2c^3x^2 - c^3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x - 1)/(a
*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c^3\left(\int 3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int -3a^4x^4e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int a^6x^6e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int -e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-3*a**4*x**4*e
xp(n*acoth(a*x)), x) + Integral(a**6*x**6*exp(n*acoth(a*x)), x) + Integral(
-exp(n*acoth(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2cx^2-c\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.737 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=81

$$\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\text{Hypergeometric2F1}\left(6,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] (64*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[6, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rubi [A] time = 0.14436, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6191, 6195, 131}

$$\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (64*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[6, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= \left(a^4c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2 x^4\,dx\\ &= -\left(\left(a^4c^2\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{2+\frac{n}{2}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}\end{aligned}$$

Mathematica [B] time = 1.30693, size = 179, normalized size = 2.21

$$\frac{c^2e^{n\coth^{-1}(ax)}\left(n\left(n^3-2n^2-16n+32\right)e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+\left(n^4-20n^2+64\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(22*n - n^3 + 120*a*x - 22*a*n^2*x + a*n^4*x - 28*a
^2*n*x^2 + a^2*n^3*x^2 - 80*a^3*x^3 + 2*a^3*n^2*x^3 + 6*a^4*n*x^4 + 24*a^5*
x^5 + E^(2*ArcCoth[a*x])*n*(32 - 16*n - 2*n^2 + n^3)*Hypergeometric2F1[1, 1
 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (64 - 20*n^2 + n^4)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/(120*a)
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2cx^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2-c\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x - 1)/(a*x + 1))^(1/2*n),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2a^2x^2e^{n\,\text{acoth}\,(ax)}\,dx+\int a^4x^4e^{n\,\text{acoth}\,(ax)}\,dx+\int e^{n\,\text{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(a**4*x**4*exp(
n*acoth(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 - c\right)^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.738 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=79

$$-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(4,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] (-16*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[4, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rubi [A] time = 0.0925007, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6191, 6195, 131}

$$-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-16*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[4, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\left(\left(a^2c\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$=\left(a^2c\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^4}\,dx,x,\frac{1}{x}\right)$$
$$=-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Mathematica [A] time = 0.704379, size = 111, normalized size = 1.41

$$-\frac{ce^{n\coth^{-1}(ax)}\left(\left(n^2-4\right)\mathrm{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1,e^{2\coth^{-1}(ax)}\right)+(n-2)ne^{2\coth^{-1}(ax)}\mathrm{Hypergeometric2F1}\left(1,\frac{n}{2}\right.}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*E^(n*ArcCoth[a*x])*(-n - 6*a*x + a*n^2*x + a^2*n*x^2 + 2*a^3*x^3 + E^(2
*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCo
th[a*x])] + (-4 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)]))/(6*a)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\left(-a^2cx^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a^2cx^2 - c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2cx^2 - c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int -e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)),
 x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2cx^2-c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.739 $\int e^{n \coth^{-1}(ax)}\,dx$

Optimal. Leaf size=78

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rubi [A] time = 0.0229457, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6170, 131}

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\ {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Mathematica [A] time = 0.175268, size = 82, normalized size = 1.05

$$\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+(n+2)\left(\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1\right.\right.\right.}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x)),x)
```

```
[Out] int(exp(n*arccoth(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x)),x)
```

```
[Out] Integral(exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.740 $\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

```
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Rubi [A] time = 0.0345959, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{n\coth^{-1}(ax)}}{acn}$$

Mathematica [A] time = 0.0493782, size = 18, normalized size = 1.

$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Maple [A] time = 0.04, size = 18, normalized size = 1.

$$\frac{e^{n \operatorname{arccoth}(ax)}}{can}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x)
```

```
[Out] exp(n*arccoth(a*x))/a/c/n
```

Maxima [A] time = 1.09939, size = 42, normalized size = 2.33

$$-\frac{e^{\left(-\frac{1}{2} n \log(ax+1)+\frac{1}{2} n \log(ax-1)\right)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -e^(-1/2*n*log(a*x + 1) + 1/2*n*log(a*x - 1))/(a*c*n)
```

Fricas [A] time = 1.67133, size = 54, normalized size = 3.

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*n)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2cx^2 - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.741 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx$

Optimal. Leaf size=72

$$\frac{2e^{n \coth^{-1}(ax)}}{ac^2n(4-n^2)} - \frac{(n-2ax)e^{n \coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

```
[Out] (2*E^(n*ArcCoth[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rubi [A] time = 0.0778262, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{2e^{n \coth^{-1}(ax)}}{ac^2n(4-n^2)} - \frac{(n-2ax)e^{n \coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^(n*ArcCoth[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)} + \frac{2\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c\left(4-n^2\right)}$$
$$= \frac{2e^{n\coth^{-1}(ax)}}{ac^2n\left(4-n^2\right)} - \frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.161797, size = 55, normalized size = 0.76

$$-\frac{\left(2a^2x^2-2anx+n^2-2\right)e^{n\coth^{-1}(ax)}}{ac^2n\left(n^2-4\right)\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(-2 + n^2 - 2*a*n*x + 2*a^2*x^2))/(a*c^2*n*(-4 + n^2)
*(-1 + a^2*x^2)))
```

Maple [A] time = 0.043, size = 55, normalized size = 0.8

$$-\frac{e^{n\operatorname{arccoth}(ax)}\left(2\,a^2x^2-2\,nax+n^2-2\right)}{\left(a^2x^2-1\right)c^2an\left(n^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -exp(n*arccoth(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [A] time = 1.73817, size = 165, normalized size = 2.29

$$-\frac{\left(2\,a^2x^2+2\,anx+n^2-2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^2n^3-4\,ac^2n-\left(a^3c^2n^3-4\,a^3c^2n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*x^2 + 2*a*n*x + n^2 - 2)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n^3 -
 4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.742 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3} dx$

Optimal. Leaf size=127

$$-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\coth^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

```
[Out] (24*E^(n*ArcCoth[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcCoth[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcCoth[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rubi [A] time = 0.129511, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\coth^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```


```
[Out] (24*E^(n*ArcCoth[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcCoth[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcCoth[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```


Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}+\frac{12\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{c\left(16-n^2\right)}$$

$$=-\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^2\left(64-20n^2+n^4\right)}$$

$$=\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(64-20n^2+n^4\right)}-\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.216089, size = 97, normalized size = 0.76

$$\frac{\left(4n^2\left(3a^2x^2-4\right)-8anx\left(3a^2x^2-5\right)+24\left(a^2x^2-1\right)^2-4an^3x+n^4\right)e^{n\coth^{-1}(ax)}}{ac^3n\left(n^2-16\right)\left(n^2-4\right)\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*x^2)^2 - 8*a*n*x*(-5 +
3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*n*(-16 + n^2)*(-4 + n^2)*(-1 +
 a^2*x^2)^2)
```

Maple [A] time = 0.043, size = 101, normalized size = 0.8

$$\frac{\left(24x^4a^4-24x^3a^3n+12a^2n^2x^2-4an^3x-48a^2x^2+n^4+40nax-16n^2+24\right){\rm e}^{n{\rm arccoth}(ax)}}{\left(a^2x^2-1\right)^2c^3a\left(n^2-16\right)\left(n^2-4\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x)
```

```
[Out] (24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-1
6*n^2+24)*exp(n*arccoth(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

Fricas [A] time = 1.66716, size = 373, normalized size = 2.94

$$-\frac{\left(24\,a^4x^4+24\,a^3nx^3+n^4+12\left(a^2n^2-4\,a^2\right)x^2-16\,n^2+4\left(an^3-10\,an\right)x+24\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+\left(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n\right)x^4-2\left(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(24*a^4*x^4 + 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 + 4*(
a*n^3 - 10*a*n)*x + 24)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^3
*n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2*(
a^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

3.743 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4} \, dx$

Optimal. Leaf size=197

$$-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\coth^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\coth^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{}{ac^4n\left(36\right.}$$

```
[Out] (720*E^(n*ArcCoth[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcCoth[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Coth[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcCoth[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rubi [A] time = 0.180955, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\coth^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\coth^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{}{ac^4n\left(36\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcCoth[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcCoth[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Coth[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcCoth[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}+\frac{30\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{c\left(36-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{360\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{c^2\left(576-52n^2+n^4\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}-\frac{360e^{n\coth^{-1}(ax)}(n-2a}{ac^4\left(4-n^2\right)\left(576-52n^2+n^4\right)}$$

$$= \frac{720e^{n\coth^{-1}(ax)}}{ac^4n\left(4-n^2\right)\left(576-52n^2+n^4\right)}-\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}$$

Mathematica [A] time = 0.300616, size = 152, normalized size = 0.77

$$-\frac{\left(10n^4\left(3a^2x^2-5\right)-120an^3x\left(a^2x^2-2\right)+8n^2\left(45a^4x^4-105a^2x^2+68\right)-48anx\left(15a^4x^4-40a^2x^2+33\right)+720\left(a^2x^2\right.\right.}{ac^4n\left(n^2-36\right)\left(n^2-16\right)\left(n^2-4\right)\left(a^2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n^6 - 6*a*n^5*x - 120*a*n^3*x*(-2 + a^2*x^2) + 720*(
-1 + a^2*x^2)^3 + 10*n^4*(-5 + 3*a^2*x^2) - 48*a*n*x*(33 - 40*a^2*x^2 + 15*
a^4*x^4) + 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a*c^4*n*(-36 + n^2)*(-1
6 + n^2)*(-4 + n^2)*(-1 + a^2*x^2)^3))
```

Maple [A] time = 0.046, size = 167, normalized size = 0.9

$$-\frac{\left(720\,x^6a^6-720\,x^5a^5n+360\,a^4n^2x^4-120\,a^3n^3x^3-2160\,x^4a^4+30\,a^2n^4x^2+1920\,x^3a^3n-6\,an^5x-840\,a^2n^2x^2+n^6+\right.}{\left(a^2x^2-1\right)^3c^4an\left(n^6-56\,n^4+784\,n^2-2304\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*
a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arccoth(a*x))/(a^2*x^2-1)^3/c^
4/a/n/(n^6-56*n^4+784*n^2-2304)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

Fricas [A] time = 1.78191, size = 683, normalized size = 3.47

$$-\frac{\left(720\,a^6x^6+720\,a^5nx^5+n^6+360\left(a^4n^2-6\,a^4\right)x^4-50\,n^4+120\left(a^3n^3-16\,a^3n\right)x^3+30\left(a^2n^4-28\,a^2n\right.\right.}{ac^4n^7-56\,ac^4n^5+784\,ac^4n^3-\left(a^7c^4n^7-56\,a^7c^4n^5+784\,a^7c^4n^3-2304\,a^7c^4n\right)x^6-2304\,ac^4n+3\left(a^5c^4n^7-56\,a^5c^4n^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -(720*a^6*x^6 + 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 +
120*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 544
```

```
*n^2 + 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x - 1)/(a*x + 1))^(1/2*n
)/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*n^5
 + 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^7 -
56*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^7 - 5
6*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

3.744 $\int e^{n \coth^{-1}(ax)} \left(c-a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=116

$$\frac{32 \left(c-a^2cx^2\right)^{3/2} \left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}} \text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] (32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(3
/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.213657, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6195, 131}

$$\frac{32 \left(c-a^2cx^2\right)^{3/2} \left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}} {}_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] (32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(3
/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
```

```
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

$$=-\frac{\left(c-a^2cx^2\right)^{3/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+\frac{n}{2}}}{x^5}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

$$=\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}\left(c-a^2cx^2\right)^{3/2}{}_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [B] time = 2.22317, size = 280, normalized size = 2.41

$$\frac{c^2\left(96a^3cx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2(n-1)e^{(n+1)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)+ax\sqrt{1-\frac{1}{a^2x^2}}(ax+n)e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c^2*(96*a^3*c*(1 - 1/(a^2*x^2))^(3/2)*x^3*(a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1
/(a^2*x^2)]*x*(n + a*x) + 2*E^((1 + n)*ArcCoth[a*x])*(-1 + n)*Hypergeometri
```

```
c2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]) - c*(-1 + a^2*x^2)*(2*E^
(n*ArcCoth[a*x])*(-1 + a^2*x^2)^2*(-(a*(-21 + n^2)*x) + 2*n*(1 - n^2 + (3 +
 n^2)*Cosh[2*ArcCoth[a*x]]) + a*(3 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*Ar
cCoth[a*x]]) + 16*a*E^((1 + n)*ArcCoth[a*x])*(-3 + 3*n - n^2 + n^3)*Sqrt[1
- 1/(a^2*x^2)]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*
x])])))/(192*a*(c - a^2*c*x^2)^(3/2))
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)} \left(-a^2 c x^2 + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2 c x^2 + c\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2 c x^2 - c\right)\sqrt{-a^2 c x^2 + c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.745 $\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=116

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a^
2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] time = 0.163098, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6195, 131}

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a^
2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
```

```
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= -\frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^3}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

$$= \frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\,{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}x}$$

Mathematica [A] time = 0.553351, size = 101, normalized size = 0.87

$$-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}e^{n\coth^{-1}(ax)}\left(2(n-1)e^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)+ax\sqrt{1-\frac{1}{a^2x^2}}(ax+n)\right)}{2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(c*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(
n + a*x) + 2*E^ArcCoth[a*x]*(-1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n
)/2, E^(2*ArcCoth[a*x])]))/(2*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.31, size = 0, normalized size = 0.

$$\int e^{n \operatorname{arccoth}(ax)} \sqrt{-a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^{2} c x^{2} + c} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\sqrt{-a^{2} c x^{2} + c} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{2} \, n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\,(ax-1)\,(ax+1)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.746 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=111

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/
2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])
/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.19688, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6195, 131}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/
2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])
/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
```

```
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-a^2cx^2}}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.176923, size = 81, normalized size = 0.73

$$-\frac{2\sqrt{c-a^2cx^2}\,e^{(n+1)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a^2cnx+a^2cx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*E^((1 + n)*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(Sqrt[1 - 1/(a^2*x^2)]*(a^2*c*x + a^2
*c*n*x))
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}\frac{1}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.747 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rubi [A] time = 0.0539141, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6184}

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.182099, size = 43, normalized size = 0.93

$$\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.041, size = 49, normalized size = 1.1

$$\frac{(ax-n)(ax-1)(ax+1)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(n^2-1\right)a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.60303, size = 153, normalized size = 3.33

$$-\frac{\sqrt{-a^2cx^2+c}(ax+n)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-\left(a^3c^2n^2-a^3c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x + n)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.748 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=102

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rubi [A] time = 0.112237, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
```

```
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c\left(9-n^2\right)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.581169, size = 110, normalized size = 1.08

$$\frac{e^{n\coth^{-1}(ax)}\left(3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)-3an^2x-2\left(n^2-1\right)n\cosh\left(2\coth^{-1}(ax)\right)+27ax+2n^3-26n\right)}{4ac^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 84, normalized size = 0.8

$$\frac{\left(6x^3a^3-6na^2x^2+3an^2x-n^3-9ax+7n\right)(ax-1)(ax+1)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a\left(n^4-10n^2+9\right)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.56895, size = 335, normalized size = 3.28

$$-\frac{\left(6\,a^3x^3+6\,a^2nx^2+n^3+3\left(an^2-3\,a\right)x-7\,n\right)\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^3n^4-10\,ac^3n^2+\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^4+9\,ac^3-2\left(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 + 6*a^2*n*x^2 + n^3 + 3*(a*n^2 - 3*a)*x - 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.749 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{120(n-ax)e^{n\coth^{-1}(ax)}}{ac^3\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\coth^{-1}(ax)}}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.176648, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{120(n-ax)e^{n\coth^{-1}(ax)}}{ac^3\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\coth^{-1}(ax)}}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}+\frac{20\int\frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{c\left(25-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{120\int\frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c^2\left(9-n^2\right)\left(25-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{120e^{n\coth^{-1}(ax)}(n-}{ac^3\left(1-n^2\right)\left(9-n^2\right)\left(25-n\right.}$$

Mathematica [A] time = 1.46125, size = 299, normalized size = 1.8

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}e^{n\coth^{-1}(ax)}\left(\frac{10n^5}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{340n^3}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{10\left(n^4-34n^2+225\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2250n}{ax\sqrt{1-\frac{1}{a^2x^2}}}-5n^4\cosh\left(5\coth^{-1}(ax)\right)+50n^2\,\mathrm{co}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(a^2*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))^(3/2)*x^3*((-10*(225 - 34*n^2 +
n^4))/Sqrt[1 - 1/(a^2*x^2)] + (2250*n)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (340*n
^3)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (10*n^5)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + 15
*(25 - 26*n^2 + n^4)*Cosh[3*ArcCoth[a*x]] - 45*Cosh[5*ArcCoth[a*x]] + 50*n^
2*Cosh[5*ArcCoth[a*x]] - 5*n^4*Cosh[5*ArcCoth[a*x]] - 125*n*Sinh[3*ArcCoth[
a*x]] + 130*n^3*Sinh[3*ArcCoth[a*x]] - 5*n^5*Sinh[3*ArcCoth[a*x]] + 9*n*Sin
h[5*ArcCoth[a*x]] - 10*n^3*Sinh[5*ArcCoth[a*x]] + n^5*Sinh[5*ArcCoth[a*x]])
)/(16*c^2*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*(c - a^2*c*x^2
)^(3/2))
```

Maple [A] time = 0.042, size = 140, normalized size = 0.8

$$\frac{\left(120\,x^5a^5-120\,na^4x^4+60\,a^3n^2x^3-20\,a^2n^3x^2-300\,x^3a^3+5\,an^4x+260\,na^2x^2-n^5-110\,an^2x+30\,n^3+225\,ax-14}{a\left(n^6-35\,n^4+259\,n^2-225\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-30
0*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*exp
(n*arccoth(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A] time = 1.69961, size = 622, normalized size = 3.75

$$-\frac{\left(120\,a^5x^5+120\,a^4nx^4+n^5+60\left(a^3n^2-5\,a^3\right)x^3-30\,n^3+20\left(a^2n^3-13\,a^2n\right)x^2+5\left(an^4-\right.}{ac^4n^6-35\,ac^4n^4+259\,ac^4n^2-\left(a^7c^4n^6-35\,a^7c^4n^4+259\,a^7c^4n^2-225\,a^7c^4\right)x^6-225\,ac^4+3\left(a^5c^4n^6-35\,a^5c^4n^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -(120*a^5*x^5 + 120*a^4*n*x^4 + n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 - 30*n^3 + 2
0*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x + 149*n)*sqrt(-a
```

```
^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 + 259
*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*c^4)
*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 - 225*
a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 225*a^3*
c^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.750 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{5040(n-ax)e^{n\coth^{-1}(ax)}}{ac^4\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)\sqrt{c-a^2cx^2}}-\frac{840(n-3ax)e^{n\coth^{-1}(ax)}}{ac^3\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{42(n-\cdots}{ac^2\left(25-n^2\right)\left(\cdots\right.}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
- (42*E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c^2*(25 - n^2)*(49 - n^2)*(c - a^2
*c*x^2)^(5/2)) - (840*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^3*(9 - n^2)*(25
- n^2)*(49 - n^2)*(c - a^2*c*x^2)^(3/2)) - (5040*E^(n*ArcCoth[a*x])*(n - a*
x))/(a*c^4*(1 - n^2)*(9 - n^2)*(25 - n^2)*(49 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.25439, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{5040(n-ax)e^{n\coth^{-1}(ax)}}{ac^4\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)\sqrt{c-a^2cx^2}}-\frac{840(n-3ax)e^{n\coth^{-1}(ax)}}{ac^3\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{42(n-\cdots}{ac^2\left(25-n^2\right)\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
- (42*E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c^2*(25 - n^2)*(49 - n^2)*(c - a^2
*c*x^2)^(5/2)) - (840*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^3*(9 - n^2)*(25
- n^2)*(49 - n^2)*(c - a^2*c*x^2)^(3/2)) - (5040*E^(n*ArcCoth[a*x])*(n - a*
x))/(a*c^4*(1 - n^2)*(9 - n^2)*(25 - n^2)*(49 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}+\frac{42\int\frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx}{c\left(49-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}-\frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{5/2}}+\frac{840\int\frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{c^2\left(25-n^2\right)\left(49-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}-\frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{840e^{n\coth^{-1}(ax)}(\cdots}{ac^3\left(9-n^2\right)\left(25-n^2\right)\left(49-\cdots\right.}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}-\frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{840e^{n\coth^{-1}(ax)}(\cdots}{ac^3\left(9-n^2\right)\left(25-n^2\right)\left(49-\cdots\right.}$$

Mathematica [A] time = 1.51471, size = 260, normalized size = 1.09

$$\frac{ax^2\left(1-\frac{1}{a^2x^2}\right)e^{n\coth^{-1}(ax)}\left(-\frac{63ax\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)}{n^2-9}+\frac{35ax\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(5\coth^{-1}(ax)\right)}{n^2-25}-\frac{7ax\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(7\coth^{-1}(ax)\right)}{n^2-49}+\frac{21anx\cdots}{\cdots}\right.}{64c^3\left(c-a^2cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))*x^2*((-35*n)/(-1 + n^2) + (35*a*x)/
(-1 + n^2) - (63*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]])/(-9 + n^2)
 + (35*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[5*ArcCoth[a*x]])/(-25 + n^2) - (7*a*S
qrt[1 - 1/(a^2*x^2)]*x*Cosh[7*ArcCoth[a*x]])/(-49 + n^2) + (21*a*n*Sqrt[1 -
 1/(a^2*x^2)]*x*Sinh[3*ArcCoth[a*x]])/(-9 + n^2) - (7*a*n*Sqrt[1 - 1/(a^2*x
^2)]*x*Sinh[5*ArcCoth[a*x]])/(-25 + n^2) + (a*n*Sqrt[1 - 1/(a^2*x^2)]*x*Sin
```

```
h[7*ArcCoth[a*x]])/(-49 + n^2)))/(64*c^3*(c - a^2*c*x^2)^(3/2))
```

Maple [A] time = 0.045, size = 218, normalized size = 0.9

$$\frac{\left(5040\,a^{7}x^{7}-5040\,na^{6}x^{6}+2520\,a^{5}n^{2}x^{5}-840\,a^{4}n^{3}x^{4}-17640\,x^{5}a^{5}+210\,a^{3}n^{4}x^{3}+15960\,a^{4}nx^{4}-42\,a^{2}n^{5}x^{2}-7140\,a^{3}n\right.}{a\left(n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(5040*a^7*x^7-5040*a^6*n*x^6+2520*a^5*n^2*x^5-840*a^4*n^3*x
^4-17640*a^5*x^5+210*a^3*n^4*x^3+15960*a^4*n*x^4-42*a^2*n^5*x^2-7140*a^3*n^
2*x^3+7*a*n^6*x+2100*a^2*n^3*x^2-n^7+22050*a^3*x^3-455*a*n^4*x-17178*a^2*n*
x^2+77*n^5+6433*a*n^2*x-1519*n^3-11025*a*x+6483*n)*exp(n*arccoth(a*x))/a/(n
^8-84*n^6+1974*n^4-12916*n^2+11025)/(-a^2*c*x^2+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

Fricas [A] time = 1.6778, size = 1030, normalized size = 4.31

$$-\frac{\left(5040\,a^{7}x^{7}+5040\,a^{6}nx^{6}+n^{7}+2520\left(a^{5}n^{2}-7\,a^{5}\right)x^{5}-77\,n^{5}+840\right.}{ac^{5}n^{8}-84\,ac^{5}n^{6}+1974\,ac^{5}n^{4}+\left(a^{9}c^{5}n^{8}-84\,a^{9}c^{5}n^{6}+1974\,a^{9}c^{5}n^{4}-12916\,a^{9}c^{5}n^{2}+11025\,a^{9}c^{5}\right)x^{8}-12916\,ac^{5}n^{2}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -(5040*a^7*x^7 + 5040*a^6*n*x^6 + n^7 + 2520*(a^5*n^2 - 7*a^5)*x^5 - 77*n^5
 + 840*(a^4*n^3 - 19*a^4*n)*x^4 + 210*(a^3*n^4 - 34*a^3*n^2 + 105*a^3)*x^3
+ 1519*n^3 + 42*(a^2*n^5 - 50*a^2*n^3 + 409*a^2*n)*x^2 + 7*(a*n^6 - 65*a*n^
4 + 919*a*n^2 - 1575*a)*x - 6483*n)*sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x +
1))^(1/2*n)/(a*c^5*n^8 - 84*a*c^5*n^6 + 1974*a*c^5*n^4 + (a^9*c^5*n^8 - 84*
a^9*c^5*n^6 + 1974*a^9*c^5*n^4 - 12916*a^9*c^5*n^2 + 11025*a^9*c^5)*x^8 - 1
2916*a*c^5*n^2 - 4*(a^7*c^5*n^8 - 84*a^7*c^5*n^6 + 1974*a^7*c^5*n^4 - 12916
*a^7*c^5*n^2 + 11025*a^7*c^5)*x^6 + 11025*a*c^5 + 6*(a^5*c^5*n^8 - 84*a^5*c
^5*n^6 + 1974*a^5*c^5*n^4 - 12916*a^5*c^5*n^2 + 11025*a^5*c^5)*x^4 - 4*(a^3
*c^5*n^8 - 84*a^3*c^5*n^6 + 1974*a^3*c^5*n^4 - 12916*a^3*c^5*n^2 + 11025*a^
3*c^5)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

3.751 $\int \frac{e^{n \coth^{-1}(ax)} x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=359

$$-\frac{2nx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(n^2+2n+2\right)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}\right.}{a(1-n)(n+1)\left(c-\right.}$$

```
[Out] -(((2 + n)*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3)/(a*(1 + n)*(c - a^2*c*x^2)^(3/2))) + ((2 + 2*n + n^2)*(1
 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*
x^3)/(a*(1 - n)*(1 + n)*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*(
1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^4)/(c - a^2*c*x^2)^(
3/2) - (2*n*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (a + x^(-1))/
(a - x^(-1))])/(a*(1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.332594, antiderivative size = 363, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6192, 6194, 129, 155, 12, 131}

$$\frac{2nx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(3-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(n^2+2n+2\right)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{a(1-n)(n+1)\left(c-a^2cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(((2 + n)*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3)/(a*(1 + n)*(c - a^2*c*x^2)^(3/2))) + ((2 + 2*n + n^2)*(1
 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*
x^3)/(a*(1 - n)*(1 + n)*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*(
1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^4)/(c - a^2*c*x^2)^(
3/2) + (2*n*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))
^((-3 + n)/2)*x^3*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(
a + x^(-1))])/(a*(3 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) && !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) && !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
```

```
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{2x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\right.}{x}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(}}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{a(1-n)(1+n)\left(c-a^2cx\right.}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{a(1-n)(1+n)\left(c-a^2cx\right.}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{a(1-n)(1+n)\left(c-a^2cx\right.}$$

Mathematica [A] time = 0.552972, size = 133, normalized size = 0.37

$$\frac{\frac{c(anx-1)e^{n\coth^{-1}(ax)}}{n^2-1}-\frac{c\left(a^2x^2-1\right)\left(\frac{2ne^{(n+1)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}+(n+1)e^{n\coth^{-1}(ax)}\right)}{n+1}}{a^4c^2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((c*E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(-1 + n^2) - (c*(-1 + a^2*x^2)*(E^(n*A
rcCoth[a*x])*(1 + n) + (2*E^((1 + n)*ArcCoth[a*x])*n*Hypergeometric2F1[1, (
1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(a*Sqrt[1 - 1/(a^2*x^2)]*x)))/(1
+ n))/(a^4*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.307, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}x^3\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.752 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) - (
2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2
)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.335145, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6189, 6192, 6195, 131}

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c(1-n^2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```


```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) - (
2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2
)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```


Rule 6189

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a
^3*c*(n^2 - 4*(p + 1)^2)), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ
```

```
[n])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}} - \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}} - \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{a^2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}} + \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{a^2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.375046, size = 127, normalized size = 0.77

$$-\frac{e^{n\coth^{-1}(ax)}\left(2(n-1)\left(a^2x^2-1\right)e^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)+ax\sqrt{1-\frac{1}{a^2x^2}}(ax-n)\right)}{a^4c(n-1)(n+1)x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-n + a*x) + 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2
*ArcCoth[a*x])]))/(a^4*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c -
a^2*c*x^2]))
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}x^2\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{n\,\operatorname{acoth}(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.753 $\int \frac{e^{n \coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] (E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0923765, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {6186}

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6186

```
Int[(E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[((1 - a*n*x)*E^(n*ArcCoth[a*x]))/(a^2*c*(n^2 - 1)*Sqrt[c + d*x
^2]), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.170015, size = 43, normalized size = 0.93

$$\frac{(anx-1)e^{n\coth^{-1}(ax)}}{a^2c\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(a^2*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.041, size = 49, normalized size = 1.1

$$-\frac{(nax-1)(ax-1)(ax+1)\,\mathrm{e}^{n\operatorname{arccoth}(ax)}}{a^2\left(n^2-1\right)}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arccoth(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.39743, size = 161, normalized size = 3.5

$$-\frac{\sqrt{-a^2cx^2+c}(anx+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-\left(a^4c^2n^2-a^4c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(a*n*x + 1)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^2*n^
2 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.754 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rubi [A] time = 0.0524908, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6184}

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.177799, size = 43, normalized size = 0.93

$$\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 49, normalized size = 1.1

$$\frac{(ax-n)(ax-1)(ax+1)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{(n^2-1)\,a}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A] time = 1.40242, size = 153, normalized size = 3.33

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax+n)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-\left(a^3c^2n^2-a^3c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x + n)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.755 $\int \frac{e^{n \coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{a^3 2^{\frac{n+1}{2}} x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} \text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-
1 + n)/2)*x^3)/((1 + n)*(c - a^2*c*x^2)^(3/2))) + (a^3*(1 - 1/(a^2*x^2))^(3
/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^3)/((1 - n^2)*(c
 - a^2*c*x^2)^(3/2)) - (2^((1 + n)/2)*a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a
*x))^((1 - n)/2)*x^3*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a
- x^(-1))/(2*a)])/((1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] time = 0.373535, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6192, 6195, 89, 79, 69}

$$-\frac{a^3 2^{\frac{n+1}{2}} x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} {}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)}{(n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```


```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-
1 + n)/2)*x^3)/((1 + n)*(c - a^2*c*x^2)^(3/2))) + (a^3*(1 - 1/(a^2*x^2))^(3
/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^3)/((1 - n^2)*(c
 - a^2*c*x^2)^(3/2)) - (2^((1 + n)/2)*a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a
*x))^((1 - n)/2)*x^3*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a
- x^(-1))/(2*a)])/((1 - n)*(c - a^2*c*x^2)^(3/2))
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\right.}{(1+n)\left(c-a^2c\right.}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.824422, size = 127, normalized size = 0.46

$$\frac{e^{n\coth^{-1}(ax)}\left(ax\sqrt{1-\frac{1}{a^2x^2}}(anx-1)-2(n-1)\left(a^2x^2-1\right)e^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-e^{2\coth^{-1}(ax)}\right)\right)}{ac(n-1)(n+1)x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*n*x) - 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(
2*ArcCoth[a*x])]))/(a*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a
^2*c*x^2])
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{e^{n\text{arccoth}(ax)}}{x}\left(-a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^5-2\,a^2c^2x^3+c^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^5 -
2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

3.756 $\int \frac{e^{n \coth^{-1}(ax)} x^4}{(c-a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=463

$$-\frac{2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(n^2+6n+15\right)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}\right.}{(1-n)(n+1)(n+3)\left(c\right.}$$

```
[Out] -(((1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3 +
n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - ((6 + n)*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) + ((15 + 6*n + n^2)*(1 - 1/(a^2*x^2))^(5/2)*(1
- 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 - n)*(1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) - ((18 + 7*n - 2*n^2 - n^3)*(1 - 1/(a^2*x^2))^(
5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2
 + n^4)*(c - a^2*c*x^2)^(5/2)) - (2*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^(
(1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5*Hypergeometric2F1[1, (-1 + n)/2,
(1 + n)/2, (a + x^(-1))/(a - x^(-1))])/((1 - n)*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.534858, antiderivative size = 467, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6192, 6195, 129, 155, 12, 131}

$$\frac{2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(3-n)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(n^2+6n+15\right)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(1-n)(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3 +
n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - ((6 + n)*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) + ((15 + 6*n + n^2)*(1 - 1/(a^2*x^2))^(5/2)*(1
- 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 - n)*(1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) - ((18 + 7*n - 2*n^2 - n^3)*(1 - 1/(a^2*x^2))^(
5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2
```

```
 + n^4)*(c - a^2*c*x^2)^(5/2)) + (2*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^(
(3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5*Hypergeometric2F1[1, (3 - n)/2, (
5 - n)/2, (a - x^(-1))/(a + x^(-1))])/((3 - n)*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
```

```
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\frac{\left(-\frac{3+n}{a}-\frac{3x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}}{x}\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3}}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3}}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3}}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3}}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$=-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3}}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 1.9755, size = 201, normalized size = 0.43

$$\frac{\left(a^2x^2-1\right)\left(-\frac{8ax\sqrt{1-\frac{1}{a^2x^2}}e^{(n+1)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)}{n+1}+\frac{e^{n\coth^{-1}(ax)}\left(-3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)+3\right.}{n^4-10n}\right.}{4a^5c\left(c-a^2cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((-1 + a^2*x^2)*((8*E^(n*ArcCoth[a*x])*(n - a*x))/(-1 + n^2) + (E^(n*ArcCot
h[a*x])*(26*n - 2*n^3 - 27*a*x + 3*a*n^2*x + 2*n*(-1 + n^2)*Cosh[2*ArcCoth[
a*x]] - 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))/(9 -
10*n^2 + n^4) - (8*a*E^((1 + n)*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(1 + n)))/(4*a^5
*c*(c - a^2*c*x^2)^(3/2))
```

Maple [F] time = 0.303, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}x^4\left(-a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^6*c^3*x
^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.757 $\int \frac{e^{n \coth^{-1}(ax)} x^3}{\left(c-a^2 c x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=330

$$-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{\left(n^2+4n+3\right)\left(c-a^2cx^2\right)^{5/2}}+\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)\left(1-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{n}}{\left(n^4-10n^2+9\right)\left(c-a\right.}$$

```
[Out] -((a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a*(1 - 1/(a^2*x^2))^(5/2
)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n + n^
2)*(c - a^2*c*x^2)^(5/2)) + (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1
- n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^
(5/2)) - (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.366695, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6192, 6195, 45, 37}

$$-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{\left(n^2+4n+3\right)\left(c-a^2cx^2\right)^{5/2}}+\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)\left(1-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{n}}{\left(n^4-10n^2+9\right)\left(c-a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a*(1 - 1/(a^2*x^2))^(5/2
)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n + n^
2)*(c - a^2*c*x^2)^(5/2)) + (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1
- n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^
(5/2)) - (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\operatorname{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\operatorname{Subst}\left(\int \left(1-\frac{x}{a}\right)^{\frac{1}{2}(-3-n)}\left(1\right.\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.654351, size = 110, normalized size = 0.33

$$-\frac{e^{n\coth^{-1}(ax)}\left(an\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)+3\left(an^3x-9anx-2n^2+10\right)-6\left(n^2-1\right)\cosh\left(2\coth^{-1}(ax)\right)\right)}{4a^4c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(E^(n*ArcCoth[a*x])*(3*(10 - 2*n^2 - 9*a*n*x + a*n^3*x) - 6*(-1 + n^2)*Cos
h[2*ArcCoth[a*x]] + a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a
*x]]))/(4*a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 93, normalized size = 0.3

$$-\frac{\left(a^{3} n^{3} x^{3}-7 n x^{3} a^{3}-3 a^{2} n^{2} x^{2}+9 a^{2} x^{2}+6 n a x-6\right) \left(a x-1\right) \left(a x+1\right) {\mathrm e}^{n \,\mathrm{arccoth}\left(a x\right)}}{a^{4} \left(n^{4}-10 n^{2}+9\right)} \left(-a^{2} c \,x^{2}+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-6
)*exp(n*arccoth(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3} \left(\frac{a x-1}{a x+1}\right)^{\frac{1}{2} \, n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.63409, size = 354, normalized size = 1.07

$$-\frac{\sqrt{-a^{2} c x^{2}+c} \left({\left(a^{3} n^{3}-7 \, a^{3} n\right)} x^{3}+6 \, a n x+3 \, {\left(a^{2} n^{2}-3 \, a^{2}\right)} x^{2}+6\right)} \left(\frac{a x-1}{a x+1}\right)^{\frac{1}{2} \, n}}{a^{4} c^{3} n^{4}-10 \, a^{4} c^{3} n^{2}+9 \, a^{4} c^{3}+{\left(a^{8} c^{3} n^{4}-10 \, a^{8} c^{3} n^{2}+9 \, a^{8} c^{3}\right)} x^{4}-2 \, {\left(a^{6} c^{3} n^{4}-10 \, a^{6} c^{3} n^{2}+9 \, a^{6} c^{3}\right)} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x + 3*(a^2*n^2 - 3*a
^2)*x^2 + 6)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 +
```

```
9*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n^4
 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.758 $\int \frac{e^{n \coth^{-1}(ax)} x^2}{\left(c-a^2 c x^2\right)^{5/2}} dx$

Optimal. Leaf size=102

$$\frac{\left(3-n^2\right)(n-ax)e^{n\coth^{-1}(ax)}}{a^3c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```

Rubi [A] time = 0.189252, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6189, 6184}

$$\frac{\left(3-n^2\right)(n-ax)e^{n\coth^{-1}(ax)}}{a^3c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```


```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```


Rule 6189

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a
^3*c*(n^2 - 4*(p + 1)^2)), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(3-n^2\right)\int\frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{a^2c\left(9-n^2\right)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{e^{n\coth^{-1}(ax)}\left(3-n^2\right)(n-ax)}{a^3c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.688619, size = 109, normalized size = 1.07

$$\frac{e^{n\coth^{-1}(ax)}\left(3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)+an^2x-2\left(n^2-1\right)n\cosh\left(2\coth^{-1}(ax)\right)-9ax-2n^3+10n\right)}{4a^3c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(10*n - 2*n^3 - 9*a*x + a*n^2*x - 2*n*(-1 + n^2)*Cosh[2
*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]
]))/(4*a^3*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.043, size = 96, normalized size = 0.9

$$\frac{\left(a^3n^2x^3-a^2n^3x^2-3\,x^3a^3+3\,nx^2a^2+2\,n^2xa-2\,n\right)(ax-1)(ax+1)\,{\rm e}^{n\,{\rm arccoth}(ax)}}{\left(n^4-10\,n^2+9\right)a^3}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2*
n)*exp(n*arccoth(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.3283, size = 356, normalized size = 3.49

$$-\frac{\sqrt{-a^2cx^2+c}\left(2\,an^2x+\left(a^3n^2-3\,a^3\right)x^3+\left(a^2n^3-3\,a^2n\right)x^2+2\,n\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+\left(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3\right)x^4-2\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 + (a^2*n^3 - 3*a^2
*n)*x^2 + 2*n)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^2
+ 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.759 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
(2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2
*c*x^2])
```

Rubi [A] time = 0.147321, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6187, 6184}

$$\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
(2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2
*c*x^2])
```

Rule 6187

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((2*(p + 1) + a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a^2
*c*(n^2 - 4*(p + 1)^2)), x] - Dist[(n*(2*p + 3))/(a*c*(n^2 - 4*(p + 1)^2)),
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
```

```
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} - \frac{(2n)\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{ac\left(9-n^2\right)}$$
$$= \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{2e^{n\coth^{-1}(ax)}n(n-ax)}{a^2c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.586381, size = 108, normalized size = 1.11

$$\frac{e^{n\coth^{-1}(ax)}\left(-a\left(n^2-1\right)nx\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)+an^3x+6\left(n^2-1\right)\cosh\left(2\coth^{-1}(ax)\right)-9anx+2n^2+6\right)}{4a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(6 + 2*n^2 - 9*a*n*x + a*n^3*x + 6*(-1 + n^2)*Cosh[2*Ar
cCoth[a*x]] - a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))
/(4*a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.041, size = 86, normalized size = 0.9

$$-\frac{\left(2a^3nx^3-2n^2x^2a^2+an^3x-3nax-n^2+3\right)(ax-1)(ax+1)\,e^{n\,\mathrm{arccoth}(ax)}}{a^2\left(n^4-10n^2+9\right)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*ar
ccoth(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.45122, size = 346, normalized size = 3.57

$$-\frac{\left(2\,a^3nx^3+2\,a^2n^2x^2+n^2+\left(an^3-3\,an\right)x-3\right)\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+\left(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3\right)x^4-2\left(a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(2*a^3*n*x^3 + 2*a^2*n^2*x^2 + n^2 + (a*n^3 - 3*a*n)*x - 3)*sqrt(-a^2*c*x^
2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2*
c^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 10*
a^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.760 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rubi [A] time = 0.106421, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
```

```
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c\left(9-n^2\right)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.199781, size = 110, normalized size = 1.08

$$\frac{e^{n\coth^{-1}(ax)}\left(3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)-3an^2x-2\left(n^2-1\right)n\cosh\left(2\coth^{-1}(ax)\right)+27ax+2n^3-26n\right)}{4ac^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A] time = 0.041, size = 84, normalized size = 0.8

$$\frac{\left(6\,x^3a^3-6\,na^2x^2+3\,an^2x-n^3-9\,ax+7\,n\right)(ax-1)(ax+1)\,{e}^{n\operatorname{arccoth}(ax)}}{a\left(n^4-10\,n^2+9\right)}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A] time = 1.51349, size = 335, normalized size = 3.28

$$-\frac{\left(6\,a^3x^3+6\,a^2nx^2+n^3+3\left(an^2-3\,a\right)x-7\,n\right)\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^3n^4-10\,ac^3n^2+\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^4+9\,ac^3-2\left(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 + 6*a^2*n*x^2 + n^3 + 3*(a*n^2 - 3*a)*x - 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.761 $\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=944

result too large to display

```
[Out] -((a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-
3 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a^5*(1 - 1/(a^2*x^2))^
(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n
+ n^2)*(c - a^2*c*x^2)^(5/2)) + (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x)
)^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c
*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1
+ 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2)) + (
4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-1
 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) + (8*a^5*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + 4*n +
n^2)*(c - a^2*c*x^2)^(5/2)) - (8*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^
((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x
^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 +
 1/(a*x))^((1 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/
(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*x^5)/
((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) + (4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1
 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2)*x^5)/((3 + n)*(c - a^2*c
*x^2)^(5/2)) - (2^((5 + n)/2)*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-
3 - n)/2)*x^5*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/2, (-1 - n)/2, (a - x^
(-1))/(2*a)])/((3 + n)*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] time = 0.632143, antiderivative size = 944, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6192, 6195, 128, 45, 37, 69}

$$-\frac{2^{\frac{n+5}{2}} a^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5 \, {}_2F_1\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3);\frac{1}{2}(-n-1);\frac{a-\frac{1}{x}}{2a}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}} x^5 \left(1-\right.}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -((a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-
3 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a^5*(1 - 1/(a^2*x^2))^
(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n
+ n^2)*(c - a^2*c*x^2)^(5/2)) + (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x)
)^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c
*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1
+ 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2)) + (
4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-1
 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) + (8*a^5*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + 4*n +
n^2)*(c - a^2*c*x^2)^(5/2)) - (8*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^
((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x
^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 +
 1/(a*x))^((1 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/
(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*x^5)/
((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) + (4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1
 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2)*x^5)/((3 + n)*(c - a^2*c
*x^2)^(5/2)) - (2^((5 + n)/2)*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-
3 - n)/2)*x^5*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/2, (-1 - n)/2, (a - x^
(-1))/(2*a)])/((3 + n)*(c - a^2*c*x^2)^(5/2))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int x^4\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx, x, \frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}} - 4a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}} + 6\right.\right.}{(c}$$

$$= -\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx, x, \frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subs}}{}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} + \frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 1.5572, size = 220, normalized size = 0.23

$$\frac{e^{n\coth^{-1}(ax)}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(5an^3x-45anx-2n^2+42\right)+6a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(2\coth^{-1}(ax)\right)-n\left(n^2-1\right)\left(a^2x^2-\right.\right.}{4ac^2(n-1)(n+1)\left(n^2-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(42 - 2*n^2 - 45*a*n*x + 5*a
*n^3*x) + 6*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[2*ArcCoth[a*x]] - n*(
-1 + n^2)*(-1 + a^2*x^2)*Cosh[3*ArcCoth[a*x]]) - 8*E^((1 + n)*ArcCoth[a*x])
*(9 - 9*n - n^2 + n^3)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 +
n)/2, -E^(2*ArcCoth[a*x])])/(4*a*c^2*(-1 + n)*(1 + n)*(-9 + n^2)*Sqrt[1 - 1
/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x}\left(-a^2cx^2+c\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^6c^3x^7-3\,a^4c^3x^5+3\,a^2c^3x^3-c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^6*c^3*x^7 -
 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

3.762 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=127

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}(n-2p),-2p,\frac{2}{x\left(a+\right.}\right.}{2p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-n/2 + p)*(1 + 1/
(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n - 2
*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rubi [A] time = 0.149586, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}{}_2F_1\left(-2p-1,\frac{1}{2}(n-2p);-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-n/2 + p)*(1 + 1/
(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n - 2
*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
```

```
Q[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Integ
ersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p} x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+p}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x\left(c-a^2cx^2\right)^p {}_2F_1\left(-1-2p, \frac{1}{2}(n-\right.}{1+2p}$$

Mathematica [A] time = 0.188116, size = 83, normalized size = 0.65

$$-\frac{\left(a^2x^2-1\right)\left(e^{2\coth^{-1}(ax)}-1\right)\left(c-a^2cx^2\right)^p e^{(n-2)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,-\frac{n}{2}-p,-\frac{n}{2}+p+2,e^{-2\coth^{-1}(ax)}\right)}{a(n-2(p+1))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((E^((-2 + n)*ArcCoth[a*x])*(-1 + E^(2*ArcCoth[a*x]))*(-1 + a^2*x^2)*(c -
a^2*c*x^2)^p*Hypergeometric2F1[1, -n/2 - p, 2 - n/2 + p, E^(-2*ArcCoth[a*x]
)])/(a*(n - 2*(1 + p))))
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int e^{n\text{arccoth}(ax)}\left(-a^2cx^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(ax-1\right)\left(ax+1\right)\right)^p e^{n\operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2 + c\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.763 $\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=51

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

```
[Out] ((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rubi [A] time = 0.123224, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6192, 6196, 37}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1+\frac{x}{a}\right)^{2p}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.0644862, size = 36, normalized size = 0.71

$$\frac{(ax+1)\left(c-a^2cx^2\right)^p e^{2p\coth^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (E^(2*p*ArcCoth[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

Maple [A] time = 0.039, size = 38, normalized size = 0.8

$$\frac{(ax+1)\,\mathrm{e}^{2\,p\,\mathrm{arccoth}(ax)}\left(-a^2cx^2+c\right)^p}{a\left(1+2\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] (a*x+1)/a/(1+2*p)*exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p
```

Maxima [A] time = 1.055, size = 49, normalized size = 0.96

$$\frac{\left(a\left(-c\right)^{p}x-\left(-c\right)^{p}\right)\left(ax-1\right)^{2p}}{a\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

Fricas [A] time = 1.401, size = 89, normalized size = 1.75

$$\frac{\left(ax-1\right)\left(-a^{2}cx^{2}+c\right)^{p}\left(\frac{ax-1}{ax+1}\right)^{p}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] (a*x - 1)*(-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^p/(2*a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax-1}{ax+1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^p, x)
```

3.764 $\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

```
[Out] ((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rubi [A] time = 0.121951, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6192, 6196, 37}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{2p}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.0621937, size = 36, normalized size = 0.69

$$\frac{(ax-1)\left(c-a^2cx^2\right)^p e^{-2p\coth^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcCoth[a*x])*(a + 2*a*p))
```

Maple [A] time = 0.037, size = 40, normalized size = 0.8

$$\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{a\left(1+2p\right)\mathrm{e}^{2p\,\mathrm{arccoth}(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x)
```

```
[Out] (a*x-1)/a/(1+2*p)*(-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x))
```

Maxima [A] time = 1.07306, size = 46, normalized size = 0.88

$$\frac{\left(a\,(-c)^{p}\,x+(-c)^{p}\right)(ax+1)^{2\,p}}{a\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

Fricas [A] time = 1.31117, size = 92, normalized size = 1.77

$$\frac{(ax+1)\left(-a^{2}cx^{2}+c\right)^{p}}{\left(2\,ap+a\right)\left(\frac{ax-1}{ax+1}\right)^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] (a*x + 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x - 1)/(a*x + 1))^p)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^p, x)
```

3.765 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rubi [A] time = 0.10418, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6141, 678, 69}

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}{}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
```

```
1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx\\
&= c^2\int (1+ax)^4\left(c-a^2cx^2\right)^{-2+p}\,dx\\
&= \left(c^2(1+ax)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\right)\int (1+ax)^{2+p}(c-acx)^{-2+p}\,dx\\
&= \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\,{}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}
\end{aligned}$$

Mathematica [A] time = 0.0226024, size = 72, normalized size = 1.14

$$-\frac{2^{p+2}(1-ax)^{p-1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p,
 -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

Maple [F] time = 0.677, size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2 \left(-a^2cx^2+c\right)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c\,(ax-1)\,(ax+1))^p\,(ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

3.766 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-2p-1,\frac{3}{2}-p,-2p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{2p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*x
))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rubi [A] time = 0.145059, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}\left(c-a^2cx^2\right)^p {}_2F_1\left(-2p-1,\frac{3}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*x
))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
```

```
Q[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Integ
ersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+p}dx,x,\frac{1}{x}\right)\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-2p,\frac{3}{2}-p;-2p;\right.}{1+2p}$$

Mathematica [A] time = 0.255989, size = 122, normalized size = 1.03

$$-\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{5\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}\text{Hypergeometric2F1}\left(p+\frac{5}{2},2p+2\right.}{2ap+5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(5*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/
2 + p, 2 + 2*p, 7/2 + p, E^(2*ArcCoth[a*x])])/((5*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

Maple [F] time = 0.372, size = 0, normalized size = 0.

$$\int \left(-a^2 c x^2 + c\right)^p \left(\frac{ax - 1}{ax + 1}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2 c x^2 + c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2 x^2 + 2\,ax + 1\right)\left(-a^2 c x^2 + c\right)^p \sqrt{\frac{ax-1}{ax+1}}}{a^2 x^2 - 2\,ax + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))
/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.767 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)/
2])/(a*p*(1 + a*x)^p)
```

Rubi [A] time = 0.0884693, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6141, 678, 69}

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)/
2])/(a*p*(1 + a*x)^p)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
```

```
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx\\ &= -\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)\\ &= -\left(\left(c(1+ax)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^{1+p}(c-acx)^{-1+p}\,dx\right)\\ &= \frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}\end{aligned}$$

Mathematica [A] time = 0.0182946, size = 67, normalized size = 1.24

$$\frac{2^{p+1}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p
, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Sympy [C] time = 9.64988, size = 651, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/2
 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(1/(a**2*x*
*2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/
a**2 + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x*
*2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,)
, 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) + Piecewise((0**
p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/
2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
- 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p)
, (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p
)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

3.768 $\int e^{\coth^{-1}(ax)} \left(c-a^2cx^2\right)^p \, dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}-p,-2p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{2p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x
))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rubi [A] time = 0.132645, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}\left(c-a^2cx^2\right)^p {}_2F_1\left(-2p-1,\frac{1}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x
))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
```

```
Q[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Integ
ersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+p}dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-2p,\frac{1}{2}-p;-2p;\frac{\ldots}{(a\ldots}\right.}{1+2p}$$

Mathematica [A] time = 0.20239, size = 122, normalized size = 1.03

$$-\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}\text{Hypergeometric2F1}\left(p+\frac{3}{2},2p+2\ldots\right.}{2ap+3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(3*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/
2 + p, 2 + 2*p, 5/2 + p, E^(2*ArcCoth[a*x])])/((3*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

Maple [F] time = 0.369, size = 0, normalized size = 0.

$$\int \left(-a^{2} c \,x^{2}+c \right)^{p} \frac{1}{\sqrt{\frac{a x -1}{a x +1}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^{2} c x^{2}+c\right)^{p}}{\sqrt{\frac{a x-1}{a x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ax+1)\left(-a^{2} c x^{2}+c\right)^{p} \sqrt{\frac{ax-1}{ax+1}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(ax-1)(ax+1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.769 $\int e^{-\coth^{-1}(ax)} \left(c - a^2cx^2\right)^p \, dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-2p-1,-p-\frac{1}{2},-2p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{2p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*x
))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2*p
, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rubi [A] time = 0.138725, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}\left(c-a^2cx^2\right)^p {}_2F_1\left(-2p-1,-p-\frac{1}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*x
))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2*p
, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
```

```
Q[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Integ
ersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$
$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+p}dx,x,\frac{1}{x}\right)$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-2p,-\frac{1}{2}-p;-2p\right.}{1+2p}$$

Mathematica [A] time = 0.220012, size = 118, normalized size = 1.

$$-\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}\text{Hypergeometric2F1}\left(p+\frac{1}{2},2p+2,\right.}{2ap+a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] -((4^(1 + p)*E^ArcCoth[a*x]*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/
(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2 +
p, 2 + 2*p, 3/2 + p, E^(2*ArcCoth[a*x])])/((a + 2*a*p)*(a*Sqrt[1 - 1/(a^2*x
^2)]*x)^(2*p)))
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int \left(-a^2 c x^2 + c\right)^p \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2 c x^2 + c\right)^p \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a^2 c x^2 + c\right)^p \sqrt{\frac{ax-1}{ax+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{ax-1}{ax+1}} \left(-c\left(ax-1\right)\left(ax+1\right)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.770 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=55

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(ax+1)\right)}{ap}$$

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x
)/2])/(a*p*(1 - a*x)^p))
```

Rubi [A] time = 0.0864525, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6142, 678, 69}

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x
)/2])/(a*p*(1 - a*x)^p))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
```

```
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx\\
&= -\left(c\int (1-ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)\\
&= -\left(\left(c(1-ax)^{-p}(c+acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{1+p}(c+acx)^{-1+p}\,dx\right)\\
&= -\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1+ax)\right)}{ap}
\end{aligned}$$

Mathematica [A] time = 0.0241235, size = 73, normalized size = 1.33

$$\frac{2^{p-1}(1-ax)^{p+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(1-p,p+2,p+3,\frac{1}{2}(1-ax)\right)}{a(p+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p, 2
+ p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p)
```

Maple [F] time = 0.554, size = 0, normalized size = 0.

$$\int \frac{\left(-a^2cx^2+c\right)^p(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(a*x-1),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(a*x-1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Sympy [C] time = 8.9077, size = 651, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/2
 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**2*x*
*2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/
a**2 - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x*
*2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,)
, 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) - Piecewise((0**
p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) + a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/
2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
+ 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p)
, (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*p
)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

3.771 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{3}{2},-2p,\frac{2}{x\left(a+\frac{1}{x}\right)}\right)}{2p+1}$$

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*x
))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rubi [A] time = 0.144269, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,-p-\frac{3}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*x
))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```


Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
```

```
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+p}dx,x,\frac{1}{x}\right.\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-2p,-\frac{3}{2}-p;-2\right.}{1+2p}$$

Mathematica [A] time = 0.267677, size = 119, normalized size = 1.01

$$\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}\text{Hypergeometric2F1}\left(p-\frac{1}{2},2p+2,p\right.}{a-2ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4^(1 + p)*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/(-1 + E^(2*ArcCot
h[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2 + p, 2 + 2*p, 1/2
+ p, E^(2*ArcCoth[a*x])])/(E^ArcCoth[a*x]*(a - 2*a*p)*(a*Sqrt[1 - 1/(a^2*x^
2)]*x)^(2*p))
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int \left(-a^{2} c\,x^{2}+c\right)^{p} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^{2} c x^{2}+c\right)^{p} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ax-1)\left(-a^{2}cx^{2}+c\right)^{p}\sqrt{\frac{ax-1}{ax+1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.772 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=342

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{7a}+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}{42a}+c^4x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-131$$

```
[Out] (-51*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (67*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(48*a) - (91*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(120*a) - (131*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) +
 (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (47*c^4*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2))/(42*a) + (8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(
a*x))^(9/2))/(7*a) + c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x + (35*c^
4*ArcCsc[a*x])/(16*a) + (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/
a
```

Rubi [A] time = 0.24551, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{7a}+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}{42a}+c^4x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-131$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-51*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (67*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(48*a) - (91*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(120*a) - (131*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) +
 (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (47*c^4*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2))/(42*a) + (8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(
a*x))^(9/2))/(7*a) + c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x + (35*c^
4*ArcCsc[a*x])/(16*a) + (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/
a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
```

```
1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^4\,\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int \frac{\left(\frac{7}{a^2}-\frac{47x}{a^3}\right)\left(1 \cdots\right.}{\cdots}\right.$$

$$= \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \cdots$$

$$= \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(\cdots\right.$$

$$= -\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4 \cdots}{\cdots}$$

$$= -\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4 \cdots}{\cdots}$$

$$= -\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4 \cdots}{\cdots}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\cdots}}{\cdots}$$

Mathematica [A] time = 0.274908, size = 120, normalized size = 0.35

$$\frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1680a^7x^7-2816a^6x^6+3045a^5x^5+1952a^4x^4-1330a^3x^3-1056a^2x^2+280ax+240\right)}{x^6}+1680a^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+3675a^6\sin^{-}\right.}{1680a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(240 + 280*a*x - 1056*a^2*x^2 - 1330*a^3*x^3 +
 1952*a^4*x^4 + 3045*a^5*x^5 - 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^6
*ArcSin[1/(a*x)] + 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7)
```

Maple [A] time = 0.165, size = 320, normalized size = 0.9

$$\frac{(ax-1)\,c^4}{1680\,a^8x^7}\left(-1680\,\sqrt{a^2x^2-1}\sqrt{a^2}x^8a^8+1680\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6+3675\,\sqrt{a^2x^2-1}\sqrt{a^2}x^7a^7+3675\,a^7x^7\sqrt{a^2}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/1680*(a*x-1)*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^8*a^8+1680*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+3675*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^7*a^7+
3675*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^
2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8-1995*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^5*a^5-1136*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4+1050*(a^2*x^2-1)^(3
/2)*(a^2)^(1/2)*x^3*a^3+816*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-280*(a^2)
^(1/2)*(a^2*x^2-1)^(3/2)*x*a-240*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a
*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A] time = 1.60309, size = 513, normalized size = 1.5

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{5355\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+31465\,c^4\left(\frac{ax-}{ax+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (5355*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 31465*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 72051*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 71801*c^4*((a*x - 1)/(
a*x + 1))^(9/2) + 4569*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 17619*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 10185*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 1995*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A] time = 1.40842, size = 489, normalized size = 1.43

$$-\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(1680\,a^8c^4x^8-1136\,a^7c^4x\right.}{1680\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 - 1136*a^7*c^4*x^7 + 229*a^6*c^4*x^6
+ 4997*a^5*c^4*x^5 + 622*a^4*c^4*x^4 - 2386*a^3*c^4*x^3 - 776*a^2*c^4*x^2 +
 520*a*c^4*x + 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15955, size = 493, normalized size = 1.44

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{1680\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{1260\,(ax-1)c^4\sqrt{\frac{a}{a}}}{ax+1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(abs(sqrt((a*x - 1)/(a*x + 1)) -
1))/a^2 + 1680*c^4*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)
) + (1260*(a*x - 1)*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 18921*(a*x -
1)^2*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 73152*(a*x - 1)^3*c^4*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 60151*(a*x - 1)^4*c^4*sqrt((a*x - 1)/(a
*x + 1))/(a*x + 1)^4 + 23380*(a*x - 1)^5*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1)^5 + 3675*(a*x - 1)^6*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^6 - 315*
c^4*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^7))*a
```

3.773 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=268

$$\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{5a}+c^3x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{60a}-\frac{31c^3\sqrt{1}}{}$$

```
[Out] (-23*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (31*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(24*a) - (43*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2))/(60*a) + (23*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*
c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(5*a) + c^3*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(7/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) + (c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.187667, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{5a}+c^3x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{60a}-\frac{31c^3\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```


```
[Out] (-23*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (31*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(24*a) - (43*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2))/(60*a) + (23*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*
c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(5*a) + c^3*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(7/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) + (c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2} x - c^3\,\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2} x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \frac{\left(\frac{5}{a^2}-\frac{23x}{a^3}\right)\right.$$

$$= \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2} x - \frac{1}{20}$$

$$= -\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(\right.$$

$$= -\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1\right.}{}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1}}{}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1}}{}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1}}{}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1}}{}$$

Mathematica [A] time = 0.204789, size = 104, normalized size = 0.39

$$\frac{c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(120a^5x^5-184a^4x^4+135a^3x^3+88a^2x^2-30ax-24\right)}{x^4}+120a^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+225a^4\sin^{-1}\left(\frac{1}{ax}\right)\right)}{120a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*(-24 - 30*a*x + 88*a^2*x^2 + 135*a^3*x^3 - 184
*a^4*x^4 + 120*a^5*x^5))/x^4 + 225*a^4*ArcSin[1/(a*x)] + 120*a^4*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5)
```

Maple [A] time = 0.141, size = 272, normalized size = 1.

$$\frac{(ax-1)\,c^3}{120\,a^6x^5}\left(-120\,\sqrt{a^2}\sqrt{a^2x^2-1}x^6a^6+120\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4+225\,\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5+225\,\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/120*(a*x-1)*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x^6*a^6+120*(a^2)^(1/
2)*(a^2*x^2-1)^(3/2)*x^4*a^4+225*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+225*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*x^5*a^5+120*ln((a^2*x+(a^2*x^2-1)^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-105*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^
3*a^3-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+30*(a^2)^(1/2)*(a^2*x^2-1)^(
3/2)*x*a+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)
*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A] time = 1.49363, size = 408, normalized size = 1.52

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+16}{\frac{4\,(ax-1)a^2}{ax+1}+\frac{5\,(ax-1)^2}{(ax+1)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
 (345*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(9/
2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 86*c^3*((a*x - 1)/(a*x + 1))^(5
/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 105*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

Fricas [A] time = 1.35892, size = 423, normalized size = 1.58

$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(120\,a^6c^3x^6-64\,a^5c^3x^5-4\right.}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 - 64*a^5*c^3*x^5 - 49*a^4*c^3*x^4 + 223*a^
3*c^3*x^3 + 58*a^2*c^3*x^2 - 54*a*c^3*x - 24*c^3)*sqrt((a*x - 1)/(a*x + 1))
)/(a^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20519, size = 401, normalized size = 1.5

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{120\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{310\,(ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+142\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/
a^2 + 120*c^3*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (
310*(a*x - 1)*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 1424*(a*x - 1)^2*c^
3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 970*(a*x - 1)^3*c^3*sqrt((a*x - 1
)/(a*x + 1))/(a*x + 1)^3 + 225*(a*x - 1)^4*c^3*sqrt((a*x - 1)/(a*x + 1))/(a
*x + 1)^4 + 15*c^3*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1
)^5))*a
```

3.774 $\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^2 \, dx$

Optimal. Leaf size=194

$$c^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}$$

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (7*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(6*a) + (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(3*a) + c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) + (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.134112, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (7*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(6*a) + (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(3*a) + c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) + (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^2\,\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{4x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(\frac{3}{a^2}-\frac{7x}{a^3}\right)\left(1+\right.}{x\sqrt{1-}}\right.$$

$$= -\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x + \frac{1}{6}\left(a\right.$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)$$

Mathematica [A] time = 0.132348, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-8a^2x^2+3ax+2\right)+6a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+9a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^2
*ArcSin[1/(a*x)] + 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x^
2)
```

Maple [A] time = 0.135, size = 224, normalized size = 1.2

$$\frac{(ax-1)c^2}{6a^4x^3}\left(-6\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+6\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+9\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)x^3a^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/6*(a*x-1)*c^2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^2*a^2+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+6*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+9*a^3*x^3*(a^2)^(1/2)*a
rctan(1/(a^2*x^2-1)^(1/2))-3*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-2*(a^2*x^2-1
)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^4/x^
3/(a^2)^(1/2)
```

Maxima [A] time = 1.52984, size = 301, normalized size = 1.55

$$-\frac{1}{3}a\left(\frac{9c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+29c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (1
5*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2) + c^
2*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

Fricas [A] time = 1.33358, size = 358, normalized size = 1.85

$$-\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^2x^4-2\,a^3c^2x^3-5\,a^2c^2x^2+\ldots\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 5*a^2*c^2*x^2 + 5*a*c^2*x + 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{2a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a**2/(x**2*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x))/a**4
```

Giac [A] time = 1.20413, size = 309, normalized size = 1.59

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c^2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{6\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{20\,(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{9\,(ax-1)^2\ldots}{(ax+\ldots}}{a^2\left(\frac{ax-1}{ax+1}+\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
 + 6*c^2*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (20*(a
*x - 1)*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 9*(a*x - 1)^2*c^2*sqrt((a
*x - 1)/(a*x + 1))/(a*x + 1)^2 + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*
x - 1)/(a*x + 1) + 1)^3))
```

3.775 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=107

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a}+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] (-2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(3/2)*x + (c*ArcCsc[a*x])/a + (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]])/a
```

Rubi [A] time = 0.0711992, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6194, 97, 154, 21, 105, 41, 216, 92, 208}

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a}+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(3/2)*x + (c*ArcCsc[a*x])/a + (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\left(c\,\mathrm{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-c\,\mathrm{Subst}\left(\int\frac{\left(\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+(ac)\,\mathrm{Subst}\left(\int\frac{-\frac{1}{a^2}+\frac{x}{a^3}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right.\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{c\,\mathrm{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2}-\frac{c\,\mathrm{Sub}}{}\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{c\,\mathrm{Subst}\left(\right.}{}\\
&= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\right.}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0782458, size = 53, normalized size = 0.5

$$
\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(ax-1)+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x]))/a
```

Maple [A] time = 0.153, size = 163, normalized size = 1.5

$$\frac{c(ax-1)}{a^2x}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2}\sqrt{a^2x^2-1}xa+\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)xa^2+ax\sqrt{a^2}\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x)
```

```
[Out] (a*x-1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))*x*a^2+a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)))/((a*x-
1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A] time = 1.51892, size = 158, normalized size = 1.48

$$-\left(\frac{4c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A] time = 1.4371, size = 248, normalized size = 2.32

$$-\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) + a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 - c)*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int-\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**2
```

Giac [A] time = 1.17049, size = 170, normalized size = 1.59

$$-a\left(\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{4\,(ax-1)c\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -a*(2*c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 + c*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4*(a*x - 1)
```

```
*c*sqrt((a*x - 1)/(a*x + 1))/((a*x + 1)*a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

3.776 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx$

Optimal. Leaf size=104

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] (-2*Sqrt[1 + 1/(a*x)])/(a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*S
qrt[1 - 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rubi [A] time = 0.0750737, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6194, 103, 21, 94, 92, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)])/(a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*S
qrt[1 - 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] time = 0.110476, size = 56, normalized size = 0.54

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax-2)}{ax-1}+\frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(-1 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]/a)/c
```

Maple [B] time = 0.141, size = 251, normalized size = 2.4

$$\frac{1}{2\left(ax-1\right)ac}\left(2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}}{\sqrt{a^{2}}}\right)x^{2}a^{3}+3\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+1\right)}x^{2}a^{2}-4\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\sqrt{\left(ax-1\right)\left(ax+\right.}}{\sqrt{a^{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/2*(2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+
3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-4*ln((a^2*x+(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-
6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2))/(a^2)^(1/2))+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^
2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.01934, size = 157, normalized size = 1.51

$$-a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^{2}c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^{2}c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*c*
sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) + l
og(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Fricas [A] time = 1.36805, size = 215, normalized size = 2.07

$$\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^{2}x^{2}-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x)/c
```

Giac [A] time = 1.14678, size = 171, normalized size = 1.64

$$a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{\frac{3(ax-1)}{ax+1}-1}{a^2c\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\sqrt{\frac{ax-1}{ax+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(abs(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/(a^2*c) - (3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)*sqrt((
a*x - 1)/(a*x + 1))/(a*x + 1) - sqrt((a*x - 1)/(a*x + 1)))))
```

3.777 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=180

$$\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] -4/(3*a*c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) - 11/(3*a*c^2*Sqrt[1 - 1
/(a*x)]*Sqrt[1 + 1/(a*x)]) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*x
)]) + x/(c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a
*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rubi [A] time = 0.115933, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```



```
[Out] -4/(3*a*c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) - 11/(3*a*c^2*Sqrt[1 - 1
/(a*x)]*Sqrt[1 + 1/(a*x)]) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*x
)]) + x/(c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a
*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{3}{a^2}+\frac{8x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{3}{a^3}-\frac{11x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right.}\right.}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^3\,\text{Sub}}{}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{\text{Subst}}{}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}}{}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\tanh^{-}}{}$$

Mathematica [A] time = 0.153513, size = 83, normalized size = 0.46

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(3a^3x^3-7a^2x^2-5ax+8\right)}{3(ax-1)^2(ax+1)} + \log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 + a
*x)^2*(1 + a*x)) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] time = 0.149, size = 530, normalized size = 2.9

$$-\frac{1}{24\,a\,(ax+1)^2\,(ax-1)^2\,c^2}\left(-45\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^5a^5-24\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^5a^6+21\,\sqrt{a^2}\,((ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/24*(-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-24*ln((a^2*x+(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+21*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^3*a^3+45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+24*ln
((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+11*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3+48*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^
3*a^4-5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-90*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2)*x^2*a^2-48*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)
^(1/2))*x^2*a^3-19*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2)*x*a-24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a
^2)^(1/2))*x*a^2+45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+24*a*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x+1)^2/(a*x
-1)^2/c^2/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.0066, size = 216, normalized size = 1.2

$$\frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^2) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Fricas [A] time = 1.37424, size = 306, normalized size = 1.7

$$\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\int\frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))),
x)/c**2
```

Giac [A] time = 1.14055, size = 231, normalized size = 1.28

$$\frac{1}{12}a\left(\frac{12\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)\left(\frac{18(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}-\frac{24\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*a*(12*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)*(18*(a*x - 1)/(a*x + 1) + 1
)/((a*x - 1)*a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) + 3*sqrt((a*x - 1)/(a*x + 1
))/(a^2*c^2) - 24*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*((a*x - 1)/(a*x + 1) -
 1)))
```

3.778 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} dx$

Optimal. Leaf size=254

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-$$

```
[Out] -6/(5*a*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) - 29/(15*a*c^3*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) - 34/(5*a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(3/2)) + (21*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sq
rt[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(5/2)*(
1 + 1/(a*x))^(3/2)) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rubi [A] time = 0.165665, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```


```
[Out] -6/(5*a*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) - 29/(15*a*c^3*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) - 34/(5*a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(3/2)) + (21*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sq
rt[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(5/2)*(
1 + 1/(a*x))^(3/2)) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{5}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{a^2\,\text{Subst}\Big(}{}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{}{c^3\left(1-\frac{1}{ax}\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{21\sqrt{1-}}{5ac^3\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{21\sqrt{1-}}{5ac^3\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{21\sqrt{1-}}{5ac^3\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{21\sqrt{1-}}{5ac^3\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{21\sqrt{1-}}{5ac^3\left(1+\right.}$$

Mathematica [A] time = 0.196055, size = 99, normalized size = 0.39

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(15a^5x^5-38a^4x^4-52a^3x^3+87a^2x^2+33ax-48\right)}{15(ax-1)^3(ax+1)^2}+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a^
4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^3*(1 + a*x)^2) + Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c^3)
```

Maple [B] time = 0.164, size = 714, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/240*(240*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^7*
a^8+525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^7*a^7-240*ln((a^2*x+(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7-285*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(3/2)*x^5*a^5-525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a^6-720*l
n((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6-83*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4-1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2)*x^5*a^5+720*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)
)*x^4*a^5+218*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+1575*(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+720*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(1/2))/(a^2)^(1/2))*x^3*a^4+342*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^
2+1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-720*ln((a^2*x+(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+3*(a^2)^(1/2)*((a*x-1)*(a*x
+1))^(3/2)*x*a-1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-240*ln((a^2
*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-243*((a*x-1)*(a*
x+1))^(3/2)*(a^2)^(1/2)-525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+240*a*l
n((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+525*(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)^3/(a*x+1)^3/c^3/((a*x-1)*(a*
x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.03109, size = 262, normalized size = 1.03

$$\frac{1}{240}\,a\left(\frac{\frac{37\,(ax-1)}{ax+1}+\frac{410\,(ax-1)^2}{(ax+1)^2}-\frac{930\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+24\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*((37*(a*x - 1)/(a*x + 1) + 410*(a*x - 1)^2/(a*x + 1)^2 - 930*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
 - 1)/(a*x + 1))^(5/2)) + 5*(((a*x - 1)/(a*x + 1))^(3/2) + 24*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A] time = 1.45363, size = 405, normalized size = 1.59

$$\frac{15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^5x^5-38\,a^4x^4-52\,a\right.}{15\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x - 48)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14955, size = 313, normalized size = 1.23

$$\frac{1}{240}a\left(\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^2\left(\frac{40(ax-1)}{ax+1}+\frac{450(ax-1)^2}{(ax+1)^2}+3\right)}{(ax-1)^2a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{480\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3\left(\frac{ax-1}{ax+1}-1\right)}+\frac{5\left(\frac{(ax-\right.}{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] 1/240*a*(240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 240*log(abs(sqr
t((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^2*(40*(a*x - 1)/(a*x + 1
) + 450*(a*x - 1)^2/(a*x + 1)^2 + 3)/((a*x - 1)^2*a^2*c^3*sqrt((a*x - 1)/(a
*x + 1))) - 480*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*((a*x - 1)/(a*x + 1) - 1
)) + 5*((a*x - 1)*a^4*c^6*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 24*a^4*c^6*
sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^9))
```

3.779 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=328

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\ldots}{21ac^4\left(\ldots\right.}$$

```
[Out] -8/(7*a*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) - 11/(7*a*c^4*(1 - 1/(
a*x))^(5/2)*(1 + 1/(a*x))^(5/2)) - 62/(21*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/
(a*x))^(5/2)) - 269/(21*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (262
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (163*Sqrt[1 - 1/(a*x)]
)/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) + ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rubi [A] time = 0.228436, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\ldots}{21ac^4\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -8/(7*a*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) - 11/(7*a*c^4*(1 - 1/(
a*x))^(5/2)*(1 + 1/(a*x))^(5/2)) - 62/(21*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/
(a*x))^(5/2)) - 269/(21*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (262
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (163*Sqrt[1 - 1/(a*x)]
)/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) + ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{48x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^2\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{}{c^4\left(1-\right.}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{}{21ac^4\sqrt{1}}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{}{21ac^4\sqrt{1}}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{}{21ac^4\sqrt{1}}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{}{21ac^4\sqrt{1}}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{}{21ac^4\sqrt{1}}$$

$$8 \qquad 11 \qquad 62$$

Mathematica [A] time = 0.26741, size = 115, normalized size = 0.35

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(105a^7x^7-281a^6x^6-559a^5x^5+965a^4x^4+715a^3x^3-1065a^2x^2-279ax+384\right)}{105(ax-1)^4(ax+1)^3}+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 + 9
65*a^4*x^4 - 559*a^5*x^5 - 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^4*(1
 + a*x)^3) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] time = 0.156, size = 898, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/13440*(33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^9*a^9-19635*(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(3/2)*x^7*a^7-33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)
*x^8*a^8-2893*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^6*a^6-53760*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-132300*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2)*x^7*a^7+27673*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*
x^5*a^5+198450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-7705*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+132300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)
*x^6*a^6+24295*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4-53760*ln((a^2*x+
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^7*a^8-13440*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^8*a^9+13440*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-37095*(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(3/2)*x^2*a^2-2637*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+5
3760*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+80
640*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6-806
40*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+3307
5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-132300*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2)*x^3*a^3-198450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+53760*
ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7+132300*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+16077*((a*x-1)*(a*x+1))^(3/2)*(
```

```
a^2)^(1/2)-33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)-13440*a*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+13440*ln((a^2*x+(a^2)^(1/2)*(
(a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^9*a^10)/a/(a^2)^(1/2)/(a*x-1)^4/(a*x
+1)^4/c^4/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A] time = 1.01233, size = 311, normalized size = 0.95

$$\frac{1}{6720}a\left(\frac{5\left(\frac{39\,(ax-1)}{ax+1}+\frac{287\,(ax-1)^2}{(ax+1)^2}+\frac{2611\,(ax-1)^3}{(ax+1)^3}-\frac{5628\,(ax-1)^4}{(ax+1)^4}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{7\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+50\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+705\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{6720\log\left(\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(5*(39*(a*x - 1)/(a*x + 1) + 287*(a*x - 1)^2/(a*x + 1)^2 + 2611*(a
*x - 1)^3/(a*x + 1)^3 - 5628*(a*x - 1)^4/(a*x + 1)^4 + 3)/(a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 7*(3*((a*x - 1
)/(a*x + 1))^(5/2) + 50*((a*x - 1)/(a*x + 1))^(3/2) + 705*sqrt((a*x - 1)/(a
*x + 1)))/(a^2*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 6
720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A] time = 1.43686, size = 602, normalized size = 1.84

$$\frac{105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax\right.}{105\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1
)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 +
 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (105
*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 - 1065*a^2
*x^2 - 279*a*x + 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 2*a^6*c^4*x
```

```
^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15102, size = 386, normalized size = 1.18

$$\frac{1}{6720}a\left(\frac{6720\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{6720\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{5\,(ax+1)^3\left(\frac{42\,(ax-1)}{ax+1}+\frac{329\,(ax-1)^2}{(ax+1)^2}+\frac{2940\,(ax-1)^3}{(ax+1)^3}+3\right)}{(ax-1)^3a^2c^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{1344\ldots}{a^2c^4\left(\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/6720*a*(6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 6720*log(abs(
sqrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - 5*(a*x + 1)^3*(42*(a*x - 1)/(a*
x + 1) + 329*(a*x - 1)^2/(a*x + 1)^2 + 2940*(a*x - 1)^3/(a*x + 1)^3 + 3)/((
a*x - 1)^3*a^2*c^4*sqrt((a*x - 1)/(a*x + 1))) - 13440*sqrt((a*x - 1)/(a*x +
 1))/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1)) + 7*(50*(a*x - 1)*a^8*c^16*sqrt((a
*x - 1)/(a*x + 1))/(a*x + 1) + 3*(a*x - 1)^2*a^8*c^16*sqrt((a*x - 1)/(a*x +
 1))/(a*x + 1)^2 + 705*a^8*c^16*sqrt((a*x - 1)/(a*x + 1)))/(a^10*c^20))
```

3.780 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=127

$$\frac{4c^5}{a^3x^2}-\frac{2c^5}{3a^4x^3}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{5a^6x^5}+\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}-\frac{c^5}{4a^9x^8}-\frac{c^5}{9a^{10}x^9}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

```
[Out] -c^5/(9*a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^7*
x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4*c
^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Rubi [A] time = 0.175185, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{4c^5}{a^3x^2}-\frac{2c^5}{3a^4x^3}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{5a^6x^5}+\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}-\frac{c^5}{4a^9x^8}-\frac{c^5}{9a^{10}x^9}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -c^5/(9*a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^7*
x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4*c
^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx\\ &= \frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}\,dx}{a^{10}}\\ &= \frac{c^5\int \frac{(1-ax)^4(1+ax)^6}{x^{10}}\,dx}{a^{10}}\\ &= \frac{c^5\int \left(a^{10}+\frac{1}{x^{10}}+\frac{2a}{x^9}-\frac{3a^2}{x^8}-\frac{8a^3}{x^7}+\frac{2a^4}{x^6}+\frac{12a^5}{x^5}+\frac{2a^6}{x^4}-\frac{8a^7}{x^3}-\frac{3a^8}{x^2}+\frac{2a^9}{x}\right)dx}{a^{10}}\\ &= -\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5}{}\end{aligned}$$

Mathematica [A] time = 0.0429798, size = 127, normalized size = 1.

$$\frac{4c^5}{a^3x^2}-\frac{2c^5}{3a^4x^3}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{5a^6x^5}+\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}-\frac{c^5}{4a^9x^8}-\frac{c^5}{9a^{10}x^9}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -c^5/(9*a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^7*
x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4*c
^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Maple [A] time = 0.055, size = 116, normalized size = 0.9

$$-\frac{c^5}{9\,a^{10}x^9}-\frac{c^5}{4\,a^9x^8}+\frac{3\,c^5}{7\,a^8x^7}+\frac{4\,c^5}{3\,a^7x^6}-\frac{2\,c^5}{5\,a^6x^5}-3\,\frac{c^5}{a^5x^4}-\frac{2\,c^5}{3\,a^4x^3}+4\,\frac{c^5}{x^2a^3}+3\,\frac{c^5}{a^2x}+c^5x+2\,\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^5,x)
```

```
[Out] -1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/a
^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/x^2/a^3+3*c^5/a^2/x+c^5*x+2*c^5*
ln(x)/a
```

Maxima [A] time = 1.0479, size = 154, normalized size = 1.21

$$c^5x + \frac{2\,c^5\log\left(x\right)}{a} + \frac{3780\,a^8c^5x^8 + 5040\,a^7c^5x^7 - 840\,a^6c^5x^6 - 3780\,a^5c^5x^5 - 504\,a^4c^5x^4 + 1680\,a^3c^5x^3 + 540\,a^2c^5x^2 - 31}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 2*c^5*log(x)/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840*
a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540*a
^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Fricas [A] time = 1.20419, size = 297, normalized size = 2.34

$$\frac{1260\,a^{10}c^5x^{10} + 2520\,a^9c^5x^9\log\left(x\right) + 3780\,a^8c^5x^8 + 5040\,a^7c^5x^7 - 840\,a^6c^5x^6 - 3780\,a^5c^5x^5 - 504\,a^4c^5x^4 + 1680\,a^3c^5x^3}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 + 5
040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 16
80*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Sympy [A] time = 0.900577, size = 124, normalized size = 0.98

$$\frac{a^{10}c^5x + 2a^9c^5\log\left(x\right) + \frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 2*a**9*c**5*log(x) + (3780*a**8*c**5*x**8 + 5040*a**7*c**5*
x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 1680
*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x**9)
)/a**10
```

Giac [A] time = 1.1442, size = 155, normalized size = 1.22

$$c^5x + \frac{2\,c^5\log\left(|x|\right)}{a} + \frac{3780\,a^8c^5x^8 + 5040\,a^7c^5x^7 - 840\,a^6c^5x^6 - 3780\,a^5c^5x^5 - 504\,a^4c^5x^4 + 1680\,a^3c^5x^3 + 540\,a^2c^5x^2 -}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] c^5*x + 2*c^5*log(abs(x))/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 -
 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 +
540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

3.781 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=90

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Rubi [A] time = 0.160054, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx \\
&= -\frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8} \\
&= -\frac{c^4\int \frac{(1-ax)^3(1+ax)^5}{x^8}\,dx}{a^8} \\
&= -\frac{c^4\int \left(-a^8+\frac{1}{x^8}+\frac{2a}{x^7}-\frac{2a^2}{x^6}-\frac{6a^3}{x^5}+\frac{6a^5}{x^3}+\frac{2a^6}{x^2}-\frac{2a^7}{x}\right) dx}{a^8} \\
&= \frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0300186, size = 90, normalized size = 1.

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Maple [A] time = 0.047, size = 83, normalized size = 0.9

$$\frac{c^4}{7\,a^8x^7}+\frac{c^4}{3\,a^7x^6}-\frac{2\,c^4}{5\,a^6x^5}-\frac{3\,c^4}{2\,a^5x^4}+3\,\frac{c^4}{x^2a^3}+2\,\frac{c^4}{a^2x}+c^4x+2\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/x^2/a
^3+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

Maxima [A] time = 1.02686, size = 109, normalized size = 1.21

$$c^4x + \frac{2\,c^4\log\left(x\right)}{a} + \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Fricas [A] time = 1.31071, size = 207, normalized size = 2.3

$$\frac{210\,a^8c^4x^8 + 420\,a^7c^4x^7\log\left(x\right) + 420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^5
*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

Sympy [A] time = 0.644833, size = 88, normalized size = 0.98

$$\frac{a^8c^4x + 2a^7c^4\log\left(x\right) + \frac{420a^6c^4x^6 + 630a^5c^4x^5 - 315a^3c^4x^3 - 84a^2c^4x^2 + 70ac^4x + 30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 + 630*a**5*c**4*x**
5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

Giac [A] time = 1.10228, size = 111, normalized size = 1.23

$$c^4x + \frac{2\,c^4\log\left(|x|\right)}{a} + \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.782 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=76

$$\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

```
[Out] -c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Rubi [A] time = 0.159287, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] -c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx\\
&= \frac{c^3\int\frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}\\
&= \frac{c^3\int\frac{(1-ax)^2(1+ax)^4}{x^6}dx}{a^6}\\
&= \frac{c^3\int\left(a^6+\frac{1}{x^6}+\frac{2a}{x^5}-\frac{a^2}{x^4}-\frac{4a^3}{x^3}-\frac{a^4}{x^2}+\frac{2a^5}{x}\right)dx}{a^6}\\
&= -\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0265082, size = 76, normalized size = 1.

$$\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] -c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Maple [A] time = 0.047, size = 71, normalized size = 0.9

$$-\frac{c^3}{5\,a^6x^5}-\frac{c^3}{2\,a^5x^4}+\frac{c^3}{3\,a^4x^3}+2\,\frac{c^3}{x^2a^3}+\frac{c^3}{a^2x}+c^3x+2\,\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/x^2/a^3+c^3/a^2/x+c^
3*x+2*c^3*ln(x)/a
```

Maxima [A] time = 1.00869, size = 95, normalized size = 1.25

$$c^3x + \frac{2\,c^3\log\left(x\right)}{a} + \frac{30\,a^4c^3x^4 + 60\,a^3c^3x^3 + 10\,a^2c^3x^2 - 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Fricas [A] time = 1.10598, size = 174, normalized size = 2.29

$$\frac{30\,a^6c^3x^6 + 60\,a^5c^3x^5\log\left(x\right) + 30\,a^4c^3x^4 + 60\,a^3c^3x^3 + 10\,a^2c^3x^2 - 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] time = 0.510331, size = 76, normalized size = 1.

$$\frac{a^6c^3x + 2a^5c^3\log\left(x\right) + \frac{30a^4c^3x^4 + 60a^3c^3x^3 + 10a^2c^3x^2 - 15ac^3x - 6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Giac [A] time = 1.12859, size = 96, normalized size = 1.26

$$c^3x + \frac{2\,c^3\log\left(|x|\right)}{a} + \frac{30\,a^4c^3x^4 + 60\,a^3c^3x^3 + 10\,a^2c^3x^2 - 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] c^3*x + 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.783 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Rubi [A] time = 0.146854, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 75}

$$\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```


```
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx\\ &= -\frac{c^2\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\ &= -\frac{c^2\int \frac{(1-ax)(1+ax)^3}{x^4}\,dx}{a^4}\\ &= -\frac{c^2\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right) dx}{a^4}\\ &= \frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0182367, size = 39, normalized size = 1.

$$\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Maple [A] time = 0.045, size = 38, normalized size = 1.

$$\frac{c^2}{3\,a^4x^3}+\frac{c^2}{x^2a^3}+xc^2+2\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/3*c^2/a^4/x^3+c^2/x^2/a^3+x*c^2+2*c^2*ln(x)/a
```

Maxima [A] time = 1.09357, size = 47, normalized size = 1.21

$$c^2x + \frac{2\,c^2\log\left(x\right)}{a} + \frac{3\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Fricas [A] time = 1.31507, size = 97, normalized size = 2.49

$$\frac{3\,a^4c^2x^4 + 6\,a^3c^2x^3\log\left(x\right) + 3\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] time = 0.345109, size = 39, normalized size = 1.

$$\frac{a^4c^2x + 2a^3c^2\log\left(x\right) + \frac{3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Giac [A] time = 1.10009, size = 49, normalized size = 1.26

$$c^2x + \frac{2\,c^2\log\left(|x|\right)}{a} + \frac{3\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] c^2*x + 2*c^2*log(abs(x))/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

3.784 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=21

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

```
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Rubi [A] time = 0.0854281, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6157, 6150, 43}

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx\\ &= \frac{c\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\ &= \frac{c\int \frac{(1+ax)^2}{x^2}\,dx}{a^2}\\ &= \frac{c\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}\\ &= -\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0136718, size = 21, normalized size = 1.

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Maple [A] time = 0.042, size = 22, normalized size = 1.1

$$-\frac{c}{a^2x}+cx+2\,\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2),x)
```

```
[Out] -c/a^2/x+c*x+2*c*ln(x)/a
```

Maxima [A] time = 1.04415, size = 28, normalized size = 1.33

$$cx+\frac{2\,c\log\left(x\right)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*x + 2*c*log(x)/a - c/(a^2*x)
```

Fricas [A] time = 1.44905, size = 57, normalized size = 2.71

$$\frac{a^2cx^2+2\,acx\log\left(x\right)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] time = 0.264319, size = 20, normalized size = 0.95

$$\frac{a^2cx+2ac\log\left(x\right)-\frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2),x)
```

```
[Out] (a**2*c*x + 2*a*c*log(x) - c/x)/a**2
```

Giac [A] time = 1.13647, size = 30, normalized size = 1.43

$$cx + \frac{2\,c\log\left(|x|\right)}{a} - \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] c*x + 2*c*log(abs(x))/a - c/(a^2*x)
```

3.785 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c + 1/(a*c*(1 - a*x)) + (2*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.159508, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c + 1/(a*c*(1 - a*x)) + (2*Log[1 - a*x])/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= \frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= \frac{a^2\int \frac{x^2}{(1-ax)^2}\,dx}{c} \\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0290702, size = 28, normalized size = 0.78

$$\frac{\frac{1}{a-a^2x}+\frac{2\log(1-ax)}{a}+x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (x + (a - a^2*x)^(-1) + (2*Log[1 - a*x])/a)/c
```

Maple [A] time = 0.044, size = 36, normalized size = 1.

$$\frac{x}{c}+2\,\frac{\ln(ax-1)}{ac}-\frac{1}{ac\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2),x)
```

```
[Out] x/c+2/c/a*ln(a*x-1)-1/a/c/(a*x-1)
```

Maxima [A] time = 1.05927, size = 47, normalized size = 1.31

$$\frac{x}{c} - \frac{1}{a^2cx - ac} + \frac{2\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/(a^2*c*x - a*c) + 2*log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.52547, size = 86, normalized size = 2.39

$$\frac{a^2x^2 - ax + 2(ax-1)\log(ax-1) - 1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

Sympy [A] time = 0.31619, size = 36, normalized size = 1.

$$a^2\left(-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\log(ax-1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

Giac [A] time = 1.08939, size = 49, normalized size = 1.36

$$\frac{x}{c} + \frac{2 \log(|ax - 1|)}{ac} - \frac{1}{(ax - 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] x/c + 2*log(abs(a*x - 1))/(a*c) - 1/((a*x - 1)*a*c)
```

3.786 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=75

$$\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Rubi [A] time = 0.176254, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\
&= -\frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.0458892, size = 75, normalized size = 1.

$$
\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Maple [A] time = 0.047, size = 65, normalized size = 0.9

$$\frac{x}{c^2}-\frac{\ln(ax+1)}{8\,ac^2}-\frac{1}{4\,ac^2\,(ax-1)^2}-\frac{7}{4\,ac^2\,(ax-1)}+\frac{17\,\ln(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2-1/8*ln(a*x+1)/a/c^2-1/4/a/c^2/(a*x-1)^2-7/4/a/c^2/(a*x-1)+17/8/a/c^2*
ln(a*x-1)
```

Maxima [A] time = 1.0519, size = 93, normalized size = 1.24

$$-\frac{7\,ax-6}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{x}{c^2}-\frac{\log(ax+1)}{8\,ac^2}+\frac{17\,\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 - 1/8*log(a*x
+ 1)/(a*c^2) + 17/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.49942, size = 211, normalized size = 2.81

$$\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log(ax+1)+17\left(a^2x^2-2\,ax+1\right)\log(ax-1)+12}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*
c^2)
```

Sympy [A] time = 0.584961, size = 73, normalized size = 0.97

$$a^4\left(-\frac{7ax-6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(-(7*a*x - 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

Giac [A] time = 1.13121, size = 77, normalized size = 1.03

$$\frac{x}{c^2}-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{17\,\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 1/8*log(abs(a*x + 1))/(a*c^2) + 17/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

3.787 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=110

$$\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}-\frac{5}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 + 1/(12*a*c^3*(1 - a*x)^3) - 5/(8*a*c^3*(1 - a*x)^2) + 39/(16*a*c^3*(
1 - a*x)) - 1/(16*a*c^3*(1 + a*x)) + (9*Log[1 - a*x])/(4*a*c^3) - Log[1 + a
*x]/(4*a*c^3)
```

Rubi [A] time = 0.199458, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}-\frac{5}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 + 1/(12*a*c^3*(1 - a*x)^3) - 5/(8*a*c^3*(1 - a*x)^2) + 39/(16*a*c^3*(
1 - a*x)) - 1/(16*a*c^3*(1 + a*x)) + (9*Log[1 - a*x])/(4*a*c^3) - Log[1 + a
*x]/(4*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx\\
&= \frac{a^6\int \frac{e^{2\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}\\
&= \frac{a^6\int \frac{x^6}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{4a^6(-1+ax)^4}+\frac{5}{4a^6(-1+ax)^3}+\frac{39}{16a^6(-1+ax)^2}+\frac{9}{4a^6(-1+ax)}+\frac{1}{16a^6(1+ax)^2}-\frac{1}{4a^6(1+ax)}\right)dx}{c^3}\\
&= \frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] time = 0.071405, size = 82, normalized size = 0.75

$$\frac{\frac{2\left(6a^5x^5-12a^4x^4-15a^3x^3+24a^2x^2+7ax-11\right)}{(ax-1)^3(ax+1)}+27\log(1-ax)-3\log(ax+1)}{12ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((2*(-11 + 7*a*x + 24*a^2*x^2 - 15*a^3*x^3 - 12*a^4*x^4 + 6*a^5*x^5))/((-1
+ a*x)^3*(1 + a*x)) + 27*Log[1 - a*x] - 3*Log[1 + a*x])/(12*a*c^3)
```

Maple [A] time = 0.052, size = 95, normalized size = 0.9

$$\frac{x}{c^{3}}-\frac{1}{16\,a c^{3}\left(a x+1\right)}-\frac{\ln\left(a x+1\right)}{4\,a c^{3}}-\frac{1}{12\,a c^{3}\left(a x-1\right)^{3}}-\frac{5}{8\,a c^{3}\left(a x-1\right)^{2}}-\frac{39}{16\,a c^{3}\left(a x-1\right)}+\frac{9\,\ln\left(a x-1\right)}{4\,a c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/16/a/c^3/(a*x+1)-1/4*ln(a*x+1)/a/c^3-1/12/a/c^3/(a*x-1)^3-5/8/a/c^3
/(a*x-1)^2-39/16/a/c^3/(a*x-1)+9/4/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.06008, size = 131, normalized size = 1.19

$$-\frac{15\,a^{3}x^{3}-12\,a^{2}x^{2}-13\,a x+11}{6\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-a c^{3}\right)}+\frac{x}{c^{3}}-\frac{\log\left(a x+1\right)}{4\,a c^{3}}+\frac{9\,\log\left(a x-1\right)}{4\,a c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
 2*a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 9/4*log(a*x - 1)
/(a*c^3)
```

Fricas [A] time = 1.60984, size = 306, normalized size = 2.78

$$\frac{12\,a^{5}x^{5}-24\,a^{4}x^{4}-30\,a^{3}x^{3}+48\,a^{2}x^{2}+14\,a x-3\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,a x-1\right)\log\left(a x+1\right)+27\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,a x-1\right)\text{lo}}{12\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-a c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*x
^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
```

```
- 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.878003, size = 102, normalized size = 0.93

$$a^{6}\left(-\frac{15a^{3}x^{3}-12a^{2}x^{2}-13ax+11}{6a^{11}c^{3}x^{4}-12a^{10}c^{3}x^{3}+12a^{8}c^{3}x-6a^{7}c^{3}}+\frac{x}{a^{6}c^{3}}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{7}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(-(15*a**3*x**3 - 12*a**2*x**2 - 13*a*x + 11)/(6*a**11*c**3*x**4 - 12*
a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(x
- 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

Giac [A] time = 1.13539, size = 108, normalized size = 0.98

$$\frac{x}{c^{3}}-\frac{\log\left(|ax+1|\right)}{4\,ac^{3}}+\frac{9\,\log\left(|ax-1|\right)}{4\,ac^{3}}-\frac{15\,a^{3}x^{3}-12\,a^{2}x^{2}-13\,ax+11}{6\,(ax+1)(ax-1)^{3}ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 9/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

3.788 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=145

$$\frac{99}{32ac^4(1-ax)}-\frac{11}{64ac^4(ax+1)}-\frac{35}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}+\frac{13}{48ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}+\frac{303\log(1-ax)}{128ac^4}$$

```
[Out] x/c^4 - 1/(32*a*c^4*(1 - a*x)^4) + 13/(48*a*c^4*(1 - a*x)^3) - 35/(32*a*c^4
*(1 - a*x)^2) + 99/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) - 11/(64
*a*c^4*(1 + a*x)) + (303*Log[1 - a*x])/(128*a*c^4) - (47*Log[1 + a*x])/(128
*a*c^4)
```

Rubi [A] time = 0.228441, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{99}{32ac^4(1-ax)}-\frac{11}{64ac^4(ax+1)}-\frac{35}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}+\frac{13}{48ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}+\frac{303\log(1-ax)}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 - 1/(32*a*c^4*(1 - a*x)^4) + 13/(48*a*c^4*(1 - a*x)^3) - 35/(32*a*c^4
*(1 - a*x)^2) + 99/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) - 11/(64
*a*c^4*(1 + a*x)) + (303*Log[1 - a*x])/(128*a*c^4) - (47*Log[1 + a*x])/(128
*a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= -\frac{a^8\int \frac{e^{2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= -\frac{a^8\int \frac{x^8}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{8a^8(-1+ax)^5}-\frac{13}{16a^8(-1+ax)^4}-\frac{35}{16a^8(-1+ax)^3}-\frac{99}{32a^8(-1+ax)^2}-\frac{303}{128a^8(-1+ax)}+\frac{1}{32a^8(1+ax)^3}-\frac{1}{64a^8(}\right.}{c^4}$$

$$= \frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}-\frac{}{64ac^4}$$

Mathematica [A] time = 0.103408, size = 98, normalized size = 0.68

$$\frac{\frac{2\left(192a^7x^7-384a^6x^6-819a^5x^5+1254a^4x^4+866a^3x^3-1258a^2x^2-275ax+400\right)}{(ax-1)^4(ax+1)^2}+909\log(1-ax)-141\log(ax+1)}{384ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(400 - 275*a*x - 1258*a^2*x^2 + 866*a^3*x^3 + 1254*a^4*x^4 - 819*a^5*x^
5 - 384*a^6*x^6 + 192*a^7*x^7))/((-1 + a*x)^4*(1 + a*x)^2) + 909*Log[1 - a*
```

```
x] - 141*Log[1 + a*x])/(384*a*c^4)
```

Maple [A] time = 0.053, size = 125, normalized size = 0.9

$$\frac{x}{c^4}+\frac{1}{64\,ac^4\,(ax+1)^2}-\frac{11}{64\,ac^4\,(ax+1)}-\frac{47\,\ln(ax+1)}{128\,ac^4}-\frac{1}{32\,ac^4\,(ax-1)^4}-\frac{13}{48\,ac^4\,(ax-1)^3}-\frac{35}{32\,ac^4\,(ax-1)^2}-\frac{9}{32\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a*x+1)-47/128*ln(a*x+1)/a/c^4-1/32/
a/c^4/(a*x-1)^4-13/48/a/c^4/(a*x-1)^3-35/32/a/c^4/(a*x-1)^2-99/32/a/c^4/(a*
x-1)+303/128/a/c^4*ln(a*x-1)
```

Maxima [A] time = 1.04468, size = 196, normalized size = 1.35

$$-\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{47\,\log(ax+1)}{128\,ac^4}+\frac{303\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x -
400)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x
^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 47/128*log(a*x + 1)/(a*c^4) + 303/128*l
og(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.59104, size = 509, normalized size = 3.51

$$\frac{384\,a^7x^7-768\,a^6x^6-1638\,a^5x^5+2508\,a^4x^4+1732\,a^3x^3-2516\,a^2x^2-550\,ax-141\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2\right.}{384\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*x
^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^
3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
+ 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2*a^6
*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 1.30888, size = 156, normalized size = 1.08

$$a^8\left(-\frac{627a^5x^5-486a^4x^4-1058a^3x^3+874a^2x^2+467ax-400}{192a^{15}c^4x^6-384a^{14}c^4x^5-192a^{13}c^4x^4+768a^{12}c^4x^3-192a^{11}c^4x^2-384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{303\log\left(x-\frac{1}{a}\right)}{128}-}{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*(-(627*a**5*x**5 - 486*a**4*x**4 - 1058*a**3*x**3 + 874*a**2*x**2 + 46
7*a*x - 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/(a**
9*c**4))
```

Giac [A] time = 1.13694, size = 130, normalized size = 0.9

$$\frac{x}{c^4}-\frac{47\log\left(|ax+1|\right)}{128\,ac^4}+\frac{303\log\left(|ax-1|\right)}{128\,ac^4}-\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 47/128*log(abs(a*x + 1))/(a*c^4) + 303/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x
 - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

3.789 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{7a}+c^4x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{14a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-\frac{303c^4}{}$$

```
[Out] (-63*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (37*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(40*a) - (303*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) -
(57*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (15*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(11/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(11/2))/(7*a) + c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x + (15*c^
4*ArcCsc[a*x])/(16*a) + (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/a
```

Rubi [A] time = 0.256587, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{7a}+c^4x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{14a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-\frac{303c^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```


```
[Out] (-63*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (37*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(40*a) - (303*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) -
(57*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (15*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(11/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(11/2))/(7*a) + c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x + (15*c^
4*ArcCsc[a*x])/(16*a) + (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
```

```
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x - c^4\operatorname{Subst}\left(\int\frac{\left(\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x}dx,x,\frac{1}{x}\right)$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x - \frac{1}{7}\left(ac^4\right)\operatorname{Subst}\left(\int\frac{\left(\frac{21}{a^2}-\frac{45}{a}\right.}{}\right.$$

$$= \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x$$

$$= -\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c$$

$$= -\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c}{}$$

$$= -\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c}{}$$

$$= -\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c}{}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{}}{}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{}}{}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{}}{}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{}}{}$$

Mathematica [A] time = 0.238405, size = 126, normalized size = 0.37

$$\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}\left(560a^7x^7-2496a^6x^6-525a^5x^5+992a^4x^4+770a^3x^3-96a^2x^2-280ax-80\right)+1680a^6x^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}\right.\right.\right.}{560a^7x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(-80 - 280*a*x - 96*a^2*x^2 + 770*a^3*x^3 + 992
*a^4*x^4 - 525*a^5*x^5 - 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1
/(a*x)] + 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

Maple [A] time = 0.188, size = 329, normalized size = 1.

$$\frac{(ax-1)^2c^4}{(560\,ax+560)\,a^8x^7}\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}x^8a^8+1680\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6+525\sqrt{a^2x^2-1}\sqrt{a^2}x^7a^7+525\,a^7x^7\sqrt{a^2}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/560*(a*x-1)^2*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^8*a^8+1680*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+525*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^7*a^7+
525*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^2
-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+35*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)*x^5*a^5-816*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-490*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*x^3*a^3+176*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+280*(a^2)^(1/2
)*(a^2*x^2-1)^(3/2)*x*a+80*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))
^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A] time = 1.61582, size = 513, normalized size = 1.5

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2205\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+13615\,c^4\left(\frac{ax-1}{ax+1}\right)}{\frac{6\,(a}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 - (2205*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 13615*c^4*((a*x - 1)/(a*x + 1)
)^(13/2) + 33621*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 39071*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 12799*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 20811*c^4*((a*x -
1)/(a*x + 1))^(5/2) - 7665*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 1155*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A] time = 1.71123, size = 485, normalized size = 1.41

$$-\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(560\,a^8c^4x^8-1936\,a^7c^4\right.}{560\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 - 1936*a^7*c^4*x^7 - 3021*a^6*c^4*x^6 +
 467*a^5*c^4*x^5 + 1762*a^4*c^4*x^4 + 674*a^3*c^4*x^3 - 376*a^2*c^4*x^2 - 3
60*a*c^4*x - 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18898, size = 493, normalized size = 1.44

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{560\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{13300\,(ax-1)c^4\sqrt{\frac{ax-1}{ax+1}}}{ax+1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1
))/a^2 + 560*c^4*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1))
+ (13300*(a*x - 1)*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 45871*(a*x - 1
)^2*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 52672*(a*x - 1)^3*c^4*sqrt(
(a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 33201*(a*x - 1)^4*c^4*sqrt((a*x - 1)/(a*
x + 1))/(a*x + 1)^4 + 11340*(a*x - 1)^5*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1)^5 + 1645*(a*x - 1)^6*c^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^6 + 1715*
c^4*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^7))*a
```

3.790 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$c^3x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{5a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{20a}-\frac{17c^3\sqrt{1-}}{}$$

```
[Out] (-27*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (17*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(8*a) - (29*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(20*a) - (21*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*c
^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(5*a) + c^3*(1 - 1/(a*x))^(3/2)*(
1 + 1/(a*x))^(9/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) + (3*c^3*ArcTanh[Sqrt[1 -
1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.191917, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^3x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{5a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{20a}-\frac{17c^3\sqrt{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```



```
[Out] (-27*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (17*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(8*a) - (29*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(20*a) - (21*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*c
^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(5*a) + c^3*(1 - 1/(a*x))^(3/2)*(
1 + 1/(a*x))^(9/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) + (3*c^3*ArcTanh[Sqrt[1 -
1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^3\,\text{Subst}\left(\int\frac{\left(\frac{3}{a}-\frac{6x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int\frac{\left(\frac{15}{a^2}-\frac{21x}{a^3}\right)\left(1\right.}{x\sqrt{1-\ldots}}\right.$$

$$= -\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x + \frac{1}{20}\left(\ldots\right.$$

$$= -\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\ldots\right.$$

$$= -\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{\ldots}}{\ldots}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\ldots}}{\ldots}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\ldots}}{\ldots}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\ldots}}{\ldots}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\ldots}}{\ldots}$$

Mathematica [A] time = 0.187653, size = 110, normalized size = 0.41

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\left(40a^5x^5-152a^4x^4-55a^3x^3+24a^2x^2+30ax+8\right)+120a^4x^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+15a^4x^4\sin^{-1}\left(\frac{1}{ax}\right)\right)}{40a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(8 + 30*a*x + 24*a^2*x^2 - 55*a^3*x^3 - 152*a^4
*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] + 120*a^4*x^4*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Maple [A] time = 0.189, size = 281, normalized size = 1.

$$\frac{(ax-1)^2 c^3}{(40\,ax+40)\,a^6x^5}\left(-120\,\sqrt{a^2}\sqrt{a^2x^2-1}x^6a^6+120\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4+15\,\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5+15\,\sqrt{a^2}\arctan\left(\frac{}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/40*(a*x-1)^2*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x^6*a^6+120*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*x^4*a^4+15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+15*(
a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*x^5*a^5+120*ln((a^2*x+(a^2*x^2-1)^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+25*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*
a^3-32*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-30*(a^2)^(1/2)*(a^2*x^2-1)^(3/
2)*x*a-8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a
*x-1)*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A] time = 1.53196, size = 408, normalized size = 1.52

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{135\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+575\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+842}{\frac{4\,(ax-1)a^2}{ax+1}+\frac{5\,(ax-1)^2}{(ax+1)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
```

```
(135*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 575*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 + 842*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 298*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 465*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

Fricas [A] time = 1.71341, size = 419, normalized size = 1.56

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(40\,a^6c^3x^6-112\,a^5c^3x^5-207\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 - 112*a^5*c^3*x^5 - 207*a^4*c^3*x^4 - 31*a^3*
c^3*x^3 + 54*a^2*c^3*x^2 + 38*a*c^3*x + 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(
a^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2177, size = 401, normalized size = 1.49

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{40\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{810\,(ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{912\,(\ldots}{\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
^2 + 40*c^3*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (81
0*(a*x - 1)*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 912*(a*x - 1)^2*c^3*s
qrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 470*(a*x - 1)^3*c^3*sqrt((a*x - 1)/(
a*x + 1))/(a*x + 1)^3 + 95*(a*x - 1)^4*c^3*sqrt((a*x - 1)/(a*x + 1))/(a*x +
 1)^4 + 145*c^3*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^5
))*a
```

3.791 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$c^2x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}+\frac{3c^2}{\cdots}$$

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (11*c^2*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/(6*a) - (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(3*a) + c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x - (c^2*ArcCsc[a*x])
/(2*a) + (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.134631, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}+\frac{3c^2}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (11*c^2*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/(6*a) - (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(3*a) + c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x - (c^2*ArcCsc[a*x])
/(2*a) + (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - c^2\,\text{Subst}\left(\int \frac{\left(\frac{3}{a}-\frac{4x}{a^2}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$= -\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x + \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(-\frac{9}{a^2}+\frac{11x}{a^3}\right)(1}{x\sqrt{1-\frac{x}{a}}}\right.$$

$$= -\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{6}\left(a^2c^2\right.$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}$$

Mathematica [A] time = 0.136157, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-16a^2x^2-9ax-2\right)+18a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 - 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x
^2*ArcSin[1/(a*x)] + 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3
*x^2)

Maple [A] time = 0.175, size = 233, normalized size = 1.2

$$\frac{(ax-1)^2 c^2}{(6\,ax+6)\,a^4x^3}\left(-18\,\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+18\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-3\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+18\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x)

[Out] 1/6*(a*x-1)^2*c^2*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+18*(a^2*x^2-1)
^(3/2)*(a^2)^(1/2)*x^2*a^2-3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+18*ln((a
^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-3*a^3*x^3*(a^2)^(1
/2)*arctan(1/(a^2*x^2-1)^(1/2))+9*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+2*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))
^(1/2)/a^4/x^3/(a^2)^(1/2)

Maxima [A] time = 1.56766, size = 301, normalized size = 1.54

$$\frac{1}{3}\,a\left(\frac{3\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+37\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+17\,c^2\left(\frac{ax-1}{ax+1}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")

[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (15
c^2((a*x - 1)/(a*x + 1))^(7/2) + 37*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 17*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 21*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))

Fricas [A] time = 1.67075, size = 362, normalized size = 1.86

$$\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4-10\,a^3c^2x^3-25\,a^2c^2x^2-\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 25*a^2*c^2*x^2 - 11*a*c^2*x - 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int -\frac{2a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4/(a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**4
```

Giac [A] time = 1.21752, size = 311, normalized size = 1.59

$$\frac{1}{3}a\left(\frac{3\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9\,c^2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{6\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}-\frac{\frac{28\,(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{9\,(ax-1)^2c^2\sqrt{\cdot}}{(ax+1)^2}}{a^2\left(\frac{ax-1}{ax+1}+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
- 6*c^2*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) - (28*(a*
x - 1)*c^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 9*(a*x - 1)^2*c^2*sqrt((a*
x - 1)/(a*x + 1))/(a*x + 1)^2 + 27*c^2*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*
x - 1)/(a*x + 1) + 1)^3))
```

3.792 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=76

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x - (3*c*ArcCsc[a*x])/a + (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.0581777, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6194, 98, 12, 105, 41, 216, 92, 208}

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x - (3*c*ArcCsc[a*x])/a + (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx=-\left(c\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)\right)$$
$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+c\,\text{Subst}\left(\int-\frac{3\sqrt{1+\frac{x}{a}}}{ax\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$
$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{(3c)\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{a}$$
$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{(3c)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{a^2}-\frac{(3c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\right.}{a}$$
$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{(3c)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{a^2}+\frac{(3c)\,\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}dx,x,\sqrt{1-\right.}{a^2}$$
$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] time = 0.0881379, size = 57, normalized size = 0.75

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(ax+1)+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) - 3*ArcSin[1/(a*x)] + 3*Log[(1 + Sqrt[1
 - 1/(a^2*x^2)])*x]))/a
```

Maple [B] time = 0.172, size = 235, normalized size = 3.1

$$-\frac{(ax-1)^2c}{(ax+1)a^2x}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2}\sqrt{a^2x^2-1}xa+\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)xa^2+3\,ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x)
```

```
[Out] -(a*x-1)^2*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)+3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2))/(a^2)^(1/2))*x*a^2+3*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-
4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1
)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A] time = 1.55693, size = 159, normalized size = 2.09

$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.65023, size = 266, normalized size = 3.5

$$\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+2\,acx+c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 + 2*
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int \frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int -\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**3*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1)), x))/a**2
```

Giac [A] time = 1.18587, size = 154, normalized size = 2.03

$$a\left(\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] a*(6*c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 - 3*c*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 4*c*sqrt
((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

3.793 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=144

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{\frac{1}{ax}+1}}{3ac\sqrt{1-\frac{1}{ax}}}-\frac{5\sqrt{\frac{1}{ax}+1}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] (-5*Sqrt[1 + 1/(a*x)])/(3*a*c*(1 - 1/(a*x))^(3/2)) - (14*Sqrt[1 + 1/(a*x)])
/(3*a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*(1 - 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rubi [A] time = 0.101411, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{\frac{1}{ax}+1}}{3ac\sqrt{1-\frac{1}{ax}}}-\frac{5\sqrt{\frac{1}{ax}+1}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```


```
[Out] (-5*Sqrt[1 + 1/(a*x)])/(3*a*c*(1 - 1/(a*x))^(3/2)) - (14*Sqrt[1 + 1/(a*x)])
/(3*a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*(1 - 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{3}{a}+\frac{2x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{a\,\text{Subst}\left(\int \frac{-\frac{9}{a^2}-\frac{5x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{a^2\,\text{Subst}\left(\int \frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] time = 0.124484, size = 69, normalized size = 0.48

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(3a^2x^2-19ax+14\right)}{(ax-1)^2}+\frac{9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 - 19*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + (9*Log[
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [B] time = 0.184, size = 346, normalized size = 2.4

$$\frac{1}{3\,(ax-1)\,ac\,(ax+1)}\left(9\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^3a^4+9\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^3a^3-27\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/3*(9*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+
9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-27*ln((a^2*x+(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
3/2)*x*a-27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+27*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+5*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)+27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-9*a*ln((a^2*x+(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-9*(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/a/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(
a*x+1))^(3/2)
```

Maxima [A] time = 1.06804, size = 180, normalized size = 1.25

$$\frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a
*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a
^2*c))
```

Fricas [A] time = 1.58576, size = 301, normalized size = 2.09

$$\frac{9\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^{3}x^{3}-16\,a^{2}x^{2}-5\,ax+14\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{2}\int\frac{x^{2}}{\frac{a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c
```

Giac [A] time = 1.195, size = 200, normalized size = 1.39

$$\frac{1}{3}\,a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}c}-\frac{(ax+1)\left(\frac{12\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^{2}c\sqrt{\frac{ax-1}{ax+1}}}-\frac{6\,\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 1/3*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(abs(sqrt((a*x -
 1)/(a*x + 1)) - 1))/(a^2*c) - (a*x + 1)*(12*(a*x - 1)/(a*x + 1) + 1)/((a*x
 - 1)*a^2*c*sqrt((a*x - 1)/(a*x + 1))) - 6*sqrt((a*x - 1)/(a*x + 1))/(a^2*c
*((a*x - 1)/(a*x + 1) - 1)))
```

3.794 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}}{5ac^2\sqrt{1-\frac{1}{ax}}}-\frac{9\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{6\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] (-6*Sqrt[1 + 1/(a*x)])/(5*a*c^2*(1 - 1/(a*x))^(5/2)) - (9*Sqrt[1 + 1/(a*x)]
)/(5*a*c^2*(1 - 1/(a*x))^(3/2)) - (24*Sqrt[1 + 1/(a*x)])/(5*a*c^2*Sqrt[1 -
1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c^2*(1 - 1/(a*x))^(5/2)) + (3*ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rubi [A] time = 0.128265, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 103, 21, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}}{5ac^2\sqrt{1-\frac{1}{ax}}}-\frac{9\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{6\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```


```
[Out] (-6*Sqrt[1 + 1/(a*x)])/(5*a*c^2*(1 - 1/(a*x))^(5/2)) - (9*Sqrt[1 + 1/(a*x)]
)/(5*a*c^2*(1 - 1/(a*x))^(3/2)) - (24*Sqrt[1 + 1/(a*x)])/(5*a*c^2*Sqrt[1 -
1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c^2*(1 - 1/(a*x))^(5/2)) + (3*ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{6\,\text{Subst}\left(\int \frac{-\frac{5}{2}-\frac{2x}{a}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{15}{2a}+\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{(2a)\,\text{Subst}\left(\int -\frac{15}{2a^2x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\right)}{5c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\right)}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-}\right)}{a^2c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

Mathematica [A] time = 0.167211, size = 78, normalized size = 0.43

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(5a^3x^3-39a^2x^2+57ax-24)}{5(ax-1)^3}+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(5*(-1
 + a*x)^3) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] time = 0.191, size = 438, normalized size = 2.4

$$\frac{1}{40\,a\,(ax-1)^2\,c^2\,(ax+1)}\left(120\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^4a^5+125\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^4a^4-480\,\ln\left(\frac{a^2x+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/40*(120*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a
^5+125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-480*ln((a^2*x+(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-85*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^2*a^2-500*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+720*ln(
(a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+148*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+750*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x
^2*a^2-480*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^
2-67*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-500*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2)*x*a+120*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))
+125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a*x
-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.11999, size = 207, normalized size = 1.14

$$\frac{1}{20}a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A] time = 1.65065, size = 385, normalized size = 2.13

$$\frac{15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(5\,a^4x^4-34\,a^3x^3+18\,a^2\right.}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20969, size = 224, normalized size = 1.24

$$\frac{1}{20}a\left(\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)^2\left(\frac{10\,(ax-1)}{ax+1}+\frac{85\,(ax-1)^2}{(ax+1)^2}+1\right)}{(ax-1)^2a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{40\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/20*a*(60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 60*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)^2*(10*(a*x - 1)/(a*x + 1) +
 85*(a*x - 1)^2/(a*x + 1)^2 + 1)/((a*x - 1)^2*a^2*c^2*sqrt((a*x - 1)/(a*x +
 1))) - 40*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.795 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=255

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+}}$$

```
[Out] -8/(7*a*c^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - 53/(35*a*c^3*(1 - 1/(a
*x))^(5/2)*Sqrt[1 + 1/(a*x)]) - 88/(35*a*c^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1
/(a*x)]) - 281/(35*a*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (176*Sqrt[1
 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(7/2)*Sqrt
[1 + 1/(a*x)]) + (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^3)
```

Rubi [A] time = 0.172283, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```


```
[Out] -8/(7*a*c^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - 53/(35*a*c^3*(1 - 1/(a
*x))^(5/2)*Sqrt[1 + 1/(a*x)]) - 88/(35*a*c^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1
/(a*x)]) - 281/(35*a*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (176*Sqrt[1
 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(7/2)*Sqrt
[1 + 1/(a*x)]) + (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^3)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{21}{a^2}+\frac{32x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{7c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^2\,\text{Subst}\left(\int \cdots\right.}{35\cdots}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\cdots}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\cdots}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\cdots}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\cdots}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\cdots}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\cdots}$$

Mathematica [A] time = 0.208061, size = 101, normalized size = 0.4

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(35a^5x^5-286a^4x^4+368a^3x^3+125a^2x^2-423ax+176\right)}{35(ax-1)^4(ax+1)}+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(176 - 423*a*x + 125*a^2*x^2 + 368*a^3*x^3 - 28
6*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)^4*(1 + a*x)) + 3*Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x])/(a*c^3)
```

Maple [B] time = 0.206, size = 714, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/1120*(3360*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^
7*a^8+3675*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^7*a^7-10080*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7-2555*(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(3/2)*x^5*a^5-11025*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a
^6+3360*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6
+1873*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4+3675*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x^5*a^5+16800*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)
)/(a^2)^(1/2))*x^4*a^5+4426*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+183
75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-16800*ln((a^2*x+(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-3350*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^2*a^2-18375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-3360*
ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-2511*(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-3675*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x^2*a^2+10080*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2
))*x*a^2+1957*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+11025*(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2)*x*a-3360*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/
(a^2)^(1/2))-3675*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a/(a^2)^(1/2)/(a*x-1
)^3/c^3/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.02702, size = 259, normalized size = 1.02

$$\frac{1}{560}\,a\left(\frac{\frac{51\,(ax-1)}{ax+1}+\frac{294\,(ax-1)^2}{(ax+1)^2}+\frac{2170\,(ax-1)^3}{(ax+1)^3}-\frac{3640\,(ax-1)^4}{(ax+1)^4}+5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}+\frac{35\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/560*a*((51*(a*x - 1)/(a*x + 1) + 294*(a*x - 1)^2/(a*x + 1)^2 + 2170*(a*x
- 1)^3/(a*x + 1)^3 - 3640*(a*x - 1)^4/(a*x + 1)^4 + 5)/(a^2*c^3*((a*x - 1)/
(a*x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 1680*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c^3) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Fricas [A] time = 1.65872, size = 468, normalized size = 1.84

$$\frac{105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(35\,a\right.}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - 105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 + 125
*a^2*x^2 - 423*a*x + 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c
^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27798, size = 277, normalized size = 1.09

$$\frac{1}{560}\,a\left(\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^3\left(\frac{56\,(ax-1)}{ax+1}+\frac{350\,(ax-1)^2}{(ax+1)^2}+\frac{2520\,(ax-1)^3}{(ax+1)^3}+5\right)}{(ax-1)^3a^2c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] 1/560*a*(1680*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(abs(s
qrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^3*(56*(a*x - 1)/(a*x +
 1) + 350*(a*x - 1)^2/(a*x + 1)^2 + 2520*(a*x - 1)^3/(a*x + 1)^3 + 5)/((a*x
 - 1)^3*a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) + 35*sqrt((a*x - 1)/(a*x + 1))/(
a^2*c^3) - 1120*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*((a*x - 1)/(a*x + 1) - 1
)))
```

3.796 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=329

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+\right.}$$

```
[Out] -10/(9*a*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)) - 29/(21*a*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)) - 208/(105*a*c^4*(1 - 1/(a*x))^(5/2)*(1
+ 1/(a*x))^(3/2)) - 1147/(315*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)
) - 1462/(105*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (2609*Sqrt[1 -
 1/(a*x)])/(315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*
a*c^4*Sqrt[1 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rubi [A] time = 0.236387, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```


```
[Out] -10/(9*a*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)) - 29/(21*a*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)) - 208/(105*a*c^4*(1 - 1/(a*x))^(5/2)*(1
+ 1/(a*x))^(3/2)) - 1147/(315*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)
) - 1462/(105*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (2609*Sqrt[1 -
 1/(a*x)])/(315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*
a*c^4*Sqrt[1 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{27}{a^2}+\frac{60x}{a^3}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{9c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{a^2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{}{c^4\left(1-\right.}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{}{315ac}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{}{315ac}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{}{315ac}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{}{315ac}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{}{315ac}$$

$$10 \qquad 29 \qquad 208$$

Mathematica [A] time = 0.273661, size = 117, normalized size = 0.36

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(315a^7x^7-2669a^6x^6+2967a^5x^5+4029a^4x^4-7399a^3x^3+339a^2x^2+4047ax-1664\right)}{315(ax-1)^5(ax+1)^2}+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1664 + 4047*a*x + 339*a^2*x^2 - 7399*a^3*x^3
+ 4029*a^4*x^4 + 2967*a^5*x^5 - 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x
)^5*(1 + a*x)^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] time = 0.192, size = 766, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/40320*(-138915*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^9*a^9-120960*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^9*a^10+98595*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^7*a^7+416745*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2)*x^8*a^8+362880*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(
1/2))*x^8*a^9-75113*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^6*a^6-240861*(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^5*a^5-1111320*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(1/2)*x^6*a^6-967680*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2
)^(1/2))*x^6*a^7+178863*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4+833490*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+725760*ln((a^2*x+(a^2)^(1/2)*((
a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+252497*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^3*a^3+833490*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+7257
60*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-1823
07*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-1111320*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x^3*a^3-967680*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
))/(a^2)^(1/2))*x^3*a^4-101271*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+7407
7*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+416745*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2)*x*a+362880*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)
)*x*a^2-138915*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)-120960*a*ln((a^2*x+(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^4/c^4/(
```

```
(a*x-1)*(a*x+1))^(1/2)/(a*x+1)^4/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A] time = 1.10717, size = 305, normalized size = 0.93

$$\frac{1}{20160}a\left(\frac{\frac{415\,(ax-1)}{ax+1}+\frac{2511\,(ax-1)^2}{(ax+1)^2}+\frac{11739\,(ax-1)^3}{(ax+1)^3}+\frac{80745\,(ax-1)^4}{(ax+1)^4}-\frac{135765\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{105\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+30\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{60480\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*((415*(a*x - 1)/(a*x + 1) + 2511*(a*x - 1)^2/(a*x + 1)^2 + 11739*
(a*x - 1)^3/(a*x + 1)^3 + 80745*(a*x - 1)^4/(a*x + 1)^4 - 135765*(a*x - 1)^
5/(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x -
 1)/(a*x + 1))^(9/2)) + 105*(((a*x - 1)/(a*x + 1))^(3/2) + 30*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4
) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A] time = 1.71006, size = 571, normalized size = 1.74

$$\frac{945\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(\right.}{315\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2
*x^2 + 4*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^7*x^7 - 2669*
a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3*x^3 + 339*a^2*x^2 + 4047*a
*x - 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*
c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27637, size = 356, normalized size = 1.08

$$\frac{1}{20160}a\left(\frac{60480\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60480\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{(ax+1)^4\left(\frac{450\,(ax-1)}{ax+1}+\frac{2961\,(ax-1)^2}{(ax+1)^2}+\frac{14700\,(ax-1)^3}{(ax+1)^3}+\frac{95445\,(a}{(ax+1}\right.}{(ax-1)^4a^2c^4\sqrt{\frac{ax-1}{ax+1}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/20160*a*(60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 60480*log(a
bs(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - (a*x + 1)^4*(450*(a*x - 1)/(
a*x + 1) + 2961*(a*x - 1)^2/(a*x + 1)^2 + 14700*(a*x - 1)^3/(a*x + 1)^3 + 9
5445*(a*x - 1)^4/(a*x + 1)^4 + 35)/((a*x - 1)^4*a^2*c^4*sqrt((a*x - 1)/(a*x
 + 1))) - 40320*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1
)) + 105*((a*x - 1)*a^4*c^8*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 30*a^4*c^
8*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^12))
```

3.797 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=116

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rubi [A] time = 0.170696, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx \\
&= -\frac{c^5\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}dx}{a^{10}} \\
&= -\frac{c^5\int\frac{(1-ax)^3(1+ax)^7}{x^{10}}dx}{a^{10}} \\
&= -\frac{c^5\int\left(-a^{10}+\frac{1}{x^{10}}+\frac{4a}{x^9}+\frac{3a^2}{x^8}-\frac{8a^3}{x^7}-\frac{14a^4}{x^6}+\frac{14a^6}{x^4}+\frac{8a^7}{x^3}-\frac{3a^8}{x^2}-\frac{4a^9}{x}\right)dx}{a^{10}} \\
&= \frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.041808, size = 116, normalized size = 1.

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Maple [A] time = 0.046, size = 105, normalized size = 0.9

$$\frac{c^5}{9\,a^{10}x^9}+\frac{c^5}{2\,a^9x^8}+\frac{3\,c^5}{7\,a^8x^7}-\frac{4\,c^5}{3\,a^7x^6}-\frac{14\,c^5}{5\,a^6x^5}+\frac{14\,c^5}{3\,a^4x^3}+4\,\frac{c^5}{x^2a^3}-3\,\frac{c^5}{a^2x}+c^5x+4\,\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x)
```

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/x^2/a^3-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Maxima [A] time = 1.00432, size = 139, normalized size = 1.2

$$c^5x + \frac{4\,c^5\log(x)}{a} - \frac{1890\,a^8c^5x^8 - 2520\,a^7c^5x^7 - 2940\,a^6c^5x^6 + 1764\,a^4c^5x^4 + 840\,a^3c^5x^3 - 270\,a^2c^5x^2 - 315\,ac^5x - 70\,c^5}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940*
a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*a*
c^5*x - 70*c^5)/(a^10*x^9)
```

Fricas [A] time = 1.54395, size = 269, normalized size = 2.32

$$\frac{630\,a^{10}c^5x^{10} + 2520\,a^9c^5x^9\log(x) - 1890\,a^8c^5x^8 + 2520\,a^7c^5x^7 + 2940\,a^6c^5x^6 - 1764\,a^4c^5x^4 - 840\,a^3c^5x^3 + 270\,a^2c^5x^2 +}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 252
0*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 270
*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

Sympy [A] time = 0.900921, size = 112, normalized size = 0.97

$$\frac{a^{10}c^5x + 4a^9c^5\log(x) - \frac{1890a^8c^5x^8 - 2520a^7c^5x^7 - 2940a^6c^5x^6 + 1764a^4c^5x^4 + 840a^3c^5x^3 - 270a^2c^5x^2 - 315ac^5x - 70c^5}{630x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 4*a**9*c**5*log(x) - (1890*a**8*c**5*x**8 - 2520*a**7*c**5*
x**7 - 2940*a**6*c**5*x**6 + 1764*a**4*c**5*x**4 + 840*a**3*c**5*x**3 - 270
*a**2*c**5*x**2 - 315*a*c**5*x - 70*c**5)/(630*x**9))/a**10
```

Giac [A] time = 1.12725, size = 248, normalized size = 2.14

$$-\frac{4\,c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(630\,c^5+\frac{4049\,c^5}{ax-1}+\frac{6201\,c^5}{(ax-1)^2}-\frac{18036\,c^5}{(ax-1)^3}-\frac{89124\,c^5}{(ax-1)^4}-\frac{160146\,c^5}{(ax-1)^5}-\frac{153090\,c^5}{(ax-1)^6}-\frac{80220}{(ax-}\right.}{630\,a\left(\frac{1}{ax-1}+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] -4*c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^5*log(abs(-1/(a*x - 1
) - 1))/a + 1/630*(630*c^5 + 4049*c^5/(a*x - 1) + 6201*c^5/(a*x - 1)^2 - 18
036*c^5/(a*x - 1)^3 - 89124*c^5/(a*x - 1)^4 - 160146*c^5/(a*x - 1)^5 - 1530
90*c^5/(a*x - 1)^6 - 80220*c^5/(a*x - 1)^7 - 21420*c^5/(a*x - 1)^8 - 2520*c
^5/(a*x - 1)^9)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^9)
```

3.798 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=100

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Rubi [A] time = 0.166298, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx\\ &= \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}\\ &= \frac{c^4\int \frac{(1-ax)^2(1+ax)^6}{x^8}dx}{a^8}\\ &= \frac{c^4\int \left(a^8+\frac{1}{x^8}+\frac{4a}{x^7}+\frac{4a^2}{x^6}-\frac{4a^3}{x^5}-\frac{10a^4}{x^4}-\frac{4a^5}{x^3}+\frac{4a^6}{x^2}+\frac{4a^7}{x}\right)dx}{a^8}\\ &= -\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0329438, size = 100, normalized size = 1.

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Maple [A] time = 0.053, size = 93, normalized size = 0.9

$$-\frac{c^4}{7\,a^8x^7}-\frac{2\,c^4}{3\,a^7x^6}-\frac{4\,c^4}{5\,a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10\,c^4}{3\,a^4x^3}+2\,\frac{c^4}{x^2a^3}-4\,\frac{c^4}{a^2x}+c^4x+4\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/x^2/a^3-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Maxima [A] time = 1.02203, size = 124, normalized size = 1.24

$$c^4x + \frac{4\,c^4\log(x)}{a} - \frac{420\,a^6c^4x^6 - 210\,a^5c^4x^5 - 350\,a^4c^4x^4 - 105\,a^3c^4x^3 + 84\,a^2c^4x^2 + 70\,ac^4x + 15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^4
*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x^7
)
```

Fricas [A] time = 1.4738, size = 231, normalized size = 2.31

$$\frac{105\,a^8c^4x^8 + 420\,a^7c^4x^7\log(x) - 420\,a^6c^4x^6 + 210\,a^5c^4x^5 + 350\,a^4c^4x^4 + 105\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x - 15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^5
*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x
- 15*c^4)/(a^8*x^7)
```

Sympy [A] time = 0.721691, size = 100, normalized size = 1.

$$\frac{a^8c^4x + 4a^7c^4\log(x) - \frac{420a^6c^4x^6 - 210a^5c^4x^5 - 350a^4c^4x^4 - 105a^3c^4x^3 + 84a^2c^4x^2 + 70ac^4x + 15c^4}{105x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 4*a**7*c**4*log(x) - (420*a**6*c**4*x**6 - 210*a**5*c**4*x**
5 - 350*a**4*c**4*x**4 - 105*a**3*c**4*x**3 + 84*a**2*c**4*x**2 + 70*a*c**4
*x + 15*c**4)/(105*x**7))/a**8
```

Giac [A] time = 1.12703, size = 216, normalized size = 2.16

$$-\frac{4\,c^4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^4\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(105\,c^4+\frac{659\,c^4}{ax-1}+\frac{1253\,c^4}{(ax-1)^2}-\frac{231\,c^4}{(ax-1)^3}-\frac{3885\,c^4}{(ax-1)^4}-\frac{5250\,c^4}{(ax-1)^5}-\frac{2730\,c^4}{(ax-1)^6}-\frac{420\,c^4}{(ax-1)^7}\right)(a}{105\,a\left(\frac{1}{ax-1}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -4*c^4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^4*log(abs(-1/(a*x - 1
) - 1))/a + 1/105*(105*c^4 + 659*c^4/(a*x - 1) + 1253*c^4/(a*x - 1)^2 - 231
*c^4/(a*x - 1)^3 - 3885*c^4/(a*x - 1)^4 - 5250*c^4/(a*x - 1)^5 - 2730*c^4/(
a*x - 1)^6 - 420*c^4/(a*x - 1)^7)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^7)
```

3.799 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rubi [A] time = 0.151695, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 75}

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx\\
&= -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}\\
&= -\frac{c^3\int \frac{(1-ax)(1+ax)^5}{x^6}\,dx}{a^6}\\
&= -\frac{c^3\int \left(-a^6+\frac{1}{x^6}+\frac{4a}{x^5}+\frac{5a^2}{x^4}-\frac{5a^4}{x^2}-\frac{4a^5}{x}\right)dx}{a^6}\\
&= \frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0231702, size = 63, normalized size = 1.

$$
\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Maple [A] time = 0.046, size = 60, normalized size = 1.

$$
\frac{c^3}{5\,a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5\,c^3}{3\,a^4x^3}-5\,\frac{c^3}{a^2x}+c^3x+4\,\frac{c^3\ln(x)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

Maxima [A] time = 1.07938, size = 80, normalized size = 1.27

$$c^3x + \frac{4\,c^3 \log(x)}{a} - \frac{75\,a^4c^3x^4 - 25\,a^2c^3x^2 - 15\,ac^3x - 3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*x
 - 3*c^3)/(a^6*x^5)
```

Fricas [A] time = 1.57881, size = 151, normalized size = 2.4

$$\frac{15\,a^6c^3x^6 + 60\,a^5c^3x^5\log(x) - 75\,a^4c^3x^4 + 25\,a^2c^3x^2 + 15\,ac^3x + 3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3*
x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

Sympy [A] time = 0.498356, size = 65, normalized size = 1.03

$$\frac{a^6c^3x + 4a^5c^3\log(x) - \frac{75a^4c^3x^4 - 25a^2c^3x^2 - 15ac^3x - 3c^3}{15x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 4*a**5*c**3*log(x) - (75*a**4*c**3*x**4 - 25*a**2*c**3*x**2
- 15*a*c**3*x - 3*c**3)/(15*x**5))/a**6
```

Giac [B] time = 1.12317, size = 184, normalized size = 2.92

$$-\frac{4\,c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(15\,c^3+\frac{107\,c^3}{ax-1}+\frac{235\,c^3}{(ax-1)^2}+\frac{170\,c^3}{(ax-1)^3}-\frac{30\,c^3}{(ax-1)^4}-\frac{60\,c^3}{(ax-1)^5}\right)(ax-1)}{15\,a\left(\frac{1}{ax-1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -4*c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^3*log(abs(-1/(a*x - 1
) - 1))/a + 1/15*(15*c^3 + 107*c^3/(a*x - 1) + 235*c^3/(a*x - 1)^2 + 170*c^
3/(a*x - 1)^3 - 30*c^3/(a*x - 1)^4 - 60*c^3/(a*x - 1)^5)*(a*x - 1)/(a*(1/(a
*x - 1) + 1)^5)
```

3.800 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Rubi [A] time = 0.147742, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx\\ &= \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\ &= \frac{c^2\int \frac{(1+ax)^4}{x^4}\,dx}{a^4}\\ &= \frac{c^2\int\left(a^4+\frac{1}{x^4}+\frac{4a}{x^3}+\frac{6a^2}{x^2}+\frac{4a^3}{x}\right)dx}{a^4}\\ &= -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0205453, size = 51, normalized size = 1.

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Maple [A] time = 0.044, size = 50, normalized size = 1.

$$-\frac{c^2}{3\,a^4x^3}-2\,\frac{c^2}{x^2a^3}-6\,\frac{c^2}{a^2x}+xc^2+4\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/x^2/a^3-6*c^2/a^2/x+x*c^2+4*c^2*ln(x)/a
```

Maxima [A] time = 1.0086, size = 62, normalized size = 1.22

$$c^2x + \frac{4\,c^2\log\left(x\right)}{a} - \frac{18\,a^2c^2x^2 + 6\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

Fricas [A] time = 1.49503, size = 122, normalized size = 2.39

$$\frac{3\,a^4c^2x^4 + 12\,a^3c^2x^3\log\left(x\right) - 18\,a^2c^2x^2 - 6\,ac^2x - c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x - c
^2)/(a^4*x^3)
```

Sympy [A] time = 0.397397, size = 51, normalized size = 1.

$$\frac{a^4c^2x + 4a^3c^2\log\left(x\right) - \frac{18a^2c^2x^2+6ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 4*a**3*c**2*log(x) - (18*a**2*c**2*x**2 + 6*a*c**2*x + c**2)
/(3*x**3))/a**4
```

Giac [B] time = 1.169, size = 151, normalized size = 2.96

$$-\frac{4\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(3\,c^2+\frac{34\,c^2}{ax-1}+\frac{66\,c^2}{(ax-1)^2}+\frac{36\,c^2}{(ax-1)^3}\right)(ax-1)}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^2*log(abs(-1/(a*x - 1
) - 1))/a + 1/3*(3*c^2 + 34*c^2/(a*x - 1) + 66*c^2/(a*x - 1)^2 + 36*c^2/(a*
x - 1)^3)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^3)
```

3.801 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=33

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rubi [A] time = 0.0933374, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6157, 6150, 88}

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx\\ &= -\frac{c\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\ &= -\frac{c\int\frac{(1+ax)^3}{x^2(1-ax)}\,dx}{a^2}\\ &= -\frac{c\int\left(-a^2+\frac{1}{x^2}+\frac{4a}{x}-\frac{8a^2}{-1+ax}\right)dx}{a^2}\\ &= \frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0199034, size = 33, normalized size = 1.

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Maple [A] time = 0.055, size = 33, normalized size = 1.

$$cx+\frac{c}{a^2x}-4\,\frac{c\ln(x)}{a}+8\,\frac{c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x)
```

```
[Out] c*x+c/a^2/x-4*c*ln(x)/a+8*c/a*ln(a*x-1)
```

Maxima [A] time = 1.02735, size = 43, normalized size = 1.3

$$cx+\frac{8\,c\log\left(ax-1\right)}{a}-\frac{4\,c\log\left(x\right)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

Fricas [A] time = 1.57583, size = 88, normalized size = 2.67

$$\frac{a^2cx^2+8\,acx\log\left(ax-1\right)-4\,acx\log\left(x\right)+c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

Sympy [A] time = 0.457601, size = 26, normalized size = 0.79

$$cx+\frac{4c\left(-\log\left(x\right)+2\log\left(x-\frac{1}{a}\right)\right)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2),x)
```

```
[Out] c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

Giac [A] time = 1.11255, size = 89, normalized size = 2.7

$$-\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{4\,c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{(ax-1)c}{a\left(\frac{1}{ax-1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -4*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4*c*log(abs(-1/(a*x - 1) -
1))/a + (a*x - 1)*c/(a*(1/(a*x - 1) + 1))
```

3.802 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rubi [A] time = 0.168444, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 77}

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= -\frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c} \\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.0337655, size = 53, normalized size = 1.

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Maple [A] time = 0.043, size = 51, normalized size = 1.

$$\frac{x}{c}-\frac{1}{ac\left(ax-1\right)^2}+4\,\frac{\ln\left(ax-1\right)}{ac}-5\,\frac{1}{ac\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x)
```

```
[Out] x/c-1/a/c/(a*x-1)^2+4/c/a*ln(a*x-1)-5/a/c/(a*x-1)
```

Maxima [A] time = 1.04098, size = 66, normalized size = 1.25

$$-\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Fricas [A] time = 1.58226, size = 140, normalized size = 2.64

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+4}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] time = 0.424671, size = 41, normalized size = 0.77

$$-\frac{5ax-4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2),x)
```

```
[Out] -(5*a*x - 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Giac [A] time = 1.13249, size = 100, normalized size = 1.89

$$\frac{ax-1}{ac} - \frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac} - \frac{\frac{5a^3c}{ax-1} + \frac{a^3c}{(ax-1)^2}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - (5*a^3*c
/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

3.803 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rubi [A] time = 0.172337, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\
&= \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= \frac{a^4\int \frac{x^4}{(1-ax)^4}\,dx}{c^2}\\
&= \frac{a^4\int \left(\frac{1}{a^4}+\frac{1}{a^4(-1+ax)^4}+\frac{4}{a^4(-1+ax)^3}+\frac{6}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0379981, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-9a^2x^2+27ax+12(ax-1)^3\log(1-ax)-13}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1 -
 a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A] time = 0.056, size = 66, normalized size = 0.9

$$\frac{x}{c^{2}}-2\,\frac{1}{ac^{2}\left(ax-1\right)^{2}}-\frac{1}{3\,ac^{2}\left(ax-1\right)^{3}}+4\,\frac{\ln\left(ax-1\right)}{ac^{2}}-6\,\frac{1}{ac^{2}\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2-2/a/c^2/(a*x-1)^2-1/3/a/c^2/(a*x-1)^3+4/a/c^2*ln(a*x-1)-6/a/c^2/(a*x-
1)
```

Maxima [A] time = 1.03598, size = 101, normalized size = 1.42

$$-\frac{18\,a^{2}x^{2}-30\,ax+13}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}+\frac{x}{c^{2}}+\frac{4\,\log\left(ax-1\right)}{ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.50434, size = 215, normalized size = 3.03

$$\frac{3\,a^{4}x^{4}-9\,a^{3}x^{3}-9\,a^{2}x^{2}+27\,ax+12\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(ax-1\right)-13}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2 +
 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x -
 a*c^2)
```

Sympy [A] time = 0.533337, size = 83, normalized size = 1.17

$$a^{4}\left(-\frac{18a^{2}x^{2}-30ax+13}{3a^{8}c^{2}x^{3}-9a^{7}c^{2}x^{2}+9a^{6}c^{2}x-3a^{5}c^{2}}+\frac{x}{a^{4}c^{2}}+\frac{4\log\left(ax-1\right)}{a^{5}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(-(18*a**2*x**2 - 30*a*x + 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2))
```

Giac [A] time = 1.12164, size = 126, normalized size = 1.77

$$\frac{ax-1}{ac^{2}}-\frac{4\log\left(\frac{|ax-1|}{(ax-1)^{2}|a|}\right)}{ac^{2}}-\frac{\frac{18\,a^{5}c^{4}}{ax-1}+\frac{6\,a^{5}c^{4}}{(ax-1)^{2}}+\frac{a^{5}c^{4}}{(ax-1)^{3}}}{3\,a^{6}c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(18*a^5*c^4/(a*x - 1) + 6*a^5*c^4/(a*x - 1)^2 + a^5*c^4/(a*x - 1)^3)/(a^6*c
^6)
```

3.804 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rubi [A] time = 0.19596, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx\\ &= -\frac{a^6\int \frac{e^{4\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}\\ &= -\frac{a^6\int \frac{x^6}{(1-ax)^5(1+ax)}\,dx}{c^3}\\ &= -\frac{a^6\int \left(-\frac{1}{a^6}-\frac{1}{2a^6(-1+ax)^5}-\frac{11}{4a^6(-1+ax)^4}-\frac{49}{8a^6(-1+ax)^3}-\frac{111}{16a^6(-1+ax)^2}-\frac{129}{32a^6(-1+ax)}+\frac{1}{32a^6(1+ax)}\right)dx}{c^3}\\ &= \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1+}{32a}\end{aligned}$$

Mathematica [A] time = 0.0627582, size = 89, normalized size = 0.8

$$\frac{2\left(48a^5x^5-192a^4x^4-45a^3x^3+660a^2x^2-701ax+224\right)+387(ax-1)^4\log(1-ax)-3(ax-1)^4\log(ax+1)}{96ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-1
+ a*x)^4)
```

Maple [A] time = 0.058, size = 95, normalized size = 0.9

$$\frac{x}{c^3}-\frac{\ln(ax+1)}{32\,ac^3}-\frac{1}{8\,ac^3\,(ax-1)^4}-\frac{11}{12\,ac^3\,(ax-1)^3}-\frac{49}{16\,ac^3\,(ax-1)^2}-\frac{111}{16\,ac^3\,(ax-1)}+\frac{129\,\ln(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/32*ln(a*x+1)/a/c^3-1/8/a/c^3/(a*x-1)^4-11/12/a/c^3/(a*x-1)^3-49/16/
a/c^3/(a*x-1)^2-111/16/a/c^3/(a*x-1)+129/32/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.0878, size = 144, normalized size = 1.3

$$-\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{x}{c^3}-\frac{\log(ax+1)}{32\,ac^3}+\frac{129\,\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a*c
^3) + 129/32*log(a*x - 1)/(a*c^3)
```

Fricas [A] time = 1.5235, size = 370, normalized size = 3.33

$$\frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log(ax+1)+387\left(a^4x^4-4\,a\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*(
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*a^4
```

```
*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] time = 0.923373, size = 114, normalized size = 1.03

$$a^{6}\left(-\frac{333a^{3}x^{3}-852a^{2}x^{2}+749ax-224}{48a^{11}c^{3}x^{4}-192a^{10}c^{3}x^{3}+288a^{9}c^{3}x^{2}-192a^{8}c^{3}x+48a^{7}c^{3}}+\frac{x}{a^{6}c^{3}}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^{7}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(-(333*a**3*x**3 - 852*a**2*x**2 + 749*a*x - 224)/(48*a**11*c**3*x**4
- 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c**3
) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

Giac [A] time = 1.12352, size = 176, normalized size = 1.59

$$\frac{ax-1}{ac^{3}}-\frac{4\log\left(\frac{|ax-1|}{(ax-1)^{2}|a|}\right)}{ac^{3}}-\frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^{3}}-\frac{\frac{333\,a^{11}c^{9}}{ax-1}+\frac{147\,a^{11}c^{9}}{(ax-1)^{2}}+\frac{44\,a^{11}c^{9}}{(ax-1)^{3}}+\frac{6\,a^{11}c^{9}}{(ax-1)^{4}}}{48\,a^{12}c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/32
*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(333*a^11*c^9/(a*x - 1) + 147*a^
11*c^9/(a*x - 1)^2 + 44*a^11*c^9/(a*x - 1)^3 + 6*a^11*c^9/(a*x - 1)^4)/(a^1
2*c^12)
```

3.805 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx$

Optimal. Leaf size=146

$$\frac{501}{64ac^4(1-ax)} - \frac{1}{64ac^4(ax+1)} - \frac{67}{16ac^4(1-ax)^2} + \frac{83}{48ac^4(1-ax)^3} - \frac{7}{16ac^4(1-ax)^4} + \frac{1}{20ac^4(1-ax)^5} + \frac{261\log(1-ax)}{64ac^4}$$

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rubi [A] time = 0.221346, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{501}{64ac^4(1-ax)} - \frac{1}{64ac^4(ax+1)} - \frac{67}{16ac^4(1-ax)^2} + \frac{83}{48ac^4(1-ax)^3} - \frac{7}{16ac^4(1-ax)^4} + \frac{1}{20ac^4(1-ax)^5} + \frac{261\log(1-ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```


```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= \frac{a^8\int \frac{e^{4\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$

$$= \frac{a^8\int \left(\frac{1}{a^8}+\frac{1}{4a^8(-1+ax)^6}+\frac{7}{4a^8(-1+ax)^5}+\frac{83}{16a^8(-1+ax)^4}+\frac{67}{8a^8(-1+ax)^3}+\frac{501}{64a^8(-1+ax)^2}+\frac{261}{64a^8(-1+ax)}+\frac{1}{64a^8(1+ax}\right.}{c^4}$$

$$= \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-\frac{}{64ac^4}$$

Mathematica [A] time = 0.0958636, size = 98, normalized size = 0.67

$$\frac{\frac{2\left(480a^7x^7-1920a^6x^6-1365a^5x^5+9300a^4x^4-6800a^3x^3-4900a^2x^2+7541ax-2384\right)}{(ax-1)^5(ax+1)}+3915\log(1-ax)-75\log(ax+1)}{960ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*a
^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log[1
```

```
- a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

Maple [A] time = 0.06, size = 125, normalized size = 0.9

$$\frac{x}{c^4} - \frac{1}{64\,ac^4\,(ax+1)} - \frac{5\,\ln(ax+1)}{64\,ac^4} - \frac{1}{20\,ac^4\,(ax-1)^5} - \frac{7}{16\,ac^4\,(ax-1)^4} - \frac{83}{48\,ac^4\,(ax-1)^3} - \frac{67}{16\,ac^4\,(ax-1)^2} - \frac{501}{64\,ac^4\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4-1/64/a/c^4/(a*x+1)-5/64*ln(a*x+1)/a/c^4-1/20/a/c^4/(a*x-1)^5-7/16/a/c
^4/(a*x-1)^4-83/48/a/c^4/(a*x-1)^3-67/16/a/c^4/(a*x-1)^2-501/64/a/c^4/(a*x-
1)+261/64/a/c^4*ln(a*x-1)
```

Maxima [A] time = 1.06622, size = 182, normalized size = 1.25

$$-\frac{3765\,a^5x^5 - 9300\,a^4x^4 + 4400\,a^3x^3 + 6820\,a^2x^2 - 8021\,ax + 2384}{480\left(a^7c^4x^6 - 4\,a^6c^4x^5 + 5\,a^5c^4x^4 - 5\,a^3c^4x^2 + 4\,a^2c^4x - ac^4\right)} + \frac{x}{c^4} - \frac{5\,\log(ax+1)}{64\,ac^4} + \frac{261\,\log(ax-1)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*
a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x - 1
)/(a*c^4)
```

Fricas [A] time = 1.28138, size = 479, normalized size = 3.28

$$\frac{960\,a^7x^7 - 3840\,a^6x^6 - 2730\,a^5x^5 + 18600\,a^4x^4 - 13600\,a^3x^3 - 9800\,a^2x^2 + 15082\,ax - 75\left(a^6x^6 - 4\,a^5x^5 + 5\,a^4x^4 - 5\,a^2\right.}{960\left(a^7c^4x^6 - 4\,a^6c^4x^5 + 5\,a^5c^4x^4 - 5\,a^3c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a^
3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*
a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 +
 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [A] time = 1.37593, size = 144, normalized size = 0.99

$$a^8\left(-\frac{3765a^5x^5-9300a^4x^4+4400a^3x^3+6820a^2x^2-8021ax+2384}{480a^{15}c^4x^6-1920a^{14}c^4x^5+2400a^{13}c^4x^4-2400a^{11}c^4x^2+1920a^{10}c^4x-480a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64}-\frac{5\log\left(x+\frac{1}{a}\right)}{64}}{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*(-(3765*a**5*x**5 - 9300*a**4*x**4 + 4400*a**3*x**3 + 6820*a**2*x**2 -
 8021*a*x + 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**13*
c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) + x/(
a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

Giac [A] time = 1.109, size = 230, normalized size = 1.58

$$\frac{(ax-1)\left(\frac{257}{ax-1}+128\right)}{128\,ac^4\left(\frac{2}{ax-1}+1\right)}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4}-\frac{5\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4}-\frac{\frac{7515\,a^{19}c^{16}}{ax-1}+\frac{4020\,a^{19}c^{16}}{(ax-1)^2}+\frac{1660\,a^{19}c^{16}}{(ax-1)^3}+\frac{420\,a^{19}c^{16}}{(ax-1)^4}+\frac{48\,a^{19}c^{16}}{(ax-1)^5}}{960\,a^{20}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/128*(a*x - 1)*(257/(a*x - 1) + 128)/(a*c^4*(2/(a*x - 1) + 1)) - 4*log(abs
(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 5/64*log(abs(-2/(a*x - 1) - 1))/(
a*c^4) - 1/960*(7515*a^19*c^16/(a*x - 1) + 4020*a^19*c^16/(a*x - 1)^2 + 166
0*a^19*c^16/(a*x - 1)^3 + 420*a^19*c^16/(a*x - 1)^4 + 48*a^19*c^16/(a*x - 1
)^5)/(a^20*c^20)
```

3.806 $\int e^{-\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^4 dx$

Optimal. Leaf size=343

$$c^4x\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{7/2}}{7a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{5/2}}{6a}+\frac{29c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{3/2}}{30a}+\frac{19c}{}$$

```
[Out] (-19*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (c^4*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(16*a) + (7*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(40*a) + (19*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(40*a) + (29*c^
4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(30*a) + (7*c^4*(1 - 1/(a*x))^(5
/2)*(1 + 1/(a*x))^(7/2))/(6*a) + (8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
7/2))/(7*a) + c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x + (35*c^4*ArcCs
c[a*x])/(16*a) - (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.256663, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^4x\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{7/2}}{7a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{5/2}}{6a}+\frac{29c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{3/2}}{30a}+\frac{19c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-19*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (c^4*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(16*a) + (7*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(40*a) + (19*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(40*a) + (29*c^
4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(30*a) + (7*c^4*(1 - 1/(a*x))^(5
/2)*(1 + 1/(a*x))^(7/2))/(6*a) + (8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
7/2))/(7*a) + c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x + (35*c^4*ArcCs
c[a*x])/(16*a) - (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x - c^4\operatorname{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{7}\left(ac^4\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{7}{a^2}-\frac{49}{a}\right.}{}\right.$$

$$= \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x -$$

$$= \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} + \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} +$$

$$= \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} + \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} + \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} +$$

$$= \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} + \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} +$$

$$= -\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} +$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} +$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} +$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} +$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} +$$

Mathematica [A] time = 0.276369, size = 120, normalized size = 0.35

$$\frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1680a^7x^7+2816a^6x^6+3045a^5x^5-1952a^4x^4-1330a^3x^3+1056a^2x^2+280ax-240\right)}{x^6}-1680a^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+3675a^6\sin^{-1}\left(\right.\right.}{1680a^7}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]

[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(-240 + 280*a*x + 1056*a^2*x^2 - 1330*a^3*x^3 - 1952*a^4*x^4 + 3045*a^5*x^5 + 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^6*ArcSin[1/(a*x)] - 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7)

Maple [A] time = 0.152, size = 320, normalized size = 0.9

$$-\frac{(ax+1)\,c^4}{1680\,a^8x^7}\sqrt{\frac{ax-1}{ax+1}}\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}x^8a^8+1680\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6-3675\sqrt{a^2x^2-1}\sqrt{a^2}x^7a^7-3675\,a^7x^7\sqrt{a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x)

[Out] -1/1680*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^8*a^8+1680*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6-3675*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^7*a^7-3675*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+1995*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5-1136*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-1050*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+816*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+280*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-240*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)

Maxima [A] time = 1.62895, size = 513, normalized size = 1.5

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{1995\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10185\,c^4\left(\frac{ax-1}{ax+1}\right.}{6\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (1995*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10185*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 17619*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 4569*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 71801*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 72051*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 31465*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 5355*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A] time = 1.45438, size = 490, normalized size = 1.43

$$-\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(1680\,a^8c^4x^8+4496\,a^7c\right.}{1680\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 + 4496*a^7*c^4*x^7 + 5861*a^6*c^4*x^6
 + 1093*a^5*c^4*x^5 - 3282*a^4*c^4*x^4 - 274*a^3*c^4*x^3 + 1336*a^2*c^4*x^2
 + 40*a*c^4*x - 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

[Out] Timed out

Giac [A] time = 1.22717, size = 707, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^4*sgn(a*x + 1)/a - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(a
)*sgn(a*x + 1) - 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
+ 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) - 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 9065*(x*abs(a) - sqrt(
a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 -
1))^8*a*c^4*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn
(a*x + 1) - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) -
 38976*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) - 6860*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) - 12992*(x*abs(a) - sqrt(a^
2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 3045*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*
abs(a)*sgn(a*x + 1) - 2816*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 -
 1))^2 + 1)^7*a*abs(a))
```

3.807 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$c^3x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{5a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{11c^3\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{12a}+\cdots$$

```
[Out] (-7*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (c^3*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(24*a) + (11*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(12*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(4*a) + (6*c^3*
(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(5*a) + c^3*(1 - 1/(a*x))^(7/2)*(1
 + 1/(a*x))^(5/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) - (c^3*ArcTanh[Sqrt[1 - 1/
(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.185821, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^3x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{5a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{11c^3\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{12a}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (c^3*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(24*a) + (11*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(12*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(4*a) + (6*c^3*
(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(5*a) + c^3*(1 - 1/(a*x))^(7/2)*(1
 + 1/(a*x))^(5/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) - (c^3*ArcTanh[Sqrt[1 - 1/
(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^3\,\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int\frac{\left(-\frac{5}{a^2}-\frac{25x}{a^3}\right.}{}\right.$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - $$

$$= \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(\right.$$

$$= \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\right.}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)}{}$$

Mathematica [A] time = 0.186756, size = 110, normalized size = 0.41

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\left(120a^5x^5+184a^4x^4+135a^3x^3-88a^2x^2-30ax+24\right)-120a^4x^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+225a^4x^4\sin^{-1}\left(\right.}{120a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(24 - 30*a*x - 88*a^2*x^2 + 135*a^3*x^3 + 184*a
^4*x^4 + 120*a^5*x^5) + 225*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 +
Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5*x^4)
```

Maple [A] time = 0.138, size = 272, normalized size = 1.

$$-\frac{(ax+1)c^3}{120\,a^6x^5}\sqrt{\frac{ax-1}{ax+1}}\left(-120\sqrt{a^2}\sqrt{a^2x^2-1}x^6a^6+120\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4-225\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5-225\sqrt{a^2}\operatorname{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/120*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(1
/2)*x^6*a^6+120*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-225*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*x^5*a^5-225*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*x^5*a^5+12
0*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+105*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-30*
(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1
)*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A] time = 1.6208, size = 408, normalized size = 1.52

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+305\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+86}{\frac{4(ax-1)a^2}{ax+1}+\frac{5(ax-}{(ax+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
```

```
(105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(9/2
) + 86*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 345*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

Fricas [A] time = 1.38954, size = 424, normalized size = 1.58

$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(120\,a^6c^3x^6+304\,a^5c^3x^5+3\right.}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 + 304*a^5*c^3*x^5 + 319*a^4*c^3*x^4 + 47*a
^3*c^3*x^3 - 118*a^2*c^3*x^2 - 6*a*c^3*x + 24*c^3)*sqrt((a*x - 1)/(a*x + 1)
))/(a^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21115, size = 532, normalized size = 1.98

$$-\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{4\,a}+\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}c^3\mathrm{sgn}\left(ax+1\right)}{a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^3*sgn(a*x + 1)/a - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) - 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) + 150
*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) - 720*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) - 1120*(x*abs(a) - sqrt(a^2*x^2 -
1))^4*a*c^3*sgn(a*x + 1) - 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*
sgn(a*x + 1) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 13
5*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) - 184*a*c^3*sgn(a*
x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

3.808 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$c^2x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}+\frac{4c^2\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}$$

```
[Out] -(c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (3*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(2*a) + (4*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2
))/(3*a) + c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) - (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.13166, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}+\frac{4c^2\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] -(c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (3*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(2*a) + (4*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2
))/(3*a) + c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) - (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx=-\left(c^2\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$=c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x-c^2\,\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x}dx,x,\frac{1}{x}\right)$$

$$=\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int\frac{\left(-\frac{3}{a^2}-\frac{9x}{a^3}\right)\cdots}{\cdots}\right.$$

$$=\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{1}{6}\left(\cdots\right.$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\cdots\right.$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\cdots\right.$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\cdots\right.$$

$$=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\cdots\right.$$

Mathematica [A] time = 0.137046, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3+8a^2x^2+3ax-2\right)-6a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+9a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^
2*ArcSin[1/(a*x)] - 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x
^2)
```

Maple [A] time = 0.146, size = 224, normalized size = 1.2

$$-\frac{(ax+1)\,c^2}{6\,a^4x^3}\sqrt{\frac{ax-1}{ax+1}}\left(-6\,\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+6\,\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2-9\,\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+6\,\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x^4*a^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-9*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x^3*a^3+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^
4-9*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+3*(a^2)^(1/2)*(a^2*x^2-
1)^(3/2)*x*a-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^4/x
^3/(a^2)^(1/2)
```

Maxima [A] time = 1.5345, size = 301, normalized size = 1.54

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+29\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3
*c^2*((a*x - 1)/(a*x + 1))^(7/2) + c^2*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

Fricas [A] time = 1.50378, size = 358, normalized size = 1.84

$$-\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^2x^4+14\,a^3c^2x^3+11\,a^2c^2x^2+\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^2*x^4 + 14*a^3*c^2*x^3 + 11*a^2*c^2*x^2 + a*c^2*x - 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int-\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-2*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**2, x))/a**4
```

Giac [A] time = 1.18411, size = 356, normalized size = 1.83

$$-\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{a}+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\mathrm{sgn}\left(ax+1\right)}{a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^2*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*
sgn(a*x + 1)/a - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
 + 1) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) - 12*(x*abs(
a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*x^2 -
 1))*c^2*abs(a)*sgn(a*x + 1) - 8*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

3.809 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=108

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}+\frac{2c\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{a}+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] (2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*(1 - 1/(a*x))^(3/2)*Sqrt[1
+ 1/(a*x)]*x + (c*ArcCsc[a*x])/a - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/a
```

Rubi [A] time = 0.0763682, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6194, 97, 154, 21, 105, 41, 216, 92, 208}

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}+\frac{2c\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{a}+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```


```
[Out] (2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*(1 - 1/(a*x))^(3/2)*Sqrt[1
+ 1/(a*x)]*x + (c*ArcCsc[a*x])/a - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - c\,\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - (ac)\,\text{Subst}\left(\int\frac{-\frac{1}{a^2}-\frac{x}{a^3}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{c\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2} + \frac{c\,\text{Subs}}{}$$

$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2} - \frac{c\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{c\csc^{-1}(ax)}{a} - \frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right.}{a}$$

Mathematica [A] time = 0.083748, size = 55, normalized size = 0.51

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(ax+1)-\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) + ArcSin[1/(a*x)] - Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x]))/a
```

Maple [A] time = 0.13, size = 166, normalized size = 1.5

$$-\frac{c\,(ax+1)}{a^2x}\sqrt{\frac{ax-1}{ax+1}}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\sqrt{a^2x^2-1}\,xa+\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-a*x*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2)))/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A] time = 1.51355, size = 158, normalized size = 1.46

$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A] time = 1.42899, size = 262, normalized size = 2.43

$$-\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2+2\,acx+c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) - a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 + 2*a*c
*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int -\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a**2
```

Giac [A] time = 1.14917, size = 163, normalized size = 1.51

$$-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}+\frac{}{\left(\left(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a*
x + 1)/a + 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a))
```

3.810 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=105

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{\frac{1}{ax}+1}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] (2*Sqrt[1 - 1/(a*x)])/(a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*Sq
rt[1 + 1/(a*x)]) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rubi [A] time = 0.0773693, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 21, 94, 92, 208}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{\frac{1}{ax}+1}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)])/(a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*Sq
rt[1 + 1/(a*x)]) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\sqrt{1+\frac{1}{ax}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

<u>Mathematica</u> [A] time = 0.113981, size = 57, normalized size = 0.54

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+2)}{ax+1} - \frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x))/(1 + a*x) - Log[(1 + Sqrt[1 - 1/(a^2*x
^2)])*x]/a)/c
```

Maple [B] time = 0.157, size = 250, normalized size = 2.4

$$-\frac{1}{2a(ax+1)c}\left(2\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)x^2a^3-3\sqrt{a^2}\sqrt{(ax-1)(ax+1)}x^2a^2+4\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)(ax}{\sqrt{a^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x)
```

```
[Out] -1/2*(2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3
-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+4*ln((a^2*x+(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((
a*x-1)/(a*x+1))^(1/2)/(a^2)^(1/2)/(a*x+1)/c/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.03992, size = 163, normalized size = 1.55

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Fricas [A] time = 1.46311, size = 161, normalized size = 1.53

$$\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \int \frac{x^2 \sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^2x^2-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] undef
```

3.811 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal. Leaf size=179

$$\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] -2/(a*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (5*Sqrt[1 - 1/(a*x)])/(3
*a*c^2*(1 + 1/(a*x))^(3/2)) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*
x)]) + x/(c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - ArcTanh[Sqrt[1 - 1/(
a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rubi [A] time = 0.11761, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -2/(a*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (5*Sqrt[1 - 1/(a*x)])/(3
*a*c^2*(1 + 1/(a*x))^(3/2)) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*
x)]) + x/(c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - ArcTanh[Sqrt[1 - 1/(
a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{1}{a^2}+\frac{4x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{3}{a^3}+\frac{5x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, \ldots\right.}{3c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^3\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\ldots}{x\sqrt{\ldots}}\right.}{\ldots}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\ldots}\right.}{\ldots}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\tanh^{-1}\left(\sqrt{1-\ldots}\right.}{ac\ldots}$$

Mathematica [A] time = 0.152771, size = 85, normalized size = 0.47

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(3a^3x^3+7a^2x^2-5ax-8\right)}{3(ax-1)(ax+1)^2} - \log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 5*a*x + 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
a*x)*(1 + a*x)^2) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] time = 0.143, size = 530, normalized size = 3.

$$-\frac{1}{24\,a\,(ax-1)^2\,(ax+1)^2\,c^2}\left(-45\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^5a^5+24\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^5a^6+21\,\sqrt{a^2}\,((ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/24*(-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+24*ln((a^2*x+(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6+21*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^3*a^3-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+24*ln
((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-11*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3-48*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^
3*a^4-5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+90*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2)*x^2*a^2-48*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)
^(1/2))*x^2*a^3+19*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2)*x*a+24*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a
^2)^(1/2))*x*a^2-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+24*a*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(1/2)/(a
*x-1)^2/(a^2)^(1/2)/(a*x+1)^2/c^2/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.02282, size = 220, normalized size = 1.23

$$-\frac{1}{12}\,a\left(\frac{3\left(\frac{9\,(ax-1)}{ax+1}-1\right)}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+18\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*a*(3*(9*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - (((a*x - 1)/(a*x + 1))^(3/2) + 18*s
qrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 12*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A] time = 1.45643, size = 267, normalized size = 1.49

$$-\frac{3\left(a^{2}x^{2}-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^{2}x^{2}-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(3\,a^{3}x^{3}+7\,a^{2}x^{2}-5\,ax-8\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}c^{2}x^{2}-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^2 - 1)*
log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*sq
rt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{4}\int\frac{x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{4}x^{4}-2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x**
*2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^2, x)
```

3.812 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} \, dx$

Optimal. Leaf size=255

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\quad}{3ac^3\left(1-\frac{1}{ax}\right.}$$

```
[Out] -4/(3*a*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) - 13/(3*a*c^3*Sqrt[1 -
 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (14*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*
x))^(5/2)) + (11*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sqr
t[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(3/2)*(1
 + 1/(a*x))^(5/2)) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rubi [A] time = 0.165558, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\quad}{3ac^3\left(1-\frac{1}{ax}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```


```
[Out] -4/(3*a*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) - 13/(3*a*c^3*Sqrt[1 -
 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (14*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*
x))^(5/2)) + (11*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sqr
t[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(3/2)*(1
 + 1/(a*x))^(5/2)) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{16x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{3c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^2\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}} +$$

Mathematica [A] time = 0.205026, size = 101, normalized size = 0.4

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(15a^5x^5+38a^4x^4-52a^3x^3-87a^2x^2+33ax+48\right)}{15(ax-1)^2(ax+1)^3}-\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 33*a*x - 87*a^2*x^2 - 52*a^3*x^3 + 38*a^4
*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^2*(1 + a*x)^3) - Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x])/(a*c^3)
```

Maple [B] time = 0.154, size = 714, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/240*(240*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^7
*a^8-525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^7*a^7+240*ln((a^2*x+(a^2)^(1
/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7+285*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^5*a^5-525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a^6-720*
ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6-83*(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4+1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2)*x^5*a^5-720*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2
))*x^4*a^5-218*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+1575*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+720*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2))*x^3*a^4+342*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a
^2-1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+720*ln((a^2*x+(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3-3*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x*a-1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2-240*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-243*((a*x-1)*(a
*x+1))^(3/2)*(a^2)^(1/2)+525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-240*a*
ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+525*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+1))^(1/2)/(a^2)^(1/2)/(a*x-1)^3/(
a*x+1)^3/c^3/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.03878, size = 266, normalized size = 1.04

$$\frac{1}{240}a\left(\frac{5\left(\frac{23(ax-1)}{ax+1}-\frac{120(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+450\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}+\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^2c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*(5*(23*(a*x - 1)/(a*x + 1) - 120*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (3
*((a*x - 1)/(a*x + 1))^(5/2) + 40*((a*x - 1)/(a*x + 1))^(3/2) + 450*sqrt((a
*x - 1)/(a*x + 1)))/(a^2*c^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2
*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A] time = 1.30839, size = 366, normalized size = 1.44

$$-\frac{15\left(a^4x^4-2a^2x^2+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^4x^4-2a^2x^2+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(15a^5x^5+38a^4x^4-52a^3x^3-87a^2x\right.}{15\left(a^5c^3x^4-2a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (15*a^5*x^5
 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x + 48)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^3, x)
```

3.813 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=329

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(\cdots\right.}$$

```
[Out] -6/(5*a*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - 31/(15*a*c^4*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)) - 28/(3*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(7/2)) + (115*Sqrt[1 - 1/(a*x)])/(21*a*c^4*(1 + 1/(a*x))^(7/2)) + (122
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (93*Sqrt[1 - 1/(a*x)])
/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
+ 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rubi [A] time = 0.225071, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```


```
[Out] -6/(5*a*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - 31/(15*a*c^4*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)) - 28/(3*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(7/2)) + (115*Sqrt[1 - 1/(a*x)])/(21*a*c^4*(1 + 1/(a*x))^(7/2)) + (122
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (93*Sqrt[1 - 1/(a*x)])
/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
+ 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{5}{a^2}+\frac{36x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{5c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{a^2\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\cdots}{c^4\left(1-\frac{1}{ax}\right)\cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\cdots}}{21ac^4\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\cdots}}{21ac^4\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\cdots}}{21ac^4\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\cdots}}{21ac^4\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\cdots}}{21ac^4\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{115\sqrt{1-\cdots}}{21ac^4\left(1+\cdots\right.}$$

$$6 \quad 31 \quad 28 \quad 115\sqrt{1-\cdots}$$

Mathematica [A] time = 0.255458, size = 117, normalized size = 0.36

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(105a^7x^7+281a^6x^6-559a^5x^5-965a^4x^4+715a^3x^3+1065a^2x^2-279ax-384\right)}{105(ax-1)^3(ax+1)^4}-\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-384 - 279*a*x + 1065*a^2*x^2 + 715*a^3*x^3 -
965*a^4*x^4 - 559*a^5*x^5 + 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^3*(
1 + a*x)^4) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] time = 0.175, size = 898, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/13440*(-33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^9*a^9-53760*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-53760*ln((a^2*x
+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^7*a^8+132300*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2)*x^7*a^7-27673*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2
)*x^5*a^5-198450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+7705*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+132300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2)*x^6*a^6+24295*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4+13440*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-37095*(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+2637*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x
*a-53760*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^
3+80640*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^6
+80640*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5-
33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+132300*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2)*x^3*a^3-198450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+13
440*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^8*a^9-537
60*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7+1323
00*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+16077*((a*x-1)*(a*x+1))^(3/2
)*(a^2)^(1/2)-33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)+13440*a*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+19635*(a^2)^(1/2)*((a*x-1)
```

```
*(a*x+1))^(3/2)*x^7*a^7-33075*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^8*a^8-2
893*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^6*a^6+13440*ln((a^2*x+(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^9*a^10)/a*((a*x-1)/(a*x+1))^(1/2)/
(a^2)^(1/2)/(a*x-1)^4/(a*x+1)^4/c^4/((a*x-1)*(a*x+1))^(1/2)
```

Maxima [A] time = 1.0368, size = 312, normalized size = 0.95

$$\frac{1}{6720}\,a\left(\frac{7\left(\frac{47\,(ax-1)}{ax+1}+\frac{655\,(ax-1)^2}{(ax+1)^2}-\frac{2625\,(ax-1)^3}{(ax+1)^3}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+329\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2940\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}-\frac{6720\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(7*(47*(a*x - 1)/(a*x + 1) + 655*(a*x - 1)^2/(a*x + 1)^2 - 2625*(a
*x - 1)^3/(a*x + 1)^3 + 3)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*(
(a*x - 1)/(a*x + 1))^(5/2)) + 5*(3*((a*x - 1)/(a*x + 1))^(7/2) + 42*((a*x -
 1)/(a*x + 1))^(5/2) + 329*((a*x - 1)/(a*x + 1))^(3/2) + 2940*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4)
 + 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A] time = 1.3448, size = 471, normalized size = 1.43

$$-\frac{105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(105\,a^7x^7+281\,a\right.}{105\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x +
 1)) + 1) - 105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 + 71
5*a^3*x^3 + 1065*a^2*x^2 - 279*a*x - 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c
^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^4, x)
```

3.814 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=90

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Rubi [A] time = 0.153422, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx\\ &= -\frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8}\\ &= -\frac{c^4\int \frac{(1-ax)^5(1+ax)^3}{x^8}\,dx}{a^8}\\ &= -\frac{c^4\int \left(-a^8+\frac{1}{x^8}-\frac{2a}{x^7}-\frac{2a^2}{x^6}+\frac{6a^3}{x^5}-\frac{6a^5}{x^3}+\frac{2a^6}{x^2}+\frac{2a^7}{x}\right) dx}{a^8}\\ &= \frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0329902, size = 90, normalized size = 1.

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Maple [A] time = 0.049, size = 83, normalized size = 0.9

$$\frac{c^4}{7\,a^8x^7}-\frac{c^4}{3\,a^7x^6}-\frac{2\,c^4}{5\,a^6x^5}+\frac{3\,c^4}{2\,a^5x^4}-3\,\frac{c^4}{x^2a^3}+2\,\frac{c^4}{a^2x}+c^4x-2\,\frac{c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/x^2/a
^3+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

Maxima [A] time = 1.03559, size = 109, normalized size = 1.21

$$c^4x - \frac{2\,c^4\log\left(x\right)}{a} + \frac{420\,a^6c^4x^6 - 630\,a^5c^4x^5 + 315\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Fricas [A] time = 1.34407, size = 207, normalized size = 2.3

$$\frac{210\,a^8c^4x^8 - 420\,a^7c^4x^7\log\left(x\right) + 420\,a^6c^4x^6 - 630\,a^5c^4x^5 + 315\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^5
*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

Sympy [A] time = 0.653665, size = 88, normalized size = 0.98

$$\frac{a^8c^4x - 2a^7c^4\log\left(x\right) + \frac{420a^6c^4x^6 - 630a^5c^4x^5 + 315a^3c^4x^3 - 84a^2c^4x^2 - 70ac^4x + 30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**8*c**4*x - 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 - 630*a**5*c**4*x**
5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

Giac [A] time = 1.12806, size = 111, normalized size = 1.23

$$c^4x - \frac{2\,c^4\log\left(|x|\right)}{a} + \frac{420\,a^6c^4x^6 - 630\,a^5c^4x^5 + 315\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.815 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

```
[Out] -c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Rubi [A] time = 0.155948, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$-\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx\\ &= \frac{c^3\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}\\ &= \frac{c^3\int \frac{(1-ax)^4(1+ax)^2}{x^6}\,dx}{a^6}\\ &= \frac{c^3\int \left(a^6+\frac{1}{x^6}-\frac{2a}{x^5}-\frac{a^2}{x^4}+\frac{4a^3}{x^3}-\frac{a^4}{x^2}-\frac{2a^5}{x}\right)dx}{a^6}\\ &= -\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0247236, size = 76, normalized size = 1.

$$-\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Maple [A] time = 0.047, size = 71, normalized size = 0.9

$$-\frac{c^3}{5\,a^6x^5}+\frac{c^3}{2\,a^5x^4}+\frac{c^3}{3\,a^4x^3}-2\,\frac{c^3}{x^2a^3}+\frac{c^3}{a^2x}+c^3x-2\,\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/x^2/a^3+c^3/a^2/x+c^
3*x-2*c^3*ln(x)/a
```

Maxima [A] time = 1.04221, size = 95, normalized size = 1.25

$$c^3x - \frac{2\,c^3\log\left(x\right)}{a} + \frac{30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Fricas [A] time = 1.15343, size = 174, normalized size = 2.29

$$\frac{30\,a^6c^3x^6 - 60\,a^5c^3x^5\log\left(x\right) + 30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] time = 0.515016, size = 76, normalized size = 1.

$$\frac{a^6c^3x - 2a^5c^3\log\left(x\right) + \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**6*c**3*x - 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Giac [A] time = 1.1096, size = 96, normalized size = 1.26

$$c^3x - \frac{2c^3\log(|x|)}{a} + \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.816 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=40

$$-\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Rubi [A] time = 0.141085, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 75}

$$-\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\ &= -\frac{c^2\int\frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\ &= -\frac{c^2\int\frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4}\\ &= -\frac{c^2\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}\\ &= \frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0193561, size = 40, normalized size = 1.

$$-\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Maple [A] time = 0.045, size = 39, normalized size = 1.

$$\frac{c^2}{3\,a^4x^3}-\frac{c^2}{x^2a^3}+xc^2-2\,\frac{c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*c^2/a^4/x^3-c^2/x^2/a^3+x*c^2-2*c^2*ln(x)/a
```

Maxima [A] time = 1.07959, size = 50, normalized size = 1.25

$$c^2x - \frac{2\,c^2\log\left(x\right)}{a} - \frac{3\,ac^2x - c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Fricas [A] time = 1.21624, size = 97, normalized size = 2.42

$$\frac{3\,a^4c^2x^4 - 6\,a^3c^2x^3\log\left(x\right) - 3\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] time = 0.347533, size = 39, normalized size = 0.98

$$\frac{a^4c^2x - 2a^3c^2\log\left(x\right) - \frac{3ac^2x - c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**4*c**2*x - 2*a**3*c**2*log(x) - (3*a*c**2*x - c**2)/(3*x**3))/a**4
```

Giac [A] time = 1.13297, size = 51, normalized size = 1.27

$$c^2x - \frac{2\,c^2\log\left(|x|\right)}{a} - \frac{3\,ac^2x - c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

3.817 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=21

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

```
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Rubi [A] time = 0.0848482, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6167, 6157, 6150, 43}

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx\\
&= \frac{c\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}\\
&= \frac{c\int \frac{(1-ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c\int \left(a^2+\frac{1}{x^2}-\frac{2a}{x}\right)dx}{a^2}\\
&= -\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0140419, size = 21, normalized size = 1.

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Maple [A] time = 0.045, size = 22, normalized size = 1.1

$$-\frac{c}{a^2x}+cx-2\,\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -c/a^2/x+c*x-2*c*ln(x)/a
```

Maxima [A] time = 0.999995, size = 28, normalized size = 1.33

$$cx - \frac{2\,c\log\left(x\right)}{a} - \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c*x - 2*c*log(x)/a - c/(a^2*x)
```

Fricas [A] time = 1.31957, size = 57, normalized size = 2.71

$$\frac{a^2cx^2 - 2\,acx\log\left(x\right) - c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] (a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] time = 0.270569, size = 20, normalized size = 0.95

$$\frac{a^2cx - 2ac\log\left(x\right) - \frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**2*c*x - 2*a*c*log(x) - c/x)/a**2
```

Giac [A] time = 1.12736, size = 30, normalized size = 1.43

$$cx - \frac{2\,c\log\left(|x|\right)}{a} - \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c*x - 2*c*log(abs(x))/a - c/(a^2*x)
```

3.818 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{ac(ax+1)}-\frac{2\log(ax+1)}{ac}+\frac{x}{c}$$

```
[Out] x/c - 1/(a*c*(1 + a*x)) - (2*Log[1 + a*x])/(a*c)
```

Rubi [A] time = 0.155279, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$-\frac{1}{ac(ax+1)}-\frac{2\log(ax+1)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] x/c - 1/(a*c*(1 + a*x)) - (2*Log[1 + a*x])/(a*c)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= \frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= \frac{a^2\int \frac{x^2}{(1+ax)^2}\,dx}{c} \\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(1+ax)^2}-\frac{2}{a^2(1+ax)}\right)dx}{c} \\
&= \frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.0309475, size = 28, normalized size = 0.8

$$\frac{-\frac{1}{a^2x+a}-\frac{2\log(ax+1)}{a}+x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (x - (a + a^2*x)^(-1) - (2*Log[1 + a*x])/a)/c
```

Maple [A] time = 0.045, size = 36, normalized size = 1.

$$\frac{x}{c}-\frac{1}{ac\,(ax+1)}-2\,\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2),x)
```

```
[Out] x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Maxima [A] time = 1.02637, size = 46, normalized size = 1.31

$$\frac{x}{c} - \frac{1}{a^2cx + ac} - \frac{2\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/(a^2*c*x + a*c) - 2*log(a*x + 1)/(a*c)
```

Fricas [A] time = 1.19498, size = 86, normalized size = 2.46

$$\frac{a^2x^2 + ax - 2(ax+1)\log(ax+1) - 1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

Sympy [A] time = 0.320006, size = 36, normalized size = 1.03

$$a^2\left(-\frac{1}{a^4cx + a^3c} + \frac{x}{a^2c} - \frac{2\log(ax+1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

Giac [A] time = 1.15606, size = 49, normalized size = 1.4

$$\frac{x}{c} - \frac{2\,\log\left(|ax+1|\right)}{ac} - \frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] x/c - 2*log(abs(a*x + 1))/(a*c) - 1/((a*x + 1)*a*c)
```

3.819 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{7}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2 + 1/(4*a*c^2*(1 + a*x)^2) - 7/(4*a*c^2*(1 + a*x)) + Log[1 - a*x]/(8*a
*c^2) - (17*Log[1 + a*x])/(8*a*c^2)
```

Rubi [A] time = 0.17431, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$-\frac{7}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```



```
[Out] x/c^2 + 1/(4*a*c^2*(1 + a*x)^2) - 7/(4*a*c^2*(1 + a*x)) + Log[1 - a*x]/(8*a
*c^2) - (17*Log[1 + a*x])/(8*a*c^2)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\
&= -\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= -\frac{a^4\int \frac{x^4}{(1-ax)(1+ax)^3}\,dx}{c^2}\\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{8a^4(-1+ax)}+\frac{1}{2a^4(1+ax)^3}-\frac{7}{4a^4(1+ax)^2}+\frac{17}{8a^4(1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}
\end{aligned}$$

Mathematica [A] time = 0.0468059, size = 70, normalized size = 0.96

$$\frac{2\left(4a^3x^3+8a^2x^2-3ax-6\right)+(ax+1)^2\log(1-ax)-17(ax+1)^2\log(ax+1)}{8a(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (2*(-6 - 3*a*x + 8*a^2*x^2 + 4*a^3*x^3) + (1 + a*x)^2*Log[1 - a*x] - 17*(1
+ a*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

Maple [A] time = 0.05, size = 65, normalized size = 0.9

$$\frac{x}{c^{2}}+\frac{1}{4\,a c^{2} \left(a x +1\right)^{2}}-\frac{7}{4\,a c^{2} \left(a x +1\right)}-\frac{17 \ln \left(a x +1\right)}{8\,a c^{2}}+\frac{\ln \left(a x -1\right)}{8\,a c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)-17/8*ln(a*x+1)/a/c^2+1/8/a/c^2*
ln(a*x-1)
```

Maxima [A] time = 1.04798, size = 93, normalized size = 1.27

$$-\frac{7\,ax+6}{4\left(a^{3}c^{2}x^{2}+2\,a^{2}c^{2}x+ac^{2}\right)}+\frac{x}{c^{2}}-\frac{17\,\log\left(ax+1\right)}{8\,ac^{2}}+\frac{\log\left(ax-1\right)}{8\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + x/c^2 - 17/8*log(a*x
 + 1)/(a*c^2) + 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A] time = 1.38272, size = 211, normalized size = 2.89

$$\frac{8\,a^{3}x^{3}+16\,a^{2}x^{2}-6\,ax-17\left(a^{2}x^{2}+2\,ax+1\right)\log\left(ax+1\right)+\left(a^{2}x^{2}+2\,ax+1\right)\log\left(ax-1\right)-12}{8\left(a^{3}c^{2}x^{2}+2\,a^{2}c^{2}x+ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*
c^2)
```

Sympy [A] time = 0.599817, size = 73, normalized size = 1.

$$a^4\left(-\frac{7ax+6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(-(7*a*x + 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

Giac [A] time = 1.14497, size = 77, normalized size = 1.05

$$\frac{x}{c^2}-\frac{17\log\left(|ax+1|\right)}{8ac^2}+\frac{\log\left(|ax-1|\right)}{8ac^2}-\frac{7ax+6}{4(ax+1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 17/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x + 6)/((a*x + 1)^2*a*c^2)
```

3.820 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{1}{16ac^3(1-ax)}-\frac{39}{16ac^3(ax+1)}+\frac{5}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3 + 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) + 5/(8*a*c^3*(1 +
 a*x)^2) - 39/(16*a*c^3*(1 + a*x)) + Log[1 - a*x]/(4*a*c^3) - (9*Log[1 + a*
x])/(4*a*c^3)
```

Rubi [A] time = 0.202373, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{1}{16ac^3(1-ax)}-\frac{39}{16ac^3(ax+1)}+\frac{5}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] x/c^3 + 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) + 5/(8*a*c^3*(1 +
 a*x)^2) - 39/(16*a*c^3*(1 + a*x)) + Log[1 - a*x]/(4*a*c^3) - (9*Log[1 + a*
x])/(4*a*c^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx\\ &= \frac{a^6\int \frac{e^{-2\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}\\ &= \frac{a^6\int \frac{x^6}{(1-ax)^2(1+ax)^4}\,dx}{c^3}\\ &= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{16a^6(-1+ax)^2}+\frac{1}{4a^6(-1+ax)}+\frac{1}{4a^6(1+ax)^4}-\frac{5}{4a^6(1+ax)^3}+\frac{39}{16a^6(1+ax)^2}-\frac{9}{4a^6(1+ax)}\right)dx}{c^3}\\ &= \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2}-\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(1+a}{4ac^3}\end{aligned}$$

Mathematica [A] time = 0.0662567, size = 104, normalized size = 0.96

$$\frac{2\left(6a^5x^5+12a^4x^4-15a^3x^3-24a^2x^2+7ax+11\right)+3(ax-1)(ax+1)^3\log(1-ax)-27(ax-1)(ax+1)^3\log(ax+1)}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) + 3*(-1
+ a*x)*(1 + a*x)^3*Log[1 - a*x] - 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])/(
12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A] time = 0.053, size = 95, normalized size = 0.9

$$\frac{x}{c^{3}}-\frac{1}{12\,a c^{3} \left(a x +1\right)^{3}}+\frac{5}{8\,a c^{3} \left(a x +1\right)^{2}}-\frac{39}{16\,a c^{3} \left(a x +1\right)}-\frac{9\,\ln \left(a x +1\right)}{4\,a c^{3}}-\frac{1}{16\,a c^{3} \left(a x -1\right)}+\frac{\ln \left(a x -1\right)}{4\,a c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3/(a*x+1)^2-39/16/a/c^3/(a*x+1)-9/4*ln(a
*x+1)/a/c^3-1/16/a/c^3/(a*x-1)+1/4/a/c^3*ln(a*x-1)
```

Maxima [A] time = 1.02748, size = 131, normalized size = 1.21

$$-\frac{15\,a^{3} x^{3}+12\,a^{2} x^{2}-13\,a x -11}{6\,\left(a^{5} c^{3} x^{4}+2\,a^{4} c^{3} x^{3}-2\,a^{2} c^{3} x - a c^{3}\right)}+\frac{x}{c^{3}}-\frac{9\,\log \left(a x +1\right)}{4\,a c^{3}}+\frac{\log \left(a x -1\right)}{4\,a c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3) + x/c^3 - 9/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)
/(a*c^3)
```

Fricas [A] time = 1.17249, size = 306, normalized size = 2.83

$$\frac{12\,a^{5} x^{5}+24\,a^{4} x^{4}-30\,a^{3} x^{3}-48\,a^{2} x^{2}+14\,a x -27\,\left(a^{4} x^{4}+2\,a^{3} x^{3}-2\,a x -1\right) \log \left(a x +1\right)+3\,\left(a^{4} x^{4}+2\,a^{3} x^{3}-2\,a x -1\right) \mathrm{l}}{12\,\left(a^{5} c^{3} x^{4}+2\,a^{4} c^{3} x^{3}-2\,a^{2} c^{3} x - a c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4*
x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x
```

```
- 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A] time = 0.887411, size = 102, normalized size = 0.94

$$a^6\left(-\frac{15a^3x^3+12a^2x^2-13ax-11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(-(15*a**3*x**3 + 12*a**2*x**2 - 13*a*x - 11)/(6*a**11*c**3*x**4 + 12*
a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x -
1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

Giac [A] time = 1.11636, size = 108, normalized size = 1.

$$\frac{x}{c^3}-\frac{9\,\log\left(|ax+1|\right)}{4\,ac^3}+\frac{\log\left(|ax-1|\right)}{4\,ac^3}-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(ax+1)^3(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 9/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

3.821 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=143

$$\frac{11}{64ac^4(1-ax)}-\frac{99}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{35}{32ac^4(ax+1)^2}-\frac{13}{48ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}+\frac{47\log(1-ax)}{128ac^4}-$$

```
[Out] x/c^4 - 1/(64*a*c^4*(1 - a*x)^2) + 11/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1
 + a*x)^4) - 13/(48*a*c^4*(1 + a*x)^3) + 35/(32*a*c^4*(1 + a*x)^2) - 99/(32
*a*c^4*(1 + a*x)) + (47*Log[1 - a*x])/(128*a*c^4) - (303*Log[1 + a*x])/(128
*a*c^4)
```

Rubi [A] time = 0.231239, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{11}{64ac^4(1-ax)}-\frac{99}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{35}{32ac^4(ax+1)^2}-\frac{13}{48ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}+\frac{47\log(1-ax)}{128ac^4}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] x/c^4 - 1/(64*a*c^4*(1 - a*x)^2) + 11/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1
 + a*x)^4) - 13/(48*a*c^4*(1 + a*x)^3) + 35/(32*a*c^4*(1 + a*x)^2) - 99/(32
*a*c^4*(1 + a*x)) + (47*Log[1 - a*x])/(128*a*c^4) - (303*Log[1 + a*x])/(128
*a*c^4)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= -\frac{a^8\int \frac{e^{-2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= -\frac{a^8\int \frac{x^8}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{32a^8(-1+ax)^3}-\frac{11}{64a^8(-1+ax)^2}-\frac{47}{128a^8(-1+ax)}+\frac{1}{8a^8(1+ax)^5}-\frac{13}{16a^8(1+ax)^4}+\frac{35}{16a^8(1+ax)^3}-\frac{99}{32a^8(1\cdots}\right.}{c^4}$$

$$= \frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)^2}-\frac{\cdots}{32ac\cdots}$$

Mathematica [A] time = 0.0946506, size = 124, normalized size = 0.87

$$\frac{2\left(192a^7x^7+384a^6x^6-819a^5x^5-1254a^4x^4+866a^3x^3+1258a^2x^2-275ax-400\right)+141(ax-1)^2(ax+1)^4\log(1-ax)\cdots}{384a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (2*(-400 - 275*a*x + 1258*a^2*x^2 + 866*a^3*x^3 - 1254*a^4*x^4 - 819*a^5*x^
5 + 384*a^6*x^6 + 192*a^7*x^7) + 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x]
```

```
- 909*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*c*x
)^4)
```

Maple [A] time = 0.062, size = 125, normalized size = 0.9

$$\frac{x}{c^4}+\frac{1}{32\,ac^4\,(ax+1)^4}-\frac{13}{48\,ac^4\,(ax+1)^3}+\frac{35}{32\,ac^4\,(ax+1)^2}-\frac{99}{32\,ac^4\,(ax+1)}-\frac{303\,\ln(ax+1)}{128\,ac^4}-\frac{1}{64\,ac^4\,(ax-1)^2}-\frac{}{64\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4+1/32/a/c^4/(a*x+1)^4-13/48/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/3
2/a/c^4/(a*x+1)-303/128*ln(a*x+1)/a/c^4-1/64/a/c^4/(a*x-1)^2-11/64/a/c^4/(a
*x-1)+47/128/a/c^4*ln(a*x-1)
```

Maxima [A] time = 1.05436, size = 196, normalized size = 1.37

$$-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{303\,\log(ax+1)}{128\,ac^4}+\frac{47\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x +
400)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x
^2 + 2*a^2*c^4*x + a*c^4) + x/c^4 - 303/128*log(a*x + 1)/(a*c^4) + 47/128*l
og(a*x - 1)/(a*c^4)
```

Fricas [A] time = 1.33578, size = 509, normalized size = 3.56

$$\frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2\right.}{384\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*x
^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^
3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A] time = 1.34274, size = 156, normalized size = 1.09

$$a^8\left(-\frac{627a^5x^5+486a^4x^4-1058a^3x^3-874a^2x^2+467ax+400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}-\cdots}{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*(-(627*a**5*x**5 + 486*a**4*x**4 - 1058*a**3*x**3 - 874*a**2*x**2 + 46
7*a*x + 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/(a**
9*c**4))
```

Giac [A] time = 1.15428, size = 130, normalized size = 0.91

$$\frac{x}{c^4}-\frac{303\log\left(|ax+1|\right)}{128\,ac^4}+\frac{47\log\left(|ax-1|\right)}{128\,ac^4}-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(ax+1)^4(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 303/128*log(abs(a*x + 1))/(a*c^4) + 47/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x
 + 400)/((a*x + 1)^4*(a*x - 1)^2*a*c^4)
```

3.822 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$c^4x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{11/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{7a}+\frac{17c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}}{14a}+\frac{11c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{10a}+\frac{5c}{}$$

```
[Out] (33*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) + (27*c^4*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(8*a) + (5*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(8*a) + (11*c
^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(10*a) + (17*c^4*(1 - 1/(a*x))^
(7/2)*(1 + 1/(a*x))^(5/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x)
)^(5/2))/(7*a) + c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x + (15*c^4*A
rcCsc[a*x])/(16*a) - (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.249705, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^4x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{11/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{7a}+\frac{17c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}}{14a}+\frac{11c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{10a}+\frac{5c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (33*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) + (27*c^4*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(8*a) + (5*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(8*a) + (11*c
^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(10*a) + (17*c^4*(1 - 1/(a*x))^
(7/2)*(1 + 1/(a*x))^(5/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x)
)^(5/2))/(7*a) + c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x + (15*c^4*A
rcCsc[a*x])/(16*a) - (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^4\,\mathrm{Subst}\left(\int\frac{\left(-\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{7}\left(ac^4\right)\mathrm{Subst}\left(\int\frac{\left(-\frac{21}{a^2}-\right.}{}\right.$$

$$= \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$= \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}$$

$$= \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}$$

$$= -\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} +$$

$$= \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4}{}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\right.}{}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\right.}{}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\right.}{}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\right.}{}$$

Mathematica [A] time = 0.241335, size = 126, normalized size = 0.37

$$\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}\left(560a^7x^7+2496a^6x^6-525a^5x^5-992a^4x^4+770a^3x^3+96a^2x^2-280ax+80\right)-1680a^6x^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}\right.\right.\right.}{560a^7x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(80 - 280*a*x + 96*a^2*x^2 + 770*a^3*x^3 - 992*
a^4*x^4 - 525*a^5*x^5 + 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1/
(a*x)] - 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

Maple [A] time = 0.148, size = 329, normalized size = 1.

$$-\frac{(ax+1)^2c^4}{(560\,ax-560)\,a^8x^7}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}x^8a^8+1680\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^6a^6-525\sqrt{a^2x^2-1}\sqrt{a^2}x^7a^7-525\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/560*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)
^(1/2)*x^8*a^8+1680*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6-525*(a^2*x^2-1)^(
1/2)*(a^2)^(1/2)*x^7*a^7-525*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)
)+1680*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8-35*(a^
2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5-816*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^
4+490*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+176*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x^2*a^2-280*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a+80*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2))/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A] time = 1.57348, size = 512, normalized size = 1.49

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{1155\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+7665\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}}{\frac{6\,(a}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 + (1155*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 7665*c^4*((a*x - 1)/(a*x + 1))
^(13/2) + 20811*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 12799*c^4*((a*x - 1)/(a*
x + 1))^(9/2) - 39071*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 33621*c^4*((a*x - 1
)/(a*x + 1))^(5/2) - 13615*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 2205*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A] time = 1.4247, size = 485, normalized size = 1.41

$$-\frac{1050\,a^{7}c^{4}x^{7}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(560\,a^{8}c^{4}x^{8}+3056\,a^{7}c^{4}\right.}{560\,a^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 + 3056*a^7*c^4*x^7 + 1971*a^6*c^4*x^6 -
 1517*a^5*c^4*x^5 - 222*a^4*c^4*x^4 + 866*a^3*c^4*x^3 - 184*a^2*c^4*x^2 - 2
00*a*c^4*x + 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23585, size = 709, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^4*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^4*sgn(a*x + 1)/a + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(
a)*sgn(a*x + 1) + 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
 - 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) + 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 945*(x*abs(a) - sqrt(a
^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) + 38080*(x*abs(a) - sqrt(a^2*x^2 - 1
))^8*a*c^4*sgn(a*x + 1) + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn(
a*x + 1) - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) + 3
2256*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 980*(x*abs(a) -
sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) + 12992*(x*abs(a) - sqrt(a^2*x
^2 - 1))^2*a*c^4*sgn(a*x + 1) - 525*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(
a)*sgn(a*x + 1) + 2496*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))
^2 + 1)^7*a*abs(a))
```

3.823 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$c^3x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{5a}+\frac{27c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}}{20a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{3c}{}$$

```
[Out] (21*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (3*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(8*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3
/2))/(4*a) + (27*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))/(20*a) + (6*c
^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2))/(5*a) + c^3*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(3/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) - (3*c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.184309, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^3x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{5a}+\frac{27c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}}{20a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{3c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (21*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (3*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(8*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3
/2))/(4*a) + (27*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))/(20*a) + (6*c
^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2))/(5*a) + c^3*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(3/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) - (3*c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - c^3\,\text{Subst}\left(\int \frac{\left(-\frac{3}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \frac{\left(-\frac{15}{a^2}-\frac{27}{a^3}\right.}{}\right.$$

$$= \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x -$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} +$$

$$= \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3}{}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\right.}{}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\right.}{}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\right.}{}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\right.}{}$$

Mathematica [A] time = 0.185723, size = 110, normalized size = 0.41

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\left(40a^5x^5+152a^4x^4-55a^3x^3-24a^2x^2+30ax-8\right)-120a^4x^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+15a^4x^4\sin^{-1}\left(\frac{1}{ax}\right)\right)}{40a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(-8 + 30*a*x - 24*a^2*x^2 - 55*a^3*x^3 + 152*a^
4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Maple [A] time = 0.135, size = 281, normalized size = 1.

$$-\frac{(ax+1)^2 c^3}{(40\,ax-40)\,a^6x^5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-120\,\sqrt{a^2}\sqrt{a^2x^2-1}x^6a^6+120\,\sqrt{a^2}\left(a^2x^2-1\right)^{3/2}x^4a^4-15\,\sqrt{a^2x^2-1}\sqrt{a^2}x^5a^5-15\,\sqrt{a^2}\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/40*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(
1/2)*x^6*a^6+120*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*x^4*a^4-15*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*x^5*a^5-15*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*x^5*a^5+120
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-25*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*x^3*a^3-32*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+30*(a
^2)^(1/2)*(a^2*x^2-1)^(3/2)*x*a-8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a*x-1)/((
a*x-1)*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A] time = 1.55652, size = 406, normalized size = 1.51

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+465\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-298}{\frac{4\,(ax-1)a^2}{ax+1}+\frac{5\,(ax-1)^2}{(ax+1)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 +
```

```
(105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 465*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 - 298*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 842*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 575*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 135*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

Fricas [A] time = 1.3351, size = 416, normalized size = 1.55

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(40\,a^6c^3x^6+192\,a^5c^3x^5+97\,a\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 + 192*a^5*c^3*x^5 + 97*a^4*c^3*x^4 - 79*a^3*c
^3*x^3 + 6*a^2*c^3*x^2 + 22*a*c^3*x - 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^
6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23083, size = 533, normalized size = 1.98

$$-\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{4\,a}+\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}c^3\mathrm{sgn}\left(ax+1\right)}{a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^3*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^3*sgn(a*x + 1)/a + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) + 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) - 10*
(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) + 720*(x*abs(a) -
sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) + 800*(x*abs(a) - sqrt(a^2*x^2 - 1)
)^4*a*c^3*sgn(a*x + 1) + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn
(a*x + 1) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 55*(x
*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 152*a*c^3*sgn(a*x +
1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

3.824 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$c^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{4c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{3a}+\frac{11c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{6a}+\frac{5c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}-\frac{3c}{}$$

```
[Out] (5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (11*c^2*(1 - 1/(a*x))^(
3/2)*Sqrt[1 + 1/(a*x)])/(6*a) + (4*c^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
])/(3*a) + c^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x - (c^2*ArcCsc[a*x])/
(2*a) - (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.131351, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{4c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{3a}+\frac{11c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{6a}+\frac{5c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}-\frac{3c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (11*c^2*(1 - 1/(a*x))^(
3/2)*Sqrt[1 + 1/(a*x)])/(6*a) + (4*c^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
])/(3*a) + c^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x - (c^2*ArcCsc[a*x])/
(2*a) - (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - c^2\,\text{Subst}\left(\int\frac{\left(-\frac{3}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a^2}-\frac{11x}{a^3}\right)(1}{x\sqrt{1+\frac{x}{a}}}\right.$$

$$= \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x - \frac{1}{6}\left(a^2c\right.$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right.$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right.$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right.$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right.$$

Mathematica [A] time = 0.133969, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3+16a^2x^2-9ax+2\right)-18a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x^
2*ArcSin[1/(a*x)] - 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*
x^2)
```

Maple [A] time = 0.151, size = 233, normalized size = 1.2

$$-\frac{(ax+1)^2c^2}{(6ax-6)a^4x^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-18\sqrt{a^2x^2-1}\sqrt{a^2}x^4a^4+18\left(a^2x^2-1\right)^{3/2}\sqrt{a^2}x^2a^2+3\sqrt{a^2x^2-1}\sqrt{a^2}x^3a^3+18\ln\left(\frac{a^2x+\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^2*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)*x^4*a^4+18*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+3*(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)*x^3*a^3+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x
^3*a^4+3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-9*(a^2)^(1/2)*(a^2
*x^2-1)^(3/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a*x-1)/((a*x-1)*(a*x+1)
)^(1/2)/a^4/x^3/(a^2)^(1/2)
```

Maxima [A] time = 1.54839, size = 302, normalized size = 1.55

$$\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-37c^2\left(\frac{ax-1}{ax+1}\right)\cdots}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21
*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 37*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

Fricas [A] time = 1.32473, size = 359, normalized size = 1.84

$$\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4+22\,a^3c^2x^3+7\,a^2c^2x^2-7}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 7*a^2*c^2*x^2 - 7*a*c^2*x + 2*c^
2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18631, size = 356, normalized size = 1.83

$$\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a}+\frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*s
```

```
gn(a*x + 1)/a + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
+ 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) + 36*(x*abs(a
) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(a^2*x^2 -
1))*c^2*abs(a)*sgn(a*x + 1) + 16*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

3.825 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=76

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] c*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x - (3*c*ArcCsc[a*x])/a - (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rubi [A] time = 0.0567363, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6194, 98, 12, 105, 41, 216, 92, 208}

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```


```
[Out] c*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x - (3*c*ArcCsc[a*x])/a - (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + c\,\text{Subst}\left(\int\frac{3\sqrt{1-\frac{x}{a}}}{ax\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x + \frac{(3c)\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - \frac{(3c)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2} + \frac{(3c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\right.}{a}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - \frac{(3c)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2} - \frac{(3c)\,\text{Subst}\left(\int\frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1}\right.}{a^2}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x - \frac{3c\csc^{-1}(ax)}{a} - \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] time = 0.0871882, size = 57, normalized size = 0.75

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(ax-1)-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/a
```

Maple [B] time = 0.132, size = 234, normalized size = 3.1

$$\frac{(ax+1)^2c}{(ax-1)a^2x}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-\sqrt{a^2x^2-1}\sqrt{a^2}x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\sqrt{a^2x^2-1}xa+\ln\left(\left(a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}\right)\frac{1}{\sqrt{a^2}}\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2
+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*x*a+ln((a^2*
x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-3*a*x*(a^2)^(1/2)*arcta
n(1/(a^2*x^2-1)^(1/2))+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a-4*ln((a^2*
x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2)/(a*x-1)/((a*x-1)
*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A] time = 1.55317, size = 159, normalized size = 2.09

$$-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A] time = 1.30446, size = 252, normalized size = 3.32

$$\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) + 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 - c)
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18923, size = 165, normalized size = 2.17

$$\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{a}+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}c\mathrm{sgn}\left(ax+1\right)}{a}-\frac{}{\left(\left(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a - 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
)
```

3.826 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=144

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] (5*Sqrt[1 - 1/(a*x)])/(3*a*c*(1 + 1/(a*x))^(3/2)) + (14*Sqrt[1 - 1/(a*x)])/
(3*a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*(1 + 1/(a*x))^(3/2)) -
 (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rubi [A] time = 0.0981175, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```


```
[Out] (5*Sqrt[1 - 1/(a*x)])/(3*a*c*(1 + 1/(a*x))^(3/2)) + (14*Sqrt[1 - 1/(a*x)])/
(3*a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*(1 + 1/(a*x))^(3/2)) -
 (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x^2\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}+\frac{2x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{5x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\,\text{Subst}\left(\int -\frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] time = 0.130275, size = 69, normalized size = 0.48

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(3a^2x^2+19ax+14\right)}{(ax+1)^2} - \frac{9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 + 19*a*x + 3*a^2*x^2))/(1 + a*x)^2 - (9*Log[(
1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [B] time = 0.131, size = 346, normalized size = 2.4

$$-\frac{1}{3\,a\,(ax+1)\,c\,(ax-1)}\left(9\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^3a^4-9\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^3a^3+27\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] -1/3*(9*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4
-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+27*ln((a^2*x+(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(3/2)*x*a-27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^2*a^2+27*ln((a^2*x+(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2+5*((a*x-1)*(a*x+1))^(3/2
)*(a^2)^(1/2)-27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x*a+9*a*ln((a^2*x+(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-9*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(1/2))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x+1)/c/((a*x-1)*(a*x+1))
^(1/2)/(a*x-1)
```

Maxima [A] time = 1.07444, size = 189, normalized size = 1.31

$$-\frac{1}{3}\,a\left(\frac{6\,\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+12\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(6*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) - (
((a*x - 1)/(a*x + 1))^(3/2) + 12*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c) + 9*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c))
```

Fricas [A] time = 1.31951, size = 235, normalized size = 1.63

$$-\frac{9(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(3a^2x^2+19ax+14\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -1/3*(9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x + 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) - (3*a^2*x^2 + 19*a*x + 14)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^2*c*x + a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15778, size = 80, normalized size = 0.56

$$\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a)) + sqrt(a^
2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

3.827 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c^2\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(5*a*c^2*(1 + 1/(a*x))^(5/2)) + (9*Sqrt[1 - 1/(a*x)])
/(5*a*c^2*(1 + 1/(a*x))^(3/2)) + (24*Sqrt[1 - 1/(a*x)])/(5*a*c^2*Sqrt[1 + 1
/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c^2*(1 + 1/(a*x))^(5/2)) - (3*ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rubi [A] time = 0.123586, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 103, 21, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c^2\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```


```
[Out] (6*Sqrt[1 - 1/(a*x)])/(5*a*c^2*(1 + 1/(a*x))^(5/2)) + (9*Sqrt[1 - 1/(a*x)])
/(5*a*c^2*(1 + 1/(a*x))^(3/2)) + (24*Sqrt[1 - 1/(a*x)])/(5*a*c^2*Sqrt[1 + 1
/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c^2*(1 + 1/(a*x))^(5/2)) - (3*ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{3x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{6\,\text{Subst}\left(\int \frac{-\frac{5}{2}+\frac{2x}{a}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{15}{2a}+\frac{9x}{2a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{5c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{(2a)\,\text{Subst}\left(\int -\frac{15}{2a^2x\sqrt{1-\frac{x}{a}}\sqrt{1}\cdots}\right.}{5c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, \ldots\right.}{ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\cdots}\right.}{a^2c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\cdots\right.}{ac^2}$$

Mathematica [A] time = 0.167981, size = 78, normalized size = 0.43

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(5a^3x^3+39a^2x^2+57ax+24)}{5(ax+1)^3}-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 57*a*x + 39*a^2*x^2 + 5*a^3*x^3))/(5*(1 +
 a*x)^3) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] time = 0.151, size = 438, normalized size = 2.4

$$-\frac{1}{40\,a\,(ax+1)^2\,c^2\,(ax-1)}\left(120\,\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}}{\sqrt{a^2}}\right)x^4a^5-125\,\sqrt{a^2}\sqrt{(ax-1)\,(ax+1)}x^4a^4+480\,\ln\left(\frac{a^2x+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/40*(120*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*
a^5-125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+480*ln((a^2*x+(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+85*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(3/2)*x^2*a^2-500*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+720*ln
((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+148*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-750*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*
x^2*a^2+480*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a
^2+67*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-500*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(1/2)*x*a+120*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)
)-125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2)^
(1/2)/(a*x+1)^2/c^2/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Maxima [A] time = 1.0334, size = 217, normalized size = 1.2

$$-\frac{1}{20}\,a\left(\frac{40\,\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+85\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/20*a*(40*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^
2) - (((a*x - 1)/(a*x + 1))^(5/2) + 10*((a*x - 1)/(a*x + 1))^(3/2) + 85*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(
a^2*c^2) - 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A] time = 1.24718, size = 315, normalized size = 1.74

$$-\frac{15\left(a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(5\,a^3x^3+39\,a^2x^2+57\,ax+24\right)\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(15*(a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*(a^2
*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (5*a^3*x^3 + 39*a^2*
x^2 + 57*a*x + 24)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 + 2*a^2*c^2*x +
a*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18331, size = 80, normalized size = 0.44

$$\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{c^2|a|}+\frac{\sqrt{a^2x^2-1}\mathrm{sgn}\left(ax+1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^2*abs(a)) + sqrt(
a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^2)
```

3.828 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=253

$$\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.}$$

```
[Out] -2/(a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (11*Sqrt[1 - 1/(a*x)])/(
7*a*c^3*(1 + 1/(a*x))^(7/2)) + (54*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x
))^(5/2)) + (71*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x))^(3/2)) + (176*Sq
rt[1 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*Sqrt[1 - 1/(a*x)]*(1
 + 1/(a*x))^(7/2)) - (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^
3)
```

Rubi [A] time = 0.16675, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```


```
[Out] -2/(a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (11*Sqrt[1 - 1/(a*x)])/(
7*a*c^3*(1 + 1/(a*x))^(7/2)) + (54*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x
))^(5/2)) + (71*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x))^(3/2)) + (176*Sq
rt[1 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*Sqrt[1 - 1/(a*x)]*(1
 + 1/(a*x))^(7/2)) - (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^
3)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
```

```
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{8x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{21}{a^3}+\frac{33x}{a^4}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,d\right.}{7c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^3\,\text{Sub}}{}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-}}{35ac^3\sqrt{1+}}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-}}{35ac^3\sqrt{1+}}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-}}{35ac^3\sqrt{1+}}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{176\sqrt{1-}}{35ac^3\sqrt{1+}}$$

Mathematica [A] time = 0.207562, size = 101, normalized size = 0.4

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(35a^5x^5+286a^4x^4+368a^3x^3-125a^2x^2-423ax-176\right)}{35(ax-1)(ax+1)^4}-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 + 2
86*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)*(1 + a*x)^4) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x])/(a*c^3)
```

Maple [B] time = 0.161, size = 714, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/1120*(3360*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x
^7*a^8-3675*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^7*a^7+10080*ln((a^2*x+(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7+2555*(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(3/2)*x^5*a^5-11025*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*
a^6+3360*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^5*a^
6+1873*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4-3675*(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(1/2)*x^5*a^5-16800*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
))/(a^2)^(1/2))*x^4*a^5-4426*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+18
375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-16800*ln((a^2*x+(a^2)^(1/2)
*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4-3350*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(3/2)*x^2*a^2+18375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+3360
*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^2*a^3+2511*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-3675*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2)*x^2*a^2+10080*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/
2))*x*a^2+1957*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-11025*(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2)*x*a+3360*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))
/(a^2)^(1/2))-3675*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+1))
^(3/2)/(a^2)^(1/2)/(a*x+1)^3/c^3/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^3
```

Maxima [A] time = 1.1127, size = 269, normalized size = 1.06

$$-\frac{1}{560}\,a\left(\frac{35\left(\frac{33\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+56\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+350\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2520\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1680\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*(35*(33*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3
/2) - a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - (5*((a*x - 1)/(a*x + 1))^(7/2) +
 56*((a*x - 1)/(a*x + 1))^(5/2) + 350*((a*x - 1)/(a*x + 1))^(3/2) + 2520*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 1680*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A] time = 1.36585, size = 416, normalized size = 1.64

$$-\frac{105\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(35\,a^5x^5+286\,a^4x^4\right.}{35\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
- 1) - (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2*x^2 - 423*a*x - 17
6)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x -
a*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3, x)
```

3.829 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=327

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{28\sqrt{1-}}{9ac^4\left(\frac{1}{ax}+\right.}$$

```
[Out] -4/(3*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - 5/(a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(9/2)) + (28*Sqrt[1 - 1/(a*x)])/(9*a*c^4*(1 + 1/(a*x))
^(9/2)) + (139*Sqrt[1 - 1/(a*x)])/(63*a*c^4*(1 + 1/(a*x))^(7/2)) + (202*Sqr
t[1 - 1/(a*x)])/(105*a*c^4*(1 + 1/(a*x))^(5/2)) + (719*Sqrt[1 - 1/(a*x)])/(
315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - (3*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rubi [A] time = 0.221861, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{28\sqrt{1-}}{9ac^4\left(\frac{1}{ax}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```


```
[Out] -4/(3*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - 5/(a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(9/2)) + (28*Sqrt[1 - 1/(a*x)])/(9*a*c^4*(1 + 1/(a*x))
^(9/2)) + (139*Sqrt[1 - 1/(a*x)])/(63*a*c^4*(1 + 1/(a*x))^(7/2)) + (202*Sqr
t[1 - 1/(a*x)])/(105*a*c^4*(1 + 1/(a*x))^(5/2)) + (719*Sqrt[1 - 1/(a*x)])/(
315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - (3*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```


Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{a^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + -$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + -$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + -$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + -$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}} + -$$

$$4 \quad 5 \quad 28\sqrt{1-\frac{1}{}} \quad 139\sqrt{1-\frac{1}{}}$$

Mathematica [A] time = 0.26325, size = 117, normalized size = 0.36

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(315a^7x^7+2669a^6x^6+2967a^5x^5-4029a^4x^4-7399a^3x^3-339a^2x^2+4047ax+1664\right)}{315(ax-1)^2(ax+1)^5}-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3 -
 4029*a^4*x^4 + 2967*a^5*x^5 + 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x)
^2*(1 + a*x)^5) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] time = 0.181, size = 766, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/40320*(120960*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2)
)*x^9*a^10-138915*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^9*a^9+362880*ln((a^
2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^8*a^9+98595*(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(3/2)*x^7*a^7-416745*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2)*x^8*a^8+75113*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^6*a^6-967680*ln((a
^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^6*a^7-240861*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^5*a^5+1111320*(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)*x^6*a^6-725760*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)
^(1/2))*x^5*a^6-178863*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^4*a^4+833490*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+725760*ln((a^2*x+(a^2)^(1/2)*((a
*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^4*a^5+252497*(a^2)^(1/2)*((a*x-1)*(a*x
+1))^(3/2)*x^3*a^3-833490*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+96768
0*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x^3*a^4+18230
7*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-1111320*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(1/2)*x^3*a^3-101271*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-36288
0*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*x*a^2-74077*(
(a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+416745*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2)*x*a-120960*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))
+138915*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2
```

```
)^(1/2)/(a*x+1)^4/c^4/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^4
```

Maxima [A] time = 1.07162, size = 312, normalized size = 0.95

$$\frac{1}{20160}a\left(\frac{105\left(\frac{29(ax-1)}{ax+1}-\frac{414(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{35\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+450\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+2961\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+14700\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+95445\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*(105*(29*(a*x - 1)/(a*x + 1) - 414*(a*x - 1)^2/(a*x + 1)^2 + 1)/(
a^2*c^4*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2))
+ (35*((a*x - 1)/(a*x + 1))^(9/2) + 450*((a*x - 1)/(a*x + 1))^(7/2) + 2961*
((a*x - 1)/(a*x + 1))^(5/2) + 14700*((a*x - 1)/(a*x + 1))^(3/2) + 95445*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A] time = 1.36328, size = 610, normalized size = 1.87

$$-\frac{945\left(a^6x^6+2a^5x^5-a^4x^4-4a^3x^3-a^2x^2+2ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6+2a^5x^5-a^4x^4-4a^3x^3-a^2x^2+2\right.}{315\left(a^7c^4x^6+2a^6c^4x^5-a^5c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (31
5*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^3*x^3 - 339
*a^2*x^2 + 4047*a*x + 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4, x)
```

3.830 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=321

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[c
 - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
- 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^
2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (c^3*Sqrt[c
- c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.149986, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[c
 - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
- 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^
2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (c^3*Sqrt[c
- c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}-\frac{a}{x^6}+\frac{3a^2}{x^5}+\frac{3a^3}{x^4}-\frac{3a^4}{x^3}-\frac{3a^5}{x^2}+\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x
\end{aligned}$$

Mathematica [A] time = 0.0733212, size = 96, normalized size = 0.3

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{3a^4}{2x^2}-\frac{a^3}{x^3}-\frac{3a^2}{4x^4}+a^7x+\frac{3a^5}{x}+a^6\log(x)+\frac{a}{5x^5}+\frac{1}{6x^6}\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(1/(6*x^6) + a/(5*x^5) - (3*a^2)/(4*x^4) - a^3/x^3
 + (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x + a^6*Log[x]))/(a^7*(1 - 1/(a^2*x^2)
)^(7/2))
```

Maple [A] time = 0.23, size = 112, normalized size = 0.4

$$\frac{\left(60\,a^{7}x^{7}+60\,a^{6}\ln\left(x\right)x^{6}+180\,x^{5}a^{5}+90\,x^{4}a^{4}-60\,x^{3}a^{3}-45\,a^{2}x^{2}+12\,ax+10\right)x}{\left(60\,ax+60\right)\left(a^{2}x^{2}-1\right)^{3}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/60*(60*a^7*x^7+60*a^6*ln(x)*x^6+180*x^5*a^5+90*x^4*a^4-60*x^3*a^3-45*a^2*
x^2+12*a*x+10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)/(a^2*x^2-1)^3/((a*x-
1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.27538, size = 216, normalized size = 0.67

$$\frac{\left(60\,a^{7}c^{3}x^{7}+60\,a^{6}c^{3}x^{6}\log\left(x\right)+180\,a^{5}c^{3}x^{5}+90\,a^{4}c^{3}x^{4}-60\,a^{3}c^{3}x^{3}-45\,a^{2}c^{3}x^{2}+12\,ac^{3}x+10\,c^{3}\right)\sqrt{a^{2}c}}{60\,a^{8}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/60*(60*a^7*c^3*x^7 + 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 + 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.831 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=236

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[
c - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*
x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2
*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x
^2)] + (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.129867, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[
c - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*
x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2
*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x
^2)] + (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^2(1+ax)^3}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5+\frac{1}{x^5}+\frac{a}{x^4}-\frac{2a^2}{x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+
\end{aligned}
$$

Mathematica [A] time = 0.0571401, size = 75, normalized size = 0.32

$$
\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{a^2}{x^2}+a^5x+\frac{2a^3}{x}+a^4\log(x)-\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(-1/(4*x^4) - a/(3*x^3) + a^2/x^2 + (2*a^3)/x + a^
5*x + a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A] time = 0.236, size = 96, normalized size = 0.4

$$\frac{\left(12\,x^5a^5+12\,a^4\ln\left(x\right)x^4+24\,x^3a^3+12\,a^2x^2-4\,ax-3\right)x}{\left(12\,ax+12\right)\left(a^2x^2-1\right)^2}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/12*(12*x^5*a^5+12*a^4*ln(x)*x^4+24*x^3*a^3+12*a^2*x^2-4*a*x-3)*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*x/(a*x+1)/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.30508, size = 166, normalized size = 0.7

$$\frac{\left(12\,a^5c^2x^5+12\,a^4c^2x^4\log\left(x\right)+24\,a^3c^2x^3+12\,a^2c^2x^2-4\,ac^2x-3\,c^2\right)\sqrt{a^2c}}{12\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(12*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 + 12*a^2*c^2*
x^2 - 4*a*c^2*x - 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2 x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.832 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=146

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c - c
/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/Sq
rt[1 - 1/(a^2*x^2)] + (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x
^2)])
```

Rubi [A] time = 0.117566, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 75}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```


```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c - c
/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/Sq
rt[1 - 1/(a^2*x^2)] + (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x
^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)(1+ax)^2}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^3-\frac{1}{x^3}-\frac{a}{x^2}+\frac{a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0449956, size = 64, normalized size = 0.44

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x+a^2\log(x)+\frac{3a^2}{2}+\frac{a}{x}+\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*((3*a^2)/2 + 1/(2*x^2) + a/x + a^3*x + a^2*Log[x])
)/(a^3*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.237, size = 80, normalized size = 0.6

$$\frac{\left(2\,x^{3}a^{3}+2\,a^{2}\ln\left(x\right)x^{2}+2\,ax+1\right)x}{\left(2\,ax+2\right)\left(a^{2}x^{2}-1\right)}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(2*x^3*a^3+2*a^2*ln(x)*x^2+2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.40766, size = 103, normalized size = 0.71

$$\frac{\left(2\,a^{3}cx^{3}+2\,a^{2}cx^{2}\log\left(x\right)+2\,acx+c\right)\sqrt{a^{2}c}}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*c*x^3 + 2*a^2*c*x^2*log(x) + 2*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.833 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.100197, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0229307, size = 39, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.213, size = 50, normalized size = 0.8

$$\frac{(ax+\ln(x))\,x}{ax+1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.39132, size = 43, normalized size = 0.64

$$\frac{\sqrt{a^2c}(ax + \log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.834 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 - a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.106363, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 - a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{-1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0303147, size = 44, normalized size = 0.61

$$
\frac{\sqrt{1-\frac{1}{a^2x^2}}(ax+\log(1-ax))}{a\sqrt{c-\frac{c}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x + Log[1 - a*x]))/(a*Sqrt[c - c/(a^2*x^2)])
```

Maple [A] time = 0.229, size = 57, normalized size = 0.8

$$\frac{(ax-1)(ax+\ln(ax-1))}{a^2x}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\frac{1}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2*(a*x+
ln(a*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.45547, size = 57, normalized size = 0.79

$$\frac{\sqrt{a^2c}(ax+\log(ax-1))}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(a*x - 1))/(a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.835 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.133429, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0726939, size = 68, normalized size = 0.39

$$
\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)\right)}{4a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*
x]))/(4*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.228, size = 102, normalized size = 0.6

$$-\frac{(ax-1)\left(-4\,a^2x^2+ax\ln(ax+1)-5\,\ln(ax-1)\,xa+4\,ax-\ln(ax+1)+5\,\ln(ax-1)+2\right)(ax+1)}{4\,a^4x^3}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\left(\frac{c\left(a^2x^2-\right.}{a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(-4*a^2*x^2+a*x*ln(a*x+1)-5*ln(a*x-1)*
x*a+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)*(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.42893, size = 155, normalized size = 0.9

$$\frac{\left(4\,a^2x^2-4\,ax-(ax-1)\log(ax+1)+5\,(ax-1)\log(ax-1)-2\right)\sqrt{a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)*sqrt(a^2*c)/(a^3*c^2*x - a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}} \sqrt{\frac{ax-1}{ax+1}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.836 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{7}{}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + Sqrt[1 - 1/(a^2*x^2)]/(a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(8*a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*
c^2*Sqrt[c - c/(a^2*x^2)]) - (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.159471, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{7}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + Sqrt[1 - 1/(a^2*x^2)]/(a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(8*a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*
c^2*Sqrt[c - c/(a^2*x^2)]) - (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{23\sqrt{1-}}{16ac}$$

Mathematica [A] time = 0.107094, size = 86, normalized size = 0.33

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(16ax+\frac{16}{1-ax}-\frac{2}{ax+1}-\frac{2}{(ax-1)^2}+23\log(1-ax)-7\log(ax+1)\right)}{16a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(16*a*x + 16/(1 - a*x) - 2/(-1 + a*x)^2 - 2/(1 + a
*x) + 23*Log[1 - a*x] - 7*Log[1 + a*x]))/(16*a*(c - c/(a^2*x^2))^(5/2))
```

Maple [A] time = 0.234, size = 175, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-16x^4a^4+7a^3x^3\ln(ax+1)-23\ln(ax-1)x^3a^3+16x^3a^3-7\ln(ax+1)a^2x^2+23\ln(ax-1)a^2x\right.}{16a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-16*x^4*a^4+7*a^3*x^3*ln(a*x
+1)-23*ln(a*x-1)*x^3*a^3+16*x^3*a^3-7*ln(a*x+1)*a^2*x^2+23*ln(a*x-1)*a^2*x^
2+34*a^2*x^2-7*a*x*ln(a*x+1)+23*ln(a*x-1)*x*a-18*a*x+7*ln(a*x+1)-23*ln(a*x-
1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.46809, size = 292, normalized size = 1.11

$$\frac{\left(16a^4x^4-16a^3x^3-34a^2x^2+18ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)+23\left(a^3x^3-a^2x^2-ax+1\right)\log(ax-1)+1\right.}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
- a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.837 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=359

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(24*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (11*Sqrt[1 - 1/(a^2*x^2)]
)/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/
(2*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (5*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.194875, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(24*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (11*Sqrt[1 - 1/(a^2*x^2)]
)/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/
(2*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (5*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^4(1+ax)^3}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{8a^7(-1+ax)^4}+\frac{11}{16a^7(-1+ax)^3}+\frac{3}{2a^7(-1+ax)^2}+\frac{51}{32a^7(-1+ax)}-\frac{1}{16a^7(1+ax)^3}+\frac{5}{16a^7(1+ax)^2}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{}{32a}$$

Mathematica [A] time = 0.147996, size = 104, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96ax+\frac{144}{1-ax}-\frac{30}{ax+1}-\frac{33}{(ax-1)^2}+\frac{3}{(ax+1)^2}-\frac{4}{(ax-1)^3}+153\log(1-ax)-57\log(ax+1)\right)}{96a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*a*x + 144/(1 - a*x) - 4/(-1 + a*x)^3 - 33/(-1
+ a*x)^2 + 3/(1 + a*x)^2 - 30/(1 + a*x) + 153*Log[1 - a*x] - 57*Log[1 + a*x
]))/(96*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] time = 0.258, size = 247, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-96x^6a^6+57\ln(ax+1)x^5a^5-153\ln(ax-1)x^5a^5+96x^5a^5-57\ln(ax+1)a^4x^4+153\ln(ax-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/96/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-96*x^6*a^6+57*ln(a*x+1)*x^5
*a^5-153*ln(a*x-1)*x^5*a^5+96*x^5*a^5-57*ln(a*x+1)*a^4*x^4+153*ln(a*x-1)*a^
4*x^4+366*x^4*a^4-114*a^3*x^3*ln(a*x+1)+306*ln(a*x-1)*x^3*a^3-222*x^3*a^3+1
14*ln(a*x+1)*a^2*x^2-306*ln(a*x-1)*a^2*x^2-338*a^2*x^2+57*a*x*ln(a*x+1)-153
*ln(a*x-1)*x*a+122*a*x-57*ln(a*x+1)+153*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.62203, size = 443, normalized size = 1.23

$$\frac{\left(96\,a^{6}x^{6}-96\,a^{5}x^{5}-366\,a^{4}x^{4}+222\,a^{3}x^{3}+338\,a^{2}x^{2}-122\,ax-57\left(a^{5}x^{5}-a^{4}x^{4}-2\,a^{3}x^{3}+2\,a^{2}x^{2}+ax-1\right)\log\left(ax+1\right)+\right.}{96\left(a^{7}c^{4}x^{5}-a^{6}c^{4}x^{4}-2\,a^{5}c^{4}x^{3}+2\,a^{4}c^{4}x^{2}+a^{3}c^{4}x-a^{2}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/96*(96*a^6*x^6 - 96*a^5*x^5 - 366*a^4*x^4 + 222*a^3*x^3 + 338*a^2*x^2 - 1
22*a*x - 57*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x +
 1) + 153*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 - a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*
x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.838 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=372

$$-\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}+\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}+\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}-\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}$$

```
[Out] (11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) - (57*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (41*a^5*(c - c/(a^2*x^2))^
(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) + (57*a^4*(c - c/(a^2*x^2))^(7/2)*x
^5)/(80*(1 - a*x)^3*(1 + a*x)) - (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 + a
*x))/(40*(1 - a*x)^3) + (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
a*x)^2) + ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) + (2*a^6*(c -
 c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (2
5*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```

Rubi [A] time = 0.542188, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}+\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}+\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}-\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] (11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) - (57*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (41*a^5*(c - c/(a^2*x^2))^
(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) + (57*a^4*(c - c/(a^2*x^2))^(7/2)*x
^5)/(80*(1 - a*x)^3*(1 + a*x)) - (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 + a
*x))/(40*(1 - a*x)^3) + (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
a*x)^2) + ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) + (2*a^6*(c -
 c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (2
5*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{9/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}\left(2a-7a^2x\right)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1-ax}(1+ax)^{7/2}(\cdots}{x^5}}{30(1-ax)^{7/2}(1+ax)^{\cdots}}$$

$$=-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}-\cdots$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\cdots\right.}{\cdots}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}+\frac{a\left(c-\cdots\right.}{15\cdots}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{\cdots}}{40(1-\cdots}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{\cdots}}{80(1-ax)^3(1+\cdots}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{\cdots}}{80(1-ax)^3(1+\cdots}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{\cdots}}{80(1-ax)^3(1+\cdots}$$

$$=\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{\cdots}}{80(1-ax)^3(1+\cdots}$$

Mathematica [A] time = 0.15939, size = 150, normalized size = 0.4

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(240a^6x^6-736a^5x^5+105a^4x^4+352a^3x^3+70a^2x^2-96ax-40\right)+480a^6x^6\log\left(\sqrt{a^2x^2-1}+a\cdots\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2 +
 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.277, size = 795, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(2016*(-c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(7/2)*x^7*a^9*c-2016*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^5*
a^9+480*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c-375*(-c/a^2)
^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^6*a^8*c+560*(-c/a^2)^(1/2)*((a*x-1)*(a*x
+1)*c/a^2)^(5/2)*x^7*a^7*c^2-105*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x
^4*a^8-2352*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^7*a^7*c^2-224*(-c/a^
2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^3*a^7+525*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(5/2)*x^6*a^6*c^2-700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x
^7*a^5*c^3+2940*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^7*a^5*c^3-630*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^2*a^6-875*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(3/2)*x^6*a^4*c^3+1050*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1
/2)*x^7*a^3*c^4-672*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x*a^5-4410*(-c
/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^7*a^3*c^4+4410*c^(9/2)*(-c/a^2)^(1/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^6*a-1050*c^(9/2)*(-c/a^2)^(1/2
)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^6*a-280*a^4*(c*
(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)+2625*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^
2)^(1/2)*x^6*a^2*c^4+2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^
2-c)/x/a^2)*x^6*c^5)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(7/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x - 1), x)
```

Fricas [A] time = 1.84289, size = 972, normalized size = 2.61

$$\left[-\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(240\,a^6c^3x^6-736\,a^5c^3x^5\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
- 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^6*c
^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x
^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240
*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 1
05*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] time = 36.1422, size = 1059, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Giac [A] time = 68.889, size = 757, normalized size = 2.03

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}c^3\operatorname{sgn}(x)}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
```

```
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

3.839 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=294

$$\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}+\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}-2a$$

```
[Out] (-5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) + (25*a^4*(c - c/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (17*a^3*(c - c/(a^2*x^2))^(5
/2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) + (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a
*x))/(6*(1 - a*x)^2) + ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x))
- (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x
)^(5/2)) - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1
+ a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rubi [A] time = 0.495301, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}+\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}-2a$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] (-5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) + (25*a^4*(c - c/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (17*a^3*(c - c/(a^2*x^2))^(5
/2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) + (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a
*x))/(6*(1 - a*x)^2) + ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x))
- (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x
)^(5/2)) - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1
+ a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}}{x^5}\,dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}(1+ax)^{5/2}\left(2a-5a^2x\right)}{x^4}\,dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1+ax)^{5/2}\left(-15a^2+\right.}{x^3\sqrt{1-ax}}}{12(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right.\right.}{}$$

$$=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-}$$

$$=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$

$$=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$

$$=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$

$$=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1-ax)}$$

Mathematica [A] time = 0.131823, size = 134, normalized size = 0.46

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4-64a^3x^3-3a^2x^2+16ax+6\right)+48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64
*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4*x
^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.187, size = 625, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7+
48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c+27*(-c/a^2)^(1/2)
*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a
^2)^(3/2)*x^5*a^5*c^2-75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+1
00*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*(
c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5-45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*
x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-150
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3-30*a^4*(c*(a^2*x^2-1)
/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2*
x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((c^(1/2)*((a*x-1)*(a*
x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^4*a+135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(1/2)*x^4*a^2*c^3+135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-
c)/x/a^2)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x - 1), x)
```

Fricas [A] time = 1.80223, size = 856, normalized size = 2.91

$$\left[-\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4c^2x^4-64\,a^3c^2x^3-3\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2
*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^2*x
^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c
^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] time = 19.4013, size = 500, normalized size = 1.7

$$c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}}\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}}\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt(c)*
```

```
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

Giac [A] time = 3.20855, size = 562, normalized size = 1.91

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\,(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}\,(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}c^2\mathrm{sgn}\,(x)}{a^2}-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) - 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

3.840 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=213

$$-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^3}$$

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x) - (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*(1
- a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rubi [A] time = 0.459027, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x) - (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*(1
- a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
```

```
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1-ax}(1+ax)^{5/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1+ax)^{3/2}(2a-3a^2x)}{x^2\sqrt{1-ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1+ax}(a^2-5a^3x)}{x\sqrt{1-ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int -}{2a(1-ax)^{3/2}(1}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}-\frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int}{2(1-ax)^{3/2}(1}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)S}{2(}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\,\mathrm{si}}{(1-ax)^{3/2}(1+ax}$$

Mathematica [A] time = 0.105637, size = 115, normalized size = 0.54

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2-4ax-1\right)+4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^2
*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]
))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.183, size = 455, normalized size = 2.1

$$\frac{x}{6\,a^2c}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{5/2}xa^5+4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5+4*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c-(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
x^3*a^3*c^2-3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*c^(5/2)*ln(x*c^(1/2)+(c*(a^2*x^2
-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-6*c^(5/2)*(-c/a^2)^(1/2)*ln((c^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a+3*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(1/2)*x^2*a^2*c^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/
2)*a^2-c)/x/a^2)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x - 1), x)
```

Fricas [A] time = 1.7508, size = 694, normalized size = 3.26

$$\left[-\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(2\,a^2cx^2-4\,acx-c\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\quad ac^{\frac{3}{2}}x\,a}{4\,a^2x},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x
*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*
x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] time = 12.2101, size = 376, normalized size = 1.77

$$c\left(\begin{cases} \frac{\sqrt{c}\sqrt{a^2x^2-1}}{a} - \frac{i\sqrt{c}\log(ax)}{a} + \frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a} + \frac{\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a} + \frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a} - \frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise} \end{cases}\right) + \frac{2c\left(\begin{cases} -\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}} + \sqrt{c}\,\mathrm{acosh}(ax) + \frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{fo} \\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}} - i\sqrt{c}\,\mathrm{asin}(ax) - \frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{ot} \end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Giac [A] time = 1.58784, size = 359, normalized size = 1.69

$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x)}{a^2}-\frac{\left(\sqrt{a^2c}x-\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 + 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) - 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

3.841 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.360192, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int}{\sqrt{1-ax}\sqrt{1+}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0780781, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] time = 0.191, size = 197, normalized size = 1.7

$$\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx\right)\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\,(\ldots}{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Fricas [A] time = 1.7825, size = 576, normalized size = 4.97

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\ldots}}{a^2cx^2\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.18531, size = 207, normalized size = 1.78

$$\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\operatorname{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\operatorname{arct}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 - 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(
a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/sq
rt(c)) - a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs(
a)
```

3.842 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=111

$$-\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) - (1 + a*x)^2/(a^2*S
qrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.307312, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 78, 50, 41, 216}

$$-\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) - (1 + a*x)^2/(a^2*S
qrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

$$= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{1+ax}}{(1-ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{2\sqrt{1-ax}\sqrt{1+ax}\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.0708311, size = 68, normalized size = 0.61

$$\frac{a^2x^2+2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-2ax-3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-3 - 2*a*x + a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] time = 0.184, size = 177, normalized size = 1.6

$$\frac{1}{x\,(ax-1)\,a}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\left(\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\sqrt{c}xa^2+2\,\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)xac-\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a\sqrt{c}-2\,a\sqrt{\frac{(ax-1)}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2)^(1/2)*((c*(a^2*x^2-1)/a^2)^(1/2)*c^(1/2)*x*a^2+2*ln(x*c
^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)
-2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^
2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 1.76919, size = 448, normalized size = 4.04

$$\left[\frac{(ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(a^2x^2-3\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac},-\frac{2\,(ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(a\right.}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x - a*c), -(2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^2*c*x - a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.843 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -(1 + a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(5 - 2*a*x)*(1 - a*x)*(
1 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 + a
*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.435983, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 98, 143, 41, 216}

$$\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```


```
[Out] -(1 + a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(5 - 2*a*x)*(1 - a*x)*(
1 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 + a
*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$$

$$= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{(1-ax)^{5/2}\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2+4ax)}{(1-ax)^{3/2}\sqrt{1+ax}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.0836096, size = 95, normalized size = 0.77

$$\frac{3a^3x^3-11a^2x^2+6(ax-1)\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-4ax+10}{3a^2cx(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

Maple [B] time = 0.184, size = 326, normalized size = 2.7

$$\frac{ax+1}{3\,a^4x^3}\left(3\,c^{3/2}\sqrt{\frac{(ax-1)\,(ax+1)\,c}{a^2}}x^3a^3+4\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}c^{3/2}x^2a^2-15\,x^2a^2c^{3/2}\sqrt{\frac{(ax-1)\,(ax+1)\,c}{a^2}}+6\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3+4*(c*(a^2*x^2-1)/a^2)^
(1/2)*c^(3/2)*x^2*a^2-15*x^2*a^2*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+6*(c
*(a^2*x^2-1)/a^2)^(1/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a
*x+1)*c/a^2)^(1/2)*x*a^2*c-4*(c*(a^2*x^2-1)/a^2)^(1/2)*c^(3/2)*x*a-6*(c*(a^
2*x^2-1)/a^2)^(1/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1
)*c/a^2)^(1/2)*a*c-2*(c*(a^2*x^2-1)/a^2)^(1/2)*c^(3/2)+12*c^(3/2)*((a*x-1)*
(a*x+1)*c/a^2)^(1/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^2
-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A] time = 1.79856, size = 591, normalized size = 4.8

$$\left[\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3-14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},-\frac{6\left(a^2x^2-2\,ax+1\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(6*
(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.844 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-a\ldots}{a^6x^5\left(c-\ldots\right.}$$

```
[Out] -(1 + a*x)^2/(5*a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)*(1 + a*x)^2)/
(3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) - (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^4*
(c - c/(a^2*x^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/(1
5*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*Arc
Sin[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.463339, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-a\ldots}{a^6x^5\left(c-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -(1 + a*x)^2/(5*a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)*(1 + a*x)^2)/
(3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) - (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^4*
(c - c/(a^2*x^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/(1
5*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*Arc
Sin[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
```

```
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{7/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+6ax)}{(1-ax)^{5/2}(1+ax)^{3/2}}\,dx}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2\left(-30a-28a^2x\right)}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x(116}{\sqrt{1-a}}}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+43ax}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+43ax}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+43ax}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.0984057, size = 105, normalized size = 0.52

$$\frac{15a^4x^4-76a^3x^3+32a^2x^2+30(ax-1)^2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+82ax-56}{15a^2c^2x(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*(-1 + a*x)^2*Sqrt
[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x
^2)]*x*(-1 + a*x)^2)
```

Maple [B] time = 0.187, size = 462, normalized size = 2.3

$$\frac{ax+1}{15\,x^5a^6}\left(15\,c^{5/2}\left(\frac{(ax-1)\,(ax+1)\,c}{a^2}\right)^{3/2}x^5a^5-45\,x^4c^{5/2}a^4\left(\frac{(ax-1)\,(ax+1)\,c}{a^2}\right)^{3/2}-16\,c^{5/2}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^4a^4-60\,c^{5/2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5-45*x^4*c^(5/2)*a^4*(
(a*x-1)*(a*x+1)*c/a^2)^(3/2)-16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-6
0*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^3+30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c+90*c^(5/2)*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*x^2*a^2+24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln(
x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^(
5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a-50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)
-6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2)
/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [A] time = 2.07318, size = 745, normalized size = 3.67

$$\left[\frac{15\left(a^4x^4 - 2a^3x^3 + 2ax - 1\right)\sqrt{c}\log\left(2a^2cx^2 + 2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - c\right) + \left(15a^5x^5 - 76a^4x^4 + 32a^3x^3 + 82a^2x^2 - 56ax\right.}{15\left(a^5c^3x^4 - 2a^4c^3x^3 + 2a^2c^3x - ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 - 76*a^4*x^
4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 - 2*a^3
*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82*a^2*x
^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.845 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=283

$$\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^{}}{5a^3x^2\left(c-\right.}$$

```
[Out] -(1 + a*x)^2/(7*a^2*(c - c/(a^2*x^2))^(7/2)*x) + (2*(1 - a*x)*(1 + a*x)^2)/
(5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) - (124*(1 - a*x)^2*(1 + a*x)^2)/(105*a^
4*(c - c/(a^2*x^2))^(7/2)*x^3) + (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(c
- c/(a^2*x^2))^(7/2)*x^4) + (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a
^2*x^2))^(7/2)*x^5) + (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*(c
 - c/(a^2*x^2))^(7/2)*x^7) - (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x]
)/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rubi [A] time = 0.502057, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^{}}{5a^3x^2\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```



```
[Out] -(1 + a*x)^2/(7*a^2*(c - c/(a^2*x^2))^(7/2)*x) + (2*(1 - a*x)*(1 + a*x)^2)/
(5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) - (124*(1 - a*x)^2*(1 + a*x)^2)/(105*a^
4*(c - c/(a^2*x^2))^(7/2)*x^3) + (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(c
- c/(a^2*x^2))^(7/2)*x^4) + (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a
^2*x^2))^(7/2)*x^5) + (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*(c
 - c/(a^2*x^2))^(7/2)*x^7) - (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x]
)/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
```

```
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{9/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+8ax)}{(1-ax)^{7/2}(1+ax)^{5/2}}\,dx}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4\left(-70a-54a^2x\right)}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{35a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^3(496}{(1-ax)}}{105a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{((1-}{}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1}{35a^6}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1}{35a^6}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1}{35a^6}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{142(1}{35a^6}$$

Mathematica [A] time = 0.12093, size = 133, normalized size = 0.47

$$\frac{105a^6x^6-562a^5x^5+74a^4x^4+1226a^3x^3-636a^2x^2+210(ax-1)^3(ax+1)\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-654ax+4}{105a^2c^3x(ax-1)^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (432 - 654*a*x - 636*a^2*x^2 + 1226*a^3*x^3 + 74*a^4*x^4 - 562*a^5*x^5 + 10
5*a^6*x^6 + 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x))
```

Maple [B] time = 0.212, size = 572, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/105*(105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7-553*x^6*c^(7/2)*a^
6*((a*x-1)*(a*x+1)*c/a^2)^(5/2)+96*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^6*a^
6-392*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^5*a^5-96*c^(7/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^5*a^5+1540*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4-24
0*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^4+210*ln(x*c^(1/2)+(c*(a^2*x^2-1)
/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^6*
c+350*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^3*a^3+240*c^(7/2)*(c*(a^2*x^2
-1)/a^2)^(5/2)*x^3*a^3-210*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)
*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*a^5*c-1470*c^(7/2)*((a*x-1)
*(a*x+1)*c/a^2)^(5/2)*x^2*a^2+180*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^2*a^2
-42*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a-180*c^(7/2)*(c*(a^2*x^2-1)/a^
2)^(5/2)*x*a+462*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)-30*c^(7/2)*(c*(a^2*x
^2-1)/a^2)^(5/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)/a^8/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [A] time = 2.60046, size = 1045, normalized size = 3.69

$$\left[\frac{105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(105\,a^7x^7-562\,a^6x^6+\right.}{105\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*a^3
*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x
+ a*c^4), -1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2
- 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^
4 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2
*a^2*c^4*x + a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.846 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqrt
[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(a
^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^4*Sqrt[c - c/(a^2*x^2)])
/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^3) + (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^4
*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.165337, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqrt
[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(a
^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^4*Sqrt[c - c/(a^2*x^2)])
/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^3) + (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^4
*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx &= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^3(1+ax)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^9-\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}+\frac{6a^4}{x^5}-\frac{6a^5}{x^4}-\frac{8a^6}{x^3}+\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2c^4\sqrt{c-\cdots}}{a^4\sqrt{1-\cdots}}
\end{aligned}$$

Mathematica [A] time = 0.0857085, size = 97, normalized size = 0.3

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{4a^6}{x^2}+\frac{2a^5}{x^3}-\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+a^9x+3a^8\log(x)+\frac{3a}{7x^7}+\frac{1}{8x^8}\right)}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*(1/(8*x^8) + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) - (3*
a^4)/(2*x^4) + (2*a^5)/x^3 + (4*a^6)/x^2 + a^9*x + 3*a^8*Log[x]))/(a^9*(1 -
 1/(a^2*x^2))^(9/2))
```

Maple [A] time = 0.231, size = 112, normalized size = 0.4

$$\frac{\left(280\,a^9x^9+840\,a^8\ln(x)\,x^8+1120\,x^6a^6+560\,x^5a^5-420\,x^4a^4-448\,x^3a^3+120\,ax+35\right)x}{280\,(ax+1)^3\left(a^2x^2-1\right)^3}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] 1/280*(280*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6+560*x^5*a^5-420*x^4*a^4-4
48*x^3*a^3+120*a*x+35)*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a*x+1)^3/(a^2*x^2-1
)^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.61214, size = 228, normalized size = 0.71

$$\frac{\left(280\,a^9c^4x^9+840\,a^8c^4x^8\log(x)+1120\,a^6c^4x^6+560\,a^5c^4x^5-420\,a^4c^4x^4-448\,a^3c^4x^3+120\,ac^4x+35\,c^4\right)\sqrt{a^2c}}{280\,a^{10}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/280*(280*a^9*c^4*x^9 + 840*a^8*c^4*x^8*log(x) + 1120*a^6*c^4*x^6 + 560*a^
5*c^4*x^5 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqrt(
a^2*c)/(a^10*x^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.847 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=324

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqr
t[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (c^3*Sqrt[c - c/(a^
2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/
(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^3*Sq
rt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.15883, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqr
t[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (c^3*Sqrt[c - c/(a^
2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/
(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^3*Sq
rt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^2(1+ax)^5}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7+\frac{1}{x^7}+\frac{3a}{x^6}+\frac{a^2}{x^5}-\frac{5a^3}{x^4}-\frac{5a^4}{x^3}+\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{5c^3\sqrt{c-}}{2a^3\sqrt{1-}}$$

Mathematica [A] time = 0.0833184, size = 94, normalized size = 0.29

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(3a^6\log(x)-\frac{-60a^7x^7+60a^5x^5-150a^4x^4-100a^3x^3+15a^2x^2+36ax+10}{60x^6}\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-(10 + 36*a*x + 15*a^2*x^2 - 100*a^3*x^3 - 150*a^
4*x^4 + 60*a^5*x^5 - 60*a^7*x^7)/(60*x^6) + 3*a^6*Log[x]))/(a^7*(1 - 1/(a^2
*x^2))^(7/2))
```

Maple [A] time = 0.247, size = 112, normalized size = 0.4

$$\frac{\left(60\,a^{7}x^{7}+180\,a^{6}\ln\left(x\right)x^{6}-60\,x^{5}a^{5}+150\,x^{4}a^{4}+100\,x^{3}a^{3}-15\,a^{2}x^{2}-36\,ax-10\right)x}{60\,\left(ax+1\right)^{3}\left(a^{2}x^{2}-1\right)^{2}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/60*(60*a^7*x^7+180*a^6*ln(x)*x^6-60*x^5*a^5+150*x^4*a^4+100*x^3*a^3-15*a^
2*x^2-36*a*x-10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)^3/(a^2*x^2-1)^2/((
a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.65373, size = 219, normalized size = 0.68

$$\frac{\left(60\,a^{7}c^{3}x^{7}+180\,a^{6}c^{3}x^{6}\log\left(x\right)-60\,a^{5}c^{3}x^{5}+150\,a^{4}c^{3}x^{4}+100\,a^{3}c^{3}x^{3}-15\,a^{2}c^{3}x^{2}-36\,ac^{3}x-10\,c^{3}\right)\sqrt{a^{2}c}}{60\,a^{8}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/60*(60*a^7*c^3*x^7 + 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 + 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2 x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.848 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=234

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[c
 - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*x^2
)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*Sq
rt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)
] + (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.142042, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 75}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[c
 - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*x^2
)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*Sq
rt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)
] + (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)(1+ax)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+$$

Mathematica [A] time = 0.053227, size = 87, normalized size = 0.37

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{3}{4}a\left(a^4x-\frac{6a^2}{x}+4a^3\log(x)-\frac{2a}{x^2}-\frac{1}{3x^3}\right)+\frac{(ax+1)^5}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*((1 + a*x)^5/(4*x^4) + (3*a*(-1/(3*x^3) - (2*a)/x^
2 - (6*a^2)/x + a^4*x + 4*a^3*Log[x]))/4))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A] time = 0.23, size = 96, normalized size = 0.4

$$\frac{\left(4\,x^5a^5+12\,a^4\ln\left(x\right)x^4-8\,x^3a^3+4\,a^2x^2+4\,ax+1\right)x}{4\,\left(ax+1\right)^3\left(a^2x^2-1\right)}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/4*(4*x^5*a^5+12*a^4*ln(x)*x^4-8*x^3*a^3+4*a^2*x^2+4*a*x+1)*(c*(a^2*x^2-1)
/a^2/x^2)^(5/2)*x/(a*x+1)^3/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.60863, size = 158, normalized size = 0.68

$$\frac{\left(4\,a^5c^2x^5+12\,a^4c^2x^4\log\left(x\right)-8\,a^3c^2x^3+4\,a^2c^2x^2+4\,ac^2x+c^2\right)\sqrt{a^2c}}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 + 4*a^2*c^2*x^2
+ 4*a*c^2*x + c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.849 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c
- c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)
/Sqrt[1 - 1/(a^2*x^2)] + (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(
a^2*x^2)])
```

Rubi [A] time = 0.128052, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c
- c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)
/Sqrt[1 - 1/(a^2*x^2)] + (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(
a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(1+ax)^3}{x^3}\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3+\frac{1}{x^3}+\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.044434, size = 59, normalized size = 0.4

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x+3a^2\log(x)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*(-1/(2*x^2) - (3*a)/x + a^3*x + 3*a^2*Log[x]))/(a^
3*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.244, size = 69, normalized size = 0.5

$$\frac{\left(2 x^{3} a^{3}+6 a^{2} \ln \left(x\right) x^{2}-6 a x -1\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \left(\frac{a x -1}{a x +1}\right)^{-\frac{3}{2}}}{2 \left(a x +1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(2*x^3*a^3+6*a^2*ln(x)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^{2} x^{2}}\right)^{\frac{3}{2}}}{\left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.61299, size = 103, normalized size = 0.7

$$\frac{\left(2 a^{3} c x^{3} + 6 a^{2} c x^{2} \log\left(x\right) - 6 a c x - c\right) \sqrt{a^{2} c}}{2 a^{4} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*c*x^3 + 6*a^2*c*x^2*log(x) - 6*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.850 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.11997, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0293966, size = 50, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+4\log(1-ax)-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] time = 0.231, size = 65, normalized size = 0.6

$$-\frac{\left(-ax+\ln\left(x\right)-4\ \ln\left(ax-1\right)\right)x\left(ax-1\right)}{\left(ax+1\right)^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.67933, size = 66, normalized size = 0.61

$$\frac{\sqrt{a^{2}c}\left(ax+4\ \log\left(ax-1\right)-\log\left(x\right)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.851 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=115

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.123677, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 77}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(1+ax)}{(-1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0475806, size = 56, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(ax+\frac{2}{1-ax}+3\log(1-ax)\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x + 2/(1 - a*x) + 3*Log[1 - a*x]))/(a*Sqrt[c - c/
(a^2*x^2)])
```

Maple [A] time = 0.243, size = 85, normalized size = 0.7

$$\frac{(ax-1)\left(a^2x^2+3\,\ln(ax-1)\,xa-ax-3\,\ln(ax-1)-2\right)}{(ax+1)\,xa^2}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\frac{1}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a
^2*(a^2*x^2+3*ln(a*x-1)*x*a-a*x-3*ln(a*x-1)-2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.54377, size = 105, normalized size = 0.91

$$\frac{\left(a^2x^2-ax+3\,(ax-1)\log(ax-1)-2\right)\sqrt{a^2c}}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c*x - a^2*c
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.852 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.145668, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{a^3(-1+ax)^3}+\frac{3}{a^3(-1+ax)^2}+\frac{3}{a^3(-1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0732523, size = 64, normalized size = 0.37

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2ax+\frac{5-6ax}{(ax-1)^2}+6\log(1-ax)\right)}{2a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(2*a*x + (5 - 6*a*x)/(-1 + a*x)^2 + 6*Log[1 - a*x]
))/(2*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.25, size = 102, normalized size = 0.6

$$\frac{(ax-1)\left(2x^3a^3+6\ln(ax-1)a^2x^2-4a^2x^2-12\ln(ax-1)xa-4ax+6\ln(ax-1)+5\right)}{2a^4x^3}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(2*x^3*a^3+6*ln(a*x-1)*a^2*x^2-4*a^2*x^
2-12*ln(a*x-1)*x*a-4*a*x+6*ln(a*x-1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.5092, size = 176, normalized size = 1.03

$$\frac{\left(2a^3x^3-4a^2x^2-4ax+6\left(a^2x^2-2ax+1\right)\log(ax-1)+5\right)\sqrt{a^2c}}{2\left(a^4c^2x^2-2a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 6*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) +
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.853 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}- \cdots$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.176567, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}- \cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^4(1+ax)}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{2a^5(-1+ax)^4}+\frac{9}{4a^5(-1+ax)^3}+\frac{31}{8a^5(-1+ax)^2}+\frac{49}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+49
\end{aligned}$$

Mathematica [A] time = 0.105571, size = 86, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48ax+\frac{186}{1-ax}-\frac{54}{(ax-1)^2}-\frac{8}{(ax-1)^3}+147\log(1-ax)-3\log(ax+1)\right)}{48a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*a*x + 186/(1 - a*x) - 8/(-1 + a*x)^3 - 54/(-1
+ a*x)^2 + 147*Log[1 - a*x] - 3*Log[1 + a*x]))/(48*a*(c - c/(a^2*x^2))^(5/2
))
```

Maple [A] time = 0.234, size = 175, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-48x^4a^4+3a^3x^3\ln(ax+1)-147\ln(ax-1)x^3a^3+144x^3a^3-9\ln(ax+1)a^2x^2+441\ln(ax-1)a\right.}{48a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*x^4*a^4+3*a^3*x^3*ln(a*x
+1)-147*ln(a*x-1)*x^3*a^3+144*x^3*a^3-9*ln(a*x+1)*a^2*x^2+441*ln(a*x-1)*a^2
*x^2+42*a^2*x^2+9*a*x*ln(a*x+1)-441*ln(a*x-1)*x*a-270*a*x-3*ln(a*x+1)+147*l
n(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.60547, size = 313, normalized size = 1.17

$$\frac{\left(48a^4x^4-144a^3x^3-42a^2x^2+270ax-3\left(a^3x^3-3a^2x^2+3ax-1\right)\log(ax+1)+147\left(a^3x^3-3a^2x^2+3ax-1\right)\log(a\right.}{48\left(a^5c^3x^3-3a^4c^3x^2+3a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(48*a^4*x^4 - 144*a^3*x^3 - 42*a^2*x^2 + 270*a*x - 3*(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)*log(a*x + 1) + 147*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2
*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.854 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=360

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}-$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^4) + Sqrt[1 - 1/(a^2*x^2)]/(2*
a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*
c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64
*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.206108, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^4) + Sqrt[1 - 1/(a^2*x^2)]/(2*
a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*
c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64
*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^5(1+ax)^2}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{4a^7(-1+ax)^5}+\frac{3}{2a^7(-1+ax)^4}+\frac{59}{16a^7(-1+ax)^3}+\frac{75}{16a^7(-1+ax)^2}+\frac{201}{64a^7(-1+ax)}+\frac{1}{32a^7(1+ax}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^4}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{}{16}$$

Mathematica [A] time = 0.126351, size = 140, normalized size = 0.39

$$\frac{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{x}{a^7}+\frac{75}{16a^8(1-ax)}-\frac{1}{32a^8(ax+1)}-\frac{59}{32a^8(1-ax)^2}+\frac{1}{2a^8(1-ax)^3}-\frac{1}{16a^8(1-ax)^4}+\frac{201\log(1-ax)}{64a^8}-\frac{9\log(ax+1)}{64a^8}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (a^7*(1 - 1/(a^2*x^2))^(7/2)*(x/a^7 - 1/(16*a^8*(1 - a*x)^4) + 1/(2*a^8*(1
- a*x)^3) - 59/(32*a^8*(1 - a*x)^2) + 75/(16*a^8*(1 - a*x)) - 1/(32*a^8*(1
+ a*x)) + (201*Log[1 - a*x])/(64*a^8) - (9*Log[1 + a*x])/(64*a^8)))/(c - c/
(a^2*x^2))^(7/2)
```

Maple [A] time = 0.249, size = 247, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-64x^6a^6+9\ln(ax+1)x^5a^5-201\ln(ax-1)x^5a^5+192x^5a^5-27\ln(ax+1)a^4x^4+603\ln(ax-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/64/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*x^6*a^6+9*ln(a*x+1)*x^5*
a^5-201*ln(a*x-1)*x^5*a^5+192*x^5*a^5-27*ln(a*x+1)*a^4*x^4+603*ln(a*x-1)*a^
4*x^4+174*x^4*a^4+18*a^3*x^3*ln(a*x+1)-402*ln(a*x-1)*x^3*a^3-618*x^3*a^3+18
*ln(a*x+1)*a^2*x^2-402*ln(a*x-1)*a^2*x^2+118*a^2*x^2-27*a*x*ln(a*x+1)+603*l
n(a*x-1)*x*a+414*a*x+9*ln(a*x+1)-201*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.82671, size = 460, normalized size = 1.28

$$\frac{\left(64\,a^6x^6-192\,a^5x^5-174\,a^4x^4+618\,a^3x^3-118\,a^2x^2-414\,ax-9\left(a^5x^5-3\,a^4x^4+2\,a^3x^3+2\,a^2x^2-3\,ax+1\right)\log\left(ax+\right.\right.}{64\left(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/64*(64*a^6*x^6 - 192*a^5*x^5 - 174*a^4*x^4 + 618*a^3*x^3 - 118*a^2*x^2 -
414*a*x - 9*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log(a
*x + 1) + 201*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log
(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 +
 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.855 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[
c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^3*Sqrt[c - c/(a^
2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a
^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
 - 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x
^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (c^3*Sqrt[c
 - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.172026, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[
c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^3*Sqrt[c - c/(a^
2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a
^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
 - 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x
^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (c^3*Sqrt[c
 - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^4(1+ax)^3}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7+\frac{1}{x^7}-\frac{a}{x^6}-\frac{3a^2}{x^5}+\frac{3a^3}{x^4}+\frac{3a^4}{x^3}-\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}
\end{aligned}$$

Mathematica [A] time = 0.0698008, size = 97, normalized size = 0.3

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{3a^4}{2x^2}-\frac{a^3}{x^3}+\frac{3a^2}{4x^4}+a^7x+\frac{3a^5}{x}-a^6\log(x)+\frac{a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/(6*x^6) + a/(5*x^5) + (3*a^2)/(4*x^4) - a^3/x^
3 - (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x - a^6*Log[x]))/(a^7*(1 - 1/(a^2*x^2
))^(7/2))
```

Maple [A] time = 0.236, size = 112, normalized size = 0.4

$$-\frac{x\left(-60\,a^{7}x^{7}+60\,a^{6}\ln\left(x\right)x^{6}-180\,x^{5}a^{5}+90\,x^{4}a^{4}+60\,x^{3}a^{3}-45\,a^{2}x^{2}-12\,ax+10\right)}{\left(60\,ax-60\right)\left(a^{2}x^{2}-1\right)^{3}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/60*((a*x-1)/(a*x+1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x*(-60*a^7*x^7+
60*a^6*ln(x)*x^6-180*x^5*a^5+90*x^4*a^4+60*x^3*a^3-45*a^2*x^2-12*a*x+10)/(a
*x-1)/(a^2*x^2-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.53244, size = 216, normalized size = 0.67

$$\frac{\left(60\,a^{7}c^{3}x^{7}-60\,a^{6}c^{3}x^{6}\log\left(x\right)+180\,a^{5}c^{3}x^{5}-90\,a^{4}c^{3}x^{4}-60\,a^{3}c^{3}x^{3}+45\,a^{2}c^{3}x^{2}+12\,ac^{3}x-10\,c^{3}\right)\sqrt{a^{2}c}}{60\,a^{8}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 - 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.856 $\int e^{-\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^{5/2} dx$

Optimal. Leaf size=238

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[c
 - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x
^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*
Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^
2)] - (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.145783, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```


```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[c
 - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x
^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*
Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^
2)] - (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5-\frac{1}{x^5}+\frac{a}{x^4}+\frac{2a^2}{x^3}-\frac{2a^3}{x^2}-\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\end{aligned}$$

Mathematica [A] time = 0.0555824, size = 77, normalized size = 0.32

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{a^2}{x^2}+a^5x+\frac{2a^3}{x}-a^4\log(x)-\frac{a}{3x^3}+\frac{1}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(1/(4*x^4) - a/(3*x^3) - a^2/x^2 + (2*a^3)/x + a^5
*x - a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A] time = 0.221, size = 96, normalized size = 0.4

$$-\frac{x\left(-12\,x^{5}a^{5}+12\,a^{4}\ln\left(x\right)x^{4}-24\,x^{3}a^{3}+12\,a^{2}x^{2}+4\,ax-3\right)}{\left(12\,ax-12\right)\left(a^{2}x^{2}-1\right)^{2}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/12*((a*x-1)/(a*x+1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x*(-12*x^5*a^5+
12*a^4*ln(x)*x^4-24*x^3*a^3+12*a^2*x^2+4*a*x-3)/(a*x-1)/(a^2*x^2-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.83784, size = 166, normalized size = 0.7

$$\frac{\left(12\,a^{5}c^{2}x^{5}-12\,a^{4}c^{2}x^{4}\log\left(x\right)+24\,a^{3}c^{2}x^{3}-12\,a^{2}c^{2}x^{2}-4\,ac^{2}x+3\,c^{2}\right)\sqrt{a^{2}c}}{12\,a^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(12*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 - 12*a^2*c^2*
x^2 - 4*a*c^2*x + 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.857 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=147

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c -
c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/S
qrt[1 - 1/(a^2*x^2)] - (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Rubi [A] time = 0.124496, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 75}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```



```
[Out] -(c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c -
c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/S
qrt[1 - 1/(a^2*x^2)] - (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*
x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^2(1+ax)}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0442541, size = 65, normalized size = 0.44

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x-a^2\log(x)-\frac{3a^2}{2}+\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*((-3*a^2)/2 - 1/(2*x^2) + a/x + a^3*x - a^2*Log[x]
))/(a^3*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.226, size = 80, normalized size = 0.5

$$-\frac{x\left(-2x^{3}a^{3}+2a^{2}\ln\left(x\right)x^{2}-2ax+1\right)}{\left(2ax-2\right)\left(a^{2}x^{2}-1\right)}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x*(-2*x^3*a^3+2*
a^2*ln(x)*x^2-2*a*x+1)/(a*x-1)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.99713, size = 103, normalized size = 0.7

$$\frac{\left(2a^{3}cx^{3}-2a^{2}cx^{2}\log\left(x\right)+2acx-c\right)\sqrt{a^{2}c}}{2a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*c*x^3 - 2*a^2*c*x^2*log(x) + 2*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}} \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.858 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.106606, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```


```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0245002, size = 41, normalized size = 0.6

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.239, size = 52, normalized size = 0.8

$$-\frac{(-ax+\ln(x))\,x}{ax-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.91127, size = 43, normalized size = 0.63

$$\frac{\sqrt{a^2c}(ax - \log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.859 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 + a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.113005, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 + a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.0301584, size = 45, normalized size = 0.62

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}(ax-\log(ax+1))}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x - Log[1 + a*x]))/(a*Sqrt[c - c/(a^2*x^2)])
```

Maple [A] time = 0.219, size = 59, normalized size = 0.8

$$-\frac{(ax+1)(-ax+\ln(ax+1))}{a^2x}\sqrt{\frac{ax-1}{ax+1}}\frac{1}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)/x/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [A] time = 1.88431, size = 57, normalized size = 0.79

$$\frac{\sqrt{a^2c}(ax-\log(ax+1))}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(a*x + 1))/(a^2*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.860 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a
*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.1429, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a
*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)(1+ax)^2}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{4a^3(-1+ax)}+\frac{1}{2a^3(1+ax)^2}-\frac{5}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0645993, size = 65, normalized size = 0.38

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax-\frac{2}{ax+1}+\log(1-ax)-5\log(ax+1)\right)}{4a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*a*x - 2/(1 + a*x) + Log[1 - a*x] - 5*Log[1 + a*
x]))/(4*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.281, size = 103, normalized size = 0.6

$$-\frac{(ax+1)\left(-4\,a^2x^2+5\,ax\ln(ax+1)-\ln(ax-1)\,xa-4\,ax+5\,\ln(ax+1)-\ln(ax-1)+2\right)(ax-1)}{4\,a^4x^3}\sqrt{\frac{ax-1}{ax+1}}\left(\frac{c\left(a^2x^2\right.}{a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-4*a^2*x^2+5*a*x*ln(a*x+1)-ln(a*x-1)*
x*a-4*a*x+5*ln(a*x+1)-ln(a*x-1)+2)*(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

Fricas [A] time = 1.97078, size = 155, normalized size = 0.9

$$\frac{\left(4\,a^2x^2+4\,ax-5\,(ax+1)\log(ax+1)+(ax+1)\log(ax-1)-2\right)\sqrt{a^2c}}{4\left(a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^2*x^2 + 4*a*x - 5*(a*x + 1)*log(a*x + 1) + (a*x + 1)*log(a*x - 1)
- 2)*sqrt(a^2*c)/(a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

3.861 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23}{}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(8*a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - Sqrt[1 - 1/(a^2*x^2)]/(a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)]) - (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.169411, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(8*a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - Sqrt[1 - 1/(a^2*x^2)]/(a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)]) - (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^2(1+ax)^3}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{8a^5(-1+ax)^2}+\frac{7}{16a^5(-1+ax)}-\frac{1}{4a^5(1+ax)^3}+\frac{1}{a^5(1+ax)^2}-\frac{23}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{7\sqrt{1-}}{16ac}$$

Mathematica [A] time = 0.118547, size = 85, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(2\left(8ax+\frac{1}{1-ax}-\frac{8}{ax+1}+\frac{1}{(ax+1)^2}\right)+7\log(1-ax)-23\log(ax+1)\right)}{16a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(2*(8*a*x + (1 - a*x)^(-1) + (1 + a*x)^(-2) - 8/(1
 + a*x)) + 7*Log[1 - a*x] - 23*Log[1 + a*x]))/(16*a*(c - c/(a^2*x^2))^(5/2)
)
```

Maple [A] time = 0.241, size = 175, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-16x^4a^4+23a^3x^3\ln(ax+1)-7\ln(ax-1)x^3a^3-16x^3a^3+23\ln(ax+1)a^2x^2-7\ln(ax-1)a^2x\right.}{16a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/16*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-16*x^4*a^4+23*a^3*x^3*ln(a*
x+1)-7*ln(a*x-1)*x^3*a^3-16*x^3*a^3+23*ln(a*x+1)*a^2*x^2-7*ln(a*x-1)*a^2*x^
2+34*a^2*x^2-23*a*x*ln(a*x+1)+7*ln(a*x-1)*x*a+18*a*x-23*ln(a*x+1)+7*ln(a*x-
1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

Fricas [A] time = 1.81731, size = 292, normalized size = 1.11

$$\frac{\left(16a^4x^4+16a^3x^3-34a^2x^2-18ax-23\left(a^3x^3+a^2x^2-ax-1\right)\log(ax+1)+7\left(a^3x^3+a^2x^2-ax-1\right)\log(ax-1)+1\right.}{16\left(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/16*(16*a^4*x^4 + 16*a^3*x^3 - 34*a^2*x^2 - 18*a*x - 23*(a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 7*(a^3*x^3 + a^2*x^2 - a*x - 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

3.862 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=358

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (5*Sqrt[1 - 1/(a^2*x^2)])
/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(24*a*c
^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (11*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^
3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(2*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.198283, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (5*Sqrt[1 - 1/(a^2*x^2)])
/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(24*a*c
^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (11*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^
3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(2*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^3(1+ax)^4}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{16a^7(-1+ax)^3}+\frac{5}{16a^7(-1+ax)^2}+\frac{19}{32a^7(-1+ax)}+\frac{1}{8a^7(1+ax)^4}-\frac{11}{16a^7(1+ax)^3}+\frac{3}{2a^7(1+ax)^2}-\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{}{32}$$

Mathematica [A] time = 0.14977, size = 104, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96ax+\frac{30}{1-ax}-\frac{144}{ax+1}-\frac{3}{(ax-1)^2}+\frac{33}{(ax+1)^2}-\frac{4}{(ax+1)^3}+57\log(1-ax)-153\log(ax+1)\right)}{96a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*a*x + 30/(1 - a*x) - 3/(-1 + a*x)^2 - 4/(1 + a
*x)^3 + 33/(1 + a*x)^2 - 144/(1 + a*x) + 57*Log[1 - a*x] - 153*Log[1 + a*x]
))/(96*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] time = 0.254, size = 247, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-96x^6a^6+153\ln(ax+1)x^5a^5-57\ln(ax-1)x^5a^5-96x^5a^5+153\ln(ax+1)a^4x^4-57\ln(ax-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/96*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-96*x^6*a^6+153*ln(a*x+1)*x^
5*a^5-57*ln(a*x-1)*x^5*a^5-96*x^5*a^5+153*ln(a*x+1)*a^4*x^4-57*ln(a*x-1)*a^
4*x^4+366*x^4*a^4-306*a^3*x^3*ln(a*x+1)+114*ln(a*x-1)*x^3*a^3+222*x^3*a^3-3
06*ln(a*x+1)*a^2*x^2+114*ln(a*x-1)*a^2*x^2-338*a^2*x^2+153*a*x*ln(a*x+1)-57
*ln(a*x-1)*x*a-122*a*x+153*ln(a*x+1)-57*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

Fricas [A] time = 1.69735, size = 443, normalized size = 1.24

$$\frac{\left(96\,a^6x^6 + 96\,a^5x^5 - 366\,a^4x^4 - 222\,a^3x^3 + 338\,a^2x^2 + 122\,ax - 153\left(a^5x^5 + a^4x^4 - 2\,a^3x^3 - 2\,a^2x^2 + ax + 1\right)\log\left(ax + 1\right)}{96\left(a^7c^4x^5 + a^6c^4x^4 - 2\,a^5c^4x^3 - 2\,a^4c^4x^2 + a^3c^4x + a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(96*a^6*x^6 + 96*a^5*x^5 - 366*a^4*x^4 - 222*a^3*x^3 + 338*a^2*x^2 + 1
22*a*x - 153*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x
+ 1) + 57*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 + a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*
x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

3.863 $\int e^{-2\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^{7/2} dx$

Optimal. Leaf size=375

$$\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}-\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}-\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}-\frac{ax^2\left(c-\right.}{15(ax}$$

```
[Out] (7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (3*a^5*(
c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) - (a*(c - c/(a^2*x^
2))^(7/2)*x^2)/(15*(1 + a*x)) - (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(1
 - a*x)^3*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*(
1 + a*x)) - (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
+ ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) - (2*a^6*(c - c/(a^2*
x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (25*a^6*(c
 - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a*
x)^(7/2)*(1 + a*x)^(7/2))
```

Rubi [A] time = 0.541287, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}-\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}-\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}-\frac{ax^2\left(c-\right.}{15(ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (3*a^5*(
c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) - (a*(c - c/(a^2*x^
2))^(7/2)*x^2)/(15*(1 + a*x)) - (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(1
 - a*x)^3*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*(
1 + a*x)) - (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
+ ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) - (2*a^6*(c - c/(a^2*
x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (25*a^6*(c
 - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a*
x)^(7/2)*(1 + a*x)^(7/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{9/2}(1+ax)^{5/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}(-2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{3/2}(-23a}{x^5}}{30(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{120}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}-\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-}{6(1+ax)}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}-\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{120(1-ax)(1+ax}$$

$$=\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}$$

$$=\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}$$

$$=\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}$$

$$=\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}$$

$$=\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}$$

Mathematica [A] time = 0.171492, size = 150, normalized size = 0.4

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(240a^6x^6+736a^5x^5+105a^4x^4-352a^3x^3+70a^2x^2+96ax-40\right)-480a^6x^6\log\left(\sqrt{a^2x^2-1}+a\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2 -
 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.263, size = 795, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(2016*(-c/a^2)^(1/2)*(c*(a^2*x^
2-1)/a^2)^(7/2)*x^7*a^9*c-2016*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^5
*a^9-480*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c+375*(-c/a^2
)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^6*a^8*c+560*(-c/a^2)^(1/2)*((a*x-1)*(a*
x+1)*c/a^2)^(5/2)*x^7*a^7*c^2+105*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*
x^4*a^8-2352*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^7*a^7*c^2-224*(-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^3*a^7-525*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^6*a^6*c^2-700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*
x^7*a^5*c^3+2940*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^7*a^5*c^3+630*(
-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^2*a^6+875*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^6*a^4*c^3+1050*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(
1/2)*x^7*a^3*c^4-672*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(9/2)*x*a^5-4410*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^7*a^3*c^4+4410*c^(9/2)*(-c/a^2)^(1
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^6*a-1050*c^(9/2)*(-c/a^2)^(1/
2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^6*a+280*a^4*(c
*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)-2625*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(1/2)*x^6*a^2*c^4-2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a
^2-c)/x/a^2)*x^6*c^5)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(7/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

Fricas [A] time = 1.81637, size = 971, normalized size = 2.59

$$\left[\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(240\,a^6c^3x^6+736\,a^5c^3x^5\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2 -
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6*c^
3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^
2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240*
(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 + 10
5*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] time = 37.5511, size = 1059, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**2 + 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 - 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Giac [A] time = 69.2752, size = 757, normalized size = 2.02

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}c^3\operatorname{sgn}(x)}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
```

```
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

3.864 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=293

$$-\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}-\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}+\frac{2a^4x^5\left(c-\right.}{(1-ax}$$

```
[Out] (-7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (a*(c -
c/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)
/((1 - a*x)^2*(1 + a*x)) - (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x
)*(1 + a*x)) + ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) + (2*a^4
*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
 - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])
/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rubi [A] time = 0.483459, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}-\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}+\frac{2a^4x^5\left(c-\right.}{(1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (-7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (a*(c -
c/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)
/((1 - a*x)^2*(1 + a*x)) - (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x
)*(1 + a*x)) + ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) + (2*a^4
*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
 - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])
/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$
$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^{7/2}(1+ax)^{3/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^{5/2}\sqrt{1+ax}(-2a-5a^2x)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{(1-ax)^{3/2}\sqrt{1+ax}(-7a^2+1}{x^3}}{12(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)}{24(1-}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-a}{4(1+ax)}$$
$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}$$
$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}$$
$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}$$
$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}$$

Mathematica [A] time = 0.136065, size = 134, normalized size = 0.46

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4+64a^3x^3-3a^2x^2-16ax+6\right)-48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 64
*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4*x
^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.188, size = 625, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7
-48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c-27*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/
a^2)^(3/2)*x^5*a^5*c^2+75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+
100*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*
(c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5+45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)
*x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-15
0*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3+30*a^4*(c*(a^2*x^2-1
)/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2
*x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((c^(1/2)*((a*x-1)*(a
*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^4*a-135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^
2)^(1/2)*x^4*a^2*c^3-135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2
-c)/x/a^2)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

Fricas [A] time = 1.69626, size = 855, normalized size = 2.92

$$\left[\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4c^2x^4+64\,a^3c^2x^3-3\,a\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2*
a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2*x^
4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c^
2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] time = 19.7811, size = 500, normalized size = 1.71

$$c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}}\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}}\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**2*Piecewise((-a*sqrt(c)*
```

```
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

Giac [A] time = 3.08876, size = 562, normalized size = 1.92

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}c^2\operatorname{sgn}(x)}{a^2}-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) + 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

3.865 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2} dx$

Optimal. Leaf size=213

$$-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^3}$$

```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x)) - (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*
(1 + a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rubi [A] time = 0.446075, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x)) - (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*
(1 + a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
```

```
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{3/2}(-2a-3a^2x)}{x^2\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1-ax}(a^2+5a^3x)}{x\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int}{2a(1-ax)^{3/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right.}{2(1-ax)^{3/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right.}{}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{(1-ax)^{3/2}(1+}$$

Mathematica [A] time = 0.108662, size = 115, normalized size = 0.54

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2+4ax-1\right)-4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^2
*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]
))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.177, size = 454, normalized size = 2.1

$$-\frac{x}{6 a^2 c}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{5/2}xa^5-4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)}{a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5-4*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c+(-c/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)
*x^3*a^3*c^2+3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*c^(5/2)*ln(x*c^(1/2)+(c*(a^2*x^
2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-6*c^(5/2)*(-c/a^2)^(1/2)*ln((c^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a-3*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(1/2)*x^2*a^2*c^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/x/a^2)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

Fricas [A] time = 1.86562, size = 693, normalized size = 3.25

$$\left[\frac{8a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)+2\left(2a^2cx^2+4acx-c\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4a^2x}, \quad ac^{\frac{3}{2}}x\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*x
^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] time = 13.2434, size = 376, normalized size = 1.77

$$c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{fo}\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{ot}\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Giac [A] time = 1.56796, size = 359, normalized size = 1.69

$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x)}{a^2}-\frac{\left(\sqrt{a^2c}x-\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 - 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) + 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

3.866 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.357486, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int}{\sqrt{1-ax}\sqrt{1}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0836818, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}-2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] time = 0.181, size = 196, normalized size = 1.7

$$-\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a^2-2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)+2*c^(1/2)*ln((c^(1/2
)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Fricas [A] time = 1.65971, size = 576, normalized size = 4.97

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\cdots}}{a^2cx\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.18484, size = 205, normalized size = 1.77

$$\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\,\mathrm{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\,\mathrm{arct}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] (2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 + 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(
a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/sq
rt(c)) + a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs(
a)
```

3.867 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=112

$$-\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{ax+1}\sqrt{1-ax}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -((1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x)) - (2*(1 - a*x)*(1 + a*x))/(a^2
*Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^
2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] time = 0.290543, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 78, 50, 41, 216}

$$-\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{ax+1}\sqrt{1-ax}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] -((1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x)) - (2*(1 - a*x)*(1 + a*x))/(a^2
*Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^
2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

$$= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{\left(\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{x\sqrt{1-ax}}{(1+ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2\sqrt{1-ax}\sqrt{1+ax}\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.0731632, size = 68, normalized size = 0.61

$$\frac{a^2x^2-2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax-3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (-3 + 2*a*x + a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A] time = 0.178, size = 176, normalized size = 1.6

$$\frac{1}{xa\left(ax+1\right)}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\left(\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\sqrt{c}xa^{2}-2\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)xac+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}a\sqrt{c}+2a\sqrt{\frac{(ax-1)}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2)^(1/2)*((c*(a^2*x^2-1)/a^2)^(1/2)*c^(1/2)*x*a^2-2*ln(x*c
^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c+(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)
+2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^
2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^{2}x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

Fricas [A] time = 1.70609, size = 447, normalized size = 3.99

$$\left[\frac{(ax+1)\sqrt{c}\log\left(2a^{2}cx^{2}-2a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)+\left(a^{2}x^{2}+3ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx+ac},\ \frac{2(ax+1)\sqrt{-c}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+\left(a^{2}\right.}{a^{2}cx+ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x + a*c), (2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*c*x + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.868 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -(1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(1 - a*x)^2*(1 + a*x)*(
5 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 + a
*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rubi [A] time = 0.422062, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 98, 143, 41, 216}

$$\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -(1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(1 - a*x)^2*(1 + a*x)*(
5 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 + a
*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$$

$$= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2-4ax)}{\sqrt{1-ax}(1+ax)^{3/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.0859912, size = 95, normalized size = 0.77

$$\frac{3a^3x^3+11a^2x^2-6(ax+1)\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-4ax-10}{3a^2cx(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (-10 - 4*a*x + 11*a^2*x^2 + 3*a^3*x^3 - 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

Maple [B] time = 0.198, size = 326, normalized size = 2.6

$$\frac{ax-1}{3\,a^4x^3}\left(3\,c^{3/2}\sqrt{\frac{(ax-1)\,(ax+1)\,c}{a^2}}x^3a^3+15\,x^2a^2c^{3/2}\sqrt{\frac{(ax-1)\,(ax+1)\,c}{a^2}}-4\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}c^{3/2}x^2a^2-6\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3+15*x^2*a^2*c^(3/2)*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)-4*(c*(a^2*x^2-1)/a^2)^(1/2)*c^(3/2)*x^2*a^2-6*(c
*(a^2*x^2-1)/a^2)^(1/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a
*x+1)*c/a^2)^(1/2)*x*a^2*c-4*(c*(a^2*x^2-1)/a^2)^(1/2)*c^(3/2)*x*a-6*(c*(a^
2*x^2-1)/a^2)^(1/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1
)*c/a^2)^(1/2)*a*c-12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+2*(c*(a^2*x^2-1
)/a^2)^(1/2)*c^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^2
-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A] time = 1.77773, size = 590, normalized size = 4.76

$$\left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3+14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)},\ \frac{6\left(a^2x^2+2\,ax+1\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), 1/3*(6*(
a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/
(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.869 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=195

$$\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[Out] -((1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x)) - (2*(1 - a*x)^3)/(5*a^3*(c
- c/(a^2*x^2))^(5/2)*x^2) + (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x
^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/
(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^
6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rubi [A] time = 0.451342, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```


```
[Out] -((1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x)) - (2*(1 - a*x)^3)/(5*a^3*(c
- c/(a^2*x^2))^(5/2)*x^2) + (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x
^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/
(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^
6*(c - c/(a^2*x^2))^(5/2)*x^5)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
```

```
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{3/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+2ax)}{\sqrt{1-ax}(1+ax)^{7/2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2\left(6a+8a^2x\right)}{\sqrt{1-ax}(1+ax)^{5/2}}\,dx}{5a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x\left(-4a^2+\dots\right.}{\sqrt{1-ax}(1\dots}}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\dots$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\dots$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\dots$$

Mathematica [A] time = 0.0976883, size = 105, normalized size = 0.54

$$\frac{15a^4x^4+76a^3x^3+32a^2x^2-30(ax+1)^2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)-82ax-56}{15a^2c^2x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (-56 - 82*a*x + 32*a^2*x^2 + 76*a^3*x^3 + 15*a^4*x^4 - 30*(1 + a*x)^2*Sqrt[
-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^
2)]*x*(1 + a*x)^2)
```

Maple [B] time = 0.184, size = 462, normalized size = 2.4

$$\frac{ax-1}{15\,x^5a^6}\left(15\,c^{5/2}\left(\frac{(ax-1)\,(ax+1)\,c}{a^2}\right)^{3/2}x^5a^5+45\,x^4c^{5/2}a^4\left(\frac{(ax-1)\,(ax+1)\,c}{a^2}\right)^{3/2}+16\,c^{5/2}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^4a^4-60\,c^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5+45*x^4*c^(5/2)*a^4*(
(a*x-1)*(a*x+1)*c/a^2)^(3/2)+16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-6
0*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^3-30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c-90*c^(5/2)*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*x^2*a^2-24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln(
x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^(
5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)
+6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2)
/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [A] time = 1.88524, size = 744, normalized size = 3.82

$$\left[\frac{15\left(a^4x^4+2a^3x^3-2ax-1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(15a^5x^5+76a^4x^4+32a^3x^3-82a^2x^2-56ax\right.}{15\left(a^5c^3x^4+2a^4c^3x^3-2a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 + 76*a^4*x^
4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 + 2*a^3*
x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2
*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*a^2*x^
2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.870 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=270

$$\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{10(1-ax)^3}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[Out] -(1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(7/2)*x) + (10*(1 - a*x)^3)/(3*a^3*(c
 - c/(a^2*x^2))^(7/2)*x^2) + (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7/2
)*x^3) + (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4) +
 (2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) + (2*(1 -
 a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (
2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/2)
*x^7)
```

Rubi [A] time = 0.489331, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{10(1-ax)^3}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```


```
[Out] -(1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(7/2)*x) + (10*(1 - a*x)^3)/(3*a^3*(c
 - c/(a^2*x^2))^(7/2)*x^2) + (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7/2
)*x^3) + (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4) +
 (2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) + (2*(1 -
 a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (
2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/2)
*x^7)
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
```

```
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{5/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+4ax)}{(1-ax)^{3/2}(1+ax)^{9/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4\left(-50a-14a^2x\right)}{\sqrt{1-ax}(1+ax)^{9/2}}\,dx}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^3\left(-144a^2\cdots\right.}{\sqrt{1-ax}(1\cdots}}{21a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{\left((1-ax)\cdots\right.}{\cdots}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax\cdots}{35a^6\left(c-\cdots\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax\cdots}{35a^6\left(c-\cdots\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax\cdots}{35a^6\left(c-\cdots\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} + \frac{2(1-ax\cdots}{35a^6\left(c-\cdots\right.}$$

Mathematica [A] time = 0.116505, size = 131, normalized size = 0.49

$$\frac{105a^6x^6+562a^5x^5+74a^4x^4-1226a^3x^3-636a^2x^2-210(ax-1)(ax+1)^3\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+654ax+4}{105a^2x(ax-1)\sqrt{c-\frac{c}{a^2x^2}}(acx+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (432 + 654*a*x - 636*a^2*x^2 - 1226*a^3*x^3 + 74*a^4*x^4 + 562*a^5*x^5 + 10
5*a^6*x^6 - 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [B] time = 0.227, size = 572, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/105*(105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7+553*x^6*c^(7/2)*a^
6*((a*x-1)*(a*x+1)*c/a^2)^(5/2)-96*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^6*a^
6-392*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^5*a^5-96*c^(7/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^5*a^5-1540*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4+24
0*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^4-210*ln(x*c^(1/2)+(c*(a^2*x^2-1)
/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^6*
c+350*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^3*a^3+240*c^(7/2)*(c*(a^2*x^2
-1)/a^2)^(5/2)*x^3*a^3-210*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)
*(a*x+1)*c/a^2)^(5/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*a^5*c+1470*c^(7/2)*((a*x-1)
*(a*x+1)*c/a^2)^(5/2)*x^2*a^2-180*c^(7/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^2*a^2
-42*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a-180*c^(7/2)*(c*(a^2*x^2-1)/a^
2)^(5/2)*x*a-462*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)+30*c^(7/2)*(c*(a^2*x
^2-1)/a^2)^(5/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)/a^8/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [A] time = 2.65497, size = 1044, normalized size = 3.87

$$\left[\frac{105\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(105\,a^7x^7+562\,a^6x^6+\right.}{105\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*a^3
*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x
+ a*c^4), 1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 +
 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4
 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*
a^2*c^4*x + a*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.871 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqr
t[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(
a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^4*Sqrt[c - c/(a^2*x^2)]
)/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sq
rt[1 - 1/(a^2*x^2)]*x^3) - (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^
4*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.155947, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqr
t[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(
a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^4*Sqrt[c - c/(a^2*x^2)]
)/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sq
rt[1 - 1/(a^2*x^2)]*x^3) - (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^
4*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^6(1+ax)^3}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^9+\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}-\frac{6a^4}{x^5}-\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2c^4\sqrt{c}}{a^4\sqrt{1-}}$$

Mathematica [A] time = 0.0809334, size = 97, normalized size = 0.3

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(-\frac{4a^6}{x^2}+\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+a^9x-3a^8\log(x)+\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*(-1/(8*x^8) + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) + (3
*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(a^9*(1
- 1/(a^2*x^2))^(9/2))
```

Maple [A] time = 0.243, size = 112, normalized size = 0.4

$$-\frac{\left(-280\,a^{9}x^{9}+840\,a^{8}\ln\left(x\right)x^{8}+1120\,x^{6}a^{6}-560\,x^{5}a^{5}-420\,x^{4}a^{4}+448\,x^{3}a^{3}-120\,ax+35\right)x}{280\,\left(ax-1\right)^{3}\left(a^{2}x^{2}-1\right)^{3}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/280*(-280*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6-560*x^5*a^5-420*x^4*a^4
+448*x^3*a^3-120*a*x+35)*x*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^3/(a^2*x^2-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.56452, size = 228, normalized size = 0.71

$$\frac{\left(280\,a^{9}c^{4}x^{9}-840\,a^{8}c^{4}x^{8}\log\left(x\right)-1120\,a^{6}c^{4}x^{6}+560\,a^{5}c^{4}x^{5}+420\,a^{4}c^{4}x^{4}-448\,a^{3}c^{4}x^{3}+120\,ac^{4}x-35\,c^{4}\right)\sqrt{a^{2}c}}{280\,a^{10}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/280*(280*a^9*c^4*x^9 - 840*a^8*c^4*x^8*log(x) - 1120*a^6*c^4*x^6 + 560*a^
5*c^4*x^5 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqrt(
a^2*c)/(a^10*x^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^{\frac{9}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.872 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=324

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqrt
[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(
3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqr
t[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*
x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^3*Sqr
t[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.156097, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqrt
[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(
3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqr
t[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*
x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^3*Sqr
t[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^5(1+ax)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}+\frac{3a}{x^6}-\frac{a^2}{x^5}-\frac{5a^3}{x^4}+\frac{5a^4}{x^3}+\frac{a^5}{x^2}-\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{5c^3\sqrt{c-}}{2a^3\sqrt{1-}}\end{aligned}$$

Mathematica [A] time = 0.0801523, size = 94, normalized size = 0.29

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{60a^7x^7-60a^5x^5-150a^4x^4+100a^3x^3+15a^2x^2-36ax+10}{60x^6}-3a^6\log(x)\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*((10 - 36*a*x + 15*a^2*x^2 + 100*a^3*x^3 - 150*a^4
*x^4 - 60*a^5*x^5 + 60*a^7*x^7)/(60*x^6) - 3*a^6*Log[x]))/(a^7*(1 - 1/(a^2*
x^2))^(7/2))
```

Maple [A] time = 0.237, size = 112, normalized size = 0.4

$$-\frac{\left(-60\,a^{7}x^{7}+180\,a^{6}\ln\left(x\right)x^{6}+60\,x^{5}a^{5}+150\,x^{4}a^{4}-100\,x^{3}a^{3}-15\,a^{2}x^{2}+36\,ax-10\right)x}{60\,\left(ax-1\right)^{3}\left(a^{2}x^{2}-1\right)^{2}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/60*(-60*a^7*x^7+180*a^6*ln(x)*x^6+60*x^5*a^5+150*x^4*a^4-100*x^3*a^3-15*
a^2*x^2+36*a*x-10)*x*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/
(a*x-1)^3/(a^2*x^2-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.62321, size = 219, normalized size = 0.68

$$\frac{\left(60\,a^{7}c^{3}x^{7}-180\,a^{6}c^{3}x^{6}\log\left(x\right)-60\,a^{5}c^{3}x^{5}-150\,a^{4}c^{3}x^{4}+100\,a^{3}c^{3}x^{3}+15\,a^{2}c^{3}x^{2}-36\,ac^{3}x+10\,c^{3}\right)\sqrt{a^{2}c}}{60\,a^{8}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 - 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.873 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=235

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[
c - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x^
2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*S
qrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2
)] - (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.138806, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 75}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[
c - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x^
2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*S
qrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2
)] - (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx &= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^4(1+ax)}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0629846, size = 81, normalized size = 0.34

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{a^2}{x^2}+a^5x-\frac{2a^3}{x}-3a^4\log(x)-\frac{5a^4}{4}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*((-5*a^4)/4 - 1/(4*x^4) + a/x^3 - a^2/x^2 - (2*a^3
)/x + a^5*x - 3*a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A] time = 0.227, size = 96, normalized size = 0.4

$$-\frac{\left(-4\,x^5a^5+12\,a^4\ln\left(x\right)x^4+8\,x^3a^3+4\,a^2x^2-4\,ax+1\right)x}{4\,\left(ax-1\right)^3\left(a^2x^2-1\right)}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/4*(-4*x^5*a^5+12*a^4*ln(x)*x^4+8*x^3*a^3+4*a^2*x^2-4*a*x+1)*x*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.6958, size = 158, normalized size = 0.67

$$\frac{\left(4\,a^5c^2x^5-12\,a^4c^2x^4\log\left(x\right)-8\,a^3c^2x^3-4\,a^2c^2x^2+4\,ac^2x-c^2\right)\sqrt{a^2c}}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 - 4*a^2*c^2*x^2
+ 4*a*c^2*x - c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.874 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c -
 c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/
Sqrt[1 - 1/(a^2*x^2)] - (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a
^2*x^2)])
```

Rubi [A] time = 0.123455, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c -
 c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/
Sqrt[1 - 1/(a^2*x^2)] - (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a
^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx &= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3-\frac{1}{x^3}+\frac{3a}{x^2}-\frac{3a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0401829, size = 59, normalized size = 0.4

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x-3a^2\log(x)-\frac{3a}{x}+\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*(1/(2*x^2) - (3*a)/x + a^3*x - 3*a^2*Log[x]))/(a^3
*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.233, size = 69, normalized size = 0.5

$$-\frac{\left(-2\,x^{3}a^{3}+6\,a^{2}\ln\left(x\right)x^{2}+6\,ax-1\right)x}{2\,\left(ax-1\right)^{3}}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*(-2*x^3*a^3+6*a^2*ln(x)*x^2+6*a*x-1)*x*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*(
(a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.57481, size = 103, normalized size = 0.7

$$\frac{\left(2\,a^{3}cx^{3}-6\,a^{2}cx^{2}\log\left(x\right)-6\,acx+c\right)\sqrt{a^{2}c}}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*c*x^3 - 6*a^2*c*x^2*log(x) - 6*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.875 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.112433, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0304425, size = 47, normalized size = 0.44

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] time = 0.239, size = 63, normalized size = 0.6

$$\frac{(ax + \ln(x) - 4\ln(ax+1))\,x\,(ax+1)}{(ax-1)^2}\sqrt{\frac{c\,(a^2x^2-1)}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] (a*x+ln(x)-4*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.71145, size = 66, normalized size = 0.62

$$\frac{\sqrt{a^2c}(ax - 4\log(ax+1) + \log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.876 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.124986, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 77}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(-1+ax)}{(1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(1+ax)^2}-\frac{3}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.048142, size = 54, normalized size = 0.48

$$
\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(ax-\frac{2}{ax+1}-3\log(ax+1)\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x - 2/(1 + a*x) - 3*Log[1 + a*x]))/(a*Sqrt[c - c/
(a^2*x^2)])
```

Maple [A] time = 0.256, size = 87, normalized size = 0.8

$$-\frac{(ax+1)\left(-a^{2}x^{2}+3\,ax\ln(ax+1)-ax+3\,\ln(ax+1)+2\right)}{(ax-1)\,xa^{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\frac{1}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)*(-a^2*x^2+3*a*x*ln(a*x+1)-a*x+3*ln
(a*x+1)+2)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{a^{2}x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [A] time = 1.63462, size = 105, normalized size = 0.93

$$\frac{\left(a^{2}x^{2}+ax-3\,(ax+1)\log(ax+1)-2\right)\sqrt{a^{2}c}}{a^{3}cx+a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (a^2*x^2 + a*x - 3*(a*x + 1)*log(a*x + 1) - 2)*sqrt(a^2*c)/(a^3*c*x + a^2*c
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.877 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.140652, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}-\frac{1}{a^3(1+ax)^3}+\frac{3}{a^3(1+ax)^2}-\frac{3}{a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0731578, size = 63, normalized size = 0.38

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2ax+\frac{-6ax-5}{(ax+1)^2}-6\log(ax+1)\right)}{2a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(2*a*x + (-5 - 6*a*x)/(1 + a*x)^2 - 6*Log[1 + a*x]
))/(2*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A] time = 0.232, size = 102, normalized size = 0.6

$$-\frac{(ax+1)\left(-2\,x^3a^3+6\,\ln(ax+1)\,a^2x^2-4\,a^2x^2+12\,ax\ln(ax+1)+4\,ax+6\,\ln(ax+1)+5\right)}{2\,a^4x^3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-2*x^3*a^3+6*ln(a*x+1)*a^2*x^2-4*a^2*
x^2+12*a*x*ln(a*x+1)+4*a*x+6*ln(a*x+1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

Fricas [A] time = 1.75579, size = 176, normalized size = 1.05

$$\frac{\left(2\,a^3x^3+4\,a^2x^2-4\,ax-6\left(a^2x^2+2\,ax+1\right)\log(ax+1)-5\right)\sqrt{a^2c}}{2\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*x^3 + 4*a^2*x^2 - 4*a*x - 6*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1) -
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.878 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}} dx$

Optimal. Leaf size=264

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{49}{}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.165807, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{49}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)(1+ax)^4}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{16a^5(-1+ax)}+\frac{1}{2a^5(1+ax)^4}-\frac{9}{4a^5(1+ax)^3}+\frac{31}{8a^5(1+ax)^2}-\frac{49}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{\sqrt{1-}}{1}$$

Mathematica [A] time = 0.101956, size = 85, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48ax-\frac{186}{ax+1}+\frac{54}{(ax+1)^2}-\frac{8}{(ax+1)^3}+3\log(1-ax)-147\log(ax+1)\right)}{48a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*a*x - 8/(1 + a*x)^3 + 54/(1 + a*x)^2 - 186/(1
+ a*x) + 3*Log[1 - a*x] - 147*Log[1 + a*x]))/(48*a*(c - c/(a^2*x^2))^(5/2))
```

Maple [A] time = 0.24, size = 175, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-48x^4a^4+147a^3x^3\ln(ax+1)-3\ln(ax-1)x^3a^3-144x^3a^3+441\ln(ax+1)a^2x^2-9\ln(ax-1)a\right.}{48a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(-48*x^4*a^4+147*a^3*x^3*ln(a
*x+1)-3*ln(a*x-1)*x^3*a^3-144*x^3*a^3+441*ln(a*x+1)*a^2*x^2-9*ln(a*x-1)*a^2
*x^2+42*a^2*x^2+441*a*x*ln(a*x+1)-9*ln(a*x-1)*x*a+270*a*x+147*ln(a*x+1)-3*l
n(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

Fricas [A] time = 1.83335, size = 313, normalized size = 1.19

$$\frac{\left(48a^4x^4+144a^3x^3-42a^2x^2-270ax-147\left(a^3x^3+3a^2x^2+3ax+1\right)\log(ax+1)+3\left(a^3x^3+3a^2x^2+3ax+1\right)\log(a\right.}{48\left(a^5c^3x^3+3a^4c^3x^2+3a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/48*(48*a^4*x^4 + 144*a^3*x^3 - 42*a^2*x^2 - 270*a*x - 147*(a^3*x^3 + 3*a^
2*x^2 + 3*a*x + 1)*log(a*x + 1) + 3*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2
*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.879 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=357

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(16*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^4) - Sqrt[1 - 1/(a^2*x^2)]/(2*a*c^3*Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^3*Sqr
t[c - c/(a^2*x^2)]*(1 + a*x)^2) - (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64*a
*c^3*Sqrt[c - c/(a^2*x^2)]) - (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.198547, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```


```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(16*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^4) - Sqrt[1 - 1/(a^2*x^2)]/(2*a*c^3*Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^3*Sqr
t[c - c/(a^2*x^2)]*(1 + a*x)^2) - (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64*a
*c^3*Sqrt[c - c/(a^2*x^2)]) - (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^2(1+ax)^5}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{32a^7(-1+ax)^2}+\frac{9}{64a^7(-1+ax)}-\frac{1}{4a^7(1+ax)^5}+\frac{3}{2a^7(1+ax)^4}-\frac{59}{16a^7(1+ax)^3}+\frac{75}{16a^7(1+ax)^2}-\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^4}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{}{32}$$

Mathematica [A] time = 0.179346, size = 105, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(2\left(32ax+\frac{1}{1-ax}-\frac{150}{ax+1}+\frac{59}{(ax+1)^2}-\frac{16}{(ax+1)^3}+\frac{2}{(ax+1)^4}\right)+9\log(1-ax)-201\log(ax+1)\right)}{64a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(2*(32*a*x + (1 - a*x)^(-1) + 2/(1 + a*x)^4 - 16/(
1 + a*x)^3 + 59/(1 + a*x)^2 - 150/(1 + a*x)) + 9*Log[1 - a*x] - 201*Log[1 +
 a*x]))/(64*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A] time = 0.263, size = 247, normalized size = 0.7

$$-\frac{(ax-1)(ax+1)\left(-64x^6a^6+201\ln(ax+1)x^5a^5-9\ln(ax-1)x^5a^5-192x^5a^5+603\ln(ax+1)a^4x^4-27\ln(ax-1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/64*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(-64*x^6*a^6+201*ln(a*x+1)*x^
5*a^5-9*ln(a*x-1)*x^5*a^5-192*x^5*a^5+603*ln(a*x+1)*a^4*x^4-27*ln(a*x-1)*a^
4*x^4+174*x^4*a^4+402*a^3*x^3*ln(a*x+1)-18*ln(a*x-1)*x^3*a^3+618*x^3*a^3-40
2*ln(a*x+1)*a^2*x^2+18*ln(a*x-1)*a^2*x^2+118*a^2*x^2-603*a*x*ln(a*x+1)+27*l
n(a*x-1)*x*a-414*a*x-201*ln(a*x+1)+9*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

Fricas [A] time = 1.65509, size = 460, normalized size = 1.29

$$\frac{\left(64\,a^{6}x^{6}+192\,a^{5}x^{5}-174\,a^{4}x^{4}-618\,a^{3}x^{3}-118\,a^{2}x^{2}+414\,ax-201\left(a^{5}x^{5}+3\,a^{4}x^{4}+2\,a^{3}x^{3}-2\,a^{2}x^{2}-3\,ax-1\right)\log\left(ax\right.}{64\left(a^{7}c^{4}x^{5}+3\,a^{6}c^{4}x^{4}+2\,a^{5}c^{4}x^{3}-2\,a^{4}c^{4}x^{2}-3\,a^{3}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(64*a^6*x^6 + 192*a^5*x^5 - 174*a^4*x^4 - 618*a^3*x^3 - 118*a^2*x^2 +
414*a*x - 201*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x + 1) + 9*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 -
 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.880 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

Optimal. Leaf size=80

$$\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.244871, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6197, 6193, 43}

$$\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```


```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.03545, size = 53, normalized size = 0.66

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^{m+1}}{m+1}+\frac{x^m}{m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^m/m + (a*x^(1 + m))/(1 + m)))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] time = 0.13, size = 63, normalized size = 0.8

$$\frac{x^{1+m} (axm+m+1)}{(ax+1)(1+m)m}\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(1+m)/m/(a*x+1)/((a*x-1)/
(a*x+1))^(1/2)
```

Maxima [A] time = 1.24026, size = 59, normalized size = 0.74

$$\frac{\left(a\sqrt{c}mx+\sqrt{c}(m+1)\right)(ax+1)x^m}{\left(m^2+m\right)a^2x+\left(m^2+m\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] (a*sqrt(c)*m*x + sqrt(c)*(m + 1))*(a*x + 1)*x^m/((m^2 + m)*a^2*x + (m^2 + m
)*a)
```

Fricas [A] time = 1.75595, size = 154, normalized size = 1.92

$$-\frac{\left(amx^2+(m+1)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-\left(am^2+am\right)x+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.881 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=76

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.260666, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6197, 6193, 43}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.024884, size = 45, normalized size = 0.59

$$\frac{x^2(2ax+3)\sqrt{c-\frac{c}{a^2x^2}}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.125, size = 53, normalized size = 0.7

$$\frac{(2\,ax+3)\,x^3}{6\,ax+6}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.57269, size = 53, normalized size = 0.7

$$\frac{\left(2\,ax^3 + 3\,x^2\right)\sqrt{a^2c}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.882 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=71

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)
]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.172292, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6197, 6193}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```



```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)
]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0233064, size = 43, normalized size = 0.61

$$\frac{\left(\frac{ax^2}{2}+x\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x + (a*x^2)/2))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.121, size = 52, normalized size = 0.7

$$\frac{(ax+2)\,x^2}{2\,ax+2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.56274, size = 47, normalized size = 0.66

$$\frac{\sqrt{a^2c}\left(ax^2 + 2\,x\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(a^2*c)*(a*x^2 + 2*x)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2 x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.883 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.0978782, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0210068, size = 39, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.217, size = 50, normalized size = 0.8

$$\frac{(ax+\ln(x))\,x}{ax+1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.6772, size = 43, normalized size = 0.64

$$\frac{\sqrt{a^2c}(ax + \log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.884 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) + (Sqrt[c - c/(a^2*x^2
)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.250164, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6197, 6193, 43}

$$\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) + (Sqrt[c - c/(a^2*x^2
)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0235748, size = 43, normalized size = 0.61

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(a\log(x)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.225, size = 50, normalized size = 0.7

$$\frac{a\ln\left(x\right)x-1}{ax+1}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] (a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.58486, size = 51, normalized size = 0.73

$$\frac{\sqrt{a^{2}c}\left(ax\log\left(x\right)-1\right)}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) - 1)/(a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2 x^2}}}{x \sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.885 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rubi [A] time = 0.243969, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6197, 6193, 37}

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

Mathematica [A] time = 0.0233752, size = 47, normalized size = 1.02

$$\frac{\left(-\frac{a}{x}-\frac{1}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/(2*x^2) - a/x))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.135, size = 53, normalized size = 1.2

$$-\frac{2\,ax+1}{2\,(ax+1)\,x}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A] time = 1.58009, size = 54, normalized size = 1.17

$$-\frac{\sqrt{a^2c}(2\,ax+1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(a^2*c)*(2*a*x + 1)/(a^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.886 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=160

$$\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) + (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.557536, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 90, 80, 50, 41, 216}

$$\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) + (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
```

```
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x^2(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(-1-2ax)(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{4a^2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}-\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{12a^2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}-\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}x\right)}{8a^2\sqrt{1-ax}}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}-$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}-$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}-$$

Mathematica [A] time = 0.0911736, size = 93, normalized size = 0.58

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(6a^3x^3+16a^2x^2+21ax+32\right)+21\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{24a^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*
a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.191, size = 196, normalized size = 1.2

$$-\frac{x}{24\,ca^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-6\,x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^4-16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c+27\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4-1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(
3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((c^(1/2)*((a*x-1
)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/
(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x - 1), x)
```

Fricas [A] time = 1.6971, size = 487, normalized size = 3.04

$$\left[\frac{2\left(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\frac{\left(6\,a^4x^4+16\,a^3x^3+21\,a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 +
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.15859, size = 173, normalized size = 1.08

$$\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3\,x\mathrm{sgn}(x)}{a^2}+\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x+\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

3.887 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=123

$$\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.472021, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 80, 50, 41, 216}

$$\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\tanh^{-1}(ax)}x\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{3a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0721543, size = 84, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(a^2x^2+3ax+5\right)+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.195, size = 173, normalized size = 1.4

$$\frac{x}{3\,a^3c}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3+3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c-3\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)+6\,c^{3/2}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((c*(a^2*x^2-1)/a^2)^(3/2)*a^3+3*(c*(a^
2*x^2-1)/a^2)^(1/2)*x*a^2*c-3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2
))+6*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+6*((a*
x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x - 1), x)
```

Fricas [A] time = 1.71404, size = 436, normalized size = 3.54

$$\left[\frac{2\left(a^3x^3+3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\frac{\left(a^3x^3+3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\sqrt{-\right.}}{3\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.20295, size = 157, normalized size = 1.28

$$\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\operatorname{sgn}(x)}{a^2}+\frac{3\operatorname{sgn}(x)}{a^3}\right)+\frac{5\operatorname{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^3|a|}+\frac{\left(3a\sqrt{c}\log(|c|)-1\right.}{a^4|a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

3.888 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=98

$$\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2*
a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rubi [A] time = 0.310332, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6167, 6159, 6129, 50, 41, 216}

$$\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2*
a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$=-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$
$$=\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$
$$=\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$
$$=\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0635101, size = 77, normalized size = 0.79

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.206, size = 147, normalized size = 1.5

$$-\frac{x}{2\,a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a^2+\sqrt{c}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2+c^(1
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-4*c^(1/2)*ln((c^(1/2)*((a*x-1)*
(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a
^2*x^2-1)/a^2)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x - 1), x)
```

Fricas [A] time = 1.75855, size = 404, normalized size = 4.12

$$\left[\frac{2\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\ \frac{\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)}{2\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.14294, size = 143, normalized size = 1.46

$$\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\operatorname{sgn}(x)}{a^2}+\frac{4\operatorname{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^2|a|}+\frac{\left(3a\sqrt{c}\log(|c|)-8\sqrt{-c}|a|\right)\operatorname{sgn}(x}{a^3|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

3.889 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.340949, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int}{\sqrt{1-ax}\sqrt{1}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0713889, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] time = 0.195, size = 196, normalized size = 1.7

$$-\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a^2-2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)-2*c^(1/2)*ln((c^(1/2
)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Fricas [A] time = 1.60418, size = 576, normalized size = 4.97

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\cdots}}{a^2cx^2\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A] time = 1.15162, size = 207, normalized size = 1.78

$$\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\operatorname{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\operatorname{arct}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 - 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(
a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/sq
rt(c)) - a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs(
a)
```

3.890 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=117

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.550538, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 157, 41, 216, 92, 208}

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```


```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-\right.}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.0818302, size = 82, normalized size = 0.7

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+ax\log\left(\sqrt{a^2x^2-1}+ax\right)-2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] - 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [B] time = 0.179, size = 306, normalized size = 2.6

$$-\frac{1}{ac}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^2a^3c+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}xa-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*x^2*a^3*c+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+c^(3/2)*ln(x*c^(1/2
)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x*a-2*c^(3/2)*(-c/a^2)^(1/2)*ln
((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x*a-2*(-c/a^2)^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c+2*(c*(a^2*x^2-1)/a^2)^(1/2)*c*x*a^2*(
-c/a^2)^(1/2)+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2
)*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x), x)
```

Fricas [A] time = 1.66008, size = 555, normalized size = 4.74

$$\left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}},-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{}}{a^2cx^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arctan(
a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*
log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) +
sqrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*(a*x - 1)), x)
```

Giac [A] time = 1.31417, size = 171, normalized size = 1.46

$$\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}+\frac{2c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2+c\right)|a|}\right)|a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] (4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
- sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) + 2*
c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs(
a)
```

3.891 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=111

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) + (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.51467, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6159, 6129, 94, 92, 208}

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) + (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} - \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+a\right.}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0737363, size = 78, normalized size = 0.7

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((4ax+1)\sqrt{a^2x^2-1}-3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((1 + 4*a*x)*Sqrt[-1 + a^2*x^2] - 3*a^2*x^2*ArcTan[1
/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.192, size = 347, normalized size = 3.1

$$-\frac{1}{2\,cx}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}xa^3+4\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*c^(3/2)
*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-4*c^(3/2)*(-c
/a^2)^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a-4
*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c+3*(-c/a^2)^(1/2)*(c
*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c+a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1
/2)+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)*x^2*c^2)
/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^2), x)
```

Fricas [A] time = 1.68254, size = 393, normalized size = 3.54

$$\left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},-\frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
```

```
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)),
x)
```

Giac [B] time = 1.57701, size = 262, normalized size = 2.36

$$\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\,(x)-\frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\mathrm{sgn}\,(x)-4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2 c^{\frac{3}{2}}|a|\mathrm{sgn}\,(x)-}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

3.892 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=137

$$a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.528254, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 96, 94, 92, 208}

$$a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^4\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2} - \frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+a}}}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2} + \frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-}\right.}{\sqrt{1-ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2} + \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0826457, size = 86, normalized size = 0.63

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(5a^2x^2+3ax+1\right)-3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{3x^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 + 3*a*x + 5*a^2*x^2) - 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.186, size = 378, normalized size = 2.8

$$-\frac{a}{3cx^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-6\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^4a^3c+6\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^2a^3+6c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*c
^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^3*a-6*c^(3/
2)*(-c/a^2)^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x
^3*a-6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1
/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(3/2)*x*a^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2
)*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^
2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^3), x)
```

Fricas [A] time = 1.64614, size = 441, normalized size = 3.22

$$\left[\frac{3a^2\sqrt{-c}x^2\log\left(-\frac{a^2cx^2+2a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)+2\left(5a^2x^2+3ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6x^2},-\frac{3a^2\sqrt{c}x^2\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(5a^2x^2+\right.}{3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)),
x)
```

Giac [A] time = 1.81442, size = 312, normalized size = 2.28

$$\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5ac\,\mathrm{sgn}\left(x\right)-3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^4c^{\frac{3}{2}}|a|\mathrm{sg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c*
```

```
x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^3)*abs(a)
```

3.893 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.556423, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
```

```
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^5\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2+16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^4}{x\sqrt{1-ax}\sqrt{1+a}}}{24\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}}}{8\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int\right.}{8\sqrt{1-}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1}\right.}{8\sqrt{1-ax}\sqrt{1+}}$$

Mathematica [A] time = 0.0934193, size = 94, normalized size = 0.6

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(32a^3x^3+21a^2x^2+16ax+6\right)-21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^
3*x^3) - 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*x^
2])
```

Maple [B] time = 0.198, size = 410, normalized size = 2.6

$$-\frac{a^2}{24\,cx^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-48\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^5a^3c+48\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^3+48\sqrt{-\frac{c}{a^2}}c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^
3+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-4
8*(-c/a^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(
1/2))*x^4*a-48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c+21*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c+27*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^2+21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/x/a^2)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+6*(c
*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^
(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^4), x)
```

Fricas [A] time = 1.71248, size = 487, normalized size = 3.12

$$\left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(32\,a^3x^3+21\,a^2x^2+16\,ax+6\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},-\frac{21\,a^3\sqrt{c}x^3\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 21*a^
2*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)),
x)
```

Giac [B] time = 2.97763, size = 427, normalized size = 2.74

$$\frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7a^2c\,\mathrm{sgn}\left(x\right)+45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

3.894 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.57542, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```


```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
```

]

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^6\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2+30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{144a^4+90a^5x}{x^2\sqrt{1-ax}\sqrt{1+ax}}}{120\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\right.}{120\sqrt{}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(3a^5\sqrt{c-\right.}{4\sqrt{}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(3a^6\sqrt{c-\right.}{}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{3a^5\sqrt{c-}}{}$$

Mathematica [A] time = 0.100253, size = 102, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4+15a^3x^3+12a^2x^2+10ax+4\right)-15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*a^
3*x^3 + 24*a^4*x^4) - 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

Maple [B] time = 0.178, size = 447, normalized size = 2.5

$$-\frac{a^2}{20\,x^4c}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-40\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\sqrt{-\frac{c}{a^2}}x^6a^4c+40\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}\sqrt{-\frac{c}{a^2}}x^4a^4+15\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] -1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*x^6*a^4*c+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^4*a^
4+15*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+40*(-c/a^2)^(1/2)*c
^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*c^
(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^5*a^2-40*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c+25*(-c/a^2)^(1/2)*(c*(
a^2*x^2-1)/a^2)^(3/2)*x^3*a^3+15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/x/a^2)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^
2*a^2+10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+4*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^5), x)
```

Fricas [A] time = 1.63842, size = 522, normalized size = 2.88

$$\left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, -\frac{15\,a^4\sqrt{c}x^4\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^4*a
rctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (24*a^
4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)),
x)
```

Giac [B] time = 3.28731, size = 489, normalized size = 2.7

$$\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\,(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}\,(x)+70\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2
*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*c
^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2 + c)^5)*abs(a)
```

3.895 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=186

$$\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a^
2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*S
qrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2)
]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.293478, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```


```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a^
2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*S
qrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2)
]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0513526, size = 71, normalized size = 0.38

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4x}{a^2}+\frac{4\log(1-ax)}{a^3}+\frac{ax^4}{4}+\frac{2x^2}{a}+x^3\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((4*x)/a^2 + (2*x^2)/a + x^3 + (a*x^4)/4 + (4*Log[1
- a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.231, size = 89, normalized size = 0.5

$$\frac{\left(x^{4}a^{4}+4\,x^{3}a^{3}+8\,a^{2}x^{2}+16\,ax+16\,\ln \left(ax-1\right)\right)x\left(ax-1\right)}{4\,a^{3}\left(ax+1\right)^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/4*(x^4*a^4+4*x^3*a^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}x^{3}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.66183, size = 111, normalized size = 0.6

$$\frac{\left(a^{4}x^{4}+4\,a^{3}x^{3}+8\,a^{2}x^{2}+16\,ax+16\,\log \left(ax-1\right)\right)\sqrt{a^{2}c}}{4\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(a^2*c
)/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.896 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=152

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rubi [A] time = 0.286066, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 77}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```


```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0425586, size = 63, normalized size = 0.41

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax\left(2a^2x^2+9ax+24\right)+24\log(1-ax)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6
*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.21, size = 82, normalized size = 0.5

$$\frac{\left(2 x^{3} a^{3}+9 a^{2} x^{2}+24 a x+24 \ln \left(a x-1\right)\right) x \left(a x-1\right)}{6 a^{2} \left(a x+1\right)^{2}} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}} \left(\frac{a x-1}{a x+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*(2*x^3*a^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/
2)*(a*x-1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^{2} x^{2}}} x^{2}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.61875, size = 97, normalized size = 0.64

$$\frac{\left(2 a^{3} x^{3}+9 a^{2} x^{2}+24 a x+24 \log \left(a x-1\right)\right) \sqrt{a^{2} c}}{6 a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(a^2*c)/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2 x^2}}\, x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.897 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=113

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^
2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])
/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.186595, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6197, 6193, 43}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^
2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])
/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.028395, size = 54, normalized size = 0.48

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(ax+6)+8\log(1-ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/
(a^2*x^2)])
```

Maple [A] time = 0.249, size = 73, normalized size = 0.7

$$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) x \left(a x-1\right)}{2 a \left(a x+1\right)^{2}} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}} \left(\frac{a x-1}{a x+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a/(
a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^{2} x^{2}}}\, x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.52516, size = 76, normalized size = 0.67

$$\frac{\left(a^{2} x^{2}+6 a x+8 \log \left(a x-1\right)\right) \sqrt{a^{2} c}}{2 a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.898 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.110757, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```



```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0291231, size = 50, normalized size = 0.46

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+4\log(1-ax)-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] time = 0.227, size = 65, normalized size = 0.6

$$-\frac{\left(-ax+\ln\left(x\right)-4\ \ln\left(ax-1\right)\right)x\left(ax-1\right)}{\left(ax+1\right)^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.70914, size = 66, normalized size = 0.61

$$\frac{\sqrt{a^{2}c}(ax+4\ \log\left(ax-1\right)-\log\left(x\right))}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.899 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (3*Sqrt[c - c/(a^2*x^2)
]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sq
rt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.271669, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (3*Sqrt[c - c/(a^2*x^2)
]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sq
rt[1 - 1/(a^2*x^2)]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0387184, size = 52, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-3a\log(x)+4a\log(1-ax)+\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^(-1) - 3*a*Log[x] + 4*a*Log[1 - a*x]))/(a*Sqrt[1
 - 1/(a^2*x^2)])
```

Maple [A] time = 0.216, size = 67, normalized size = 0.6

$$-\frac{\left(3 a \ln \left(x \right) x -4 \ln \left(a x -1\right) x a -1\right) \left(a x -1\right)}{\left(a x +1\right)^{2}}\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(\frac{a x -1}{a x +1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(3*a*ln(x)*x-4*ln(a*x-1)*x*a-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x
+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^{2} x^{2}}}}{x \left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.64533, size = 82, normalized size = 0.76

$$\frac{\sqrt{a^{2} c}\left(4 \, a x \log \left(a x - 1\right) - 3 \, a x \log \left(x\right) + 1\right)}{a^{2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.900 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2*
x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[
1 - 1/(a^2*x^2)] + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2
*x^2)]
```

Rubi [A] time = 0.276782, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```


```
[Out] Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2*
x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[
1 - 1/(a^2*x^2)] + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2
*x^2)]
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0464821, size = 66, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^2\log(x)+4a^2\log(1-ax)+\frac{3a}{x}+\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*x^2) + (3*a)/x - 4*a^2*Log[x] + 4*a^2*Log[1 -
a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.23, size = 82, normalized size = 0.6

$$-\frac{\left(8\,a^{2}\ln\left(x\right)x^{2}-8\,\ln\left(ax-1\right)a^{2}x^{2}-6\,ax-1\right)\left(ax-1\right)}{2\,\left(ax+1\right)^{2}x}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*ln(a*x-1)*a^2*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x-1)/(a*x+1)^2/x/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.57251, size = 194, normalized size = 1.32

$$\frac{8\,a^{3}\sqrt{c}x^{2}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+\sqrt{a^{2}c}(6\,ax+1)}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
1)*sqrt(c) + a*c)/(a*x^2 - x)) + sqrt(a^2*c)*(6*a*x + 1))/(a^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.901 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=188

$$\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2*
x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^
2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.288252, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```


```
[Out] Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2*
x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^
2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0489094, size = 76, normalized size = 0.4

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)+\frac{3a}{2x^2}+\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(3*x^3) + (3*a)/(2*x^2) + (4*a^2)/x - 4*a^3*Log[x
] + 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.275, size = 90, normalized size = 0.5

$$-\frac{\left(24\,a^{3}\ln\left(x\right)x^{3}-24\,\ln\left(ax-1\right)x^{3}a^{3}-24\,a^{2}x^{2}-9\,ax-2\right)\left(ax-1\right)}{6\,\left(ax+1\right)^{2}x^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*ln(a*x-1)*x^3*a^3-24*a^2*x^2-9*a*x-2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.66606, size = 213, normalized size = 1.13

$$\frac{24\,a^{4}\sqrt{c}x^{3}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+\left(24\,a^{2}x^{2}+9\,ax+2\right)\sqrt{a^{2}c}}{6\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2 x^2}}}{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.902 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=222

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^2
)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) + (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (
4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^3*Sqrt[c -
 c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.294734, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```


```
[Out] Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^2
)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) + (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (
4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^3*Sqrt[c -
 c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a}{}\end{aligned}$$

Mathematica [A] time = 0.0608056, size = 81, normalized size = 0.36

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)+\frac{a}{x^3}+\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(4*x^4) + a/x^3 + (2*a^2)/x^2 + (4*a^3)/x - 4*a^4
*Log[x] + 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.224, size = 98, normalized size = 0.4

$$-\frac{\left(16\,a^{4}\ln\left(x\right)x^{4}-16\,\ln\left(ax-1\right)a^{4}x^{4}-16\,x^{3}a^{3}-8\,a^{2}x^{2}-4\,ax-1\right)\left(ax-1\right)}{4\,\left(ax+1\right)^{2}x^{3}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(16*a^4*ln(x)*x^4-16*ln(a*x-1)*a^4*x^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.68253, size = 230, normalized size = 1.04

$$\frac{16\,a^{5}\sqrt{c}x^{4}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+\left(16\,a^{3}x^{3}+8\,a^{2}x^{2}+4\,ax+1\right)\sqrt{a^{2}c}}{4\,a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*sqrt
(a^2*c))/(a^2*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2 x^2}}}{x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.903 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=264

$$\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\cdots}{\sqrt{1-\cdots}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2*
x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt[
1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)
]*x^2) + (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*S
qrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^2
*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.304403, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\cdots}{\sqrt{1-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2*
x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt[
1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)
]*x^2) + (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*S
qrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^2
*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^6(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^6}-\frac{3a}{x^5}-\frac{4a^2}{x^4}-\frac{4a^3}{x^3}-\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^4\sqrt{c}}{\sqrt{}}
\end{aligned}
$$

Mathematica [A] time = 0.0721837, size = 90, normalized size = 0.34

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{240a^4x^4+120a^3x^3+80a^2x^2+45ax+12}{60x^5}-4a^5\log(x)+4a^5\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((12 + 45*a*x + 80*a^2*x^2 + 120*a^3*x^3 + 240*a^4*x
^4)/(60*x^5) - 4*a^5*Log[x] + 4*a^5*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]
)
```

Maple [A] time = 0.237, size = 106, normalized size = 0.4

$$-\frac{\left(240\,a^5\ln(x)\,x^5-240\,\ln(ax-1)\,x^5a^5-240\,x^4a^4-120\,x^3a^3-80\,a^2x^2-45\,ax-12\right)(ax-1)}{60\,(ax+1)^2\,x^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] -1/60*(240*a^5*ln(x)*x^5-240*ln(a*x-1)*x^5*a^5-240*x^4*a^4-120*x^3*a^3-80*a
^2*x^2-45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^4/((a*x
-1)/(a*x+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A] time = 1.62598, size = 257, normalized size = 0.97

$$\frac{240\,a^6\sqrt{c}x^5\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\left(240\,a^4x^4+120\,a^3x^3+80\,a^2x^2+45\,ax+12\right)\sqrt{a^2c}}{60\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
 - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (240*a^4*x^4 + 120*a^3*x^3 + 80*a^2*x^2
 + 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{x^5 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.904 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

Optimal. Leaf size=81

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a
^2*x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.249416, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 43}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a
^2*x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0407592, size = 50, normalized size = 0.62

$$\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax}{m+1}-\frac{1}{m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m*(-m^(-1) + (a*x)/(1 + m)))/(a*Sqrt[1 - 1/(a^2*x^
2)])
```

Maple [A] time = 0.114, size = 65, normalized size = 0.8

$$\frac{x^{1+m} (axm - m - 1)}{(1 + m) m (ax - 1)} \sqrt{\frac{c (a^2x^2 - 1)}{a^2x^2}} \sqrt{\frac{ax - 1}{ax + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(
1+m)/m/(a*x-1)
```

Maxima [A] time = 1.21221, size = 62, normalized size = 0.77

$$\frac{\left(a\sqrt{c}mx - \sqrt{c}(m + 1)\right)(ax - 1)x^m}{\left(m^2 + m\right)a^2x - \left(m^2 + m\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] (a*sqrt(c)*m*x - sqrt(c)*(m + 1))*(a*x - 1)*x^m/((m^2 + m)*a^2*x - (m^2 + m
)*a)
```

Fricas [A] time = 1.6933, size = 154, normalized size = 1.9

$$-\frac{\left(amx^2 - (m + 1)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2 - \left(am^2 + am\right)x + m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}x^m \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.905 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=76

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2
*x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.262506, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 43}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2
*x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0272598, size = 45, normalized size = 0.59

$$\frac{x^2(2ax-3)\sqrt{c-\frac{c}{a^2x^2}}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(-3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.115, size = 53, normalized size = 0.7

$$\frac{(2\,ax-3)\,x^3}{6\,ax-6}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*
x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.63313, size = 53, normalized size = 0.7

$$\frac{\left(2\,ax^3 - 3\,x^2\right)\sqrt{a^2c}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.906 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=72

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.176238, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6197, 6193}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.0234619, size = 42, normalized size = 0.58

$$\frac{x(ax-2)\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-2 + a*x))/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.116, size = 52, normalized size = 0.7

$$\frac{(ax-2)\,x^2}{2\,ax-2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.51924, size = 47, normalized size = 0.65

$$\frac{\sqrt{a^2c}\left(ax^2-2\,x\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(a^2*c)*(a*x^2 - 2*x)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.907 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.0994955, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0202002, size = 41, normalized size = 0.6

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.203, size = 52, normalized size = 0.8

$$
-\frac{(-ax+\ln(x))\,x}{ax-1}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A] time = 1.47676, size = 43, normalized size = 0.63

$$\frac{\sqrt{a^2c}(ax - \log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax - 1}{ax + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.908 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (Sqrt[c - c/(a^2*x^2)]*
Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.25567, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 43}

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (Sqrt[c - c/(a^2*x^2)]*
Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.023077, size = 41, normalized size = 0.59

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(a\log(x)+\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.212, size = 50, normalized size = 0.7

$$\frac{a\ln\left(x\right)x+1}{ax-1}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] (a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [A] time = 1.68361, size = 51, normalized size = 0.74

$$\frac{\sqrt{a^{2}c}\left(ax\log\left(x\right)+1\right)}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) + 1)/(a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.909 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=47

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rubi [A] time = 0.246109, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 37}

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```


```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}\end{aligned}$$

Mathematica [A] time = 0.0239413, size = 47, normalized size = 1.

$$\frac{\left(\frac{1}{2x^2}-\frac{a}{x}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*x^2) - a/x))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.121, size = 53, normalized size = 1.1

$$-\frac{2\,ax-1}{(2\,ax-2)\,x}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A] time = 1.54959, size = 54, normalized size = 1.15

$$-\frac{\sqrt{a^2c}(2\,ax - 1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(a^2*c)*(2*a*x - 1)/(a^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.910 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=163

$$\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) + (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.547207, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 90, 80, 50, 41, 216}

$$\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) + (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
```

```
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}(-1+2ax)}{\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}-\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{12a^2\sqrt{1-ax}\sqrt{1+ax}}\\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}-\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{8a^2\sqrt{1-a}}\\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}\\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}\\
&= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0896265, size = 93, normalized size = 0.57

$$
\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(6a^3x^3-16a^2x^2+21ax-32\right)+21\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{24a^3\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.19, size = 196, normalized size = 1.2

$$-\frac{x}{24\,ca^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-6\,x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^4+16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}xa^2c+27\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4+1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(
3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((c^(1/2)*((a*x-1
)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/
(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1), x)
```

Fricas [A] time = 1.72642, size = 487, normalized size = 2.99

$$\left[\frac{2\left(6\,a^4x^4-16\,a^3x^3+21\,a^2x^2-32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\frac{\left(6\,a^4x^4-16\,a^3x^3+21\,a^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 -
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.14038, size = 173, normalized size = 1.06

$$\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\operatorname{sgn}(x)}{a^2}-\frac{8\operatorname{sgn}(x)}{a^3}\right)+\frac{21\operatorname{sgn}(x)}{a^4}\right)x-\frac{32\operatorname{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

3.911 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=124

$$\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.470053, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 80, 50, 41, 216}

$$\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```


```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\tanh^{-1}(ax)}x\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{x(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2} + \frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{3a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

<u>Mathematica</u> [A] time = 0.0688948, size = 84, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(a^2x^2-3ax+5\right)-3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A] time = 0.174, size = 173, normalized size = 1.4

$$\frac{x}{3\,a^{3}c}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}}a^{3}-3\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}xa^{2}c+3\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)-6\,c^{3/2}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a^
2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2
))-6*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+6*((a*
x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{a^{2}x^{2}}}\,x^{2}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

Fricas [A] time = 1.66724, size = 436, normalized size = 3.52

$$\left[\frac{2\left(a^{3}x^{3}-3\,a^{2}x^{2}+5\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}+3\sqrt{c}\log\left(2\,a^{2}cx^{2}-2\,a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)}{6\,a^{3}},\frac{\left(a^{3}x^{3}-3\,a^{2}x^{2}+5\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}+3\sqrt{-c}}{3\,a^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.18048, size = 158, normalized size = 1.27

$$\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}-\frac{\left(3\,a\sqrt{c}\log(|c|)+1\right.}{a^4|a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

3.912 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=99

$$-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2
*a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rubi [A] time = 0.305347, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6167, 6159, 6129, 50, 41, 216}

$$-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2
*a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a} - \frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a} - \frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a} - \frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a} - \frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0632169, size = 100, normalized size = 1.01

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{ax+1}\left(a^2x^2-5ax+4\right)-6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt[1 -
 a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.172, size = 147, normalized size = 1.5

$$-\frac{x}{2\,a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}a^2+\sqrt{c}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2+c^(1
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-4*c^(1/2)*ln((c^(1/2)*((a*x-1)*
(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a
^2*x^2-1)/a^2)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

Fricas [A] time = 1.72474, size = 404, normalized size = 4.08

$$\left[\frac{2\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\ \frac{\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\cdots}}{a^2cx^2-\cdots}\right.}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.17384, size = 143, normalized size = 1.44

$$\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\operatorname{sgn}(x)}{a^2}-\frac{4\operatorname{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^2|a|}+\frac{\left(3a\sqrt{c}\log\left(|c|\right)+8\sqrt{-c}|a|\right)\operatorname{sgn}(x}{a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

3.913 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.339978, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int}{\sqrt{1-ax}\sqrt{1}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.075918, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}-2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A] time = 0.197, size = 197, normalized size = 1.7

$$\frac{x}{a^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\ln\left(\frac{1}{\sqrt{c}}\left(\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx\right)\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Fricas [A] time = 1.687, size = 576, normalized size = 4.97

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{}}{a^2cx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A] time = 1.17392, size = 205, normalized size = 1.77

$$\left(\frac{2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{2\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}+\frac{\sqrt{a^2cx^2-c}\,\mathrm{sgn}(x)}{a^2}-\frac{\left(2\sqrt{c}|a|\,\mathrm{arct}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] (2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 + 2*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(
a)) + sqrt(a^2*c*x^2 - c)*sgn(x)/a^2 - (2*sqrt(c)*abs(a)*arctan(sqrt(-c)/sq
rt(c)) + a*sqrt(c)*log(abs(c)) + sqrt(-c)*abs(a))*sgn(x)/(a^2*abs(a)))*abs(
a)
```

3.914 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=117

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.525426, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 157, 41, 216, 92, 208}

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```



```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-\right.}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.0766057, size = 82, normalized size = 0.7

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+ax\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [B] time = 0.186, size = 306, normalized size = 2.6

$$-\frac{1}{ac}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(-\sqrt{-\frac{c}{a^{2}}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}x^{2}a^{3}c+a^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^{2}}}+c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)\sqrt{-\frac{c}{a^{2}}}xa-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*x^2*a^3*c+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+c^(3/2)*ln(x*c^(1/2
)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x*a-2*c^(3/2)*(-c/a^2)^(1/2)*ln
((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x*a+2*(-c/a^2)^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c-2*(c*(a^2*x^2-1)/a^2)^(1/2)*c*x*a^2*(
-c/a^2)^(1/2)-2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2
)*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

Fricas [A] time = 1.74179, size = 554, normalized size = 4.74

$$\left[-\sqrt{-c}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)+\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}},2\,\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}}{}}}{a^{2}cx^{2}-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arctan(a
*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*l
og(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + s
qrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x*(a*x + 1)), x)
```

Giac [A] time = 1.31971, size = 171, normalized size = 1.46

$$-\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2\,c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2+c\right)|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] -(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 + sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) - 2
*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs
(a)
```

3.915 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) +
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.523391, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6159, 6129, 94, 92, 208}

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) +
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+\right.}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0635413, size = 78, normalized size = 0.7

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((4ax-1)\sqrt{a^2x^2-1}+3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*ArcTan
[1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.182, size = 348, normalized size = 3.1

$$\frac{1}{2\,cx}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}xa^3+4\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^
(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*c^(3/2)*
ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^2*a-4*c^(3/2)*(-c/
a^2)^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a+4*
(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c-3*(-c/a^2)^(1/2)*(c*
(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c-a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/
2)-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2)*x^2*c^2)/
(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

Fricas [A] time = 1.7116, size = 393, normalized size = 3.51

$$\left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},-\frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
```

```
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)),
x)
```

Giac [B] time = 1.51355, size = 262, normalized size = 2.34

$$\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\operatorname{sgn}(x)+4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2 c^{\frac{3}{2}}|a|\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

3.916 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=140

$$a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.533698, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 96, 94, 92, 208}

$$a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^4\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} + \frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} - \frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{a-}\right.}{\sqrt{1-ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} - \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}x\tanh^{-1}\left(\sqrt{1-}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.0752232, size = 86, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(5a^2x^2-3ax+1\right)+3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{3x^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Maple [B] time = 0.18, size = 378, normalized size = 2.7

$$-\frac{a}{3\,c x^{2}}\sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\left(-6\sqrt{-\frac{c}{a^{2}}}\sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}}x^{4} a^{3} c+6\sqrt{-\frac{c}{a^{2}}}\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{3/2} x^{2} a^{3}+6\,c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*c
^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*x^3*a-6*c^(3/
2)*(-c/a^2)^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x
^3*a+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1
/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(3/2)*x*a^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/x/a^2
)*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^
2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^{2} x^{2}}}}{(ax+1)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^3), x)
```

Fricas [A] time = 1.69421, size = 440, normalized size = 3.14

$$\left[\frac{3\,a^{2}\sqrt{-c}x^{2}\log\left(-\frac{a^{2} c x^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2} c x^{2}-c}{a^{2} x^{2}}}-2\,c}{x^{2}}\right)+2\left(5\,a^{2} x^{2}-3\,ax+1\right)\sqrt{\frac{a^{2} c x^{2}-c}{a^{2} x^{2}}}}{6\,x^{2}},\ \frac{3\,a^{2}\sqrt{c}x^{2}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2} c x^{2}-c}{a^{2} x^{2}}}}{a^{2} c x^{2}-c}\right)+\left(5\,a^{2} x^{2}-3\right.}{3\,x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)),
x)
```

Giac [A] time = 1.89716, size = 312, normalized size = 2.23

$$-\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5 ac\mathrm{sgn}\left(x\right)+3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^4 c^{\frac{3}{2}}|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
```

```
*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

3.917 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.553579, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```


```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
```

```
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^5\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2-16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^4}{x\sqrt{1-ax}\sqrt{\cdots}}}{24\sqrt{1-ax}\sqrt{1+a\cdots}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\cdots}{x\sqrt{1-\cdots}}}{8\sqrt{1-ax}\sqrt{1+\cdots}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\cdots}{8\sqrt{1\cdots}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\cdots\right.}{8\sqrt{1-ax}\sqrt{1\cdots}}$$

Mathematica [A] time = 0.0835905, size = 94, normalized size = 0.6

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(32a^3x^3-21a^2x^2+16ax-6\right)+21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2 + 32*
a^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*
x^2])
```

Maple [B] time = 0.207, size = 410, normalized size = 2.6

$$\frac{a^2}{24\,cx^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-48\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^5a^3c+48\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}x^3a^3+48\sqrt{-\frac{c}{a^2}}c^{3/2}\ln\left(x\sqrt{c}+\sqrt{\frac{c\,(a}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^3
+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-48
*(-c/a^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1
/2))*x^4*a+48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c-21*(-c
/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c-27*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(3/2)*x^2*a^2-21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/x/a^2)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)-6*(c*
(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(
1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^4), x)
```

Fricas [A] time = 1.63999, size = 487, normalized size = 3.12

$$\left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(32\,a^3x^3-21\,a^2x^2+16\,ax-6\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},-\frac{21\,a^3\sqrt{c}x^3\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 21*a^
2*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)),
x)
```

Giac [B] time = 2.98079, size = 427, normalized size = 2.74

$$\frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)-\frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7a^2c\,\mathrm{sgn}\left(x\right)+45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

3.918 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.585254, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```


```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```


Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
```

```
]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{(1-ax)^{3/2}}{x^6\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{36a^2-30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}x\right)\int \frac{144a^4-90a^5x}{x^2\sqrt{1-ax}\sqrt{1+ax}}}{120\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\right.}{120}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(3a^5\sqrt{c-\right.}{4}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(3a^6\sqrt{c-\right.}{}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{3a^5\sqrt{c-}}{}$$

Mathematica [A] time = 0.0890427, size = 102, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4-15a^3x^3+12a^2x^2-10ax+4\right)+15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a^
3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

Maple [B] time = 0.198, size = 447, normalized size = 2.5

$$-\frac{a^2}{20\,x^4c}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-40\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\sqrt{-\frac{c}{a^2}}x^6a^4c+40\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{3/2}\sqrt{-\frac{c}{a^2}}x^4a^4-15\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] -1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*x^6*a^4*c+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^4*a^
4-15*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+40*(-c/a^2)^(1/2)*c
^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*c^
(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^5*a^2+40*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c-25*(-c/a^2)^(1/2)*(c*(
a^2*x^2-1)/a^2)^(3/2)*x^3*a^3-15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/x/a^2)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^
2*a^2-10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+4*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^5), x)
```

Fricas [A] time = 1.65055, size = 521, normalized size = 2.88

$$\left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, \frac{15\,a^4\sqrt{c}x^4\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4*ar
ctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24*a^4
*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)),
x)
```

Giac [B] time = 3.14559, size = 489, normalized size = 2.7

$$-\frac{1}{10}\left(15\,a^{3}\sqrt{c}\arctan\left(-\frac{\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{15\left(\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}\right)^{9}a^{3}c\operatorname{sgn}(x)+70\left(\sqrt{a^{2}c}x-\sqrt{a^{2}cx^{2}-c}\right)^{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*
c^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2 + c)^5)*abs(a)
```

3.919 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal. Leaf size=186

$$\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a
^2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*
Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2
)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.289775, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a
^2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*
Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2
)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}} \\
&= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0510524, size = 69, normalized size = 0.37

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax\left(a^3x^3-4a^2x^2+8ax-16\right)+16\log(ax+1)\right)}{4a^4\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(-16 + 8*a*x - 4*a^2*x^2 + a^3*x^3) + 16*Log[1
+ a*x]))/(4*a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.219, size = 89, normalized size = 0.5

$$\frac{\left(x^{4}a^{4}-4\,x^{3}a^{3}+8\,a^{2}x^{2}-16\,ax+16\,\ln\left(ax+1\right)\right)x\left(ax+1\right)}{4\,a^{3}\left(ax-1\right)^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/4*(x^4*a^4-4*x^3*a^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{a^{2}x^{2}}}x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.62731, size = 111, normalized size = 0.6

$$\frac{\left(a^{4}x^{4}-4\,a^{3}x^{3}+8\,a^{2}x^{2}-16\,ax+16\,\log\left(ax+1\right)\right)\sqrt{a^{2}c}}{4\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(a^2*c
)/a^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.920 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal. Leaf size=151

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rubi [A] time = 0.286942, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 77}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0434261, size = 62, normalized size = 0.41

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax\left(2a^2x^2-9ax+24\right)-24\log(ax+1)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6
*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.237, size = 82, normalized size = 0.5

$$-\frac{\left(-2\,x^{3}a^{3}+9\,a^{2}x^{2}-24\,ax+24\,\ln\left(ax+1\right)\right)x\left(ax+1\right)}{6\,a^{2}\left(ax-1\right)^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(-2*x^3*a^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{a^{2}x^{2}}}x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.67213, size = 97, normalized size = 0.64

$$\frac{\left(2\,a^{3}x^{3}-9\,a^{2}x^{2}+24\,ax-24\,\log\left(ax+1\right)\right)\sqrt{a^{2}c}}{6\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(a^2*c)/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.921 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal. Leaf size=112

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x]
)/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.188849, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6197, 6193, 43}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```


```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x]
)/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0303431, size = 53, normalized size = 0.47

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(ax-6)+8\log(ax+1))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1
/(a^2*x^2)])
```

Maple [A] time = 0.224, size = 73, normalized size = 0.7

$$\frac{\left(a^{2}x^{2}-6ax+8\ \ln\left(ax+1\right)\right)x\left(ax+1\right)}{2a\left(ax-1\right)^{2}}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a
*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{a^{2}x^{2}}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.69876, size = 76, normalized size = 0.68

$$\frac{\left(a^{2}x^{2}-6ax+8\ \log\left(ax+1\right)\right)\sqrt{a^{2}c}}{2a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.922 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] time = 0.111038, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0253328, size = 47, normalized size = 0.44

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A] time = 0.223, size = 63, normalized size = 0.6

$$\frac{(ax+\ln(x)-4\ln(ax+1))\,x\,(ax+1)}{(ax-1)^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] (a*x+ln(x)-4*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A] time = 1.74814, size = 66, normalized size = 0.62

$$\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.923 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) - (3*Sqrt[c - c/(a^2*x
^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])
/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.272152, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) - (3*Sqrt[c - c/(a^2*x
^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])
/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0356927, size = 53, normalized size = 0.49

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1
 - 1/(a^2*x^2)])
```

Maple [A] time = 0.23, size = 67, normalized size = 0.6

$$-\frac{(3\,a\ln(x)\,x-4\,ax\ln(ax+1)+1)\,(ax+1)}{(ax-1)^2}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] -(3*a*ln(x)*x-4*a*x*ln(a*x+1)+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*
x-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [A] time = 1.66644, size = 82, normalized size = 0.76

$$\frac{\sqrt{a^2c}(4\,ax\log(ax+1)-3\,ax\log(x)-1)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.924 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=146

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2
*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt
[1 - 1/(a^2*x^2)] - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^
2*x^2)]
```

Rubi [A] time = 0.278066, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```


```
[Out] -Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2
*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt
[1 - 1/(a^2*x^2)] - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^
2*x^2)]
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0460094, size = 65, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/(2*x^2) + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 +
 a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.229, size = 82, normalized size = 0.6

$$\frac{\left(8\,a^{2}\ln\left(x\right)x^{2}-8\,\ln\left(ax+1\right)a^{2}x^{2}+6\,ax-1\right)\left(ax+1\right)}{2\,\left(ax-1\right)^{2}x}\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] 1/2*(8*a^2*ln(x)*x^2-8*ln(a*x+1)*a^2*x^2+6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A] time = 1.69383, size = 194, normalized size = 1.33

$$\frac{8\,a^{3}\sqrt{c}x^{2}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)+\sqrt{a^{2}c}(6\,ax-1)}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
1)*sqrt(c) + a*c)/(a*x^2 + x)) + sqrt(a^2*c)*(6*a*x - 1))/(a^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.925 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=187

$$-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2
*x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
 - 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x
^2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.279813, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```


```
[Out] -Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2
*x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
 - 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x
^2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\mathrm{lo}}{\sqrt{1-\frac{}{a^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0467831, size = 75, normalized size = 0.4

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(ax+1)+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/(3*x^3) + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[
x] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.23, size = 90, normalized size = 0.5

$$-\frac{\left(24 a^{3} \ln \left(x\right) x^{3}-24 a^{3} x^{3} \ln \left(a x+1\right)+24 a^{2} x^{2}-9 a x+2\right) \left(a x+1\right)}{6 \left(a x-1\right)^{2} x^{2}} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*a^3*x^3*ln(a*x+1)+24*a^2*x^2-9*a*x+2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^{2} x^{2}}} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A] time = 1.56929, size = 213, normalized size = 1.14

$$\frac{24 a^{4} \sqrt{c} x^{3} \log \left(\frac{2 a^{3} c x^{2}+2 a^{2} c x+\sqrt{a^{2} c}(2 a x+1) \sqrt{c}+a c}{a x^{2}+x}\right)-\left(24 a^{2} x^{2}-9 a x+2\right) \sqrt{a^{2} c}}{6 a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.926 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=221

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^
2)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) - (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(
a^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) +
(4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] - (4*a^3*Sqrt[c
- c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.287234, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```


```
[Out] -Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^
2)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) - (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(
a^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) +
(4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] - (4*a^3*Sqrt[c
- c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```


Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-
\end{aligned}$$

Mathematica [A] time = 0.0518255, size = 80, normalized size = 0.36

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2a^2}{x^2}+\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(ax+1)+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/(4*x^4) + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^
4*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A] time = 0.246, size = 98, normalized size = 0.4

$$\frac{\left(16\,a^4\ln\left(x\right)x^4-16\,\ln\left(ax+1\right)a^4x^4+16\,x^3a^3-8\,a^2x^2+4\,ax-1\right)\left(ax+1\right)}{4\,\left(ax-1\right)^2x^3}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] 1/4*(16*a^4*ln(x)*x^4-16*ln(a*x+1)*a^4*x^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)*(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A] time = 1.58758, size = 230, normalized size = 1.04

$$\frac{16\,a^5\sqrt{c}x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\left(16\,a^3x^3-8\,a^2x^2+4\,ax-1\right)\sqrt{a^2c}}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*sqrt
(a^2*c))/(a^2*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.927 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=263

$$-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\ldots}}{\sqrt{1\ldots}}$$

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2
*x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt
[1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2
)]*x^2) - (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*
Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^
2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] time = 0.295275, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\ldots}}{\sqrt{1\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2
*x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt
[1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2
)]*x^2) - (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*
Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^
2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^6(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^6}-\frac{3a}{x^5}+\frac{4a^2}{x^4}-\frac{4a^3}{x^3}+\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^4
\end{aligned}$$

Mathematica [A] time = 0.0711244, size = 89, normalized size = 0.34

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{240a^4x^4-120a^3x^3+80a^2x^2-45ax+12}{60x^5}-4a^5\log(x)+4a^5\log(ax+1)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(12 - 45*a*x + 80*a^2*x^2 - 120*a^3*x^3 + 240*a^4*
x^4)/(60*x^5) - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)
])
```

Maple [A] time = 0.237, size = 106, normalized size = 0.4

$$-\frac{\left(240\,a^5\ln(x)\,x^5-240\,\ln(ax+1)\,x^5a^5+240\,x^4a^4-120\,x^3a^3+80\,a^2x^2-45\,ax+12\right)(ax+1)}{60\,(ax-1)^2\,x^4}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] -1/60*(240*a^5*ln(x)*x^5-240*ln(a*x+1)*x^5*a^5+240*x^4*a^4-120*x^3*a^3+80*a
^2*x^2-45*a*x+12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(
3/2)/(a*x-1)^2/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A] time = 1.71001, size = 257, normalized size = 0.98

$$\frac{240\,a^6\sqrt{c}x^5\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)-\left(240\,a^4x^4-120\,a^3x^3+80\,a^2x^2-45\,ax+12\right)\sqrt{a^2c}}{60\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x
 + 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (240*a^4*x^4 - 120*a^3*x^3 + 80*a^2*x^2
 - 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.928 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=154

$$\frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}-\frac{c2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\right.}{a(2-n)}$$

```
[Out] (4*c*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2
, 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n)) - (2^(1 + n/2)*
c*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^
(-1))/(2*a)])/(a*(2 - n))
```

Rubi [C] time = 0.0652096, antiderivative size = 81, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6194, 136}

$$-\frac{c2^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+4}{2}}F_1\left(\frac{n+4}{2};\frac{n-2}{2},2;\frac{n+6}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -((2^(2 - n/2)*c*(1 + 1/(a*x))^((4 + n)/2)*AppellF1[(4 + n)/2, (-2 + n)/2,
2, (6 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(4 + n)))
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx=-\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$=-\frac{2^{2-\frac{n}{2}}c\left(1+\frac{1}{ax}\right)^{\frac{4+n}{2}}F_1\left(\frac{4+n}{2};\frac{1}{2}(-2+n),2;\frac{6+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(4+n)}$$

Mathematica [A] time = 0.239939, size = 123, normalized size = 0.8

$$\frac{ce^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+(n+2)\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n}{2}+1\right.}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(2*a*x + a*n*x + E^(2*ArcCoth[a*x])*n*Hypergeometric2
F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])] + 4*E^(2*ArcCoth[a*x])*Hypergeometric2F1[
2, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]))/(a*(2 + n))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int e^{n\text{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2cx^2 - c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(\int a^2 e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int -\frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^2}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x**2,
x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.929 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=150

$$\frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}-\frac{(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{acn}+\frac{x\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\right.}{c}$$

```
[Out] -((((1 + n)*(1 + 1/(a*x))^(n/2))/(a*c*n*(1 - 1/(a*x))^(n/2))) + ((1 + 1/(a*x
))^(n/2)*x)/(c*(1 - 1/(a*x))^(n/2)) + (2*(1 + 1/(a*x))^(n/2)*Hypergeometric
2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a*c*(1 - 1/(a*x))^(n/2)
)
```

Rubi [A] time = 0.123679, antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6194, 129, 155, 12, 131}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac(2-n)}-\frac{(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{acn}+\frac{x\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -((((1 + n)*(1 + 1/(a*x))^(n/2))/(a*c*n*(1 - 1/(a*x))^(n/2))) + ((1 + 1/(a*x
))^(n/2)*x)/(c*(1 - 1/(a*x))^(n/2)) + (2*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(
a*x))^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (a - x^(-1))/(a +
 x^(-1))])/(a*c*(2 - n))
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} - \frac{a\,\text{Subst}\left(\int \frac{n^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{a^2x}\,dx, x\right.}{cn}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} - \frac{n\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx, x,\right.}{ac}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(\right.}{ac(2-n)}$$

Mathematica [A] time = 0.252392, size = 94, normalized size = 0.63

$$\frac{e^{n\coth^{-1}(ax)}\left(n^2e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+(n+2)\left(n\text{Hypergeometric2F1}\left(1,\frac{n}{2},\right.\right.\right.}{acn(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + n*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int e^{n\,\text{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int \frac{x^2e^{n\operatorname{acoth}(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c - \frac{c}{a^2x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

3.930 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}+\frac{\left(-n^3-n^2+4n+6\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{ac^2(2-n)n(n+2)}-\frac{\left(n^2+\right.}{}$$

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(a*c^2*(2 +
 n))) + ((6 + 4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 +
 n)/2))/(a*c^2*(2 - n)*n*(2 + n)) - ((6 + 4*n + n^2)*(1 + 1/(a*x))^((-2 + n
)/2))/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)) + ((1 - 1/(a*x))^(-1 - n/2)*(1
+ 1/(a*x))^((-2 + n)/2)*x)/c^2 + (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1
, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a*c^2*(1 - 1/(a*x))^(n/2))
```

Rubi [A] time = 0.250866, antiderivative size = 303, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6194, 129, 155, 12, 131}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2(2-n)}+\frac{\left(-n^3-n^2+4n+6\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{ac^2(2-n)n(n+2)}-\frac{\left(n^2+4n+6\right)\left(\frac{1}{ax}\right.}{ac^2n}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(a*c^2*(2 +
 n))) + ((6 + 4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 +
 n)/2))/(a*c^2*(2 - n)*n*(2 + n)) - ((6 + 4*n + n^2)*(1 + 1/(a*x))^((-2 + n
)/2))/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)) + ((1 - 1/(a*x))^(-1 - n/2)*(1
+ 1/(a*x))^((-2 + n)/2)*x)/c^2 + (2*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))
^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-
1))])/(a*c^2*(2 - n))
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
```

```
1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{3x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} - \frac{a\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-}}{}\right.}{c^2(2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} - \frac{\left(6+4n+n^2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1\right.}{c}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(6+4n-n^2-n^3\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{\left(6+4n+\right.}{}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(6+4n-n^2-n^3\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{\left(6+4n+\right.}{}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(6+4n-n^2-n^3\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{\left(6+4n+\right.}{}$$

Mathematica [A] time = 0.431414, size = 142, normalized size = 0.49

$$\frac{e^{n\coth^{-1}(ax)}\left(2(n-2)n^2e^{2\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n}{2}+1,\frac{n}{2}+2,e^{2\coth^{-1}(ax)}\right)+2\left(n^2-4\right)n\text{Hypergeometric2F}\right.}{2ac^2(n-2)n(n+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(12 - 3*n^2 - 8*a*n*x + 2*a*n^3*x - n^2*Cosh[2*ArcCoth[
a*x]] + 2*E^(2*ArcCoth[a*x])*(-2 + n)*n^2*Hypergeometric2F1[1, 1 + n/2, 2 +
```

```
n/2, E^(2*ArcCoth[a*x])] + 2*n*(-4 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])] + 2*n*Sinh[2*ArcCoth[a*x]]))/(2*a*c^2*(-2 + n)*n*(2
+ n))
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int e^{n\operatorname{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^4x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4 \int \frac{x^4 e^{n \operatorname{acoth}(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*exp(n*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

3.931 $\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{a^2x^2}}\, dx$

Optimal. Leaf size=295

$$\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2^{\frac{n+1}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeom}}{a(1-n)\sqrt{1-}}$$

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[1 - 1/(a^2*x^2)] + (2*n*Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[1 - 1/(a^2*x^2)]) - (2^((1 + n)
/2)*Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*Hypergeometric2F1[(1 -
n)/2, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a)])/(a*(1 - n)*Sqrt[1 - 1/(a^2
*x^2)])
```

Rubi [C] time = 0.15462, antiderivative size = 111, normalized size of antiderivative = 0.38, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6194, 136}

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+3}{2}}F_1\left(\frac{n+3}{2};\frac{n-1}{2},2;\frac{n+5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+3)\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((2^(3/2 - n/2)*Sqrt[c - c/(a^2*x^2)]*(1 + 1/(a*x))^((3 + n)/2)*AppellF1[(
3 + n)/2, (-1 + n)/2, 2, (5 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(3
 + n)*Sqrt[1 - 1/(a^2*x^2)]))
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}F_1\left(\frac{3+n}{2};\frac{1}{2}(-1+n),2;\frac{5+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(3+n)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.463257, size = 146, normalized size = 0.49

$$\frac{ax^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}e^{n\coth^{-1}(ax)}\left(2e^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-e^{2\coth^{-1}(ax)}\right)+2ne^{\coth^{-1}(ax)}\text{Hypergeo}\right.}{(n+1)\left(a^2x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2*(a*(1
 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E^ArcCoth[a*x]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, -E^(2*ArcCoth[a*x])] + 2*E^ArcCoth[a*x]*n*Hypergeometric2F
1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]))/((1 + n)*(-1 + a^2*x^2))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c - \frac{c}{a^2x^2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.932 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=183

$$\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[c - c/(a^2*x^2)] + (2*n*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] time = 0.165345, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6197, 6194, 96, 131}

$$\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[c - c/(a^2*x^2)] + (2*n*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[c - c/(a^2*x^2)])
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\left(n\sqrt{1-\frac{1}{a^2x^2}}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\,{}_2F_1\left(1, \frac{1-n}{2}; \frac{3-n}{2}; \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right.}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.340543, size = 112, normalized size = 0.61

$$\frac{\left(a^2x^2-1\right)e^{n\coth^{-1}(ax)}\left(2ne^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},e^{2\coth^{-1}(ax)}\right)+a(n+1)x\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3(n+1)x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)*(a*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E
^ArcCoth[a*x]*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x
])]))/(a^3*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2)
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int e^{n\,\text{arccoth}(ax)}\frac{1}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

3.933 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=116

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}F_1\left(\frac{n}{2}+p+1;\frac{1}{2}(n-2p),2;\frac{n}{2}+p+2;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

```
[Out] -((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF1
[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x)]
)/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rubi [A] time = 0.114846, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6194, 136}

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}F_1\left(\frac{n}{2}+p+1;\frac{1}{2}(n-2p),2;\frac{n}{2}+p+2;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF1
[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x)]
)/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+p}}{x^2}dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}F_1\left(1+\frac{n}{2}+p;\frac{1}{2}(n-2p),2;2+\frac{n}{2}+p;\frac{a+}{2}\right.}{a(2+n+2p)}\end{aligned}$$

Mathematica [F] time = 0.473303, size = 0, normalized size = 0.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} \left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{n\,\text{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

3.934 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p\text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1-\frac{1}{ax}\right)}{a(2p+1)}$$

```
[Out] ((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rubi [A] time = 0.0850554, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6197, 6194, 65}

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(2,2p+1;2(p+1);1-\frac{1}{ax}\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}}{x^2}dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1-\frac{1}{ax}\right)}{a(1+2p)}\end{aligned}$$

Mathematica [F] time = 0.454987, size = 0, normalized size = 0.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]

[Out] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]), x]
```

Maple [F] time = 0.189, size = 0, normalized size = 0.

$$\int\frac{1}{e^{2p\,\mathrm{arccoth}(ax)}}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)

[Out] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p/((a*x - 1)/(a*x + 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x - 1)/(a*x + 1))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p/((a*x - 1)/(a*x + 1))^p, x)
```

3.935 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=75

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(2,2p+1,2(p+1),\frac{1}{ax}+1\right)}{a(2p+1)}$$

```
[Out] -(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p
, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rubi [A] time = 0.0841536, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6197, 6194, 65}

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(2,2p+1;2(p+1);1+\frac{1}{ax}\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p
, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx \\
&= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{2p}}{x^2}dx,x,\frac{1}{x}\right)\right) \\
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1+\frac{1}{ax}\right)}{a(1+2p)}
\end{aligned}
$$

Mathematica [F] time = 0.462574, size = 0, normalized size = 0.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F] time = 0.182, size = 0, normalized size = 0.

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

```
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^p \left(\frac{ax-1}{ax+1}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^p \left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{2p\,\text{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c - \frac{c}{a^2x^2}\right)^p \left(\frac{ax-1}{ax+1}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^p, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
   flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```